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# Chapter 1

## Introduction to Machine Learning

Machine Learning is undeniably one of the most influential and powerful technologies in today's world. It is a tool for turning information into knowledge. In the past 50 years, there has been an explosion of data. This mass of data is useless unless we analyse it and find the patterns hidden within. Machine learning techniques are used to automatically find the valuable underlying patterns within complex data that we would otherwise struggle to discover. The hidden patterns and knowledge about a problem can be used to predict future events and perform all kinds of complex decision making.

There are multiple forms of Machine Learning; supervised, unsupervised, semi-supervised and reinforcement learning. Each form of Machine Learning has differing approaches, but they all follow the same underlying process and theory. I'll cover supervised and unsupervised learning for my report.

### 1.1 Supervised and Unsupervised Learning

The most basic thing to remember is that we already know what our correct output should look like in Supervised Learning. But, we have little or no idea about what our results should look like.

#### **Supervised Learning:**

- Classification: Spam/Not-spam.
- Regression: Predicting age.

#### **Unsupervised Learning:**

- Clustering: Grouping based on different variables.
- Non Clustering: Finding structure in chaotic environment.

# Chapter 2

## Linear Regression

### 2.1 Linear Regression with one variable

Regression being a part of Supervised Learning is used for estimating data (Real-valued output).

#### 2.1.1 Cost Function

This function measures the performance of a Machine Learning model for given data.

**Hypothesis:**  $h_{\theta}(x) = \theta_0 + \theta_1 x$

**Parameters:**  $\theta_0, \theta_1$

**Cost Function:**

$$J(\theta_0, \theta_1) = 1/2m \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 \quad (2.1)$$

**Goal:** Minimize cost function with  $\theta_0, \theta_1$  as parameters.

#### 2.1.2 Gradient Descent

**Basic idea:**

- Start with some  $\theta_0, \theta_1$
- Keep changing  $\theta_0, \theta_1$  to reduce  $J(\theta_0, \theta_1)$  until we end up at minima.

**Algorithm:** repeat until convergence:

$$\theta_j := \theta_j - \alpha \frac{\partial J(\theta_0, \theta_1)}{\partial \theta_j} \quad (2.2)$$

(for  $j = 0, 1$ , here).

**Intuition:** If  $\alpha$  is too small, descent can be slow and if too large, descent may fail to converge or even diverge. Gradient descent can converge to a local minimum, even with fixed learning rate  $\alpha$ . As we approach local minimum, gradient descent will automatically take smaller steps. So, no need to decrease  $\alpha$  over time.

### 2.1.3 Gradient Descent for linear regression

Combining gradient descent algorithm with linear regression model, we get:

$$j = 0 : \frac{\partial J(\theta_0, \theta_1)}{\partial \theta_0} = 1/2 \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \quad (2.3)$$

$$j = 1 : \frac{\partial J(\theta_0, \theta_1)}{\partial \theta_1} = 1/2 \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x^{(i)} \quad (2.4)$$

Now, we can repeat 2.3 and 2.4 until convergence to obtain the minima.

"Batch" gradient descent: Each step of gradient descent uses all the training examples. For eq. "m" batches in equation 2.1.

## 2.2 Multivariate Linear Regression

Linear regression involving more than one variable. For eq., Predicting price of a house based on parameters "Plot Area", "No. of Floors", "Connectivity with markets", etc.

### 2.2.1 Multiple Features

The multivariable form of the hypothesis is as follows:

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \dots + \theta_n x_n. \quad (2.5)$$

This hypothesis function can be concisely represented as:

$$h_{\theta}(x) = \theta^T x \quad (2.6)$$

where,  $\theta^T$  is a  $1 \times n$  matrix consisting of  $\theta_0, \theta_1, \theta_2, \dots, \theta_n$ .

### 2.2.2 Gradient Descent for Multiple Variables

Gradient descent formula for Multiple variable will be similar to that of single variable.

$$\theta_j = \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)} \quad (2.7)$$

Repeating this equation until convergence will give the minima. <sup>1</sup>

---

<sup>1</sup>  $x_0 = 1$  in equation 2.7

## Feature Scaling

Feature Scaling is used to reduce the number of iterations in Gradient Descent. Basic idea of feature scaling is to bring all the features on the same scale. (in general we try to approximate every feature in the range  $-1 < x_i < 1$ )

Reducing the number of iteration doesn't mean making computation of each step easier. And also it does not effect comtational efficiency of Normal Equation.

## Mean Normalisation

Mean Normalisation makes features to have approximately zero mean.

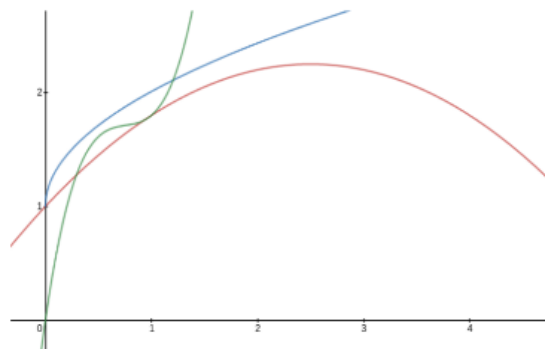
## Learning Rate

If  $\alpha$  is too small: slow convergence.

if  $\alpha$  is too large:  $J(\theta)$  may not decrease on every iteration, or may not converge.

