$\cancel{H} = H_0 + H'$

$\psi_n = \psi_{n0} + \psi_{n1} + \psi_{n2} + \dots$

$E_n = E_{n0} + E_{n1} + E_{n2} + \dots$

$$E_{n1} = \langle \psi_{n0} | H' | \psi_{n0} \rangle$$

$$\psi_{n1} = \sum_{m \neq n} \frac{\langle \psi_{m0} | H' | \psi_{n0} \rangle}{E_{n0} - E_{m0}} \psi_{m0}$$

if there's degeneracy $E_{n0} = E_{m0}$
then this fails

Degenerate Perturbation Theory

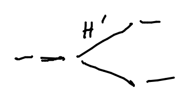
Suppose

$H_0 \psi_{a0} = E_0 \psi_{a0}$ $\quad \langle \psi_{a0} | \psi_{b0} \rangle = 0$

$H_0 \psi_{b0} = E_0 \psi_{b0}$

then $\alpha \psi_{a0} + \beta \psi_{b0}$ is also an eigenstate

Usually, H' will break the degeneracy
(i.e. eigenvalues will be different)



$H = H_0 + \lambda H'$

as you increase λ $0 \to 1$
watch energies divide

if $A|\psi\rangle = a|\psi\rangle$
then
$\langle\psi|A^\dagger = \langle\psi|a^*$

then, if you decrease λ back to 0.
eigenstates will return to two
specific linear combinations of ψ_{a0} & ψ_{b0}
these are the "good" eigenstates

Given some wavefunction $\psi = \alpha\psi_a + \beta\psi_b$

$$\langle \psi_{a0} | \Big(H_0 |\psi_1\rangle + H' |\psi_0\rangle = E_0 |\psi_1\rangle + E_1 |\psi_0\rangle \Big)$$

$$\cancel{\overbrace{\langle \psi_{a0} | H_0}^{=\langle \psi_{a0}| E_0}} |\psi_1\rangle + \langle \psi_{a0} | H' | \psi_0 \rangle = \cancel{\langle \psi_{a0} | E_0 | \psi_1 \rangle} + E_1 \langle \psi_{a0} | \psi_0 \rangle$$

$$\langle \psi_{a0} | H' | \psi_0 \rangle = E_1 \langle \psi_{a0} | \psi_0 \rangle$$

substitute $|\psi_0\rangle = \alpha |\psi_{a0}\rangle + \beta |\psi_{b0}\rangle$

$$\alpha \langle \psi_{a0} | H' | \psi_{a0} \rangle + \beta \langle \psi_{a0} | H' | \psi_{b0} \rangle + E_1 \alpha \overset{1}{\cancel{\langle \psi_{a0} | \psi_{a0} \rangle}}$$

$$\alpha W_{aa} + \beta W_{ab} = \alpha E_1$$

where $W_{ij} \equiv \langle \psi_{i0} | H' | \psi_{j0} \rangle$ $\quad i,j = \{a,b\}$

$W_{ji} = W_{ij}^*$

Do the same thing with $\langle \psi_{b0} |$

$$\alpha W_{ba} + \beta W_{bb} = \beta E_1$$

$$\begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E_1 \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

What is E_1? Eigenvalues of W.

$$E_{1\pm} = \frac{1}{2}\left[W_{aa} + W_{bb} \pm \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2} \right]$$

two different values, so
perturbation creates two nondegenerate states
$E_0 + E_{1+}$ & $E_0 + E_{1-}$

"good eigenstates" are the eigenvectors $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ of W

$E_0 + E_{1+} : \alpha_+ \psi_{a0} + \beta_+ \psi_{b0}$ $\quad E_0 + E_{1-} : \alpha_- \psi_{a0} + \beta_- \psi_{b0}$

If $W_{ab} = 0$, then ψ_a & ψ_b are the "good" eigenstates.

Suppose we can find a hermitian operator A

$$[A, H_0] = [A, H'] = 0,$$

Suppose $A\psi_{a0} = \mu \psi_{a0}$ $\quad A\psi_{b0} = \nu \psi_{b0}$ $\quad \mu \neq \nu$

then

$$\langle \psi_{a0} | [A, H'] | \psi_{b0} \rangle = 0$$

$$\langle \psi_{a0} | A H' | \psi_{b0} \rangle - \langle \psi_{a0} | H' A | \psi_{b0} \rangle = 0$$

$$= \mu \langle \psi_{a0} | H' | \psi_{b0} \rangle - \langle \psi_{a0} | H' | \psi_{b0} \rangle \nu = 0$$

$$(\mu - \nu) \langle \psi_{a0} | H' | \psi_{b0} \rangle = 0$$

$$(\mu - \nu) W_{ab} = 0.$$

$$\mu \neq \nu \rightarrow W_{ab} = 0$$

$\therefore$ ψ_{a0} & ψ_{b0} are "good" eigenstates

n-fold degeneracy (n states, same energy)

E_1's are eigenvalues of $W_{ij} = \langle \psi_{i0} | H' | \psi_{j0} \rangle$