Chapter 3 McIntyre : time evolution of states

$|\psi(t)\rangle$

Schrodinger equation $\quad i\hbar \frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$

$H(t)$: Hamiltonian operator corresponds to total energy of the system

H is an observable : Hermitian,

$$\underbrace{H|E_n\rangle}_{\text{energy eigenstates}} = \underbrace{E_n}_{\text{energy eigenvalues}}|E_n\rangle \qquad \langle E_k|E_n\rangle = \delta_{kn}$$

Any state $|\psi\rangle$ can be written

$$|\psi(t)\rangle = \sum_n \underbrace{c_n(t)}_{\text{constants}}|E_n\rangle$$

If H is independent of time,

$$i\hbar\frac{d}{dt}\sum_n c_n(t)|E_n\rangle = H\sum c_n(t)|E_n\rangle$$

$$i\hbar\sum_n \frac{dc_n(t)}{dt}|E_n\rangle = \sum c_n(t)\,H|E_n\rangle$$

$\langle E_k|$ $\qquad\qquad \langle E_k| = \sum c_n(t)E_n|E_n\rangle$

$$\sum_n i\hbar\frac{dc_n(t)}{dt}\langle E_k|E_n\rangle$$

$$i\hbar\frac{dc_k(t)}{dt} = c_k(t)E_k$$

$$\dot{c}_k = \frac{-iE_k}{\hbar}c_k$$

$$c_k(t) = e^{-\frac{iE_kt}{\hbar}}c_k(0)$$

$$|\psi(t)\rangle = \sum_n c_n(t)|E_n\rangle$$

$$\boxed{|\psi(t)\rangle = \sum_n c_n(0)\,e^{-iE_nt/\hbar}|E_n\rangle}$$

1) Write $|\psi(0)\rangle$ in energy basis.
2) Multiply each term by $e^{-iE_nt/\hbar}$.

e.g. $|\psi(0)\rangle = |E_1\rangle$

$$|\psi(t)\rangle = e^{-iE_1t/\hbar}|E_1\rangle$$

Stationary state – does not change with time (overall phase doesn't matter)

$|\psi(0)\rangle = a|E_1\rangle + b|E_2\rangle$

$$\mathcal{P}_{E_1}(t) = |\langle E_1|\psi(t)\rangle|^2$$

$$= \left|\langle E_1|\left(a e^{-iE_1 t/\hbar}|E_1\rangle + b e^{-iE_2 t/\hbar}|E_2\rangle\right)\right|^2$$

$$= \left| a e^{-iE_1 t/\hbar}\right|^2$$

$$= |a|^2 \quad \text{independent of } t$$

$$\langle E\rangle = \sum_n \mathcal{P}_{E_n} E_n \quad \text{independent of } t$$

How about $\langle A\rangle$?

$$\mathcal{P}_{A_n}(t) = |\langle A_n|\psi(t)\rangle|^2$$

if A & H have same eigenstates

then $\mathcal{P}_{A_n}(t)$ is independent of time (& $\langle A\rangle$ too)

which is true if $[A,H]=0$. (compatible)

— — —

if $[A,H] \neq 0$, then $|a_1\rangle, |a_2\rangle$, etc $\neq |E_i\rangle$

suppose 2-dim. space E_1, E_2

$$|a_1\rangle = c|E_1\rangle + d|E_2\rangle$$

$$\mathcal{P}_{a_1} = |\langle a_1|\psi(t)\rangle|^2$$

$$= \left|\left(c^*\langle E_1| + d^*\langle E_2|\right)\left(a e^{-iE_1 t/\hbar}|E_1\rangle + b e^{-iE_2 t/\hbar}|E_2\rangle\right)\right|^2$$

$$= \left| c^* a e^{-iE_1 t/\hbar} + d^* b e^{-iE_2 t/\hbar}\right|^2$$

$\mathrm{Re}\, z = \frac{z+z^*}{2}$

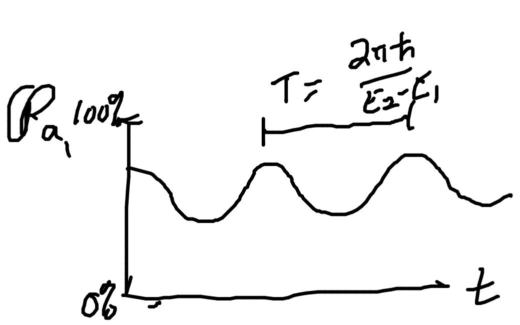
$$= |ac|^2 + |bd|^2 + 2\,\mathrm{Re}\left(a b^* c^* d\, e^{-i(E_2-E_1)t/\hbar}\right)$$

time dependence

$e^{-i\omega t}$

$\omega = \frac{E_2-E_1}{\hbar}$

Bohr frequency



Consider a single magnetic dipole (electron)

$$\vec{\mu} = g \frac{q}{2m_e} \vec{S}$$

g = gyromagnetic ratio

$g = 2$

put in a magnetic field $\vec{B}$

$$H = -\vec{\mu} \cdot \vec{B} = -g \frac{\overset{-e}{\downarrow}\!\!q}{2m_e} \vec{S} \cdot \vec{B} = \frac{e}{m_e} \vec{S} \cdot \vec{B}$$

Suppose $\vec{B} = B_0 \hat{z}$

$$H = \underbrace{\frac{e}{m_e} B_0}_{\omega_0 = \frac{eB_0}{m_e}} S_z$$

← operator

$$H = \omega_0 S_z \qquad [H, S_z] = 0.$$

H has same eigenstates $|\uparrow\rangle$ & $|\downarrow\rangle$ as S_z.

& eigenvalues $E_\pm = \pm \frac{\hbar}{2} \omega_0$

$|\uparrow\rangle$ —— $+\frac{\hbar\omega_0}{2}$

$|\downarrow\rangle$ —— $-\frac{\hbar\omega_0}{2}$

In $\uparrow\downarrow$ basis

$$H \doteq \frac{\hbar\omega_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$