## Polynomial Regression

Selecting proper polynomial for fitting data is very important.



**Red:** Quadratic

**Blue:** Square root funtion  $\theta_0 + \theta_1 x + \theta_2 \sqrt{x}$

**Green:** Cubic function

## 2.3 Normal equation

Normal Equation is a method to solve for  $\theta_T$  analytically, by creating a  $m \times (n + 1)$  matrix  $X$  and another  $m \times 1$  matrix  $Y$ .<sup>2</sup>

---

<sup>2</sup>Every element of first column of matrix  $X$  is 1 and other are the feature's coefficient

Mathematically  $\theta$  is given as:

$$\theta = (X^T X)^{-1} X^t y \quad (2.8)$$

Gradient Descent	Normal Equation
Need to choose $\alpha$	No need to choose $\alpha$
Needs many iteration	Don't need to iterate
Works well with large n	Slow for large n

### Reasons for non-invertibility of $X^T X$

- Redundant features (linear dependence) <sup>3</sup>
- Too many features ( $m \leq n$ )

---

<sup>3</sup>Eg. Using both  $m^2$  &  $(feet)^2$  features

# Chapter 3

## Logistic Regression

### 3.1 Classification and Representation

#### 3.1.1 Classification

The classification problem is just like the regression problem, except that the values we now want to predict take on only a small number of discrete values. For now, we'll discuss binary classification problem.

#### 3.1.2 Hypothesis Representation

We may use our old regression algorithm by classifying data on the basis of a threshold. But it will have very poor performance.

We will introduce "Sigmoid Function", also called the "Logistic Function":

$$h_{\theta}(x) = g(\theta^T x) \quad (3.1)$$

$$z = \theta^T x \quad (3.2)$$

$$g(z) = \frac{1}{1 + e^{-z}} \quad (3.3)$$

This is how the Sigmoid Function looks like:



Figure 3.1: Sigmoid Function 3.3



### 3.1.3 Decision Boundary

The decision boundary is the line that separates the area where  $y=0$  and where  $y=1$ . It is similar to the decision boundary for linear regression, the only difference is distribution of values (linear and sigmoid)

## 3.2 Logistic Regression Model

### 3.2.1 Cost Function

Cost function for logistic regression looks like:

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_{\theta}(x^{(i)}), y^{(i)}) \quad (3.4)$$

$$\text{Cost}(h_{\theta}(x), y) = -\log(h_{\theta}(x)) \quad \text{if } y = 1$$

$$\text{Cost}(h_{\theta}(x), y) = -\log(1 - h_{\theta}(x)) \quad \text{if } y = 0$$

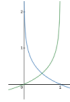


Figure 3.2: Cost Function

### Simplified Cost Function

This cost function can be compressed into a single function:

$$\text{Cost}(h_{\theta}(x), y) = -y \log(h_{\theta}(x)) - (1 - y) \log(1 - h_{\theta}(x)) \quad (3.5)$$

A vectorised implementation is:

$$h = g(X\theta) \\ J(\theta) = \frac{1}{m} \cdot (-y^T \log h - (1 - y)^T \log(1 - h))$$

Vectorised implementation for Gradient Descent:

$$\theta := \theta - \frac{\alpha}{m} X^T (g(X\theta) - y)$$

## 3.3 Multiclass Classification

### 3.3.1 One-vs-all

This approach is when data has more than two categories. We divide our problem into  $n^1$  binary classification problems, in each one, we predict the probability considering one of the category to be +ve and all other to be -ve. Repeating this for all other categories will finally give us all the decision boundaries.

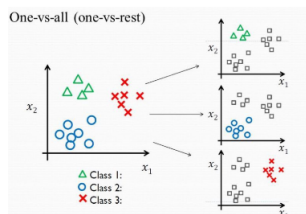
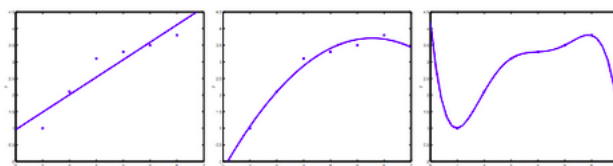


Figure 3.3: One vs all classification method

## 3.4 The Problem of Overfitting

Consider the problem of predicting  $y$  from  $x \in \mathbb{R}$ . The leftmost figure below shows the result of fitting a  $y = \theta_0 + \theta_1 x$  to a dataset. We see that the data doesn't really lie on straight line, and so the fit is not very good.



Instead, if we had added an extra feature  $x^2$ , and fit  $t = \theta_0 + \theta_1 x + \theta_2 x^2$ , then we obtain a slightly better fit to the data (See middle figure). Naively, it might seem that the more features we add, the better. However, there is also a danger in adding too many features: The rightmost figure is the result of fitting a 5<sup>th</sup> order polynomial  $y = \sum_{j=0}^5 \theta_j x^j$ . We see that even though the fitted curve passes through the data perfectly, we would not expect this to be a very good predictor of, say, housing prices ( $y$ ) for different living areas ( $x$ ). Without formally defining what these terms mean, we'll say the figure on the left shows an instance of **underfitting**—in which the data clearly shows structure not captured by the model—and the figure on the right is an example of **overfitting**.

