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# Chapter 1

## Week 1

### 1.1 Supervised and Unsupervised Learning

The most basic thing to remember is that we already know what our correct output should look like in Supervised Learning. But, we have little or no idea about what our results should look like.

**Supervised Learning:**

- Classification: Spam/Not-spam.
- Regression: Predicting age.

**Unsupervised Learning:**

- Clustering: Grouping based on different variables.
- Non Clustering: Finding structure in chaotic environment.

### 1.2 Linear Regression with one variable

Regression being a part of Supervised Learning is used for estimating data (Real-valued output).

#### 1.2.1 Cost Function

This function measures the performance of a Machine Learning model for given data.

**Hypothesis:**  $h_{\theta}(x) = \theta_0 + \theta_1 x$

**Parameters:**  $\theta_0, \theta_1$

**Cost Function:**

$$J(\theta_0, \theta_1) = 1/2m \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 \quad (1.1)$$

**Goal:** Minimize cost function with  $\theta_0, \theta_1$  as parameters.

## 1.2.2 Gradient Descent

Basic idea:

- Start with some  $\theta_0, \theta_1$
- Keep changing  $\theta_0, \theta_1$  to reduce  $J(\theta_0, \theta_1)$  until we end up at minima.

**Algorithm:** repeat until convergence:

$$\theta_j := \theta_j - \alpha \frac{\partial J(\theta_0, \theta_1)}{\partial \theta_j} \quad (1.2)$$

(for  $j = 0, 1$ , here).

**Intution:** If  $\alpha$  is too small, descent can be slow and if too large, descent may fail to converge or even diverge. Gradient descent can converge to a local minimum, even with fixed learning rate  $\alpha$ . As we approach local minimum, gradient descent will automatically take smaller steps. So, no need to decrease  $\alpha$  over time.

## 1.2.3 Gradient Descent for linear regression

Combining gradient descent algorithm with linear regression model, we get:

$$j = 0 : \frac{\partial J(\theta_0, \theta_1)}{\partial \theta_0} = 1/2 \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \quad (1.3)$$

$$j = 1 : \frac{\partial J(\theta_0, \theta_1)}{\partial \theta_1} = 1/2 \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x^{(i)} \quad (1.4)$$

Now, we can repeat 1.3 and 1.4 until convergence to obtain the minima.

"Batch" gradient descent: Each step of gradient descent uses all the training examples. For eq. "m" batches in equation 1.1.

# Chapter 2

## Week 2

### 2.1 Multivariate Linear Regression

Linear regression involving more than one variable. For eq., Predicting price of a house based on parameters "Plot Area", "No. of Floors", "Connectivity with markets", etc.

#### 2.1.1 Multiple Features

The multivariable form of the hypothesis is as follows:

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \dots + \theta_n x_n. \quad (2.1)$$

This hypothesis function can be concisely represented as:

$$h_{\theta}(x) = \theta^T x \quad (2.2)$$

where,  $\theta^T$  is a  $1 \times n$  matrix consisting of  $\theta_0, \theta_1, \theta_2, \dots, \theta_n$ .

#### 2.1.2 Gradient Descent for Multiple Variables

Gradient descent formula for Multiple variable will be similar to that of single variable.

$$\theta_j = \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)} \quad (2.3)$$

Repeating this equation until convergence will give the minima. <sup>1</sup>

#### Feature Scaling

Feature Scaling is used to reduce the number of iterations in Gradient Descent. Basic idea of feature scaling is to bring all the features on the same scale. (in general we try

---

<sup>1</sup> $x_0 = 1$  in equation 2.3

to approximate every feature in the range  $-1 < x_i < 1$ )

Reducing the number of iteration doesn't mean making computation of each step easier. And also it does not effect comtational efficiency of Normal Equation.

## Mean Normalisation

Mean Normalisation makes features to have approximately zero mean.

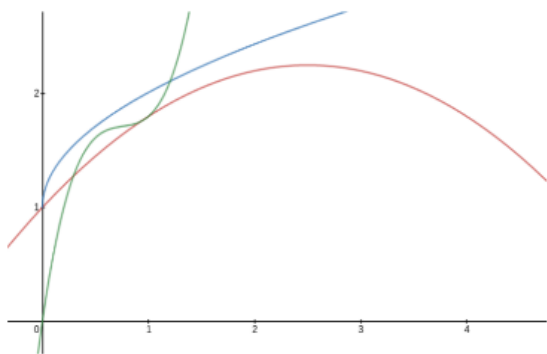
## Learning Rate

If  $\alpha$  is too small: slow convergence.

if  $\alpha$  is too large:  $J(\theta)$  may not decrease on every iteration, or may not converge.

