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RL HW-2

1. $p(\text{even}) = 2\,p(\text{odd})$, Gaussian $(i, 1)$

1. 10 items / arms 10 times

$E[\text{total reward after 10 pulls}] = 10 \times E[q_*(a)]$

and, $E[q_*(a)]$

$= \sum p(a) \cdot q_*(a)$

$= \frac{2}{15}(2+4+6+8+10) + \frac{1}{15}(1+3+7+5+9)$

$= 4 + \frac{5}{3} = \frac{17}{3}$

$\left\{ p[a] = \begin{cases} 2/15, & a \in \text{even} \\ 1/15, & a \in \text{odd} \end{cases} \right.$ b/w $[1, 10]$ $\in I$.

Hence, $10\,E[q_*(a)]$

$= \frac{170}{3} = 56.66$

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2. $\left.\begin{array}{l} R = 0 \mid p = 1/2 \\ R = 1 \mid p = 1/2 \end{array}\right\} \forall a \in \{1, 2, 4, 5, 7, 9, 10\}$

$\left.\begin{array}{l} R = 0 \mid p = 3/10 \\ R = 0.2 \mid p = 3/10 \\ R = 1 \mid p = 4/10 \end{array}\right\} \forall a \in \{3, 6, 8\}$

Now, $q_*^{(1)}(a) = 0(0.5) + 1(0.5) = \underline{0.5}$

and, $q_*^{(2)}(a) = 0(0.3) + 0.2(0.3) + 1(0.4)$

$= \underline{0.46}$

Now, for optimal ~~stoc~~ stochastic policies we will make 6 combinations of arms with $q_*(a) = 0.5$.

1. Arms(1, 2, ~~3~~ 4, 5, 7) with ~~prob~~ 0.2 each
2. Arms (1, 2, 4, 5) with 0.25 each
3. Arms (1, 2, 4) with 1/3 each
4. Arms (1, 2) with 1/2 each
5. Arms (1, 2, 4, 5, 7, 9) with 1/6 each
6. Arms (1) with 0.4 and Arm (2) with 0.6.

4. Table for $p(s', r | s, a)$

s	a	s'	r'	$p(s, r \| s, a)$
high	search	high	r search	α
high	search	low	r search	$(1-\alpha)$
low	search	high	-3	$(1-\beta)$
low	search	low	r search	β
high	wait	high	r wait	1
low	wait	low	r wait	1
low	recharge	high	0	1

★. This table was derived by MDP table given in Example 3.3. and the transition graph.

★. As ~~the~~ rewards have no dependence on prob. distribution we can say that $p(s', r | s, a) = p(s' | s, a)$.

3. $r \in \{0,1\}$. explore at $t = 1, 3, 5$ (over non-greedy only)

$Q_1(a)$, $a \in \{1,2,3\}$ exploit at $t = 2, 4, 6$

Sample Mean: Let $Q_1(1) = 0.2$

$Q_1(2) = 0.3$

At $t=1$: $Q_1(3) = 0.5$

let $A_1 = 1$, $R_1 = 1$

$Q_2(1) = \frac{0.2+1}{2} = 0.6$, $Q_2(2) = 0.3$, $Q_2(3) = 0.5$

At $t=2$:

(Greedy) let $A_2 = 1$, $R_2 = 0$

$Q_3(1) = \frac{0.6+0}{3} = 0.2$, $Q_3(2) = 0.3$; $Q_3(3) = 0.5$ ∴

At $t=3$:

let $A_3 = 2$, $R_3 = 0$

$Q_4(1) = 0.2$, $Q_4(2) = \frac{0.3+0}{2} = 0.15$, $Q_4(3) = 0.5$

At $t=4$:

(Greedy) let $A_4 = 3$, $R_4 = 1$

$Q_5(1) = 0.2$, $Q_5(2) = 0.15$, $Q_5(3) = \frac{0.5+1}{2} = 0.75$

At $t=5$:

let $A_5 = 1$, $R_5 = 1$

$Q_6(1) = \frac{0.2+1}{4} = 0.3$, $Q_6(2) = 0.15$, $Q_6(3) = 0.75$

(Greedy) At $t=6$:

let $A_6 = 3$, $R_6 = 0$

$Q_7(1) = 0.3$, $Q_7(2) = 0.15$, $Q_7(3) = \frac{0.75+0}{3} = 0.25$

Not req.

6. <u>3.15</u>

The signs of rewards is not important, only the intervals between them are as the difference of rewards matters.

Generally, if ~~[illegible]~~ good reward $= -1$ or 5
and bad reward corresponding $= -3$, 3
They will ~~be~~ have same results.

Now,

$$\text{Eq. } 3.8 \Rightarrow G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$
$$= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} \text{ —— } \infty$$

Adding a const. c to all rewards:

$$\Rightarrow G_t = \sum_{0}^{\infty} \gamma^k \left[R_{t+k+1} + c \right]$$

$$\therefore G_t = \frac{1}{1-\gamma} \left(R_{t+k+1} + c \right)$$

As every state reward gets an additional const. term, added to it, $v_c = \mathbb{E} \left[\sum_{0}^{\infty} \gamma^k c \right]$

$$\therefore v_c = \frac{c}{1-\gamma}$$

Here, it does not affect relative values in any state as $v_c = \frac{c}{1-\gamma}$ in terms of c and γ.

3.16

In an episodic task, adding a const. to all rewards does affect the agent because the cumulative reward is dep. on the length of the episode.

let us say if $r = -1$, the lenght of the episode decreases as the agent wants to find the ~~exit~~ quickly. (in maze runner)

But if say $r = +1$ because of some +ve value of c, the lenght of episode increases as agent does not want to find the exit, in maze runner.

