PH2103: Physics Laboratory IV

Calculation of gravitational acceleration using a pendulum and the Doppler effect

Satvik Saha*
19MS154

Indian Institute of Science Education and Research, Kolkata,
Mohanpur, West Bengal, 741246, India.

March 17, 2021

Abstract

In this experiment, we use two principles to measure the local gravitational acceleration, namely the relationship between the frequency of oscillation of a simple pendulum with its length ℓ and gravitational acceleration g, as well as the Doppler effect.

1 Experimental setup

The pendulum consists of a platform suspended by four strings of equal length ℓ by its corners. A smartphone, which acts as an acoustic sensor, is placed on it, and the pendulum is allowed to oscillate with a small angular displacement. A stationary source of sound of frequency $\nu_0 = 1000\,\mathrm{Hz}$ is placed such that the smartphone moves precisely towards and away from the source as it oscillates. The frequency ν measured by the smartphone thus varies in time with the oscillations, which is used to measure the frequency of oscillation. This in turn is used to calculate g.

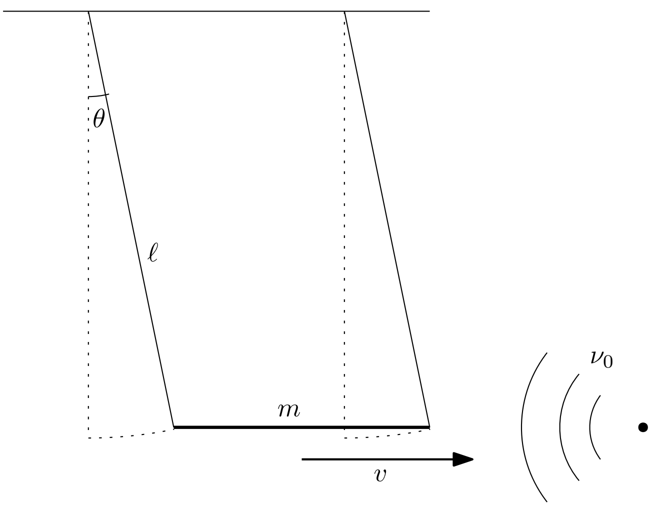

Figure 1: A schematic of the experimental setup.

*Email: ss19ms154@iiserkol.ac.in

2 Theory

The motion of the pendulum can be approximated to be equivalent to that of a simple pendulum. Note that the platform does not rotate in space, and hence contributes no angular momentum. The Lagrangian is thus

$$\mathcal{L} = T - V = \frac{1}{2}m\ell^2\dot{\theta}^2 + mg\ell\cos\theta,$$

which is identical to that of a simple pendulum. Solving the Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right) = \frac{\partial\mathcal{L}}{\partial\theta},$$

and using $\sin\theta \approx \theta$ for small oscillations, we recover

$$\ddot{\theta} + \frac{g}{\ell}\theta = 0,$$

whence

$$\theta(t) = \theta_0\cos(\omega t),$$

$\omega^2 = g/l$. The time period of oscillation is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}.$$

Rearranging for g, and setting the frequency of oscillation $f = 1/T$,

$$g = 4\pi^2\ell f^2. \tag{$\star$}$$

Note that

$$\dot{\theta}(t) = -\omega\theta_0\sin(\omega t),$$

so the velocity $v \approx \ell\dot{\theta}$ of the pendulum in the direction of our sound source also oscillates with frequency f. Since the frequency of sound detected by our smartphone varies as

$$\nu = \nu_0\left(1 + \frac{v}{v_0}\right),$$

the frequency ν also oscillates with frequency f. This means that we can extract f from the detected waveform, either by counting the number of oscillations over some time or by performing a Fourier transform. This along with ℓ will give a value for g.

3 Measurements

The effective length of the pendulum has been approximated as the length of the strings, recorded using a tape measure and averaged. The frequency of oscillation has been calculated by observing 30 oscillations in ν over a period of 40 s, and has been confirmed by performing a Fourier transform on the data.

