Physics 4310 Homework #9

5 problems

Due by Wednesday, April 6

▷ **1.**
Prove that if f is an eigenfunction of L_z with eigenvalue μ, then $L_\pm f$ is also an eigenfunction of L_z, but with eigenvalue $\mu + \hbar$.

▷ **2.**
The raising and lowering operators for spin are $S_\pm = S_x \pm iS_y$. Using the S_z matrix notation from Macintyre,
(a) ...prove that applying the raising operator to $|\uparrow\rangle$, or the lowering operator to $|\downarrow\rangle$, gives you zero
(b) show that $S_+|\downarrow\rangle \propto |\uparrow\rangle$ and $S_-|\uparrow\rangle \propto |\downarrow\rangle$

▷ **3.**
For a pair of spin–1/2 particles, prove that $s = 1$ for $|\uparrow\uparrow\rangle$ and $s = 0$ for $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$. Use the fact that $S^2\chi = \hbar^2 s(s+1)\chi$, and write $S^2 = (\vec{S}_1 + \vec{S}_2)\cdot(\vec{S}_1 + \vec{S}_2)$.

▷ **4.**
Consider a spin–1/2 particle and a spin–3/2 particle. Their total angular momentum is measured to be $s = 1$ and $m = 0$.
(a) What is the probability that the first particle is spin-up ($m_2 = +1/2$)?
(b) What other value(s) could s take?

▷ **5.**
(Griffiths 5.1) Typically, the interaction potential only depends on the vector $\vec{r} \equiv \vec{r}_1 - \vec{r}_2$. In that case the Schrodinger equation separates, if we change variables from $\vec{r}_1, \vec{r}_2$ to

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \quad \text{and} \quad \vec{R} \equiv = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

(the latter is the center of mass).
(a) If

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

is the reduced mass of the system, show that

$$\vec{r}_1 = \vec{R} + \frac{\mu}{m_1}\vec{r} \qquad \vec{r}_2 = \vec{R} - \frac{\mu}{m_2}\vec{r} \qquad \nabla_1 = \frac{\mu}{m_2}\nabla_R + \nabla_r \quad \text{and} \quad \nabla_2 = \frac{\mu}{m_1}\nabla_R - \nabla_r$$

(b) Show that the energy eigenstate equation (aka the time-independent Schrodinger equation) becomes

$$-\frac{\hbar^2}{2(m_1 + m_2)}\nabla_R^2\psi - \frac{\hbar^2}{2\mu}\nabla_r^2\psi + V(\vec{r})\psi = E\psi$$

(c) Separate the variables, letting $\psi(\vec{R}, \vec{r}) = \psi_R(\vec{R})\psi_r(\vec{r})$. Show that ψ_R satisfies the one-particle Schrodinger equation, with the *total* mass, potential zero, and some energy E_R. Show that ψ_r satisfies the one-particle Scrondinger equation with the *reduced* mass, potential $V(\vec{r})$, and some energy E_r, so that $E = E_R + E_r$.

What this tells us is that the center of mass moves like a free particle, and the relative motion is the same as if we had a single particle with the reduced mass, subject to the potential V. We can do the same thing in classical mechanics, reducing the two-body problem to an equivalent one-body problem.