THE GALAXY SURVEY WITH THE SKA RESULTS FOR DIFFERENT SENSITIVITIES

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This results are obtained using these parameters, $H_0 = 73.0$, $c = 3 \times 10^5$, $\Omega_{m_0} = 0.24$, $\Omega_{K_0} = 0.0$, $\Omega_{\Lambda_0} = 0.76$, $w_0 = -1$, $w_a = 0$ This results marginalize the $\ln D_A(z)$ and H(z) over $\{w_0, w_a, \Omega_m, \Omega_k, H_0\}$.

1. Results

• Results for $0\mu Jy$ sensitivity

```
=== combined ===
Vsur =
         116.33371916142235
                                   h^-3 Gpc^3
Ngal =
          1738478.9380518293
                                   millions
                                   10^-3 h^3 Mpc^-3
ngal =
         14943.895463700861
Err[lnDa](\%) = 0.13409
Err[lnH](\%) =
                0.29200
r(lnDa, lnH) = 0.40829
Err[lnR](\%) = 0.09720
*** Error on w***
Err[w]
                   = 0.00426
*** Error on w, with Omega_m marginalized over ***
                   2x2 = 25.759044210285058
Figure of Merit
Err[w]
                   = 0.06983
     Err[wa]
                   = 0.42639
   Err[wa_w]
                   = -0.02448
    Err[Omega_m] %=
                          NaN
 Err[ln Omega_b] %=
    Err[Omega_m_w] = -0.00026
      Err[Omega_k]%= 4.39764
    Err[Omega_k_w] = -0.00217
```

Date: March 28, 2014.

```
Err[H_0](\%) = 1.33068
```

```
<< CMB Priors Added (Edit da_accuracy & om_accuracy in "add_cmb") >>
  Err[lnDa(z=1090)](\%) = 0.20000
    Err[Omega_m](%)
                     = 2.00000
  *** Error on w***
  Err[w]
                  = 0.00415
  *** Error on w, with Omega_m marginalized over ***
  Figure of Merit 2x2 = 26.795164050762256
  Err[w]
                  = 0.04433
      Err[wa]
                  = 0.39181
     Err[wa_w]
                 = -0.01540
      Err[Omega_m] %= 1.22075
   Err[ln Omega_b] %= 0.02377
      Err[Omega_m_w] = 0.00034
      Err[Omega_k]%= 0.70934
      Err[Omega_k_w] = 0.00017
       Err[H_0](\%) = 0.47957
 Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$
 ahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$ python FoM_O
 ***************** Coe 2009 FoM DETF*****************
 FOM DETF (Coe2009) for OmuJy SKA = 9.56247594673
 FoM DETF (Coe 2009) for OmuJy SKA + CMB = 20.1775166362
 ****************** SKA FoM **************************
 FoM of MOmJy 2x2 = 34.768519192
 FoM of MOmJy\_cmb 2x2 = 40.4546226351
 ***********************************
• Results for 1\mu Jy sensitivity
```

```
=== combined ===
Vsur = 116.33371916142235
                                 h^-3 Gpc^3
Ngal = 160925.53224535252
                                 millions
ngal =
        1383.3094429144439
                                 10^-3 h^3 Mpc^-3
Err[lnDa](\%) = 0.13424
Err[lnH](\%) = 0.29223
r(lnDa, lnH) = 0.40830
```

FoM of 1 $MOmJy_cmb$ 2x2 = 40.3992191386

```
Err[lnR](\%) = 0.09730
*** Error on w***
Err[w]
                 = 0.00426
*** Error on w, with Omega_m marginalized over ***
Figure of Merit 2x2 = 25.726334489912613
Err[w]
                 = 0.06986
     Err[wa]
                 = 0.42663
                = -0.02450
   Err[wa_w]
    Err[Omega_m] %=
 Err[ln Omega_b] %=
                        NaN
    Err[Omega_m_w] = 0.00348
     Err[Omega_k]%= 4.40178
    Err[Omega_k_w] = -0.00217
      Err[H_0](\%) = 1.33135
 << CMB Priors Added (Edit da_accuracy & om_accuracy in "add_cmb") >>
Err[lnDa(z=1090)](\%) = 0.20000
  Err[Omega_m](%)
                   = 2.00000
*** Error on w***
                 = 0.00416
Err[w]
*** Error on w, with Omega_m marginalized over ***
Figure of Merit 2x2 = 26.763623963042058
Err[w]
                = 0.04435
     Err[wa]
                = 0.39205
   Err[wa_w]
                = -0.01542
    Err[Omega_m] %= 1.22168
 Err[ln Omega_b] %= 0.02377
    Err[Omega_m_w] = 0.00034
     Err[Omega_k]\% = 0.70988
    Err[Omega_k_w] = 0.00017
      Err[H_0](\%) = 0.47990
Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$
Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$ python FoM_1mJy_14
****************** Coe 2009 FoM DETF***********************
FOM DETF (Coe2009) for 1muJy SKA = 9.54949619831
FoM DETF (Coe 2009) for 1muJy SKA + CMB = 20.1734672793
****************** SKA FoM *********************************
FoM of 1 M0mJy 2x2 = 34.7222673388
```

