Vectors and Operations on Vectors

1. What is a Vector?

A vector is an ordered list of numbers that can represent different things, depending on the context. Each number in the vector is called a 'component'. A vector can be thought of as an arrow that has direction and magnitude.

Real-Life Example

In real life, if you're going on a hike:
- If you walk 5 miles east, that could be represented as a vector: v = [5] (since it's in one direction).
- If you walk 5 miles east and 3 miles north, your journey can be represented as a 2D vector: v = [5, 3].

2. Operations on Vectors

2.1 Vector Addition

When you add two vectors, you simply add the corresponding components together.

Formula:
If v1 = [x1, y1] and v2 = [x2, y2], then:
v1 + v2 = [x1 + x2, y1 + y2]

Real-Life Example:
Drone 1 flies 4 miles east and 3 miles north: v1 = [4, 3]
Drone 2 flies 2 miles east and 1 mile north: v2 = [2, 1]
The combined movement would be v_total = [6, 4] (6 miles east and 4 miles north).

2.2 Scalar Multiplication

This is where you multiply a vector by a scalar (a single number).

Formula:
If v = [x, y] and k is a scalar, then:
k * v = [k * x, k * y]

Real-Life Example:
If you are running at 2 miles per hour (east), represented as v = [2], and you triple your speed, the new vector is:
3 * v = [6] (running 6 miles per hour east).

2.3 Magnitude of a Vector

The magnitude (or length) of a vector measures how far the vector reaches.

Formula:
For a 2D vector v = [x, y], the magnitude is:
|v| = sqrt(x^2 + y^2)

Real-Life Example:
Walking 3 miles east and 4 miles north gives a total straight-line distance of 5 miles.

2.4 Dot Product

The dot product gives a way to multiply two vectors and get a scalar result, showing how much one vector acts in the direction of another.

Formula:
If v1 = [x1, y1] and v2 = [x2, y2], then:
v1 . v2 = x1 * x2 + y1 * y2

Real-Life Example:
If you push a box at an angle, the dot product helps quantify how much of your push is moving the box in its desired direction.

3. Example with Python

Here's how you can perform basic vector operations using Python and the numpy library.

```
import numpy as np

# Defining vectors
v1 = np.array([4, 3])
v2 = np.array([2, 1])

# Vector addition
v_total = v1 + v2
print(f"Vector Addition: {v_total}")
```

```
# Scalar multiplication
scalar = 3
scaled_v = scalar * v1
print(f"Scalar Multiplication: {scaled_v}")

# Magnitude of vector
magnitude_v1 = np.linalg.norm(v1)
print(f"Magnitude of v1: {magnitude_v1}")

# Dot product
dot_product = np.dot(v1, v2)
print(f"Dot Product: {dot_product}")

Output:
Vector Addition: [6 4]
Scalar Multiplication: [12  9]
Magnitude of v1: 5.0
Dot Product: 11
```