Computer Project

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"Writing code a computer can understand is science. Writing code other programmers can understand is an art."

— **Jason Gorman**

"I am rarely happier than when spending an entire day programming my computer to perform automatically a task that would otherwise take me a good ten seconds to do by hand."

— **Douglas Adams**

Problem 1 An n digit integer $(a_1 a_2 \dots a_n)$, where each digit $a_i \in \{0, 1, \dots, 9\}$, is said to have *unique digits* if no digits are repeated, i.e., there is no i, j such that $a_i = a_j$ $(i \neq j)$.

Verify whether an inputted number has *unique digits*.

Solution The problem involves simply counting the number of occurrences of each digit in the given number and checking whether any of them exceed 1.

`main (number:Integer)`

1. Initialize an integer array `digits` of length `10`, indexed with integers from `[0]` to `[9]` with all elements set to `0`.
2. If `number` exceeds `0`, proceed. Otherwise, jump to (3).
   (a) Store the last digit[1] of `number` in a temporary variable `d`.
   (b) Increment the integer at the `d` index of `digits`.
   (c) If `digits[d]` exceeds `1`, the number does not have *unique digits*. Display a suitable message, and **exit**.
   (d) Discard the last digit of `number` by performing an integer division by `10` and storing the result back in `number`.
   (e) Jump to (2).
3. The number has *unique digits*. Display a suitable message.
4. **Exit**

[1]The last digit of an integer n is simply n mod 10

Source Code

```java
public class Unique {
	public static void main (String[] args) {
		try {
			/* Parse the first command line argument as the number
			   to check for unique digits */
			long number = Long.parseLong(args[0]);
			if (isUnique(number)) {
				System.out.println("Unique Number!");
			} else {
				System.out.println("Not a Unique Number!");
			}
		} catch (NumberFormatException | IndexOutOfBoundsException e) {
			/* Handle missing or incorrectly formatted arguments */
			System.out.println("Enter 1 argument (number[integer])!");
		}
	}

	public static boolean isUnique (long number) {
		/* Keep track of the number of occurrences of each digit */
		int[] count = new int[10];
		for (long n = Math.abs(number); n > 0; n /= 10) {
			/* Extract the last digit of the number */
			int digit = (int) n % 10;
			count[digit]++;
			if (count[digit] > 1){
				return false;
			}
		}
		return true;
	}
}
```

Variable Description

`Unique::main(String[])`		
`long`	`number`	The inputted number

`Unique::isUnique(long)`		
`long`	`number`	The number to check for uniqueness
`int[]`	`count`	The number of occurrences of each digit
`long`	`n`	Counter, temporarily stores the value of `number`
`int`	`digit`	The last digit in `n`

"Elegance is not a dispensable luxury but a factor that decides between success and failure."

— **Edsger W. Dijkstra**

Problem 2 A *partition* of a positive integer n is defined as a collection of other positive integers such that their sum is equal to n. Thus, if $(a_1, a_2, \ldots, a_k)$ is a partition of n,

$$n \;=\; a_1 + a_2 + \cdots + a_k \qquad (a_i \in \mathbb{Z}^+)$$

Display every *unique partition* of an inputted number.

Solution This problem can be solved elegantly using *recursion*[2]. Note that when partitioning a number n, we can calculate the partitions of $(n-1)$ and append 1 to each solution. Similarly, we can append 2 to partitions of $(n-2)$, 3 to partitions of $(n-3)$, and so on. By continuing in this fashion, all cases will be reduced to the single *base case*[3] of finding the partitions of 0, of which there are trivially none.[citation needed]

There is a slight flaw in this algorithm — partitions are often repeated. This can be overcome by imposing the restriction that each new term has to be of a lesser magnitude than the previous. In this way, repeated partitions will be automatically discarded.

`main (target:Integer)`

1. Call `partition(target, target, "")`.
2. **Exit**

`partition (target:Integer, previousTerm:Integer, suffix:String)`

1. If `target` is `0`, display `suffix` and **return**.
2. Initialize a counter `i` to `1`.
3. If `i` is less than or equal to both the `target` and `previousTerm`, proceed. Otherwise, jump to (4).
   (a) Call `partition(target - i, i, suffix + " " + i)`.
   (b) Increment `i` by `1`.
   (c) Jump to (3).
4. **Return**

[2]Recursion occurs when a thing is defined in terms of itself or of its type.

[3]A base case is a case for which the answer is known and can be expressed without recursion.

Source Code

```java
public class Partition {
	public static void main (String[] args) {
		try {
			/* Parse the first command line argument as the target sum */
			int target = Integer.parseInt(args[0]);
			if (target < 1) {
				throw new NumberFormatException();
			}
			partition(target);
		} catch (NumberFormatException | IndexOutOfBoundsException e) {
			/* Handle missing or incorrectly formatted arguments */
			System.out.println("Enter 1 argument (number[natural
			    number])!");
		}
	}

	/* Wrapper method for displaying partitions of a number */
	public static void partition (int target) {
		partition(target, target, "");
	}

	/* Display the partitions of the target */
	public static void partition (int target, int previousTerm, String suffix) {
		/* Base case : '0' has no partitions */
		if (target == 0)
			System.out.println(suffix);
		/* Recursively solve for partitions by diminishing the target,
		   adding that difference to the solution, and partitioning the
		   remaining sum */
		for (int i = 1; i <= target && i <= previousTerm; i++)
			partition(target - i, i, suffix + " " + i);
	}
}
```

Variable Description

`Partition::main(String[])`		
`int`	`target`	The inputted number

`Partition::partition(int)`		
`int`	`target`	The number to be partitioned

`Partition::partition(int, int, String)`		
`int`	`target`	The number to be partitioned
`int`	`previousTerm`	The previous term in the partition sequence
`String`	`suffix`	Terms in the sequence calculated so far
`int`	`i`	Counter variable, stores the next term in the sequence

"Simplicity is the ultimate sophistication."

— **Leonardo da Vinci**

Problem 3 A *Caesar cipher* is a type of monoalphabetic substitution cipher in which each letter in the plaintext is replaced by a letter some fixed number of positions down the alphabet. The positions are circular, i.e., after reaching Z, the position wraps around to A. For example, following is some encrypted text, using a right shift of 5.

```
Plain:   ABCDEFGHIJKLMNOPQRSTUVWXYZ
Cipher:  FGHIJKLMNOPQRSTUVWXYZABCDE
```

Thus, after mapping the alphabet according to the scheme $A \mapsto 0, B \mapsto 1, \ldots, Z \mapsto 25$, we can define an encryption function E_n, in which a letter x is shifted rightwards by n as follows.

$$E_n(x) \;=\; (x + n) \mod 26$$

The corresponding decryption function D_n is simply

$$D_n(x) \;=\; (x - n) \mod 26$$

Implement a simple version of a *Caesar cipher*, encrypting capitalized plaintext by shifting it by a given value. Interpret positive shifts as rightwards, negative as leftwards.

Solution This problem can be solved simply by exploiting the fact that Unicode characters are already arranged in order, with successive alphabets encoded by consecutive numbers. In addition, the encryption function can be defined exactly as given in the question — characters can be converted to their corresponding codes, manipulated by addition of the `shift`, and converted back into alphabetic form.

`main (shift:Integer, plainText:String)`

1. Normalize `plainText` to uppercase.
2. Normalize `shift` by replacing it with `shift` mod 26.
3. Initialize an empty String `cipherText`.
4. Initialize a counter `i` to `0`.
5. If `i` is less than the length of `plainText`, proceed. Otherwise, jump to (6).
   (a) Store the character in `plainText` at position `i` in a variable `plain`.
   (b) Initialize an empty character `crypt`.
   (c) If `plain` is not an alphabet, assign `plain` to `crypt` and jump to (5g).
   (d) Convert `plain` into a number, such that `A` is mapped to `0`, `B` to `1` and so on. Store this in a temporary variable `n`.

(e) Add `shift` to `n`, calculate its least residue modulo 26[4], and store the result in `n`.
(f) Convert `n` back into a character and store the result in `crypt`.
(g) Append `crypt` to `cipherText`.
(h) Increment `i` by `1` and jump to (5).

6. Display `cipherText`.
7. **Exit**

Source Code

```
public class CaesarShift {
	public static void main (String[] args) {
		try {
			/* Parse the first command line argument as the shift */
			int shift = Integer.parseInt(args[0]) % 26;
			/* Parse the second command line argument as the text to
			    encrypt */
			String plaintext = args[1].toUpperCase();
			String ciphertext = "";
			for (int i = 0; i < plaintext.length(); i++) {
				char plain = plaintext.charAt(i);
				char crypt = ' ';
				if ('A' <= plain && plain <= 'Z') {
					/* Only shift letters of the alphabet */
					crypt = numToChar(charToNum(plain) + shift);
				} else {
					/* Keep special characters intact */
					crypt = plain;
				}
				/* Append the encrypted character to the cipherText */
				ciphertext += crypt;
			}
			System.out.println(ciphertext);
		} catch (NumberFormatException | IndexOutOfBoundsException e) {
			/* Handle missing or incorrectly formatted arguments */
			System.out.println("Enter 2 arguments (shift[integer],
			    plaintext[text])!");
		}
	}

	/* Map letters to numbers */
	public static int charToNum (char letter) {
```

[4]The set of integers $K = \{0, 1, 2, \ldots, n-1\}$ is called the least residue system modulo n. The number k such that $k \in K$ and $a \equiv k \pmod{n}$ is called the least residue of a modulo n.

```
31                return Character.toUpperCase(letter) - 'A';
32        }
33
34        /* Map numbers to letters */
35        public static char numToChar (int number) {
36                return (char) ('A' + Math.floorMod(number, 26));
37        }
38  }
```

Variable Description

CaesarShift::main(String[])		
int	shift	The inputted 'shift'
String	plainText	The text to encrypt
String	cipherText	The encrypted text
int	i	Counter variable, stores the position in plainText
char	plain	The character to encrypt
char	crypt	The encrypted form of plain

CaesarShift::charToNum(char)		
char	letter	The character to convert to an integer

CaesarShift::numToChar(int)		
int	number	The number to convert to a character

"There are 2 hard problems in computer science: cache invalidation, naming things, and off-by-1 errors."

— Leon Bambrick

Problem 4 A *palindrome* is a sequence of characters which reads the same backwards as well as forwards. For example, `madam`, `racecar` and `kayak` are words which are palindromes. Similarly, the sentence "`A man, a plan, a canal -- Panama!`" is also a palindrome.

Analyze a sentence of input and display all *words* which are palindromes. If the entire *sentence* is also a palindrome, display it as well.

(A word is an unbroken sequence of characters, separated from other words by whitespace. Ignore single letter words such as I *and* a*. Ignore punctuation, numeric digits, whitespace and case while analyzing the entire sentence.)*

Solution The main challenge here is intelligently dividing a *sentence* into its component *words*. Verifying whether a sequence of characters is a palindrome is fairly simple — extracting those characters from a string of alphabets, numbers, punctuation and whitespace is not.

The main idea behind isolating words from sentences is to define two *markers* — a `start` to keep track of the boundary between whitespace and letters, and an `end` to mark the boundary between letters and whitespace. In this way, the markers can inch their way along the sentence, isolating words in the process. Managing the order of condition checking and incrementing of counters does require some careful manoeuvring in order to avoid any *off-by-1 errors*[5] — any of which would inevitably result in incorrect, hence undesirable output.[citation needed]

`main ()`

1. Accept a string as input, store it in a variable `sentence`.
2. Call `checkWords(sentence)` and `checkSentence(sentence)`. Store the returned values in booleans.
   (a) If either of them is `true`, set a boolean `foundPalindrome` to `true`, otherwise set it to `false`.
3. Display a suitable message if `foundPalindrome` is `false`.
4. **Exit**

[5] An off-by-one error often occurs in computer programming when an iterative loop iterates one time too many or too few.

`checkWords (sentence:String)`

1. Initialize a boolean `foundPalindrome` to false.
2. Initialize two integer counters: `start` to `-1`, `end` to `0`.
3. If `end` is less than the length of `sentence`, proceed. Otherwise, jump to (4).
   (a) Increment `start` as long as the character at the `[start + 1]` position in `sentence` is whitespace.
   (b) Assign `end` to `start`.
   (c) Increment `end` as long as it does not exceed the length of `sentence` and the character at the `[end]` position in `sentence` is not whitespace.
   (d) Assign the string of characters between `start` and `end` from `sentence` (inclusive, exclusive) to a variable `word`.
   (e) Call `isPalindrome(word)`. If `word` is a palindrome:
      i. Set `foundPalindrome` to `true`.
      ii. Display `word`.
   (f) Assign `end - 1` to `start`.
   (g) Jump to (3)
4. **Return** `foundPalindrome`

`checkSentence (sentence:String)`

1. Call `isPalindrome(sentence)`. If `sentence` is a palindrome:
   (a) Display `word`.
   (b) **Return** `true`.
2. **Return** `false`.

`isPalindrome (text:String)`

1. Normalize `text` by converting it into uppercase and removing all non-alphabetic characters.
2. Let the length of `text` be labeled temporarily as `t`.
3. Initialize two integer counters: `i` to `0`, `j` to `l - 1`.
4. If `i` is less than `j`, proceed. Otherwise, jump to (5).
   (a) If the characters at positions `i` and `j` in `text` are not equal, **return** `false`.
   (b) Increment `i` by `1`.
   (c) Decrement `j` by `1`.
   (d) Jump to (4)
5. **Return** `true` only if `text` is longer than one character. Otherwise, **return** `false`.

