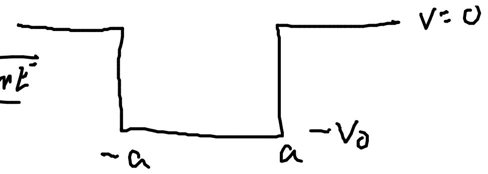
Scattering states $E > 0$



$x < -a$: $\psi(x) = \underbrace{Ae^{ikx}}_{\text{incident}} + \underbrace{Be^{-ikx}}_{\text{reflected}}$ $\quad k = \frac{\sqrt{2mE}}{\hbar}$

$x > a$: $\psi(x) = \underbrace{Fe^{ikx}}_{\text{transmitted}}$ $\quad T = \frac{|F|^2}{|A|^2} \quad R = \frac{|B|^2}{|A|^2}$

$-a < x < a$: $C \sin lx + D \cos lx$ $\quad l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$

B.C. $\psi_-(-a) = \psi_+(-a) \Rightarrow Ae^{-ika} + Be^{ika} = -C\sin la + D\cos la$

$\psi_-'(-a) = \psi_+'(-a) \Rightarrow ik[Ae^{-ika} - Be^{ika}] = l[C\cos la + D\sin la]$

$x = +a$ $\quad C\sin la + D\cos la = Fe^{ika}$

$l[C\cos la - D\sin la] = ikFe^{ika}$

DO MATH

$$\frac{|A|^2}{|F|^2} = \frac{1}{T} = 1 + \frac{V_0^2}{4E(E+V_0)} \sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right)$$

if $\sin^2 = 0$, then $T = 1$

for certain values of E, there is no reflection
the potential is "transparent"

To get transparency, $\frac{2a}{\hbar}\sqrt{2m(E+V_0)} = n\pi$

$$\underbrace{E_n + V_0}_{\text{distance above bottom of well}} = \frac{n^2\pi^2\hbar^2}{2m(2a)^2}$$

width $= 2a$

Infinite square well

$$E_n = \frac{n^2\pi^2\hbar^2}{2m(\text{width})^2}$$

All energies can exist for scattering state,
but energies that match the allowed energies
of an ∞ $\square$ well, will have $T=1$ & $R=0$.

On other hand, if $\sin^2 = 1$

$$\frac{1}{T} = 1 + \frac{V_0^2}{4E(E+V_0)} \rightarrow T = \frac{4E(E+V_0)}{4E(E+V_0) + V_0^2},$$

as E goes up,
$T \rightarrow 1$ anyway -

Chapter 3

Uncertainty Principle

$$\sigma_A \sigma_B \geq \left| \frac{1}{2i} \langle [A,B] \rangle \right|$$

e.g. $[x,p] = x \frac{\hbar}{i}\frac{d}{dx} f - \frac{\hbar}{i}\frac{d}{dx} x f$

$= \frac{\hbar}{i}\left[x f' - (f + x f') \right]$

$= \frac{\hbar}{i}(-f)$

$= i\hbar$

$\sigma_x \sigma_p \geq \left| \frac{1}{2i} \langle i\hbar \rangle \right|$

$\geq \left| \frac{i\hbar}{2i} \right|$

$\sigma_x \sigma_p \geq \frac{\hbar}{2}$ Heisenberg Uncertainty Principle

Compare $\sigma_{S_z} \sigma_{S_y}$ for $|\uparrow\rangle$

$\sigma_{S_z} \sigma_{S_y} \geq \left| \frac{1}{2i} \langle [S_z, S_y] \rangle \right|$

$\geq \left| \frac{1}{2i} \langle i\hbar(-S_x) \rangle \right|$

$\geq \frac{\hbar}{2} \left| \langle S_x \rangle \right|$

≥ 0

$\langle S_x \rangle = 0$

($\frac{1}{2}$ come out $+\frac{\hbar}{2}$
$\frac{1}{2}$ " " $-\frac{\hbar}{2}$)

So we can have $\sigma_{S_z} = 0$

But $\sigma_x \sigma_p \geq \frac{\hbar}{2}$

so x is never perfectly known
& p " " " "

Position & Momentum Eigenstates are not real states.

$k = \frac{2\pi}{\lambda}$

$\lambda = \frac{h}{p}$ deBroglie

$k = \frac{2\pi}{h} p$

$\Delta x \Delta p \geq \frac{\hbar}{2}$

position eigenstate at $x = x_0$ is $\delta(x - x_0)$
which is not a function
does not describe a real thing

momentum eigenstate is e^{ikx} $k = \frac{2\pi}{h} p$

$$\int_{-\infty}^{\infty} |\psi|^2 dx = \int_{-\infty}^{\infty} |e^{ikx}|^2 dx = \int_{-\infty}^{\infty} 1 \, dx = \infty,$$

non-normalizable

$\Delta E \Delta t \geq \hbar/2$