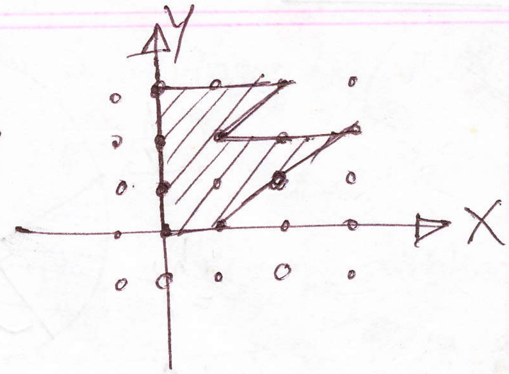
Geometry

21-06-07

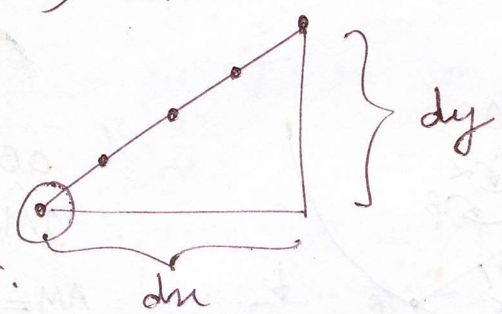
10888

Picks Theorem

points have int co-ordinates.



$$I = A - B/2 + 1$$
$$= (2A - B)/2 + 1$$



~~g = gcd(dx, dy)~~

এটি শুরু next Edge এর.

3 cases:

1) $dx > 0$, $dy > 0$
   (border points + 1 end point) $= \gcd(dx, dy)$

2) $dx > 0$, $dy = 0$
   $B = dx$

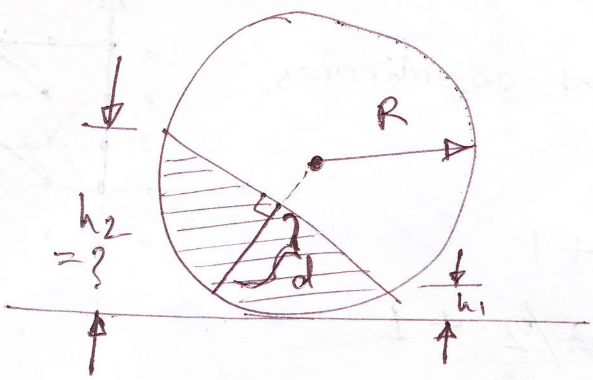
3) $dy > 0$, $dx = 0$
   $B = dy$

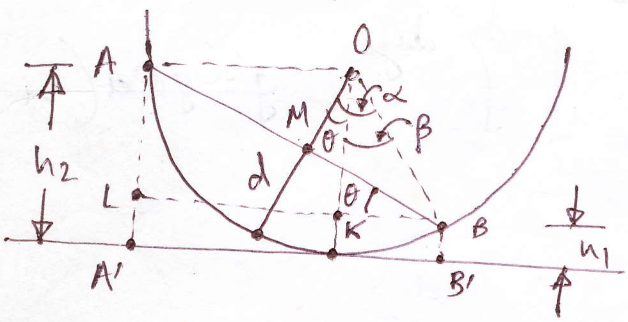
$$\boxed{\begin{array}{l} 4)\ dx = 0,\ dy = 0 \\ B = 0 \end{array}}$$

(দরকার নাই)

Later → Sp. Picks theorem,
11017 এ লাগবে
অন্য রকম।

10792 Laurel-Hardy





$OB = OA = R$

$OM = R - d$

$AM = BM = \sqrt{OA^2 - OM^2}$

$= \sqrt{R^2 - (R-d)^2}$

$= \sqrt{2Rd - d^2}$

in ΔALB, $\sin\theta = \dfrac{AL}{AB} = \dfrac{h_2 - h_1}{2\,AM} = \dfrac{h_2 - h_1}{2\sqrt{2Rd - d^2}}$

$\Rightarrow h_2 = h_1 + 2\sqrt{2Rd - d^2}\cdot\sin\theta$

---

from,

ΔOMB, $\cos\alpha = \dfrac{OM}{OB} = \dfrac{R-d}{R} \Rightarrow \alpha = \cos^{-1}\left(\dfrac{R-d}{R}\right)$

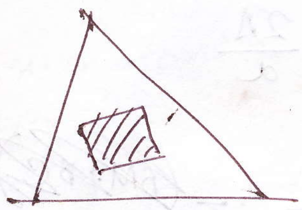
ΔOKB, $\cos\beta = \dfrac{OK}{OB} = \dfrac{R-h_1}{R} \Rightarrow \beta = \cos^{-1}\left(\dfrac{R-h_1}{R}\right)$

$\theta = \alpha - \beta = \ldots$

$h_2 = h_1 + \ldots$

~~10725~~ 10725 tri-sq.

