

# Computer Project

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Roll number: 24

*“Writing code a computer can understand is science. Writing code  
other programmers can understand is an art.”*

— **Jason Gorman**

*“I am rarely happier than when spending an entire day programming my computer to perform automatically a task that would otherwise take me a good ten seconds to do by hand.”*

— Douglas Adams

**Problem 1** An  $n$  digit integer  $(a_1a_2 \dots a_n)$ , where each digit  $a_i \in \{0, 1, \dots, 9\}$ , is said to have *unique digits* if no digits are repeated, i.e., there is no  $i, j$  such that  $a_i = a_j$  ( $i \neq j$ ).

Verify whether an inputted number has *unique digits*.

**Solution** The problem involves simply counting the number of occurrences of each digit in the given number and checking whether any of them exceed 1.

**main** (**number**:Integer)

1. Initialize an integer array **digits** of length 10, indexed with the numbers 0 to 9 with all elements set to 0.
2. If **number** exceeds 0, proceed. Otherwise, jump to (3).
  - (a) Store the last digit<sup>1</sup> of **number** in a temporary variable **d**.
  - (b) Increment the integer at the **d** index of **digits**.
  - (c) If **digits[d]** exceeds 1, the number does not have *unique digits*. Display a suitable message, and **exit**.
  - (d) Discard the last digit of **number** by performing an integer division by 10 and storing the result back in **number**.
  - (e) Jump to (2).
3. The number has *unique digits*. Display a suitable message.
4. **Exit**

---

<sup>1</sup>The last digit of an integer  $n$  is simply  $n \bmod 10$

## Source Code

```
1 public class Unique {
2     public static void main (String[] args) {
3         try {
4             long number = Long.parseLong(args[0]);
5             if (isUnique(number)) {
6                 System.out.println("Unique Number!");
7             } else {
8                 System.out.println("Not a Unique Number!");
9             }
10        } catch (NumberFormatException | IndexOutOfBoundsException e) {
11            System.out.println("Enter 1 argument (number[integer])!");
12        }
13    }
14
15    public static boolean isUnique (long number) {
16        int[] count = new int[10];
17        for (long n = Math.abs(number); n > 0; n /= 10) {
18            int digit = (int) n % 10;
19            count[digit]++;
20            if (count[digit] > 1){
21                return false;
22            }
23        }
24        return true;
25    }
26 }
```

*“Elegance is not a dispensable luxury but a factor that decides between success and failure.”*

— Edsger W. Dijkstra

**Problem 2** A *partition* of a positive integer  $n$  is defined as a collection of other positive integers such that their sum is equal to  $n$ . Thus, if  $(a_1, a_2, \dots, a_k)$  is a partition of  $n$ ,

$$n = a_1 + a_2 + \dots + a_k \quad (a_i \in \mathbb{Z}^+)$$

Display every *unique partition* of an inputted number.

**Solution** This problem can be solved elegantly using *recursion*<sup>2</sup>. Note that when partitioning a number  $n$ , we can calculate the partitions of  $(n - 1)$  and append 1 to each solution. Similarly, we can append 2 to partitions of  $(n - 2)$ , 3 to partitions of  $(n - 3)$ , and so on. By continuing in this fashion, all cases will be reduced to the single *base case* of finding the partitions of 0, of which there are trivially none.

There is a slight flaw in this algorithm — partitions are often repeated. This can be overcome by imposing the restriction that each new term has to be of a lesser magnitude than the previous. In this way, repeated partitions will be automatically discarded.

`main (target:Integer)`

1. Call `partition(target, target, "")`.
2. **Exit**

`partition (target:Integer, previousTerm:Integer, suffix:String)`

1. If `target` is 0, display `suffix` and **return**.
2. Initialize a counter `i` to 1.
3. If `i` is less than or equal to both the `target` and `previousTerm`, proceed. Otherwise, jump to (4).
  - (a) Call `partition(target - i, i, suffix + " " + i)`.
  - (b) Increment `i` by 1.
  - (c) Jump to (3).
4. **Return**

---

<sup>2</sup>Recursion occurs when a thing is defined in terms of itself or of its type.

## Source Code

```
1
2 public class Partition {
3     public static void main (String[] args) {
4         try {
5             int target = Integer.parseInt(args[0]);
6             if (target < 1) {
7                 throw new NumberFormatException();
8             }
9             partition(target);
10        } catch (NumberFormatException | IndexOutOfBoundsException e) {
11            System.out.println("Enter 1 argument (number[natural
12                                number])!");
13        }
14    }
15    public static void partition (int target) {
16        partition(target, target, "");
17    }
18    public static void partition (int target, int previousTerm, String suffix) {
19        if (target == 0)
20            System.out.println(suffix);
21        for (int i = 1; i <= target && i <= previousTerm; i++)
22            partition(target - i, i, suffix + " " + i);
23    }
```

*“Simplicity is the ultimate sophistication.”*

— Leonardo da Vinci

**Problem 3** A *Caesar cipher* is a type of monoalphabetic substitution cipher in which each letter in the plaintext is replaced by a letter some fixed number of positions down the alphabet. The positions are circular, i.e., after reaching *Z*, the position wraps around to *A*. For example, following is some encrypted text, using a right shift of 5.

