Physics 3410 Homework #10

4 problems

Due by April 20

▷ **1.**

Fun with standard deviations. We showed in class that $\langle E \rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$. Now

(a) Show that, for a system in equilibrium with a reservoir at temperature T, the average value of E^2 is

$$\left\langle E^2 \right\rangle = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2}$$

(b) The standard deviation of the energy (that is, the size of its fluctuations around the average) is defined as

$$\sigma_E^2 = \left\langle E^2 \right\rangle - \langle E \rangle^2$$

Prove that the standard deviation is related to the heat capacity $C = \frac{\partial \langle E \rangle}{\partial T}$ by the formula

$$\sigma_E = kT\sqrt{C/k}$$

▷ **2.**

Starting from the partition function for the ideal gas that we derived in class, by way of the Helmholtz free energy, derive

(a) The ideal gas law

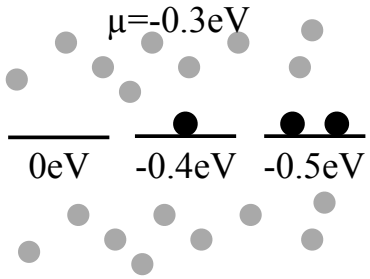
(b) The Sackur-Tetrode equation

▷ **3.**

Write the partition function of an ideal gas of carbon monoxide. Treat the vibrations in the bond between the atoms as a harmonic oscillator (see last week's homework), and assume that the temperature is high enough so that all modes are active.

▷ **4.**

Consider a system (like a bonding site on a molecule) which is surrounded by a reservoir of particles having a chemical potential of $\mu = -0.3\,\text{eV}$ and temperature $T = 300\,\text{K}$. The system can be empty, or can contain one or two particles. if the system is empty, its energy is $0\,\text{eV}$. If it holds one particle, the system has energy $-0.4\,\text{eV}$, and if it holds two particles, it has energy $-0.5\,\text{eV}$.



(a) Find the Gibbs factor for each of the three possible states

(b) Find the grand partition function $\mathcal{Z}$

(c) What is the probability that the system is empty?

(d) What is the average occupancy $\bar{n}$ of the system? That is, on average how many particles does it contain?