MA2202: Probability I

Random variables

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Definition 3.1 (Random variable)**.** Given a probability space $(\Omega, \mathcal{E}, P)$, a function $X : \Omega \to \mathbb{R}$ is called a random variable if $X^{-1}(r, \infty) \in \mathcal{E}$ for all $r \in \mathbb{R}$.

Remark. For some $S \subseteq \mathbb{R}$, we denote

$$P(X \in S) = P(\{\omega \in \Omega \colon X(\omega) \in S\}).$$

Definition 3.2 (Discrete random variable)**.** A random variable which can assume only a countably infinite number of values is called a discrete random variable.

Example. Let $X : \Omega \to \mathbb{R}$ denote the number of heads obtained when a fair coin is tossed thrice. Note that $\Omega = \{0, 1, 2, 3\}$. Thus, $P(X = 0) = P(X = 4) = 1/8$ and $P(X = 1) = P(X = 2) = 3/8$.

Definition 3.3 (Probability distribution)**.** The probability distribution of a random variable X is the set of pairs $(X(A), P(A))$ for all $A \in \mathcal{E}$.

Definition 3.4 (Probability mass function)**.** Let X be a discrete random variable. The probability mass function of X is the function $p_X : \mathbb{R} \to [0, 1]$,

$$p_X(\alpha) = P(X = \alpha).$$

Remark. Since X is a discrete random variable, the set $S = \{x \in \mathbb{R} \colon p_X(\alpha) > 0\}$ is countable, and

$$\sum_{x \in S} p_X(x) = 1.$$