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<sup>1</sup> $n$  = no of categories in dataset

## How to address this issue?

1. Reduce the number of features:
  - Manually select which features to keep.
  - Use a model selection algorithm.<sup>2</sup>
2. Regularisation:
  - Keep all the features, but reduce the magnitude of parameters  $\theta_j$ .
  - Regularization works well when we have a lot of slightly useful features.

### 3.4.1 Regularized Cost Function

To solve this problem of overfitting, we can eliminate the influence of  $\theta_3 x^3$  and  $\theta_4 x^4$ . Without actually getting rid of these features or changing the form of our hypothesis, we can instead modify our cost function:

$$J_\theta = \min \text{ of } \left[ \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^2 \theta_j^2 \right] \quad (3.6)$$

These extra terms will inflate the cost of extra parameters.

The  $\lambda$  is called the **regularisation parameter**/ It determines how much the costs of our theta parameters are inflated.

### 3.4.2 Regularized Linear Regression

Repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} + \frac{\lambda}{m} \theta_j \quad j \in \{1, 2, \dots, n\}$$

}

The term  $\frac{\lambda}{m} \theta_j$  performs our regularization. With some manipulation our update rule can also be represented as:

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<sup>2</sup>we'll cover it later

$$\theta_j := \theta_j(1 - \alpha \frac{\lambda}{m}) - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})x_j^{(i)}$$

The first term in the above equation,  $\alpha \frac{\lambda}{m}$  will always be less than 1. Intuitively you can see it as reducing the value of  $j\theta_j$  by some amount on every update.

### Normal Eequation

This will be the non-iterative approach for regularazation.

To add in regularization, we'll just add another term:

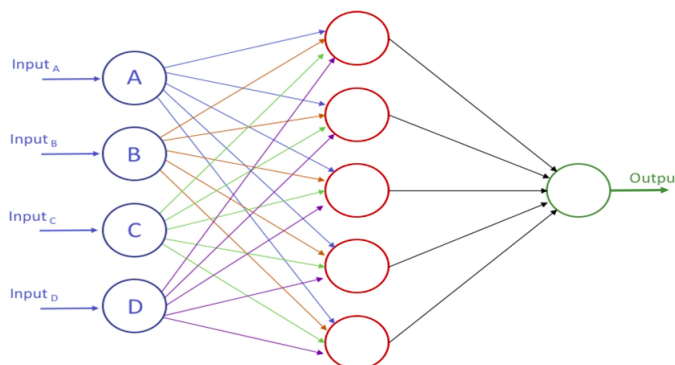
$$\theta = X^T X + \lambda.L^{-1}X^T y \tag{3.7}$$

where, L is  $(n + 1) \times (n + 1)$  matrix with 0 at the top lest, 1's down the diagonal and all other element '0'.

# Chapter 4

## Neural Networks

At a very simple level, neurons are basically computational units that take inputs (**dendrites**) as electrical inputs (called "spikes") that are channeled to outputs (**axons**). In our model, our dendrites are like the input features  $x_1...x_n$ , and the output is the result of our hypothesis function. In this model our  $x_0$  input node is sometimes called the "bias unit." It is always equal to 1.



Our input nodes (layer 1), also known as the "input layer", go into another node (layer 2), which finally outputs the hypothesis function, known as the "output layer". We can have intermediate layers of nodes between the input and output layers called the "hidden layers."

These "hidden layer" nodes are called as "activation units". The values for each activation nodes are represented as:

$$\begin{array}{c} x_0 \quad a_1^{(2)} \\ x_1 \rightarrow a_2^{(2)} \rightarrow h_{\theta}(x) \\ x_2 \quad a_3^{(2)} \\ x_3 \end{array}$$

$$a_1^{(2)} = g(\theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3) \quad (4.1)$$

$$a_2^{(2)} = g(\theta_{20}^{(1)} x_0 + \theta_{21}^{(1)} x_1 + \theta_{22}^{(1)} x_2 + \theta_{23}^{(1)} x_3) \quad (4.2)$$

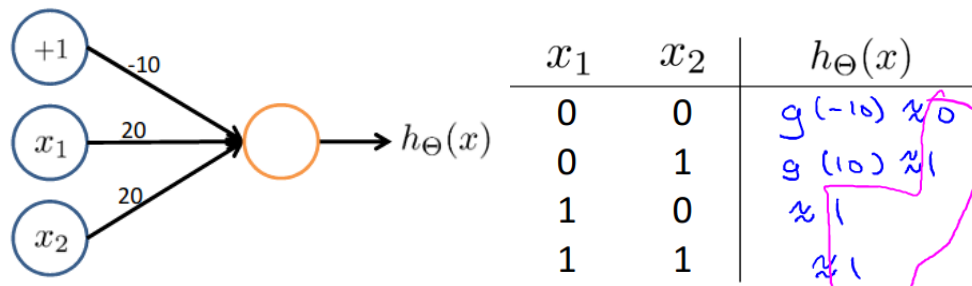
$$a_3^{(2)} = g(\theta_{30}^{(1)} x_0 + \theta_{31}^{(1)} x_1 + \theta_{32}^{(1)} x_2 + \theta_{33}^{(1)} x_3) \quad (4.3)$$

$$h_\theta(x) = a_1^{(3)} = a_1^{(2)} = g(\theta_{10}^{(2)} x_0 + \theta_{11}^{(2)} x_1 + \theta_{12}^{(2)} x_2 + \theta_{13}^{(2)} x_3) \quad (4.4)$$

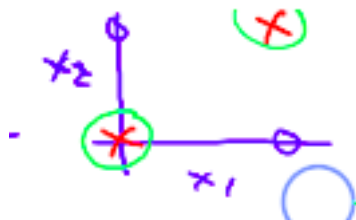
From these equations, we can conclude that we will get a matrix for each layer to calculate the weight for the second layer.

