

~~we 0~~

① Structure

first

③ Distance $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$

② Doubles

EPS

Point

```
struct Point{
    int x, y;
} A;
```

accessing

A.x
A.y

pointer

$A \rightarrow x$

$\bullet A \rightarrow y$

Line:

$$① ax + by + c = 0 \Rightarrow a, b, c$$

$$② \text{Line } \frac{a}{\text{point type}} + \frac{d}{t} \cdot (x, y)$$

$$\frac{a}{\text{point type}} + \frac{d}{t}$$

$$(x, y)$$

from 2 points: Line Segment:

P Q

$$① a = Q.x - P.x$$

$$b = P.y - Q.y$$

$$c = P.x \cdot Q.y - Q.x \cdot P.y$$

struct Line {
Point P, Q;
int a, b, c;}

② $a = \underline{P} - \underline{Q}$ $b = \underline{Q} - \underline{P}$
or $a = \underline{Q} - \underline{P}$ $b = \underline{P} - \underline{Q}$

struct Line {
Point a, d;
};

then \rightarrow Line segment
 \rightarrow ray
 \rightarrow line

$$t = [0, 1]$$

$$t = [0, \infty)$$

$$t = (-\infty, \infty)$$

Line (perpen) passing through b.

$$[b - (a + dt)] \cdot d = 0 \Rightarrow t = \frac{a \cdot d - b \cdot d}{d^2}$$

$$c = \cancel{a} + dt$$

Circle:

(*) x, y, r

(*) Point c;
int r;

$$\text{Line } \therefore b + (c - b)t$$

Line/L.S. intersection:

Type 1:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

some catch:

$$\begin{array}{l} ax + by + c > 0 \\ \text{the line } \therefore ax + by + c = 0 \\ ax + by + c < 0 \end{array}$$

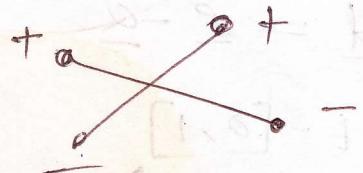
Line 1: (P_1, Q_1)

Line 2: (P_2, Q_2)

$$a_1 = Q_1 \cdot x - P_1 \cdot x \quad b_1 = P_1 \cdot y - Q_1 \cdot y$$

$$c_1 = P_1 \cdot x \times Q_1 \cdot y - Q_1 \cdot x \times P_1 \cdot y$$

$$a_2 = \dots \quad b_2 = \dots \quad c_2 = \dots$$



$$f_{P_2} = a_1(P_2 \cdot x) + b_1(Q_2 \cdot y) + c_1$$

$$f_{Q_2} = a_1(Q_2 \cdot x) + b_1(P_2 \cdot y) + c_1$$

$$f_{P_1} = a_2(P_1 \cdot x) + b_2(Q_1 \cdot y) + c_2$$

$$f_{Q_1} = a_2(Q_1 \cdot x) + b_2(P_1 \cdot y) + c_2$$

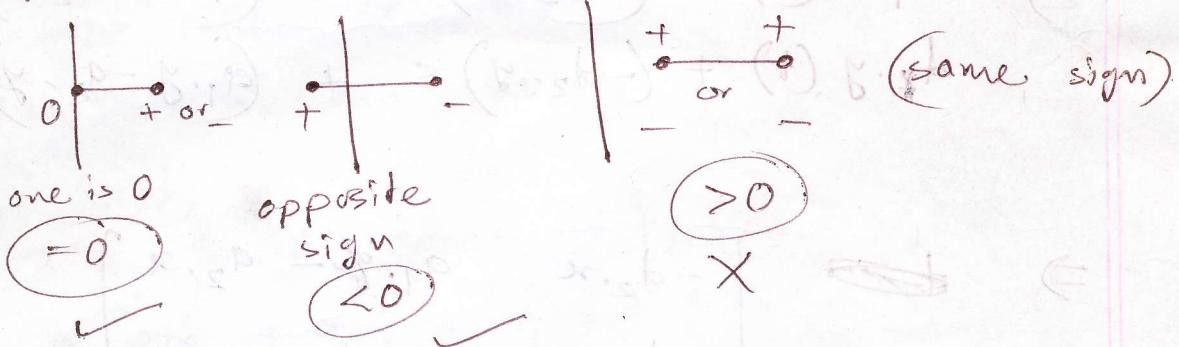
\rightarrow if $P_1 Q_1$ is a L.S. then,

