-

A —— —— B ε

G —— 0

2 Bosons

E_s		
0	GG	1
ε	$\frac{1}{\sqrt{2}}(GA + AG)$	$e^{-\beta\varepsilon}$
ε	GB	$e^{-\beta\varepsilon}$
2ε	AA	$e^{-2\beta\varepsilon}$
2ε	AB	$e^{-2\beta\varepsilon}$
2ε	BB	$+\ e^{-2\beta\varepsilon}$

$$Z = \sum_s e^{-\beta E_s}$$

$$Z = 1 + 2e^{-\beta\varepsilon} + 3e^{-2\beta\varepsilon}$$

Fermions

GA

GB

AB

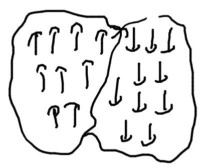
$$Z = 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$$

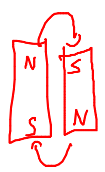
~~Chapter~~ Section 8.2 Ising Model of a Ferromagnet

- paramagnet : dipoles are independent
- ferromagnet : dipoles influence one another
  dipoles want to be aligned with their neighbors

  ↑↑↑↑↑

  regions called domains where dipoles all point alike





- Antiferromagnets: (Cr, NiO, FeO) dipoles like to point opposite their neighbors

  ↓↑↓↑
  ↑↓↑↓

Ising Model

each dipole i points ↑ or ↓

$s_i = 1$ or -1

for every pair of neighbors i & j: ↑↑ or ↓↓ : energy $-\varepsilon$ $s_i s_j = 1$

↑↓ or ↓↑ : energy $+\varepsilon$ $s_i s_j = -1$

$$U = \sum_{\substack{i,j \\ \text{neighbors}}} (-\varepsilon s_i s_j) = -\varepsilon \sum_{\text{neighbors}} s_i s_j$$

$$Z = \sum_{\{s_i\}} e^{-\beta U}$$

2^N terms

One dimension

↑↓↓↓↑↓ . . .
1 2 3 4 5 6

$$U = -\epsilon (s_1 s_2 + s_2 s_3 + s_3 s_4 + \cdots + s_{N-1} s_N)$$

$$Z = \sum_{s_1=-1,1} \sum_{s_2=\pm 1} \cdots \sum_{s_N=\pm 1} e^{\beta\varepsilon s_1 s_2} e^{\beta \varepsilon s_2 s_3} \cdots e^{\beta \varepsilon s_{N-1} s_N}$$

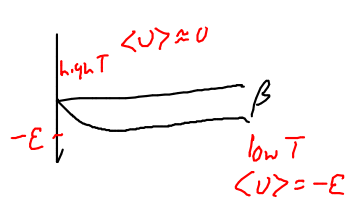
$$\sum_{s_N} e^{\beta\varepsilon(s_{N-1} s_N)} = e^{+\beta\varepsilon} + e^{-\beta\varepsilon} = 2\cosh\beta\varepsilon$$

$$Z = 2^N (\cosh\beta\varepsilon)^{N-1} \underset{N \gg 1}{\approx} (2\cosh\beta\varepsilon)^N$$

$$\langle U \rangle = -\frac{\partial}{\partial\beta} \ln Z = -\frac{\partial}{\partial\beta}\left[N(\cancel{\ln 2} + \ln\cosh\beta\varepsilon)\right]$$

$$= -N \frac{1}{\cosh\beta\varepsilon} \varepsilon \sinh\beta\varepsilon$$

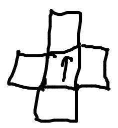
$$\langle U \rangle = -N\varepsilon \tanh\beta\varepsilon$$



↑↑↑↑↑ or ↓↓↓↓↓ symmetry breaking

Same as paramagnet in external field

Mean-Field Approximation



Consider one dipole & its n neighbors

Suppose dipole points up

$$E_\uparrow = -\epsilon \sum_{\text{neighbors}} s_{\text{neighbor}} = -\epsilon n \bar{s}_{\text{neighbor}}$$

$\bar{s}$: average alignment of neighbors

$$E_\downarrow = +\epsilon n \bar{s}$$

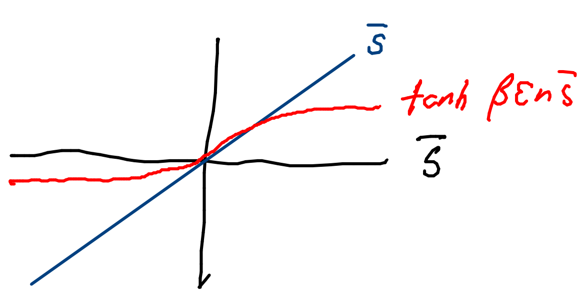
$$Z_i = e^{\beta\epsilon n \bar{s}} + e^{-\beta \epsilon n \bar{s}} = 2\cosh(\beta\epsilon n \bar{s})$$

$$\bar{s}_i = 1\,\underbrace{\frac{e^{\beta\epsilon n\bar{s}}}{Z_i}}_{\text{prob. pointing }\uparrow} + -1\,\underbrace{\frac{e^{-\beta\epsilon n\bar{s}}}{Z_i}}_{\text{prob. pointing }\downarrow} = \frac{\sinh \beta\epsilon n\bar{s}}{\cosh \beta\epsilon n \bar{s}} = \tanh \beta\epsilon n\bar{s}$$

Mean-Field Approximation: $\bar{s}_i = \bar{s}$

assumes spatial uniformity

$$\bar{s} = \tanh \beta \epsilon n \bar{s}$$

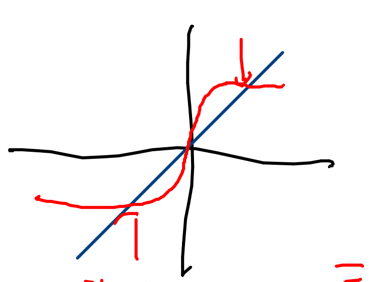


solution: $\bar{s} = 0$

dipoles just as likely to point ↑ or ↓

paramagnet

if $\beta n \epsilon > 1$



stable solutions

$\bar{s} \neq 0$

ferromagnet phase

$$\beta n\epsilon = 1 \longrightarrow kT_c = n\epsilon$$

when paramagnet goes to ferromagnet

phase transition

T_c : Curie temperature