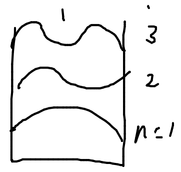
Photons: bosons with $\mu = 0$
created or destroyed freely

1D In a 1D box of length L

$\lambda = \frac{2L}{n}$ $\quad p = \frac{hn}{2L}$



energy $\mathcal{E} = pc = \frac{hcn}{2L}$ relativistic!

in 3D, $\mathcal{E} = |\vec{p}|c = \frac{hc}{2L}\sqrt{n_x^2+n_y^2+n_z^2} = \frac{hc|\vec{n}|}{2L}$

polarization; energy of state; # photons in that state

total energy $U = 2\sum_{n_x}\sum_{n_y}\sum_{n_z} \frac{hc|\vec{n}|}{2L} \frac{1}{e^{hcn/2LkT}-1}$

$= \int_0^\infty \int_0^{\pi/2} \int_0^{\pi/2} \frac{hcn}{L} \frac{1}{e^{hcn/2LkT}-1} n^2 \sin\theta \, d\phi \, d\theta \, dn$

first octant $\frac{1}{8}$ of sphere

$= \frac{4\pi}{8}\int_0^\infty \frac{hcn^3}{L} \frac{1}{e^{hcn/2LkT}-1} dn$

Let $\mathcal{E} = \frac{hcn}{2L} \rightarrow n = \frac{2L\mathcal{E}}{hc}$

$= \frac{4\pi}{8}\int_0^\infty \frac{hc}{L}\left(\frac{2L}{hc}\right)^3 \mathcal{E} \frac{1}{e^{\mathcal{E}/kT}-1} \frac{2L}{hc} d\mathcal{E}$

L^3

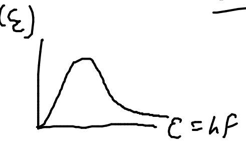
$U = V\int_0^\infty \frac{8\pi \, \mathcal{E}^3/(hc)^3}{e^{\mathcal{E}/kT}-1} d\mathcal{E}$

$\frac{U}{V} = \int_0^\infty u(\mathcal{E}) \, d\mathcal{E}$

$u(\mathcal{E})$: energy density per photon energy

$u(hf)$: energy density per frequency

<u>blackbody spectrum</u>



Let $x = \mathcal{E}/kT$ $\quad dx = \frac{d\mathcal{E}}{kT}$

$\frac{U}{V} = \int_0^\infty \frac{8\pi}{(hc)^3} \frac{\mathcal{E}^3}{e^{\mathcal{E}/kT}-1} d\mathcal{E}$

$= \frac{8\pi}{(hc)^3}\int_0^\infty \frac{x^3 (kT)^3}{e^x - 1} kT \, dx$

$\frac{U}{V} = \frac{8\pi}{(hc)^3}(kT)^4 \int_0^\infty \frac{x^3}{e^x-1} dx$ $\leftarrow 6.5$

$\frac{U}{V} \propto T^4$ Stefan-Boltzmann Law

$u(x) = \frac{8\pi}{(hc)^3}(kT)^4 \frac{x^3}{e^x-1}$

peaks at $u'(x) = 0 \rightarrow x = 2.82$

peak $\mathcal{E} = xkT = 2.82kT$ ← Wien's Law

at low T (300K, e.g.) $\mathcal{E}_{peak}$ is infrared

higher T. $\mathcal{E}_{peak} \rightarrow$ red $\rightarrow$ yellow $\rightarrow$ blue $\rightarrow$ UV $\rightarrow$ etc.

Sound Waves in a Solid

Model solid as a 3D set of harmonic oscillators

"Einstein solid"

N oscillators

q quanta of energy

$\gg 1$

$$\Omega = \binom{N+q-1}{q}$$

$$S = k \ln \Omega = k \ln \binom{N+q-1}{q}$$

$q = \frac{U}{\varepsilon}$ $\frac{dq}{dU} = \frac{1}{\varepsilon}$

ε: energy/quantum

$$\approx k\left[N \ln(N+q) + q \ln(N+q) - N\ln N - q\ln q\right]$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{\partial S}{\partial q}\frac{\partial q}{\partial U} = k\frac{1}{\varepsilon}\left[\ln \frac{N+q}{q}\right]$$

$$e^{\varepsilon/kT} = \frac{N+q}{q} = \frac{N}{q} + 1$$

$$\frac{1}{e^{\varepsilon/kT}-1} = \frac{q}{N} \leftarrow$$ # of quanta per oscillator

occupancy of oscillator

these quanta are bosons with $\mu = 0$

<u>phonons</u>

Phonons are different from photons in 3 ways:

1) much slower: c_s speed of sound ($c_s \simeq$ constant)

2) in solid, sound can be longitudinally polarized <u>or</u> transversely polarized

<u>3</u> polarizations

3) Sound waves have minimum $\lambda_{min} = 2d$