## 4.1 Intutions

### 4.1.1 OR Function



### 4.1.2 Important Note



For any prediction which involve a straight line as decision boundary, we can represent it with a neural network without any hidden layer but otherwise we'll have to include few hidden layers. An important point to note is that we can represent almost any distribution with cenrain arrangement of neural network.

## 4.2 Multiclass Classification

To classify data into multiple classes, we'll have to define out set of resulting classes as  $y$ :

$$y^{(i)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

## 4.3 Cost Function

Let's first define a few variables that we'll need to use:

- $L$  = total number of layers in the network
- numbers of units (non counting bias unit) in layer 1
- $K$  = numbet of output unit/classes

Cost function for neural networks will be slightly more complicated as it involves few other factors of ' $K$ ' and ' $L$ ' defined earlier.

$$J(\theta) = -\frac{1}{m} \sum_{i=1}^m \sum_{k=1}^K y_k^{(i)} \log((h_{\theta}(x^{(i)}))_k) + (1 - y_k^{(i)}) \log(1 - (h_{\theta}(x^{(i)}))_k) + \frac{\lambda}{2m} \sum_{l=1}^{L-1} \sum_{i=1}^{s_l} \sum_{j=1}^{s_{l+1}} (\theta_{j,i}^{(L)})^2 \quad (4.5)$$

**Note:**

- the double sum simply adds up the logistic regression costs calculated for each cell in the output layer.
- the triple sum simply adds up the squares of all the individual  $\theta$ s in the entire network.
- the  $i$  in the triple sum does not refer to training example  $i$ .

## 4.4 Backpropagation Algorithm

Given training set  $(x^{(1)}, y^{(1)}) \dots (x^{(m)}, y^{(m)})$

- Set  $\Delta_{i,j}^{(l)} := 0$  for all  $(l,i,j)$ , (hence you end up having a matrix full of zeros)

For training example  $t=1$  to  $m$ :

1. Set  $a^{(1)} := x^{(t)}$
2. Perform forward propagation to compute  $a^{(l)}$  for  $l = 2, 3, \dots, L$

### Gradient Computation

3. Using  $y^{(t)}$ , compute  $\delta^{(L)} = a^{(L)} - y^{(L)}$
4. Compute  $\delta^{(l)}$
5.  $\Delta^{(L)} := \Delta^{(L)} + \delta^{(l+1)}(a^{(l)})^T$

Hence we update our new  $\Delta$  matrix.

- $D_{i,j}^{(l)} := \frac{1}{m} \Delta_{i,j}^{(l)} + \lambda \Theta_{i,j}^{(l)}$ , if  $j \neq 0$ .
- $D_{i,j}^{(l)} := \frac{1}{m} \Delta_{i,j}^{(l)}$  if  $j = 0$

## 4.5 Gradient Checking

Gradient checking will assure that our backpropagation works as intended. We can approximate the derivative of our cost function with:

$$\frac{\partial}{\partial \Theta} J(\Theta) \approx \frac{J(\Theta + \epsilon) - J(\Theta - \epsilon)}{2\epsilon}$$

And for multiple features,

$$\frac{\partial}{\partial \Theta_j} J(\Theta) \approx \frac{J(\Theta_1, \dots, \Theta_j + \epsilon, \dots, \Theta_n) - J(\Theta_1, \dots, \Theta_j - \epsilon, \dots, \Theta_n)}{2\epsilon}$$

## 4.6 Putting it Together

First, pick a network architecture; choose the layout of your neural network, including how many hidden units in each layer and how many layers in total you want to have.

- Number of input units = dimension of features  $x^{(i)}$
- Number of output units = number of classes



- Number of hidden units per layer = usually more the better (must balance with cost of computation as it increases with more hidden units)
- Defaults: 1 hidden layer. If you have more than 1 hidden layer, then it is recommended that you have the same number of units in every hidden layer.

## Training a Neural Network

1. Randomly initialize the weights<sup>3</sup>
2. Implement forward propagation to get  $h_{\theta}(x^{(i)})$  for any  $x^{(i)}$
3. Implement the cost function
4. Implement backpropagation to compute partial derivatives
5. Use gradient checking to confirm that your backpropagation works. Then disable gradient checking.
6. Use gradient descent or a built-in optimization function to minimize the cost function with the weights in theta.

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<sup>3</sup>A good choice for  $e_{init} = \frac{\sqrt{6}}{\sqrt{L_{in}+L_{out}}}$

# Chapter 5

## Improving Neural Networks

What to try next for improving our neural networks?

