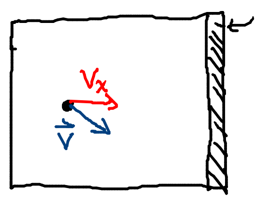
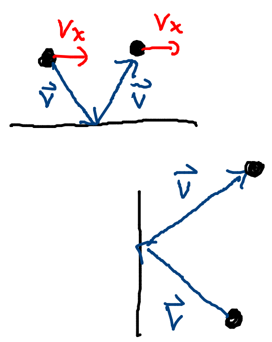
Proof: Ideal Gas Law

- consider 1 particle in a box with a ideal piston



assume elastic collisions
smooth walls
ignore gravity



$|\vec{v}|$ constant

v_x constant

$|\vec{v}|$ constant $\quad |\vec{v}|^2 = v_x^2 + v_y^2$

v_y constant

$|v_x|$ constant

pressure on piston $\quad P = \dfrac{\langle F_x \rangle}{A}$ ← time average

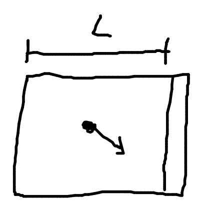
F_x = force on wall, also force on particle (N3L)

$F_x = 0$ most of the time

$$F_x = m a_x = m \frac{\Delta v_x}{\Delta t}$$

$$\langle F_x \rangle = m \left\langle \frac{\Delta v_x}{\Delta t} \right\rangle$$

Average over time it takes to go from left wall → piston → left wall



$$\Delta t = \frac{2L}{|v_x|}$$

$$\Delta v_x = v_{xf} - v_{xi} = -|v_x| - |v_x| = -2|v_x|$$

$$\langle F_x \rangle = m \frac{2|v_x|}{2L/|v_x|} = \frac{m|v_x|^2}{L}$$

minus sign doesn't matter

$$P = \frac{\langle F_x \rangle}{A} = \frac{m|v_x|^2}{AL} \Rightarrow PV = m v_x^2$$

volume V

For N independent particles,

$$PV = m_1 v_{1x}^2 + m_2 v_{2x}^2 + m_3 v_{3x}^2 + \dots m_N v_{Nx}^2$$

if $m_1 = m_2 = m$,

over all particles

$$PV = m N \langle v_x^2 \rangle$$

$\langle v_x^2 \rangle \neq \langle v_x \rangle^2$

$$= 2N \langle \tfrac{1}{2} m v_x^2 \rangle$$

↓ equipartition theorem

$U = \frac{1}{2} m v_{1x}^2 + \frac{1}{2} m v_{1y}^2 + \frac{1}{2} m v_{1z}^2 \dots$

$$PV = 2N \left(\tfrac{1}{2} k_B T\right)$$

$$PV = N k_B T \quad \blacksquare$$

Chapter 2

- Counting outcomes or states

  - possible outcomes of an event

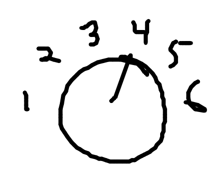
    "flip a coin" $\Omega = 2$ ← standard variable for counts

    "roll a six-sided die" $\Omega = 6$

- Counting possible states of a system

  What is a "state"?

  - light bulb: on or off $\Omega = 2$
  - one coin on a table: H or T $\Omega = 2$
  - volume dial



  - set of coins

    eg. 3 coins

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

$$\Omega = 8 = 2^3$$

system of N independent objects
each with Ω_1 possible states
then $\boxed{\Omega = \Omega_1^N}$

other examples of states

- position of notebook on a table

$$\Omega = \Omega_{\substack{\text{what}\\\text{page is}\\\text{it on}}} \; \Omega_{\substack{x\\\text{position}}} \; \Omega_{\substack{y\\\text{position}}} \; \Omega_{\text{orientation}}$$

- molecules in this room
  - position of each molecule & velocity of each molecule

Rearranging N items ABCDE

e.g. $N = 5$ ABCDE

choose 1st in line: $5 \times$

choose 2nd in line: $4 \times$

$3 \times$

$2 \times$

1

$\Omega = 5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$

$N! = N(N-1)(N-2) \ldots 3(2)(1)$

Can rearrange ABC $3! = 6$ ways

ABC	ABB
ACB	ABB
BAC	BAB
BCA	BBA
CAB	BAB
CBA	BBA

What if some ~~two~~ objects are identical or indistinguishable?

e.g. ABB

$3!$ overcounts by a factor of $2!$

$\Omega = \frac{3!}{2!}$

e.g. AAAB

AABA

ABAA

BAAA

$4! = 24$ ways to rearrange 4 objects

each pattern of AAAB will show up $3! = 6$ times in this list of 24 (rearrange A's)

$\therefore \Omega = \frac{4!}{3!} = 4$

In general, if N objects and there are n_1 duplicates of 1, n_2 duplicates of another etc

$$\Omega = \frac{N!}{n_1! n_2! \ldots}$$

e.g. AABBC

$$\Omega = \frac{5!}{\underset{\#As}{2!}\,\underset{\#Bs}{2!}\,1!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2} = 30$$