Assignment 2a

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2024-09-10

Answer 1

(a)

```r
print(seq(0, 20, by = 0.2))
```

```
##   [1]  0.0  0.2  0.4  0.6  0.8  1.0  1.2  1.4  1.6  1.8  2.0  2.2  2.4  2.6  2.8
##  [16]  3.0  3.2  3.4  3.6  3.8  4.0  4.2  4.4  4.6  4.8  5.0  5.2  5.4  5.6  5.8
##  [31]  6.0  6.2  6.4  6.6  6.8  7.0  7.2  7.4  7.6  7.8  8.0  8.2  8.4  8.6  8.8
##  [46]  9.0  9.2  9.4  9.6  9.8 10.0 10.2 10.4 10.6 10.8 11.0 11.2 11.4 11.6 11.8
##  [61] 12.0 12.2 12.4 12.6 12.8 13.0 13.2 13.4 13.6 13.8 14.0 14.2 14.4 14.6 14.8
##  [76] 15.0 15.2 15.4 15.6 15.8 16.0 16.2 16.4 16.6 16.8 17.0 17.2 17.4 17.6 17.8
##  [91] 18.0 18.2 18.4 18.6 18.8 19.0 19.2 19.4 19.6 19.8 20.0
```

(b)

```r
print(runif(1, min = 0, max = 10))
```

```
## [1] 1.205211
```

(c)

```r
print(runif(10, min = 0, max = 10))
```

```
##  [1] 0.3874994 5.3239424 2.2474365 1.2084094 0.7240667 1.8308729 0.9932816
##  [8] 9.6483512 1.6905259 3.7214616
```

Answer 2

(a)

```r
a <- c("Magnus Carlsen", "Hilary Hahn", "Linus Torvalds", "Persi Diaconis")
```

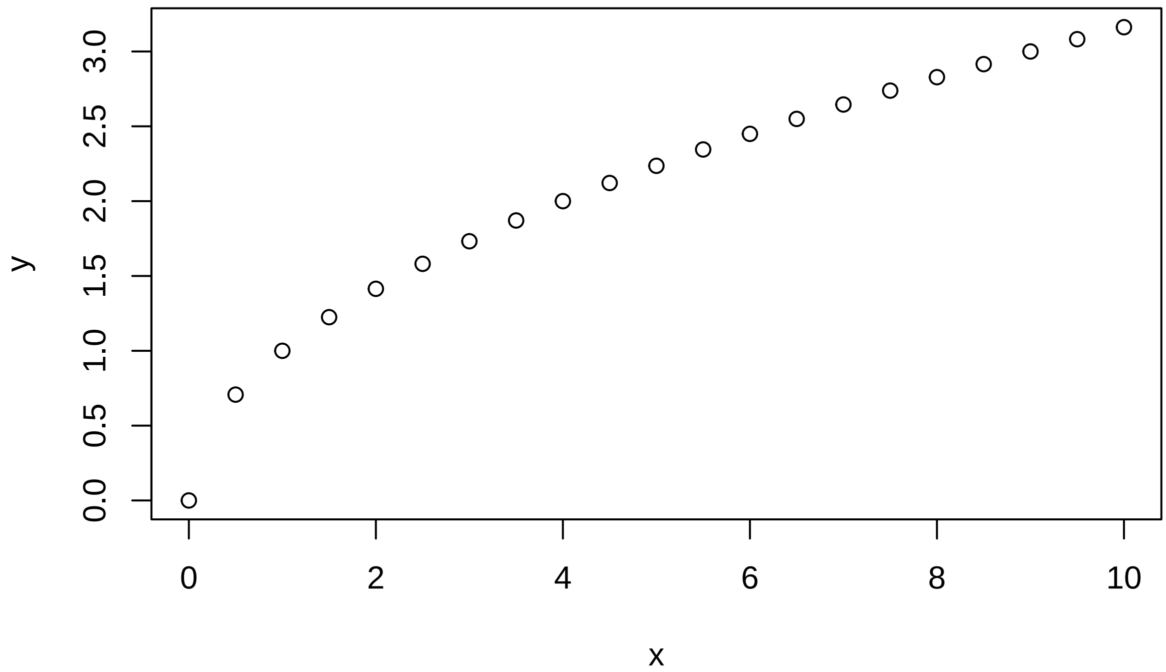
(b)

```r
sample(a, 1)
```

```
## [1] "Magnus Carlsen"
```