Source Code

```
import java.util.Scanner;

public class Palindrome {
	public static void main (String[] args) {
		System.out.print("Enter your sentence : ");
		String sentence = (new Scanner(System.in)).nextLine().trim();
		/* Keep track of whether palindromes have been found */
		boolean foundPalindrome = false;
		System.out.println("Palindromes : ");
		foundPalindrome |= checkWords(sentence);
		foundPalindrome |= checkSentence(sentence);
		if (!foundPalindrome) {
			System.out.println("(No palindromes found!)");
		}
	}

	/* Slice a sentence into words and check each individually */
	public static boolean checkWords (String sentence) {
		boolean foundPalindrome = false;
		int start = -1;
		int end = 0;
		while (end < sentence.length()) {
			while (Character.isWhitespace(sentence.charAt(++start)));
			end = start;
			while (end < sentence.length() &&
			    !Character.isWhitespace(sentence.charAt(end++)));
			String word = sentence.substring(start, end).trim();
			if (isPalindrome(word)) {
				foundPalindrome = true;
				System.out.println(getAlphabets(word));
			}
			start = end - 1;
		}
		return foundPalindrome;
	}

	/* Check the sentence as a whole */
	public static boolean checkSentence (String sentence) {
		if (isPalindrome(sentence)) {
			System.out.println("The sentence '" + sentence + "' is a
			    palindrome.");
			return true;
		}
		return false;
```

```
43          }
44
45          /* Check whether a piece of text is identical forward as well as backwards */
46          public static boolean isPalindrome (String text) {
47                  String rawText = getAlphabets(text).toUpperCase();
48                  for (int i = 0, j = rawText.length() - 1; i < j; i++, j--) {
49                          if (rawText.charAt(i) != rawText.charAt(j)) {
50                                  return false;
51                          }
52                  }
53                  /* Make sure that the text is not just one letter */
54                  return (rawText.length() > 1);
55          }
56
57          /* Strip a piece of text of all characters except alphabetic ones */
58          public static String getAlphabets (String text) {
59                  String rawText = "";
60                  for (int i = 0; i < text.length(); i++) {
61                          if (Character.isAlphabetic(text.charAt(i))) {
62                                  rawText += text.charAt(i);
63                          }
64                  }
65                  return rawText;
66          }
67  }
```

Variable Description

Palindrome::main(String[])		
String	sentence	Stores the text to check for palindromes
boolean	foundPalindrome	Stores whether palindromes have been found

Palindrome::checkWords(String)		
String	sentence	Stores the sentence to divide into words
boolean	foundPalindrome	Stores whether palindromes have been found
int	start	Counter variable, stores the index of the start of a word
int	end	Counter variable, stores the index of the end of a word
String	word	Stores words in sentence, extracted between start and end

Palindrome::checkSentence(String)		
String	sentence	Stores the sentence to divide into words

Palindrome::isPalindrome(String)		
String	text	Stores the text to check
String	rawText	Stores only alphabets from text
int	i	Counter variable, stores the current index in text

Palindrome::getAlphabets(String)		
String	text	Stores the text to extract alphabets from
String	rawText	Stores only alphabets from text
int	i	Counter variable, stores the current index in text

"In programming the hard part isn't solving problems, but deciding what problems to solve."

— **Paul Graham**

Problem 5 A *prime number* (or a *prime*) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

Display all primes upto a given limit, along with their number.

Solution This problem can be tackled in a multitude of ways.[citation needed] We could define a function for checking the primality of a given number, then iterate through all numbers in the required range. A common way of checking for primality is *trial division*. It consists of testing whether the number n is a multiple of any integer between 2 and $\sqrt{n}$. Although this works well enough for small numbers, repeating this consecutively for very large inputs is tedious and inefficient. Since the problem consists of identifying primes in a *range*, and not individually, we can make use of more efficient methods.

The *Sieve of Eratosthenes* is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite the multiples of each prime, starting with the first prime number, 2. As a result, when a prime p is found, none of its multiples will be tested further for primality — they are eliminated early on. In comparison, *trial division* has worse theoretical complexity than that of the *Sieve of Eratosthenes* in generating ranges of primes. When testing each prime, the optimal trial division algorithm uses all prime numbers not exceeding its square root, whereas the Sieve of Eratosthenes produces each composite only from its prime factors.

`main (upperLimit:Integer)`

1. Create a new `SieveOfEratosthenes`, pass it `upperLimit` and assign it to `sieve`.
2. Call `sieve->sievePrimes()`.
3. Display the indices which correspond to `true` in the boolean array `sieve->primes`.
4. **Exit**

`SieveOfEratosthenes (upperLimit:Integer)`

1. Initialize a boolean array `primes`, indexed with integers from `[0]` to `[upperLimit - 1]`, with all elements set to `true`.
2. Set `primes[0]` and `primes[1]` to `true`.
3. **Define** the function `SieveOfEratosthenes::sievePrimes()` and **return** the resultant object.

`SieveOfEratosthenes::sievePrimes ()`

1. Initialize an integer variable `prime` to 2.
2. If `prime` is less than the square root of `upperLimit`, proceed. Otherwise, **return**.
   (a) Initialize an integer variable `multiple` to the square of `prime`.
   (b) If `multiple` is less than `upperLimit`, proceed. Otherwise, jump to (2c).
      i. Set `primes[multiple]` to `false`.
      ii. Increment `multiple` by `prime`.
      iii. Jump to (2b)
   (c) Increment `prime` until `primes[prime]` is `true`.
   (d) Jump to (2).
3. **Return**

Source Code

```
1  public class SieveOfEratosthenes {
2         private final int upperLimit;
3         private boolean[] primes;
4
5         /* Initialize the list of numbers using an upper limit */
6         public SieveOfEratosthenes (int upperLimit) {
7                 this.upperLimit = upperLimit;
8                 this.initPrimes();
9         }
10
11        public boolean[] getPrimes () {
12                return primes;
13        }
14
15        /* Initialize all value to 'prime' by default */
16        public void initPrimes () {
17                this.primes = new boolean[upperLimit];
18                /* Mark known values as 'not prime' */
19                primes[0] = false;
20                primes[1] = false;
21                for (int i = 2; i < upperLimit; i++)
22                        primes[i] = true;
23        }
24
25        /* Iteratively sieve the numbers to leave primes behind */
26        public void sievePrimes () {
27                /* Start with the first prime */
28                int prime = 2;
```

```
29              while ((prime * prime) < upperLimit) {
30                      /* Start with the first multiple not crossed off */
31                      int multiple = prime * prime;
32                      while (multiple < upperLimit) {
33                              /* Cross multiples of a prime off the list */
34                              primes[multiple] = false;
35                              multiple += prime;
36                      }
37                      /* Skip forward to the next prime */
38                      while (!primes[++prime]);
39              }
40      }
41  }
```

```
1   public class Primes {
2       public static void main (String[] args) {
3               try {
4                       /* Parse the first command line argument as the upper limit
5                          on primes to calculate */
6                       int upperLimit = Integer.parseInt(args[0]);
7                       if (upperLimit < 2) {
8                               throw new NumberFormatException();
9                       }
10                      SieveOfEratosthenes sieve = new
                            SieveOfEratosthenes(upperLimit);
11                      sieve.sievePrimes();
12                      showPrimes(sieve.getPrimes());
13              } catch (NumberFormatException | IndexOutOfBoundsException e) {
14                      /* Handle missing or incorrectly formatted arguments */
15                      System.out.println("Enter 1 argument (limit[integer, >1])!");
16                      System.out.println("(Primes will be displayed up to, not
                            including 'limit')");
17              }
18      }
19
20      /* Display all primes calculated */
21      public static void showPrimes (boolean[] primes) {
22              int primeCount = 0;
23              /* Format all number to the same width */
24              int maxLength = Integer.toString(primes.length).length();
25              for (int i = 0; i < primes.length; i++) {
26                      /* If 'i' is prime, primes[i] will be marked 'true' */
27                      if (primes[i]) {
28                              System.out.printf("%" + maxLength + "d ", i);
29                              primeCount++;
```

```
30                      }
31                }
32                System.out.println("\nTotal number of primes : " + primeCount);
33        }
34  }
```

Variable Description

SieveOfEratosthenes		
int	upperLimit	The number of integers to sieve
boolean[]	primes	Primes, with contents indicating the primality of the index

SieveOfEratosthenes::initPrimes()		
int	i	Counter variable

SieveOfEratosthenes::sievePrimes()		
int	prime	Counter variable, stores current primes found
int	multiple	Counter variable, stores the multiples of prime

Primes::main(String[])		
int	upperLimit	The highest integer to check for primality (exclusive)
SieveOf Eratosthenes	sieve	An object capable of sieving primes

Primes::showPrimes(boolean[])		
boolean[]	primes	Primes, with contents indicating the primality of the index
int	primeCount	The number of primes found
int	maxLength	The length of the longest number to display
int	i	Counter variable, stores the current integer to check for primality

"Any fool can use a computer. Many do."

— **Ted Nelson**

Problem 6 Design a simple interface for an examiner which can format and display marks scored by a group of students in a particular examination. Calculate the percentage scored by each candidate and display the list of students and percentages in an ASCII bar chart, arranged alphabetically.

Solution This problem calls for a fairly straightforward flow of logic. The main goal is to present the user with a simple way of providing input, along with nicely formatted output.

`main (upperLimit:Integer)`

1. Input the maximum marks allotted for the examination as a floating point. Store it as `maxMarks`.
2. Input the total number of students whose marks are to be recorded as an integer. Store it as `numberOfStudents`.
3. Create a new `Marksheet`, pass it `maxMarks`, `numberOfStudents` and assign it to `sheet`.
4. Initialize an integer counter `i` to `0`;
5. If `i` is less than `numberOfStudents`, proceed. Otherwise, jump to (6).
   (a) Input a student's name as a string. Store it as `name`.
   (b) Input the student's marks as a floating point. Store it as `marks`.
   (c) Call `sheet->addMarks(name, marks)`.
   (d) Jump to (5).
6. Call `sheet->sortByName()`.
7. Call `sheet->displayChart()`.
8. Call `sheet->sortMaxScorers()`.
9. **Exit**

`Marksheet (maxMarks:FloatingPoint, numberOfStudents:Integer)`

1. Initialize a string array `names`, indexed with integers from `[0]` to `[numberOfStudents - 1]`.
2. Initialize a floating point array `marks`, indexed with integers from `[0]` to `[numberOfStudents - 1]`.
3. Initialize an integer counter `lastStudent` to `-1`.
4. **Define** the functions:

(a) `Marksheet::addMarks(name, score)`
(b) `Marksheet::sortByName()`
(c) `Marksheet::displayChart()`
(d) `Marksheet::displayMaxScorers()`

5. **Return** the resultant object.

`Marksheet::addMarks (name:String, score:FloatingPoint)`

1. Increment `lastStudent` by `1`.
2. Set the `names[lastStudent]` to `name`.
3. Set the `marks[lastStudent]` to `score`.
4. **Return**

`Marksheet::sortByName ()`

1. Assign `lastStudent` to `right`.
2. If `right` exceeds `0`, proceed. Otherwise, **return**.
   (a) Initialize an integer counter `i` to `1`.
   (b) If `i` is less than or equal to `right`, proceed. Otherwise, jump to (2c).
      i. If `names[i-1]` comes lexicographically after `names[i]`:
         A. Swap the elements at `names[i-1]` and `names[i]`.
         B. Swap the elements at `marks[i-1]` and `marks[i]`.
      ii. Jump to (2b).
   (c) Jump to (2).

`Marksheet::displayChart ()`

1. For every string `name` in `names`:
   (a) Calculate the length of the bar in the chart as a fraction of the screen width. Store the calculated number of characters to display as `points`.
   (b) Display `name`, a string of suitable characters for the bar of length `points`, along with the percentage scored.
2. **Return**

`Marksheet::displayMaxScorers ()`

1. Calculate the maximum floating point in `marks` and store it as `maxScore`.
2. For every integer `i` between `0` and `numberOfStudents` (inclusive, exclusive) such that `marks[i]` is equal to the `maxScore`, display `names[i]`.
3. **Return**

Source Code

```
public class Marksheet {
	public static final int SCREEN_WIDTH = 100;
	private final double maxMarks;
	private final int numberOfStudents;
	private int lastStudent;
	private String[] names;
	private double[] marks;

	/* Initialize some final data */
	public Marksheet (double maxMarks, int numberOfStudents) {
		this.maxMarks = maxMarks;
		this.numberOfStudents = numberOfStudents;
		this.names = new String[numberOfStudents];
		this.marks = new double[numberOfStudents];
		this.lastStudent = -1;
	}

	/* Add names and marks to the stack */
	public boolean addMarks (String name, double score) {
		try {
			names[++lastStudent] = name;
			marks[lastStudent] = score;
			return true;
		} catch (IndexOutOfBoundsException e) {
			return false;
		}
	}

	/* Display the names and percentages in a bar chart */
	public void displayChart () {
		System.out.println(Marksheet.multiplyString("-",
		    Marksheet.SCREEN_WIDTH));
		for (int i = 0; i <= lastStudent; i++) {
			/* Calculate the fraction of marks earned */
			double fraction = marks[i] / maxMarks;
			String name = (names[i].length() < 16)
					? names[i]
					: (names[i].substring(0,13) + "...");
			int points = (int) (fraction * (SCREEN_WIDTH - 34));
			/* Generate and pad the bar to display */
			String bar = multiplyString("*", points)
				+ multiplyString(" ", SCREEN_WIDTH - 34 - points);
			System.out.printf("| %16s | %s | %6.2f %% |%n"
									, name
```

```
                                                , bar
                                                , fraction * 100);
        }
        System.out.println(Marksheet.multiplyString("-",
            Marksheet.SCREEN_WIDTH));
    }

    /* Display the name of students with the highest score */
    public void displayMaxScorers () {
        String maxScorers = "";
        double maxScore = getMaxScore();
        for (int i = 0; i <= lastStudent; i++) {
            if (marks[i] == maxScore) {
                maxScorers += ", " + names[i];
            }
        }
        System.out.println(maxScorers.substring(1)
                           + " scored the highest ("
                           + maxScore + "/"
                           + maxMarks + ")");
    }

    /* Sort the names and associated marks lexicographically */
    public void sortByName () {
        for (int right = lastStudent; right > 0; right--)
            for (int i = 1; i <= right; i++)
                if (names[i-1].compareToIgnoreCase(names[i]) > 0)
                    swapRecords(i, i - 1);

    }

    /* Get the value of the highest score */
    public double getMaxScore () {
        double max = Integer.MIN_VALUE;
        for (int i = 0; i <= lastStudent; i++) {
            max = Math.max(max, marks[i]);
        }
        return max;
    }

    /* Utility function to swap student records */
    private void swapRecords (int x, int y) {
        String tempName = names[x];
        double tempMark = marks[x];
        names[x] = names[y];
        marks[x] = marks[y];
```

```
89                names[y] = tempName;
90                marks[y] = tempMark;
91        }
92
93        /* Utility function for repeating strings */
94        public static String multiplyString (String s, int n) {
95                String out = "";
96                while (n --> 0)
97                        out += s;
98                return out;
99        }
100 }
```

```
1  import java.util.Scanner;
2  import java.util.InputMismatchException;
3
4  public class ScoreRecorder {
5         public static void main (String[] args) {
6                 /* Create an object capable of managing input */
7                 Scanner inp = new Scanner(System.in);
8                 double maxMarks = 0.0;
9                 int numberOfStudents = 0;
10                try {
11                        System.out.print("Enter the maximum marks allotted for each
                              student : ");
12                        maxMarks = inp.nextDouble();
13                        System.out.print("Enter the total number of students : ");
14                        numberOfStudents = inp.nextInt();
15                        /* Check for any erroneous data */
16                        if (maxMarks <= 0) {
17                               System.out.println("Maximum marks must be positive!");
18                               System.exit(0);
19                        }
20                        if (numberOfStudents <= 0) {
21                               System.out.println("Number of students must be
                                     positive!");
22                               System.exit(0);
23                        }
24                        /* Create an object capable of recording scoresheets */
25                        Marksheet sheet = new Marksheet(maxMarks, numberOfStudents);
26                        System.out.println("Enter " + numberOfStudents + " students'
                              names and marks : ");
27                        /* Accept student data */
28                        for (int i = 0; i < numberOfStudents; i++) {
29                               String name = "";
```

```
30                              while (!inp.hasNextDouble()) {
31                                      name += inp.next() + " ";
32                              }
33                              double marks = inp.nextDouble();
34                              if (marks <= 0 || marks > maxMarks) {
35                                      System.out.println("Marks must be within 0.0 and
                                            " + maxMarks + "!");
36                                      System.exit(0);
37                              }
38                              sheet.addMarks(name.trim(), marks);
39                      }
40                      /* Sort and display */
41                      sheet.sortByName();
42                      sheet.displayChart();
43                      sheet.displayMaxScorers();
44              } catch (InputMismatchException e) {
45                      /* Handle missing or incorrectly formatted arguments */
46                      System.out.println("Invalid Input!");
47                      System.exit(0);
48              }
49      }
50  }
```

