# Physics 4310 Homework #11

**4 problems**
**Due by Friday, April 29**

▷ **1.**
[Ch 6] Consider an infinite square well with a slightly tilted floor:

$$V(x) = \begin{cases} \epsilon x & 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

Find expressions for the approximate (first-order) ground state energy *and* eigenstate of this potential; write them in closed form if possible.

▷ **2.**
[Ch 6] Consider a two-dimensional infinite square well, with potential $V(x, y) = 0$ if $0 \leq x \leq a$ and $0 \leq y \leq a$ and $\infty$ otherwise. The energy eigenstates are

$$\psi_{n_x n_y}(x, y) = \left(\frac{2}{a}\right) \sin\left(\frac{n_x \pi}{a} x\right) \sin\left(\frac{n_y \pi}{a} y\right),\ n_x, n_y = 1, 2, 3, \ldots$$

with energy $E = \frac{\pi^2 \hbar^2}{2ma^2}(n_x^2 + n_y^2)$. Notice that the second-lowest energy $E = \frac{\pi^2 \hbar^2}{2ma^2}(4 + 1)$ has a twofold degeneracy: $(n_x, n_y) = (1, 2)$ and $(2, 1)$. To break this degeneracy, we add a perturbation to the Hamiltonian:

$$H' = \begin{cases} V_0, & 0 \leq x \leq \frac{a}{2} \quad \text{and} \quad 0 \leq y \leq \frac{a}{2} \\ 0, & \text{otherwise} \end{cases}$$

Find the approximate energies $E_\pm$ and eigenstates $\psi_\pm$ once the degeneracy is broken.

▷ **3.**
[Ch 7] Use a gaussian trial function (Eq. 7.2) to obtain the lowest upper bound you can on the ground state energy of the linear potential $V(x) = \alpha|x|$ and the quartic potential $V(x) = \alpha x^4$. Compare your bounds to the exact ground state energy of the potential $V(x) = \alpha x^2$.

▷ **4.**
[Ch 8] Consider the potential

$$V(x) = \begin{cases} V_0(1 - \frac{x^2}{a^2}), & -a < x < a \\ 0, & \textit{otherwise} \end{cases}$$

where $V_0$ and $a$ are constants. Use the WKB approximation to find the scattering solution to the Schrodinger equation for this potential, with $E \gg V_0$.