$$\underline{J(y)} = \sum_{i=1}^{N} (y - x_i)^2$$

$$\frac{\partial J(y)}{\partial y} = \sum_{i=1}^{N} \cancel{2}(y - x_i) = 0$$

$$Ny = \sum_{i=1}^{N} x_i \longrightarrow y = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$J_2(y) = \sum_{i=1}^{N} |y - x_i|$$

$$\frac{\partial J_2(y)}{\partial y} = \sum_{i=1}^{N} \underbrace{\mathrm{sign}}_{-1 \text{ or } +1}(y - x_i) = 0$$

$$\frac{d|x|}{dx} = \mathrm{sign}(x) \quad \text{if } x \neq 0$$

1 2 (3 ' 4) 5 6

1 2 3

$x_1, x_2 \cdots\cdots x_N$

$x_{(1)} \le x_{(2)} \le \cdots\cdots \le x_{(N-1)} \le x_{(N)}$

$$SAD(\alpha) = \sum_{i=1}^{N} |x_{(i)} - \alpha| \quad \text{non diff if } \alpha = x_{(i)} \text{ for any } i.$$

$x_{(k)} \le \alpha \le x_{(k+1)} \longrightarrow$ suppose

$$SAD(\alpha) = \sum_{i=1}^{k} (\alpha - x_{(i)}) + \sum_{i=k+1}^{N} (x_{(i)} - \alpha)$$

for simplicity, initially consider $N = 2$

x_1 and x_2 $\qquad x_1 < x_2$

$(x_1 \le \alpha \le x_2)$

$$SAD(\alpha) = \cancel{\alpha} - x_1 + x_2 - \cancel{\alpha} = x_2 - x_1$$

Now consider $\alpha \notin [x_1, x_2]$

$$\alpha < x_1 \longrightarrow SAD(\alpha) = x_1 - \alpha + x_2 - \alpha$$
$$= x_1 + x_2 - 2\alpha$$
$$> x_1 + x_2 - 2x_1$$
$$= x_2 - x_1$$

$$\alpha > x_2 \longrightarrow SAD(\alpha) = \alpha - x_1 + \alpha - x_2$$
$$= 2\alpha - x_1 - x_2$$
$$> 2x_2 - x_1 - x_2$$
$$= x_2 - x_1$$

If $SAD(\alpha)$ is to be minimized then $\alpha \in [x_1, x_2]$

$x_{(1)} \quad x_{(2)} \quad x_{(c)} \quad x_{(n-1)} \quad x_{(n)}$

Nested intervals

$[x(1), x(n)], [x(2), x(n-1)] \ldots [x(i), x(n+1-i)]$

$1 \le i \le c$

If n is even, then $c = n/2$

$\rightarrow [x(n/2), x(n/2+1)]$

If n is odd, then $c = \frac{n+1}{2}$

single number

$\rightarrow \left[x_{\frac{n+1}{2}}, x_{\frac{n+1}{2}}\right]$

We want to minimize SAD for entire array

= sum of SAD values for every interval

We want to consider $\alpha \in \bigcap_{i=1}^{c} [x(i), x(n+1-i)]$

$\therefore$ α must lie in the innermost interval.

If n is even, the innermost interval is

$\alpha \in [x(n/2), x(n/2+1)]$. $\therefore$ α must be the median.

If n is odd, innermost interval is $\left(x_{\frac{n+1}{2}}\right)$

QED