## Polynomial Regression

Selecting proper polynomial for fitting data is very important.



**Red:** Quadratic

**Blue:** Square root funtion  $\theta_0 + \theta_1 x + \theta_2 \sqrt{x}$

**Green:** Cubic function

## 2.2 Normal equation

Normal Equation is a method to solve for  $\theta_T$  analytically, by creating a  $m \times (n + 1)$  matrix  $X$  and another  $m \times 1$  matrix  $Y$ .<sup>2</sup>

Mathematically  $\theta$  is given as:

$$\theta = (X^T X)^{-1} X^T y \quad (2.4)$$

---

<sup>2</sup>Every element of first column of matrix  $X$  is 1 and other are the feature's coefficient

Gradient Descent	Normal Equation
Need to choose $\alpha$	No need to choose $\alpha$
Needs many iteration	Don't need to iterate
Works well with large n	Slow for large n

### Reasons for non-invertibility of $X^T X$

- Redundant features (linear dependence) <sup>3</sup>
- Too many features ( $m \leq n$ )

## 2.3 Octave/MATLAB commands

### Basic Operations

```
octave:1> a = pi
a = 3.1416
octave:2> disp(sprintf('6 decimals: %0.6f', a))
6 decimals: 3.141593
octave:3> a
a = 3.1416
octave:4> format long
octave:5> a
a = 3.141592653589793
octave:6> format short
octave:7> a
a = 3.1416
octave:8> v = 1:0.1:2
v =
```

```

    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000
1.7000    1.8000    1.9000    2.0000
```

```
octave:9> v = 1:0.1:2
v =
```

Columns 1 through 8:

```

    1.0000    1.1000    1.2000    1.3000    1.4000    1.5000    1.6000
1.7000
```

Columns 9 through 11:

---

<sup>3</sup>Eg. Using both  $m^2$  &  $(feet)^2$  features

```
1.8000    1.9000    2.0000
```

```
octave:10> v = 1:6  
v =
```

```
1    2    3    4    5    6
```

```
octave:11> zeros(1,3)  
ans =
```

```
0    0    0
```

```
octave:12> rand(1,3)  
ans =
```

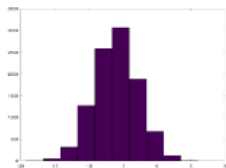
```
0.43623    0.76554    0.23635
```

```
octave:13> randn(1,3)  
ans =
```

```
0.5602642   -0.0043628    0.1344922
```

```
octave:14> w = -6 + sqrt(10)*(randn(1,10000))
```

```
octave:15> hist(w)
```



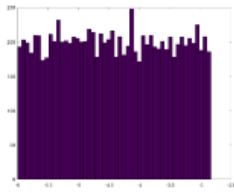
```
octave:1> w = -6 + sqrt(10)*(rand(1,10000));
```

```
octave:2> hist(w,50)
```

## Moving Data around

```
octave:1> A = [1,2;3,4;4,5]  
A =
```

```
1    2
```



```

3     4
4     5

```

```

octave:2> size(A)
ans =

```

```

3     2

```

```

octave:3> sz = size(A)
sz =

```

```

3     2

```

```

octave:4> size(sz)
ans =

```

```

1     2

```

```

octave:5> size(A,1)
ans = 3
octave:6> size(A,2)
ans = 2
octave:7> length(A)
ans = 3
octave:8> length([1,2,3,4,5])
ans = 5
octave:9>
octave:9>
octave:9> pwd
ans = /home/sahasra
octave:10> cd /home/sahasra/
octave:11> pwd
ans = /home/sahasra
octave:12> ls
Android          Documents        Music            Public           Videos
AndroidStudioProjects Downloads        MyPaint          snap
Desktop          examples.desktop Pictures          Templates
octave:13> who

```



Variables in the current scope:

A     ans   sz

octave:14> whos

Variables in the current scope:

Attr	Name	Size	Bytes	Class
	A	3x2	48	double
	ans	1x13	13	char
	sz	1x2	16	double

Total is 21 elements using 77 bytes

octave:15> clear

octave:16> whos

octave:17> A = [1,2;3,4;5,6]

