

MA3103

Introduction to Graph Theory and Combinatorics

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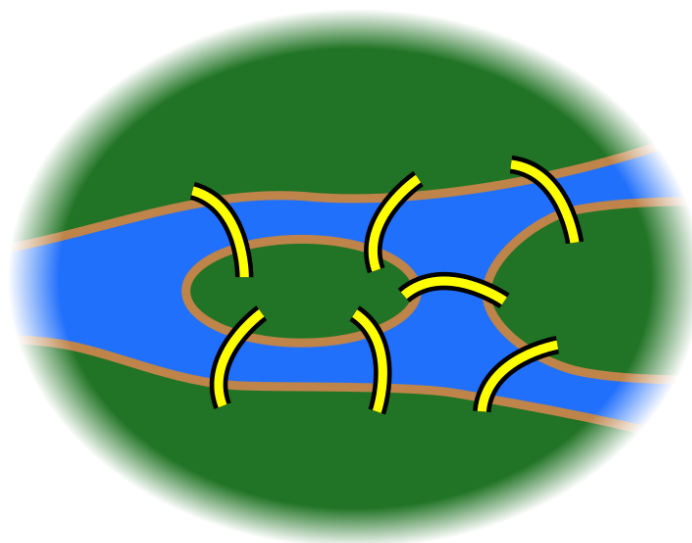
Contents

1 Introduction	1
1.1 The Seven Bridges of Königsberg	1

1 Introduction

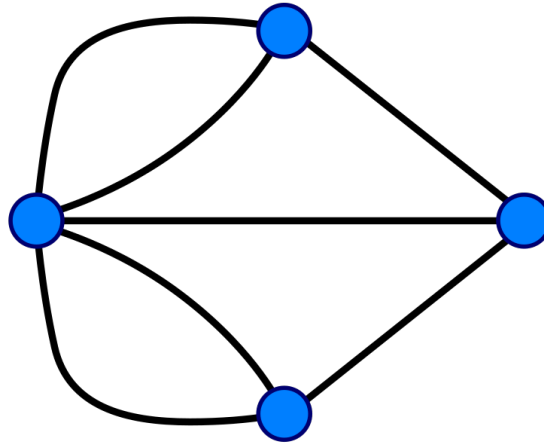
1.1 The Seven Bridges of Königsberg

The diagram below depicts a region in the city of Königsberg, Prussia. There are two islands, connected with the mainland and to each other via seven bridges. The Seven Bridges Problem is posed thus: is it possible to walk through the entire city, visiting each one of the four landmasses by crossing each of the bridges exactly once?



Leonhard Euler showed that this is impossible; no such walk exists. The techniques he developed in doing so laid the foundations of *graph theory*.

The first thing to note is that the exact shape of the walk/trail is immaterial; all that matters is the sequence of landmasses visited and bridges crossed. Thus, each landmass can be compacted to a single point or *vertex*, and each bridge a line or *edge* connecting two such points. The resulting figure is a graph. Note that the orientations or placements of the points and lines are irrelevant, as long as the connections are undisturbed.



Now, examine a landmass which is on the trail but is neither our starting point, nor our ending point. In order to reach this landmass, we must enter via a bridge; but we cannot stay in the landmass, so we must leave via another a bridge. Thus, for each time we pass through this landmass, we can cross off two bridges joined to it. Once we are done, no bridge may remain unused; this means that we must have started with an even number of bridges joined to this landmass.

However, all four vertices in our graph connect to an odd number of edges. Since we require at least two vertices to act as intermediate points on our path, the desired walk is impossible.