Triangular square is within a triangle. (NOT ~~strict~~ necessarily strictly inside)



what is the max possible ~~for~~ ~~area of~~ area of t-s for a give Δ?

Observation:

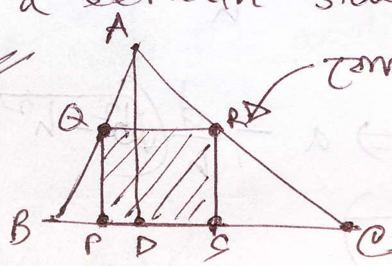
sq এর এক দিক Tri এর এক দিকে থাকবে।

for a certain side.

case I

$B < 90^\circ$

$C \leq 90^\circ$



কোনার দুটো কোণে থাকলে optimal

$BC = a,\ CA = b,\ AB = c$

$PQ = QR = RS = SP = d.$

$AD = h = \frac{2\Delta}{a}$

$$BD = \sqrt{AB^2 - AD^2} = \sqrt{c^2 - h^2}$$

$$\frac{BP}{BD} = \frac{PQ}{AD} \Rightarrow BP = \frac{BD \times PQ}{AD} = \frac{\sqrt{c^2-h^2}\cdot d}{h} = \frac{d}{h}\sqrt{c^2-h^2}$$

$$PD = BD - BP = \sqrt{c^2-h^2} - \frac{d}{h}\sqrt{c^2-h^2} = \sqrt{c^2-h^2}\cdot\left(1 - \frac{d}{h}\right)$$

similarly,

$$SD = \sqrt{b^2-h^2}\left(1 - \frac{d}{h}\right)$$

now,

$$PD + SD = PS = d.$$

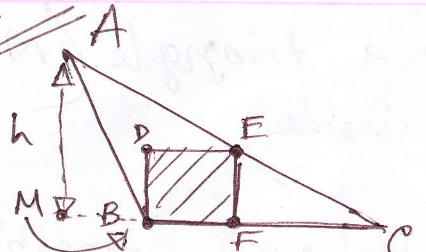
$$\Rightarrow \sqrt{c^2-h^2}\left(1-\frac{d}{h}\right) + \sqrt{b^2-h^2}\left(1-\frac{d}{h}\right) = d$$

$$\Rightarrow \sqrt{c^2-h^2} + \sqrt{b^2-h^2} = \frac{d}{h}\left(h + \sqrt{b^2-h^2} + \sqrt{c^2-h^2}\right)$$

$$\Rightarrow d = \frac{a * h}{h + \sqrt{b^2-h^2} + \sqrt{c^2-h^2}}$$

$\sqrt{b^2-h^2} + \sqrt{c^2-h^2} = a$

case II



$B > 90^\circ$

$C \le 90^\circ$

যেকোনো corner এ set

$BC = a,\ CA = b,\ AB = c$

$BD = DE = EF = BF = d$

$AM = h = \frac{2\Delta}{a}$

$$\frac{CF}{CM} = \frac{EF}{AM} \Rightarrow CF = \frac{CM * EF}{AM} = \frac{(\cancel{BM + BC}) * d}{h}$$

$$\Rightarrow CF = \frac{\left(\sqrt{AC^2 - AM^2} + \cancel{a}\right) * d}{h} = \frac{\left(\sqrt{b^2 - h^2} - \cancel{a}\right) d}{h}$$

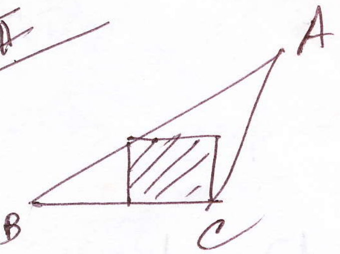
now,

$$BF = d \Rightarrow BC - CF = d \Rightarrow a - \frac{d}{h}\left(\sqrt{b^2 - h^2}\ \cancel{a}\right) = d$$

$$\Rightarrow a\cancel{b} = \frac{d}{h}\left(h + \cancel{a}\sqrt{b^2 - h^2}\right)$$

$$\Rightarrow d = \frac{ah}{h + \sqrt{b^2 - h^2}}$$

case III



$B \le 90^\circ$

$C > 90^\circ$

একই রকম.

