MA3101 : Introduction to Graph Theory and Combinatorics

Satvik Saha, `19MS154` August 28, 2021

Exercise 1 Show that if $n+1$ integers are chosen from the set $\{1, 2, \ldots, mn\}$, then there are always two which differ by less than m.

Solution Consider the set $S_a = \{am+b : 1 \leq b \leq m\}$ for each $0 \leq a < n$. These are disjoint, and their union is precisely $\{1, 2, \ldots, mn\}$. Since there are a total of n sets, the Pigeonhole Principle guarantees that upon choosing $n+1$ integers from $\{1, 2, \ldots, mn\}$, there are at least two which belong to the same set; say $p, q \in S_a$ for some a. Thus write $p = am + b_p$, $q = am + b_q$, let $p > q$ without loss of generality, and note that $p - q = b_p - b_q \leq m - 1 < m$.