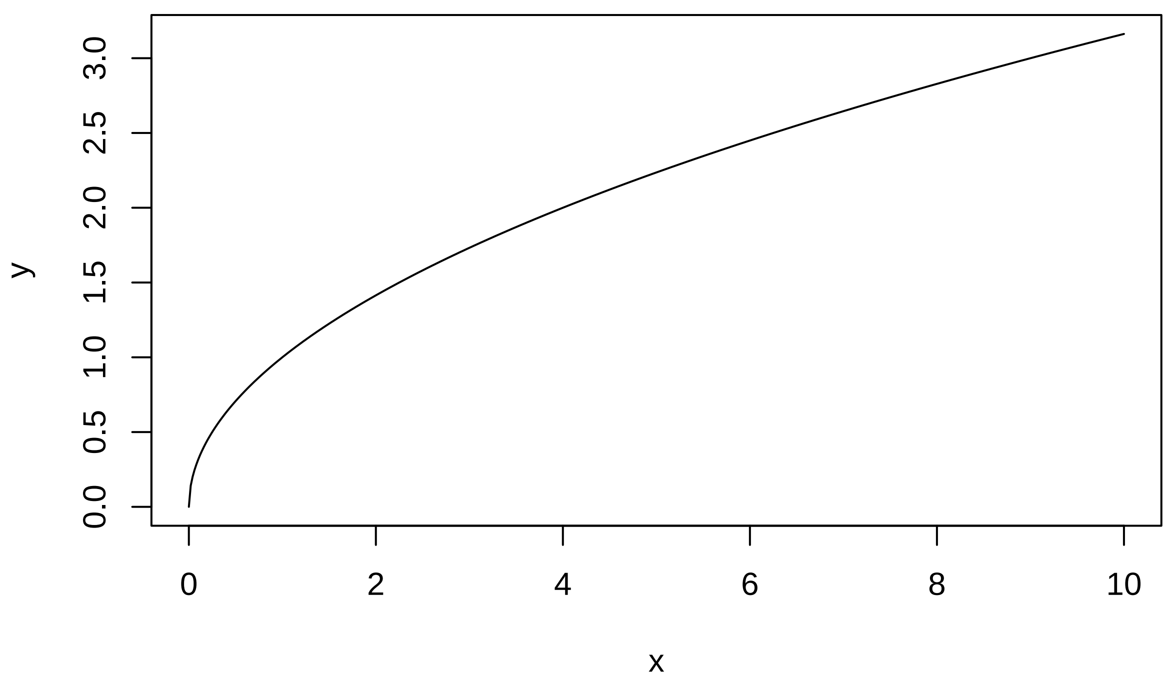
(c)

```r
x <- seq(0, 10, by = 0.5)
y <- sqrt(x)
plot(x, y)
```



(d)