$$d = \frac{ah}{h + \sqrt{c^2 - h^2}}$$

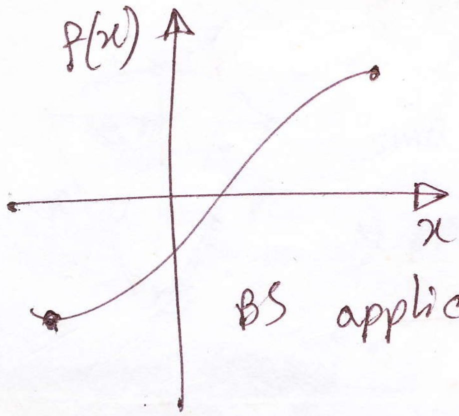
প্রতি বাহুর জন্য 1 রকম d আসবে।

পরপর d_a, d_b, d_c এর Max টা print করতে হবে

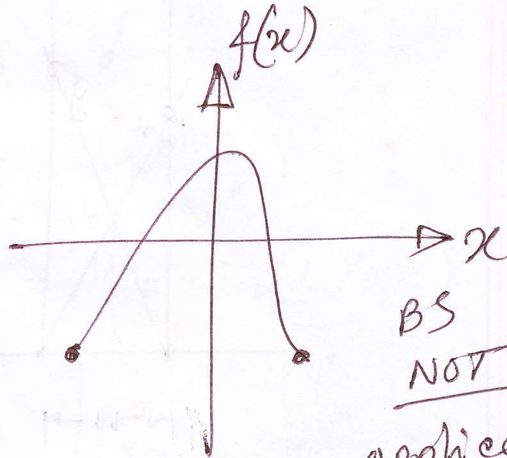
Binary Search

Bisection Method

monotonic response.



BS applicable.



BS **NOT** applicable

why BS?

solve করা ~~কর~~ যায় না / ঝামেলার.

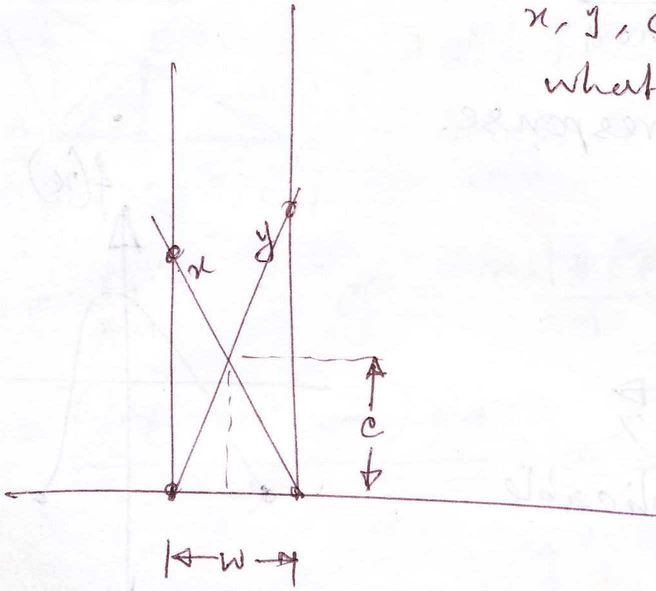
say,

$e^x + x = 3$, or $\sin x + 2x = 3$, etc.

*

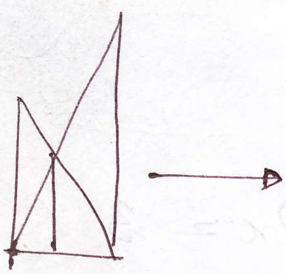
$\boxed{10566}$

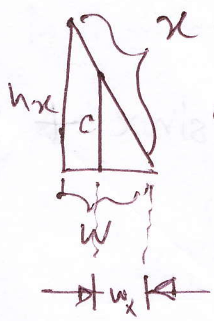
$x, y, c > 0$ and given what is w?



