Physics 3410 Exam 1 Solutions

February 29, 2016

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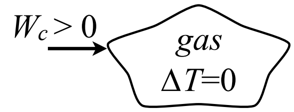
1. If positive compression work is done on an ideal gas, and its temperature remains constant,



$\boxed{3}$ (a) **B** The internal energy of the gas

A) increases **B)** stays constant **C)** decreases

$\boxed{3}$ (b) **B** What direction does heat flow during this process?

A) into the gas **B)** out of the gas **C)** neither

$\boxed{3}$ (c) **A** The gas's volume is

A) decreasing **B)** increasing

$\boxed{3}$ 2. **D** In laser cooling, physicists are able to bring atoms very close to absolute zero by using lasers to slow the motion of atoms. The atoms' temperature drops because . . . the atoms.

A) heat flows into **B)** heat flows out of

C) work flows into **D)** work flows out of

3. An ideal gas of point particles has volume $V = 1\,\mathrm{m}^3$, pressure $P = 1.2\,\mathrm{atm}$ ($1\,\mathrm{atm} = 10^5\,\mathrm{Pa}$), and temperature $T = 300\,\mathrm{K}$.

$\boxed{3}$ (a) Find the internal energy U of the gas. (Hint: find NkT first using the ideal gas law.)

$NkT = PV = (1.2\times 10^5\,\mathrm{Pa})(1\,\mathrm{m}^3) = 1.2\times 10^5\,\mathrm{J}$, and so

$$U = \frac{3}{2}NkT = \boxed{1.8\times 10^5\,\mathrm{J}}$$

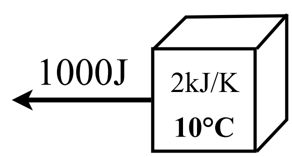
$\boxed{3}$ (b) **D** Suppose the gas is sealed in a box (so its volume remains constant), and it is heated to 400 K. The pressure of the gas after it has been heated is

A) 0.3 atm **B)** 0.9 atm **C)** 1.2 atm **D)** 1.6 atm **E)** 4.8 atm

The pressure is linearly proportional to the temperature ($PV = NkT$), so $\frac{P_f}{P_i} = \frac{T_f}{T_i}$, or

$$P_f = P_i\frac{T_f}{T_i} = (1.2\,\mathrm{atm})\frac{400\,\mathrm{K}}{300\,\mathrm{K}} = \boxed{1.6\,\mathrm{atm}}$$

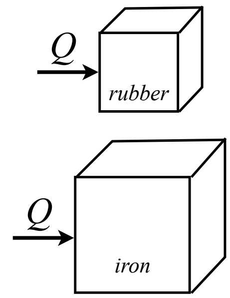
$\boxed{3}$ 4. One kilogram of rubber has a heat capacity of 2000 J/K and an initial temperature of 10 °C. If 1000 J of heat flows out of the rubber, what is the temperature of the rubber?



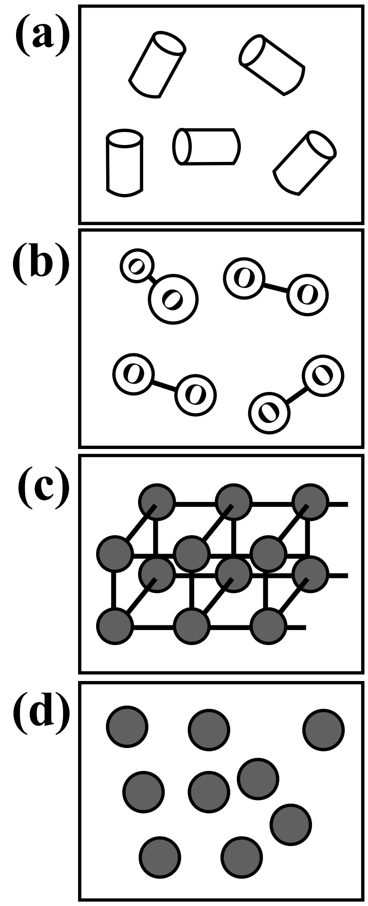
$$\Delta T = \frac{Q}{C} = \frac{1000\,\mathrm{J}}{2000\,\mathrm{J/K}} = 0.5\ \mathrm{C}^\circ$$

Heat flows out so the temperature drops by half a degree, to $\boxed{9.5\,^\circ\mathrm{C}}$.

