$\underline{Pf:}$ Let $\{X_t\}_{t \in \mathbb{N}}$ be a TS

Mean: $\bar{X} = \frac{1}{n} \sum_{t=1}^{T} X_t$, $\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^{T} (\bar{X} - X_t)^2$ } The version defined in the lecture

$\hat{S}^2 = \frac{1}{T-1} \sum_{t=1}^{T} (\bar{X} - X_t)^2$ } Unbiased version

$$\boxed{(a-b)^2 = ((-1)\cdot(b-a))^2 = (-1)^2 \cdot (b-a)^2}$$

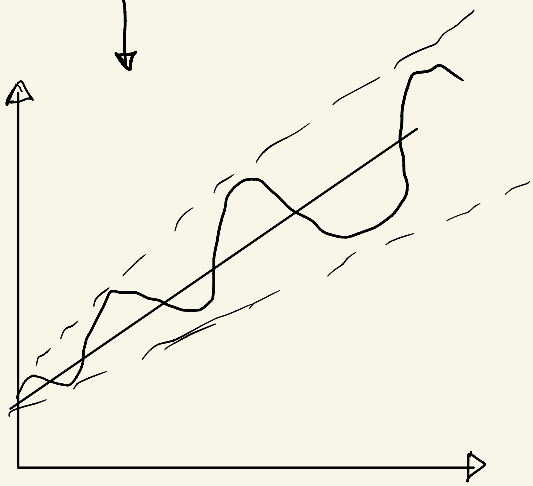
$$\hat{\gamma} : \mathbb{Z} \to \mathbb{R}, \quad \hat{\gamma}(h) = \frac{1}{T} \sum_{t=T_1}^{T_2} (X_{t+h} - \bar{X})(X_t - \bar{X})$$

$$\hat{\rho}(h) = \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)} \quad , \quad \hat{\rho} \in [-1, 1]$$

P2:

Box-Cox: Assumption: $\sigma_t = k \cdot \mu_t^{\alpha}$, where $\mu_t = E(X_t)$

$\sigma_t^2 = Var(X_t)$



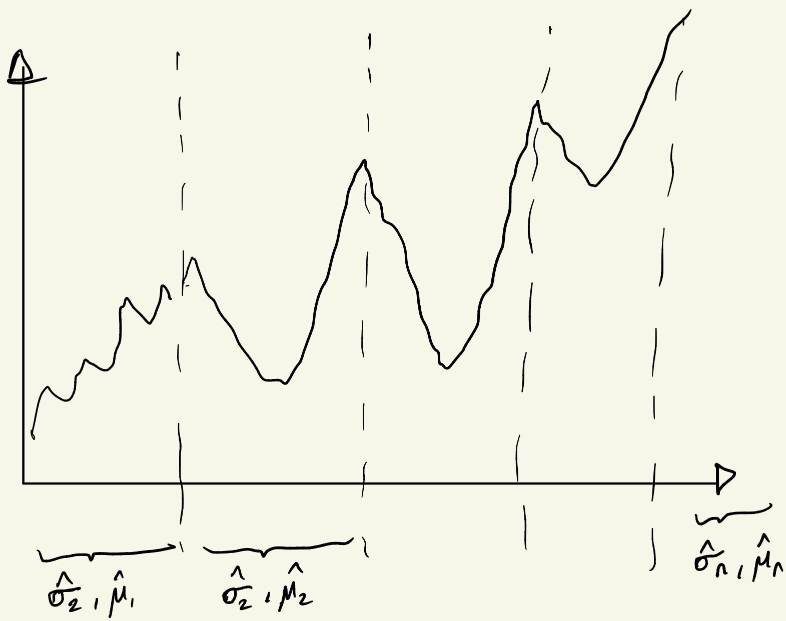
Goal: Stabilize the variance

Transformation:

$$X_t^{(\lambda)} = \begin{cases} \dfrac{(X_t + c)^{\lambda} - 1}{\lambda} & \lambda \neq 0 \\ \log(X_t) & \lambda = 0 \end{cases}$$

$$\log(\sigma_t) = \log(k \cdot \mu_t^{\alpha}) = \underbrace{\log(k)}_{=\beta_0} + \underbrace{\alpha}_{=\beta_1} \underbrace{\log(\mu_t)}_{=\ln}$$

Now we estimate α with OLS, before we estimate μ_t and σ_t on different time period



$$\Longrightarrow \hat{\lambda} = 1 - \hat{\alpha}$$

<u>TREND ESTIMATION</u>: Assumption $X_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon_t$

$$= (1 \; t \; t^2) \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \varepsilon_t$$

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ \vdots \\ X_T \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & \vdots & \vdots \\ 1 & \vdots & \vdots \\ 1 & \vdots & \vdots \\ 1 & T & T^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \vdots \\ \varepsilon_T \end{pmatrix} \overset{\text{OLS}}{\underset{\hat{\beta}}{\Longrightarrow}} \hat{\beta} = (A^T A)^{-1} A^T X$$

$= A$ $\qquad\qquad \beta$

SEASONALITY : $y_t = \delta_1 \cdot s_{1,t} + \cdots + \delta_{12} \cdot s_{12,t} + \varepsilon_t$

$$\sum_{i=1}^{12} \delta_i = 0$$