Physics 4310 Homework #2

3 problems

Due by January 29

▷ **1.**

Given $A \doteq \begin{pmatrix} 1 & 2 \\ 3i & 4 \end{pmatrix}$ and $|\psi\rangle \doteq \begin{pmatrix} i \\ -1 \end{pmatrix}$, find the matrix representations of

(a) $A|\psi\rangle$

(b) $\langle\psi|A$

(c) $\langle\psi|A\dagger$

▷ **2.**

Suppose B is a Hermitian operator which is represented in the S_z basis by the matrix

$$\begin{pmatrix} 1 & 2-i \\ a & 2 \end{pmatrix}$$

(a) What is a?

(b) If I apply this measurement to a system in arbitrary state $|\psi\rangle$, what are the possible outcomes of this measurement? What could the final state of the system be?

(c) Find the probability of the outcomes, if $|\psi\rangle = |\uparrow\rangle$.

▷ **3.**

Consider the operator $S_{\hat{n}}$ for a Stern-Gerlach device oriented along the vector

$$\hat{n} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

(a) Prove that it is represented (in the S_z basis) by the matrix

$$\frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}$$

(b) Prove that $\cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}e^{i\phi}|\downarrow\rangle$ is an eigenvector of $S_{\hat{n}}$, and find the corresponding eigenvalue.

(c) For what values of θ and ϕ does $S_{\hat{n}} = S_y$?