Q4)

a) Let $Y = \min \{X_i\}_{i=1}^{n}$ where

$X_i \sim \text{Bernoulli}(p)$.

$$P(Y=1) = \prod_{i=1}^{n} P(X_i = 1) = p^n$$

$$P(Y=0) = 1 - P(Y=1) = 1 - p^n.$$

So Y is a Bernoulli random variable with parameter p^n.

b) $Y = \min \{X_i\}_{i=1}^{n}$ where $X_i \sim \text{Geometric}(p)$.

Then
$$P(Y \geq y) = \prod_{i=1}^{n} P(X_i \geq y)$$
$$= \left((1-p)^y\right)^n$$
$$= \left[(1-p)^n\right]^y$$

$$\therefore P(Y < y) = 1 - \left[(1-p)^n\right]^y$$

which is the CDF of a geometric r.v. with parameter $1-(1-p)^n$.

Q5 The key is to realise that S is directly proportional to the sample std. dev.

Here is how

arithmetic mean

$$\sum_{i,j}(x_i - x_j)^2 = \sum_i \sum_j (x_i - m + m - x_j)^2$$

$$= \sum_i \sum_j (x_i - m)^2 + (x_j - m)^2 + 2(x_i - m)(x_j - m)$$

$$= n\sum_i (x_i - m)^2 + n\sum_j (x_j - m)^2 + 0$$

as $\sum_i x_i - m = 0$

$$= 2n \sum_i (x_i - m)^2 = 2n(n-1)\,\frac{\sum_i (x_i - m)^2}{n-1}$$

$$= 2n(n-1) \times (\text{std. dev.})^2$$

Thus std deviation

$$= \left(\frac{S}{2n(n-1)}\right)^{1/2}$$