• Results for 7.3μ Jy senstivity

```
=== combined ===
Vsur = 116.33371916142235
                                  h^-3 Gpc^3
Ngal =
         11788.925144398419
                                  millions
ngal =
        101.33712933255700
                                  10^-3 h^3 Mpc^-3
Err[lnDa](\%) = 0.13499
Err[lnH](\%) = 0.29358
r(lnDa, lnH) = 0.40828
Err[lnR](\%) = 0.09782
*** Error on w***
                  = 0.00428
Err[w]
*** Error on w, with Omega_m marginalized over ***
                            25.529367943450627
Figure of Merit 2x2 =
Err[w]
                  = 0.07002
     Err[wa]
                  = 0.42799
                = -0.02462
   Err[wa_w]
    Err[Omega_m] %=
                         NaN
 Err[ln Omega_b] %=
                         NaN
    Err[Omega_m_w] = -0.00706
     Err[Omega_k]%= 4.42037
    Err[Omega_k_w] = -0.00219
      Err[H_0](\%) = 1.33466
<< CMB Priors Added (Edit da_accuracy & om_accuracy in "add_cmb") >>
Err[lnDa(z=1090)](\%) = 0.20000
  Err[Omega_m](%)
                    = 2.00000
*** Error on w***
Err[w]
                  = 0.00418
*** Error on w, with Omega_m marginalized over ***
Figure of Merit 2x2 = 26.572443203439267
Err[w]
                 = 0.04444
     Err[wa]
                  = 0.39337
   Err[wa_w]
                = -0.01550
    Err[Omega_m] %= 1.22674
 Err[ln Omega_b] %= 0.02377
    Err[Omega_m_w] = 0.00034
     Err[Omega_k]%= 0.71279
```

```
THE GALAXY SURVEY WITH THE SKA RESULTS FOR DIFFERENT SENSITIVITIES
     Err[Omega_k_w] = 0.00017
      Err[H_0](\%) = 0.48155
 Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$
 Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$ python FoM_7point3
 ***************** Coe 2009 FoM DETF******************
 FOM DETF (Coe2009) for 7point3muJy SKA = 9.48586185491
 FoM DETF (Coe 2009) for 7point3muJy SKA + CMB = 20.049268645
 FoM of 7.3 M0mJy 2x2 = 34.4681421735
 FoM of 7.3 MOmJy\_cmb 2x2 = 40.0991553131
 Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$
• Results of 23\mu Jy sensitivity
  === combined ===
  Vsur = 116.33371916142235
                             h^-3 Gpc^3
  Ngal = 1106.0338963935974
                             millions
  ngal = 9.5074231647222316
                             10^-3 h^3 Mpc^-3
  Err[lnDa](\%) = 0.14512
  Err[lnH](\%) = 0.31470
  r(lnDa, lnH) = 0.40802
```

```
Err[lnR](\%) = 0.10511
*** Error on w***
               = 0.00458
*** Error on w, with Omega_m marginalized over ***
Figure of Merit 2x2 = 22.862327309799355
Err[w]
               = 0.07236
    Err[wa]
               = 0.45010
  Err[wa_w]
               = -0.02649
   Err[Omega_m] %=******
 Err[ln Omega_b] %=*******
   Err[Omega_m_w] = 0.00101
    Err[Omega_k]%= 4.72216
   Err[Omega_k_w] = -0.00243
     Err[H_0](\%) = 1.38215
```