Plain: ABCDEFGHIJKLMNOPQRSTUVWXYZ  
Cipher: FGHIJKLMNOPQRSTUVWXYZABCDE

Thus, after mapping the alphabet according to the scheme  $A \mapsto 0, B \mapsto 1, \dots, Z \mapsto 23$ , we can define an encryption function  $E_n$ , in which a letter  $x$  is shifted rightwards by  $n$  as follows.

$$E_n(x) = (x + n) \mod 26$$

The corresponding decryption function  $D_n$  is simply

$$D_n(x) = (x - n) \mod 26$$

Implement a simple version of a *Caesar cipher*, encrypting capitalized plaintext by shifting it by a given value. Interpret positive shifts as rightwards, negative as leftwards.

**Solution** This problem can be solved simply by exploiting the fact that Unicode characters are already arranged in order, with successive alphabets encoded by consecutive numbers. In addition, the encryption function can be defined exactly as given in the question — characters can be converted to their corresponding codes, manipulated by addition of the `shift`, and converted back into alphabetic form.

```
main (shift:Integer, plainText:String)
```

1. Normalize `plainText` to uppercase.
2. Normalize `shift` by replacing it with `shift mod 26`.
3. Initialize an empty String `cipherText`.
4. Initialize a counter `i` to 0.
5. If `i` is less than the length of `plainText`, proceed. Otherwise, jump to (6).
  - (a) Store the character in `plainText` at position `i` in a variable `plain`.
  - (b) Initialize an empty character `crypt`.
  - (c) If `plain` is not an alphabet, assign `plain` to `crypt` and jump to (5g).
  - (d) Convert `plain` into a number, such that `A` is mapped to 0, `B` to 1 and so on. Store this in a temporary variable `n`.

- (e) Add `shift` to `n`, calculate its least residue modulo  $26^3$ , and store the result in `n`.
  - (f) Convert `n` back into a character and store the result in `crypt`.
  - (g) Append `crypt` to `cipherText`.
  - (h) Increment `i` by 1 and jump to (5).
6. Display `cipherText`.
7. **Exit**

---

<sup>3</sup>The set of integers  $K = \{0, 1, 2, \dots, n-1\}$  is called the least residue system modulo  $n$ . The number  $k$  such that  $k \in K$  and  $a \equiv k \pmod{n}$  is called the least residue of  $a$  modulo  $n$ .



## Source Code

```
1 public class CaesarShift {
2     public static void main (String[] args) {
3         try {
4             int shift = Integer.parseInt(args[0]) % 26;
5             String plaintext = args[1].toUpperCase();
6             String ciphertext = "";
7             for (int i = 0; i < plaintext.length(); i++) {
8                 char plain = plaintext.charAt(i);
9                 char crypt = ' ';
10                if ('A' <= plain && plain <= 'Z') {
11                    crypt = numToChar(charToNum(plain) + shift);
12                } else {
13                    crypt = plain;
14                }
15                ciphertext += crypt;
16            }
17            System.out.println(ciphertext);
18        } catch (NumberFormatException | IndexOutOfBoundsException e) {
19            System.out.println("Enter 2 arguments (shift[integer],
20                               plaintext[text])!");
21        }
22    }
23    public static int charToNum (char letter) {
24        return Character.toUpperCase(letter) - 'A';
25    }
26    public static char numToChar (int number) {
27        return (char) ('A' + Math.floorMod(number, 26));
28    }
29 }
30 }
```

*“There are 2 hard problems in computer science: cache invalidation,  
naming things, and off-by-1 errors.”*

— Leon Bambrick

**Problem 4** A *palindrome* is a sequence of characters which reads the same backwards as well as forwards. For example, `madam`, `racecar` and `kayak` are words which are palindromes. Similarly, the sentence “A man, a plan, a canal -- Panama!” is also a palindrome.

Analyze a sentence of input and display all *words* which are palindromes. If the entire *sentence* is also a palindrome, display it as well.

*(A word is an unbroken sequence of characters, separated from other words by whitespace. Ignore single letter words such as I and a. Ignore punctuation, numeric digits, whitespace and case while analyzing the entire sentence.)*

**Solution** The main challenge here is intelligently dividing a *sentence* into its component *words*. Verifying whether a sequence of characters is a palindrome is fairly simple — extracting those characters from a string of alphabets, numbers, punctuation and whitespace is not.

The main idea behind isolating words from sentences is to define two *markers* — a **start** to keep track of the boundary between whitespace and letters, and an **end** to mark the boundary between letters and whitespace. In this way, the markers can inch their way along the sentence, isolating words in the process. Managing the order of condition checking and incrementing of counters does require some careful maneuvering in order to avoid any *off-by-1 errors*<sup>4</sup>.

**main** ()

1. Accept a string as input, store it in a variable `sentence`.
2. Call `checkWords(sentence)` and `checkSentence(sentence)`. Store the returned values in booleans.
  - (a) If either of them is `true`, set a boolean `foundPalindrome` to `true`, otherwise set it to `false`.
3. Display a suitable message if `foundPalindrome` is `false`.
4. **Exit**

---

<sup>4</sup>An off-by-one error often occurs in computer programming when an iterative loop iterates one time too many or too few.