$$f_{P_1} * f_{Q_1} \leq 0$$

if $P_2 Q_2$ is a L.S. then

$$f_{P_2} * f_{Q_2} \leq 0$$

why is so?



if $a_1 * b_2 - a_2 * b_1 = 0$ then {
parallel}

if $a_1 * c_2 - a_2 * c_1 = 0$ then

coincident.

If not parallel, intersection $\Rightarrow I$

$$I.x = \frac{\begin{vmatrix} b_1 & a_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$I.y = \frac{\begin{vmatrix} a_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Type 2:

Line 1: $\underline{a_1} + \underline{d_1(t)}$

Line 2: $\underline{a_2} + \underline{d_2(s)}$

for intersect,

$$\underline{a_1} + \underline{d_1 t} = \underline{a_2} + \underline{d_2 s}$$

$$\Rightarrow \underline{d_1 t} - \underline{d_2 s} + (\underline{a_1} - \underline{a_2}) = 0$$

$$\Rightarrow d_1 \cdot x(t) + (-d_2 \cdot x) s + (a_1 \cdot x - a_2 \cdot x) = 0$$

$$d_1 \cdot y(t) + (-d_2 \cdot y) s + (a_1 \cdot y - a_2 \cdot y) = 0$$

$$\Rightarrow t = \frac{\begin{vmatrix} -d_2 \cdot x & a_1 \cdot x - a_2 \cdot x \\ -d_2 \cdot y & a_1 \cdot y - a_2 \cdot y \end{vmatrix}}{\begin{vmatrix} d_1 \cdot x & -d_2 \cdot x \\ d_1 \cdot y & -d_2 \cdot y \end{vmatrix}}$$

$$s = \frac{\begin{vmatrix} a_1 \cdot x - a_2 \cdot x & d_1 \cdot x \\ a_1 \cdot y - a_2 \cdot y & d_1 \cdot y \end{vmatrix}}{\begin{vmatrix} d_1 \cdot x & -d_2 \cdot x \\ d_1 \cdot y & -d_2 \cdot y \end{vmatrix}}$$

if $\begin{vmatrix} d_1 \cdot x & -d_2 \cdot x \\ d_1 \cdot y & -d_2 \cdot y \end{vmatrix} = 0$ then parallel.

Now, t, s must satisfy the line/ray/ls criteria. Then ans is:

$$\underline{s} = \underline{a}_1 + \underline{d}_1 t$$

Line in 3D:

Type 1 is not possible (planar not)

Type 2 remain same,

for intersect,

3 equ will come like,

$$p_1 t + q_1 s + r_1 = 0$$

$$p_2 t + q_2 s + r_2 = 0$$

$$p_3 t + q_3 s + r_3 = 0$$

for

Step 1:

Solve with those 2 equations which

$$p_1 q_2 \neq p_2 q_1$$

~~then we get t, s wi~~
if all are no 2 are found then, no solution

Step 2:

s, t must satisfy the 3rd equ

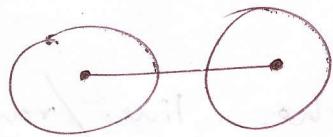
if not, no solt.

