$\langle A \rangle$: expectation value of A

$\langle \psi | A | \psi \rangle$

Functions as Vectors

square-integrable functions $\int |\psi(x)|^2 \, dx < \infty$

$$\langle f | g \rangle = \int f^*(x) \, g(x) \, dx$$

A set of functions f_n form a basis if

- orthonormal $\langle f_m | f_n \rangle = \delta_{mn}$
- Any function $f(x) = \sum_{n=1}^{\infty} c_n f_n(x)$

$$c_n = \langle f_n | f \rangle$$

e.g. $\sin nx$ and $\cos nx$ over $-\pi \leq x \leq \pi$ form a basis

$$\int_{-\pi}^{\pi} \sin nx \, \sin mx \, dx = \delta_{nm}$$

$$\int_{-\pi}^{\pi} \sin nx \, \cos mx \, dx = 0,$$

basis

e.g. $e^{-2\pi i k x}$ $\quad k \in \mathbb{R}$ $\quad -\infty < x < \infty$

$$f(x) = \int_{-\infty}^{\infty} f(k) \, e^{-2\pi i k x} \, dk$$

Fourier transform.

linear combination

Operator A takes vectors to vectors, or functions to functions

e.g. $A = x$ $\quad A f(x) = x f(x)$

$A f(x) = f(x)^2$

$A = \frac{d}{dx}$ $\quad A f(x) = \frac{d}{dx} f(x)$

$$\langle A \rangle = \langle \psi | A | \psi \rangle = \int \psi^*(x)\, A \psi(x)\, dx$$

Operators have eigenfunctions

$$A f(x) = \lambda f(x)$$

e.g. $A = \frac{d}{dx}$ $\quad e^{\alpha x}$ is a eigenfunction of A

$$A e^{\alpha x} = \alpha e^{\alpha x}$$

∞ # of eigenstates! $\quad$ ∞-dimensional vector space

$\hat{p} = \frac{\hbar}{i} \frac{\partial}{\partial x}$ $\quad$ momentum operator

$\hat{x} = x$ $\quad$ position operator

e.g. $\langle x \rangle = \langle \psi | \hat{x} | \psi \rangle = \int \psi^* x \psi \, dx$

$\langle p \rangle = \langle \psi | \hat{p} | \psi \rangle = \int \psi^* \frac{\hbar}{i} \frac{\partial}{\partial x} \psi \, dx$

$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}$ $\quad$ $\hat{V}\psi = V\psi$

KE $\quad$ PE

$= \frac{1}{2m} \frac{\hbar}{i} \frac{\partial}{\partial x} \frac{\hbar}{i} \frac{\partial}{\partial x} + V$

$= -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V$

Time-Dependent Schrodinger equation $\quad$ $i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi = -\frac{\hbar^2}{2m} \frac{d^2 \Psi}{dx^2} + V\Psi$

to find $\Psi(x,t)$; given $\Psi(x,0) = \psi(x)$,

1) write $\Psi(x,0)$ in energy basis
2) apply schrodinger factors

Need energy eigenstates

$$\hat{H} \psi = E \psi$$

$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$ $\quad$ Time-independent Schrodinger's equation

energy eigenstates or stationary states

Infinite Square Well (summary)

$$V(x) = \begin{cases} 0, & 0 \le x \le a \\ \infty, & \text{otherwise} \end{cases}$$

When $0 \le x \le a$, $V = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

Is energy eigenstate equation

$$\psi'' = -\frac{2mE}{\hbar^2}\psi = -k^2\psi \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

$$\rightarrow \psi(x) = A\sin kx + B\cos kx$$

Boundary Conditions

$$\psi(0) = \psi(a) = 0$$

$$\psi(0) = B = 0$$

$$\psi(a) = A\sin ka = 0 \rightarrow ka = n\pi$$

$$\therefore k = \frac{n\pi}{a} \qquad n \in \mathbb{Z}^+$$

(This is probably more detail than I'll give in future summaries.)