**checkWords** (sentence:String)

1. Initialize a boolean **foundPalindrome** to false.
2. Initialize two integer counters: **start** to -1, **end** to 0.
3. If **end** is less than the length of **sentence**, proceed. Otherwise, jump to (4).
  - (a) Increment **start** as long as the character at the [**start** + 1] position in **sentence** is whitespace.
  - (b) Assign **end** to **start**.
  - (c) Increment **end** as long as it does not exceed the length of **sentence** and the character at the [**end**] position in **sentence** is not whitespace.
  - (d) Assign the string of characters between **start** and **end** from **sentence** (inclusive, exclusive) to a variable **word**.
  - (e) Call **isPalindrome(word)**. If **word** is a palindrome:
    - i. Set **foundPalindrome** to true.
    - ii. Display **word**.
  - (f) Assign **end** - 1 to **start**.
  - (g) Jump to (3)
4. **Return** **foundPalindrome**

**checkSentence** (sentence:String)

1. Call **isPalindrome(sentence)**. If **sentence** is a palindrome:
  - (a) Display **word**.
  - (b) **Return** true.
2. **Return** false.

**isPalindrome** (text:String)

1. Normalize **text** by converting it into uppercase and removing all non-alphabetic characters.
2. Let the length of **text** be labeled temporarily as **t**.
3. Initialize two integer counters: **i** to 0, **j** to **t** - 1.
4. If **i** is less than **j**, proceed. Otherwise, jump to (5).
  - (a) If the characters at positions **i** and **j** in **text** are not equal, **return** false.
  - (b) Increment **i** by 1.
  - (c) Decrement **j** by 1
  - (d) Jump to (4)
5. **Return** true only if **text** is longer than one character. Otherwise, **return** false.

## Source Code

```
1 import java.util.Scanner;
2
3 public class Palindrome {
4     public static void main (String[] args) {
5         System.out.print("Enter your sentence : ");
6         String sentence = (new Scanner(System.in)).nextLine().trim();
7         boolean foundPalindrome = false;
8         System.out.println("Palindromes : ");
9         foundPalindrome |= checkWords(sentence);
10        foundPalindrome |= checkSentence(sentence);
11        if (!foundPalindrome) {
12            System.out.println("(No palindromes found!)");
13        }
14    }
15
16    public static boolean checkWords (String sentence) {
17        boolean foundPalindrome = false;
18        int start = -1;
19        int end = 0;
20        while (end < sentence.length()) {
21            while (Character.isWhitespace(sentence.charAt(++start)));
22            end = start;
23            while (end < sentence.length() &&
24                !Character.isWhitespace(sentence.charAt(end++)));
25            String word = sentence.substring(start, end).trim();
26            if (isPalindrome(word)) {
27                foundPalindrome = true;
28                System.out.println(getAlphabets(word));
29            }
30            start = end - 1;
31        }
32        return foundPalindrome;
33    }
34
35    public static boolean checkSentence (String sentence) {
36        if (isPalindrome(sentence)) {
37            System.out.println("The sentence '" + sentence + "' is a
38                palindrome.");
39            return true;
40        }
41        return false;
42    }
43
44    public static boolean isPalindrome (String text) {
```

```

43     String rawText = getAlphabets(text).toUpperCase();
44     for (int i = 0, j = rawText.length() - 1; i < j; i++, j--) {
45         if (rawText.charAt(i) != rawText.charAt(j)) {
46             return false;
47         }
48     }
49     return (rawText.length() > 1);
50 }
51
52 public static String getAlphabets (String text) {
53     String rawText = "";
54     for (int i = 0; i < text.length(); i++) {
55         if (Character.isAlphabetic(text.charAt(i))) {
56             rawText += text.charAt(i);
57         }
58     }
59     return rawText;
60 }
61 }

```

*“In programming the hard part isn’t solving problems, but deciding what problems to solve.”*

— Paul Graham

**Problem 5** A *prime number* (or a *prime*) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

Display all primes upto a given limit, along with their number.

**Solution** This problem could be tackled in a number of ways. We could define a function for checking the primality of a given number, then iterate through all numbers in the required range. A common way of checking for primality is *trial division*. It consists of testing whether the number  $n$  is a multiple of any integer between 2 and  $\sqrt{n}$ . Although this works well enough for small numbers, repeating this consecutively for very large inputs is tedious and inefficient. Since the problem consists of identifying primes in a *range*, and not individually, we can make use of more efficient methods.

The *Sieve of Eratosthenes* is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite the multiples of each prime, starting with the first prime number, 2. As a result, when a prime  $p$  is found, none of its multiples will be tested further for primality — they are eliminated early on. In comparison, *trial division* has worse theoretical complexity than that of the *Sieve of Eratosthenes* in generating ranges of primes. When testing each prime, the optimal trial division algorithm uses all prime numbers not exceeding its square root, whereas the Sieve of Eratosthenes produces each composite from its prime factors only.