A =

```
1  2
3  4
5  6
```

octave:18> A(3,2)

ans = 6

octave:19> A(2,:)

ans =

```
3  4
```

octave:20> A(:,2)

ans =

```
2
4
6
```

octave:21> A([1,3], :)

ans =

```
1  2
5  6
```

octave:22> A([2,3], :)

```
ans =
```

3	4
5	6

```
octave:23> A(:,2) = [10;11;12]
```

```
A =
```

1	10
3	11
5	12

```
octave:24> A = [A, [5;6;7]]
```

```
A =
```

1	10	5
3	11	6
5	12	7

```
octave:25> A(:)
```

```
ans =
```

1
3
5
10
11
12
5
6
7

```
octave:26> A
```

```
A =
```

1	10	5
3	11	6
5	12	7

```
octave:27> B = [45;46;47]
```

```
B =
```

45
46
47

```
octave:28> C = [A,B]
C =
```

1	10	5	45
3	11	6	46
5	12	7	47

### Computing on Data

```
octave:1> A = [1 2;3 4;5 6]
A =
```

1	2
3	4
5	6

```
octave:2> B = [11 12; 13 14; 15 16]
B =
```

11	12
13	14
15	16

```
octave:3> C = [1 1; 2 2]
C =
```

1	1
2	2

```
octave:4> A*C
ans =
```

5	5
11	11
17	17

```
octave:5> A .* B % A .* B gives element wise operation
ans =
```

11	24
39	56
75	96

```
octave:6> 1 ./ A
ans =
```

1.00000	0.50000
0.33333	0.25000
0.20000	0.16667

```
octave:7> v = [1;2;3]
v =
```

```
1
2
3
```

```
octave:8> log(v)
ans =
```

```
0.00000
0.69315
1.09861
```

```
octave:9> exp(v)
ans =
```

```
2.7183
7.3891
20.0855
```

```
octave:10> abs([-1; 2; -3])
ans =
```

```
1
2
3
```

```
octave:11> A
A =
```

```
1 2
3 4
5 6
```

```
octave:12> A' % A' = A transpose
ans =
```

```
1 3 5
2 4 6
```

```

octave:13> val = max([1;2;3;6;7])
val = 7
octave:14> max(A)
ans =

    5    6

octave:15> A
A =

    1    2
    3    4
    5    6

octave:16> a = [1;4;6;7;9]
a =

    1
    4
    6
    7
    9

octave:17> a < 3
ans =

    1
    0
    0
    0
    0

octave:18> find(a<3)
ans = 1
octave:19> A = magic(3) % Magic Square
A =

    8    1    6
    3    5    7
    4    9    2

octave:20> [r,c] = find(a >= 7)
r =

```

```

    4
    5

c =

    1
    1

octave:21> a
a =

    1
    4
    6
    7
    9

octave:22> a = a'
a =

    1    4    6    7    9

octave:23> sum(a)
ans = 27
octave:24> rand(3)
ans =

    0.272471    0.059338    0.757392
    0.414497    0.174242    0.354694
    0.811891    0.935437    0.956667

octave:25> A
A =

    8    1    6
    3    5    7
    4    9    2

octave:26> max(A,[],1)
ans =

    8    9    7

octave:27> max(A,[],2)
ans =

```

```

8
7
9

octave:28> max(max(A))
ans = 9
octave:29> A
A =

    8    1    6
    3    5    7
    4    9    2

octave:30> pinv(A)
ans =

    0.147222   -0.144444    0.063889
   -0.061111    0.022222    0.105556
   -0.019444    0.188889   -0.102778

octave:31> temp = pinv(A)
temp =

    0.147222   -0.144444    0.063889
   -0.061111    0.022222    0.105556
   -0.019444    0.188889   -0.102778

octave:32> temp * A
ans =

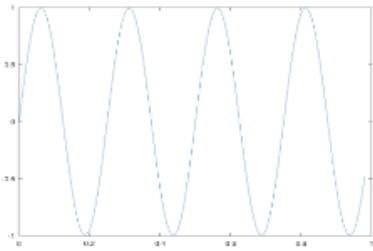
    1.0000e+00    2.0817e-16   -3.1641e-15
   -6.1062e-15    1.0000e+00    6.2450e-15
    3.0531e-15    4.1633e-17    1.0000e+00

octave:33> % this is the 3x3 Identity matrix,
           % not having exact values beacuse of variable overflow

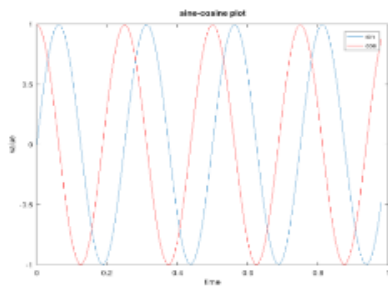
octave:1> t = [0:0.01:0.98];
octave:2> y1 = sin(2*pi*4*t);
octave:3> plot(t,y1)

octave:4> y2 = cos(2*pi*4*t);
octave:5> plot(t,y1);

```



```
octave:6> hold on;
octave:7> plot(t, y2, 'r');
octave:8> xlabel('time')
octave:9> ylabel('value')
octave:10> legend('sin', 'cos')
octave:11> title('sine-cosine plot')
octave:12> print -dpng 'myPlot.png'
```