Variable Description

Marksheet		
int	SCREEN_WIDTH	Number of characters to use in the display width
double	maxMarks	The maximum marks allotted for the examination
int	numberOf Students	The number of students whose marks are to be recorded
int	lastStudent	The index number of the last student added to the marksheet
String[]	names	The names of the students
double[]	marks	The marks of the students

Marksheet::addMarks(String, double)		
String	name	The name of the student to be added
double	score	The marks of the student to be added

Marksheet::displayChart()		
int	i	Counter variable
double	fraction	The fraction on marks scored over the maximum marks

String	name	Temporarily stores a formatted version of a student's name
int	points	The number of characters to display in the bar chart
String	bar	The bar in the chart, along with whitespace padding

Marksheet::displayMaxScorers()		
String	maxScorers	The list of highest scoring students
double	maxScore	The highest score
int	i	Counter variable

Marksheet::sortByName()		
int	right	Counter variable
int	i	Counter variable

Marksheet::getMaxScore()		
double	max	The maximum score in marks
int	i	Counter variable

Marksheet::swapRecords(int, int)		
int	x, y	The indices of the records to swap
String	tempName	Temporary storage of a name
double	tempMark	Temporary storage of a mark

Marksheet::multiplyString(String, int)		
String	s	The string to multiply
int	n	The number of times to multiply s
String	out	The string containing n copies of s

ScoreRecorder::main(String[])		
Scanner	inp	The input managing object
double	maxMarks	The maximum marks allotted for the examination
int	numberOf Students	The number of students whose marks are to be recorded
Marksheet	sheet	An object capable of managing student records
int	i	Counter variable
String	name	The name of the student to be added
double	marks	The marks of the student to be added

"To iterate is human, to recurse divine"

— L. Peter Deutsch

Problem 7 The *determinant* of a square matrix $A_{n,n}$ is defined recursively as follows.

$$det(A_{n,n}) \;=\; \begin{vmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{vmatrix} \;=\; \sum_{j=1}^{n} (-1)^{i+j} a_{i,j} \cdot det(M_{i,j})$$

where $M_{i,j}$ is defined as the minor of $A_{n,n}$, an $(n-1) \times (n-1)$ matrix formed by removing the ith row and jth column from $A_{n,n}$.

The determinant of a (2×2) matrix is simply given by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \;=\; ad \;-\; bc$$

For example, the determinant of a (3×3) matrix is given by the following expression.

$$\begin{aligned} \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \;&=\; a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ &=\; aei + bfg + cdh - ceg - bdi - afh \end{aligned}$$

Calculate the *determinant* of an inputted $(n \times n)$ square matrix.

Solution This problem offers the opportunity to showcase the power of recursive functions. Here, the complex task of calculating the determinant of a large matrix can be subdivided into multiple smaller tasks. In fact, each of these tasks is precisely the same as the larger one — the only difference is the size of the matrices. Eventually, the problem reduces to finding the determinants of multiple (2×2) matrices. The values thus obtained can be pieced together to form the final answer.

`main ()`

1. Input the size (number of rows/columns) of the square matrix. Store it as `size`.
2. Create a new `SquareMatrix`, pass it `size`, and assign it to `matrix`.
3. For each `i` $\in \{1, 2, \ldots,$ `size`$\}$:

(a) For each `j` $\in \{1, 2, \ldots,$ `size`$\}$:
   i. Input an integer as `n`.
   ii. Set the element at `[i, j]` of `matrix` to `n`.
4. Call `matrix->getDeterminant()` and display the returned value.
5. **Exit**

`Matrix (rows:Integer, columns:Integer)`
1. Initialize an integer array of integer arrays `elements`, indexed with integers from `[1]` to `[rows]`, with each contained integer array indexed with integers from `[1]` to `[columns]` .
2. **Return** the resultant object.

`SquareMatrix (size:Integer)`
1. **Define** the functions:
   (a) `SquareMatrix::getDeterminant()`
   (b) `SquareMatrix::getMinorMatrix(row, column)`
2. **Return** a `Matrix`, with both `rows` and `columns` set to `size`.

`SquareMatrix::getDeterminant ()`
1. If the `size` is `1`, **return** the only element (`elements[1, 1]`).
2. If the `size` is `2`, **return** (`elements[1, 1]`$\times$`elements[2, 2]`) $-$ (`elements[1, 2]`$\times$`elements[2, 1]`).
3. Initialize an integer variable `determinant` to `0`.
4. For each `i` $\in \{1, 2, \ldots,$ `size`$\}$:
   (a) Call `this->getMinorMatrix(i, i)->getDeterminant()`. Store the result in `d`.
   (b) Add $((-1)^{\texttt{i}+1} \times$ `matrix[1, i]` $\times$ `d`) to `determinant`.
5. **Return** `determinant`.

`SquareMatrix::getMinorMatrix (row:Integer, column:Integer)`
1. Create a new `SquareMatrix`, pass it `(size - 1)`, and assign it to `minor`.
2. Copy all elements from `this` to `minor`, except for those at position `[row, *]` or `[*, column]`.
3. **Return** `minor`.

Source Code

```
public class Matrix {
	protected final int rows;
	protected final int columns;
	protected int[][] elements;

	/* Initialize a matrix of a given order */
	public Matrix (int rows, int columns) {
		this.rows = rows;
		this.columns = columns;
		this.elements = new int[rows][columns];
	}

	public int getRows () {
		return this.rows;
	}

	public int getColumns () {
		return this.columns;
	}

	/* Set elements in the matrix using natural indices */
	public void setElementAt (int element, int row, int column) {
		if (row < 1 || row > rows || column < 1 || column > columns)
			return;
		elements[row-1][column-1] = element;
	}

	/* Get elements from the matrix using natural indices */
	public int getElementAt (int row, int column) {
		if (row < 1 || row > rows || column < 1 || column > columns)
			return Integer.MIN_VALUE;
		return elements[row-1][column-1];
	}
}
```

```
public class SquareMatrix extends Matrix {
	protected int size;

	/* Initialize the matrix with the same number of rows and columns */
	public SquareMatrix (int size) {
		super(size, size);
		this.size = size;
	}

```

```
10          public int getSize () {
11                  return this.size;
12          }
13
14          /* Recursively calculate the determinant of the matrix */
15          public int getDeterminant () {
16                  /* Base cases */
17                  if (this.size == 1)
18                          return getElementAt(1, 1);
19                  if (this.size == 2)
20                          return (getElementAt(1, 1) * getElementAt(2, 2))
21                                  - (getElementAt(1, 2) * getElementAt(2, 1));
22                  int determinant = 0;
23                  /* Accumulate the determinants of minors with alternating signs */
24                  for (int i = 1; i <= size; i++)
25                          determinant += ((int) Math.pow(-1, 1+i))
26                                          * getElementAt(1, i)
27                                          * getMinorMatrix(1, i).getDeterminant();
28                  return determinant;
29          }
30
31          /* Get the minor matrix by removing a row and a column */
32          public SquareMatrix getMinorMatrix (int row, int column) {
33                  /* Check bounds */
34                  if (row < 1 || row > size || column < 1 || column > size)
35                          return null;
36                  if (this.size <= 1)
37                          return new SquareMatrix(0);
38                  SquareMatrix minor = new SquareMatrix(this.size - 1);
39                  for (int i = 1, p = 1; p < size; i++, p++) {
40                          /* Skip 'row' */
41                          if (i == row)
42                                  i++;
43                          for (int j = 1, q = 1; q < size; j++, q++) {
44                                  /* Skip 'column' */
45                                  if (j == column)
46                                          j++;
47                                  /* Copy values into the new matrix */
48                                  minor.setElementAt(this.getElementAt(i, j), p, q);
49                          }
50                  }
51                  return minor;
52          }
53  }
```

```java
import java.util.Scanner;

public class Determinant {
	public static void main (String[] args) {
		/* Create an object for managing input */
		Scanner inp = new Scanner(System.in);
		try {
			System.out.print("Enter the size of the (size X size) square
			    matrix : ");
			int size = inp.nextInt();
			/* Create a square matrix which has suitable methods for
			    calculation */
			SquareMatrix matrix = new SquareMatrix(size);
			System.out.println("Enter " + (size * size) + " integers : ");
			for (int i = 1; i <= size; i++)
				for (int j = 1; j <= size; j++)
					matrix.setElementAt(inp.nextInt(), i, j);
			System.out.println("\nThe determinant is : " +
			    matrix.getDeterminant());
		} catch (Exception e) {
			/* Handle missing or incorrectly formatted arguments */
			System.out.println("Invalid Input!");
		}
	}

	/* Display the matrix in a neat format */
	public static void showMatrix (Matrix m) {
		for (int i = 1; i <= m.getRows(); i++) {
			for (int j = 1; j <= m.getColumns(); j++) {
				System.out.printf("%4d ", m.getElementAt(i, j));
			}
			System.out.println();
		}
	}
}
```

Variable Description

Matrix		
int	rows	Number of rows in the matrix
int	columns	Number of columns in the matrix
int[][]	elements	The array of integer arrays, storing the elements of the matrix

SquareMatrix		
int	size	Number of both rows and columns in the matrix

SquareMatrix::getDeterminant()		
int	determinant	The determinant of the SquareMatrix
int	i	Counter variable

SquareMatrix::getMinorMatrix(int, int)		
int	row	The row to remove from the matrix
int	column	The column to remove from the matrix
SquareMatrix	minor	The matrix obtained by removing row and column
int	i, j	Counter variables

Determinant::main(String[])		
Scanner	inp	The input managing object
int	size	Number of both rows and columns in the matrix
SquareMatrix	matrix	The matrix whose determinant is to be calculated
int	i, j	Counter variables

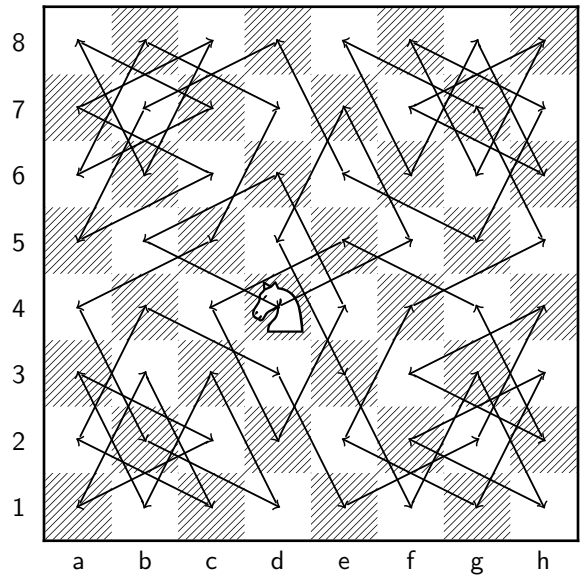
Determinant::showMatrix(Matrix)		
Matrix	m	The matrix to display
int	i, j	Counter variables

"My project is 90% done. I hope the second half goes as well."

— Scott W. Ambler

Problem 8 A *Knight's Tour* is a sequence of moves of a knight on a chessboard such that the *knight* visits every square only once. If the knight ends on a square that is one knight's move from the beginning square, the tour is *closed* forming a closed loop, otherwise it is *open*.

There are many ways of constructing such paths on an empty board. On an 8×8 board, there are no less than $26,534,728,821,064$ *directed*[6] *closed* tours. Below is one of them.

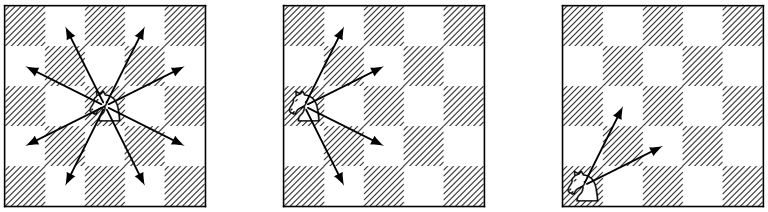


Construct a *Knight's Tour* (*open* or *closed*) on an $n \times n$ board, starting from a given square.

(Mark each square with the move number on which the knight landed on it. Mark the starting square 1*.)*

[6]Two tours along the same path that travel in opposite directions are counted separately, as are rotations and reflections.

Solution A knight on a chessboard can move to a square that is two squares away horizontally and one square vertically, or two squares vertically and one square horizontally.



The mobility of a knight can make varies greatly with its position on the board — near the centre, it can jump to one of 8 squares while when in a corner, it can jump to only 2. On the other hand, the number of possible *sequences* of squares a knight can traverse grows extremely quickly. Although it may seem that a simple *brute force* search can quickly find one of *trillions* of solutions, there are approximately 4×10^{51} different paths to consider on an 8×8 board. For even larger boards, iterating through every possible path is clearly impractical.[citation needed]

This problem calls for implementing a *backtracking*[7] *algorithm*, coupled with some *heuristic*[8] to speed up the search. One such heuristic is *Warnsdorf's Rule*.