## Source Code

```
1 public class SieveOfEratosthenes {
2     private final int upperLimit;
3     private boolean[] primes;
4
5     public SieveOfEratosthenes (int upperLimit) {
6         this.upperLimit = upperLimit;
7         this.initPrimes();
8     }
9
10    public boolean[] getPrimes () {
11        return primes;
12    }
13
14    public void initPrimes () {
15        this.primes = new boolean[upperLimit];
16        primes[0] = false;
17        primes[1] = false;
18        for (int i = 2; i < upperLimit; i++)
19            primes[i] = true;
20    }
21
22    public void sievePrimes () {
23        int prime = 2;
24        while ((prime * prime) < upperLimit) {
25            int multiple = prime * prime;
26            while (multiple < upperLimit) {
27                primes[multiple] = false;
28                multiple += prime;
29            }
30            while (!primes[++prime]);
31        }
32    }
33 }

```

```
1 public class Primes {
2     public static void main (String[] args) {
3         try {
4             int upperLimit = Integer.parseInt(args[0]);
5             if (upperLimit < 2) {
6                 throw new NumberFormatException();
7             }
8             SieveOfEratosthenes sieve = new
9                 SieveOfEratosthenes(upperLimit);
10            sieve.sievePrimes();
11        }
12    }
13 }
```

```

10         showPrimes(sieve.getPrimes());
11     } catch (NumberFormatException | IndexOutOfBoundsException e) {
12         System.out.println("Enter 1 argument (limit[integer, >1])!");
13         System.out.println("(Primes will be displayed up to, not
            including 'limit')");
14     }
15 }
16
17 public static void showPrimes (boolean[] primes) {
18     int primeCount = 0;
19     int maxLength = Integer.toString(primes.length).length();
20     for (int i = 0; i < primes.length; i++) {
21         if (primes[i]) {
22             System.out.printf("%" + maxLength + "d ", i);
23             primeCount++;
24         }
25     }
26     System.out.println("\nTotal number of primes : " + primeCount);
27 }
28 }

```



*“Any fool can use a computer. Many do.”*

— **Ted Nelson**

**Problem 6** Design a simple interface for an examiner which can format and display marks scored by a group of students in a particular examination. Calculate the percentage scored by each candidate and display the list of students and percentages in an ASCII bar chart, arranged alphabetically.

**Solution**

## Source Code

```
1 public class Marksheet {
2     public static final int SCREEN_WIDTH = 100;
3     private final double maxMarks;
4     private final int numberOfStudents;
5     private int lastStudent;
6     private String[] names;
7     private double[] marks;
8
9     public Marksheet (double maxMarks, int numberOfStudents) {
10         this.maxMarks = maxMarks;
11         this.numberOfStudents = numberOfStudents;
12         this.names = new String[numberOfStudents];
13         this.marks = new double[numberOfStudents];
14         this.lastStudent = -1;
15     }
16
17     public boolean addMarks (String name, double score) {
18         try {
19             names[++lastStudent] = name;
20             marks[lastStudent] = score;
21             return true;
22         } catch (IndexOutOfBoundsException e) {
23             return false;
24         }
25     }
26
27     public void displayChart () {
28         System.out.println(Marksheet.multiplyString("-",
29             Marksheet.SCREEN_WIDTH));
30         for (int i = 0; i <= lastStudent; i++) {
31             double fraction = marks[i] / maxMarks;
32             String name = (names[i].length() < 16)
33                 ? names[i]
34                 : (names[i].substring(0,13) + "...");
35             int points = (int) (fraction * (SCREEN_WIDTH - 34));
36             String bar = multiplyString("*", points)
37                 + multiplyString(" ", SCREEN_WIDTH - 34 - points);
38             System.out.printf("| %16s | %s | %6.2f %% |%n"
39                 , name
40                 , bar
41                 , fraction * 100);
42         }
43         System.out.println(Marksheet.multiplyString("-",
44             Marksheet.SCREEN_WIDTH));
45     }
46 }
```

```

43     }
44
45     public void displayMaxScorers () {
46         String maxScorers = "";
47         double maxScore = getMaxScore();
48         for (int i = 0; i <= lastStudent; i++) {
49             if (marks[i] == maxScore) {
50                 maxScorers += ", " + names[i];
51             }
52         }
53         System.out.println(maxScorers.substring(1)
54                             + " scored the highest ("
55                             + maxScore + "/"
56                             + maxMarks + ")");
57     }
58
59     public void sortByName () {
60         for (int right = lastStudent; right > 0; right--)
61             for (int i = 1; i <= right; i++)
62                 if (names[i-1].compareToIgnoreCase(names[i]) > 0)
63                     swapRecords(i, i - 1);
64
65     }
66
67     public double getMaxScore () {
68         double max = Integer.MIN_VALUE;
69         for (int i = 0; i <= lastStudent; i++) {
70             max = Math.max(max, marks[i]);
71         }
72         return max;
73     }
74
75     private void swapRecords (int x, int y) {
76         String tempName = names[x];
77         double tempMark = marks[x];
78         names[x] = names[y];
79         marks[x] = marks[y];
80         names[y] = tempName;
81         marks[y] = tempMark;
82     }
83
84     public static String multiplyString (String s, int n) {
85         String out = "";
86         while (n --> 0)
87             out += s;
88         return out;