- Getting more training examples
- Trying smaller sets of features
- Trying additional features
- Trying polynomial features
- Increasing or decreasing  $\lambda$

### 5.1 Evaluating a Hypothesis

A hypothesis may have a low error for training examples but still be inaccurate (because of overfitting). Thus, to evaluate a hypothesis, given a dataset of training examples, we can split up the data into two sets: a training set and a test set. Typically, the training set consists of 70% of your data and the test set is the remaining 30%.

The test set error

1. For linear regression:  $J_{test}(\Theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_{\Theta}(x_{test}^{(i)}) - y_{test}^{(i)})^2$
2. For classification ~ Misclassification error (aka 0/1 misclassification error):

$$err(h_{\Theta}(x), y) = \begin{cases} 1 & \text{if } h_{\Theta}(x) \geq 0.5 \text{ and } y = 0 \text{ or } h_{\Theta}(x) < 0.5 \text{ and } y = 1 \\ 0 & \text{otherwise} \end{cases}$$

This gives us a binary 0 or 1 error result based on a misclassification. The average test error for the test set is:

$$\text{Test Error} = \frac{1}{m_{test}} \sum_{i=1}^{m_{test}} err(h_{\Theta}(x_{test}^{(i)}), y_{test}^{(i)})$$

This gives us the proportion of the test data that was misclassified.

### 5.1.1 Model Selection

Just because a learning algorithm fits a training set well, that does not mean it is a good hypothesis. It could over fit and as a result your predictions on the test set would be poor.

Given many models with different polynomial degrees, we can use a systematic approach to identify the 'best' function. In order to choose the model of your hypothesis, you can test each degree of polynomial and look at the error result.

We usually break down our dataset into three sets:

- Training set: 60%
- Cross validation set: 20%
- Test set: 20%

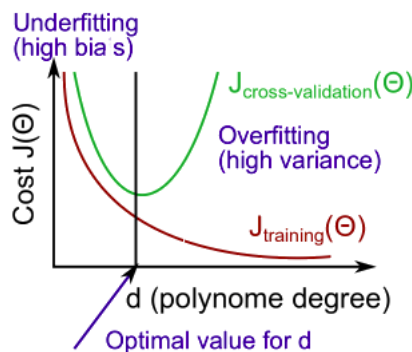
Now to improve our training:

1. Optimize the parameters in  $\theta$  using training set.
2. Find the polynomial degree  $d$  with the least error using the cross validation set.
3. Estimate the generalization error using the test set with  $J_{test}(\Theta^{(d)})$  ( $d = \theta$  from polynomial with lower error)

## 5.2 Bias vs. Variance

**High bias (underfitting):** both  $J_{train}(\theta)$  and  $J_{CV}(\theta)$  will be high and also similar.

**High variance (overfitting):**  $J_{train}(\theta)$  will be low but  $J_{CV}(\theta)$ .



### 5.2.1 Regularization

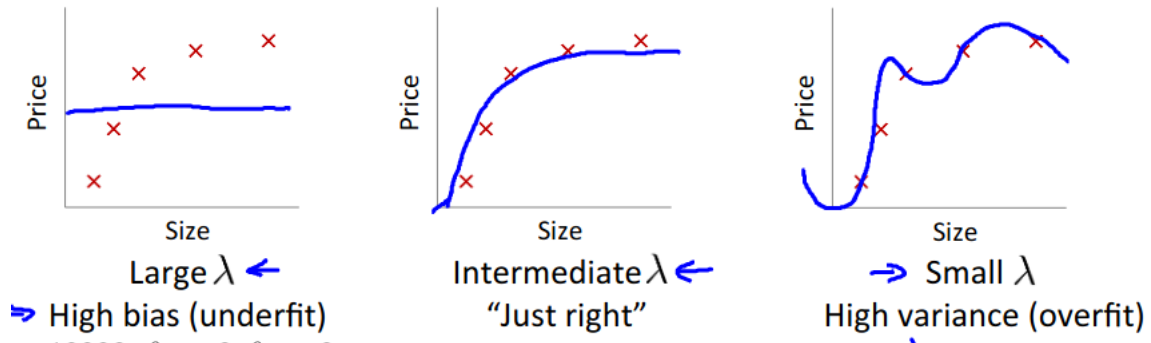
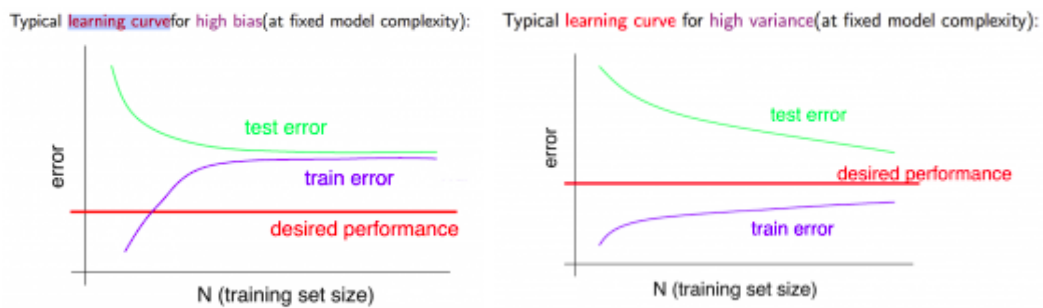


Figure 5.1: Fitting of data with  $\lambda$

In the figure above, we see that as  $\lambda$  increases, our fit becomes more rigid. On the other hand, as  $\lambda$  approaches 0, we tend to overfit the data. So how do we choose our parameter  $\lambda$  to get it 'just right'?

