Sliding down Chords of a Circle

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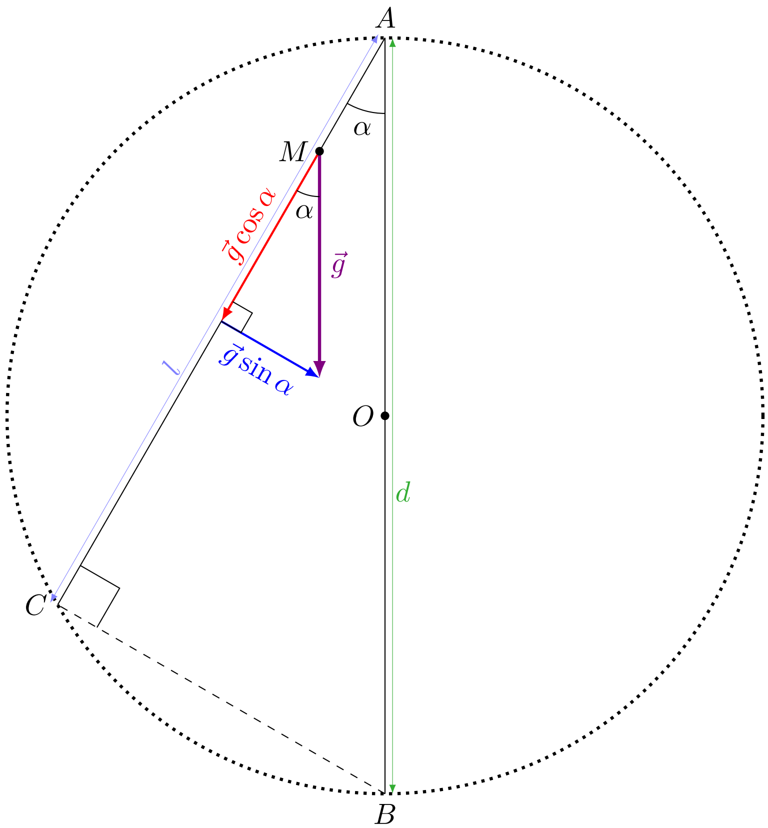
Points A, B, C lie on a circle. A particle at rest at point A falls under the influence of gravity along AC, reaching point C in time t. The line segment AC is inclined from AB, a vertical diameter, by an angle α. The following figure illustrates the given problem.



Note that ΔABC is right-angled at C. Therefore, we have :

$$l = d\cos\alpha$$
$$a_l = -g\cos\alpha$$

Where a_l is the net acceleration of the particle along AC. The equations of uniformly accelerated motion along a straight line along AC give :

$$
\begin{aligned}
-l &= v_0 t \, + \, \frac{1}{2} a_l t^2, \\
-d \, \cos\alpha &= -\frac{1}{2} \, g \, \cos\alpha \, t^2,
\end{aligned}
$$

$$t = \sqrt{\frac{2d}{g}}$$

Clearly, the time taken to reach C from A along a straight line is independent of the angle α made with the vertical. Thus, time taken for a particle to slide from the highest point of a circle down to any other point on the circumference via a chord is a constant.