> The knight is moved so that it always proceeds to the square from which the knight will have the *fewest* onward moves.

This allows us to define a ranking algorithm for each possible path — the positions which result in the smallest number of further moves, or is furthest away from the board's centre will be investigated first. In case of a tie, we can either proceed without making any changes to the already existing positions, or introduce a random element. This has the effect of producing different results on successive executions, giving a variety of solutions.

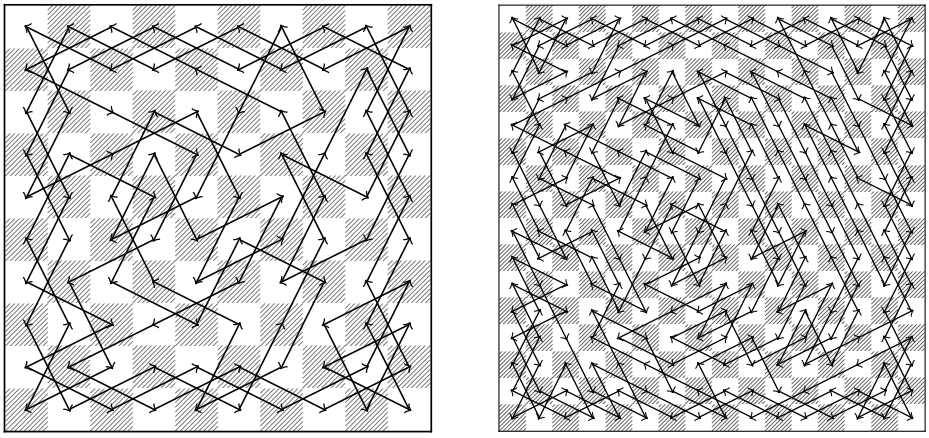
One drawback of resolving ties randomly is that an early "wrong" choice in the position tree can force the calculation of every resulting path without reaching a solution, effectively reducing the algorithm to a brute force search. This is especially problematic

[7] Backtracking is a general algorithm for finding some or all solutions to some computational problems that incrementally builds candidates to the solutions, and abandons each partial candidate ("backtracks") as soon as it determines that the candidate cannot possibly be completed to a valid solution.

[8] A heuristic technique is any approach to problem solving that employs a practical method not guaranteed to be optimal or perfect, but sufficient for the immediate goals. Where finding an optimal solution is impossible or impractical, heuristic methods can be used to speed up the process of finding a satisfactory solution.

for large boards, where it may take hours to backtrack and reach a solution. Thus, the "randomness factor" should be adjusted according to the board size.

A high randomness can be useful for searching specifically for *closed tours*, as a randomness of 0 simply produces the same solution every time (which may or may not be closed). Below are some tours generated by the program.



The tendency of the path to remain close to the edges of the board, where the mobility of the knight is restricted, is clearly evident.

`main (boardSize:Integer, initSquare:Position, randomness:FloatingPoint)`

1. Create a new `TourSolver`, pass it `boardSize`, `initSquare`, `randomness`, and assign it to `t`.
2. Call `t->getSolution()`. Store the returned move stack as `solution`.
3. Display the board obtained by calling `t->getBoard()` along with the moves in `solution`.
4. **Exit**

`TourSolver (size:Integer, initSquare:Position, randomness:FloatingPoint)`

1. Initialize an integer arrays of integer arrays indexed with integers from `[1]` to `[size]`, simulating a chessboard. Store it as `board`, which records the move numbers on which the knight lands on it.
2. Initialize a `Position` stack `path`, along with methods to add and remove `Position`'s from it.
3. Set an integer counter `numberOfMoves` to `0`, as part of the `path` stack.
4. **Define** the functions:
   (a) `TourSolver::solve(p)`

   (b) `TourSolver::getPossibleMoves(p)`
5. **Return** the resultant object.

`TourSolver::solve (p:Position)`

1. If the `path` stack is full, **return** `true`, indicating that the tour has been solved.
2. Call `this->getPossibleMoves(p)`. Store the returned list of possible legal moves as `moves`.
3. Sort `moves`, ranking each possible position according to *Warnsdorf's Rule*.
4. For every `move` in the list `moves`:
   (a) Push `move` onto the `path` stack and `board`.
   (b) If the call `this->solve(move)` returns `true`, **return** `true`. Otherwise, pop `move` from the `path` stack and `board` (*backtrack*).
5. If the list `moves` has been exhausted, **return** `false`, indicating that there are no solutions from the position `p` for that particular move stack.

`TourSolver::getPossibleMoves (p:Position)`

1. Initialize a list of moves `possibleMoves`.
2. For every possible square `move` a knight can jump to from `p` (on an empty board):
   (a) If `move` is currently a legal move, without falling outside the board or on a previously traversed square, add it to `possibleMoves`.
3. **Return** `possibleMoves`

Source Code

```
public class TourSolver {
	private final int size;
	private Position[] path;
	private int numberOfMoves;
	private int[][] board;
	private int[][] degreesOfFreedom;
	private Position initPosition;
	private double tieBreakRandomness;

	/* Store the list of possible changes in the 'x' and 'y' coordinates of
	   a knight on an empty board */
	private static final int[][] KNIGHT_MOVES = {
		{-1, -2}, {-1, 2}, {1, -2}, {1, 2},
		{-2, -1}, {-2, 1}, {2, -1}, {2, 1}
	};

	/* Initialize the board and move stack */
	public TourSolver (int size, Position initPosition, double randomness) {
		this.size = size;
		this.initPosition = initPosition;
		this.tieBreakRandomness = randomness / 2.0;
		this.path = new Position[size * size];
		this.numberOfMoves = 0;
		this.initBoard();
		this.initDegreesOfFreedom();
	}

	/* Reset the board */
	public void resetSolution () {
		this.path = new Position[size * size];
		this.numberOfMoves = 0;
		this.initBoard();
	}

	/* Initialize a blank board */
	private void initBoard () {
		board = new int[size][size];
		for (int i = 0; i < size; i++)
			for (int j = 0; j < size; j++)
				board[i][j] = 0;
	}

	/* Calculate the mobility of a knight on each square */
	private void initDegreesOfFreedom () {
```

```
45              degreesOfFreedom = new int[size][size];
46              for (int i = 0; i < size; i++)
47                      for (int j = 0; j < size; j++)
48                              degreesOfFreedom[i][j] = getPossibleMovesCount(new
                                    Position(i, j));
49      }
50
51      /* Push a move onto the move stack, add it to the board */
52      public boolean addMove (Position p) {
53              if (numberOfMoves < (size * size)) {
54                      path[numberOfMoves++] = p;
55                      board[p.getX()][p.getY()] = numberOfMoves;
56                      return true;
57              }
58              return false;
59      }
60
61      /* Pop a move from the move stack, remove it from the board */
62      public boolean removeMove () {
63              if (numberOfMoves > 0) {
64                      Position p = path[numberOfMoves - 1];
65                      /* Empty squares are marked '0' */
66                      board[p.getX()][p.getY()] = 0;
67                      path[--numberOfMoves] = null;
68                      return true;
69              }
70              return false;
71      }
72
73      public int[][] getBoard () {
74              return board;
75      }
76
77      /* Get the stack of moves comprising a knight's tour */
78      public Position[] getSolution () {
79              if (size < 5)
80                      return null;
81              addMove(initPosition);
82              if(solve(initPosition))
83                      return path;
84              return null;
85      }
86
87      /* Recursively solve a tour from a given position */
88      public boolean solve (Position p) {
89              /* If the move stack is full, the tour has been solved */
```

```
90              if (numberOfMoves == (size * size))
91                      return true;
92              /* Get every legal move and rank them using Warnsdorf's Rule */
93              Position[] possibleMoves = getPossibleMoves(p);
94              if (possibleMoves[0] == null)
95                      return false;
96              sortMoves(possibleMoves);
97              for (Position move : possibleMoves) {
98                      if (move != null) {
99                              /* Try a move */
100                             addMove(move);
101                             if (solve(move))
102                                     return true;
103                             /* Backtrack */
104                             removeMove();
105                     }
106             }
107             return false;
108     }
109
110     /* Sort a list of positions using Warnsdorf's Rule */
111     public void sortMoves (Position[] moves) {
112             int count = 0;
113             for (Position p : moves)
114                     if (p != null)
115                             count++;
116             for (int right = count; right > 0; right--)
117                     for (int i = 1; i < right; i++)
118                             if (compareMoves(moves[i-1], moves[i]) > 0)
119                                     swapMoves(i-1, i, moves);
120     }
121
122     /* Compare 2 moves using Warnsdorf's Rule */
123     public int compareMoves (Position a, Position b) {
124             /* Compare the mobilities of the knight */
125             int aCount = getPossibleMovesCount(a);
126             int bCount = getPossibleMovesCount(b);
127             if (aCount != bCount)
128                     return aCount - bCount;
129             /* Compare the mobilities of the knight on an empty board */
130             int aFree = degreesOfFreedom[a.getX()][a.getY()];
131             int bFree = degreesOfFreedom[b.getX()][b.getY()];
132             if (aFree != bFree)
133                     return aFree - bFree;
134             /* Resolve ties using a pre-decided element of randomness */
135             return (Math.random() < tieBreakRandomness)? 1 : -1;
```

```
136         }
137
138         /* Utility function to swap moves in the list of possible moves */
139         private static void swapMoves (int x, int y, Position[] moves) {
140                 Position t = moves[x];
141                 moves[x] = moves[y];
142                 moves[y] = t;
143         }
144
145         /* Get the list of all possible, legal moves not touching a previously
146            traveled square from a given position */
147         public Position[] getPossibleMoves (Position start) {
148                 Position[] possibleMoves = new Position[KNIGHT_MOVES.length];
149                 int i = 0;
150                 for (int[] move : KNIGHT_MOVES) {
151                         /* Generate a new */
152                         int x = start.getX() + move[0];
153                         int y = start.getY() + move[1];
154                         /* Check the legality of that move */
155                         if (isWithinBoard(x, y) && board[x][y] == 0) {
156                                 possibleMoves[i++] = new Position(x, y);
157                         }
158                 }
159                 return possibleMoves;
160         }
161
162         /* Get the number of legal moves */
163         public int getPossibleMovesCount (Position start) {
164                 int i = 0;
165                 for (Position p : getPossibleMoves(start))
166                         if (p != null)
167                                 i++;
168                 return i;
169         }
170
171         /* Check whether a position lies within the board */
172         public boolean isWithinBoard (int x, int y) {
173                 return (x >= 0 && x < size && y >= 0 && y < size);
174         }
175 }
```

```java
public class Position {
	private final int x;
	private final int y;

	/* Initialize using the coordinates on the board */
	public Position (int x, int y) {
		this.x = x;
		this.y = y;
	}

	/* Initialize using the position in algebraic notation */
	public Position (String s) {
		int x = 0;
		int i = 0;
		while (i < s.length() && Character.isAlphabetic(s.charAt(i))) {
			x = (x * 26) + Character.toLowerCase(s.charAt(i)) - 'a' + 1;
			i++;
		}
		int y = Integer.parseInt(s.substring(i));
		this.x = x - 1;
		this.y = y - 1;
	}

	public int getX () {
		return x;
	}

	public int getY () {
		return y;
	}

	public boolean equals (Position p) {
		return (p != null)
			&& (this.getX() == p.getX()) && (this.getY() == p.getY());
	}

	@Override
	public String toString () {
		return xToString(this.x) + (this.y + 1);
	}

	/* Convert a file number to its algebraic notation form */
	public static String xToString (int n) {
		int x = n + 1;
		String letters = "";
		while (x > 0) {
```

```
                letters = (char) ('a' + (--x % 26)) + letters;
                x /= 26;
            }
            return letters;
        }
}
```

```
public class KnightTour {
        public static void main (String[] args) {
            try {
                /* Parse the first command line argument as the size of the
                    board */
                int boardSize = Integer.parseInt(args[0]);
                if (boardSize <= 0)
                    throw new NumberFormatException();
                /* Parse the second command line argument as the starting
                    square
                  of the knight, written in algebraic notation */
                String initSquare = (args.length > 1)? args[1] : "a1";
                /* Parse the third command line argument as the degree of
                  randomness to be used while resolving ties */
                double randomness = (args.length > 2)?
                    Double.parseDouble(args[2])
                                                : Math.pow(0.8, boardSize) * 2;
                /* Create an object capable of solving knight's tours */
                TourSolver t = new TourSolver(boardSize, new
                    Position(initSquare), randomness);
                Position[] solution = t.getSolution();
                if (solution != null) {
                    showBoard(t.getBoard());
                    showMoves(solution);
                    if (isClosed(solution))
                        System.out.println("\nThe tour is Closed!");
                } else {
                    System.out.println("No Knight's Tours found!");
                }
            } catch (Exception e) {
                /* Handle missing or incorrectly formatted arguments */
                System.out.print("Enter an integer (> 1) as the first
                    argument, ");
                System.out.println("and a well formed chessboard coordinate as
                    the second!");
                System.out.println("                          (size,
                    startSquare * , randomness * )");
                System.out.println();
```

```
32                  System.out.println("(size     -> Solve a Tour on a (size x
                        size) board)");
33                  System.out.println("(startSquare * -> A square in algebraic
                        chess notation of the form 'fr',");
34                  System.out.println("            where f = the letter
                        representing the file(column)");
35                  System.out.println("            and  r = the number
                        representing the rank(row).)");
36                  System.out.println("(startSquare is set to 'a1' by default)");
37                  System.out.println("(randomness * -> A number between 0(no
                        randomness) and 1(even chances),");
38                  System.out.println("            determining the randomness in
                        ranking positions of");
39                  System.out.println("            the same weightage while
                        searching. A randomness of 0 will");
40                  System.out.println("            produce the same tour every
                        time, for a specific size and");
41                  System.out.println("            startSquare. Keep extremely
                        small values of randomness for");
42                  System.out.println("            very large boards.)");
43                  System.out.println("(randomness is set to 2 * (0.8)^boardSize
                        by default)");
44                  System.out.println();
45                  System.out.println("                                        <
                        * = optional arguments >");
46          }
47      }
48
49      /* Display the board, with each square marked with the move number on which
50         the knight landed on it */
51      public static void showBoard (int[][] board) {
52              String hLine = "  " + multiplyString("+-----", board.length) + "+";
53              System.out.println(hLine);
54              for (int column = board.length - 1; column >= 0; column--) {
55                      System.out.printf(" %2d ", column + 1);
56                      for (int row = 0; row < board.length; row++) {
57                              System.out.printf("| %3d ", board[row][column]);
58                      }
59                      System.out.printf("|%n%s%n", hLine);
60              }
61              System.out.print("  ");
62              for (int i = 0; i < board.length; i++) {
63                      System.out.printf(" %2s ", Position.xToString(i));
64              }
65              System.out.println();
66      }
```

```
67
68          /* Display the list of moves in the tour in algebraic notation */
69          public static void showMoves (Position[] moves) {
70                  System.out.print("\nMoves : ");
71                  String movesOut = "";
72                  for (int i = 1; i < moves.length; i++) {
73                          movesOut += (moves[i-1] + "-" + moves[i] + ", ");
74                  }
75                  System.out.println(movesOut.substring(0, movesOut.length() - 2));
76          }
77
78          /* Utility function for repeating strings */
79          public static String multiplyString (String s, int n) {
80                  String result = "";
81                  while (n --> 0)
82                          result += s;
83                  return result;
84          }
85
86          /* Check whether a tour is closed or not */
87          public static boolean isClosed (Position[] path) {
88                  int l = path.length - 1;
89                  int dX = Math.abs(path[0].getX() - path[l].getX());
90                  int dY = Math.abs(path[0].getY() - path[l].getY());
91                  return (dX == 1 && dY == 2) || (dX == 2 && dY == 1);
92          }
93  }
```