```

```

89     }
90 }

1  import java.util.Scanner;
2  import java.util.InputMismatchException;
3
4  public class ScoreRecorder {
5      public static void main (String[] args) {
6          Scanner inp = new Scanner(System.in);
7          double maxMarks = 0.0;
8          int numberOfStudents = 0;
9          try {
10             System.out.print("Enter the maximum marks allotted for each
11                 student : ");
12             maxMarks = inp.nextDouble();
13             System.out.print("Enter the total number of students : ");
14             numberOfStudents = inp.nextInt();
15             if (maxMarks <= 0) {
16                 System.out.println("Maximum marks must be positive!");
17                 System.exit(0);
18             }
19             if (numberOfStudents <= 0) {
20                 System.out.println("Number of students must be
21                     positive!");
22                 System.exit(0);
23             }
24             Marksheet sheet = new Marksheet(maxMarks, numberOfStudents);
25             System.out.println("Enter " + numberOfStudents + " students'
26                 names and marks : ");
27             for (int i = 0; i < numberOfStudents; i++) {
28                 String name = "";
29                 while (!inp.hasNextDouble()) {
30                     name += inp.next() + " ";
31                 }
32                 double marks = inp.nextDouble();
33                 if (marks <= 0 || marks > maxMarks) {
34                     System.out.println("Marks must be within 0.0 and
35                         " + maxMarks + "!");
36                     System.exit(0);
37                 }
38                 sheet.addMarks(name.trim(), marks);
39             }
40             sheet.sortByName();
41             sheet.displayChart();
42             sheet.displayMaxScorers();

```

```
39         } catch (InputMismatchException e) {
40             System.out.println("Invalid Input!");
41             System.exit(0);
42         }
43     }
44 }
```

“To iterate is human, to recurse divine”

— L. Peter Deutsch

**Problem 7** The *determinant* of a square matrix  $A_{n,n}$  is defined recursively as follows.

$$\det(A_{n,n}) = \begin{vmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{vmatrix} = \sum_{j=1}^n (-1)^{i+j} a_{i,j} \cdot \det(M_{i,j})$$

where  $M_{i,j}$  is defined as the minor of  $A_{n,n}$ , an  $(n-1) \times (n-1)$  matrix formed by removing the  $i$ th row and  $j$ th column from  $A_{n,n}$ .

The determinant of a  $(2 \times 2)$  matrix is simply given by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

For example, the determinant of a  $(3 \times 3)$  matrix is given by the following expression.

$$\begin{aligned} \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} &= a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ &= aei + bfg + cdh - ceg - bdi - afh \end{aligned}$$

Calculate the *determinant* of an inputted  $(n \times n)$  square matrix.

**Solution**

## Source Code

```
1 public class Matrix {
2     protected final int rows;
3     protected final int columns;
4     protected int[][] elements;
5
6     public Matrix (int rows, int columns) {
7         this.rows = rows;
8         this.columns = columns;
9         this.elements = new int[rows][columns];
10    }
11
12    public int getRows () {
13        return this.rows;
14    }
15
16    public int getColumns () {
17        return this.columns;
18    }
19
20    public void setElementAt (int element, int row, int column) {
21        if (row < 1 || row > rows || column < 1 || column > columns)
22            return;
23        elements[row-1][column-1] = element;
24    }
25
26    public int getElementAt (int row, int column) {
27        if (row < 1 || row > rows || column < 1 || column > columns)
28            return Integer.MIN_VALUE;
29        return elements[row-1][column-1];
30    }
31 }

```

```
1 public class SquareMatrix extends Matrix {
2     protected int size;
3     public SquareMatrix (int size) {
4         super(size, size);
5         this.size = size;
6     }
7
8     public int getSize () {
9         return this.size;
10    }
11
12    public int getDeterminant () {
```

```

13         if (this.size == 1)
14             return getElementAt(1, 1);
15         if (this.size == 2)
16             return (getElementAt(1, 1) * getElementAt(2, 2))
17                 - (getElementAt(1, 2) * getElementAt(2, 1));
18         int determinant = 0;
19         for (int i = 1; i <= size; i++)
20             determinant += ((int) Math.pow(-1, 1+i)) * getElementAt(1, i)
21                             * getMinorMatrix(1,
22                                     i).getDeterminant();
23     }
24
25     public SquareMatrix getMinorMatrix (int row, int column) {
26         if (row < 1 || row > size || column < 1 || column > size)
27             return null;
28         if (this.size <= 1)
29             return new SquareMatrix(0);
30         SquareMatrix minor = new SquareMatrix(this.size - 1);
31         for (int i = 1, p = 1; p < size; i++, p++) {
32             if (i == row)
33                 i++;
34             for (int j = 1, q = 1; q < size; j++, q++) {
35                 if (j == column)
36                     j++;
37                 minor.setElementAt(this.getElementAt(i, j), p, q);
38             }
39         }
40         return minor;
41     }
42 }

1  import java.util.Scanner;
2
3  public class Determinant {
4      public static void main (String[] args) {
5          Scanner inp = new Scanner(System.in);
6          try {
7              System.out.print("Enter the size of the (size X size) square
7                  matrix : ");
8              int size = inp.nextInt();
9              SquareMatrix matrix = new SquareMatrix(size);
10             System.out.println("Enter " + (size * size) + " integers : ");
11             for (int i = 1; i <= size; i++)
12                 for (int j = 1; j <= size; j++)

