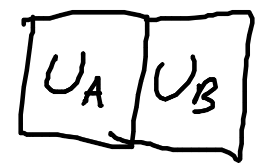
$$\Omega = f(N)\, V^N\, U^{3N/2}$$

If I have two ideal gases, each with N particles,



$\underset{\text{fixed}}{U} = \underbrace{U_A + U_B}_{\text{variable}}$

$\underset{\text{of both}}{\Omega(U_A)} = \Omega_A \Omega_B$

$$= f(N) V_A^N U_A^{3N/2} \quad f(N) V_B^N U_B^{3N/2}$$

$$= f(N)^2 (V_A V_B)^N (U_A U_B)^{3N/2}$$

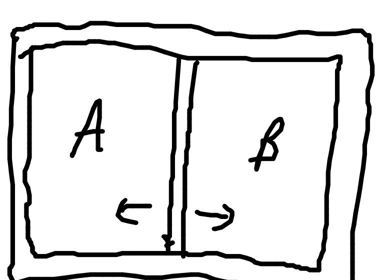
compare Einstein solid result $\Omega(q_A) = \left(\frac{e}{N}\right)^N (q_A q_B)^N$

maximized when $q_A = q_B$

Gaussian distribution $\sigma \sim \frac{1}{\sqrt{N}}$

$\therefore$ ideal gases will reach equilibrium at $U_A = U_B$ and fluctuations $\sigma \sim \frac{1}{\sqrt{N}}$

Same result if gases can exchange volume with each other $V_A + V_B = V$ constant



Piston can move back & forth so gases can exchange volume.

Why is indistinguishability important?

Consider N balls labelled $1 \ldots N$

e.g. $N=10$ ① ② ③ ④ ⑤ | ⑥ ⑦ ⑧ ⑨ ⑩

all in a row, but balls can swap places at will

Initially, there's a barrier halfway through

$$\Omega = \left(\tfrac{N}{2}\right)!\left(\tfrac{N}{2}\right)!$$

$$S = k\ln\Omega = 2k\ln\left(\tfrac{N}{2}\right)! \overset{N\gg1}{\approx} 2k\left[\tfrac{N}{2}\ln\tfrac{N}{2} - \tfrac{N}{2}\right]$$

$$= Nk\left[\ln\tfrac{N}{2} - 1\right]$$

- Now remove barrier

$$\Omega = N!$$

$$S = k\ln N! = Nk[\ln N - 1]$$

$$\Delta S = S_f - S_i = Nk[\ln N - \cancel{1}] - Nk\left[\ln\tfrac{N}{2} - \cancel{1}\right]$$

$$= Nk\ln 2$$

- Replace barrier

$$\Delta S = -Nk\ln 2$$

I can reduce entropy of Universe by replacing a simple barrier? Hmm, unlikely.

Solution: suppose particles are indistinguishable

• • • • • | • • • • • $\quad \Omega = 1 \quad S = 0$

swapping particles does not change state of the system

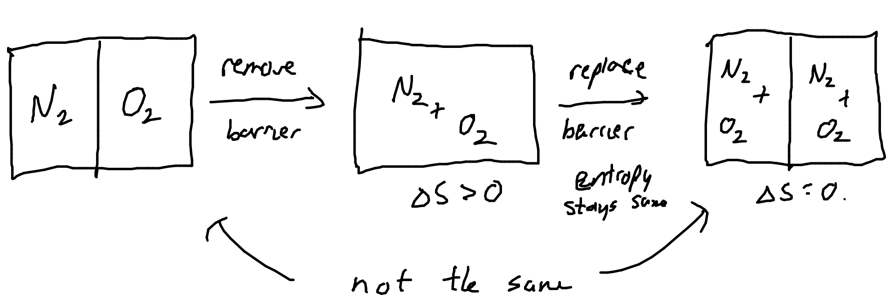
remove the barrier: • • • • • • • • • • $\quad \Omega = 1 \quad S = 0$

$\Delta S = 0$

replace the barrier: $\Delta S = 0$,

That is reasonable, & so ideal gas molecules are indistinguishable (if same type of gas)

Two Types of Gas



cf Maxwell's Demon

— — END OF EXAM 1 MATERIAL — — —

Chapter 3:

Thermal Equilibrium $\begin{cases} \text{same temperature} \\ \text{entropy maximized} \end{cases}$ what is the relationship?