problem 1 :

① Let $\{y_t\}_{t\in\mathbb{Z}}$ be a TS, a linear filter

$$W = \sum_{j=-\infty}^{\infty} w_j B^j$$ acts on $\{y_t\}$ as follows

W, $y_t = \sum_{j=-\infty}^{\infty} w_j y_{t-j}$, is a linear combination

of the $\{y_t\}$

② $\Delta = 1 - B^1$ causal ?

③ Forecasting → Non-causal filters are useless on Forecasting

④ z transform $f_W : \mathbb{C} \to \mathbb{C}$, $f_W(z) = W(z)$

Characteristic Polynomial $P_W : \mathbb{C} \to \mathbb{C}$, $p(z) = z^q$, $f_W(z)$

(5) Example: $V = 0.2B^{-2} + 0.6 + 0.2B$, $f_V(z) = 0.2z^{-2} + 0.6 + 0.2z$

$q=1 \implies P_W(z) = 0.2 + 0.6z + 0.2z^2$

This could also be the CP of

$V_1 = 0.2 + 0.6B + 0.2B^2$

$V_2 = 0.2 + B^{-1000} + 0.6B^{-359} + 0.2B^{-998}$

(6) $y_t = 18$, applying eq., $\Delta = 2 - B$, yields, 0

$y_t = t$, ", Δ^2, ", $(\Delta y_t = t - (t-1) = 1)$

$y_t = t^r$, ", Δ^{r+1}, ", $y_t = (-1)^t$

$r \in \mathbb{N}$

$\frac{1}{3}B^{-2} + \frac{1}{3} + \frac{1}{3}B = t$

$\frac{1}{3}(t+1) + \frac{1}{3}t + \frac{1}{3}(t-1)$

$= t$

problem 2

ⓐ $W = \Delta_5 = 1 - B^5$ } This can be used for seasonality with periodicity 5

$U = 0.2B^{-1} + 0.6 + 0.2B$ } This can be used to filter short time fluctuations

ⓑ The order does not differ the result would be the same

ⓒ Find u such that $u = W \circ V$. We can always U as

$$U = \sum_{k=-\infty}^{\infty} u_k B^k \quad \text{where} \quad u_k = \sum_{j=-\infty}^{\infty} w_j v_{k-j}$$

$$(I)\quad u_1 = \sum_{k=-\infty}^{\infty} w_j \cdot v_{1-j} = w_0 v_1 + w_5 v_{-4} = 0.2 + 0 = 0.2$$

$$w_0 = 1,\quad w_1 = w_2 = w_3 = w_4 = 0,\quad w_5 = -1 \quad w_6 = w_7 = \cdots = w_{-1} = w_{-2} = \cdots = 0$$

$W_m \backslash V_l$	$V_{-1} = \frac{1}{5}$	$V_0 = \frac{3}{5}$	$V_1 = \frac{1}{5}$
$W_0 = 1$	-1	0	1
$W_5 = -1$	4	5	6

$$
u_k = \begin{cases} 1/5 & k = -2 \\ 3/5 & k = 0 \\ 1/5 & k = 1 \\ -2/5 & k = 4 \\ +3/5 & k = 5 \\ -1/5 & k = 6 \\ 0 & \text{else} \end{cases}
$$

(II) $u = W \circ V = (1 - B^5)(0.2B^{-1} + 0.6 + 0.2B)$

$$= 0.2B^{-1} + 0.6 + 0.2B - 0.2B^4 - 0.6B^5 - 0.2B^6$$

(d) Find $\tilde{W}$ such that $W = \Delta \circ \tilde{W} \Leftrightarrow \Delta^{-1} \circ W = \tilde{W}$

$\Delta = 1 - B$, $W = 1 - B^5$ $\qquad W = \Delta \circ \tilde{W}$

$$\tilde{W} = \Delta^{-1} \circ W = \left(\sum_{j=0}^{\infty} B^j \right) (1 - B^5) = \underbrace{\sum_{j=0}^{\infty} B^j}_{B^0 + B^1 + \cdots B^6} - \underbrace{\sum_{j=0}^{\infty} B^{j+5}}_{B^5 + B^6 + B^7}$$

$$= 1 + B + B^2 + B^3 + B^4$$

problem 3 :

