No class Mar 30, Apr 1, Apr 4

Video lecture on Chapter 4 will be posted on the website next week.

HW9 will be due Wednesday, April 6th.

Office Hours on Tuesday Mar 29 & Tue April 5th 10-11am.

Next quiz (two weeks from today) will include questions from Chapter 4.

Exam #2 will be 2nd week of April - day TBD. Suggestions? Chapters 3-5.

$$\langle E \rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} \quad \text{or} \quad = -\frac{\partial \ln Z}{\partial \beta} \qquad \beta = \frac{1}{kT}$$

Paramagnet $\quad \uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow$

$N = 1$: single dipole with 2 states

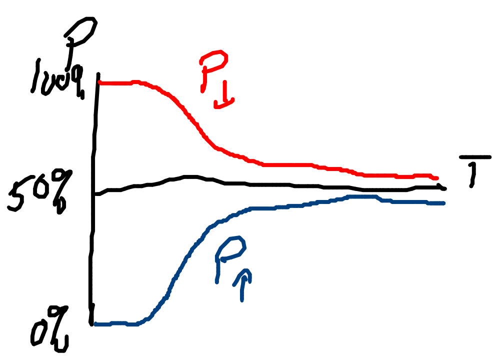
$$E_\uparrow = +\mu B \qquad E_\downarrow = -\mu B$$

$$Z = \sum_s e^{-\beta E_s} = e^{-\beta(\mu B)} + e^{-\beta(-\mu B)}$$

$$P_\downarrow = \frac{e^{-\beta E_\downarrow}}{Z} = \frac{e^{+\beta\mu B}}{e^{-\beta\mu B} + e^{\beta\mu B}} = \frac{1}{1 + e^{-2\beta\mu B}}$$

If T low, β high
$P_\downarrow = 1$

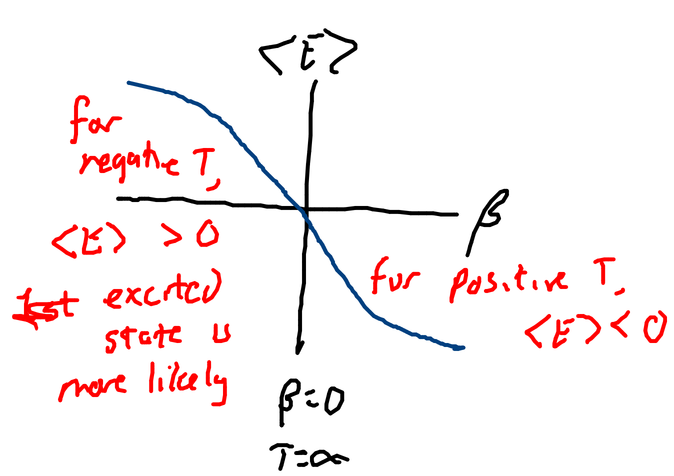
If T high, β low
$P_\downarrow = \frac{1}{2}$

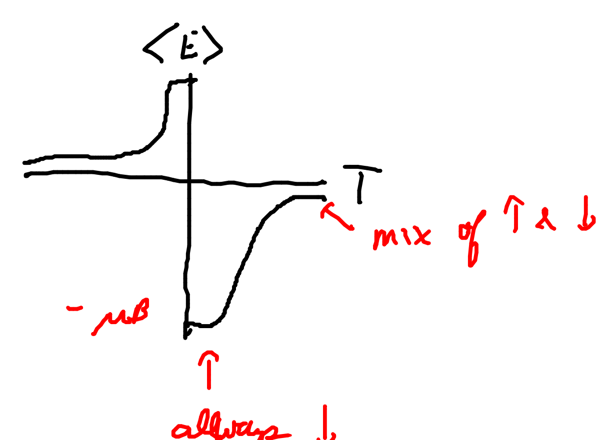


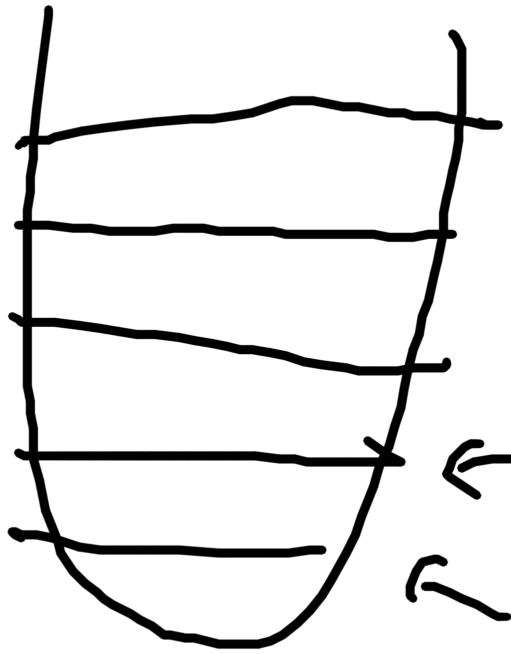
$$Z = e^{\beta\mu B} + e^{-\beta\mu B} = 2\cosh \beta\mu B$$

$$\langle E \rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{\cancel{2}\cosh\beta\mu B}\left(\cancel{2}\mu B \sinh \beta\mu B\right)$$

