PH2201

Basic Quantum Mechanics

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Satvik Saha
19MS154

*Indian Institute of Science Education and Research, Kolkata,
Mohanpur, West Bengal, 741246, India.*

Contents

1 Introduction

The quantum world differs from the classical world in many aspects, most of which we seldom encounter in our daily lives and are hence unintuitive.

- The physical world is not deterministic; uncertainty is intrinsic to the quantum world. This is sometimes illustrated by the Schrödinger's cat thought experiment.
- Both light and matter exhibit characteristics of waves as well as those of particles. However, a single object cannot exhibit both of these properties simultaneously.
- Physical quantities may be quantized – they may be constrained to have discrete values rather than vary continuously.

1.1 Blackbody radiation

A blackbody is an object which absorbs all radiation incident on it, and reflects none. It also emits radiation of all frequencies.

Kirchoff's Law says that the rates of emission and absorption of radiation of a body in thermal equilibrium will be equal. By thermal equilibrium, we mean that the temperatures of the body and its surroundings are equal.

Proposition 1.1 (Stefan-Boltzmann Law)**.** *The power emitted by a blackbody is given by*

$$P = \sigma A T^4.$$

Here, $\sigma \approx 5.67 \times 10^{-8}\,\mathrm{Js^{-1}m^{-2}K^{-4}}$ *is called the Stefan-Boltzmann constant.*

We may break down the total energy density $\rho \propto T^4$ in terms of the contributions from each frequency, so

$$\rho = \int_0^\infty \rho(\nu)\, d\nu.$$

It turns out that $\rho(\nu)$ is non-monotonic. This cannot be explained by classical mechanics (Rayleigh-Jean's Law), which predicts that $\rho(\nu)$ is unbounded with increasing frequency – the famous ultraviolet catastrophe.

Proposition 1.2 (Wien's Law)**.** *The positions of the peaks in $\rho(\nu)$ are described by*

$$\lambda_{peak} = \frac{w}{T}.$$

Here, $w \approx 2.9 \times 10^{-3}\,\mathrm{mK}$.

Note that at $T \approx 300\,\mathrm{K}$, the peak wavelength λ_{peak} is in the infrared range: this is why night vision googles are useful.

Consider a collection of electromagnetic waves in a blackbody cavity, with temperature T. This can be seen as the superposition of normal modes. The classical approach to the blackbody problem is to suppose that the energy density at a particular frequency is given by

$$\rho(\nu) = \overline{E} n(\nu),$$

where $n(\nu)$ is the number density of wave modes with frequency ν, and E is the average energy of the radiation.

The classical law of equipartition of energy gives

$$\overline{E} = k_B T,$$

where k_B is the Boltzmann constant.

The wavenumber of for modes within the cavity is given by

$$\boldsymbol{k} = \frac{2\pi}{L}\boldsymbol{n},$$

where $\boldsymbol{n} = (n_x, n_y, n_z)$ with integral components. Now,

$$\nu = \frac{c}{\lambda} = \frac{c}{L} n.$$

Treating n as a continuous variable and using $dV = 4\pi n^{\cdot} dn$, we write

$$n(\nu)\, d\nu = \frac{8\pi}{c^3} \nu^2\, d\nu.$$

This leads to the Rayleigh-Jean Law,

$$\rho(\nu)\, d\nu = \overline{E} n(\nu)\, d\nu = \frac{8\pi k_B T}{c^3} \nu^2\, d\nu.$$

Planck looked at the probability distribution for the energy,

$$P(E) = \frac{1}{k_B T} e^{-E/k_B T}.$$

This is the Boltzmann distribution. It can be shown that

$$\overline{E} = \frac{\int_0^\infty EP(E)\,dE}{\int_0^\infty P(E)\,dE} = k_B T,$$

which recovers the Rayleigh-Jean Law.

Planck's idea was to restrict E to discrete values; integral multiples of the frequency ν. This leads to

$$\overline{E} = \frac{\sum EP(E)}{\sum P(E)} = \frac{h\nu}{e^{h\nu/k_B T} - 1}.$$

This gives us the Planck distribution.

Proposition 1.3 (Planck's Law)**.** *The spectral energy density of radiation emitted by a blackbody in thermal equilibrium is described by the distribution*

$$\rho(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/k_B T} - 1}\,d\nu.$$

Here, $h \approx 6.626 \times 10^{-34}\,\mathrm{J\,s}$ *is called Planck's constant.*

When $h\nu \ll 1$, we recover the Rayleigh-Jean limit. When $h\nu \gg 1$, we get the Wien limit.

Now we calculate,

$$\rho = \int_0^\infty \rho(\nu)\,d\nu = \frac{8\pi^5 k_B^4}{15c^3h^3}T^4,$$

which recovers the Stefan-Boltzmann Law with

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3}.$$

Also, the maxima of the Planck distribution recovers Wien's Law, with

$$\nu_{max} \approx 2.8\frac{k_B T}{h}.$$

1.2 Photoelectric effect

This reveals the dual nature of light. Classical optics relies on the wave nature of light, thus explaining phenomena such as interference and diffraction. This culminates in Maxwell's equations, which predict the wave nature of light as the propagation of oscillating electric and magnetic fields.

The photoelectric effect in the phenomenon in which light shining on a metal surface ejects electrons from it, thus producing a current. Suppose that the incident light has frequency ν, intensity I and this produces a current i. We can calculate the maximum kinetic energy of the emitted electrons $E_{max} = eV_0$ by adjusting an opposing potential V.

It turns out that for a constant intensity I, the photocurrent saturates at the same value. However, different frequencies ν produces different stopping potentials V_0; the greater the frequency, the greater the magnitude of the stopping potential. This turns out to have a linear relationship, with

$$E_{max} = eV_0 = h(\nu - \nu_0) = h\nu - \phi.$$

The slope h is universal for all metals, while $\phi = h\nu_0$ varies between different metals. This shows that below a certain frequency ν_0, we obtain no photocurrent, regardless of the intensity! This appears strange from a classical perspective, where the energy delivered by an electromagnetic wave is related to its intensity, not its frequency.

Einstein proposed that light strikes the metal in bundles of energy, all integral multiples of $h\nu$. There is also a minimum binding energy which must be overcome to liberate electrons from the metal surface – this cannot be paid in a continuous manner, since any partial energy given to an electron will be lost before the arrival of the next energy bundle. Thus, each energy bundle must carry a minimum energy $h\nu_0$ in order to liberate electrons and produce a photocurrent.

This establishes a particle-like mature of light. Each energy bundle, or particle of light, is called a photon.

1.3 Matter waves

Louis de Broglie further hypothesised that matter also has a wave nature, with an associated wavelength of

$$\lambda_{matter} = \frac{h}{p}.$$

This has been demonstrated by Davisson and Germer, where a stream of electrons exhibits diffraction.

This has some amazing implications. In the classical world, knowledge of a particle's position and momentum is enough to pinpoint its trajectory with arbitrary precision. However, the wavelike nature of matter would imply that we can no longer talk of a definite, localized position when we have knowledge of the particle's momentum! This uncertainty is inherent to quantum mechanics.

Another consequence is the phenomenon of quantum tunnelling.