Variable Description

TourSolver		
int	size	Number of files/ranks in the chessboard
Position[]	path	Stack of moves which are part of the solved tour
int	numberOfMoves	Counter variable, number of moves made in the solved tour
int[][]	board	An integer array of integer arrays, representing a chessboard, with each square marked with the move number at which the knight lands on it
int[][]	degreesOf Freedom	An integer array of integer arrays, representing a chessboard, with each square marked with the number of possible knight moves from it (on an empty board)
Position	initPosition	The position on the board the knight starts from
double	tieBreak Randomness	The degree to which a move in the path is randomly decided
int[][]	KNIGHT_MOVES	List of legal changes in the x and y positions of a knight
TourSolver::initBoard()		
int	i, j	Counter variables
TourSolver::initDegreesOfFreedom()		
int	i, j	Counter variables
TourSolver::addMove(Position)		
Position	p	The new position to add to the path stack
TourSolver::removeMove()		
Position	p	The position popped from the path stack
TourSolver::solve()		
Position[]	possible Moves	List of possible moves that can be added to the path stack
Position	move	Current move to evaluate in the path
TourSolver::sortMoves(Position[])		
Position[]	moves	List of moves to rank using Warnsdorf's heuristic
int	count	Total number of moves in moves
int	right	Counter variable
int	i	Counter variable

TourSolver::compareMoves(Position, Position)		
Position	a, b	Positions/moves to compare using Warnsdorf's heuristic
int	aCount, bCount	Respective number of possible legal moves for a and b
int	aFree, bFree	Respective number of possible legal moves on an empty board for a and b

TourSolver::swapMoves(int, int, Position[])		
int	x, y	The indices of the moves to swap
Position[]	moves	Array of moves containing the moves to be swapped

TourSolver::getPossibleMoves(Position)		
Position	start	Position from where possible moves are to be generated
int	i	Counter variable
int[]	move	Pair of legal changes in the x and y positions of a knight
int	x, y	New x and y positions of the knight

TourSolver::getPossibleMovesCount(Position)		
Position	start	Position from where possible moves are to be generated
Position	p	Possible position

TourSolver::isWithinBoard(int, int)		
int	x, y	The x and y positions on the board to verify

Position		
int	x, y	The x and y coordinates on the board encoded by the Position

Position::this(String)		
String	s	Chess position written in algebraic notation
int	x, y	The x and y coordinates on the board
int	i	Counter variable

Position::xToString(int)		
int	n	File (x position) to convert to algebraic notation
String	letters	n expressed as a base 26 number, digits starting from (a)

`int`	`x`	Counter variable, temporarily stores the file to convert

`KnightTour::main(String[])`		
`int`	`boardSize`	Number of files/ranks in the chessboard
`String`	`initSquare`	The position on the board the knight starts from (algebraic notation)
`double`	`randomness`	The degree to which a move in the path is randomly decided
`TourSolver`	`t`	An object capable of generating *knight's tours*
`Position[]`	`solution`	The solved sequence of moves in the *knight's tour*

`KnightTour::showBoard(int[][])`		
`int[][]`	`board`	An integer array of integer arrays, representing a chessboard, with each square marked with the move number at which the knight lands on it
`String`	`hline`	Horizontal line drawn to represent board squares
`int`	`row, column, i`	Counter variables

`KnightTour::showMoves(Position[])`		
`Position[]`	`moves`	The sequence of moves to display
`int`	`i`	Counter variable

`KnightTour::multiplyString(String, int)`		
`String`	`s`	The string to multiply
`int`	`n`	The number of times to multiply `s`
`String`	`out`	The string containing `n` copies of `s`

`KnightTour::isClosed(Position[])`		
`Position[]`	`path`	The solved sequence of moves in the *knight's tour*
`int`	`l`	Index of last move in `path`
`int`	`dX, dY`	Differences in x and y coordinates of the knight between the first and last moves

"Curiosity begins as an act of tearing to pieces, or analysis."

— **Samuel Alexander**

Problem 9 Calculate the *square root* of a given positive number, using only *addition*, *subtraction*, *multiplication* and *division*.

Solution The problem of finding the *square root* of a positive real number k is equivalent to finding a positive root of the function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$

$$f(x) \;=\; x^2 - k$$

This problem can be solved using *Newton's method*. *Newton's method* is an iterative process for finding a root of a general function $f : \mathbb{R} \to \mathbb{R}$ by creating an initial guess, then improving upon it.

Let f' denote the derivative of the function f. Thus, the equation of the tangent to the curve $f(x)$, drawn through the point $(x_n, f(x_n))$ is given by the following equation.

$$y \;=\; f'(x_n)(x - x_n) + f(x_n)$$

The idea here is that the *x-intercept* of this tangent will be a better approximation to the root of the function f. Setting $y = 0$, solving for x and renaming it to x_{n+1} yields the following expression.

$$x_{n+1} \;=\; x_n \;-\; \frac{f(x_n)}{f'(x_n)}$$

Plugging in the required function for this problem, we have

$$x_{n+1} \;=\; x_n \;-\; \frac{x_n^2 - k}{2x_n}$$

Simplifying, we arrive at our expression for the term x_{n+1} in our iterative process.

$$x_{n+1} \;=\; \frac{1}{2}\left(x_n \;+\; \frac{k}{x_n}\right)$$

This is the sort of simple expression we have been looking for, involving only one addition and two multiplications per iteration. As n becomes very large, the term x_n approaches the *square root* of k.

`main (number:FloatingPoint, maxIterations:Integer)`

1. Call `squareRoot(number, maxIterations)`. Store the result in `root`.
2. Display `root`, along with the error from the value calculated by the library function `Math->sqrt(number)`.
3. **Exit**

`squareRoot (n:FloatingPoint, maxIterations:Integer)`

1. Store the initial guess `n / 2` in the variable `x`.
2. For `maxIterations` times:
   (a) Calculate `0.5 * (x + (n / x))`. Store the result back in `x`.
3. **Return** `x`

Source Code

```
1  public class SquareRoot {
2        public static void main (String[] args) {
3              /* Parse the first command line argument as the number to square root
                   */
4              double number = Double.parseDouble(args[0]);
5              /* Parse the second conmand line argument as the number of iterations.
6                Default to 100 */
7              int maxIterations = (args.length > 1)? Integer.parseInt(args[1]) :
                   100;
8
9              double root = squareRoot(number, maxIterations);
10             double library_root = Math.sqrt(number);
11
12             /* Display the calculated root, along with a comparison with the
13               library calculated value */
14             System.out.printf("Calculated square root :  %f%n", root);
15             System.out.printf("System library square root : %f%n", library_root);
16             System.out.printf("Error : %f%n", (root - library_root));
17       }
18
19       public static double squareRoot (double n, int maxIterations) {
20             /* Handle edge cases, ignore negative values */
21             if (n < 0)
22                   return Double.NaN;
23             if (n == 0)
24                   return 0.0;
25             /* Start by guessing half of the number */
26             double x = n / 2;
```

```
27              for (int i = 0; i < maxIterations; i++) {
28                      x = 0.5 * (x + (n / x));
29              }
30              return x;
31      }
32  }
```

Variable Description

SquareRoot::main(String[])		
double	number	Stores the number whose square root is to be extracted
int	maxIterations	Stores the number of iterations for which Newton's method is to be applied
double	root	Stores the calculated square root of number
double	library_root	Stores the square root of number given by the Java library

SquareRoot::squareRoot(double, int)		
double	x	Stores the results of successive iterations of Newton's method
int	i	Counter variable

"Objects are abstractions of processing. Threads are abstractions of schedule."

— **James O. Coplien**

Problem 10 Let a *fraction* here be restricted to the ratio of two integers, m and n, where $n \neq 0$. Thus, a fraction $\frac{m}{n}$ is said to be reduced its *lowest terms* when m and n are relatively prime.

Implement this model of *fractions*, such that they are *immutable* and reduced to their *lowest terms* by default. Also implement a simple method for adding two *fractions*.

Solution The problem of reducing a fraction $\frac{m}{n}$ to its lowest terms can be solved simply by dividing the numerator and the denominator by their *greatest common divisor*, i.e., $\gcd(m, n)$. This works as $\gcd(p, q) = 1$ if and only if p and q are relatively prime. Fraction addition can also be implemented using the following formula.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

The *greatest common divisor* of two integers can be calculated recursively using *Euclid's algorithm*.

$$\gcd(a, b) = \gcd(b, a \bmod b)$$

`main ()`

1. Create 2 `Fraction` objects `a` and `b` using data supplied by the user.
2. Call `Fraction->addFractions(a, b)`. Store the result in another `Fraction` object `sum`.
3. Display `a`, `b` and `sum`.
4. **Exit**

`Fraction (numerator:Integer, denominator:Integer)`

1. Set internal variables `numerator` and `denominator`, keeping them private.
2. Reduce the fraction to its lowest form.
   (a) Calculate the *greatest common divisor* of `numerator` and `denominator`, then divide each by the result.
   (b) Shift any negative sign in `denominator` to `numerator`.
3. **Define** the function `Fraction::addFractions(fraction1, fraction2)`, and **return** the resultant object.

`Fraction::addFractions (fraction1:Fraction, fraction2:Fraction)`

1. Calculate the `numerator` and `denominator` of the sum using the formula discussed above.
2. Create a new `Fraction` object using the calculated `numerator` and `denominator`, then **return** it.

Source Code

```
1  public class Fraction {
2
3         /* Store the numerator and denominator */
4         protected int numerator;
5         protected int denominator;
6
7         public Fraction (int numerator, int denominator) {
8                 /* Handle invalid fractions */
9                 if (denominator == 0)
10                        throw new ArithmeticException("Division by zero!");
11
12                this.numerator = Math.abs(numerator);
13                this.denominator = Math.abs(denominator);
14
15                if (numerator != 0) {
16                        /* Reduce to lowest terms */
```

```
17                      int g = gcd(this.numerator, this.denominator);
18                      this.numerator /= g;
19                      this.denominator /= g;
20                      /* Make sure that the sign is on the numerator */
21                      this.numerator *= Math.signum(numerator) *
                            Math.signum(denominator);
22              } else {
23                      /* Make sure all 'zero fractions' are the same */
24                      this.denominator = 1;
25              }
26      }
27
28      public int getNumerator () {
29              return this.numerator;
30      }
31
32      public int getDenominator () {
33              return this.denominator;
34      }
35
36      /* Return a String representation of the Fraction for display */
37      public String toString () {
38              /* Format all fractions with denominator '1' as simple integers */
39              if (this.denominator == 1)
40                      return this.numerator + "";
41              return this.numerator + " / " + this.denominator;
42      }
43
44      /* Add 2 Fraction objects */
45      public static Fraction addFractions (Fraction a, Fraction b) {
46              int sumNumerator = (a.getNumerator() * b.getDenominator()) +
                    (a.getDenominator() * b.getNumerator());
47              int sumDenominator = a.getDenominator() * b.getDenominator();
48              return new Fraction(sumNumerator, sumDenominator);
49      }
50
51      /* Calculate the greatest common divisor of integers, using Euclid's method
            recursively */
52      private static int gcd (int p, int q) {
53              return (p < q)? gcd(q, p) : ((p % q) == 0)? q : gcd(q, p % q);
54      }
55  }
```

```java
import java.util.Scanner;

public class FractionAdder {
	public static void main (String[] args) {
		Scanner inp = new Scanner(System.in);
		try {
			/* Get the two fractions from user input */
			System.out.print("Enter the numerator and denominator [integer
			    integer] of the first fraction : ");
			Fraction a = new Fraction(inp.nextInt(), inp.nextInt());
			System.out.print("Enter the numerator and denominator [integer
			    integer] of the second fraction : ");
			Fraction b = new Fraction(inp.nextInt(), inp.nextInt());

			/* Calculate and display the sum of the fractions.
			   Here, we take advantage of the toString() method defined for
			       Fractions */
			Fraction sum = Fraction.addFractions(a, b);
			System.out.printf("%n(%s) + (%s) = (%s) %n", a, b, sum);
		} catch (ArithmeticException e) {
			System.out.println("Invalid fraction - division by zero!");
		}
	}
}
```

Variable Description

`Fraction`		
`int`	`numerator`	Stores the numerator of the fraction
`int`	`denominator`	Stores the denominator of the fraction

`Fraction(int, int)`		
`int`	`g`	Stores the greatest common divisor of `numerator` and `denominator`

`Fraction::addFractions(Fraction, Fraction)`		
`Fraction`	`a, b`	The two fractions to be added
`int`	`sumNumerator`	The numerator of the sum
`int`	`sumDenominator`	The denominator of the sum

`FractionAdder::main(String[])`		
`Scanner`	`inp`	The input managing object
`Fraction`	`a, b`	The two fractions to be added
`Fraction`	`sum`	The sum of the fractions `a` and `b`

"Dividing one number by another is mere computation; knowing what to divide by what is mathematics."

— **Jordan Ellenberg**

Problem 11 A rational number q can be broken down into a *simple continued fraction* in the form given below.