BS on w, $lo = 0$, $hi = \min(x, y)$

when $c = hi$ / when $c = 0$





and

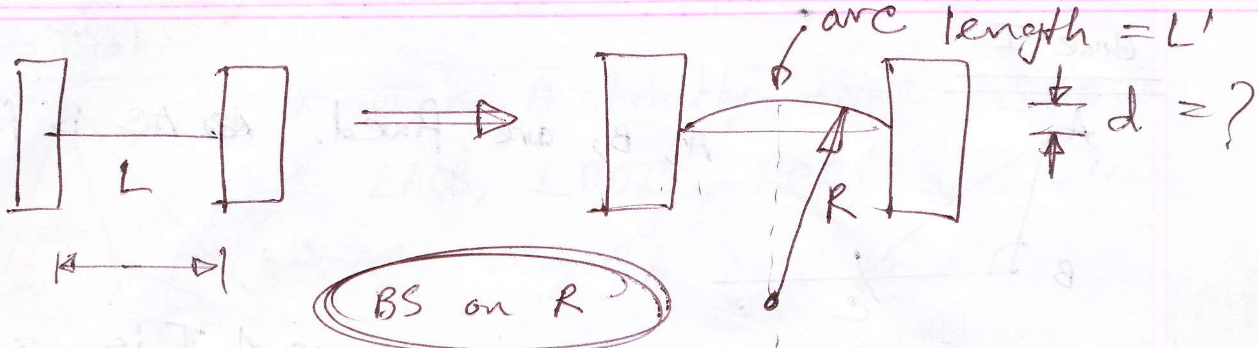
$h_x = \sqrt{x^2 - w^2}$ $\qquad$ $h_y = \sqrt{y^2 - w^2}$

$w_x = \dfrac{c \cdot w}{h_x}$ $\qquad$ $w_y = \dfrac{c \cdot w}{h_y}$

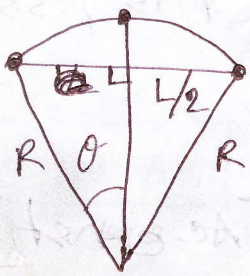
$\boxed{\text{BS Logic}}$

$w = w_x + w_y$

✓ 10668



BS on R



$\sin\theta = \frac{L/2}{R} \Rightarrow \theta = \sin^{-1}\left(\frac{L}{2R}\right)$

$L_{BS} = R(2\theta) = 2R\sin^{-1}(L/2R)$

$\boxed{\text{BS Logic}}$

$lo = L/2$

$hi = \infty$

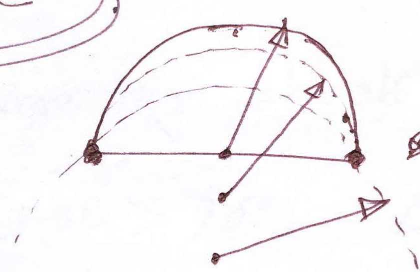
$L_{BS} = L'$

$L_{BS} > L'$
$R = lo$

$L_{BS} < L'$
$R = hi$

Logic

দুইটি Point এ সবচেয়ে বড় arc
আঁকা যাবে যখন rad = length/2



Points are fixed.
if $R{+}{+} \Rightarrow L{-}{-}$

—— o ——

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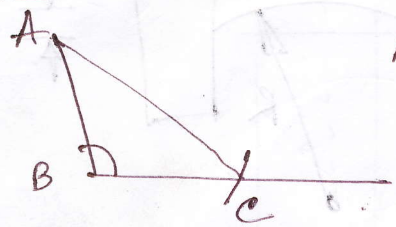
10695

Case-I



A, B, are fixed. ~~AB~~ AC is fixed.

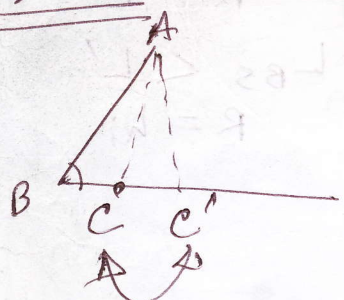
if $\angle ABC \geq 90°$ then only 1 T is possible.

AND:

1) AC++ $\Rightarrow$ BC++ [AB const]
2) AB++ ~~AC~~ $\Rightarrow$ BC-- [AC const]

SO, BS applicable.

Case-II



Ambiguity.

$\angle ABC < 90°$ then

i) 2 T are possible when $AC < AB$

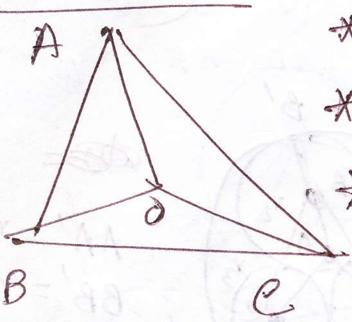
ii) 1 T is possible when $AC \geq AB$

so, BS is not possible in this case.