We report

$$\ell \approx 44\,\text{cm}, \qquad f \approx 0.75\,\text{Hz}.$$

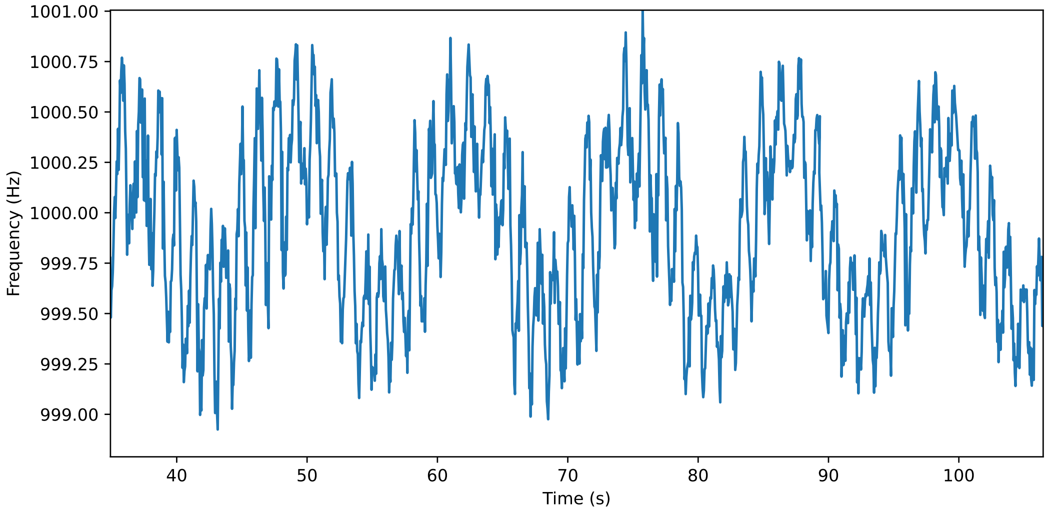

Figure 2: A sample of the data collected, zoomed in for clarity.

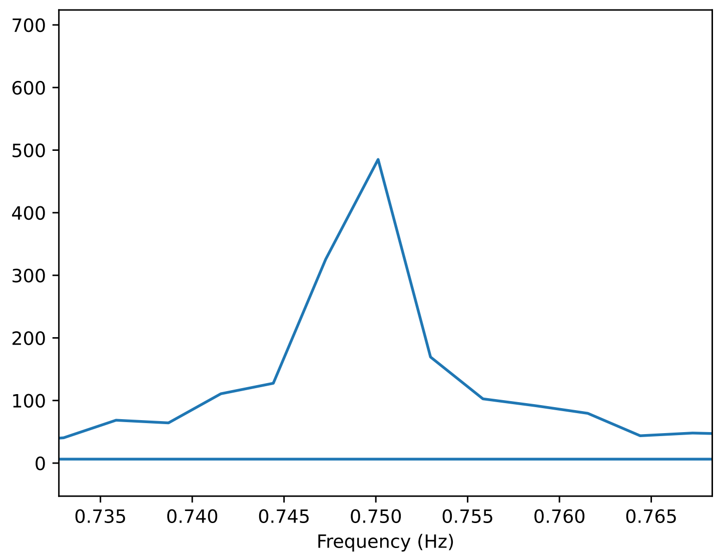

Figure 3: The Fourier transform of the frequency data, around $f = 0.75\,\mathrm{Hz}$.

4 Calculations

Using our working formula ($\star$),

$$g = 4\pi^2 \ell f \approx 4\pi^2 \cdot 0.44\,\mathrm{m} \cdot \left(0.75\,\mathrm{s}^{-1}\right)^2 \approx 9.77\,\mathrm{m/s}^2.$$

5 Error analysis

To get an upper estimate on the error in g, we differentiate ($\star$) and write

$$\frac{\delta g}{g} \approx \frac{\delta \ell}{\ell} + 2\,\frac{\delta f}{f}.$$

We estimate $\delta\ell = 0.5\,\text{cm}$ and $\delta f = 0.005$. Thus, we claim a relative error of

$$\frac{\delta g}{g} \approx 0.025 = 2.5\,\%,$$

with a reported value of

$$g \;=\; 9.77 \pm 0.24\ \text{m/s}^2.$$

Given that the standard value of gravitational acceleration is around $9.79\,\text{m/s}^2$, we have an absolute error of $-0.02\,\text{m/s}^2$, with a percentage error of $-0.2\,\%$. This of course ignores factors such as elevation and local features, whose effect may be considered negligible.

6 Discussion

We have measured the local acceleration due to gravity with some accuracy.

The frequency curve in Figure. 2 displays oscillations apart from the one caused by the Doppler effect, specifically the large sinusoidal oscillation with a time period of around $15\,\text{s}$. This persists even when the smartphone is placed on a stationary surface, and is likely an artefact of the speaker-microphone combination.