The Range of a Projctile

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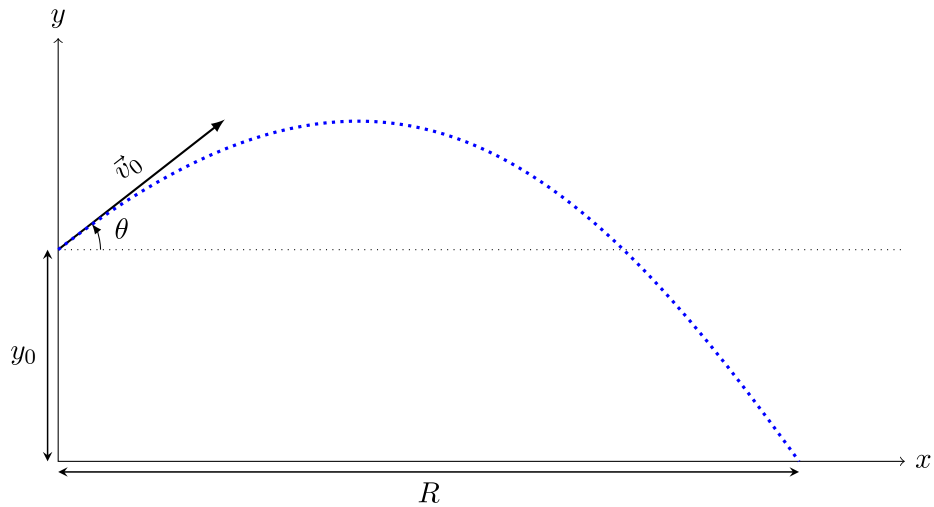

Consider the equations of motion of a projectile, launched at an elevation θ from a height y_0, experiencing uniform acceleration $-g$ along the y-axis.

$$x(t) = v_0 \cos\theta \tag{1}$$

$$y(t) = y_0 + v_0 t \sin\theta - \frac{1}{2} g t^2 \tag{2}$$

When the projectile hits the ground, we see that $y(t) = 0$. Let this time be $tflight$ and the corresponding horizontal displacement be R.

$$0 = y_0 + v_0 \sin\theta - \frac{1}{2} g t^2$$

$$t_{flight} = \frac{1}{g}(v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gy_0})$$

$$R = \frac{1}{g}(v_0 \cos\theta)(v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gy_0}) \tag{3}$$

For $R = R_{max}$, we have $\frac{d}{d\theta}R = 0$