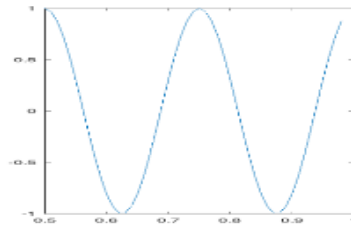
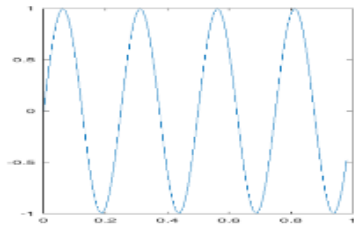
```
octave:13> close
octave:14> figure(1); plot(t, y1);
octave:15> figure(2); plot(t, y2);
octave:16> subplot(1,2,1); % Divides plot a 1x2 grid
octave:17> plot(t,y1);
octave:18> subplot(1,2,2)
octave:19> plot(t,y2);
octave:20> axis([0.5 1 -1 1])
```

## Control Statements: for, while, if, else-if ...

```
octave:1> v = zeros(10,1)
v =
```

0





```
0
0
0
0
0
0
0
0
0
0
0
```

```
octave:2> for i=1:10,
> v(i) = 2^i;
> end;
```

```
octave:3> v
v =
```

```
2
4
8
16
32
64
128
256
512
1024
```

```
octave:4> i=1;
octave:5> while i <= 5,
> v(i) = 100;
> i = i+1;
> end;
octave:6> v
v =
```

100  
100  
100  
100  
100  
64  
128  
256  
512  
1024

```
octave:7> i = 1;  
octave:8> while true,  
> v(i) = 999;  
> i = i+1;  
> if i == 6,  
>     break;  
> end;  
> end;
```

```
octave:9> v  
v =
```

999  
999  
999  
999  
999  
64  
128  
256  
512  
1024

# Chapter 3

## Week 3

### 3.1 Classification and Representation

#### 3.1.1 Classification

The classification problem is just like the regression problem, except that the values we now want to predict take on only a small number of discrete values. For now, we'll discuss binary classification problem.

#### 3.1.2 Hypothesis Representation

We may use our old regression algorithm by classifying data on the basis of a threshold. But it will have very poor performance.

We will introduce "Sigmoid Function", also called the "Logistic Function":

$$h_{\theta}(x) = g(\theta^T x) \quad (3.1)$$

$$z = \theta^T x \quad (3.2)$$

$$g(z) = \frac{1}{1 + e^{-z}} \quad (3.3)$$

This is how the Sigmoid Function looks like:



Figure 3.1: Sigmoid Function 3.3

### 3.1.3 Decision Boundary

The decision boundary is the line that separates the area where  $y=0$  and where  $y=1$ . It is similar to the decision boundary for linear regression, the only difference is distribution of values (linear and sigmoid)

## 3.2 Logistic Regression Model

### 3.2.1 Cost Function

Cost function for logistic regression looks like:

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_{\theta}(x^{(i)}), y^{(i)}) \quad (3.4)$$

$$\text{Cost}(h_{\theta}(x), y) = -\log(h_{\theta}(x)) \quad \text{if } y = 1$$

$$\text{Cost}(h_{\theta}(x), y) = -\log(1 - h_{\theta}(x)) \quad \text{if } y = 0$$

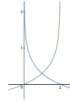


Figure 3.2: Cost Function

### Simplified Cost Function

This cost function can be compressed into a single function:

$$\text{Cost}(h_{\theta}(x), y) = -y \log(h_{\theta}(x)) - (1 - y) \log(1 - h_{\theta}(x)) \quad (3.5)$$

A vectorised implementation is:

$$h = g(X\theta) \\ J(\theta) = \frac{1}{m} \cdot (-y^T \log h - (1 - y)^T \log(1 - h))$$

Vectorised implementation for Gradient Descent:

$$\theta := \theta - \frac{\alpha}{m} X^T (g(X\theta) - y)$$

## 3.3 Multiclass Classification

### 3.3.1 One-vs-all

This approach is when data has more than two categories. We divide our problem into  $n^1$  binary classification problems, in each one, we predict the probability considering one of the category to be +ve and all other to be -ve. Repeating this for all other categories will finally give us all the decision boundaries.

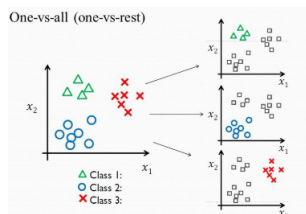


Figure 3.3: One vs all classification method

---

<sup>1</sup> $n$  = no of categories in dataset