

## Physics 4310 Homework #6

4 problems

Due by March 14 (after break)

▷ **1.**

Evaluate the following integrals. (These are easy, but you need to be a *little* careful.)

(a)  $\int_{-3}^{+1} (x^3 - 3x^2 + 2x - 1)\delta(x + 2) dx$

(b)  $\int_0^{-\infty} [\cos(3x) + 2]\delta(x - \pi) dx$

(c)  $\int_{-1}^{+1} \exp(|x| + 3)\delta(x - 2) dx$

▷ **2.**

Delta functions are actually “distributions” because they live under integral signs. Two distributions ( $D_1(x)$  and  $D_2(x)$ ) are said to be equal if

$$\int_{-\infty}^{\infty} D_1(x)f(x) dx = \int_{-\infty}^{\infty} D_2(x)f(x) dx$$

for every (ordinary) function  $f(x)$ .

(a) Show that

$$\delta(cx) = \frac{1}{|c|}\delta(x)$$

where  $c$  is a real constant. (Be sure to check the case where  $c$  is negative.)

(b) Let  $\theta(x)$  be the *step function*:

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

Show that  $\frac{d\theta}{dx} = \delta(x)$ . Hint: Use integration by parts, along with the definition of “equal distributions” above.

▷ **3.**

In class we discussed the *even* bound state wave functions for the finite square well. I want you to analyze the *odd* bound state wave functions now. Derive the transcendental equation for the allowed energies, and solve it graphically. Show that there is always at least one bound state.

▷ **4.**

Consider the “step” potential:

$$V(x) = \begin{cases} 0, & x \leq 0 \\ V_0, & x > 0 \end{cases}$$

(a) Calculate the reflection coefficient for the case  $E < V_0$ , assuming the incident wave comes in from the left (as in the finite square well).

(b) Calculate the reflection coefficient for the case  $E > V_0$ .