$$|\odot\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$|\otimes\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$$

Superposition states not mixtures of $|\uparrow\rangle_s$ & $|\downarrow\rangle_s$.

Representing States as Matrices

We can represent kets as column vectors if we specify a basis

e.g. in a S_z basis

$$|\uparrow\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\downarrow\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\doteq$ means "represented by"

$$|\odot\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) \doteq \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|\otimes\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$|\psi\rangle \doteq \begin{pmatrix} a \\ b \end{pmatrix} \doteq a|\uparrow\rangle + b|\downarrow\rangle$$

$$a = \langle\uparrow|\psi\rangle \qquad b = \langle\downarrow|\psi\rangle$$

$$|\psi\rangle \doteq \begin{pmatrix} \langle\uparrow|\psi\rangle \\ \langle\downarrow|\psi\rangle \end{pmatrix}$$

$$\langle\uparrow|\uparrow\rangle = 1 \qquad \langle\uparrow| \doteq \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$|\psi\rangle \doteq \begin{pmatrix} a \\ b \end{pmatrix} \qquad \langle\psi| = \begin{pmatrix} a^* & b^* \end{pmatrix}$$

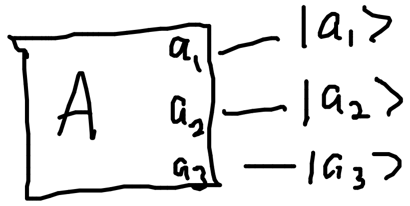
$$\langle\psi|\psi\rangle = \begin{pmatrix} a^* & b^* \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = |a|^2 + |b|^2$$

General quantum systems may have more than 2 dimensions

e.g. a spin-1 system can have $S_z = \hbar, 0, -\hbar$

e.g. a harmonic oscillator can have $E = \frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega, \frac{5}{2}\hbar\omega, \ldots$

If an observable A yields quantized measurement results a_n.



Kronecker delta

orthonormal $\quad \langle a_i|a_j\rangle = \delta_{ij} \equiv \begin{cases} 1, & i=j \\ 0, & i\neq j \end{cases}$

completeness $\quad |\psi\rangle = \sum_{i=1}^{n} \big(\langle a_i|\psi\rangle\big)|a_i\rangle$

$$= \sum_{i=1}^{n} \langle a_i|\psi\rangle |a_i\rangle$$

e.g. spin 1 system $\quad |+\rangle, |0\rangle, |-\rangle$

$$|\psi\rangle = 2i|+\rangle - |0\rangle + |-\rangle$$

What is probability that a S_z measurement will give outcome $|+\rangle$?

$$\langle\psi|\psi\rangle = \big(-2i\langle +| - \langle 0| + \langle -|\big)\big(2i|+\rangle - |0\rangle + |-\rangle\big)$$

$$= 4 + 1 + 1 = 6$$

$$|\psi'\rangle = \frac{2i}{\sqrt{6}}|+\rangle - \frac{1}{\sqrt{6}}|0\rangle + \frac{1}{\sqrt{6}}|-\rangle$$

$$P_+ = |\langle +|\psi\rangle|^2 = \left|\frac{2i}{\sqrt{6}}\right|^2 = \frac{4}{6} = \frac{2}{3}$$

future

wavefunction $\quad \psi(x) = \langle x|\psi\rangle$

Chapter 2

Physical observables (e.g. S_z) are represented by <u>operators</u>.

Operator acts on a ket and returns a ket.

$$A|\psi\rangle = |\phi\rangle$$

Operators have special kets which do not change under operation except for a multiplicative constant.

$$\exists\ |\psi\rangle \qquad \underset{\text{operator}}{A}|\psi\rangle = \underset{\text{number}}{a}|\psi\rangle$$

These $|\psi\rangle$'s are eigenkets or <u>eigenstates</u> of A and corresponding numbers a are <u>eigenvalues</u>.

e.g. eigenstates of S_z

$S_z|\psi\rangle$ means "run $|\psi\rangle$ through a S_z analyzer"

eigenstates of S_z are $|\uparrow\rangle$ & $|\downarrow\rangle$

$$|\uparrow\rangle \rightarrow \boxed{S_z} \rightarrow |\uparrow\rangle$$

$$|\downarrow\rangle \rightarrow \boxed{S_z} \rightarrow |\downarrow\rangle$$

$$S_z|\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$$

$$S_z|\downarrow\rangle = -\frac{\hbar}{2}|\downarrow\rangle$$

(sometimes we'll choose units so $\frac{\hbar}{2} = 1$)

$$\boxed{\text{When you measure an observable } A\text{, only possible outcomes are the eigenvalues of } A\text{, and the system will end up in corresponding eigenstate of } A.}$$

Operators are ~~written~~ represented ~~as~~ by matrices.

In the S_z basis

$$S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_z|\uparrow\rangle = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$S_z|\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$$

An operator is always diagonal in its own basis with eigenvalues on the diagonal.