<< CMB Priors Added (Edit da_accuracy & om_accuracy in "add_cmb") >>

```
Err[lnDa(z=1090)](\%) = 0.20000
  Err[Omega_m](%)
                = 2.00000
*** Error on w***
Err[w]
                = 0.00445
*** Error on w, with Omega_m marginalized over ***
Figure of Merit 2x2 = 24.038217192902810
Err[w]
                = 0.04566
     Err[wa]
               = 0.41482
              = -0.01668
   Err[wa_w]
    Err[Omega_m] %= 1.31689
 Err[ln Omega_b] %= 0.02377
    Err[Omega_m_w] = 0.00037
     Err[Omega_k]%= 0.76525
    Err[Omega_k_w] = 0.00019
     Err[H_0](\%) = 0.50566
Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$
Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$ python FoM_
****************** Coe 2009 FoM DETF********************
FOM DETF (Coe2009) for 23 muJy SKA = 9.48586185491
FoM DETF (Coe 2009) for 23 muJy SKA + CMB =
                                      20.049268645
FoM of 23 MOmJy 2x2 = 31.0642918259
FoM of 23 MOmJy\_cmb 2x2 = 36.0851026084
***********************************
Sahbas-MacBook-Pro:w0_wa_marginlized_over_omega_K_Omega_m_h_Ob sahba$
```

2. Summary

The main subroutines that been used to produce this results are

TABLE 1. Different values of the $S_{\rm rms}$ and the corresponded values of σ_{w_a} , σ_{w_0} and FoM, marginalized over $\{\Omega_m, \Omega_K, H_0\}$

Uncertainties/ setups	$S_{\rm rms}$ (Jy)	σ_{w_0}	σ_{w_a}	$\sigma_{\Omega_m}\%$	σ_{Ω_b}	$\sigma_{\Omega_K}\%$	$\sigma_{H_0}\%$	FoM	DETF FoM
SKA	0μ	0.06983	0.42639	-	-	4.39764	1.33068	127	99
	1μ	0.06986	0.42663	_	_	4.40178	1.33135	124	96
	$7.3~\mu$	0.07002	0.42799	-	-	4.42037	1.33466	112	85
	23μ	0.07236	0.45010	-	-	4.72216	1.38215	55	35
	0μ	0.04433	0.39181	1.22075	2.01440	0.70934	0.47957	319	306
SKA + CMB	1μ	0.04435	0.39205	1.22168	2.01457	0.70988	0.47990	316	299
	$7.3~\mu$	0.04444	0.39337	1.22674	2.01652	0.71279	0.48155	299	279
	23μ	0.04566	0.41482	1.31689	2.1980	0.76525	0.50566	213	182

```
SUBROUTINE report_result(z,bias,npara,fis)
  IMPLICIT none
  integer, intent(IN) :: npara
 double precision, intent(IN) :: fis(npara,npara), z,bias
 double precision, allocatable, dimension(:,:) :: cov
  double precision, allocatable, dimension(:)
  double precision :: r12,err_lnda,err_lnh,err_lnR,beta, linear_pk,dgdlna,g
  integer :: i,j
  external linear_pk,dgdlna,g
  ALLOCATE(cov(2,2),work(2))
  cov=fis
 CALL DVERT(cov,2,2,work)
 beta=(1d0+dgdlna(z)/g(z))/bias
 r12=cov(1,2)/sqrt(cov(1,1)*cov(2,2))
 err_lnda=sqrt(cov(1,1))
  err_lnh=sqrt(cov(2,2))
  err_lnR=err_lnda*sqrt((1d0-r12**2d0) &
       /(1d0+2d0*r12*err_lnda/err_lnh+(err_lnda/err_lnh)**2d0))
  open(12,file='Fisher_1mJy_diff_14bins.txt', status='unknown')
 write(12,'(4F18.5)') err_lnda,err_lnh,err_lnR
  open(13,file='output_1mJy_diff_14bins.txt', status='unknown')
 write(13,'(5F18.5)') z, err_lnda*1d2,err_lnh*1d2,err_lnR*1d2, beta
 print'(1A15,1F9.5)','Err[lnDa](%) =',err_lnda*1d2
 print'(1A15,1F9.5)','Err[lnH](%) =',err_lnh*1d2
 print'(1A15,1F9.5)','r(lnDa,lnH) =',r12
 print'(1A15,1F9.5)','Err[lnR](%) =',err_lnR*1d2
 DEALLOCATE(cov,work)
 return
```

