energy states

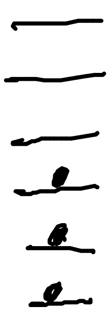
$\bar{n}$ average occupancy
i.e. how many particles are in this microstate on average

Fermions

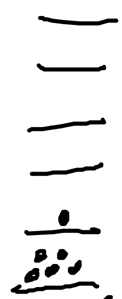
$$\bar{n} = \frac{1}{e^{\beta(\varepsilon-\mu)}+1}$$

Bosons

$$\bar{n}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}-1}$$



$\leftarrow \mu$: energy where $\bar{n} = \frac{1}{2}$



$\leftarrow \mu$ always below ground state.

The more particles there are, the closer μ gets to ground state.

μ is related to N:
higher N, higher μ.

If $\varepsilon \gg \mu$, $e^{\beta(\varepsilon-\mu)} \gg 1$

$$\frac{1}{e^{\beta(\varepsilon-\mu)}+1} \approx \frac{1}{e^{\beta(\varepsilon-\mu)}-1} \approx \frac{1}{e^{\beta(\varepsilon-\mu)}}$$

Maxwell-Boltzmann distribution for noninteracting particles

At $\varepsilon \gg \mu$, $\bar{n}$ is small
& only one particle at most usually occupies any energy microstate at those higher energy levels → no interaction
& it doesn't matter if fermions or bosons

Fermi gas at low temperatures

e.g. conduction electrons in a metal

at high T, we could model as an ideal gas

$\hookrightarrow V_Q \ll \frac{V}{N}$

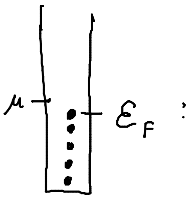
At 300K, electrons have $V_Q \gg (2Å)^3$

ideal gas Z breaks down

use ~~a~~ "low Temperature" model

Start at $T=0$

$$\bar{n}(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/kT}+1} = \begin{cases} 0, & \varepsilon > \mu \\ 1, & \varepsilon < \mu \end{cases}$$

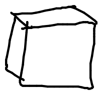


ε_F : Fermi energy

$\varepsilon_F \lesssim \mu$

In 3D space, what are the energy states?

In a $L \times L \times L$ box



States are 3D standing waves

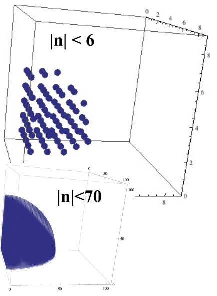
with wavelength $\lambda_{n_x} = \frac{2L}{n_x}$

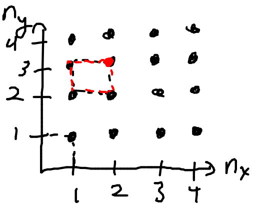
$$p_x = \frac{h}{\lambda_{n_x}} = \frac{h n_x}{2L}$$

$$p_y = \frac{h n_y}{2L} \qquad p_z = \frac{h n_z}{2L}$$

$$n_x, n_y, n_z = 1, 2, 3, 4, \ldots$$

$$E(n_x, n_y, n_z) = \frac{p^2}{2m} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$





Each dot is a state.

Can hold 2 electrons.

Each dot has a volume of 1 (in n-space) associated with it.

So each electron gets a volume of $\frac{1}{2}$.

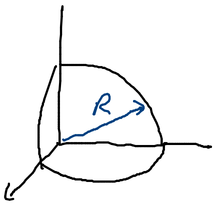
If I fill these ~~states~~ states with N electrons starting from smallest energies

I will create a "volume" $N/2$.

If $N \gg 1$, they will form an $\frac{1}{8}$th of a sphere with radius R

sphere will have "volume" $\frac{1}{8}\left(\frac{4}{3}\pi R^3\right) = \frac{N}{2}$

$$\rightarrow R^3 = \frac{3}{\pi} N$$



$$E = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) = \frac{h^2}{8mL^2}|\vec{n}|^2$$

$$\varepsilon_F = \frac{h^2}{8mL^2} R^2$$

$$= \frac{h^2}{8mL^2}\left(\frac{3N}{\pi}\right)^{2/3}$$

$L^3 = V$

$$\boxed{\varepsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}}$$