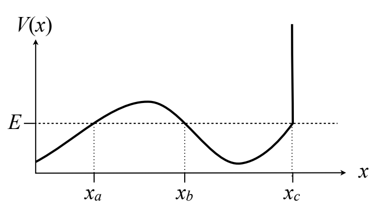
Physics 4310 Name:________________________

April 15, 2016 **Exam 2** 16 questions, 67 points

- *This exam is due back by Friday, April 22nd, at 1:00pm. It should not take you that long; it would probably be prudent to aim to finish it by Wednesday's class, so you don't have to worry about flat tires, oversleeping, etc.*
- *You may use a calculator, your textbook, your notes, anything I've posted on the website, and a symbolic algebra system like Mathematica, Maple, WolframAlpha, etc.*
- *You may not use any other quantum mechanics books or the Internet other than for WolframAlpha as mentioned above, without prior authorization from me.*
- *You may not consult with anyone other than me about the exam.*
- *Partial credit is available* everywhere. *Go ahead and explain any answer you wish.*
- *Anywhere you see* [O/X], *that means to circle all correct answers, and X out the answers that are not correct.*
- *Go ahead and attach extra paper to the end of this exam if you need it; make sure any work is clearly labelled with the number of the problem it corresponds to. You have plenty of time so please submit* neat, succinct *work and my partial credit pen will look more kindly towards it.*
- *Good luck!*

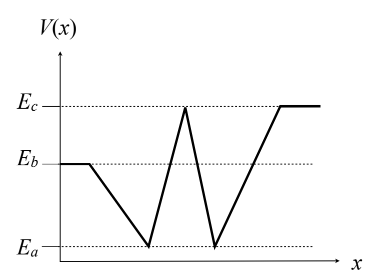
$\boxed{3}$ 1. Given the potential $V(x)$ and a particle with energy E as shown, in which regions does $\psi(x) = 0$? [O/X]

$x < x_a$ $\quad$ $x_a < x < x_b$ $\quad$ $x_b < x < x_c$ $\quad$ $x > x_c$



$\boxed{3}$ 2. For which ranges of E does this potential have scattering state solutions? [O/X]

$E < E_a$ $\quad$ $E_a < E < E_b$ $\quad$ $E_b < E < E_c$ $\quad$ $E > E_c$



$\boxed{3}$ 3. A particle in the potential

$$V(x) = \begin{cases} 0, & 0 \le x \le a \\ \infty, & \text{otherwise} \end{cases}$$

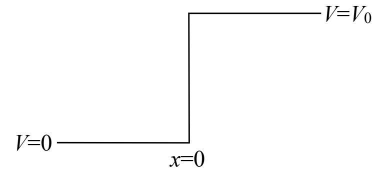
has, at time $t = 0$, the wavefunction

$$\Psi(x, 0) = \frac{1}{\sqrt{a}} \left[\sin\left(\frac{\pi x}{a}\right) - \sin\left(\frac{3\pi x}{a}\right) \right]$$

Write $\Psi(x, t)$.

$\boxed{3}$ 4. A delta-function well $V(x) = -\alpha\delta(x)$ $(\alpha > 0)$ has exactly one bound-state solution: $E = -\frac{m\alpha^2}{2\hbar^2}$. For what values of α does it also have the solution $E = +\frac{m\alpha^2}{2\hbar^2}$?

5. One energy eigenstate of the Hamiltonian with potential



$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x > 0 \end{cases}$$

has the form

$$\psi(x) = \begin{cases} \frac{2}{3} G e^{-ikx} & x < 0 \\ F e^{+ik'x} + G e^{-ik'x} & x > 0 \end{cases}$$

where $k = \frac{\sqrt{2mE}}{\hbar}$, $k' = \frac{\sqrt{2m(E-V_0)}}{\hbar}$, and $E = \frac{4}{3}V_0$.

$\boxed{3}$ (a) Find F in terms of G.

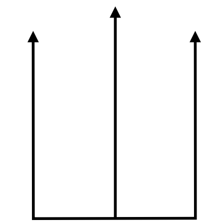
$\boxed{2}$ (b) _____ This solution $\psi(x)$ models an incident ray travelling to the $\ldots$, along with its reflection and transmission at $x = 0$.

A) left $\leftarrow$ **B)** right $\rightarrow$

$\boxed{3}$ (c) Find the reflection coefficient R. (The transmission coefficient is a little trickier.)

