

$$J(y) = \sum_{i=1}^N (y - x_i)^2$$

$$\frac{\partial J(y)}{\partial y} = \sum_{i=1}^N 2(y - x_i) = 0$$

$$N y = \sum_{i=1}^N x_i \rightarrow y = \frac{1}{N} \sum_{i=1}^N x_i$$

$$J_2(y) = \sum_{i=1}^N |y - x_i|$$

$$\frac{\partial J_2(y)}{\partial y} = \sum_{i=1}^N \text{sign}(y - x_i) = 0$$

$$\left| \begin{array}{l} \frac{d|x|}{dx} \\ = \text{sign}(x) \\ \text{if } x \neq 0 \end{array} \right.$$

-1 or +1

1 2 (3 4) 5 6

1 (2) 3

$$x_1, x_2, \dots, x_N$$

$$x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(N-1)} \leq x_{(N)}$$

$$SAD(\alpha) = \sum_{i=1}^N |x_{(i)} - \alpha| \quad \text{non diff if } \alpha = x_{(i)} \text{ for any } i.$$

$$x_{(k)} \leq \alpha \leq x_{(k+1)} \rightarrow \text{suppose}$$

$$SAD(\alpha) = \sum_{i=1}^k (\alpha - x_{(i)}) + \sum_{i=k+1}^N (x_{(i)} - \alpha)$$

for simplicity, initially consider  $N = 2$

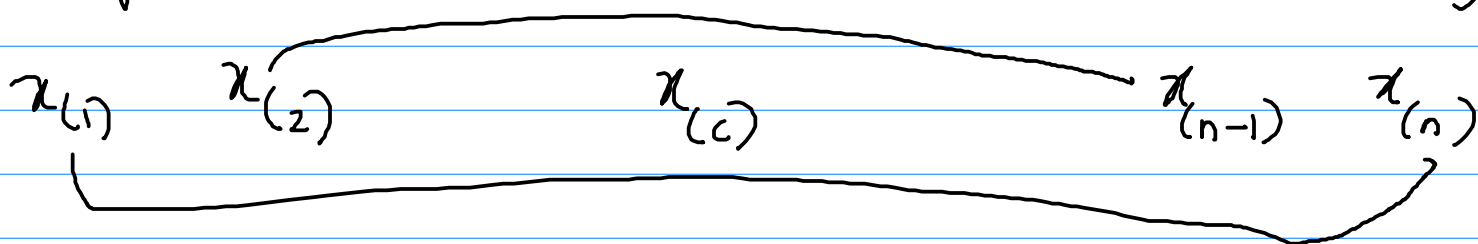
$$\begin{array}{ccc} x_1 & \text{and } x_2 & x_1 < x_2 \\ (x_1 \leq \alpha \leq x_2) & & \\ SAD(\alpha) = & \cancel{\alpha - x_1} + x_2 - \cancel{\alpha} & = x_2 - x_1 \end{array}$$

Now consider  $\alpha \notin [x_1, x_2]$

$$\begin{aligned} \alpha < x_1 &\rightarrow SAD(\alpha) = x_1 - \alpha + x_2 - \alpha \\ &= x_1 + x_2 - 2\alpha \\ &> x_1 + x_2 - 2x_1 \\ &= x_2 - x_1 \end{aligned}$$

$$\begin{aligned} \alpha > x_2 &\rightarrow SAD(\alpha) = \alpha - x_1 + \alpha - x_2 \\ &= 2\alpha - x_1 - x_2 \\ &> 2x_2 - x_1 - x_2 \\ &= x_2 - x_1 \end{aligned}$$

If  $SAD(\alpha)$  is to be minimized then  $\alpha \in [x_1, x_2]$



Nested intervals

$$[x(1), x(n)], [x(2), x(n-1)] \dots [x(i), x(n+1-i)]$$

If  $n$  is even, then  $C = n/2$

$$\downarrow$$
$$\rightarrow [x_{(n/2)}, x_{(n/2+1)}]$$

If  $n$  is odd, then  $C = \frac{n+1}{2}$

$$\rightarrow \overset{\text{single number}}{[x_{\frac{n+1}{2}}, x_{\frac{n+1}{2}}]}$$

We want to minimize SAD for entire array  
= sum of SAD values for every interval

We want to consider  $\alpha \in \bigcap_{i=1}^C [x(i), x(n+1-i)]$

$\therefore \alpha$  must lie in the innermost interval.

If  $n$  is even, the innermost interval is  
 $\alpha \in [x_{(n/2)}, x_{(n/2+1)}]$ .  $\therefore \alpha$  must be the median.

If  $n$  is odd, innermost interval is  $(x_{\frac{n+1}{2}})$

(QED)