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \cfrac{1}{a_n}}}}$$

This may be represented by the abbreviated notation $[a_0; a_1, a_2, \ldots, a_n]$. For example, $[0; 1, 1, 2, 1, 4, 2]$ is shorthand for the following.

$$\frac{42}{73} = 0 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2}}}}}}$$

Calculate the *simple continued fraction* expression for a given, positive fraction.

Solution We can thus solve this problem recursively by noting that the following holds.

$$\frac{p}{q} = \underbrace{\left\lfloor \frac{p}{q} \right\rfloor}_{\text{Integer part}} + \underbrace{\frac{p \bmod q}{q}}_{\text{Fractional part}}$$

Thus, by defining $f(\frac{p}{q})$ as the continued fraction representation of the fraction $\frac{p}{q}$, we can write

$$f\left(\frac{p}{q}\right) = \left\lfloor \frac{p}{q} \right\rfloor + f\left(\frac{q}{p \bmod q}\right)$$

Here, we are going to use the `Fraction` class defined in the solution to **Problem 10**, in order to take advantage of the reduced form and sign checks it carries out.

`main (numerator:Integer, denominator:Integer)`

1. Pack `numerator` and `denominator` into a `Fraction` object. Store it as `f`.
2. Call `getContinuedFraction(f)`. Display the returned `String`.
3. **Exit**

`getContinuedFraction (Fraction f)`

1. Unpack `numerator` and `denominator` from `f`.
2. Call `getContinuedFraction(numerator, denominator)`. Store the returned `String` in the variable `expansion`.
3. Replace the first comma (`,`) in `expansion` with a semicolon (`;`).
4. **Return** `expansion`

`getContinuedFraction (numerator:Integer, denominator:Integer)`

1. **If** `denominator` is `1`, **return** `numerator`.
2. Calculate the integer part of `numerator / denominator`. Store it in `x`.
3. Call `getContinuedFraction(denominator, numerator % denominator)`. Store the result in `y`.
4. **Return** `x + y`

Source Code

```
1  public class ContinuedFraction {
2        public static void main (String[] args) {
3              try {
4                     /* Parse command line arguments as the numerator and
                          denominator
5                        of the fraction */
6                     int numerator = Integer.parseInt(args[0]);
7                     int denominator = Integer.parseInt(args[1]);
8                     System.out.println(getContinuedFraction(new
                          Fraction(numerator, denominator)));
9              } catch (ArithmeticException e) {
10                    System.out.println("Invalid fraction - division by zero!");
11             } catch (Exception e) {
12                    System.out.println("Enter 2 arguments! ([numerator]
                          [denominator])");
13             }
14       }
15
16       /* Return the String representation of the continued fraction */
17       public static String getContinuedFraction (Fraction f) {
```

```
18              String expansion = "[" + getContinuedFraction(f.getNumerator(),
                    f.getDenominator());
19              /* By convention, the first comma is replaced with a semicolon */
20              return expansion.replaceFirst(",", ";");
21      }
22
23      /* Recursively calculate the continued fraction representation */
24      public static String getContinuedFraction (int numerator, int denominator) {
25              /* Base case : the fraction is now irreducible */
26              if (denominator == 1)
27                      return numerator + "]";
28              /* Pull out the integer part, invert the fraction and recurse */
29              return (numerator / denominator) + ", " +
                    getContinuedFraction(denominator, numerator % denominator);
30      }
31  }
```

Variable Description

ContinuedFraction::main(String[])		
int	numerator	Stores the numerator of the fraction to evaluate
int	denominator	Stores the denominator of the fraction to evaluate

ContinuedFraction::getContinuedFraction(Fraction)		
Fraction	f	Stores the fraction to evaluate
String	expansion	Stores the continued fraction representation of f

"Intelligence is the ability to avoid doing work, yet getting the work done."

— **Linus Torvalds**

Problem 12 The *binomial coefficient* [9] of two integers $n \geq k \geq 0$ is defined as follows.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Here, $n!$ is the *factorial* of n, defined as follows.

$$n! = 1 \times 2 \times 3 \times \cdots \times (n-2) \times (n-1) \times n$$

Compute the binomial coefficient for two given integers.

Solution Note that we can rewrite the definition of the binomial by cancelling out common factors from the factorials.

$$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-(k-1))}{k(k-1)(k-2)\cdots 1}$$

Now that we have this definition, it is easy to see that we can separate the term $\frac{n}{k}$ and leave behind a smaller binomial coefficient. Thus, we arrive at the recursive formula

$$\binom{n}{k} = \frac{n}{k} \cdot \binom{n-1}{k-1}$$

Coupled with the observation that $\binom{n}{0} = 1$, we can solve this problem recursively.

We can introduce a small optimisation by observing that $\binom{n}{k} = \binom{n}{n-k}$. Thus, for $k > \frac{n}{2}$, we can replace k with $n-k$ to reduce the number of recursive calls.

[9] They are given this name as they describe the coefficients of the expansion of powers of a binomial, according to the *binomial theorem*.

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

`main (n:Integer, k:Integer)`

1. Call and display `binomial(n, k)`.
2. **Exit**

`binomial (n:Integer, k:Integer)`

1. **If** `k` is zero, **return** 1.
2. **If** `k` exceeds half of `n`, call `binomial(n, n - k)`.
3. **Return** `binomial(n - 1, k - 1) * (n / k)`.

Source Code

```
1  public class Binomial {
2  	public static void main (String[] args) {
3  		try {
4  			/* Parse the command line arguments as the terms in the
   			    binomial coefficient */
5  			long n = Long.parseLong(args[0]);
6  			long k = Long.parseLong(args[1]);
7  			System.out.println(binomial(n, k));
8  
9  		} catch (NumberFormatException | IndexOutOfBoundsException e) {
10 			System.out.println("Enter 2 arguments! ([+integer]
   			    [+integer])");
11 		} catch (Exception e) {
12 			System.out.println("Invalid 'k'! (0 <= k <= n)");
13 		}
14 	}
15 
16 	/* Recursively calculate the binomial coeffient n choose k */
17 	public static long binomial (long n, long k) throws Exception {
18 		/* Invalid case */
19 		if (k > n)
20 			throw new Exception();
21 		/* Base case : n choose 0 is 1 */
22 		if (k == 0)
23 			return 1;
24 		/* Optimisation to reduce the number of recursive steps by reflecting
25 		   k along the middle of n */
26 		if (k > (n / 2))
27 			return binomial(n, n - k);
28 		/* Recurse by unfolding the multiplication */
29 		return (n * binomial(n - 1, k - 1) / k);
30 	}
31 }
```

Variable Description

`Binomial::main(String[])`		
`long`	`n, k`	The arguments for calculating the binomial coefficient

`Binomial::binomial(long, long)`		
`long`	`n, k`	The arguments for calculating the binomial coefficient

"If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is."

— **John von Neumann**

Problem 13 Palindromes can be generated in many ways. One of them involves picking a number, reversing the order of its digits and adding the result to the original. For example, we have

$$135 + 531 \;=\; 666$$

Not all numbers will yield a palindrome after one step. Instead, we can repeat the above process, using the sum obtained as as the new number to reverse.

$$\begin{aligned} 963 + 369 &= 1332 \\ 1332 + 2331 &= 3663 \end{aligned}$$

This process is often called the *196-algorithm*. Some numbers seem never to yield a palindrome even after millions of iterations. These are called *Lychrel numbers*. The smallest of these in base 10 is conjectured to be the number 196, although none have been mathematically proven to exist.

Generate the steps and final palindrome of the *196-algorithm*, given a natural number as a *seed* [10].

Solution This problem can be solved without much complication. We can either create a loop, or use *tail recursion* [11] to roll up the process. The only problem here is that the numbers involved grow very large, very fast. Thus, care must be taken while dealing with such cases. Here, a library method for addition has been used to identify integer overflow.

[10] A *seed* is an initial number, from which subsequent numbers are generated.

[11] *Tail recursion* involves the use of *tail calls*. These are simply recursive function calls which appear as the last statement of the function body. Most programming languages can optimize tail recursion internally into a simple loop, thus avoiding the addition of stack frames on each recursive call.

`main (number:Integer)`

1. Call `generatePalindrome(number, 0)`.
2. **Exit**

`generatePalindrome (n:Integer, step:Integer)`

1. Reverse the digits in `n`. Store the result in `r`.
2. **If** `n` is equal to `r`:
   (a) Display `n` as a palindrome, along with `step`.
   (b) **Return**
3. Add `n` and `r`. Store the sum in the variable `sum`.
4. Call `generatePalindrome(sum, step + 1)`

Source Code

```
class PalindromeGenerator {
	public static void main (String[] args) {
		/* Parse the first command line argument as the seed */
		long n = Long.parseLong(args[0]);
		generatePalindrome(n, 0);
	}

	public static void generatePalindrome (long n, int step) {
		long r = reverse(n);
		if (n == r) {
			/* Base case : palindrome reached */
			System.out.printf("%d is a palindrome (%d step%s)%n", n, step,
			    ((step == 1)? "" : "s"));
		} else {
			try {
				/* Use a library method to add. This will throw an
				   Exception in case of overflow, which would have
				   otherwise been ignored */
				long sum = Math.addExact(n, r);
				System.out.printf("%d + %d = %d%n", n, r, sum);
				/* Recurse via tail recursion, simply incrementing the
				    step value */
				generatePalindrome(sum, step + 1);
			} catch (ArithmeticException e) {
				/* Stop if the numbers become too big */
				System.out.printf("Long Overflow - Sum exceeded maximum
				    size at step %d%n", step);
			}
```

```
26              }
27      }
28
29      /* Reverse the integer supplied */
30      public static long reverse (long n) {
31              long r = 0;
32              while (n > 0) {
33                      /* Pull out the last digit and accumulate it on another
                            variable */
34                      r = (r * 10) + (n % 10);
35                      n /= 10;
36              }
37              return r;
38      }
39  }
```

Variable Description

PalindromeGenerator::main(String[])		
long	n	Stores the *seed* for the palindrome generation

PalindromeGenerator::generatePalindrome(long, int)		
long	n	Stores the current number to generate a palindrome from
long	r	Stores the reverse of n
int	step	Stores the step of the generation currently executing
long	sum	Stores the sum of n and r

PalindromeGenerator::reverse(long)		
long	r	Stores the reverse of n

"Over thinking leads to problems that don't even exist in the first place."

— **Jayson Engay**

Problem 14 Compute the *prime factorization* of a given natural number.

Solution This solution is meant to showcase the drawbacks of using *recursion* in some problems.

Let $f(n)$ denote the expansion of the *prime factorization* of the natural number n. We *could* observe that if we can find naturals p and q such that $n = pq$, we can write

$$f(pq) \;=\; f(p) + f(q)$$

Using this, we can wrap up the iteration over the naturals into a recursive function.

The problem with this approach is that for moderately large numbers, the number of nested calls grows rapidly. For large enough numbers, the default memory allocated for the *call stack* by the *Java Virtual Machine* falls woefully short. As a result, it becomes necessary to manually set the size of the *thread stack size* by passing the `-Xss<size>` option to the *JVM* during program execution.

`main (number:Integer)`

1. Call and display `factorize(number, 2)`.
2. **Exit**

`factorize (n:Integer, next:Integer)`

1. **If** `n` is one, **return** an empty `String`.
2. **If** `next` exceeds, or is equal to, `n`, **return** `next`.
3. **If** `next` divides `n`:
   (a) Append `next` to the `String` returned by the call `factorize(n / next, next)`.
   (b) **Return** the above value.
4. **Return** `factorize(n, next + 1)`

Source Code

```
1 public class Factorize {
2       public static void main (String[] args) {
3             /* Parse the first command line argument as the number to factorize */
4             int number = Integer.parseInt(args[0]);
5             /* Start from 2 */
```

```
6               System.out.println(factorize(number, 2));
7       }
8
9       /* Return the String representation of the prime factorization of an integer
            */
10      public static String factorize (int n, int next) {
11              /* Base case 1 : nothing to factorize */
12              if (n == 1)
13                      return "";
14              /* Base case 2 : reached a prime */
15              if (next >= n)
16                      return next + "";
17              /* Check for a factor */
18              if ((n % next) == 0)
19                      return next + " " + factorize(n / next, next);
20              /* Recurse by incrementing the next 'factor' to check */
21              return factorize(n, next + 1);
22      }
23  }
```

Variable Description

Factorize::main(String[])		
int	number	Stores the number to be factorized

Factorize::main(String[])		
int	n	Stores the current number to be factorized
int	next	Stores the next number to check for divisibility

"Meaning lies as much
in the mind of the reader
as in the Haiku."

— **Douglas Hofstadter**

Problem 15 A *codebook* is a document which stores a *lookup table* for coding and decoding text – each word has a different word, phrase or string to replace it. Design a system which, when given a *codebook* written in plaintext, translates a given sentence into its encoded form.

Solution Solving this problem requires careful reading of the supplied codebook. Here, the following format is assumed.

```
word          codeword
next_word     other_codeword
.             .
.             .
```

Thus, this data can be transformed into an *array*, which can then be searched for strings appearing in the supplied input.

`main (codebook:String)`

1. Create a `CodeSubstituter` object, pass it the filename `codebook`, and assign it to `cs`.
2. Get a line of user input. Store it in `sentence`.
3. Split `sentence` along whitespace into the `String` array `words`.
4. For each `word` in `words`:
   (a) Call `cs->getEncodedText(word)`. Store the result in `encodedText`.
   (b) Display `encodedText`.
5. **Exit**

`CodeSubstituter (codebook:String)`

1. Open the file pointed to by `codebook`. Start from the beginning in read mode.
2. On the first pass through `codebook`, count the number of lines and store the result in `numberOfLines`.
3. Close, and reopen `codebook`. Start at the beginning.
4. Initialize a `2` column `String` array, with `numberOfLines` as the number of rows. Assign it to `wordMap`.

5. Start reading `codebook` again. For each line, stored in `line` and each row in `wordMap` :
   (a) Split `line` along whitespace.
   (b) Store the first half in the first column of `wordMap`, and the second half in the second column of the same.
6. Close the file `codebook`.
7. **Define** the function `CodeSubstituter::getEncodedText(word)` and **return** the resultant object.

