**If you understand these solutions you should try the rest:-**

**N:B:- The solutions of the problems 13-15 are really tricky. So if you are very comfortable in solving rest of the problems only then I would suggest you to look at them.**

1. **How many different ways 5 people can stand in a line?[Level-1]**

5!.

1. **How many different ways 5 people can sit in a round table? [Level-2]**

4!. (Rotation means the same arrangement)

1. **How many different ways a necklace with 5 beads can be made? [Level-2]**

4!/2. (Here we can also make reflection making 2 different arrangements same).

1. **7. Suppose you want to arrange 7 people, A, B, C, D, E, F, and G in seats at a movie theater. But A, B, and C have been best friends since first grade and insist on sitting together (although not necessarily in the order ABC). How many ways can they be seated? [Level-3]**

The trick is to first think of the group ABC as a single person, and figure out how many arrangements are possible under that assumption. If you think of the group ABC as a single person, then instead of arranging 7 people in order, you're now arranging 5 people in order, and the number of ways to do that is 5! or 120.

Except, that number doesn't take into account the number of different ways that the ABC-group can arrange themselves within their own group. For each arrangement in our original list, such as:

G F D [ABC-group] E

there are actually 6 possible arrangements:

G F D **A B C** E

G F D **A C B** E

G F D **B A C** E

G F D **B C A** E

G F D **C A B** E

G F D **C B A** E

because the number of ways that the 3-member group ABC can be arranged, of course, is 3!, or 6. So to get the true number of possible arrangements, we have to take our intermediate answer of 5! and multiply by 3! , and the answer is 5! \* 3! = 120 x 6 = 720.

**11. Same Problem(Problem-9), subject to the rules:**

**i)A, B, and C are sitting next to each other**

**ii)D is sitting next to the ABC-group [Level-3]**

Now we have 4 "objects" to sort: ABC-D-group, E, F, and G, so they can be arranged in 4! ways. For each of those arrangements, within the ABC-D-group, the two sub-groups -- the ABC-group and the element D -- can be ordered in 2! ways. And finally, within the ABC-group itself, its members can be ordered in 3! ways. So the number of arrangements such that:

A, B, and C are sitting next to each other

D is sitting next to the ABC-group

is 4! x 3! x 2 = 288.

**12. How many ways 16 distinct balls can be grouped in 5 distinct baskets? [Level-3]**

**516 .** For every ball there are 5 boxes. So we can put every ball to any of the 5 boxes independently to each other. There are 5 different possibilities for every ball. So for the first ball we have 5 ways, for the second ball we have another 5 ways(5\*5 in total), for the third we have another 5(5\*5\*5 in total) and so on. So from the multiplication principle we can deduce for 16 distinct ball the number of ways is **516 .**

**13. How many ways 16 identical balls can be grouped in 5 distinct baskets?**

**[Level-5]**

Before solving this problem let us look at the following equation

x1+x2+x3+…………+xr=n.

Now the problem is how many non-negative integer solutions does this equation have? More precisely it says find the number of solutions where x1>=0, x2>=0, x3>=0,……..,xr>=0 and all of them are integer numbers.

This is a generalized version of the problem-16(where r=5 and n=18).

The thing is this problem has a very interesting and outstanding formula. Before proving this formula we should first concentrate of solving something smaller(This is a common approach for solving combinatorics problems). Let us make an equation.

x1+x2+x3=6.[r=3 and n=6]

We do some manual solutions.

x1 x2 x3 Binaryrepresentation

0 0 6 00111111

0 1 5 01011111

0 2 4 01101111

0 3 3 01110111

………………………………………………………………...

2 3 1 11011101

2 4 0 11011110

………………………………………………………………..

3 2 1 11101101

………………………………………………………………..

6 0 0 11111100

The real confusion now is what is Binaryrepresentation(this is the core of the solution) column doing here. If you analyze carefully here consecutive **1’s** means the value of a variable(x1,x2,x3) and 0 means a ‘+’ or it means we all now go to next variable(if possible) and assign it the consecutive **1’s**.

For example the solution(2,4,0) is represented by 11011110 . The first 2 consecutive 1’s means x1=2, then we find a 0. So the next 4 consecutive **1’s** means x2=4. Then we find the next 0 but there is no 1 after this. So x3=0.

Now you see the length of all the binary numbers is 8 and every number has 2 **0’s** and 6 **1’s.** The main thing is every valid solutions can be expressed by unique binary representation of length 8(2 **0’s** and 6 **1’s**) and all such binary representation also represent different solutions. So we can make a new equivalent problem for the equation here. The problem is

**What is the number of different binary numbers of length 8(2 0’s and 6 1’s)?** [tricky isn’t it]

**The answer is 8C2/8C6.**

So we now recall the main generalized equation. So the length of the binary number for generalized equation is **n+r-1**(**n 1’s** and **(r-1)** number of 0’**s**).

So the new equivalent problem for the equation is

**What is the number of different binary numbers of length n+r-1((r-1) 0’s and n 1’s)?**

**The answer is n+r-1Cr-1/ n+r-1Cn.**

We have come thus far. Now we concentrate on the main problem. Here all distinct baskets mean distinct variables. As the balls are identical the number of balls in first basket is denoted by x1, number of balls in second basket is x2, and so on. So the equivalent equation for this problem is

x1+x2+x3+x4+x5=16.[n=16 and r=5]

The answer is **16+5-1C5-1.**

**14. How many ways 16 distinct balls can be grouped in 5 identical baskets?**

**[Level-5]**

Before solving this problem we should have some idea about **Stirling number of the second kind(**btw there are 2 kinds of **Stirling Number).**

Stirling number of the second kind is the number of ways we can partition a set of **n(distinct)** objects with **k** non-empty subsets(subsets can be viewed as identical). That means we have to group these **n** object and make exactly **k** non-empty groups. There is a very beautiful!!! formula for the Stirling numbers.

(**n** objects **k** groups)=S*2*(**n,k**) [S2 means starling number of 2nd kind]

= **(1/k!) *\** (** *j=0***∑***j=k***(-1)***k-j* ***\**** *k***C***j* ***\** j***n***).** [summation of 0 to k]

N:B:- For more information visit <http://en.wikipedia.org/wiki/Stirling_numbers_of_the_second_kind> ]

Now we come back to the problem. The problem is grouping 16 distinct balls into 5 identical boxes(or equivalently groups). So we should use Stirling number. But there is a problem. Here the boxes can be empty. So what is the solution (as we can only use Stirling number for only non-empty boxes or groups)?

The solution is rather easy. If we carefully observe we can have only 1,2,3,4 or 5 non-empty groups for different scenarios. So we can make upto 5 non-empty groups. So the answer is now solving the problems with each number of non-empty groups.

i.e.

Ways of grouping 16 balls into 1 non-empty box(groups)= S*2*(**n,1**).

Ways of grouping 16 balls into 2 non-empty box(groups)= S*2*(**n,2**).

Ways of grouping 16 balls into 3 non-empty box(groups)= S*2*(**n,3**).

Ways of grouping 16 balls into 4 non-empty box(groups)= S*2*(**n,4**).

Ways of grouping 16 balls into 5 non-empty box(groups)= S*2*(**n,5**).

So the total ans = S*2*(**n,1**)+ S*2*(**n,2**)+ S*2*(**n,3**)+ S*2*(**n,4**) +S*2*(**n,5**).

Do the calculation yourself. :P

N:B: You can use recursion to solve this problem with some extra time and memory complexity(ha ha).

**15. How many ways 16 identical balls can be grouped in 5 identical basket.?**

I am answering it later.