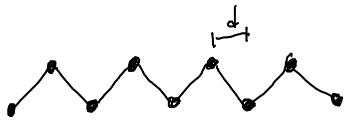
Phonons



minimum wavelength $\lambda_{min} = 2d$

# of bumps $\approx N$

Each dot has a corresponding harmonic oscillator

$$q = N\bar{n} = \frac{N}{e^{\epsilon/kT}-1}$$

avg number of quanta per oscillator

ALSO avg number of quanta per standing wave state

because # of standing wave states $= N$

1, 2, 3, . . . , max # of bumps.

Heat Capacity of a Solid, given this model

$U = q\epsilon$

$$C = \frac{\partial U}{\partial T} = \frac{\partial U}{\partial q}\frac{\partial q}{\partial T} = -\epsilon\frac{N}{(e^{\epsilon/kT}-1)^2}e^{\epsilon/kT}\left(-\frac{\epsilon}{kT^2}\right)$$

$$= \frac{N\epsilon^2 e^{\epsilon/kT}}{kT^2(e^{\epsilon/kT}-1)^2}$$

As $T \to 0$, $C \sim e^{-\epsilon/kT}$

this goes to zero which is good!
but too fast: exponentially.

Experiment: $C \propto T^3$ at low T.

Freezes out quickly because all oscillators are treated as independent — freeze out at same time.

If we think in terms of standing waves & phonons, then high-frequency waves freeze out first, but low-freq. waves stick around

Proof

polarization

sound has max. frequency

$$U = 3\sum_{n_x=1}^{\sqrt[3]{N}}\sum_{n_y=1}^{\sqrt[3]{N}}\sum_{n_z=1}^{\sqrt[3]{N}}\epsilon\,\bar{n}(\epsilon)$$

Peter Debye: assume this is really an $\frac{1}{8}$th of sphere with same "volume" in n-space: N.

sphere has radius n_{max}

$$\frac{1}{8}\cdot\frac{4}{3}\pi n_{max}^3 = N \longrightarrow n_{max} = \left(\frac{6N}{\pi}\right)^{1/3}$$

$$U \approx 3\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{n_{max}}\epsilon\,\bar{n}(\epsilon)\,n^2\sin\theta\,dn\,d\theta\,d\phi$$

$$U = 3\,\frac{4\pi}{8}\int_0^{n_{max}}\left(\frac{hc_s}{2L}n\right)\frac{1}{e^{hc_s n/2LkT}-1}\,n^2\,dn$$

- At high T, shape doesn't matter, only total # of states N
- At low T, phonons are all close to ground state & boundary is unoccupied

Let $x = \frac{hc_s n}{2LkT}$ $\quad dx = \frac{hc_s}{2LkT}dn$

$$x_{max} = \frac{hc_s}{2LkT}n_{max} = \frac{hc_s}{2kT}\left(\frac{6N}{\pi V}\right)^{1/3} \equiv \frac{T_D}{T}$$

$(L^3)^{1/3}$

Debye temperature

algebra happens

$$U = \frac{9NkT^4}{T_D^3}\int_0^{T_D/T}\frac{x^3}{e^x-1}\,dx$$

high-temperature

$T \gg T_D$ $\quad x \ll 1$ $\quad e^x - 1 \approx (1+x) - 1 = x$

$$U = \frac{9NkT^4}{T_D^3}\int_0^{T_D/T}\frac{x^3}{x}\,dx$$

high-temperature

$T \gg T_D \qquad x \ll 1 \qquad e^x - 1 \simeq (1+x) - 1 = x$

$$U = \frac{9NkT^4}{T_D^3} \int_0^{T_D/T} \frac{x^3}{x}\, dx$$

$$= \frac{\cancel{9}^{3} NkT^{\cancel{4}}}{\cancel{T_D}^3} \frac{1}{\cancel{3}} \left(\frac{\cancel{T_D}}{\cancel{T}} \right)^3$$

$$= 3NkT$$

equipartition theorem with $f = 6$!

low-temperature

$T \ll T_D \qquad \frac{T_D}{T} \gg 1$

$$U = \frac{9NkT^4}{T_D^3} \int_0^{T_D/T} \frac{x^3}{e^x - 1}\, dx$$

$$\simeq \frac{9NkT^4}{T_D^3} \int_0^{\infty} \frac{x^3}{e^x - 1}\, dx$$

(integrand decaying exponentially)

$$\simeq \frac{9NkT^4}{T_D^3} \frac{\pi^4}{15} = \frac{3\pi^4}{5} \frac{NkT^4}{T_D^3} \qquad T \ll T_D$$

$$C = \frac{dU}{dT} = \frac{12\pi^4}{5} Nk \frac{1}{T_D^3} T^3$$

agrees beautifully with low-T experiments for almost all <u>solids</u>

metals

$$C = \sim T + \sim T^3$$

electrons phonons