```



```

13         matrix.setElementAt(inp.nextInt(), i, j);
14         System.out.println("\nThe determinant is : " +
        matrix.getDeterminant());
15     } catch (Exception e) {
16         System.out.println("Invalid Input!");
17     }
18 }
19
20 public static void showMatrix (Matrix m) {
21     for (int i = 1; i <= m.getRows(); i++) {
22         for (int j = 1; j <= m.getColumns(); j++) {
23             System.out.printf("%4d ", m.getElementAt(i, j));
24         }
25         System.out.println();
26     }
27 }
28 }

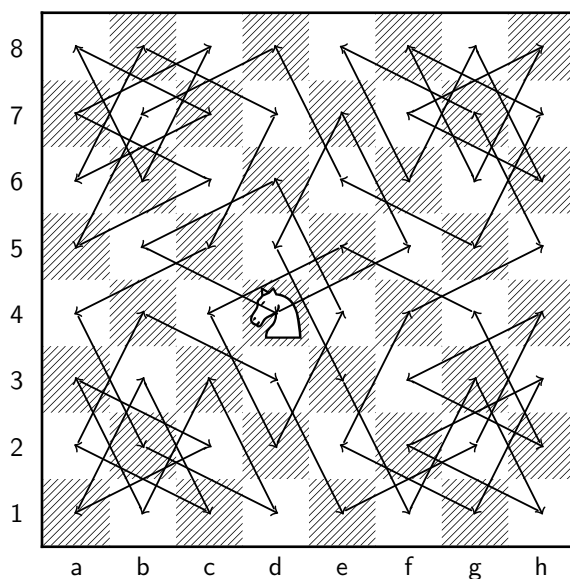
```

*“My project is 90% done. I hope the second half goes as well.”*

— Scott W. Ambler

**Problem 8** A *Knight’s Tour* is a sequence of moves of a knight on a chessboard such that the *knight* visits every square only once. If the knight ends on a square that is one knight’s move from the beginning square, the tour is *closed* forming a closed loop, otherwise it is *open*.

There are many ways of constructing such paths on an empty board. On an  $8 \times 8$  board, there are no less than 26,534,728,821,064 *directed*<sup>5</sup> *closed* tours. Below is one of them.



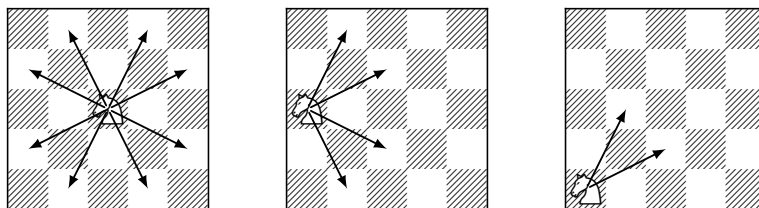
Construct a *Knight’s Tour* (*open* or *closed*) on an  $n \times n$  board, starting from a given square.

(Mark each square with the move number on which the knight landed on it. Mark the starting square 1.)

---

<sup>5</sup>Two tours along the same path that travel in opposite directions are counted separately, as are rotations and reflections.

**Solution** A knight on a chessboard can move to a square that is two squares away horizontally and one square vertically, or two squares vertically and one square horizontally.



The mobility of a knight can vary greatly with its position on the board — near the centre, it can jump to one of 8 squares while when in a corner, it can jump to only 2. On the other hand, the number of possible *sequences* of squares a knight can traverse grows extremely quickly. Although it may seem that a simple *brute force* search can quickly find one of *trillions* of solutions, there are approximately  $4 \times 10^{51}$  different paths to consider on an  $8 \times 8$  board. For even larger boards, iterating through every possible path is clearly impractical.<sup>[citation needed]</sup>

This problem calls for implementing a *backtracking*<sup>6</sup> *algorithm*, coupled with some *heuristic*<sup>7</sup> to speed up the search. One such heuristic is *Warnsdorf's Rule*.

The knight is moved so that it always proceeds to the square from which the knight will have the *fewest* onward moves.

This allows us to define a ranking algorithm for each possible path — the positions which result in the smallest number of further moves, or is furthest away from the board's centre will be investigated first. In case of a tie, we can either proceed without making any changes to the already existing positions, or introduce a random element. This has the effect of producing different results on successive executions, giving a variety of solutions.

One drawback of resolving ties randomly is that an early “wrong” choice in the position tree can force the calculation of every resulting path without reaching a solution, effectively reducing the algorithm to a brute force search. This is especially problematic

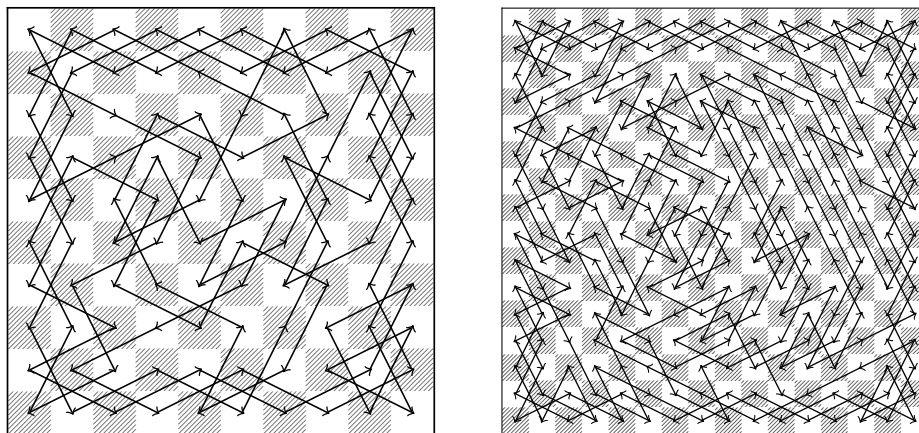
---

<sup>6</sup>Backtracking is a general algorithm for finding some or all solutions to some computational problems that incrementally builds candidates to the solutions, and abandons each partial candidate (“backtracks”) as soon as it determines that the candidate cannot possibly be completed to a valid solution.