END SUBROUTINE report_result

```
SUBROUTINE report_result3x3(fis)
 IMPLICIT none
 double precision, intent(IN) :: fis(5,5)
 double precision :: work(5),cov(5,5), A(5,5), M55DET, DET5x5,DET2x2,A05(5,5)
 integer :: i, j
 print*,'*** Error on w, ignoring Omega_k and Omega_m ***'
 print'(1A20,1F9.5)','Err[w]
                                     =',1d0/dsqrt(fis(1,1))
 cov=fis
 A05=SQRT(fis)
 A = fis
 DET5x5 = M55DET(SQRT(fis))
 DET2x2 = A05(1,1)*A05(2,2) - A05(1,2)*A05(2,1)
 !open(14,file='Fis5x5_OmJy_diff_14bins.txt', status='unknown')
  !write(14,'(4F18.5)') A
  CALL DVERT(cov,5,5,work)
 print*, '*** Error on w, with Omega_m marginalized over ***'
 !print*,'The Fisher Matrix = ', '[', A(1,1), A(1,2), A(1,3), A(1,4), A(1,5), ';'
                                          , A(2,1) , A(2,2) , A(2,3) , A(2,4) , A(2,4)
                                          , A(3,1), A(3,2), A(3,3), A(3,4), A(3,5),
 !
                                           A(4,1), A(4,2), A(4,3), A(4,4), A(4,5), ';
                                         , A(5,1), A(5,2), A(5,3), A(5,4), A(5,5),
! print*,'The Fisher Matrix **0.5 = ', '([[', A05(1,1)', ',', A05(1,2), ',', A05(1,3)]
                                          ',','[', A05(2,1) , ',', A05(2,2) , ',',A
!
 !
                                          '[', A05(3,1), ',',A05(3,2), ',',A05(3,3)
                                          '[', A05(4,1), ',',A05(4,2), ',',A05(4,3),
   !
                                         '[', A05(5,1), ',',A05(5,2), ',',A05(5,3),
  1
 print*, 'Figure of Merit
                          2x2 = ', abs(DET2x2)
! print*, 'Figure of Merit
                                =', abs(DET5x5)
                          5x5
 print'(1A20,1F9.5)','Err[w]
                                     =',dsqrt(cov(1,1))
 print'(1A20,1F9.5)','Err[wa]
                                =',dsqrt(cov(2,2))
 print'(1A20,1F9.5)','Err[wa_w]
                                  =',(cov(1,2))
 !print'(1A20,1F9.5)','Err[wa_w]
                                   =',(cov(2,1))
 print'(1A20,1F9.5)', 'Err[Omega_m](%) =',dsqrt(cov(5,5))*1d2
```

```
print'(1A20,1F9.5)','Err[Omega_k](%) =',dsqrt(cov(3,3))*1d2
  print'(1A20,1F9.5)','Err[H_0](%) =',dsqrt(cov(4,4))*1d2
 return
END SUBROUTINE report_result3x3
SUBROUTINE transform_fisher(z,fisDH,fis3x3)
 USE cosmo
  ! transform 2x2 Fisher matrix for (Da,H) to 3x3 matrix for
  ! (w,Omega_k,ln(Omega_m)), and accumulate it [e.g., Eq.(30) of Shoji et al.]
