Physics 3410 Homework #9

5 problems

Solutions

▷ **1.**

If I take air ($f = 5$) at room temperature ($T_i = 300\,\mathrm{K}$) and compress it adiabatically to a tenth of its initial value, what is its temperature T_f after the compression?

Answer:

During an adiabatic process, PV^γ remains constant where $\gamma = \frac{f+2}{f} = \frac{7}{5}$ is the adiabatic exponent. In this problem we're interested in temperature rather than pressure, so we can use the ideal gas law to rewrite PV^γ as

$$PV^\gamma = \frac{NkT}{V}V^\gamma = NkTV^{\gamma-1}$$

so $TV^{\gamma-1} = TV^{2/5}$ must be constant. If we use the subscripts i and f to refer to initial and final, and we note that $V_f = \frac{1}{10}V_i$, then

$$\begin{aligned}
T_i V_i^{2/5} &= T_f V_f^{2/5} \\
\implies T_f &= T_i \left(\frac{V_i}{V_f}\right)^{2/5} \\
&= (300\,\mathrm{K})(10)^{2/5} \\
&= (300\,\mathrm{K})(2.51) = \boxed{754\,\mathrm{K}}
\end{aligned}$$

▷ **2.**

Consider an engine which operates between room temperature $T_c = 300\,\mathrm{K}$ and some hot reservoir T_h. What is the minimum value of T_h so that the engine can have an efficiency of 50%?

Answer:

The efficiency of an engine has the limit

$$\eta \leq 1 - \frac{T_c}{T_h}$$
$$\frac{T_c}{T_h} \leq 1 - \eta$$
$$T_h \geq \frac{T_c}{1-\eta}$$
$$T_h \geq \frac{300\,\mathrm{K}}{1-0.5}$$
$$T_h \geq 600\,\mathrm{K}$$

Thus the hot reservoir has to be at least $\boxed{600\,\mathrm{K}}$ (or 327° C).

▷ **3.**
Suppose a power plant produces 1 GW of electricity at 40% efficiency, taking in steam at a temperature of 500° C. The waste heat is expelled into the environment at 20° C. At what rate (in watts) is heat expelled into the environment?

Answer:____________________________________
The power put out by the power plant is $W = 10^9\,\mathrm{W}$, and I want to know Q_{out}. The heat into the power plant $Q_{in} = W + Q_{out}$ (the same equation works for power as for energy, if we think of the variables as measuring the amount of energy in one second), and the efficiency is

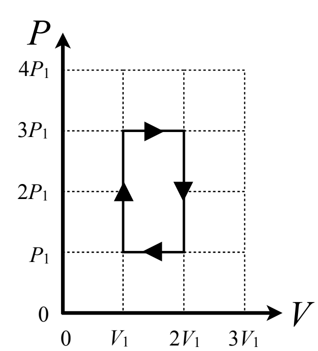
$$\eta = \frac{W}{Q_{in}} = \frac{W}{W + Q_{out}} \implies W = (W + Q_{out})\eta \implies W\frac{1-\eta}{\eta} = Q_{out}$$

Because the efficiency is $\eta = 0.4$, the heat out is

$$Q_{out} = \frac{1-0.4}{0.4}(1\,\mathrm{GW}) = \boxed{1.5\,\mathrm{GW}}$$

(No, the temperature didn't matter.)

▷ **4.**
The figure shows a heat engine which involves constant-pressure and constant-volume processes, where the pressure goes between some value P_1 and $3P_1$, and the volume goes between some value V_1 to $2V_1$. The engine uses air, which has $f = 5$.
(a) Calculate the internal energy U at each of the four corners.
(b) Calculate the work flow during each of the four processes.
(c) Calculate the heat flow during each of the four processes.
(d) Calculate the efficiency η of this engine.



Answer:______________________________________

(a) The thermal energy is $U = \frac{f}{2}NkT = \frac{f}{2}PV = \frac{5}{2}PV$, so the energies of the four corners are

$$\begin{matrix} 7.5P_1V_1 & 15P_1V_1 \\ 2.5P_1V_1 & 5P_1V_1 \end{matrix}$$

(b) The work flow is the integral $W = -\int P\,dV$. The left and right branches have no change in volume, so the work done during those processes is zero. ($W_l = W_r = 0$). The work done by the bottom process is $W_b = -P_1(V_1 - 2V_1) = P_1V_1$, and the work done by the top process is $W_t = -3P_1V_1$.

(c) To calculate heat flow during a process, we need to refer to the first law of thermodynamics: $Q = \Delta U - W$. We can calculate ΔU from the results of part (a) (remembering that ΔU means final minus initial).

- Top: $\Delta U = 15P_1V_1 - 7.5P_1V_1 = 7.5P_1V_1$
$W_t = -3P_1V_1$
$Q_t = 7.5P_1V_1 - (-3P_1V_1) = 10.5P_1V_1$
- Right: $\Delta U = 5P_1V_1 - 15P_1V_1 = -10P_1V_1$
$W_r = 0$
$Q_r = -10P_1V_1 - 0 = -10P_1V_1$
- Bottom: $\Delta U = 2.5P_1V_1 - 5P_1V_1 = -2.5P_1V_1$
$W_b = P_1V_1$
$Q_b = -2.5P_1V_1 - P_1V_1 = -3.5P_1V_1$
- Left: $\Delta U = 7.5P_1V_1 - 2.5P_1V_1 = 5P_1V_1$
$W_l = 0$
$Q_l = 5P_1V_1$

(d) The efficiency of the engine is

$$\eta = \frac{W_e}{Q_{in}}$$

The total work done on the gas is $W = W_t + W_b = -3P_1V_1 + P_1V_1 = -2P_1V_1$. This is the work that flows into the gas; the work done by the engine W_e is actually the negative of this, or $W_e = +2P_1V_1$.

Each process above has an associated heat flow, but only the positive ones are heat flow *in*:

$$Q_{in} = Q_t + Q_l = 10.5P_1V_1 + 5P_1V_1 = 15.5P_1V_1$$

(The heat flow out is $Q_{out} = -(Q_b + Q_r) = 13.5P_1V_1$, and it's good to check that $Q_{in} = W_e + Q_{out}$.)

Thus the efficiency is

$$\eta = \frac{2P_1V_1}{15.5P_1V_1} = \boxed{13\%}$$

▷ **5.**
Suppose that heat leaks into a typical kitchen refrigerator (without freezer) at an average rate of 400 W. What is the minimum amount of power it needs to draw from the wall? (Refrigerators generally keep food at 40° F.)

Answer:
We know that the COP of a refrigerator is no better than

$$\frac{T_c}{T_h - T_c}$$

In this case $T_h \approx 300\,\mathrm{K}$ and $T_c = 40^\circ\,\mathrm{F} = 4^\circ\,\mathrm{C} = 277\,\mathrm{K}$; thus the maximum COP is $\frac{277}{300-277} = 12.0$. The amount of energy a refrigerator can pull from its cold reservoir is the COP times the amount of work input. If the refrigerator has to remove $400\,\mathrm{W}$ from its cold reservoir, then it must draw $400\,\mathrm{W}/12 = \boxed{33\,\mathrm{W}}$ from the wall.