<sup>7</sup>A heuristic technique is any approach to problem solving that employs a practical method not guaranteed to be optimal or perfect, but sufficient for the immediate goals. Where finding an optimal solution is impossible or impractical, heuristic methods can be used to speed up the process of finding a satisfactory solution.

for large boards, where it may take hours to backtrack and reach a solution. Thus, the “randomness factor” should be adjusted according to the board size.

A high randomness can be useful for searching specifically for *closed tours*, as a randomness of 0 simply produces the same solution every time (which may or may not be closed). Below are some tours generated by the program.



The tendency of the path to remain close to the edges of the board is clearly evident.

## Source Code

```
1 public class TourSolver {
2     private final int size;
3     private Position[] path;
4     private int numberOfMoves;
5     private int[][] board;
6     private int[][] degreesOfFreedom;
7     private Position initPosition;
8     private double tieBreakRandomness;
9
10    private static final int[][] KNIGHT_MOVES = {
11        {-1, -2}, {-1, 2}, {1, -2}, {1, 2},
12        {-2, -1}, {-2, 1}, {2, -1}, {2, 1}
13    };
14
15    public TourSolver (int size, Position initPosition, double randomness) {
16        this.size = size;
17        this.initPosition = initPosition;
18        this.tieBreakRandomness = randomness / 2.0;
19        this.path = new Position[size * size];
20        this.numberOfMoves = 0;
21        this.initBoard();
22        this.initDegreesOfFreedom();
23    }
24
25    public void resetSolution () {
26        this.path = new Position[size * size];
27        this.numberOfMoves = 0;
28        this.initBoard();
29    }
30
31    private void initBoard () {
32        board = new int[size][size];
33        for (int i = 0; i < size; i++)
34            for (int j = 0; j < size; j++)
35                board[i][j] = 0;
36    }
37
38    private void initDegreesOfFreedom () {
39        degreesOfFreedom = new int[size][size];
40        for (int i = 0; i < size; i++)
41            for (int j = 0; j < size; j++)
42                degreesOfFreedom[i][j] = getPossibleMovesCount(new
43                    Position(i, j));
44    }
```

```

44
45     public boolean addMove (Position p) {
46         if (numberOfMoves < (size * size)) {
47             path[numberOfMoves++] = p;
48             board[p.getX()][p.getY()] = numberOfMoves;
49             return true;
50         }
51         return false;
52     }
53
54     public boolean removeMove () {
55         if (numberOfMoves > 0) {
56             Position p = path[numberOfMoves - 1];
57             board[p.getX()][p.getY()] = 0;
58             path[--numberOfMoves] = null;
59             return true;
60         }
61         return false;
62     }
63
64     public int[][] getBoard () {
65         return board;
66     }
67
68     public Position[] getSolution () {
69         if (size < 5)
70             return null;
71         addMove(initPosition);
72         if(solve(initPosition))
73             return path;
74         return null;
75     }
76
77     public boolean solve (Position p) {
78         if (numberOfMoves == (size * size))
79             return true;
80         Position[] possibleMoves = getPossibleMoves(p);
81         if (possibleMoves[0] == null)
82             return false;
83         sortMoves(possibleMoves);
84         for (Position move : possibleMoves) {
85             if (move != null) {
86                 addMove(move);
87                 if (solve(move))
88                     return true;
89                 removeMove();

```

```

90         }
91     }
92     return false;
93 }
94
95 public void sortMoves (Position[] moves) {
96     int count = 0;
97     for (Position p : moves)
98         if (p != null)
99             count++;
100     for (int right = count; right > 0; right--)
101         for (int i = 1; i < right; i++)
102             if (compareMoves(moves[i-1], moves[i]) > 0)
103                 swapMoves(i-1, i, moves);
104 }
105
106 public int compareMoves (Position a, Position b) {
107     int aCount = getPossibleMovesCount(a);
108     int bCount = getPossibleMovesCount(b);
109     if (aCount != bCount)
110         return aCount - bCount;
111     int aFree = degreesOfFreedom[a.getX()][a.getY()];
112     int bFree = degreesOfFreedom[b.getX()][b.getY()];
113     if (aFree != bFree)
114         return aFree - bFree;
115     return (Math.random() < tieBreakRandomness)? 1 : -1;
116 }
117
118 private static void swapMoves (int x, int y, Position[] moves) {
119     Position t = moves[x];
120     moves[x] = moves[y];
121     moves[y] = t;
122 }
123
124 public Position[] getPossibleMoves (Position start) {
125     Position[] possibleMoves = new Position[KNIGHT_MOVES.length];
126     int i = 0;
127     for (int[] move : KNIGHT_MOVES) {
128         int x = start.getX() + move[0];
129         int y = start.getY() + move[1];
130         if (isWithinBoard(x, y) && board[x][y] == 0) {
131             possibleMoves[i++] = new Position(x, y);
132         }
133     }
134     return possibleMoves;
135 }