 IMPLICIT none
  integer :: a,b,i,j
 double precision, intent(IN)
                                 :: z,fisDH(2,2)
 double precision, intent(INOUT) :: fis3x3(5,5)
 double precision :: dpdq(5,2)
  double precision :: chi,h2,func0,func1,func2,func3,func4,rombint,fz
  external h2,func0,func1,func2,func3,func4,rombint
  chi=rombint(func0,0d0,z,1d-7)
  ! ORIGINAL CODE
  !dpdq(1,1)=-1.5d0*ode0*rombint(func1,0d0,z,1d-7)/chi
                                                               ! dlnDa/dw
  !dpdq(1,2) = 1.5d0*ode0*dlog(1d0+z)/h2(z)
                                                               ! dlnH/dw
   !dpdq(2,1)=-1.5d0*ode0*rombint(func4,0d0,z,1d-7)/chi
                                                               ! dlnDa/dwa
  !dpdq(2,2) = 1.5d0*ode0*(dlog(1d0+z) - z/(1d0+ z))/h2(z)
                                                                                  ! dlnH/dwa
  !dpdq(3,1) = -0.5d0*rombint(func2,0d0,z,1d-7)/chi+chi**2d0/6d0 ! dlnDa/d0mega_k
  !dpdq(3,2) = 0.5d0*(1d0+z)**2d0/h2(z)
                                                               ! dlnH/dOmega_k
  !dpdq(5,1)=-0.5d0*om0*rombint(func3,0d0,z,1d-7)/chi
                                                               ! dlnDa/dln(Omega_m)
  !dpdq(5,2) = 0.5d0*om0*(1d0+z)**3d0/h2(z)
                                                               ! dlnH/dln(Omega_m)
  ! MODYFIED CODE
  fz = (1d0 + z)**(3d0*(1d0 + w0+ w_a)) * exp(-3d0 * w_a *(z/(1d0+ z)))
  dpdq(1,1)=-1.5d0*ode0*rombint(func1,0d0,z,1d-7)/chi
                                                             ! dlnDa/dw
  dpdq(1,2) = 1.5d0*ode0*dlog(1d0+z)*fz/h2(z)
                                                                 ! dlnH/dw
  dpdq(2,1)=-1.5d0*ode0*rombint(func4,0d0,z,1d-7)/chi
                                                             ! dlnDa/dwa
 dpdq(2,2) = 1.5d0*ode0*(dlog(1d0+z) - z/(1d0+ z))*fz/h2(z)
                                                                                    ! dlnH/dwa
 dpdq(3,1)=-0.5d0*rombint(func2,0d0,z,1d-7)/chi+chi**2d0/6d0! dlnDa/d0mega_k
 dpdq(3,2) = 0.5d0*((1d0+z)**2d0 - fz)/h2(z)
                                                                     ! dlnH/dOmega_k
 dpdq(4,1) = -1d0/H_0 ! dlnDa/dH_0
 dpdq(4,2) = c/(2998d0*H_0)! dlnH/dH_0
 dpdq(5,1)=-0.5d0*rombint(func3,0d0,z,1d-7)/chi
                                                     ! dlnDa/d(Omega_m)
```