* This case will go onto ∞.

$s' = p(s', r | s, a)$

$(s', a') = \pi(a', s') \cdot \sum_{r} p(s', r, s, a)$

8. v_{*} in terms of q_{*}:

$$v_{*}(s) = \max_{a \in A(s)} q_{\pi *}(s, a)$$

$$= \max_{a \in A(s)} \sum_{s', r} p(s', r | s, a) \left[r + \gamma \max_{a \in A(s')} q_{*}(s', a) \right]$$

11. RV R_{t+2} dependent on S_t, A_t = ?

$p[R_{t+2} | s', a']$

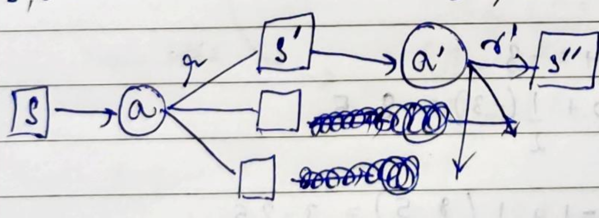
$= \sum_{s''} p(R_{t+2}, s'' | s', a')$

$\cancel{p(R_{t+2}, s, a)}$ π $\cancel{(a', s')\, p(\ldots)}$

• Now, $p(R_{t+2} | s, a) = \sum_{s, a} p(s', a' | s, a) \cdot p(R_{t+2} | s', a')$

$= \sum_{s', a'} \left[\pi(a' | s') \cdot \sum_{r} p(s', r | s, a) \right] \cdot \sum_{s''} \left[p(R_{t+2}, s'' | s', a) \right]$

$= \sum_{s', r} p(s', r | s, a) \cdot \sum_{s'', a'} \pi(a' | s')\, p(s'', r' | s', a)$



12. $E[R_{t+2} | S_t = s, A_t = a]$

$= \sum_{r'} r' \, P(r = R_{t+2} | S_t = s, A_t = a)$

(from above)

$= \sum_{r'} r' \sum_{s', r} P(s', r | s, a) \sum_{r', s''} \pi(a' | s') \, P(s'', r' | s', a)$

↳ MDP's PMF

13. $v_\pi(s) = E[G_t \mid S_t = s]$

$= E_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s]$

$= E_\pi[R_{t+1} \mid S_t = s] + \gamma E_\pi[G_{t+1} \mid S_t = s]$

$= E_\pi[R_{t+1} \mid S_t = s] + \gamma E_\pi[R_{t+2} + \gamma G_{t+2} \mid s]$ — recursively

$= E_\pi[R_{t+1} \mid s] + \gamma E_\pi[E_\pi[G_{t+1}](s') \mid s]$

$= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a)[r + \gamma E_\pi[G_{t+1} \mid S_{t+1} = s']]$

$= \sum_a \pi(a|s) \sum_{s', r} p(s', r|s, a)[r + \gamma v_\pi(s')]$, $\forall s \in S$.

Bellman Equation for $v_\pi(s)$.

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14. $R_1 = 2$; $R_2 = -1$, $R_3 = 10$, $R_4 = -3$, $\gamma = 0.5$

Discounted reward for each timestep:

$G_0 = R_1 + \gamma G_1$

$G_1 = R_2 + \gamma G_2$

$G_2 = R_3 + \gamma G_3$

$G_3 = R_4 + \gamma \cancel{G_4}^{0}$ $\quad \therefore G_3 = R_4 = -3$

$\Rightarrow G_2 = 10 + \frac{1}{2}(-3) = 8.5$

$\Rightarrow G_1 = -1 + \frac{1}{2}(8.5) = 3.25$

$\Rightarrow G_0 = 2 + \frac{1}{2}(3.250) = 3.625$

New $G_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2}$ — ∞

$= c + \gamma c + \gamma^2 c$ — ∞

$= c\left(\frac{1}{1-\gamma}\right) = \frac{c}{1-\gamma}$

15. Given: $v_*(s)\ \forall s \in S$

We know, $\pi_*(s) = \max_a q_*(s,a)$

$\Rightarrow \pi_*(s) = \max_a \sum_{s',r} p(s',r|s,a)[r + \gamma v_*(s')]$

So, in this case we take the best action that returns the highest value to maintain optimality by iterating over all ~~states~~ actions that can be chosen over a state, greedily.

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17. Given: $\pi'(s) \geq \pi(s)$
Prove: $v_{\pi'}(s) \geq v_\pi(s)$

We know, $v_\pi(s) \leq q_\pi(s, \pi'(s))$

$= \mathbb{E}[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid s, \pi'(s)]$

$= \mathbb{E}_{\pi'}[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid s]$

From here

$\leq \mathbb{E}_{\pi'}[R_{t+1} + \gamma q_\pi(S_{t+1}, \pi'(S_{t+1})) \mid s]$

$= \mathbb{E}_{\pi'}[R_{t+1} + \gamma \mathbb{E}[R_{t+2} + \gamma v_\pi(S_{t+2})] \mid S_{t+1}] \mid S_t = s, A_{t+1} = \pi'(S_{t+1})]$

$= \mathbb{E}_{\pi'}[R_{t+1} + \gamma R_{t+2} \text{---} \gamma^2 v_\pi(S_{t+k}) \mid s]$

$= \mathbb{E}_{\pi'}[R_{t+1} + \gamma R_{t+2} \text{---} \gamma^5 R_{t+6} \mid s]$

$= \mathbb{E}_{\pi'}[G_t \mid S_t = s]$

$= v_{\pi'}(s)$

Hence, $\boxed{v_\pi(s) \leq v_{\pi'}(s)}$