Physics 3410 Homework #4

5 problems

Due by February 15

▷ **1.**

Find the entropy of

(a) an N–state paramagnet with energy $U = N_\uparrow$, if N and U are both large.

(b) an N–oscillator Einstein solid with energy q, when both variables are large numbers.

▷ **2.**

Two three-oscillator Einstein solids ($N_A = N_B = 3$) can exchange energy with each other; together they have energy $q = 5$.

(a) What is the probability that $q_A = 2$?

(b) What is the probability that $q_A = 5$?

▷ **3.**

Two Einstein solids are in contact with each other, the first having N_A oscillators and the second N_B oscillators. The total energy of both is q, with solid A having energy q_A and solid B having energy q_B. (In other words, same as in class.) In the high-temperature limit $q_A \gg N_A \gg 1$ and $q_B \gg N_B \gg 1$, prove that the most likely energy macrostate q_A is the one where

$$\frac{q_A}{q_B} = \frac{N_A}{N_B}$$

Hints: Find the value of q_A which maximizes the multiplicity $\Omega(q_A)$. Use the chain rule, the product rule, and the fact that $\frac{\partial q_A}{\partial q_B} = -1$. And if you end up seeing $N_A - 1$, resist the urge to drop the 1.

▷ **4.**

What is the surface area of a 12–dimensional sphere with radius $R = 2\,\mathrm{m}$?

▷ **5.**

Consider an ideal gas with $N = 10^{23}$ particles and an internal energy of $U = 100\,\mathrm{J}$. If the gas triples in volume, but its entropy remains constant, what is the internal energy after it expands?