$$= -\mu B \tanh \beta\mu B = -\mu B \tanh \frac{\mu B}{kT}$$

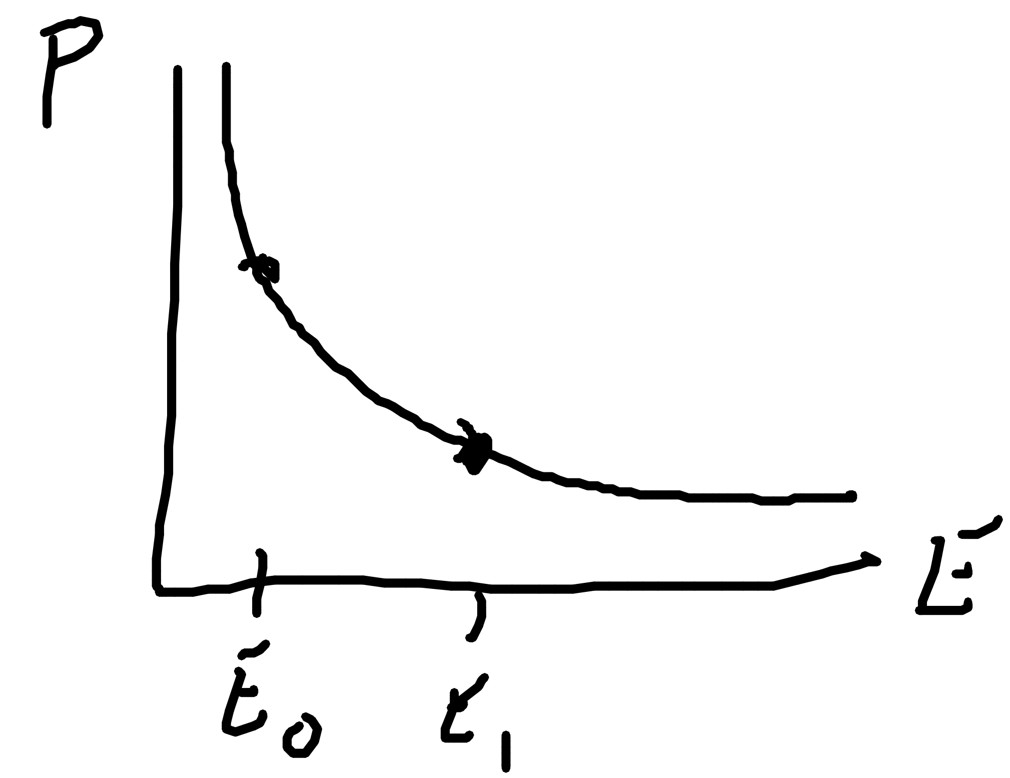






$\leftarrow P = e^{-E_1/kT}$

$\leftarrow P = e^{-E_0/kT}$



lower energy microstates are <u>always</u> more likely than higher energy microstates

10% 10% 10% 10% $-3.4\,eV$

4-fold degeneracy

20% $-13.6\,eV$

$E = -3.4\,eV$ is more likely than $E = -13.6\,eV$ because we're talking about <u>macrostates</u>.

e.g. Rotational motion of carbon monoxide

energy levels are quantized

$$E_j = j(j+1)\varepsilon \qquad j = 0, 1, 2, \ldots$$

degeneracy of j is $g_j = 2j+1$

"density of states" (especially ~~when~~ in cases where energy is continuous)

$\vdots$

— — — — — $j=2,\ E_2 = 6\varepsilon,\ g_2 = 5$

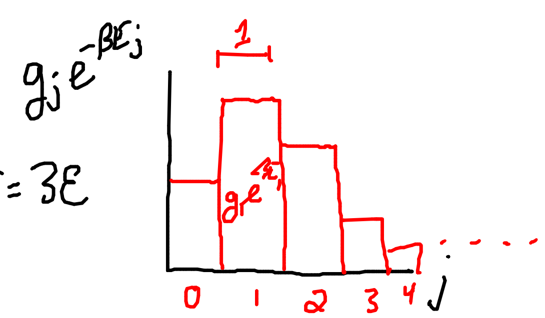
— — — $j=1,\ E_1 = 2\varepsilon,\ g_1 = 3$

— $j=0,\ E_0 = 0,\ g_0 = 1$

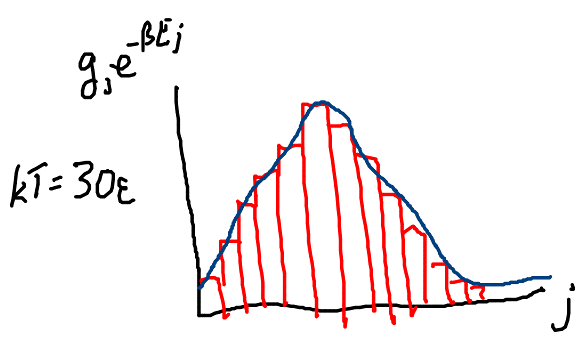
Probability of a given microstate in row j $\quad P = \dfrac{e^{-\beta E_j}}{Z}$

Probability of a given <u>macrostate</u> j $\quad P = g_j \dfrac{e^{-\beta E_j}}{Z}$

$$Z = \sum_s e^{-\beta E_s} = \sum_{j=0}^{\infty} g_j e^{-\beta E_j}$$



Z = area under bars



Z can be approximated by an integral

for high T,

$$Z \approx \int_0^{\infty} g_j e^{-\beta E_j}\, dj$$

$$= \int_0^{\infty} (2j+1)\, e^{-\beta j(j+1)\varepsilon}\, dj$$

$x = j(j+1) = j^2 + j$

$dx = 2j+1\ dj$

$$= \int_0^{\infty} e^{-\beta x \varepsilon}\, dx = \frac{1}{\beta\varepsilon} = \frac{kT}{\varepsilon}$$

$$\langle E \rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -(\beta\varepsilon)\left(-\frac{1}{\varepsilon}\beta^{-1}\right) = \frac{1}{\beta} = kT.$$

Equipartition Theorem with $f = 2$ $\qquad c = 0$