MA2202: Probability I

Random vectors

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Definition 4.1 (Random vector)**.** A random vector $\boldsymbol{X} \colon \Omega \to \mathbb{R}^n$ is a tuple of random variables $X_i \colon \Omega \to \mathbb{R}$.

Definition 4.2 (Joint cumulative distribution function)**.** The joint cumulative distribution function of a random vector $\boldsymbol{X}$ is the map $F_{\boldsymbol{X}} \colon \mathbb{R}^n \to [0, 1]$, given as

$$F_{\boldsymbol{X}}(\boldsymbol{s}) = P(X_1 \leq s_1, \dots, X_n \leq s_n).$$

Definition 4.3 (Joint probability mass function)**.** If X_i are discrete random variables, their joint probability mass function is the map $p_{\boldsymbol{X}} \colon \mathbb{R}^n \to [0, 1]$,

$$p_{\boldsymbol{X}}(\boldsymbol{s}) = P(X_1 = s_1, \dots, X_n = s_n).$$

Definition 4.4 (Joint probability density function)**.** Suppose that

$$F_{\boldsymbol{X}}(\boldsymbol{s}) = \int_{-\infty}^{s_n} \cdots \int_{-\infty}^{s_1} f_{\boldsymbol{X}}(t_1, \dots, t_n)\, dt_1 \dots\, dt_n,$$

then $f_{\boldsymbol{X}} \colon \mathbb{R}^n \to [0, 1]$ is the probability density function corresponding to the joint cumulative distribution function $F_{\boldsymbol{X}}$.

Remark. If $f_{\boldsymbol{X}}$ is continuous, then

$$f_{\boldsymbol{X}} = \frac{\partial F_{\boldsymbol{X}}(t_1, \dots, t_n)}{\partial t_1 \dots \partial t_n}.$$

Definition 4.5 (Joint moment generating function)**.** Let $\boldsymbol{X}$ be a random vector. Then, its joint moment generating function is defined as

$$M_{\boldsymbol{X}}(\boldsymbol{t}) = E\left[e^{\boldsymbol{t}^\top \boldsymbol{X}}\right] = E\left[e^{t_1 X_1 + \cdots + t_n X_n}\right].$$

Remark. If $X_1, \ldots, X_n$ are independent, then

$$M_{\boldsymbol{X}}(\boldsymbol{t}) = \prod M_{X_i}(t_i).$$