1. Create a list of lambdas.
2. Iterate through the  $\lambda$ s and for each, go through all the models to learn some  $\theta$
3. Compute the cross validation error.
4. Select the best combo of  $\theta$  &  $\lambda$ .

### 5.2.2 Learning Curves



# Chapter 6

## Unsupervised Learning

Unsupervised learning is a type of machine learning that looks for previously undetected patterns in a data set with no pre-existing labels and with a minimum of human supervision. In contrast to supervised learning that usually makes use of human-labeled data, unsupervised learning, also known as self-organization allows for modeling of probability densities over inputs. It forms one of the three main categories of machine learning, along with supervised and reinforcement learning. Semi-supervised learning, a related variant, makes use of supervised and unsupervised techniques.

### 6.1 K-Means Clustering Algorithm

Kmeans algorithm is an iterative algorithm that tries to partition the dataset into K pre-defined distinct non-overlapping subgroups (clusters) where each data point belongs to only one group. It tries to make the intra-cluster data points as similar as possible while also keeping the clusters as different (far) as possible. It assigns data points to a cluster such that the sum of the squared distance between the data points and the cluster's centroid (arithmetic mean of all the data points that belong to that cluster) is at the minimum. The less variation we have within clusters, the more homogeneous (similar) the data points are within the same cluster.

The way kmeans algorithm works is as follows:

1. Specify number of clusters K.
2. Initialize centroids by first shuffling the dataset and then randomly selecting K data points for the centroids without replacement.
3. Keep iterating until there is no change to the centroids. i.e assignment of data points to clusters isn't changing:

- Compute the sum of the squared distance between data points and all centroids.
- Assign each data point to the closest cluster (centroid).
- Compute the centroids for the clusters by taking the average of the all data points that belong to each cluster.

### 6.1.1 The Elbow Method: Choosing the number of clusters

For the k-means clustering method, the most common approach for choosing the number of clusters is the so-called **elbow method**. It involves running the algorithm multiple times over a loop, with an increasing number of cluster choice and then plotting a clustering score as a function of the number of clusters.

What is the score or metric which is being plotted for the elbow method? Why is it called the ‘elbow’ method?

A typical plot looks like following,

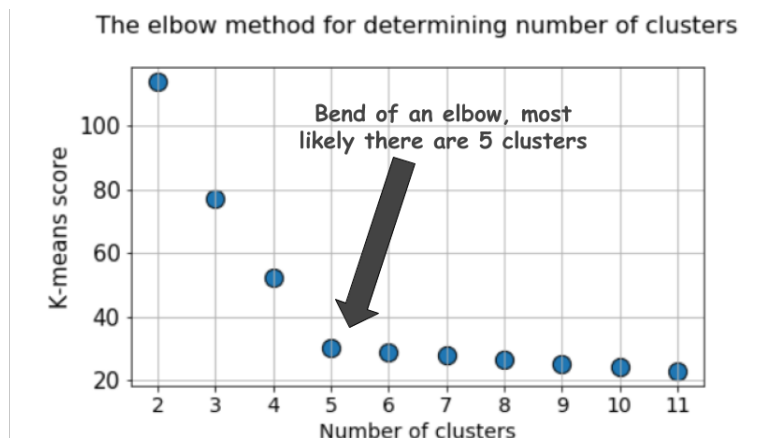


Figure 6.1: Elbow Method: Score plot

The score is, in general, a measure of the input data on the k-means objective function i.e. **some form of intra-cluster distance relative to inner-cluster distance**.

### 6.1.2 Image compression with K-means

In a straightforward 24-bit color representation of an image, each pixel is represented as three 8-bit unsigned integers (ranging from 0 to 255) that specify the red, green and blue intensity values. This encoding is often referred to as the RGB encoding. Our image contains thousands of colors, and in this part of the exercise, you will reduce the number of colors to 16 colors. By making this reduction, it is possible to represent

(compress) the photo in an efficient way. Specifically, you only need to store the RGB values of the 16 selected colors, and for each pixel in the image you now need to only store the index of the color at that location (where only 4 bits are necessary to represent 16 possibilities). In this exercise, you will use the K-means algorithm to select the 16 colors that will be used to represent the compressed image. Concretely, you will treat every pixel in the original image as a data example and use the K-means algorithm to find the 16 colors that best group (cluster) the pixels in the 3- dimensional RGB space. Once you have computed the cluster centroids on the image, you will then use the 16 colors to replace the pixels in the original image.

