Ladder in Equilibrium

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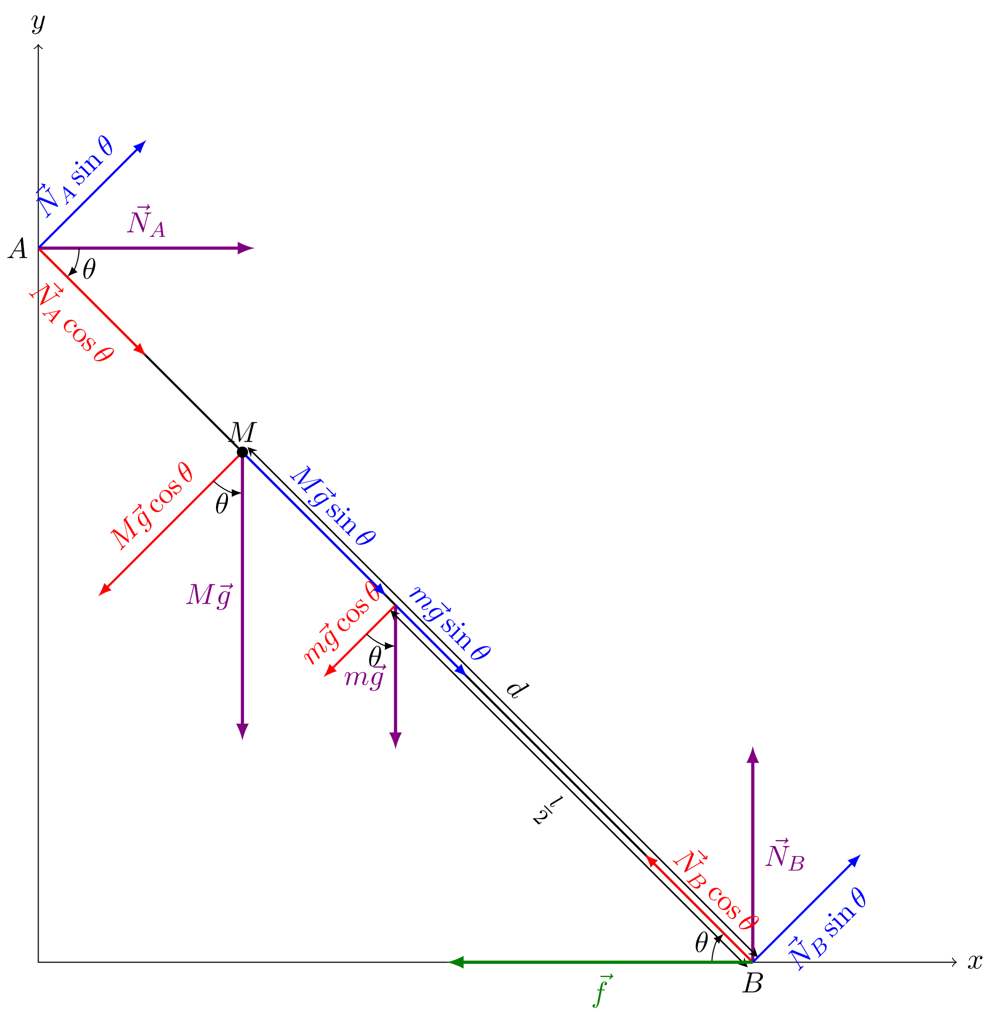
Question A man of mass M climbs up a ladder of length l and mass m, resting against a wall and inclined against the floor at an angle θ. If the coefficient of static friction between the floor and ladder is μ, what is the maximum distance the d the man can ascend along the ladder from it's base before it slips?



Solution Draw all the forces acting on the ladder : $\vec{N}_A$ (normal force from the wall), $\vec{N}_B$ (normal force from the floor), $\vec{f}$ (static friction from the floor), $m\vec{g}$ (weight of the ladder) and $M\vec{g}$ (weight of the man).

Note that the ladder is in equilibrium. Thus, we can write

$$\begin{aligned}\vec{\tau}_{net} &= 0\\ \vec{F}_{net,x} &= 0\\ \vec{F}_{net,y} &= 0 \qquad (1)\end{aligned}$$

Also note that when the man is at the maximum height at which the ladder stays in equilibrium, the static friction also attains its maximum value, i.e. , $f = \mu N_B$.

Considering the net torque about point B, we have

$$\begin{aligned}0 &= Mgd\cos\theta + \frac{1}{2}mgl - N_A l\sin\theta\\ N_A l\sin\theta &= (Mgd + \frac{1}{2}mgl)\cos\theta\\ N_A &= \left(\frac{1}{l}Mgd + \frac{1}{2}mgl\right)\cot\theta \qquad (2)\end{aligned}$$

Considering the net force along the x-axis, we have

$$\begin{aligned}0 &= N_A - f\\ \mu N_B &= N_A \qquad (3)\end{aligned}$$

Considering the net force along the y-axis, we have

$$\begin{aligned}0 &= N_B - mg - Mg\\ (m+M)g &= \frac{1}{\mu}\left(\frac{1}{l}Mgd + \frac{1}{2}mgl\right)\cot\theta \qquad \text{(From (2) and (3))}\\ \mu(m+M)g\tan\theta &= \frac{1}{l}Mgd + \frac{1}{2}mgl\end{aligned}$$

$$\boxed{d = \frac{l}{M}\left[\mu(m+M)\tan\theta - \frac{1}{2}m\right]} \qquad (4)$$