Problem 1

Let X_t be a Time Series such that

$$X_t = a \cdot X_{t-1} + \varepsilon_t \,, \quad \varepsilon_t \overset{iid}{\sim} N(0,1) \,, \quad |a| < 1$$

$\varepsilon_t \overset{iid}{\sim} N(0,1)$: we need i.i.d for MLE set up

$|a|<1$: for stationarity.

For MLE we need the LL:

$$L(x_1, x_2, \dots, x_t; \theta) = f_{X_t, \dots, X_1}(x_t, \dots, x_1; \theta) = f(x_t | x_{t-1}, \dots, x_1) \cdot f(x_{t-1}, \dots, x_1)$$

(θ: a)

$$\dots = \prod_{i=2}^{t} f_{X_i | X_{i-1}, \dots, X_1}(x_i | x_{i-1}, \dots, x_1) \cdot f_{X_1}(x_1)$$

$$X_{17} | X_{16}, \dots X_1 \sim N(aX_{16}, 1)$$

The mean of $X_t | (X_{t-1}, \dots X_1) = (X_{t-1}, \dots X_1) = aX_{t-1} + \varepsilon_t$

Mean of X_t is $E[X_t] = a \cdot X_{t-1} + 0$

$$f_{X_{17} | X_{16}, X_{15}, \dots X_1}(x_{17} | x_{16}, \dots x_1) \overset{=}{\underset{N(aX_{16}, 1)}{f}}(x_{17} - aX_{16}) = f_{N(0,1)}(\varepsilon_{17})$$

$$f_{X_{17} | X_{16}, \dots X_1}(0 | 0, 1, 2)$$

This part is Random variables

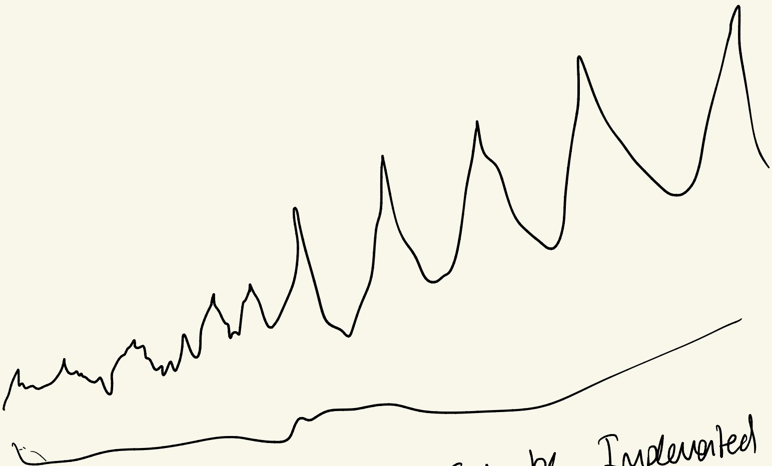
This part is realization little x of R.V $X_{17} | X_{16}, \dots X_1$

AR(2): $X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \varepsilon_t \qquad \varepsilon_t \sim N(0,1)$

$\downarrow$

$X_t | X_{t-1}, X_{t-2} \sim N(\Phi_1 X_{t-1} + \Phi_2 X_{2-2}, 1)$

$$\underbrace{f_{X_t | X_{t-1}, \ldots, X_t}(x_t | x_{t-1}, \ldots, x_1) = f_{N(0,1)}(\overbrace{x_{t-1} - \Phi_1 x_{t-1} - \Phi_2 x_{2-1}}^{\varepsilon_t})}_{\text{density}}$$



Box-Cox Can be Implemented

Problem 3

AIC = Akaike Information Criterion

$-2 \log LL + 2k$

- $\log LL$ → Likelyhood
- k → # of parameters

increasing coefficient cause overfitting this formula gives the best coefficient to add by penalizing putting more parameters and maximizing log likelyhood

Fit for different Models and find lowest AIC

There Are three ways to test best models

1-) Info Criteria

2-) Periodogram and ACF PACF ← looking

3-) Fitting different models and doing statistical tests to parameters if they are significantly different than "0"

Difference btw SARIMA and modelling seasonality with dummy variables

↓

SARIMA models seasonal part of stochastic variable.