যাবে 20

< কিছু checking বাদ দিলাম >

The Prob:



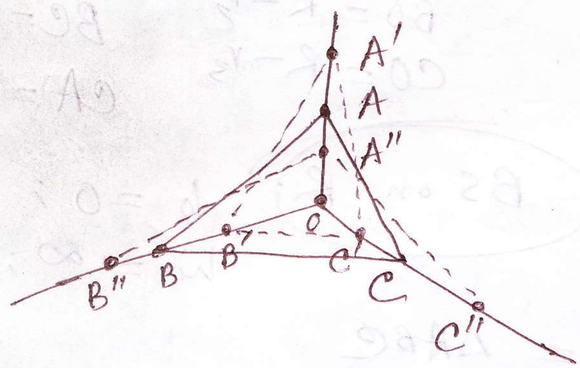
* O is strictly inside ΔABC
* $\angle AOB$, $\angle BOC$, $\angle COA$ are given
* AB, BC, CA are given

find AO, BO, CO

$\boxed{\text{BS on AO:}}$

BS part

$lo = 0$ $hi = \min(AB, AC)$ ~~min~~



$OA{+}{+} \Rightarrow BC_{BS}{-}{-}$

$OA{-}{-} \Rightarrow BC_{BS}{+}{+}$

$\boxed{\text{Logic}}$

$BC_{BS} = BC$

1) place AO
2) cut AB, AC from OB, OC
3) get BC and compare it with original BC.

আরও কিছু check লাগবে। সেটা কি??

$\angle AOB > 90^\circ$, $\angle AOC > 90^\circ$ ✓

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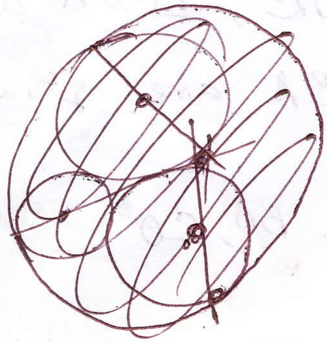
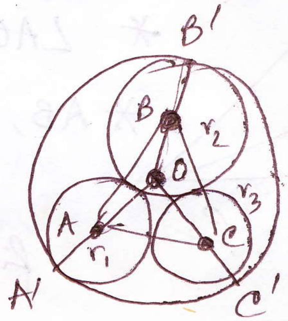
10322 <SubProblem>

Circum Circle of 3 touching circles (r_1, r_2, r_3) (R)



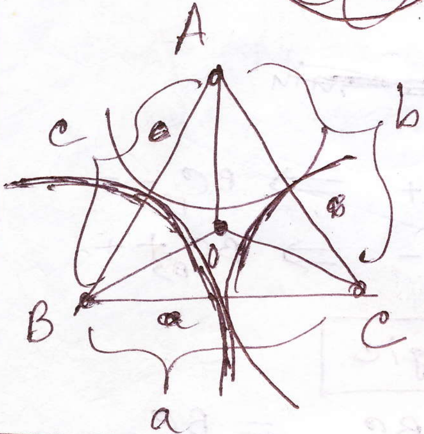


$AA' = r_1$

$BB' = r_2$

$CC' = r_3$

$\boxed{A'O = B'O = C'O = R}$



$AO = R - r_1$

$BO = R - r_2$

$CO = R - r_3$

$AB = r_1 + r_2 = c$

$BC = r_2 + r_3 = a$

$CA = r_3 + r_1 = b$

BS on R

$lo = 0;$

$hi = \infty;$

Logic

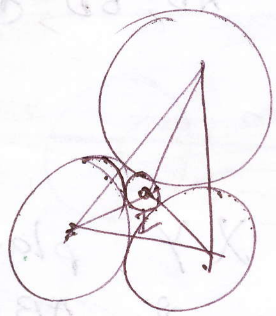
$\Delta AOB + \Delta BOC + \Delta COA = \Delta ABC$

