McIntyre Chapter 4

Einstein – Podolsky – Rosen (EPR) Paradox

Einstein: properties of physical objects have independent reality

$\vec{S}$ $\nearrow$ QM: S_z can be well-defined but if so, S_x & S_y are not

Einstein: nonsense!

Suppose a spin-0 particle decays into two spin-$\frac{1}{2}$ particles $\pi^0 \rightarrow e^- + e^+$

conservation of ang. mom: 1 spin up & 1 spin down

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$A \quad e^- \longleftarrow \pi^0 \longrightarrow e^+ \quad B$$

Observer A measures e^- with $S_{1z}|\psi\rangle$

outcomes { 50% chance $\uparrow$, 50% chance $\downarrow$

Observer B measures a little later

outcome guaranteed to be the opposite

even if A & B are light-years apart & B measures seconds after A.

Not too weird yet.

What if, instead, Alice measures with $S_{1x}|\psi\rangle$?

If her particle were pre-selected as $\uparrow$, she would get 50% chance of $\odot$ or $\otimes$

Ditto B, but again results will be opposite.

It's as if two electrons are communicating through an instantaneous channel.

Einstein: Nonsense!

"violates locality"

"Spooky action-at-a distance"

These two electrons are <u>entangled</u>

"perfectly entangled": know nothing about their individual states but <u>do</u> know their global state

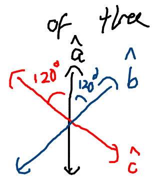
Einstein's solution: <u>hidden variables</u>

"random" outcomes not random but result of some hidden value that QM doesn't account for

1964: John Bell: hidden variables are incompatible with QM

Suppose we have our singlet state.

A & B can make measurements along one of three different axes.



Each observer chooses a measurement $(\hat{a}, \hat{b}, \hat{c})$ at random.

We record each observer's outcome as $+$ or $-$.

Hidden Variable Theory: each particle has instructions for how those measurements should turn out.

Possible Instruction Sets

	A: $\hat{a}$	$\hat{b}$	$\hat{c}$	B: $\hat{a}$	$\hat{b}$	$\hat{c}$
N_1:	+	+	+	−	−	−
N_2	+	+	−	−	−	+
N_3	+	−	+	−	+	−
N_4	+	−	−	−	+	+
N_5	−	+	+	+	−	−
N_6	−	+	−	+	−	+
N_7	−	−	+	+	+	−
N_8	−	−	−	+	+	+

Possible Measurement Choices:

aa, ab, ac, ba, ..., cc

Probability that A & B get opposite results?

for types 1 & 8, $P_{opp} = 100\%$ $P_{same} = 0$.

" " 2−7, $P_{opp} = 5/9$ $P_{same} = 4/9$

$$P_{opp} = \frac{N_1 + \frac{5}{9}(N_2 + N_3 + N_4 + N_5 + N_6 + N_7) + N_8}{N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 + N_8} \geq \frac{5}{9}$$

$$P_{same} \leq \frac{4}{9}$$

Bell inequalities

QM

Suppose A records a $+$ along some direction let's call that $\hat{z}$.

B measures along $\hat{n}$ at some θ w.r.t. z-axis. (0 or 120° or 240°)

$$P_{++} = |\langle +_{\hat{z}} +_{\hat{n}} | \psi\rangle|^2 \qquad |\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$= \left|\left(\cos\frac{\theta}{2}\langle + + | + e^{-i\phi}\sin\frac{\theta}{2}\langle + - |\right)\frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)\right|^2$$

$$= \left|\frac{1}{\sqrt{2}} e^{-i\phi}\sin\frac{\theta}{2}\right|^2 = \frac{1}{2}\sin^2\frac{\theta}{2}$$

$$P_{same} = P_{++} + P_{--} = \sin^2\frac{\theta}{2}$$

1/3 of time, $\theta = 0°$

2/3 of time, $\theta = 120°$

$$P_{same} = \frac{1}{3}\sin^2 0° + \frac{2}{3}\sin^2 60° = \frac{2}{3}\frac{3}{4} = \frac{1}{2}$$

Bell's Inequality: $P_{same} \leq \frac{4}{9}$

∴ No hidden instruction sets

No hidden variables.

Experiment : QM wins! Einstein loses!

Entanglement $\longrightarrow$ instantaneous quantum communication!