Physics 4310 Name:________________________

due May 6, 2016 at 5pm **Final Exam** 19 questions, 77 points

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- *This exam is due back by Friday, May 6th, at 5:00pm. Please slide it under my office door (in MH4019).*
- *You may use a calculator, your textbook, your notes, anything I've posted on the website, and a symbolic algebra system like Mathematica, Maple, WolframAlpha, etc.*
- *You may not use any other quantum mechanics books or the Internet other than for WolframAlpha as mentioned above, without prior authorization from me.*
- *You may not consult with anyone other than me about the exam.*
- *Partial credit is available* everywhere. *Go ahead and explain any answer you wish.*
- *Go ahead and attach extra paper to the end of this exam if you need it; make sure any work is clearly labelled with the number of the problem it corresponds to. You have plenty of time so please submit* neat, succinct *work and my partial credit pen will look more kindly towards it.*
- *Good luck!*

$\boxed{3}$ 1. _____ A particle in an infinite square well of width a starts off in the ground state, with energy E_0. The well is then slowly stretched to twice its initial width. After the stretching is done, the average energy of the particle is
A) $\frac{1}{4}E_0$ **B)** $\frac{1}{2}E_0$ **C)** E_0 **D)** $2E_0$ **E)** $4E_0$

$\boxed{3}$ 2. _____ Two particles, at positions x_1 and x_2, have the wavefunction

$$\psi(x_1, x_2) = A(x_1 + x_2)e^{(x_1-x_2)^2/2}$$

These particles are
A) bosons **B)** fermions **C)** neither of these

3. Consider the once-ionized Helium atom (two protons, one electron). To a first approximation, the solutions are the same as that of the hydrogen atom, except with e^2 replaced with $2e^2$. (For example, $V(r) = -\frac{2e^2}{4\pi\epsilon_0}\frac{1}{r}$.)

$\boxed{2}$ (a) Write the ground state wavefunction of the helium atom, in terms of the standard Bohr radius $a = 4\pi\epsilon_0\hbar^2/me^2$.

$\boxed{4}$ (b) This first approximation assumes that the two protons are in the same location. If they are in different locations, then their potential has additional terms, of which the largest is the dipole term:

$$V(r) = -\frac{2e^2}{4\pi\epsilon_0}\frac{1}{r} + \frac{\epsilon}{r^2}$$

where ϵ is a small constant. (The potential wouldn't be radially symmetric, but let's ignore that.) Find the ground-state energy with this correction. *Remember to integrate in three dimensions.*

$\boxed{5}$ 4. Consider a free particle in a potential $V(x) = 0$, but subject to periodic boundary conditions $\psi(x+L) = \psi(x)$. (For instance, the particle might be constrained to slide around a ring with circumference L.) The energy eigenstates are the usual free particle solutions

$$\psi(x) = \frac{1}{\sqrt{L}} e^{\pm ikx}$$

where $k = \sqrt{2mE}/\hbar$, except the wavefunction is normalizable, *and* the boundary condition quantizes the available energies:

$$k_n = \frac{2\pi n}{L} \quad \text{and} \quad E_n = \frac{4\pi^2\hbar^2}{2mL^2} n^2$$

where $n = 0, \pm 1, \pm 2, \ldots$. Suppose we introduce a perturbation to the Hamiltonian:

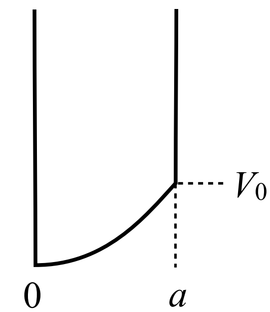
$$H' = V_0,\ 0 < x < L/4$$

This Hamiltonian breaks the degeneracy of the states $n = +1$ and $n = -1$, splitting the one energy into two. Find the size ΔE of the split, in terms of L and V_0. *(Note: remember that $\langle \psi_1 | A | \psi_2 \rangle = \int \psi_1^* A \psi_2 \, dx$: don't forget the complex conjugate.)*

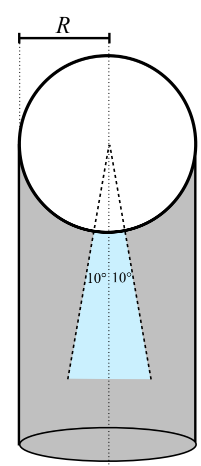
$\boxed{3}$ 5. Consider the infinite square well potential with a rounded bottom:

$$V(x) = \begin{cases} V_0(x/a)^2, & 0 < x < a \\ \infty, & \text{otherwise} \end{cases}$$

Use the WKB approximation to find an integral expression for the allowed energies E_n in this well. (Don't do the integral.)



$\boxed{3}$ 6. A uniform beam of particles of radius R is shot at a hard sphere with radius R, as shown. What fraction of the particles are reflected back at a 10° angle from the beam's axis? (This is a completely classical problem, by the way.)



7. Consider two electrons in the singlet state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$.

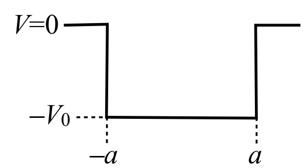
(a) ________________ If I measure the first electron with a S_x operator, what is the probability that the measurement gives the outcome $|\odot\rangle$ (i.e. $+\hbar/2$)?

A) 0% **B)** 50% **C)** 100% **D)** None of these.