Co-linear Test: $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

$$\frac{x_1 - x_2}{y_1 - y_2} = \frac{x_1 - x_3}{y_1 - y_3} \quad \text{or} \quad (x_1 - x_2)(y_1 - y_3) \\ = (x_1 - x_3)(y_1 - y_2)$$

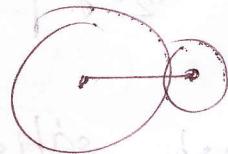
Circle-Circle Thing

5 cases: ~~etc.~~



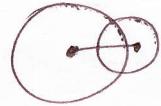
$$d > r_1 + r_2$$

~~(overlap)~~ (outside)



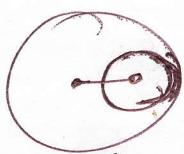
$$d = r_1 + r_2$$

(touch ex)



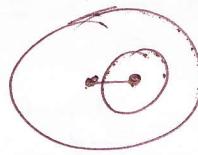
$$|r_1 - r_2| < d < r_1 + r_2$$

(intersect)



$$d = |r_1 - r_2|$$

(touch internally)



$$0 \leq d < |r_1 - r_2|$$

(inside)

Finding angle

" $\frac{\cos^{-1}}{a \cos}$ " is better than

$\begin{matrix} \downarrow \\ (0, \pi) \end{matrix}$

(\sin^{-1})

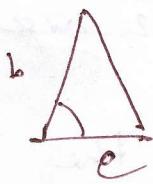
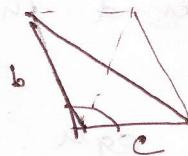
$\begin{matrix} \downarrow \\ (-\frac{\pi}{2}, \frac{\pi}{2}) \end{matrix}$

you know a, b, c ~~how~~ how would you angle

$$\Delta ? \quad \Delta = \sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2}bc \sin C$$

$$\text{so, } C = \arcsin \left(\frac{2\Delta}{bc} \right)$$

but



will have same A
but different A

Δ
(as well as a)

Line Division



$$C = \frac{am(B) + n(A)}{m+n}$$

$$c.x = (m B.x + n A.x) / (m+n)$$

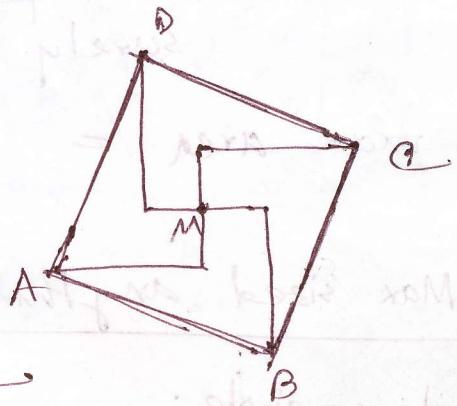
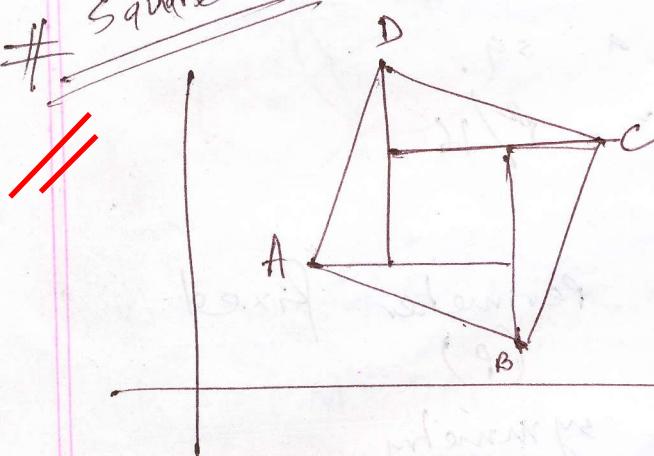
$$c.y = (m B.y + n A.y) / (m+n)$$

so use,

$$A = \cos B \left(\frac{b^2 + c^2 - a^2}{2bc} \right)$$

[cosine rule]

