Physics 3410 Homework #1

4 problems

Solutions

▷ **1.**

An ideal gas with 10^{23} molecules is at temperature $T = 300\,\mathrm{K}$. What is its total thermal energy U if the gas is
(a) neon (Ne)?
(b) hydrogen (H_2)?
(c) water (H_2O)?
Assume that vibrational degrees of freedom are frozen out.

Answer:

The relationship between thermal energy and temperature is given by the formula

$$U = N\frac{f}{2}kT$$

In this case

$$U = (10^{23})f(0.5)(1.38 \times 10^{-23}\,\mathrm{J/K})(300\,\mathrm{K}) = (207\,\mathrm{J})f$$

where f varies depending on the type of gas.

(a) Neon atoms have spherical symmetry, so only has translational degrees of freedom. $f = 3$, so $\boxed{U = 621\,\mathrm{J}}$.

(b) Hydrogen molecules have cylindrical symmetry, so has 2 degrees of freedom: $f = 3 + 2 = 5$, so $\boxed{U = 1035\,\mathrm{J}}$.

(c) Water molecules have no continuous symmetry, so $f = 3 + 3 = 6$, and $\boxed{U = 1242\,\mathrm{J}}$.

▷ **2.**

When $300\,\mathrm{J}$ of heat are added to a certain block of metal, its temperature increases by $10\,\mathrm{C}^\circ$.

(a) What is the metal's heat capacity?
(b) How many metal atoms are in this block? (Hint: what is f for a solid? Assume that no modes are "frozen out".)

Answer:

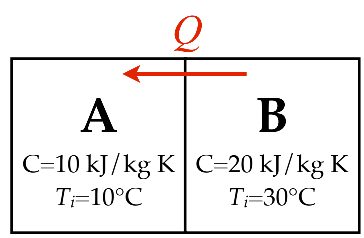
(a) The heat capacity is from the equation $Q = C\Delta T$, or $C = \frac{Q}{\Delta T}$. If $Q = 300\,\mathrm{J}$ of heat can cause the temperature to rise by $\Delta T = 10\,\mathrm{C}^\circ$, then $C = 300\,\mathrm{J}/10\,\mathrm{C}^\circ = \boxed{30\,\mathrm{J/K}}$.

(b) A solid has $f = 6$ degrees of freedom, and we saw in class that $C = N\frac{f}{2}k$, provided that no work is done when heat is added to the block. (In practice, the block may expand a little bit when heated, which does a *little* bit of work on the surrounding air, but we can ignore that.) Solving this equation for N:

$$N = \frac{2C}{fk} = \frac{2(30\,\mathrm{J/K})}{6(1.38\times10^{-23}\,\mathrm{J/K})} = \boxed{7.2\times10^{23}}$$

▷ **3.**

Block A, with heat capacity $C_A = 10\,\mathrm{kJ/kg/K}$ and initial temperature $T = 10°\,\mathrm{C}$ is placed next to Block B, which has heat capacity $C_B = 20\,\mathrm{kJ/kg/K}$ and initial temperature $T = 30°\,\mathrm{C}$. What is the final temperature T_f of both blocks, when they reach equilibrium, assuming that no heat is exchanged with the environment? Hint: start by calculating the total heat $-Q$ that flows out of B as a function of T_f, and then the heat Q that flows into A. That gives us two functions in two variables, Q and T_f.



Answer:________________________________
When heat Q flows into block A, its temperature change is related to it by $Q = C_A\Delta T$, so

$$Q = (10\,\mathrm{kJ/kg/K})(T_f - 10°\,\mathrm{C})$$

The same heat Q flows out of block B, and so

$$-Q = (20\,\mathrm{kJ/kg/K})(T_f - 30°\,\mathrm{C})$$

(I've made the heat negative because the heat is flowing out of B. You could have instead written $Q = (20\,\mathrm{kJ/kg/K})(30°\,\mathrm{C} - T_f)$ so that $30 - T_f$ is positive.)

We can add both equations together:

$$\begin{aligned}
0 &= (10)(T_f - 10) + (20)(T_f - 30) \\
&= 10T_f - (10)(10) + 20T_f - (20)(30) \\
(10)(10) + (20)(30) &= (10+20)T_f \\
\implies T_f &= \frac{(10)(10)+(20)(30)}{10+20} \\
&= \frac{700}{30} = \boxed{23.3°\,\mathrm{C}}
\end{aligned}$$

Look at the second-to-last line, and notice that it has the form

$$\frac{C_A T_{iA} + C_B T_{iB}}{C_A + C_B}$$

This has the form of a weighted average of the temperatures, where the heat capacities serve as the weights. Because B has the higher heat capacity, the final temperature is closer to its initial temperature than it is to A's. This result generalizes to multiple systems, as well.

▷ **4.**

Suppose I have $1\,\mathrm{kg}$ of liquid water at $370\,\mathrm{K}$ (just below the boiling point). How much water ice (in kilograms) at $273\,\mathrm{K}$ do I have to add to bring the temperature down to $300\,\mathrm{K}$? (Remember that you have to melt the ice and warm it up to $300\,\mathrm{K}$.) The specific heat of liquid water is $c = 4.2\,\mathrm{kJ/kg/K}$ and the latent heat of melting is $L = 333\,\mathrm{kJ/kg}$.

Answer:____________________________________

The hot water will cool from $370\,\mathrm{K}$ to $300\,\mathrm{K}$, which means that

$$Q = mc\Delta T = (1\,\mathrm{kg})(4.2\,\mathrm{kJ/kg/K})(70\,\mathrm{K}) = 294\,\mathrm{kJ}$$

of heat leaves the hot water, and enters the ice, where it must melt it *and* raise the resulting cold water from $273\,\mathrm{K}$ to $300\,\mathrm{K}$. If m is the mass of the ice, then

$$\begin{aligned}
294\,\mathrm{kJ} &= mL + mc\Delta T = m(333\,\mathrm{kJ/kg} + m(4.2\,\mathrm{kJ/kg/K})(300\,\mathrm{K} - 273\,\mathrm{K}) \\
\implies m &= \frac{294\,\mathrm{kJ}}{333\,\mathrm{kJ/kg} + 113.4\,\mathrm{kJ/kg}} \\
&= \boxed{0.66\,\mathrm{kg}}
\end{aligned}$$

Notice that if we started with cold water at the freezing point, then the denominator would just be $113.4\,\mathrm{kJ/kg}$, and so we would need $2.6\,\mathrm{kg}$ of cold water instead—four times as much. Ice is great for cooling things!