PH2201

Basic Quantum Mechanics

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1 Introduction

The quantum world differs from the classical world in many aspects, most of which we seldom encounter in our daily lives and are hence unintuitive.

- The physical world is not deterministic; uncertainty is intrinsic to the quantum world. This is sometimes illustrated by the Schrödinger's cat thought experiment.
- Both light and matter exhibit characteristics of waves as well as those of particles. However, a single object cannot exhibit both of these properties simultaneously.
- Physical quantities may be quantized – they may be constrained to have discrete values rather than vary continuously.

1.1 Blackbody radiation

A blackbody is an object which absorbs all radiation incident on it, and reflects none. It also emits radiation of all frequencies.

Kirchoff's Law says that the rates of emission and absorption of radiation of a body in thermal equilibrium will be equal. By thermal equilibrium, we mean that the temperatures of the body and its surroundings are equal.

Proposition 1.1 (Stefan-Boltzmann Law)**.** *The power emitted by a blackbody is given by*

$$P = \sigma A T^4.$$

Here, $\sigma \approx 5.67 \times 10^{-8}\,\mathrm{Js^{-1}m^{-2}K^{-4}}$ *is called the Stefan-Boltzmann constant.*

We may break down the total energy density $\rho \propto T^4$ in terms of the contributions from each frequency, so

$$\rho = \int_0^\infty \rho(\nu)\, d\nu.$$

It turns out that $\rho(\nu)$ is non-monotonic. This cannot be explained by classical mechanics (Rayleigh-Jean's Law), which predicts that $\rho(\nu)$ is unbounded with increasing frequency – the famous ultraviolet catastrophe.

Proposition 1.2 (Wien's Law)**.** *The positions of the peaks in $\rho(\nu)$ are described by*

$$\lambda_{peak} = \frac{w}{T}.$$

Here, $w \approx 2.9 \times 10^{-3}\,\mathrm{mK}$.

Note that at $T \approx 300\,\mathrm{K}$, the peak wavelength λ_{peak} is in the infrared range: this is why night vision googles are useful.

Consider a collection of electromagnetic waves in a blackbody cavity, with temperature T. This can be seen as the superposition of normal modes. The classical approach to the blackbody problem is to suppose that the energy density at a particular frequency is given by

$$\rho(\nu) = \overline{E}n(\nu),$$

where $n(\nu)$ is the number density of wave modes with frequency ν, and E is the average energy of the radiation.

The classical law of equipartition of energy gives

$$\overline{E} = k_B T,$$

where k_B is the Boltzmann constant.

The wavenumber of for modes within the cavity is given by

$$\boldsymbol{k} = \frac{2\pi}{L}\boldsymbol{n},$$

where $\boldsymbol{n} = (n_x, n_y, n_z)$ with integral components. Now,

$$\nu = \frac{c}{\lambda} = \frac{c}{L}n.$$

Treating n as a continuous variable and using $dV = 4\pi n^{\cdot} dn$, we write

$$n(\nu)\,d\nu = \frac{8\pi}{c^3}\nu^2\,d\nu.$$

This leads to the Rayleigh-Jean Law,

$$\rho(\nu)\,d\nu = \overline{E}n(\nu)\,d\nu = \frac{8\pi k_B T}{c^3}\nu^2\,d\nu.$$

Planck looked at the probability distribution for the energy,

$$P(E) = \frac{1}{k_B T}e^{-E/k_B T}.$$

This is the Boltzmann distribution. It can be shown that

$$\overline{E} = \frac{\int_0^\infty EP(E)\,dE}{\int_0^\infty P(E)\,dE} = k_B T,$$

which recovers the Rayleigh-Jean Law.

Planck's idea was to restrict E to discrete values; integral multiples of the frequency ν. This leads to

$$\overline{E} = \frac{\sum EP(E)}{\sum P(E)} = \frac{h\nu}{e^{h\nu/k_B T} - 1}.$$

This gives us the Planck distribution.

Proposition 1.3 (Planck's Law). *The spectral energy density of radiation emitted by a blackbody in thermal equilibrium is described by the distribution*

$$\rho(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/k_BT}-1}\,d\nu.$$

Here, $h \approx 6.626 \times 10^{-34}\,\mathrm{J\,s}$ *is called Planck's constant.*

When $h\nu \ll 1$, we recover the Rayleigh-Jean limit. When $h\nu \gg 1$, we get the Wien limit.

Now we calculate,

$$\rho = \int_0^\infty \rho(\nu)\,d\nu = \frac{8\pi^5 k_B^4}{15c^3h^3}T^4,$$

which recovers the Stefan-Boltzmann Law with

$$\sigma \;=\; \frac{2\pi^5 k_B^4}{15c^2h^3}.$$

Also, the maxima of the Planck distribution recovers Wien's Law, with

$$\nu_{max} \;\approx\; 2.8\frac{k_BT}{h}.$$