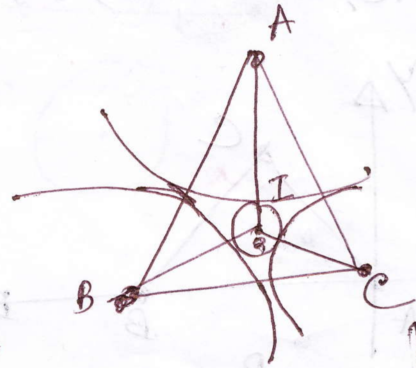
$\Rightarrow \Delta(r_1+r_2, R-r_1, R-r_2) + \Delta(r_2+r_3, R-r_2, R-r_3) + \Delta(r_3+r_1, R-r_3, R-r_1)$ (LHS) $= \Delta(r_1+r_2, r_2+r_3, r_3+r_1)$ (RHS)

if any invalid T or LHS < RHS,
R ~~বাড়াতে~~ বাড়াতে হবে। so, $lo = R;$

else, $hi = R;$ (কমাতে হবে)

Incircle of 3 touching Circles (r_1, r_2, r_3)





$AI = r_1 + sr$

$BI = r_2 + sr$

$CI = r_3 + sr$

$\boxed{\text{BS on } Sr}$

$lo = 0$

$hi = \min(r_1, r_2, r_3)$

similar logic,

$\Delta AIB + \Delta BIC + \Delta CIA = \Delta ABC$, 2D

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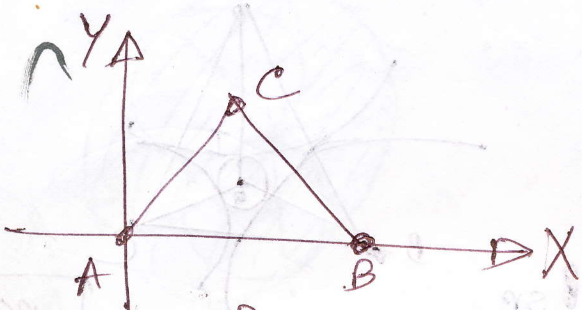
Vol^m of Pyramid

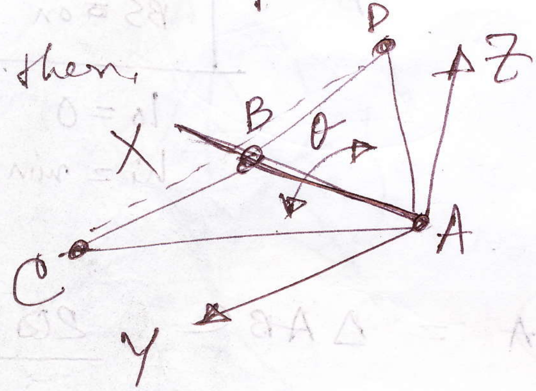
ABCD Pyr.

~~AB, AC, AD~~ AB, BC, CA , AD, BD, CD



XY plane
& ABC plot,

then,



$\boxed{\text{BS on } \theta}$

lo = 0
hi = π [in radian]

get co-ordinate of D,
then calc. CD.
compare,
CD_{BS} vs. CD.

$\theta{++} \longrightarrow CD_{BS}{++}$.

then, CD_{BS} = CD 2(at),

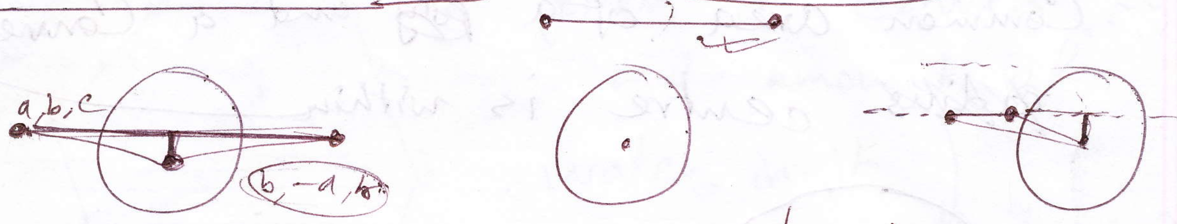
Ans = $\frac{1}{3}\Delta(ABC) * (D.z)$

struct Point {
~~int~~ double x,y,z;
}

$\boxed{\frac{1}{3} \times \text{Base} \times \text{height}}$

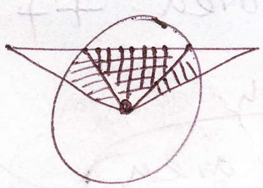
Convex Poly ~~Vs~~ And. Circle

Circle Vs. Line Seg. Intersection

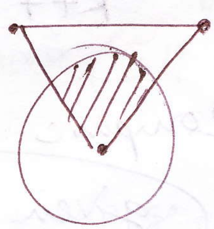


$$\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$$

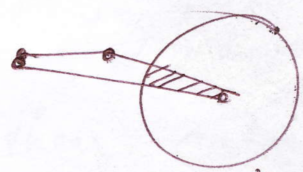
Circle Vs. Triangle (one point is centre) (Common Area) with the Line