$U = 1 - 0.8B + 0.15B^2$

$W = 1 - B^5$

Causal inverse linear filter, with absolutely summable weights, exists if the roots of the z-transform of W are bigger than 1 in abs value

$$f_u(z) = 1 - 0.8z + 0.15z^2 = 0$$

$$\Leftrightarrow 0 = z^2 - \frac{80}{15}z + \frac{20}{3} \Rightarrow z_{1/2} = \frac{80}{30} \pm \sqrt{\left(\frac{80}{30}\right)^2 - \frac{20}{20}}$$

$\Rightarrow z_1 = \frac{10}{3}$ & $z_2 = 2 \Rightarrow$ proposition applies

$f_w(z) = 1 - z^5 = 0$ we know that we can write

$\uparrow$ there is 5 solutions for this equation

$$z = |z| \cdot e^{i \cdot \varphi}$$

$$\Rightarrow \underbrace{1 - |z|^5 \cdot e^{5 \cdot i \cdot \varphi} = 0}$$

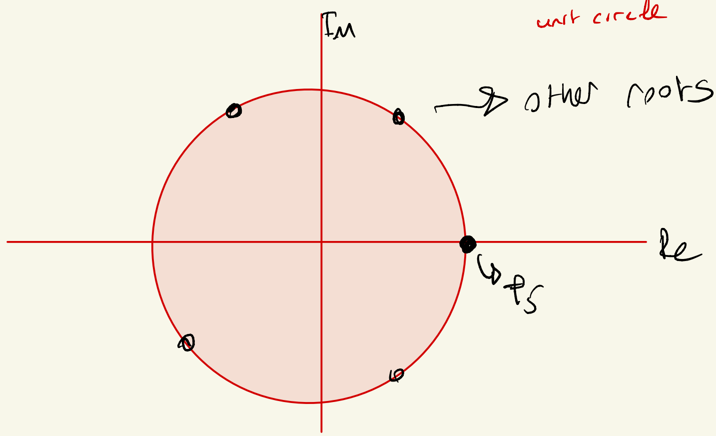
$$\Rightarrow e^{k \cdot 2\pi \cdot i} = 1, k \in \mathbb{Z}$$

$$5 \cdot i \cdot \varphi = k \cdot 2\pi \cdot i \Leftrightarrow \varphi = \frac{k \cdot 2\pi}{5}$$

$\downarrow$

$$\varphi_1 = \frac{2\pi}{5}, \quad \varphi_2 = \frac{4\pi}{5}, \quad \varphi_3 = \frac{6\pi}{5}$$

$$\varphi_4 = \frac{8\pi}{5}, \quad \varphi_5 = \underbrace{\frac{10\pi}{5}}_{= 2\pi = 1 \text{ on unit circle}}, \quad \varphi_6 = 0$$



(b) $u = 1 - \frac{4}{5}B + \frac{3}{20}B^2 = \underbrace{\left(1 - \frac{1}{2}B\right)}_{u_1}\underbrace{\left(1 - \frac{3}{10}B\right)}_{u_2}$

$$u^{-1} = u_2^{-1} \cdot u_1^{-1} = \left(\sum_{j=0}^{\infty}\left(\frac{3}{10}\right)^{j} B^{j}\right)\left(\sum_{j=0}^{\infty}\left(\frac{1}{2}\right)^{j'} B^{j'}\right)$$

We write $u^{-1} = \sum_{j=0}^{\infty} a_j \cdot B^j$, where

$$a_j = \sum_{l=-\infty}^{\infty} u_{1,l} \cdot u_{2,j-l} = \sum_{l=0}^{\infty}\left(\frac{1}{2}\right)^{l}\left(\frac{3}{10}\right)^{j-l}$$

$$= \left(\frac{3}{10}\right)^{j} \sum_{l=0}^{j}\left(\frac{5}{3}\right)^{l}$$

$$= \left(\frac{3}{10}\right)^{j} \frac{1 - \left(\frac{5}{3}\right)^{j+1}}{1 - \frac{5}{3}}$$