Refer to slides on hypergeometric distr. in ecology.

We know r = # leopards in 1st catch
n = # leopards in 2nd catch
i = # marked leopards in 2nd catch.

We need to determine N = # leopards in SGNP.

This is an MLE problem and uses the fact that the r.v. X, which stands for the number of marked animals in the 2nd catch, has a hypergeometric distribution.

$$P(X=i;N) = \frac{C(r,i)\, C(N-r, n-i)}{C(N,n)}$$

For what N does this likelihood $P(X=i; N)$ attain its maximum value? For this we consider to find N such that

$$P(X=i; N) > P(X=i; N-1) \longrightarrow (1)$$

and

$$P(X=i; N) > P(X=i; N+1) \longrightarrow (2)$$

For (1) we have

$$\frac{\cancel{C(r,i)}\, C(N-r, n-i)}{C(N,n)} > \frac{\cancel{C(r,i)}\, C(N-r-1, n-i)}{C(N-1,n)}$$

$$\therefore \quad \frac{\dfrac{(N-r)!}{\cancel{(n-i)!}\,(N-r-n+i)!}}{\dfrac{N!}{\cancel{n!}\,(N-n)!}} > \frac{\dfrac{(N-r-1)!}{\cancel{(n-i)!}\,(N-r-1-n+i)!}}{\dfrac{(N-1)!}{\cancel{n!}\,(N-n-1)!}}$$

$$\therefore \quad \frac{\dfrac{N-r}{N-r-n+i}}{\dfrac{N}{N-n}} > 1$$

$$\longrightarrow \quad N^2 - Nn - Nr + rn > N^2 - Nr - Nn + Ni$$

$$\longrightarrow \quad N < rn/i$$

From ② we have

$$\frac{\cancel{C(r,i)}\,C(N-r,n-i)}{C(N,n)} > \frac{\cancel{C(r,i)}\,C(N+1-r,n-i)}{C(N+1,n)}$$

$$\therefore \quad \frac{\dfrac{(N-r)!}{\cancel{(n-i)!}\,(N-r-n+i)!}}{\dfrac{N!}{\cancel{n!}\,(N-n)!}} > \frac{\dfrac{(N-r+1)!}{\cancel{(n-i)!}\,(N-r-n+i+1)!}}{\dfrac{(N+1)!}{\cancel{n!}\,(N-n+1)!}}$$

$$1 > \frac{\dfrac{N-r+1}{N-r-n+i+1}}{\dfrac{N+1}{N-n+1}}$$

$$\frac{N+1}{N-n+1} > \frac{N-r+1}{N-r-n+i+1}$$

$$\therefore \cancel{N^2} - \cancel{Nr} - \cancel{Nn} + Ni + \cancel{2N} - \cancel{r} - \cancel{n} + i + \cancel{1} > \cancel{N^2} - \cancel{Nn} + \cancel{2N} - \cancel{rN} + rn - \cancel{r} - \cancel{n} + \cancel{1}$$

$$\longrightarrow N > \frac{rn}{i} - 1$$

$\therefore\ N < \frac{rn}{i}$ and $N > \frac{rn}{i} - 1$

$\therefore\ N = \frac{rn}{i}$ or its floor (to be an integer)

This is the MLE of N given r, n, i.
Notice that we are not using any calculus here.