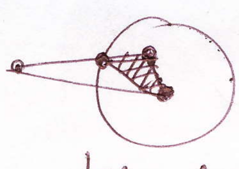
pie slice
+
triangle
+
pie slice



pie slice



pie slice



triangle

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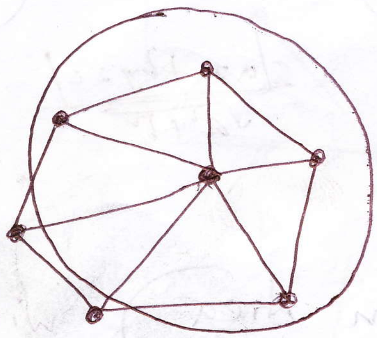
11177

Prob:

Common area of a ~~Poly~~ circle and a Convex Poly.

~~radius~~ centre is within



divide into subprob. prev.

BS on R

R++ , common area ++

compare with

given c, area

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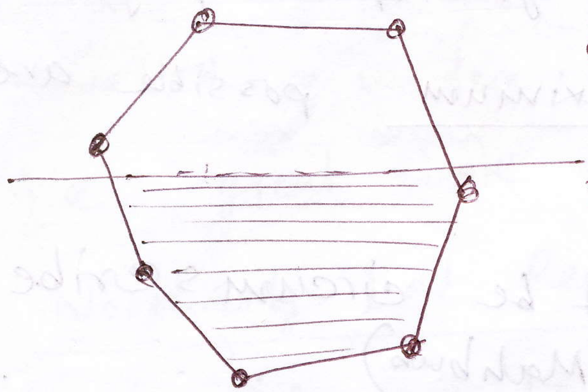
Water Level



given a convex poly, and amount of water in it.

$A \le A(\text{poly})$

determine water level.

BS on level.

lo = min (all y of poly)

hi = max (all y of poly.

Polygon intersect. then calc. its area.

compare with $\underline{A}$

***

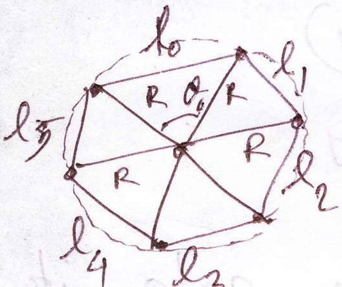
Prob → ZJU ? Number ?

* given a set of lengths of a poly. what is the maximum possible area?

soln

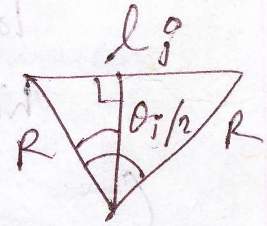
the poly must be circumscribe. (Proof by Mahbub)