`CodeSubstituter::getEncodedText (word:String)`

1. For each row in `wordMap`:
   (a) If the first column entry matches `word`, **return** the second column entry.
2. **Return** `word`

Source Code

```
1  import java.io.IOException;
2  import java.io.FileReader;
3  import java.io.BufferedReader;
4
5  public class CodeSubstituter {
6         protected String filename;
7
8         protected int numberOfLines;
9         protected String[][] wordMap;
10
11        /* Create a codebook from a supplied file */
12        public CodeSubstituter (String filename) throws IOException {
13               this.filename = filename;
14               countNumberOfLines();
15               initWordMap();
16        }
17
18        /* Calculate the number of lines to store on the first pass */
19        private void countNumberOfLines () throws IOException {
20               FileReader fileReader = new FileReader(filename);
21               BufferedReader bufferedReader = new BufferedReader(fileReader);
22
23               numberOfLines = 0;
24               /* Keep incrementing the accumulator while lines are available */
25               while (bufferedReader.readLine() != null)
26                      numberOfLines++;
27
```

```
28              bufferedReader.close();
29              fileReader.close();
30      }
31
32      /* Initialize the map/dictionary by reading the file on the second pass */
33      private void initWordMap () throws IOException {
34              wordMap = new String[numberOfLines][2];
35
36              FileReader fileReader = new FileReader(filename);
37              BufferedReader bufferedReader = new BufferedReader(fileReader);
38
39              for (int i = 0; i < numberOfLines; i++) {
40                      /* Split a line along whitespace */
41                      String[] words = bufferedReader.readLine().split("\\s+");
42                      if (words.length >= 2) {
43                              wordMap[i][0] = words[0];
44                              wordMap[i][1] = words[1];
45                      } else {
46                              /* Ignore empty lines */
47                              wordMap[i][0] = wordMap[i][1] = "";
48                      }
49              }
50
51              bufferedReader.close();
52              fileReader.close();
53      }
54
55      /* Returns the codeword, given a plain word */
56      public String getEncodedText (String word) {
57              /* Iterate through all entries */
58              for (int i = 0; i < numberOfLines; i++) {
59                      if (wordMap[i][0].equalsIgnoreCase(word)) {
60                              return wordMap[i][1];
61                      }
62              }
63              /* Reflect the original back if not found in the codebook */
64              return word;
65      }
66  }
```

```java
import java.util.Scanner;
import java.io.IOException;
import java.io.FileNotFoundException;

public class TextEncoder {
	public static void main (String[] args) throws Exception {
		try {
			/* Parse the first command line argument as the path to the
			    codebook */
			CodeSubstituter cs = new CodeSubstituter(args[0]);

			/* Get a sentence to encode, and extract the individual words
			    */
			System.out.print("Enter a sentence to encode : ");
			String sentence = (new Scanner(System.in)).nextLine();
			String[] words = sentence.split("\\s+");

			System.out.print("Encoded sentence     : ");
			/* Iterate through each word, replacing it with the codeword
			    in the codebook */
			for (int i = 0; i < words.length; i++) {
				String encodedText =
				    cs.getEncodedText(words[i].toLowerCase().replaceAll("[^a-z]",
				    ""));
				System.out.print(encodedText + " ");
			}
			System.out.println();
		} catch (ArrayIndexOutOfBoundsException e) {
			System.out.println("Enter 1 argument ([codebook_filename])");
		} catch (FileNotFoundException e) {
			System.out.println("Codebook not found! Enter a valid
			    filename.");
		} catch (IOException e) {
			e.printStackTrace();
		}
	}
}
```

Variable Description

CodeSubstituter		
String	filename	Stores the path of the file containing the codebook
int	numberOfLines	Stores the number of lines in the file filename
String[][]	wordMap	A table of plain words and their corresponding codewords

CodeSubstituter::countNumberOfLines()		
FileReader	fileReader	An object for reading character based files
Buffered Reader	bufferedReader	An object for buffering character streams

CodeSubstituter::initWordMap()		
FileReader	fileReader	An object for reading character based files
Buffered Reader	bufferedReader	An object for buffering character streams
String[]	words	Temporarily stores the parts of a line in the codebook

TextEncoder::main(String[])		
Code Substituter	cs	An object for accessing a codebook
String	sentence	Stores a line of user input to be encoded
String[]	words	Stores the list of words in sentence

"Hofstadter's Law: It always takes longer than you expect, even when you take into account Hofstadter's Law."

— Douglas Hofstadter

Problem 16 Analyse the frequency of each letter in the English alphabet appearing in a given file. Store the results in a different file.

Solution All that has to be done here is reading the contents of a file, counting the occurrences of each character, then tabulating the results before writing them to another file. Here, the characters have also been sorted based on their frequencies.

`main (fromFile:String, toFile:String)`

1. Create a `CharacterCounter` object, pass it `fromFile`, and assign it to `cc`.
2. Call `cc->writeReportToFile(toFile)`.
3. **Exit**

`CharacterCounter (fromFile:String)`

1. Read all the lines from the file `fromFile` and store the resultant `String` in `fileData`.
2. Initialize a `26` row `Character` array `letters`, as well as a `26` row `Integer` array `letterCount`.
3. For each letter `c` $\in \{a, b, \ldots, z\}$:
   (a) Store `c` in an empty row in `letters`.
   (b) Count the number of occurrences of `c` in `fileData`. Store the result in the corresponding row in `letterCount`.
   (c) Move to a new row in `letters` and `letterCount`.
4. Store the sum of all entries in `letterCount` in the variable `totalLetters`.
5. Sort the entries in `letters` and `letterCount`, in descending order of the entries in `letterCount` using *bubble sort*.
6. **Define** the function `CharacterCounter::writeReportToFile(toFile)` and **return** the resultant object.

`CharacterCounter::writeReportToFile (toFile:String)`

1. Open the file pointed to by `toFile`. Start from the beginning in write mode.
2. Write all entries in `letters` and `letterCount`, formatted to include the ratio of the entry in `letterCount` to `totalLetters`.

3. Write `totalLetters` to `toFile`, along with any entry in `letters` whose corresponding entry in `letterCount` is zero.
4. Close the file `toFile`.
5. **Return**

Source Code

```
1  import java.io.IOException;
2  import java.io.FileReader;
3  import java.io.FileWriter;
4  import java.io.BufferedReader;
5  import java.io.BufferedWriter;
6  import java.io.PrintWriter;
7
8  public class CharacterCounter {
9         protected String filename;
10
11        protected String fileData;
12        protected char[] letters;
13        protected int[] letterCount;
14        protected int totalLetters;
15
16        /* Create a table of letter counts in a given file */
17        public CharacterCounter (String filename) throws IOException {
18               this.filename = filename;
19               this.fileData = "";
20               this.letterCount = new int[26];
21               this.letters = new char[26];
22               this.totalLetters = 0;
23               getFileData();
24               countAllLetters();
25               sortLetters();
26        }
27
28        /* Read all lines in the file and store them in a String */
29        private void getFileData () throws IOException {
30               FileReader fileReader = new FileReader(filename);
31               BufferedReader bufferedReader = new BufferedReader(fileReader);
32
33               String line = "";
34               while ((line = bufferedReader.readLine()) != null)
35                      fileData += line.toLowerCase();
36
37               bufferedReader.close();
38               fileReader.close();
```

```
39          }
40
41          /* Return the number of occurrences of a character in the file */
42          public int getCountOf (char c) {
43                  int count = 0;
44                  for (int i = 0; i < fileData.length(); i++) {
45                          if (fileData.charAt(i) == c) {
46                                  count++;
47                          }
48                  }
49                  return count;
50          }
51
52          /* Compile the counts of all letters in the file */
53          public void countAllLetters () {
54                  for (char c = 'a'; c <= 'z'; c++) {
55                          letters[c - 'a'] = c;
56                          letterCount[c - 'a'] = getCountOf(c);
57                          totalLetters += letterCount[c - 'a'];
58                  }
59          }
60
61          /* Sort the entries by frequency (bubble sort) */
62          private void sortLetters () {
63                  for (int right = 26; right > 0; right--)
64                          for (int i = 1; i < right; i++)
65                                  if (letterCount[i] > letterCount[i-1])
66                                          swap(i, i-1);
67          }
68
69          /* Utility swapping method */
70          private void swap (int i, int j) {
71                  char tmpChar = letters[i];
72                  int tmpCount = letterCount[i];
73                  letters[i] = letters[i-1];
74                  letterCount[i] = letterCount[i-1];
75                  letters[i-1] = tmpChar;
76                  letterCount[i-1] = tmpCount;
77          }
78
79          /* Create and write the final report to a file */
80          public void writeReportToFile (String toFilename) throws IOException {
81                  FileWriter fileWriter = new FileWriter(toFilename);
82                  BufferedWriter bufferedWriter = new BufferedWriter(fileWriter);
83                  PrintWriter printWriter = new PrintWriter(bufferedWriter);
84
```

```
85              /* Make sure the frequencies all fit, aligned in the same column */
86              int l = (totalLetters + "").length();
87              String unusedLetters = "";
88              for (int i = 0; i < 26; i++) {
89                      /* Show the letter, frequency and percentage out of the total
                           */
90                      char c = letters[i];
91                      int count = letterCount[i];
92                      double percent = (count * 100.0) / totalLetters;
93                      if (count > 0) {
94                              printWriter.printf("%c : %5.2f%% (%" + l + "d) %n",
95                                              c, percent, count);
96                      } else {
97                              /* Separate unused letters */
98                              unusedLetters += c + " ";
99                      }
100             }
101             printWriter.printf("Total letters : %d%n", totalLetters);
102             if (unusedLetters.length() == 0)
103                     unusedLetters = "(none)";
104             printWriter.printf("Unused letters : %s%n", unusedLetters);
105
106             printWriter.close();
107             bufferedWriter.close();
108             fileWriter.close();
109     }
110 }
```

```
1  import java.io.IOException;
2  import java.io.FileNotFoundException;
3
4  public class AnalyseCharacterFrequency {
5          public static void main (String[] args) {
6                  try {
7                          /* Parse the commnd line arguments as the file to analyse and
                               the
8                            file to pipe the results into */
9                          String fromFile = args[0];
10                         String toFile = args[1];
11
12                         /* Create and write the report */
13                         CharacterCounter cc = new CharacterCounter(fromFile);
14                         cc.writeReportToFile(toFile);
15                 } catch (ArrayIndexOutOfBoundsException e) {
16                         System.out.println("Enter 2 arguments! ([filename_from]
```

```
                        [filename_to])");
17              } catch (FileNotFoundException e) {
18                      System.out.println("Enter a valid filename!");
19              } catch (IOException e) {
20                      e.printStackTrace();
21              }
22      }
23  }
```

Variable Description

CharacterCounter		
String	filename	Stores the path of the file to analyse
String	fileData	Stores all character data from the file
char[]	letters	The list of all letters, in order of frequency
int[]	letterCount	The frequencies of each corresponding letter in letters
int	totalLetters	Stores the total number of letters in fileData
CharacterCounter::getFileData()		
FileReader	fileReader	An object for reading character based files
BufferedReader	bufferedReader	An object for buffering character streams
String	line	Stores a line of text in the file
CharacterCounter::getCountOf(char)		
char	c	The character whose frequency is to be found in fileData
int	count	The frequency of c in fileData
CharacterCounter::countAllLetters()		
char	c	The character whose frequency is to be found
CharacterCounter::sortLetters()		
int	right, i	Counter variables
CharacterCounter::swap(int, int)		
int	i, j	Indices of letters and letterCount whose entries are to be swapped
CharacterCounter::writeReportToFile(String)		
String	toFilename	Stores the path of the file to write the report to
FileWriter	fileWriter	An object for writing character based files

`BufferedWriter`	`bufferedWriter`	An object for buffering character streams being written to a file
`PrintWriter`	`printWriter`	An object for writing data to an output stream
`int`	`l`	Stores the number of digits in `totalLetters`
`String`	`unusedLetters`	Stores the list of letters not present in `fileData`
`char`	`c`	Stores the current character being written
`int`	`count`	Stores the frequency of `c`
`double`	`percent`	Stores the percentage of `count` out of `totalLetters`

`AnalyseCharacterFrequency::main(String[])`		
`String`	`fromFile`	Stores the path of the file to analyse
`String`	`toFile`	Stores the path of the file to write the report to
`CharacterCounter`	`cc`	An object for analysing the frequencies of letters in files

"If Java had true garbage collection, most programs would delete themselves upon execution."

— **Robert Sewell**

Problem 17 The classical *Möbius function* $\mu(n)$ is an important function in number theory and combinatorics. For positive integers n, $\mu(n)$ is defined as the sum of the primitive n^{th} roots of unity. It attains the following values.

$\mu(1) = +1$
$\mu(n) = -1$ if n is a square-free positive integer with an odd number of prime factors.
$\mu(n) = \ \ 0$ if n has a squared prime factor.
$\mu(n) = +1$ if n is a square-free positive integer with an even number of prime factors.

Compute the $\mu(n)$ for positive integers n within a specified range.

Solution For any given $n \in \mathbb{N}$, all we have to do is search for factors by trial-division, and find their multiplicity. If this is greater than 1, we can stop here since we have found squared prime factors. Otherwise, we can reduce the problem by dividing out these factors from n and repeating. By trying factors in ascending order and then discarding them from n, we are guaranteed to hit only prime factors, and can thus skip primality checks.