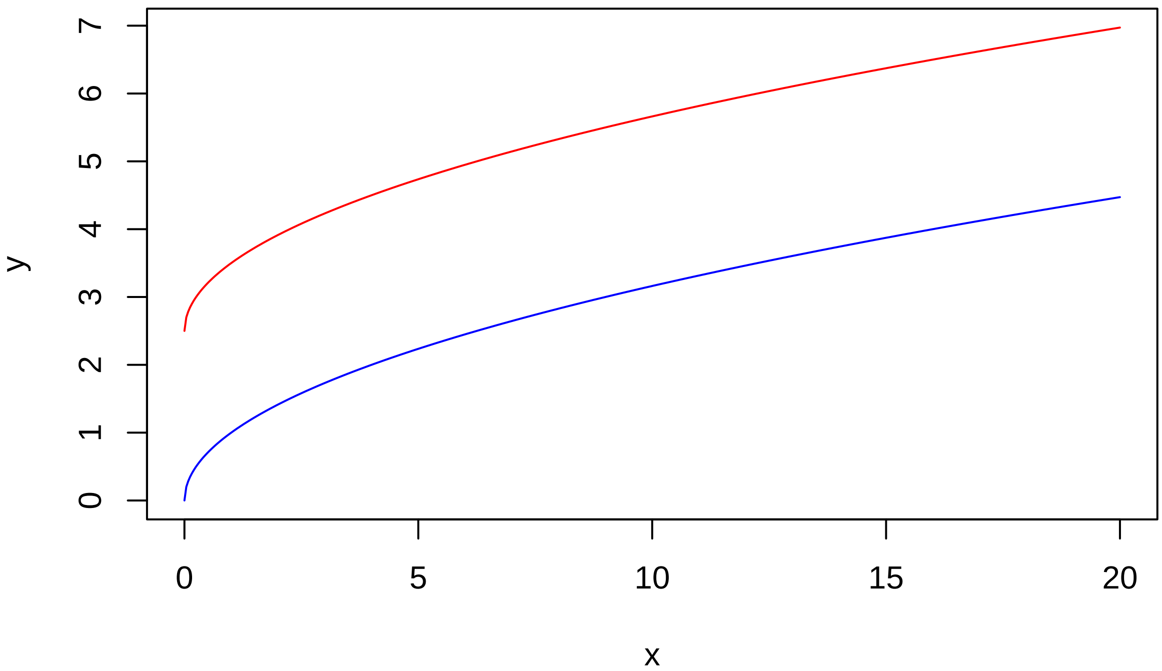
```
x <- seq(0, 10, length.out = 500)
y <- sqrt(x)
plot(x, y, type = "l")
```



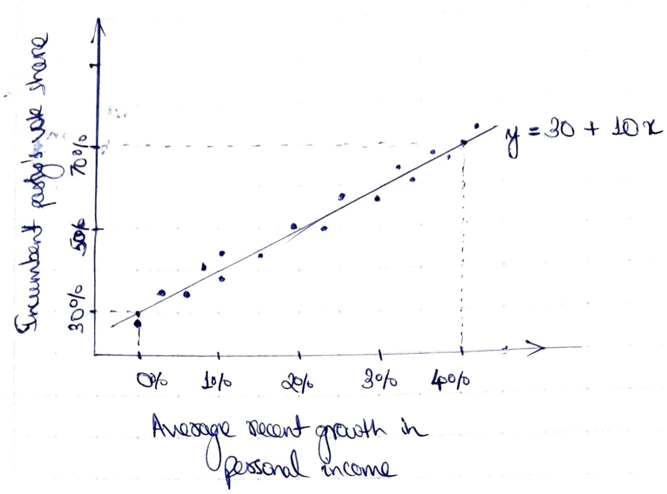
(e)

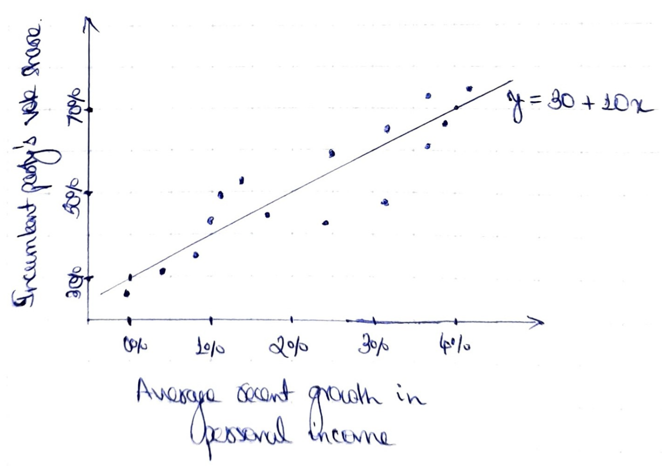
```
x <- seq(0, 20, length.out = 500)
y <- data.frame(
  y1 = sqrt(x),
  y2 = 2.5 + sqrt(x)
```

```r
)
matplot(x, y, type = "l", col = c("blue", "red"), lty = 1)
```