Square



If 2 consecutive points given (A, B)

$$\Delta x = B.x - A.x$$

$$\Delta y = B.y - A.y$$

$$c.x = B.x - \Delta y$$

$$c.y = B.y + \Delta x$$

$$B.x = A.x + \Delta x$$

$$B.y = A.y + \Delta y$$

$$d.x = A.x - \Delta y$$

$$d.y = A.y + \Delta x$$

If 2 opposite points are given (A, C)

$$M.x = (A.x + C.x)/2; \quad M.y = (A.y + C.y)/2$$

~~$$\Delta x = M.x - A.x$$~~

$$\Delta y = M.y - A.y$$

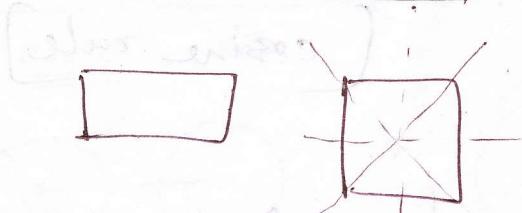
$$B.x = M.x + \Delta y$$

$$B.y = M.y - \Delta x$$

$$d.x = M.x - \Delta y$$

$$d.y = M.y + \Delta x$$

Max-sized Rect? Perimeter fixed.



(P)

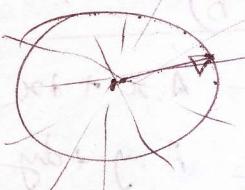
4 way symmetry.

surely \rightarrow a sq.

$$\text{max area} = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$$

Max sized anything? Perimeter fixed
(P)

it is circle: ∞ -way symmetry,



$$2\pi r = P$$
$$\Rightarrow r = \frac{P}{2\pi}$$

$$A = \pi r^2 = \pi \cdot \left(\frac{P}{2\pi}\right)^2 = \frac{P^2}{4\pi}$$

$$\text{Now } \frac{1}{\pi} < 4$$

$$\Rightarrow \frac{1}{4} < \frac{1}{\pi}$$

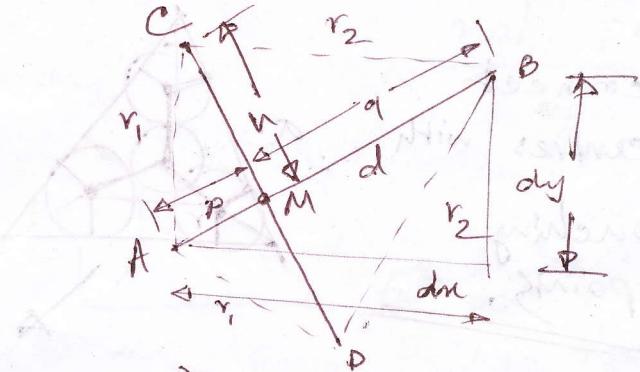
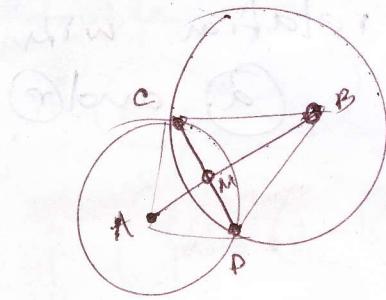
$$\Rightarrow \frac{1}{16} < \frac{1}{4\pi}$$

$$\Rightarrow \frac{P^2}{16} < \frac{P^2}{4\pi}$$

so, clearly circle has greater area.

"More symmetry more optimal"

Cir-Cir Intersection → calc'g:



$$\Delta = \text{area of tri. of } (r_1, r_2, d)$$

$$h = CM = DM = \frac{2\Delta}{d}$$

$$p = AM = \sqrt{r_1^2 - h^2}$$

$$q = BM = \sqrt{r_2^2 - h^2}$$

$$M.x = \frac{p(B.x) + q(A.x)}{p+q}$$

$$M.y = \frac{p(B.y) + q(A.y)}{p+q}$$

$$dx = B.x - A.x$$

$$dy = B.y - A.y$$

$$C.x = M.x - \cancel{dy} \cdot h/d$$

$$C.y = M.y + \cancel{dx} \cdot h/d$$

$$D.x = M.x + dy \cdot h/d$$

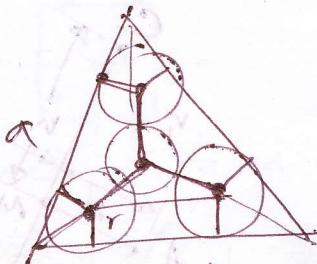
$$D.y = M.y - \cancel{dx} \cdot h/d$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}$$