$\boxed{2}$ (b) If S_x operators are applied to both electrons at the same time, what can you say about the outcome of the measurements?

$\boxed{3}$ 8. Given the potential

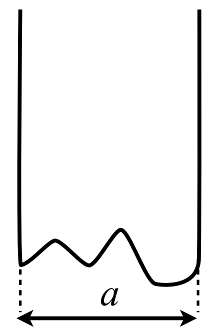
$$V(x) = \begin{cases} -V_0 & -a < x < a \\ 0 & |x| > a \end{cases}$$



If $V_0 = \dfrac{\pi^2\hbar^2}{16ma^2}$, what are the possible positive energies $E > 0$?

$\boxed{3}$ 9. _____ If a particle sits in an infinite square well of width a with an arbitrarily bumpy bottom, which of these is the best lower bound on the uncertainty Δp in its momentum? (Use Heisenberg, not WKB. No integrals necessary.)

A) $\Delta p \geq \frac{\hbar}{2a}$ **B)** $\Delta p \geq \frac{\hbar}{a}$ **C)** $\Delta p \geq \frac{2\hbar}{a}$



$\boxed{3}$ 10. _____ Consider two electrons contained in some sort of well. In which spin state χ will the electrons be farther apart in space?

A) $\chi = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ **B)** $\chi = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$\boxed{4}$ 11. Match the following expressions to one of these.

A) 0 **B)** $\sqrt{2}\hbar Y_1^0$ **C)** $-\hbar Y_1^{-1}$ **D)** $\hbar Y_1^{-1}$ **E)** $2\hbar^2 Y_1^{-1}$ **F)** $\hbar Y_2^{-1}$

(a) _____ $L^2 Y_1^{-1}$

(b) _____ $L_z Y_1^{-1}$

(c) _____ $L_+ Y_1^{-1}$

(d) _____ $L_- Y_1^{-1}$

12. Consider the harmonic oscillator potential

$$V(x) = \frac{1}{2}kx^2$$

A particle is in the state

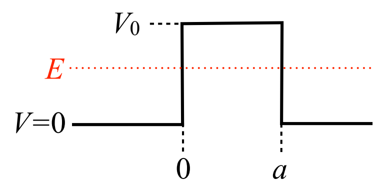
$$\Psi(x,0) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left[\frac{3}{5} + i\frac{4}{5}\sqrt{\frac{2m\omega}{\hbar}}x\right]e^{-\frac{m\omega}{2\hbar}x^2}$$

at time $t = 0$.

$\boxed{3}$ (a) Find $\Psi(x,t)$.

$\boxed{3}$ (b) If I measure the energy of this particle, what outcomes E could I get, and with what probabilities?

13. Consider a finite square barrier of height V_0, as shown. A particle has an energy E which lies below the top of the barrier, and $k = \sqrt{2mE}/\hbar$ while $\kappa = \sqrt{2m(V_0 - E)}/\hbar$. The wavefunction is



$$\psi(x) = \begin{cases} 5Ae^{ikx} + 3Ae^{-ikx}, & x < 0 \\ Ce^{\kappa x} + De^{-\kappa x}, & 0 < x < a \\ 4iAe^{ikx}, & x > a \end{cases}$$

$\boxed{3}$ (a) Find the transmission coefficient T through the barrier.

$\boxed{3}$ (b) Find C/A in terms of k and κ. (Look at $x = 0$.)

$\boxed{3}$ 14. A particle is in a three-dimensional potential

$$V(\vec{r}) = Ar^3$$

The particle is in an energy eigenstate

$$\psi(r,\theta,\phi) = \frac{A}{r}u(r)Y_2^1(\theta,\phi)$$

with energy E. What differential equation does $u(r)$ satisfy?

$\boxed{3}$ 15. What is the probability that the electron in the ground state $\psi_{100}(r,\theta,\phi)$ of the hydrogen atom is in the range $0 < r < a$?

$\boxed{3}$ 16. Suppose ψ_i are simultaneously (orthonormal) eigenstates of H and the Hermitian operator A, with eigenvalues E_i and a_i accordingly. Simplify

$$\langle\psi_1|HA|\psi_2\rangle + \langle\psi_1|AH|\psi_1\rangle$$

17. A spin–1/2 particle has z–component $m_1 = -1/2$, and a spin–1 particle has $m_2 = -1$.

$\boxed{3}$ (a) What are the possible values of the total z–component m of both particles?

$\boxed{3}$ (b) What are the possible values of the total spin s of both particles?

$\boxed{3}$ 18. Given the wavefunction

$$\psi(x) = \frac{1}{\sqrt{a}}\left[\sin\left(\frac{\pi x}{a}\right) + i\sin\left(\frac{2\pi x}{a}\right)\right]$$

in the infinite square well potential with width a. Find the expectation value $\langle p \rangle$ of its momentum.

19. Consider two operators in the S_z basis, the Hamiltonian H and another A:

$$H = \begin{pmatrix} 1 & -i \\ i & 3 \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} 2 & i \\ -i & 1 \end{pmatrix}$$

$\boxed{3}$ (a) If the energy of a state $|\psi\rangle$ is measured, what possible values could it have?

$\boxed{3}$ (b) What is the average energy of the state $|\uparrow\rangle$?

$\boxed{3}$ (c) If $|\psi\rangle = |\odot\rangle$ at time $t = 0$, find $\dfrac{\partial \langle A \rangle}{\partial t}$ at $t = 0$.