

STATISTICAL DEPTH FUNCTIONS

In the multivariate and functional setting

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MASTER'S THESIS

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ABSTRACT

DEDICATION

DECLARATION

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INTRODUCTION

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