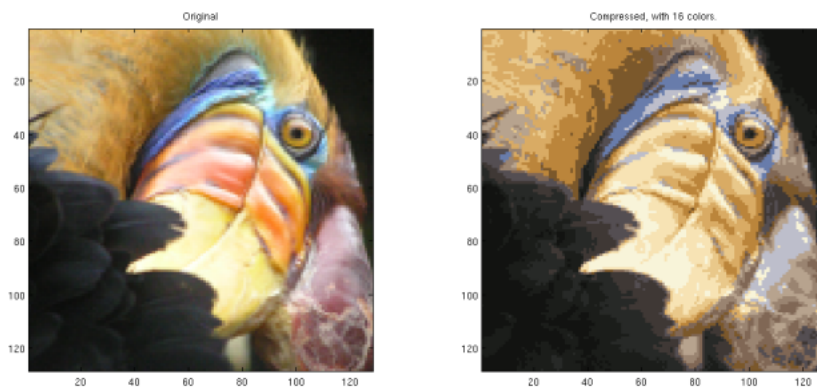


Figure 6.2: Original and Reconstructed Image using K-means

# Chapter 7

## Octave/MATLAB Commands

### Basic Operations

```
octave:1> a = pi
a = 3.1416
octave:2> disp(sprintf('6 decimals: %0.6f', a))
6 decimals: 3.141593
octave:3> a
a = 3.1416
octave:4> format long
octave:5> a
a = 3.141592653589793
octave:6> format short
octave:7> a
a = 3.1416
octave:8> v = 1:0.1:2
v =

    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000
    1.6000    1.7000    1.8000    1.9000    2.0000

octave:9> v = 1:0.1:2
v =

Columns 1 through 8:

    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000
    1.6000    1.7000

Columns 9 through 11:

    1.8000    1.9000    2.0000
```



```

octave:10> v = 1:6
v =

    1    2    3    4    5    6

octave:11> zeros(1,3)
ans =

    0    0    0

octave:12> rand(1,3)
ans =

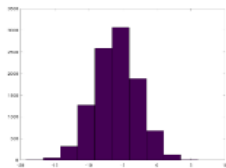
    0.43623    0.76554    0.23635

octave:13> randn(1,3)
ans =

    0.5602642   -0.0043628    0.1344922

octave:14> w = -6 + sqrt(10)*(randn(1,10000))
octave:15> hist(w)

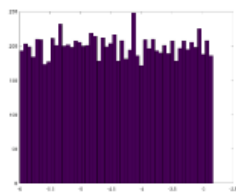
```



```

octave:1> w = -6 + sqrt(10)*(rand(1,10000));
octave:2> hist(w,50)

```



## Moving Data around

```
octave:1> A = [1,2;3,4;4,5]
A =

     1     2
     3     4
     4     5

octave:2> size(A)
ans =

     3     2

octave:3> sz = size(A)
sz =

     3     2

octave:4> size(sz)
ans =

     1     2

octave:5> size(A,1)
ans = 3
octave:6> size(A,2)
ans = 2
octave:7> length(A)
ans = 3
octave:8> length([1,2,3,4,5])
ans = 5
octave:9>
octave:9>
octave:9> pwd
ans = /home/sahasra
octave:10> cd /home/sahasra/
octave:11> pwd
ans = /home/sahasra
octave:12> ls
Android          Documents      Music          Public          Videos
AndroidStudioProjects Downloads     MyPaint       snap
Desktop          examples.desktop Pictures      Templates
octave:13> who
Variables in the current scope:
```

```
A      ans    sz
```

```
octave:14> whos
```

```
Variables in the current scope:
```

Attr	Name	Size	Bytes	Class
	A	3x2	48	double
	ans	1x13	13	char
	sz	1x2	16	double

```
Total is 21 elements using 77 bytes
```

```
octave:15> clear
```

```
octave:16> whos
```

```
octave:17> A = [1,2;3,4;5,6]
```

```
A =
```

```
1    2
3    4
5    6
```

```
octave:18> A(3,2)
```

```
ans = 6
```

```
octave:19> A(2,:) 
```

```
ans =
```

```
3    4
```

```
octave:20> A(:,2)
```

```
ans =
```

```
2
4
6
```

```
octave:21> A([1,3], :)
```

```
ans =
```

```
1    2
5    6
```

```
octave:22> A([2,3], :)
```

```
ans =
```

```

      3      4
      5      6

octave:23> A(:,2) = [10;11;12]
A =

```

```

      1      10
      3      11
      5      12

```

```

octave:24> A = [A, [5;6;7]]
A =

```

```

      1      10      5
      3      11      6
      5      12      7

```

```

octave:25> A(:)
ans =

```

```

      1
      3
      5
     10
     11
     12
      5
      6
      7

```

```

octave:26> A
A =

```

```

      1      10      5
      3      11      6
      5      12      7

```

```

octave:27> B = [45;46;47]
B =

```

```

     45
     46
     47

```

```
octave:28> C = [A,B]
C =
```

1	10	5	45
3	11	6	46
5	12	7	47

## Computing on Data

```
octave:1> A = [1 2;3 4;5 6]
A =
```

1	2
3	4
5	6

```
octave:2> B = [11 12; 13 14; 15 16]
B =
```

11	12
13	14
15	16

```
octave:3> C = [1 1; 2 2]
C =
```

1	1
2	2

```
octave:4> A*C
ans =
```

5	5
11	11
17	17

```
octave:5> A .* B % A .* B gives element wise operation
ans =
```

