Exam 2 covers Griffiths Ch 2, 4, little bit of 3 (see outline)

take-home exam

handed out end of Friday's class

due following Friday

Free Electron Gas

electrons allowed to roam freely in a volume V

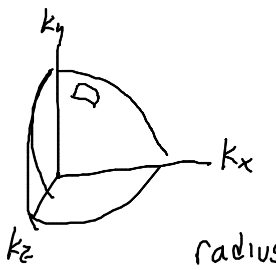
available states are defined by

$$\vec{k} = \frac{n_x \pi}{l_x}\hat{x} + \frac{n_y \pi}{l_y}\hat{y} + \frac{n_z \pi}{l_z}\hat{z}$$

wave vector

$n_x, n_y, n_z = 1, 2, 3, \ldots$

lowest energy state



$$E(\vec{k}) = \frac{\hbar^2 |\vec{k}|^2}{2m}$$

radius of sphere is

$$k_F = \left(3 \frac{N}{V} \pi^2\right)^{1/3} \qquad N = \text{\# of electrons}$$

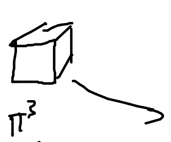
$$E_F = \frac{\hbar^2 k_F^2}{2m}$$ Fermi energy of a free electron gas

Total energy:

think of a spherical shell in k-space with thickness dk

volume $\frac{1}{8} 4\pi k^2\, dk$

\# of electron states in that shell



$$2 \times \frac{\text{volume of shell}}{\text{volume of 1 state}} = 2\, \frac{\frac{1}{2}\pi k^2\, dk}{\pi^3/V} = \frac{V}{\pi^2} k^2\, dk$$

all states have same k

& same energy $E = \frac{\hbar^2 k^2}{2m}$

Energy of shell = \# states × energy per state

$$= \frac{V}{\pi^2} k^2\, dk \; \frac{\hbar^2 k^2}{2m}$$

$$\text{total energy} = \int_{0}^{k_F} \frac{V}{\pi^2} k^2 \frac{\hbar^2 k^2}{2m}\, dk = \frac{\hbar^2 V}{2\pi^2 m} \frac{1}{5} k_F^5$$

all shells

$$\rightarrow E = \frac{\hbar^2 (3\pi^2 N)^{5/3}}{10\pi^2 m} V^{-2/3}$$

if V gets smaller, E goes up → pressure

$E = C V^{-2/3}$

$$P = -\frac{dE}{dV} = -C\left(-\frac{2}{3} V^{-5/3}\right) = \frac{\frac{2}{3} C V^{-2/3}}{V} = \frac{2}{3}\frac{E}{V}$$

$$PV = \frac{2}{3} E$$

(compare ideal gas pt particles $E = \frac{3}{2} NkT$, $\frac{2}{3} E = NkT$)

degeneracy or exclusion pressure

why solids exist

Add a periodic potential to free electron gas

$$V(x+a) = V(x) \quad \forall x$$

$$-\frac{\hbar^2}{2m}\psi'' + V(x)\psi = E\psi$$

solutions must satisfy $\psi(x+a) = e^{iKa}\psi(x)$

Bloch's theorem

K : some number

Periodic Boundary Conditions

$$\psi(x + Na) = \psi(x) \qquad N \gg 1, \; N \in \mathbb{Z}$$

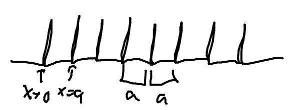
$$e^{iNKa}\psi(x) = \psi(x)$$

$$iNKa = 2\pi n \qquad n \in \mathbb{Z}$$

$$K = \frac{2\pi n}{Na}$$

If we know $\psi(x)$ in $0<x<a$, we're done.

e.g. Suppose $V(x) = \alpha \sum_{j=0}^{N-1} \delta(x - ja)$



In region $0<x<a$,

$$-\frac{\hbar^2}{2m}\psi'' = E\psi$$

$$\psi(x) = A\sin kx + B\cos kx \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

in $-a<x<0$:

$$\psi(x) = e^{-iKa}\left(A\sin k(x+a) + B\cos k(x+a)\right)$$

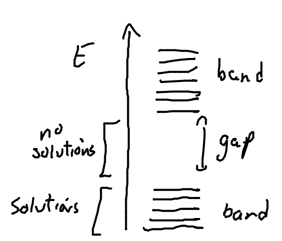
Look at B.C. at $x=0$,

$$\cos Ka = \cos ka + \frac{m\alpha}{\hbar^2 k}\sin ka$$

$$= \cos z + \beta\frac{\sin z}{z} \qquad z = ka, \quad \beta = \frac{m\alpha a}{\hbar^2}$$

not necessarily between -1 & 1

so values of k ($\& E$) with no solution



1 state for every atom

N states

if 1 electron per atom,

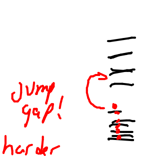
first band will be half full (2 electrons/state)

electrons can gain energy easily

conductor

2 electrons per atom,

first band completely full



insulator