$$\lim_{n\to\infty} \frac{n!}{i!\,(n-i)!}\left(\frac{\lambda}{n}\right)^{i}\left(1-\frac{\lambda}{n}\right)^{n-i}$$

$$= \lim_{n\to\infty} \frac{n(n-1)\cdots(n-i+1)}{i!}\,\frac{\lambda^{i}}{n^{i}}\,\frac{(1-\lambda/n)^{n}}{(1-\lambda/n)^{i}}$$

$$= \frac{\lambda^{i}}{i!}\lim_{n\to\infty}\left(\underbrace{\frac{n(n-1)\cdots\cdots(n-i+1)}{n^{i}}}_{\to 1}\right)\frac{\overbrace{(1-\lambda/n)^{n}}^{e^{-\lambda}}}{\underbrace{(1-\lambda/n)^{i}}_{\to 1}}$$

$$= \frac{e^{-\lambda}\lambda^{i}}{i!} = P(X=i)$$

$$\varphi_X(t) = e^{\lambda(e^t - 1)}$$

$$\varphi'_X(t) = e^{\lambda(e^t - 1)} \lambda e^t$$

$$\varphi''_X(t) = e^{\lambda(e^t-1)} \lambda e^t + (\lambda e^t)^2 e^{\lambda(e^t-1)}$$

$$\varphi''_X(0) = 1\lambda + \lambda^2 \cdot 1 = \lambda^2 + \lambda$$

$$= E(X^2)$$