MA2202: Probability I

Conditional probability

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Suppose a fair die is rolled and the outcome is even. The probability that the outcome was prime is $1/3$. This is because the only even prime is 2, and we know that we have rolled one of $2, 4$ or 6, all of which are equally likely.

Definition 2.1 (Conditional probability)**.** Suppose a random experiment is performed, and we know that an event B occurred. Let $A \in \mathcal{E}$. The probability of occurrence of A given B is called the conditional probability $P(A \,|\, B)$.

Remark. Note that the conditional probability A given B is equivalent to the occurrence of $A \cap B$ computed relative to the probability of the occurrence of B.

$$P(A \,|\, B)\, P(B) = P(A \cap B).$$

Theorem 2.1 (Bayes' Theorem)**.** *If $A, B \in \mathcal{E}$ with $P(B) \neq 0$, we have*

$$P(A \,|\, B) = \frac{P(A)}{P(B)} P(B \,|\, A).$$

Proof. This follows directly from

$$P(A \cap B) = P(A \,|\, B)\, P(B) = P(B \,|\, A)\, P(A).$$ □

Example. Suppose a fair die is rolled and the outcome is even. The probability that the outcome is prime is $1/3$. This is because the only even prime is 2, the available even numbers are $2, 4, 6$, and the available primes are $2, 3, 5$. Thus,

$$P(\text{prime} \,|\, \text{even}) = \frac{P(\text{even})}{P(\text{prime})} P(\text{even} \,|\, \text{prime}) = \frac{3/6}{3/6} \cdot \frac{1}{3} = \frac{1}{3}.$$

Definition 2.2 (Independent events)**.** If $A, B \in \mathcal{E}$ such that $P(A), P(B) \neq 0$, and we have

$$P(A \mid B) = P(A), \qquad P(B \mid A) = P(B),$$

then we say that A and B are independent.

Equivalently, $P(A \cap B) = P(A)P(B)$.