A circle Related Prob:

connect
centres with
touching
points.



relation with

d and r

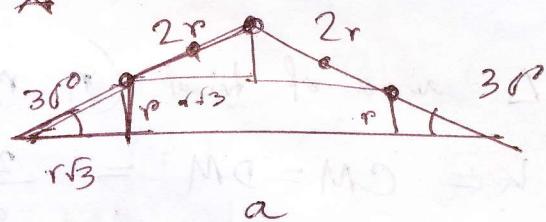
area of sector

$$\frac{1}{2} \alpha r^2$$

area of

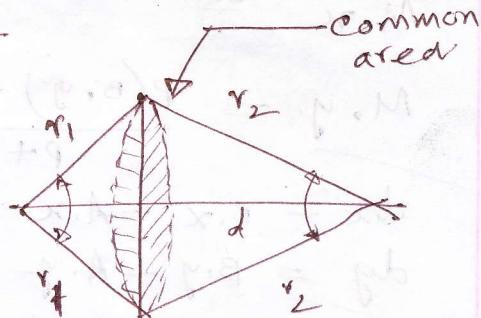
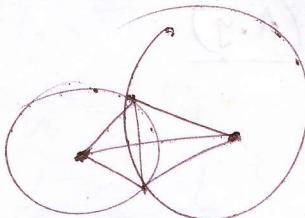
$$\frac{1}{2} \alpha r^2 = \frac{1}{2} r^2 \sin \alpha$$

$$= \frac{1}{2} r^2 (\alpha - \sin \alpha)$$



$$a = 4r\sqrt{3}$$

Overlapped area of Circles:

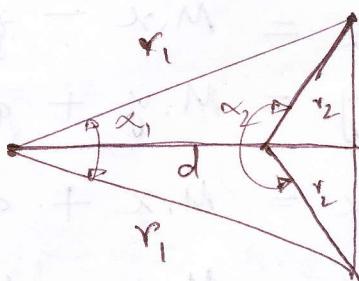


$$\alpha_1 = 2 \arccos \left(\frac{(r_1^2 + d^2 - r_2^2)}{2r_1 d} \right)$$

$$\alpha_2 = 2 \arccos \left(\frac{(r_2^2 + d^2 - r_1^2)}{2r_2 d} \right)$$

common area,

$$\frac{1}{2} r_1^2 (\alpha_1 - \sin \alpha_1) + \frac{1}{2} r_2^2 (\alpha_2 - \sin \alpha_2)$$



we must use

\arccos
because $> 180^\circ$ is
possible.

$[x, y] \rightarrow m$

$[x, y] \rightarrow b$

How to handle doubles? EPS?

double a, b

if $(a \neq b)$ wrong.

if $(\text{fabs}(a - b) < \text{EPS})$

doubles are
not precise

$1e^{-8} \sim 1e^{-12}$

(not) reasonable.

- ① Firstly equality check depends on problem sometimes.
- ② then less/greater check.

Rounding:

say, to 3 digits:

double round(double x) {

return floor($(x * 1000 + 0.5) / 1000$);

for rounding.