$\boxed{3}$ 5. **B** A block of rubber and a larger block of iron begin at the same temperature, and receive the same amount of heat. After the application of heat, the block of iron is warmer than the block of rubber. Which block has the larger heat capacity C?
A) the iron **B)** the rubber **C)** it cannot be determined



6. For each of the following systems, choose the correct thermal energy U, assuming *all* degrees of freedom are activated and none are frozen out.



$\boxed{2}$ (a) **C** A gas of N cylinders in three dimensions.
A) $\frac{1}{2}NkT$ **B)** $\frac{3}{2}NkT$ **C)** $\frac{5}{2}NkT$ **D)** $3NkT$

$\boxed{2}$ (b) **D** N molecules of oxygen in three dimensions.
A) $\frac{3}{2}NkT$ **B)** $\frac{5}{2}NkT$ **C)** $3NkT$ **D)** $\frac{7}{2}NkT$

$\boxed{2}$ (c) **C** A solid of N atoms in three dimensions.
A) $\frac{1}{2}NkT$ **B)** $\frac{3}{2}NkT$ **C)** $3NkT$ **D)** $\frac{9}{2}NkT$

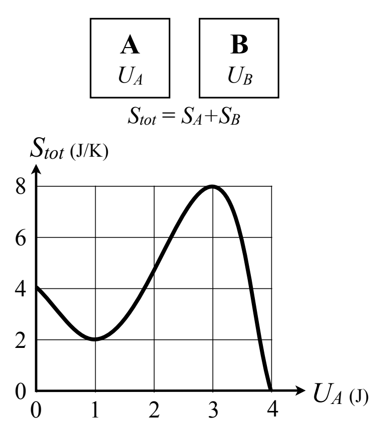
$\boxed{2}$ (d) **B** A gas of N circles in two dimensions.
A) $\frac{1}{2}NkT$ **B)** NkT **C)** $\frac{3}{2}NkT$ **D)** $2NkT$

$\boxed{3}$ 7. **D** If N is a large number, we can approximate $N(2^{2N})$ best as
A) $N(2^N)$ **B)** $N(2^{2N})$ **C)** 2^N **D)** 2^{2N}

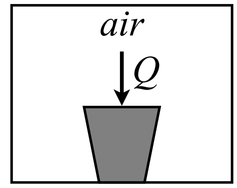
$\boxed{3}$ 8. Write an expression for the number of ways you can rearrange 5 A's, 2 B's, and 3 C's. You needn't simplify, so long as your expression is correct.

$$\frac{10!}{5!2!3!} = 2520$$

$\boxed{3}$ 9. **D** Two different systems A and B are allowed to exchange energy with each other, with the total energy $U = U_A + U_B = 4\,\mathrm{J}$. The graph shows the total entropy S_{tot} of both systems, as a function of the energy U_A in system A. When the systems are in thermal equilibrium, how much energy is in system A?
A) 0 J **B)** 1 J **C)** 2 J **D)** 3 J **E)** 4 J **F)** 8 J



10. A cup of cold water warms up in a room, as heat passes from the air into the water. We can assume the air's temperature does not change (because its mass is so much greater).



$\boxed{2}$ (a) **C** The change in the air's entropy, ΔS_a, is

A) positive **B)** zero **C)** negative

Heat flows out of the air, so the entropy drops by $\Delta S_w = Q/T$.

$\boxed{2}$ (b) **A** The change in the water's entropy, ΔS_w, is

A) positive **B)** negative

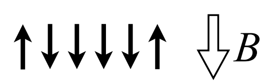
$\boxed{2}$ (c) **A** How do the magnitudes of the entropy changes compare?

A) $|\Delta S_w| > |\Delta S_a|$ **B)** $|\Delta S_w| = |\Delta S_a|$ **C)** $|\Delta S_w| < |\Delta S_a|$

The total entropy is increasing, so the increase in the water must be more than the decrease in the air.

$\boxed{3}$ 11. **A** The figure shows a paramagnet of $N = 6$ spins in a magnetic field, in a particular energy macrostate. What is the multiplicity $\Omega(U)$ of that energy macrostate?