```

```

136
137     public int getPossibleMovesCount (Position start) {
138         int i = 0;
139         for (Position p : getPossibleMoves(start))
140             if (p != null)
141                 i++;
142         return i;
143     }
144
145     public boolean isWithinBoard (int x, int y) {
146         return (x >= 0 && x < size && y >= 0 && y < size);
147     }
148 }

1 public class Position {
2     private final int x;
3     private final int y;
4
5     public Position (int x, int y) {
6         this.x = x;
7         this.y = y;
8     }
9
10    public Position (String s) {
11        int x = 0;
12        int i = 0;
13        while (i < s.length() && Character.isAlphabetic(s.charAt(i))) {
14            x = (x * 26) + Character.toLowerCase(s.charAt(i)) - 'a' + 1;
15            i++;
16        }
17        int y = Integer.parseInt(s.substring(i));
18        this.x = x - 1;
19        this.y = y - 1;
20    }
21
22    public int getX () {
23        return x;
24    }
25
26    public int getY () {
27        return y;
28    }
29
30    public boolean equals (Position p) {
31        return (p != null)

```



```

32         && (this.getX() == p.getX()) && (this.getY() == p.getY());
33     }
34
35     @Override
36     public String toString () {
37         return xToString(this.x) + (this.y + 1);
38     }
39
40     public static String xToString (int n) {
41         int x = n + 1;
42         String letters = "";
43         while (x > 0) {
44             letters = (char) ('a' + (--x % 26)) + letters;
45             x /= 26;
46         }
47         return letters;
48     }
49 }

1 public class KnightTour {
2     public static void main (String[] args) {
3         try {
4             int boardSize = Integer.parseInt(args[0]);
5             if (boardSize <= 0)
6                 throw new NumberFormatException();
7             String initSquare = (args.length > 1)? args[1] : "a1";
8             double randomness = (args.length > 2)?
9                 Double.parseDouble(args[2])
10                : Math.pow(0.8, boardSize) * 2;
11             TourSolver t = new TourSolver(boardSize, new
12                 Position(initSquare), randomness);
13             Position[] solution = t.getSolution();
14             if (solution != null) {
15                 showBoard(t.getBoard());
16                 showMoves(solution);
17                 if (isClosed(solution))
18                     System.out.println("\nThe tour is Closed!");
19             } else {
20                 System.out.println("No Knight's Tours found!");
21             }
22         } catch (Exception e) {
23             System.out.print("Enter an integer (> 1) as the first
24                 argument, ");
25             System.out.println("and a well formed chessboard coordinate as
26                 the second!");

```

```

23         System.out.println("                                (size,
           startSquare * , randomness * )");
24     System.out.println();
25     System.out.println("(size      -> Solve a Tour on a (size x
           size) board)");
26     System.out.println("(startSquare * -> A square in algebraic
           chess notation of the form 'fr',");
27     System.out.println("                                where f = the letter
           representing the file(column)");
28     System.out.println("                                and  r = the number
           representing the rank(row).)");
29     System.out.println("(startSquare is set to 'a1' by default)");
30     System.out.println("(randomness * -> A number between 0(no
           randomness) and 1(even chances),");
31     System.out.println("                                determining the randomness in
           ranking positions of");
32     System.out.println("                                the same weightage while
           searching. A randomness of 0 will");
33     System.out.println("                                produce the same tour every
           time, for a specific size and");
34     System.out.println("                                startSquare. Keep extremely
           small values of randomness for");
35     System.out.println("                                very large boards.)");
36     System.out.println("(randomness is set to 2 * (0.8)^boardSize
           by default)");
37     System.out.println();
38     System.out.println("                                <
           * = optional arguments >");
39     }
40 }
41
42 public static void showBoard (int[][] board) {
43     String hLine = " " + multiplyString("+-----", board.length) + "+";
44     System.out.println(hLine);
45     for (int column = board.length - 1; column >= 0; column--) {
46         System.out.printf(" %2d ", column + 1);
47         for (int row = 0; row < board.length; row++) {
48             System.out.printf("| %3d ", board[row][column]);
49         }
50         System.out.printf("|%n%s%n", hLine);
51     }
52     System.out.print(" ");
53     for (int i = 0; i < board.length; i++) {
54         System.out.printf(" %2s ", Position.xToString(i));
55     }
56     System.out.println();

```

```

57     }
58
59     public static void showMoves (Position[] moves) {
60         System.out.print("\nMoves : ");
61         String movesOut = "";
62         for (int i = 1; i < moves.length; i++) {
63             movesOut += (moves[i-1] + "-" + moves[i] + ", ");
64         }
65         System.out.println(movesOut.substring(0, movesOut.length() - 2));
66     }
67
68     public static String multiplyString (String s, int n) {
69         String result = "";
70         while (n --> 0)
71             result += s;
72         return result;
73     }
74
75     public static boolean isClosed (Position[] path) {
76         int l = path.length - 1;
77         int dX = Math.abs(path[0].getX() - path[l].getX());
78         int dY = Math.abs(path[0].getY() - path[l].getY());
79         return (dX == 1 && dY == 2) || (dX == 2 && dY == 1);
80     }
81 }

```

This project was compiled with Xe<sub>La</sub>TeX.

All files involved in the making of this project can be found at  
<https://github.com/sahasatvik/Computer-Project/tree/master/XI>

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