If so then, order doesn't matter



$\boxed{\text{BS on } R}$

calc,



$$\cancel{R}\ \frac{l_i/2}{R} = \cancel{\tan}\sin\frac{\theta_i}{2}$$

$$\Rightarrow \theta_i = 2\sin^{-1}\left(\frac{l_i}{2R}\right)$$

$\boxed{\text{Logic}}$

$$\sum \theta_i = 2\pi$$

if so, then break,

~~calc~~

ans = $\sum$ tri_area$_i$

R++, → $\sum\theta < 2\pi$

R--, → $\sum\theta > 2\pi$

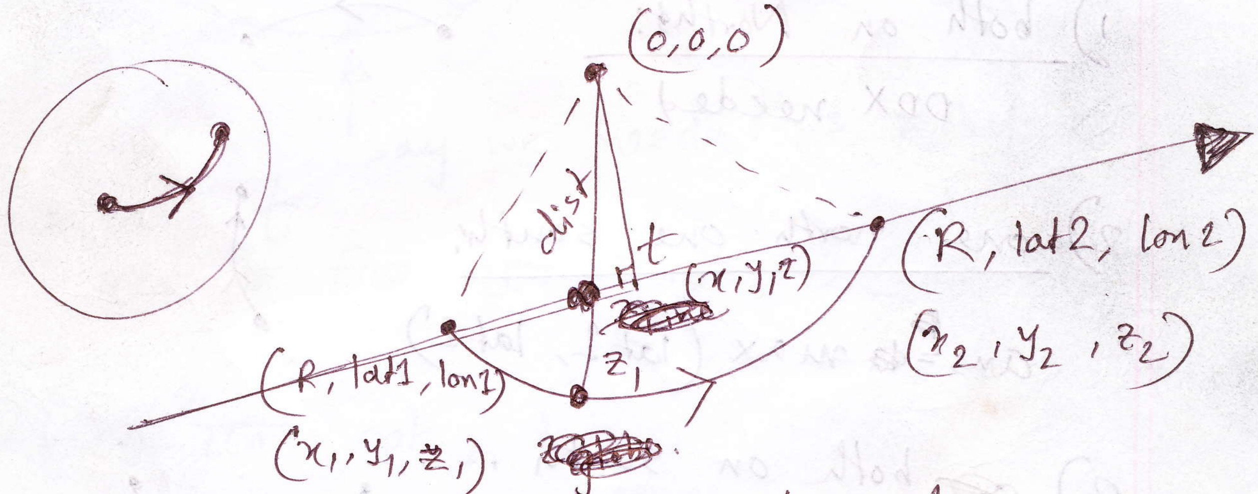
*** *

10809 — Great Circle.

Given 2 Points (lat, long), and they travelled under ~~paths,~~ shortest path, i.e. great circle distance, what is the

Northerly Lat Reached.

i.e. Z is maximized.



parameter of the line is: t

$$x = x_1 + (x_2 - x_1)t$$

$$y = y_1 + (y_2 - y_1)t$$

$$z = z_1 + (z_2 - z_1)t$$

now,

$$\frac{z_1}{z} = \frac{R}{dist} = \frac{R}{\sqrt{x^2+y^2+z^2}} = \frac{R}{f(t)}$$

$$\Rightarrow z_1 = \frac{R.z}{f(t)} = \frac{R \cdot g(t)}{f(t)} = F(t)$$

now,

Z_1 becomes function of t.

diff. t to get maximum vale.

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if the path points are on,

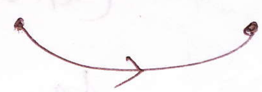
1) both on North:

DDX needed

2) one north one south:

ans = max (lat 1, lat 2)

3) ~~one~~ both on south.

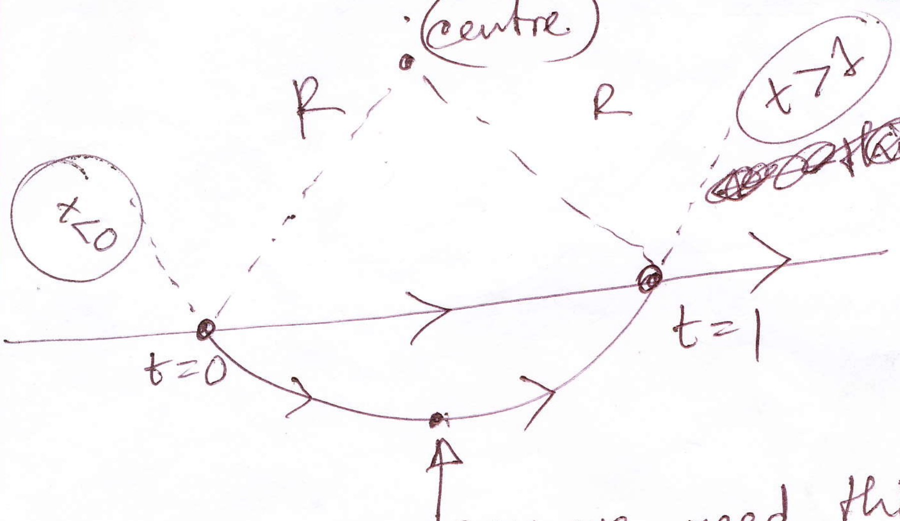


ans = max (lat 1, lat 2)

4) if one is lat = 90N

Undefined.

Advantage on taking the parameter



~~this point are not optimal at all~~

say we need this point

এখন t নিতে হবে মধ্যে $0 \le t \le 1$

*) $t<0$ হলে opt, $t=0$

*) $t>1$ হলে opt, $t=1$