`main (lo:Integer, hi:Integer)`

1. Assert that the integers in the range `[lo, hi)` are all positive.
2. For each `i` $\in \{$`lo`$, $`lo`$ + 1, \ldots,$ `hi`$ - 1\}$:
   (a) Call and display `mobius(i)`.
3. **Exit**

`mobius (n:Integer)`

1. If `n` is one, **return** `1`.
2. Initialize an integer variable `mob` to one.
3. For `i` $\in \{2, 3, \ldots, n\}$:
   (a) Initialize an integer `multiplicity` to zero.
   (b) While `i` divides `n`, assign `n / i` to `n` and increment `multiplicity`.
   (c) If `multiplicity` is one, flip the sign of `mob`.
   (d) If `multiplicity` is greater than one, **return** `0`.
4. **Return** `mob`

Source Code

```
public class Mobius {
	public static final String[] graph =
		{"*       ",
		 "   *    ",
		 "       *"};
	public static void main (String[] args) {
		try {
			int lo = Integer.parseInt(args[0]);
			int hi = Integer.parseInt(args[1]);
			if (lo < 1 || hi <= lo)
				throw new NumberFormatException();
			for (int i = lo; i < hi; i++) {
				int m = mobius(i);
				System.out.printf(" (%d)\t\t = %2d%24s\n", i, m, graph[m
					+ 1]);
			}
		} catch (NumberFormatException | IndexOutOfBoundsException e) {
			System.out.println("Enter 2 arguments (lower_limit[integer,
				>0], upper_limit[integer, >lower_limit])!");
		}
	}

	public static int mobius (int n) {
		if (n < 1)
			return 0;
		if (n == 1)
			return 1;
		int mob = 1;
		for (int i = 2; i <= n; i++) {
			int multiplicity = 0;
			while ((n % i) == 0) {
				n /= i;
				multiplicity++;
			}
			if (multiplicity == 1) {
				mob = -mob;
			} else if (multiplicity > 1) {
				return 0;
			}
		}
		return mob;
	}
}
```

Variable Description

`Mobius::main(String[])`		
`int`	`lo`	Lower bound of integers to evalute
`int`	`hi`	Upper bound of integers to evalute
`int`	`i`	Counter variable, stores the integer to be evaluated

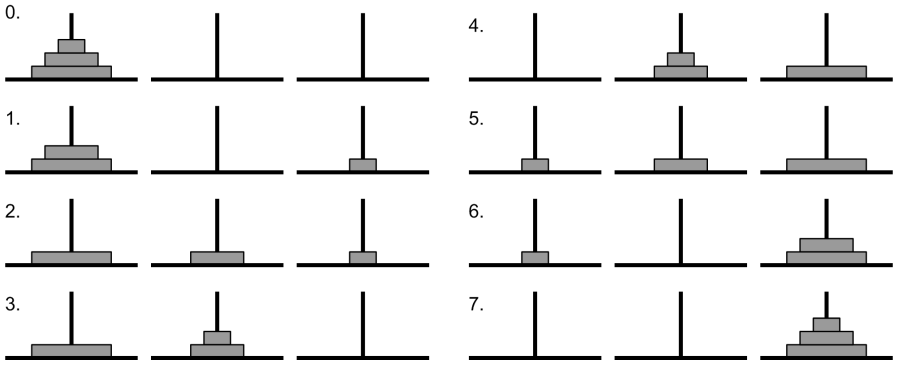
`Mobius::mobius(int)`		
`int`	`n`	The number where the mobius function is to be evaluated
`int`	`mob`	Sign of the value of the mobius function
`int`	`i`	Counter variable, stores the current factor to be tested
`int`	`multiplicity`	The power of `i` in the factorisation of `n`

"In order to understand recursion, one must first understand recursion."

— **Anonymous**

Problem 18 The *Tower of Hanoi* is a mathematical puzzle, consisting of three rods and a number of disks of different sizes which can slide onto any rod. The puzzle starts with all disks, in ascending order of size, on one rod. The objective of the puzzle is to move the entire stack to another rod, obeying the foolowing rules.

1. Only one disk can be moved at a time.
2. Each move consists of taking the upper disk from one stack and placing it on the top of another stack or empty rod.
3. No disk can be placed on a smaller disk.



Solution to the Towers of Hanoi with 3 disks.

Solve the *Tower of Hanoi* puzzle for an arbitrary number of disks, enumerating the required moves.

Solution The main insight here is that the problem involving n disks can be reduced to one with $n-1$ disks. Labelling the rods A, B and C, and the disks with numerals 1 through n (smallest to largest), our aim is to move the entire stack from A to C. If we can solve the problem with $n-1$ disks, all we have to do is to move the topmost $n-1$ disks from A to B, move the remaining disk on A to C, and again move the $n-1$ disks on B to C. The base case for this recursive solution is moving 1 disk, which is trivial.

Clearly, if the problem with n disks takes k_n number of moves, the problem with $n+1$ moves will take $k_n + 1 + k_n = 2k_n + 1$ moves. For the base case with one disk,

$k_1 = 1$. With this infromation, we see that the *Tower of Hanoi* with n disks can be solved in exactly $2^n - 1$ moves.

`main (disks:Integer)`

1. Call `solveHanoi(disks, "A", "C", "B")`.
2. **Exit**

`solveHanoi (disk:Integer, source:String, destination:String, spare:String)`

1. If `disk` is zero, **return**.
2. Call `solveHanoi(disk - 1, source, spare, destination)`.
3. Move disk number `disk` has to be moved from `source` to `destination`.
4. Call `solveHanoi(disk - 1, spare, destination, source)`.
5. **Return**

Source Code

```
1  public class TowersOfHanoi {
2        public static void main (String[] args) {
3              try {
4                     int disks = Integer.parseInt(args[0]);
5                     if (disks < 1)
6                           throw new NumberFormatException();
7                     solveHanoi(disks, "A", "C", "B");
8              } catch (NumberFormatException | IndexOutOfBoundsException e) {
9                     System.out.println("Enter 1 argument
                          (number_of_disks[integer])!");
10             }
11       }
12
13       public static void solveHanoi (int disk, String source, String destination,
             String spare) {
14             if (disk == 0)
15                    return;
16             solveHanoi(disk - 1, source, spare, destination);
17             System.out.printf("(%d) : %s -> %s%n", disk, source, destination);
18             solveHanoi(disk - 1, spare, destination, source);
19       }
20 }
```

Variable Description

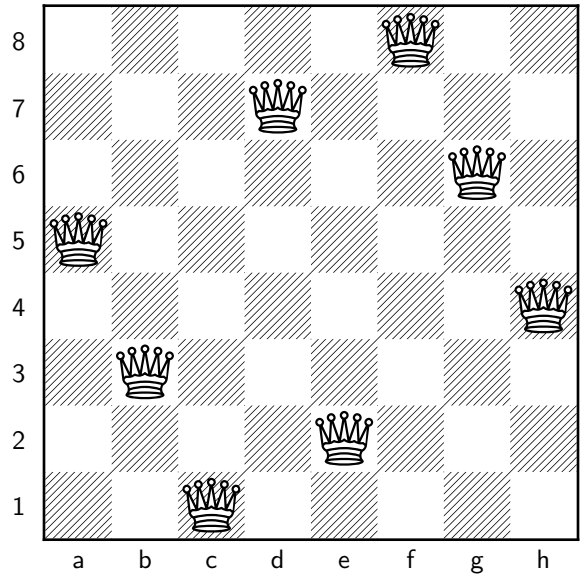
TowersOfHanoi::main(String[])		
int	disks	The number of disks in the problem

TowersOfHanoi::solveHanoi(int, String, String, String)		
int	disk	The current disk to be moved
String	source	The rod from which the stack is to be moved
String	destination	The rod to which the stack is to be moved
String	spare	The additional rod, where the remaining n-1 disks are temporarily moved

"Chess is the gymnasium of the mind."

— Blaise Pascal

Problem 19 The *8 queens puzzle* involves placing 8 queens on an 8×8 chessboard such that no two queens threaten each other, i.e. no two queens share the same rank, file or diagonal. It was first published by the chess composer *Max Bezzel* in 1848. This puzzle has 92 solutions, including reflections and rotations. Below is one of them.



The *n queens puzzle* is an extension of this puzzle, involving n queens on an $n \times n$ chessboard. Count the total number of solutions for the *n queens puzzle*, including reflections and rotations.

Solution This problem can be solved with *recursion* and *backtracking*. Starting from the topmost row of the chessboard, we can place a queen and for each available choice, place a queen on the next row, and so on, recursively shrinking the chessboard to solve. Invalid solutions can thus be discarded as they are formed without brute-forcing every possible permutation of queens on the board.

Finally, by noting that exactly one queen must occupy each row, we can optimize the board by storing only the column numbers of queens on each row in an array, instead of simulating a full 2D board.

Source Code

```
1  public class NQueens {
2          private final int size;
3          private int[] board;
4          private int numberOfSolutions;
5          private final boolean drawSolutions;
6
7          public NQueens (int size, boolean drawSolutions) {
8                  this.size = size;
9                  this.drawSolutions = drawSolutions;
10                 this.initBoard();
11         }
12
13         public int countSolutions () {
14                 solveNQueens(0);
15                 return numberOfSolutions;
16         }
17
18         private void initBoard () {
19                 this.board = new int[size];
20                 this.numberOfSolutions = 0;
21                 for (int i = 0; i < size; i++)
22                         board[i] = -1;
23         }
24
25         private boolean isThreatened (int row) {
26                 for (int i = 0; i < row; i++) {
27                         if ((board[row] == board[i])
28                             || ((board[row] - board[i]) == (row - i))
29                             || ((board[row] - board[i]) == (i - row))) {
30                                 return true;
31                         }
32                 }
33                 return false;
34         }
35
36         private void solveNQueens (int row) {
37                 if (row == size) {
38                         numberOfSolutions++;
39                         if (drawSolutions) {
40                                 drawBoard();
```

```
41                              System.out.println();
42                      }
43                      return;
44              }
45              for (board[row] = 0; board[row] < size; board[row]++) {
46                      if (!isThreatened(row)) {
47                              solveNQueens(row + 1);
48                      }
49              }
50      }
51
52      public void drawBoard () {
53              for (int i = 0; i < size; i++) {
54                      for (int j = 0; j < size; j++) {
55                              System.out.print(((board[i] == j)? "Q" : "-") + " ");
56                      }
57                      System.out.println();
58              }
59      }
60
61      public static void main (String[] args) {
62              try {
63                      int size = Integer.parseInt(args[0]);
64                      boolean drawSolutions = (args.length > 1)?
                            Boolean.parseBoolean(args[1]) : false;
65                      if (size < 1)
66                              throw new NumberFormatException();
67
68                      NQueens q = new NQueens(size, drawSolutions);
69                      System.out.println(q.countSolutions());
70              } catch (NumberFormatException | IndexOutOfBoundsException e) {
71                      System.out.println("Enter at least 1 argument
                            (size_of_board[integer], <show_solutions>[true/false])!");
72                      System.out.println("(show_solutions defaults to false)");
73              }
74      }
75  }
```

Variable Description

"Computers are useless. They can only give you answers."

— Pablo Picasso

Problem 20 *Reverse Polish Notation (RPN)* or *postfix notation* is a mathematical notation for writing arithmetic expresssions in which operators follow their operands. Thus, as long as each operator has a fixed number of operands, the use of parentheses or rules of precedence are no longer required to write unambiguous expressions. For example, the expression `2 3 * 3 2 ^ 2 - *` evaluates to `42`.

Create a program capable of evaluating *RPN* expressions which use the following operators.

`+`	Addition
`-`	Subtraction
`*`	Multiplication
`/`	Division
`^`	Exponentiation

Solution The nature of *RPN* lends itself to a very simple implementation with a stack for pushing operands into as they appear in an expression. When an operator is encountered, the required number of operands are popped from the stack, the operation is carried out, and the result is popped back into the stack. This continued until the entire expression has been parsed, leaving only the evaluated result in the stack.

`main (expression:String)`

1. Call `evaluateRPNExpression(expression)` and display the returned value.
2. **Exit**

`evaluateRPNExpression (expression:String)`

1. Split `expression` along whitespace into an array of tokens. Call it `tokens`.
2. Create a stack of floating points large enough to hold all elements in `tokens`. Call it `operandStack`.
3. For each string `token` $\in$ `tokens`:
   (a) If `token` is a floating point:
      i. Push `token` onto `operandStack`.
      ii. Get the next `token` from `tokens`.
      iii. Jump back to (3a).
   (b) Pop an operand from `operandStack` and call it `rightOperand`.
   (c) Pop another operand from `operandStack` and call it `leftOperand`.

(d) Depending on which operator `token` represents, evaluate the operation with `token` as the operator and `leftOperand` and `rightOperand` as the respective operands. Call it `result`.

(e) Push `result` onto `operandStack`.

4. Pop and operand from `operandStack` and **return** it.

Source Code

```
1  import java.util.Scanner;
2
3  public class RPNCalculator {
4         private static double[] operandStack;
5         private static int top;
6
7         public static void main (String[] args) {
8                 System.out.printf("Reverse Polish Expression : ");
9                 String expression = (new Scanner(System.in)).nextLine();
10                double result = evaluateRPNExpression(expression);
11                System.out.printf("Evaluated Expression  :   %s %n",
                      Double.toString(result));
12        }
13
14        public static double evaluateRPNExpression (String expression) {
15                String[] tokens = expression.split("\\s+");
16                top = -1;
17                operandStack = new double[tokens.length];
18
19                for (String token : tokens) {
20                        if (isDouble(token)) {
21                                pushOperand(Double.parseDouble(token));
22                                continue;
23                        }
24
25                        double rightOperand = popOperand();
26                        double leftOperand = popOperand();
27                        double result = 0.0;
28                        switch (token.charAt(0)) {
29                                case '+' :    result = leftOperand + rightOperand;
30                                              break;
31                                case '-' :    result = leftOperand - rightOperand;
32                                              break;
33                                case '*' :    result = leftOperand * rightOperand;
34                                              break;
35                                case '/' :    result = leftOperand / rightOperand;
36                                              break;
```

```
37                              case '^' :    result = Math.pow(leftOperand,
                                    rightOperand);
38                                            break;
39                              default :     System.out.printf("Unknown operator
                                    (%s)!\n", token);
40                                            System.exit(0);
41                      }
42                      pushOperand(result);
43              }
44              return popOperand();
45      }
46
47      private static void pushOperand (double n) {
48              operandStack[++top] = n;
49      }
50
51      private static double popOperand () {
52              if (top < 0) {
53                      System.out.println("Insufficient operands!");
54                      System.exit(0);
55              }
56              return operandStack[top--];
57      }
58
59      private static boolean isDouble (String n) {
60              try {
61                       Double.parseDouble(n);
62                       return true;
63              } catch (NumberFormatException e) {}
64              return false;
65      }
66  }
```

Variable Description

RPNCalculator		
double[]	operandStack	The stack of operands in order of appearance.
int	top	The index of the topmost element of operandStack

RPNCalculator::main(String[])		
String	expression	The expression in RPN to be evaluated
double	result	The evaluated form of expression

RPNCalculator::evaluateRPNExpression(String)		
String	expression	The expression in RPN to be evaluated
String[]	tokens	The individual tokens in expression, separated by whitespace
String	token	An individual token from tokens
double	rightOperand	The operand to be taken on the right side of the operator
double	leftOperand	The operand to be taken on the left side of the operator
double	result	The result on evaluating the operator token on rightOperand and leftOperand

RPNCalculator::pushOperand(double)		
double	n	The operand to be pushed into operandStack

RPNCalculator::isDouble(String)		
String	n	The string to be tested on whether it is a floating point or not

This project was compiled with XeLaTeX.

All files involved in the making of this project can be found at
https://github.com/sahasatvik/Computer-Project/tree/master/ISC

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