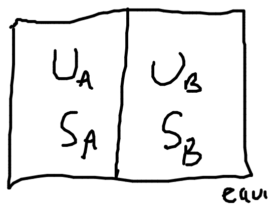
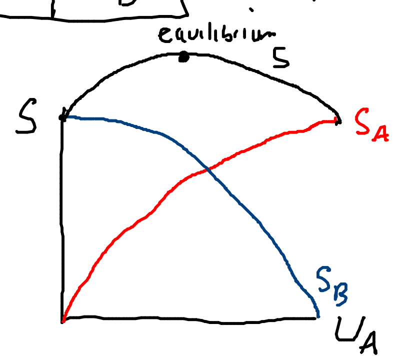


$U_A + U_B = U \leftarrow$ constant

$S_A + S_B = S \leftarrow$ not constant



$U_B = U - U_A$

$\frac{\partial U_B}{\partial U_A} = -1$

$$\frac{\partial S}{\partial U_A} = 0 \quad \text{at equilibrium}$$

$$= \frac{\partial S_A}{\partial U_A} + \frac{\partial S_B}{\partial U_A}$$

$$= \frac{\partial S_A}{\partial U_A} + \frac{\partial S_B}{\partial U_B}\underbrace{\frac{\partial U_B}{\partial U_A}}_{-1}$$

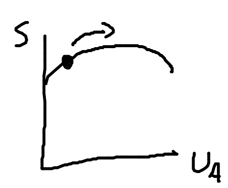
$$\frac{\partial S}{\partial U_A} = \frac{\partial S_A}{\partial U_A} - \frac{\partial S_B}{\partial U_B}$$

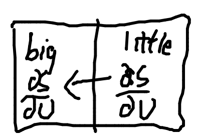
At equilibrium, $\frac{\partial S_A}{\partial U_A} = \frac{\partial S_B}{\partial U_B}$

Is that temperature? No.

$\frac{\partial S_A}{\partial U_A} > \frac{\partial S_B}{\partial U_B}$

energy flows $B \rightarrow A$





energy flows to larger $\frac{\partial S}{\partial U}$

$$\left[\frac{\partial S}{\partial U}\right] = \frac{J/K}{J} = \frac{1}{K}$$

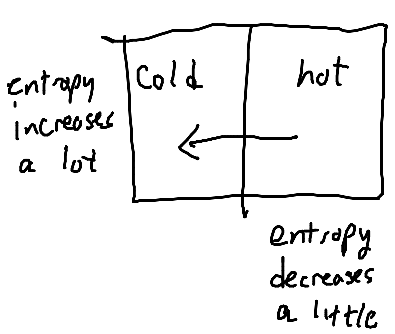
$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N} \quad \text{OR} \quad T = \left(\frac{\partial U}{\partial S}\right)_{V,N}$$

At constant V, N : $\frac{1}{T} = \frac{dS}{dU} \longrightarrow dS = \frac{dU}{T}$ ← small change in U

dS ↑ small change in S

if T small, then entropy changes a lot

if T large, then entropy changes a little



total entropy increases when energy (heat) flows from hot to cold

S is "quasi-conserved" : never destroyed, but can be created

Heat flow is a major method of entropy increase & the only method for a system to lose entropy.

e.g. Einstein Solid $q \gg N \gg 1$

$$\Omega = \left(\frac{eq}{N}\right)^N \rightarrow S = k\ln\Omega = kN\left[1 + \ln\frac{q}{N}\right]$$

Let $U = qu$ u: energy per quantum

$$S = kN\left[1 + \ln\frac{U/u}{N}\right]$$

$$= kN\left[1 + \ln U - \ln u - \ln N\right]$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{kN}{U} \qquad T = \frac{U}{Nk} \rightarrow U = NkT$$

equipartition theorem

$f = 2$

(vibrational d.o.f.)

e.g. ideal gas

$$S = Nk\left[\ln\frac{V}{N} + \frac{3}{2}\ln\frac{U}{N} + c\right]$$

$$= \frac{3}{2}Nk\ln U + V, N \text{ stuff}$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{3}{2}\frac{Nk}{U} \rightarrow U = \frac{3}{2}NkT$$

e.p.t. with $f = 3$

(point particles, translational only)

generally, at const V, N

if $U = N\frac{f}{2}kT = N\frac{f}{2}k\frac{dU}{dS}$

$$\rightarrow \int dS = \int N\frac{f}{2}k\frac{dU}{U}$$

$$S = \frac{Nf}{2}k\ln U + f(V, N)$$

$$= \underbrace{\frac{f}{2}Nk\ln U}_{\text{If entropy has this term, you can read off \# d.o.f.}} + \ldots$$

$S = k\ln\Omega$

$$\Omega = e^{S/k} = e^{\frac{f}{2}N\ln U} * \ldots = C\,U^{Nf/2}$$

e.g. ideal gas of O_2 $f = 5$ at room temperature

$$\Omega \propto U^{N\frac{5}{2}}$$