# Physics 4310 Homework #10

**3 problems**

**Due by Friday, April 15**

▷ **1.**

Write the wavefunction $\psi(x_1, x_2)$ of two non-interacting particles in a harmonic oscillator (see chapter 2.3, particularly equations 2.59 and 2.62) in the lowest possible energy eigenstate, if the particles are

**(a)** ... distinguishable particles

**(b)** ... bosons

**(c)** ... fermions.

▷ **2.**

Suppose I have two noninteracting particles of mass $m$ in the infinite square well. How much larger is the square separation distance $\langle (x_1 - x_2)^2 \rangle$ if they are identical fermions, than if they are distinguishable particles?

▷ **3.**

Consider the potential of evenly-spaced delta functions $V(x) = \alpha \sum_{j=0}^{N-1} \delta(x - ja)$ in a system with periodic boundary conditions $\psi(x + Na) = \psi(x)$. In class we said/will say that

$$\psi(x) = A \sin kx + B \cos kx, \qquad 0 < x < a$$

and that $\psi(x + a) = e^{iKa}\psi(x)$ where $K = \frac{2\pi n}{Na}$ for some integer $n$.

**(a)** Write the wavefunction $\psi(x)$ for the region $-a < x < 0$. *(This is in the book but it's worth working it out on your own, with the book as reference.)*

**(b)** Use the boundary conditions at $x = 0$ to show that

$$\cos Ka = \cos z + \beta \frac{\sin z}{z}$$

where $\beta = \frac{m\alpha a}{\hbar^2}$ and $z = ka$.