or truncate

trunc

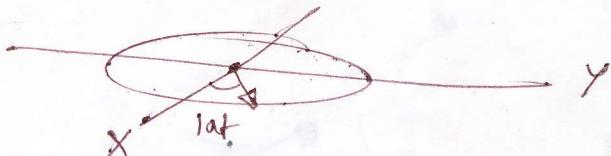
$$\boxed{0.999 \rightarrow 1}$$

Ray shooting notes

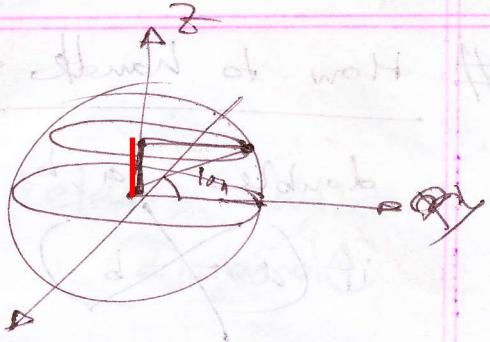


$$\text{lon} \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$$

$$\text{lat} \rightarrow [-\pi, \pi]$$



Geodesic Distance



$$x = R \cos(\text{lat}) \cos(\text{lon})$$

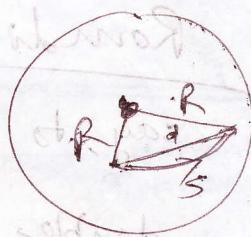
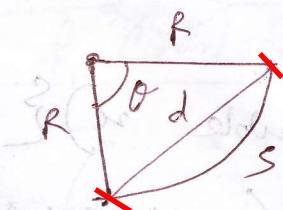
$$y = R \sin(\text{lat}) \cos(\text{lon})$$

$$z = R \sin(\text{lon})$$

$$\text{Point } A(\text{lat}_1, \text{lon}_1) \rightarrow A(x_1, y_1, z_1)$$

$$\text{Point } B(\text{lat}_2, \text{lon}_2) \rightarrow B(x_2, y_2, z_2)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



$$\cos \theta = \frac{\sqrt{R^2 + R^2 - d^2}}{2R \cdot R} = \frac{\sqrt{2R^2 - d^2}}{2R^2} = \frac{1 - d^2/2R^2}{2R^2}$$

$$\Rightarrow \theta = \arccos(1 - d^2/2R^2)$$

$$[s = R * \theta]$$

Polygons

area:

```

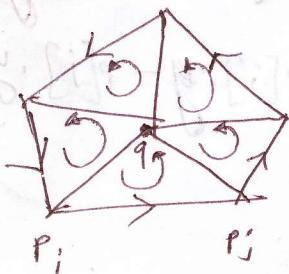
for (i = 0; i < n; i++) {
    j = (i + 1) % n
    A += p[i].x * p[j].y - p[j].x * p[i].y
}
  
```

$$\left| \begin{array}{l} \\ A \end{array} \right| = 2.$$

cw / ccw ?

- * $A > 0$ ccw
- * $A < 0$ ~~ccw~~
- * $A = 0$??

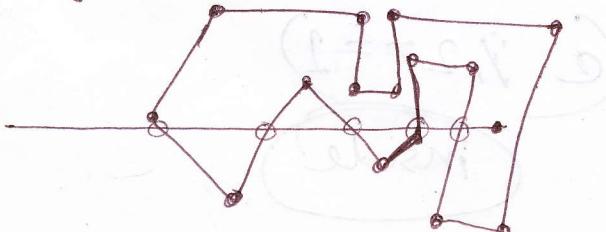
Point in Poly? (Convex)



so, for all i [and $j = (i + 1) \% n$]
 q, p_i, p_j will form a
 ccw triangle if q lies
 inside Poly.

Point in Poly (Any)

Ray shooting method



~~z~~ zfr

fig 3.25.25.25

~~z~~ zfr = zfr CRT 25.25.25

inside

$$x_1 = \frac{q.y - p_i.x}{p_i.x - p_j.x} = \frac{q.y - p_i.y}{p_i.y - p_j.y}$$

① check if they are it is online
with any ~~two~~ → decide accordingly
to problem.

② $c = 0;$

for ($i = 0; i < n; i++$) {

$$j = (i+1) \% n;$$

if $((p[i].y \leq q.y \text{ and } q.y < p[j].y) \text{ or }$

$(p[j].y \leq q.y \text{ and } q.y < p[i].y)) \{$

$$ct = p[i].x + p[j].x$$

$$x_1 = p[i].x + (p[i].x - p[j].x)$$

$$\times (q.y - p[i].y)$$

$$/(p[i].y - p[j].y)$$

if $(x_1 < q.x)$

$ct++;$

}

}

return c if $(c \geq 2)$

inside

