# Flashlight Tracking Aircraft

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Question   A flashlight is aimed at an aircraft flying at an altitude h with a constant velocity $\vec{v}$. Its beam continues to point directly towards the position of the aircraft as it moves. At $t = 0$, its instantaneous angular velocity is ω_0. Express the instantaneous angular velocity ω of the rotation of the flashlight's beam as a function of time.
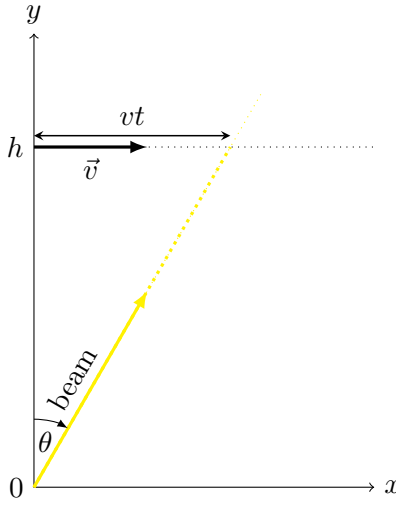
Solution   We have

$$\tan\theta \;=\; \frac{vt}{h}$$

$$\theta \;=\; \tan^{-1}\frac{vt}{h}$$

$$\frac{d\theta}{dt} \;=\; \omega(t) \;=\; \frac{1}{1 + (\frac{vt}{h})^2} \cdot \frac{v}{h} \tag{1}$$

Note that $\omega(0) = \omega_0 = \frac{v}{h}$. Thus

$$\boxed{\omega(t) \;=\; \frac{\omega_0}{1 + \omega_0^2 t^2}} \tag{2}$$