Answer 3





Research homework assignment

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Suppose that X and Y are independent (real) random variables admitting densities f_X and f_Y, and let $Z = X/Y$. We will say that Z has Cauchy tails when $f_Z(z) \sim C/z^2$ as $|z| \to \infty$ for some $C > 0$.

Proposition 1: Suppose that $\mathbb{E}[|X|] < \infty$, and that f_Y is bounded. If f_Y is continuous at 0 with $f_Y(0) > 0$, then $Z = X/Y$ has Cauchy tails with

$$f_Z(z) \sim \frac{f_Y(0)\,\mathbb{E}[|X|]}{z^2} \quad \text{as } |z| \to \infty.$$

Proof: Observe that the density f_Z of Z is given by

$$f_Z(z) = \int_{\mathbb{R}} |y| f_X(zy) f_Y(y)\, dy = \frac{1}{z^2} \int_{\mathbb{R}} |x| f_X(x) f_Y\left(\frac{x}{z}\right)\, dx.$$

This can be seen by applying change of variables on (X, Y) using the map $(x, y) \mapsto (x/y, y)$, whose inverse is $(z, y) \mapsto (zy, y)$, and computing the marginal.

Let $f_Y < M$, so

$$|x| f_X(x) f_Y\left(\frac{x}{z}\right) < |x| f_X(x) M \in L^1(\mathbb{R})$$

via $\mathbb{E}[|X|] < \infty$. By the Dominated Convergence Theorem, we have

$$\lim_{z\to\infty} z^2 f_Z(z) = \lim_{z\to\infty} \int_{\mathbb{R}} |x| f_X(x) f_Y\left(\frac{x}{z}\right)\, dx = \int_{\mathbb{R}} |x| f_X(x) f_Y(0)\, dx = f_Y(0)\, \mathbb{E}[|X|]. \qquad \square$$

Remark: In the special case that $X, Y \sim N(0, 1)$, we have $X/Y \sim \text{Cauchy}(0, 1)$.

Note that Proposition 1 does not cover situations such as $Y \sim \text{Uniform}(0, 1)$. The continuity assumption on f_Y can be relaxed slightly by dealing with the left and right tails of Z separately.

Proposition 2: Suppose that $\mathbb{E}[|X|] < \infty$, and that f_Y is bounded. If f_Y has both left and right limits at 0, then $Z = X/Y$ has a Cauchy right tail with

$$f_Z(z) \sim \frac{f_Y(0^-)\, \mathbb{E}[X^-] + f_Y(0^+)\, \mathbb{E}[X^+]}{z^2} \quad \text{as } z \to +\infty,$$

and a Cauchy left tail with

$$f_Z(z) \sim \frac{f_Y(0^+)\, \mathbb{E}[X^-] + f_Y(0^-)\, \mathbb{E}[X^+]}{z^2} \quad \text{as } z \to -\infty.$$

Proof: Following the same argument as in Proposition 1, we use the Dominated Convergence Theorem on

$$z^2 f_Z(z) = \int_{-\infty}^{0} |x| f_X(x) f_Y\left(\frac{x}{z}\right)\, dx + \int_{0}^{+\infty} |x| f_X(x) f_Y\left(\frac{x}{z}\right)\, dx. \qquad \square$$

Remark: Note that $Z = (1/Y)/(1/X)$, so Proposition 2 has an analogue which demands $\mathbb{E}[1/|Y|] < \infty$ and that the density of $1/X$ have both left and right limits at 0.