11	24
39	56
75	96

```
octave:6> 1 ./ A
ans =
```

```

    1.00000    0.50000
    0.33333    0.25000
    0.20000    0.16667

octave:7> v = [1;2;3]
v =

    1
    2
    3

octave:8> log(v)
ans =

    0.00000
    0.69315
    1.09861

octave:9> exp(v)
ans =

    2.7183
    7.3891
   20.0855

octave:10> abs([-1; 2; -3])
ans =

    1
    2
    3

octave:11> A
A =

    1    2
    3    4
    5    6

octave:12> A' % A' = A transpose
ans =

    1    3    5
    2    4    6

```

```

octave:13> val = max([1;2;3;6;7])
val = 7
octave:14> max(A)
ans =

    5    6

octave:15> A
A =

    1    2
    3    4
    5    6

octave:16> a = [1;4;6;7;9]
a =

    1
    4
    6
    7
    9

octave:17> a < 3
ans =

    1
    0
    0
    0
    0

octave:18> find(a<3)
ans = 1
octave:19> A = magic(3) % Magic Square
A =

    8    1    6
    3    5    7
    4    9    2

octave:20> [r,c] = find(a >= 7)
r =

    4

```

```

5

c =

    1
    1

octave:21> a
a =

    1
    4
    6
    7
    9

octave:22> a = a'
a =

    1    4    6    7    9

octave:23> sum(a)
ans = 27
octave:24> rand(3)
ans =

    0.272471    0.059338    0.757392
    0.414497    0.174242    0.354694
    0.811891    0.935437    0.956667

octave:25> A
A =

    8    1    6
    3    5    7
    4    9    2

octave:26> max(A,[],1)
ans =

    8    9    7

octave:27> max(A,[],2)
ans =

```



```

8
7
9

octave:28> max(max(A))
ans = 9
octave:29> A
A =

    8    1    6
    3    5    7
    4    9    2

octave:30> pinv(A)
ans =

    0.147222   -0.144444    0.063889
   -0.061111    0.022222    0.105556
   -0.019444    0.188889   -0.102778

octave:31> temp = pinv(A)
temp =

    0.147222   -0.144444    0.063889
   -0.061111    0.022222    0.105556
   -0.019444    0.188889   -0.102778

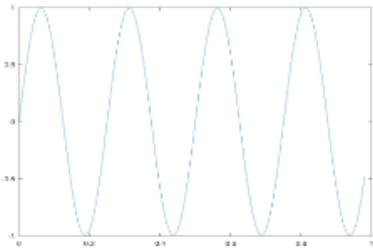
octave:32> temp * A
ans =

    1.0000e+00    2.0817e-16   -3.1641e-15
   -6.1062e-15    1.0000e+00    6.2450e-15
    3.0531e-15    4.1633e-17    1.0000e+00

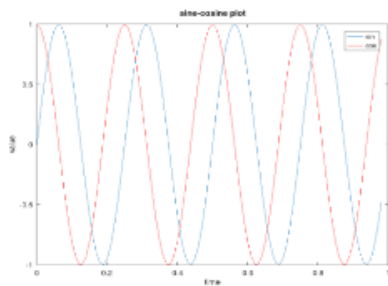
octave:33> % this is the 3x3 Identity matrix,
           % not having exact values beacuse of variable overflow

octave:1> t = [0:0.01:0.98];
octave:2> y1 = sin(2*pi*4*t);
octave:3> plot(t,y1)

octave:4> y2 = cos(2*pi*4*t);
octave:5> plot(t,y1);
octave:6> hold on;
```



```
octave:7> plot(t, y2, 'r');
octave:8> xlabel('time')
octave:9> ylabel('value')
octave:10> legend('sin', 'cos')
octave:11> title('sine-cosine plot')
octave:12> print -dpng 'myPlot.png'
```

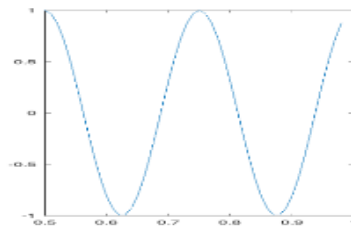
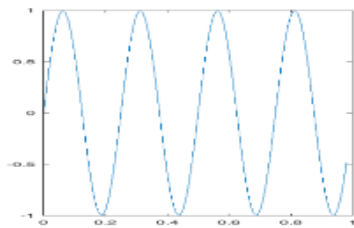


```
octave:13> close
octave:14> figure(1); plot(t, y1);
octave:15> figure(2); plot(t, y2);
octave:16> subplot(1,2,1); % Divides plot a 1x2 grid
octave:17> plot(t,y1);
octave:18> subplot(1,2,2)
octave:19> plot(t,y2);
octave:20> axis([0.5 1 -1 1])
```

**Control Statements: for, while, if, else-if ...**

```
octave:1> v = zeros(10,1)
v =

    0
    0
```



```

0
0
0
0
0
0
0
0
0
0

octave:2> for i=1:10,
> v(i) = 2^i;
> end;

octave:3> v
v =

    2
    4
    8
   16
   32
   64
  128
  256
  512
 1024

octave:4> i=1;
octave:5> while i <= 5,
> v(i) = 100;
> i = i+1;
> end;
octave:6> v
v =

```

```

100
100
100
100
100
  64
128
256
512
1024

octave:7> i = 1;
octave:8> while true,
> v(i) = 999;
> i = i+1;
> if i == 6,
>   break;
> end;
> end;

octave:9> v
v =

  999
  999
  999
  999
  999
   64
  128
  256
  512
 1024

```