 $dpdq(5,2) = 0.5d0*((1d0+z)**3d0 - fz)/h2(z) ! dlnH/d0mega_m$

do a=1.5

do b=1,5

do i=1,2

```
do j=1,2
             ! transform and accumulate fis3x3
             fis3x3(a,b)=fis3x3(a,b)+dpdq(a,i)*dpdq(b,j)*fisDH(i,j)
       enddo
    enddo
  enddo
 return
END SUBROUTINE transform_fisher
SUBROUTINE add_cmb(fis3x3)
 USE cosmo
  ! Add the distance information from CMB [e.g., Eq.(38) of Shoji et al.]
 IMPLICIT none
 integer :: a,b,i,j
 double precision :: da_accuracy=0.2d0 ! Percent error in Da(zcmb)
 double precision :: om_accuracy=2d0
                                      ! Percent error in Omega_m
 double precision, intent(INOUT) :: fis3x3(5,5)
 double precision :: dpdq(5),zcmb=1090d0
 double precision :: chi,h2,func0,func1,func2,func3,func4,rombint
  external h2,func0,func1,func2,func3,func4,rombint
 chi=rombint(func0,0d0,zcmb,1d-7)
 dpdq(1)=-1.5d0*ode0*rombint(func1,0d0,zcmb,1d-7)/chi
                                                            ! dlnDa/dw
 dpdq(2)=-1.5d0*ode0*rombint(func4,0d0,zcmb,1d-7)/chi
                                                            ! dlnDa/dwa
 dpdq(3)=-0.5d0*rombint(func2,0d0,zcmb,1d-7)/chi+chi**2d0/6d0 ! dlnDa/d0mega_k
 dpdq(4) = -1d0/H_0
                       ! dlnDa/dH_0
 dpdq(5)=-0.5d0*rombint(func3,0d0,zcmb,1d-7)/chi
                                                      ! dlnDa/dOmega_m
  !dpdq(5)=-0.5d0*rombint(func2,0d0,zcmb,1d-7)/chi+chi**2d0/6d0 !dlnDa/dh
  do a=1.5
    do b=1,5
       ! add Da(z=1090)
       fis3x3(a,b)=fis3x3(a,b)+dpdq(a)*dpdq(b)*1d4/da_accuracy**2d0
    enddo
  enddo
  ! add Omega_matter
 fis3x3(5,5)=fis3x3(5,5)+1d4/(om_accuracy*om0)**2d0
 print*,'<< CMB Priors Added (Edit da_accuracy & om_accuracy in "add_cmb") >>'
 print'(1A23,1F9.5)','Err[lnDa(z=1090)](%) =',da_accuracy
 print'(1A23,1F9.5)','Err[Omega_m](%) =',om_accuracy
 return
```

```
END SUBROUTINE add_cmb
DOUBLE PRECISION FUNCTION h2(redshift)
 USE cosmo
 ! h2(z) = Omega_matter(1+z)^3+Omega_lambda
 IMPLICIT none
 DOUBLE PRECISION, intent(IN) :: redshift
 DOUBLE PRECISION :: fz,ok0
   fz = (1d0 + redshift)**(3d0*(1d0 + w0+ w_a)) * exp(-3d0 * w_a * (redshift/(1d0+ redshift)))
 h2 = om0*(1d0+redshift)**3d0+ ok0 * (1d0+redshift)**2d0+ode0* fz
 return
END FUNCTION h2
DOUBLE PRECISION FUNCTION funcO(redshift)
 ! func0(z) = 1/[h2(z)]^0.5
 IMPLICIT none
 DOUBLE PRECISION, intent(IN) :: redshift
 DOUBLE PRECISION :: h2
 external :: h2
 func0 = 1d0/dsqrt(h2(redshift))
 return
END FUNCTION funcO
DOUBLE PRECISION FUNCTION func1(redshift)
 ! func1(z) = ln(1+z)/[h2(z)]^1.5
 IMPLICIT none
 DOUBLE PRECISION, intent(IN) :: redshift
 DOUBLE PRECISION :: h2, fz
 external :: h2
 fz = (1d0 + redshift)**(3d0*(1d0 + w0+ w_a)) * exp(-3d0 * w_a * (redshift/(1d0+ redshift)))
 func1 = dlog(1d0+redshift) * fz /h2(redshift)**1.5d0
 return
END FUNCTION func1
DOUBLE PRECISION FUNCTION func2(redshift)
 USE cosmo
 ! func2(z) = (1+z)^2/[h2(z)]^1.5
 IMPLICIT none
 DOUBLE PRECISION, intent(IN) :: redshift
 DOUBLE PRECISION :: h2,fz
```

external :: h2

```
fz = (1d0 + redshift)**(3d0*(1d0 + w0+ w_a)) * exp(-3d0 * w_a * (redshift/(1d0+ redshift)))
      func2 = ( (1d0+redshift)**2d0 - fz)/h2(redshift)**1.5d0
      return
END FUNCTION func2
DOUBLE PRECISION FUNCTION func3(redshift)
       ! func3(z) = (1+z)^3/[h2(z)]^1.5
      IMPLICIT none
      DOUBLE PRECISION, intent(IN) :: redshift
      DOUBLE PRECISION :: h2, fz
      external :: h2
         fz = (1d0 + redshift)**(3d0*(1d0 + w0+ w_a)) * exp(-3d0 * w_a * (redshift/(1d0+ redshift))*
      func3 = ((1d0 + redshift) **3d0 - fz) /h2(redshift) **1.5d0
      return
END FUNCTION func3
DOUBLE PRECISION FUNCTION func4(redshift)
       ! func4(z) = ln(1+z) - z/1+z / [h2(z)]^1.5
      IMPLICIT none
      DOUBLE PRECISION, intent(IN) :: redshift
      DOUBLE PRECISION :: h2, fz
      external :: h2
      fz = (1d0 + redshift)**(3d0*(1d0 + w0+ w_a)) * exp(-3d0 * w_a * (redshift/(1d0+ redshift)) * exp(-3d0 * w_a * (redshift)) 
      func4 = fz * (dlog(1d0+redshift) - (redshift/(1d0+ redshift)))/h2(redshift)**1.5d0
      return
END FUNCTION func4
```