A) 15 **B)** 30 **C)** 64 **D)** 360 **E)** 720



$\boxed{3}$ 12. **A** An Einstein solid has $N = 10^9$ oscillators and $q = 10^6$ quanta of energy. The multiplicity Ω of the solid is approximately equal to

A) $\left(\frac{10^9 e}{10^6}\right)^{10^6}$ **B)** $\left(\frac{10^9 e}{10^6}\right)^{10^9}$ **C)** $\left(\frac{10^6 e}{10^9}\right)^{10^6}$ **D)** $\left(\frac{10^6 e}{10^9}\right)^{10^9}$

This is a low-temperature limit $q \ll N$, so

$$\Omega \approx \left(\frac{eN}{q}\right)^q = \left(\frac{10^9 e}{10^6}\right)^{10^6}$$

13. The multiplicity Ω of an ideal gas of $N = 4$ particles, with volume V and internal energy U, is proportional to a d–dimensional sphere with radius R.

$\boxed{2}$ (a) **D** What is d?

A) 3 **B)** 4 **C)** 6 **D)** 12

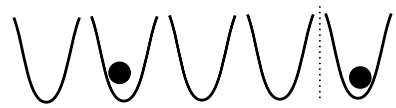
$2mU = p_{1x}^2 + p_{1y}^2 + p_{1z}^2 + p_{2x}^2 + p_{2y}^2 + p_{2z}^2 + p_{3x}^2 + p_{3y}^2 + p_{3z}^2 + p_{4x}^2 + p_{4y}^2 + p_{4z}^2$

This describes a 12-dimensional sphere.

$\boxed{2}$ (b) What is R?

$\boxed{\sqrt{2mU}}$

14. Consider an Einstein solid of 5 oscillators, containing $q = 2$ quanta of energy.



3 (a) **C** What is the total number of ways Ω that the two quanta of energy can be distributed?

A) 6 **B)** 10 **C)** 15 **D)** 21 **E)** None of these

$$\binom{N+q-1}{q} = \binom{5+2-1}{2} = \frac{6!}{4!2!} = \frac{6 \cdot 5}{2} = \boxed{15}$$

3 (b) **B** How many of those microstates have exactly one unit of energy in the last oscillator? (Hint: Think of the last oscillator as a separate solid.)

A) 1 **B)** 4 **C)** 10 **D)** 15 **E)** None of these

If we restrict one unit of energy in the last oscillator, then we have two Einstein solids: A with $N = 4$ and $q = 1$, and B with $N = 1$ and $q = 1$.

$$\Omega = \Omega_L \Omega_R = \binom{4+1-1}{1}\binom{1+1-1}{1} = (4)(1) = \boxed{4}$$

2 (c) What is the probability that there is one unit of energy in the last oscillator?

The probability is

$$P = \frac{4}{15} = \boxed{27\%}$$

3 15. **D** Consider an ideal gas of N point particles. The energy U of the gas doubles, while its volume stays the same. The increase ΔS of the gas's entropy is
A) $2Nk$ **B)** $3Nk$ **C)** $Nk\ln 2$ **D)** $\frac{3}{2}Nk\ln 2$

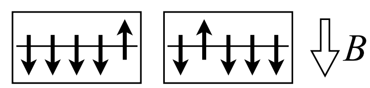
The Sackur-Tetrode equation is

$$S = kN\ln V + \frac{3}{2}kN\ln U + N \text{ stuff}$$

If U doubles, then

$$\Delta S = S_f - S_i = \frac{3}{2}kN\ln 2U - \frac{3}{2}kN\ln U = \frac{3}{2}kN\ln 2$$

3 XC 16. **Extra Credit:** Two paramagnets, each with 5 dipoles, are placed in a downward-pointing magnetic field, so that the energy of the paramagnet is equal to the number of dipoles that point upward ($U = N_\uparrow$). The total energy in both paramagnets combined is $U = 2$, and the paramagnets can exchange energy with each other. What is the *probability* that half of the energy will be found in each paramagnet?



The total number of microstates in the paramagnet with $U = 2$, is

$$\Omega_{all} = \binom{N}{U} = \binom{10}{2} = \frac{10!}{8!2!} = 45$$

The number of microstates where the energy is evenly split is

$$\Omega = \Omega_L\Omega_R = \binom{5}{1}\binom{5}{1} = 25$$

and so the probability that the energy is evenly split is $P = \dfrac{\Omega}{\Omega_{all}} = \dfrac{25}{45} = \boxed{56\%}$