6. Consider an infinite square well with a delta barrier in the middle.

$$V(x) = \begin{cases} \alpha\delta(x), & -\frac{a}{2} < x < \frac{a}{2} \\ \infty, & \text{otherwise} \end{cases}$$



$\boxed{3}$ (a) Write wavefunctions $\psi_L(x)$ and $\psi_R(x)$ for either side of the delta barrier, as a function of $k = \frac{\sqrt{2mE}}{\hbar}$. I recommend writing ψ_L in terms of $x + \frac{a}{2}$ and ψ_R in terms of $x - \frac{a}{2}$.

$\boxed{3}$ (b) Use boundary conditions to find an equation for k. It will be transcendental and have a tangent in it; don't try to solve it.

$\boxed{2}$ (c) _____ The overall eigenstates $\psi_n(x) = \begin{cases} \psi_L(x) & -\frac{a}{2} < x < 0 \\ \psi_R(x) & 0 < x < \frac{a}{2} \end{cases}$ are

A) all even ($\psi(x) = \psi(-x)$) **B)** all odd ($\psi(x) = -\psi(-x)$)
C) Alternate between even and odd

$\boxed{3}$ 7. For the harmonic oscillator, the operator $N = a_+ a_-$ is Hermitian, and it commutes with the Hamiltonian, so the energy eigenstates ψ_n of H are also eigenstates of N. What is the eigenvalue of N that corresponds to ψ_n?

$\boxed{3}$ 8. Calculate $\langle xp \rangle$ for a harmonic oscillator in energy eigenstate ψ_n. Use raising and lowering operators. (Warning: this isn't a Hermitian operator so your answer may seem a little... *imaginary*.)

9. Consider a wavefunction

$$\psi(x) = \sum_{j=0}^{\infty} c_j x^j$$

where $x > 0$. The coefficients obey the recursion relation

$$c_{j+1} = \frac{j - K}{(j+1)(j+2)} c_j$$

$\boxed{3}$ (a) Explain why K must be an integer. Be specific.

$\boxed{3}$ (b) If $K = 2$, what is $\psi(x)$ (in terms of c_0)?

$\boxed{3}$ 10. _____ Consider the wavefunction

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx - ck^4 t)}\, dk$$

for some constant c. What is v_g/v_p, the ratio of its group velocity to its phase velocity?
A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** c

$\boxed{3}$ 11. Find the normalized ground state wavefunction $\psi(r, \theta, \phi)$ for a harmonic oscillator in three dimensions and no angular momentum, with potential

$$V(x) = \frac{1}{2}kr^2$$

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12. Suppose particle A is a spin–3 particle, and particle B is a spin–1 particle. Their total m is $m = +1$.

$\boxed{3}$ (a) Which of these are the possible values for the total spin s? **[O/X]**

-2 $\quad -1$ $\quad 0$ $\quad 1$ $\quad 2$ $\quad 3$ $\quad 4$ $\quad 5$

$\boxed{3}$ (b) Which of the following could be the values of m_A and m_B? **[O/X]**

$m_A = 3$	$m_A = 1$	$m_A = -2$	$m_A = 1$
$m_B = -2$	$m_B = 0$	$m_B = 1$	$m_B = 1$

$\boxed{3}$ 13. Suppose a spin–1/2 particle is placed in a constant magnetic field $\vec{B} = B\hat{x}$, where B is a constant. Its Hamiltonian is $H = -\vec{\mu} \cdot \vec{B}$ or

$$H = -\gamma B S_x$$

Find $\dfrac{d\langle S_y \rangle}{dt}$ for the $|\uparrow\rangle$ state (with $m = 1/2$).

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14. Write the following without the L operator.

$\boxed{3}$ (a) $L_z Y_3^1 =$

$\boxed{3}$ (b) $L^2 Y_3^1 =$

$\boxed{3}$ 15. _____ If $\vec{L}$ is the usual angular momentum operator, then $(L_x - iL_y)Y_3^1(\theta,\phi)$ is equal to which of these (not counting normalization):

A) 0
B) $Y_2^0(\theta,\phi)$
C) $Y_2^1(\theta,\phi)$
D) $Y_3^0(\theta,\phi)$
E) $Y_3^1(\theta,\phi)$
F) $Y_2^1(\theta,\phi)$
G) $Y_3^2(\theta,\phi)$

$\boxed{3}$ 16. _____ For a hydrogen atom, which of the following are legitimate sets of values for n, l, & m? [O/X]

$n = 5$	$n = 6$	$n = 7$	$n = 8$
$l = 5$	$l = 1/2$	$l = 6$	$l = 2$
$m = 0$	$m = 1/2$	$m = -5$	$m = 3$