Physics 4310 Homework #7

5 problems

Due by March 21

▷ **1.**

Show that the eigenstates of the momentum operator p in one dimension are travelling waves with the deBroglie wavelength

$$\lambda = \frac{2\pi\hbar}{p}$$

Hint: If you're a little rusty with waves, remember that the definition of wavelength is the smallest λ *for which* $\psi(x) = \psi(x+\lambda)$ *for all* x*.*

▷ **2.**

Use the equation

$$\frac{d\langle Q\rangle}{dt} = \frac{i}{\hbar}\langle [H,Q]\rangle + \left\langle \frac{dQ}{dt} \right\rangle$$

to find $\frac{d\langle p\rangle}{dt}$.

▷ **3.**

(Griffiths 4.2) Use separation of variables in *cartesian* coordinates to solve the infinite *cubical* well (a "particle in a box"):

$$V(x,y,z) = \begin{cases} 0 & \text{if } x,\ y, \text{ and } z \text{ are all between 0 and } a \\ \infty & \text{otherwise} \end{cases}$$

(a) Find the energy eigenstates, and their corresponding eigenvalues.
(b) Call the *distinct* energies $E_1 < E_2 < E_3 < \ldots$. Find E_1, E_2, E_3, E_4, E_5, and E_6. Determine their degeneracies—that is, the number of different states that share the same energy. (In one dimension, degenerate bound states do not occur (Problem 2.45), but in three dimensions they are very common.)

▷ **4.**

(a) Find Y_0^0 and Y_2^1 Show that they are normalized and orthogona.
(b) Find Y_l^l. Check that it satisfies the angular equation

$$\sin\theta \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial Y}{\partial\theta}\right) + \frac{\partial^2 Y}{\partial\phi^2} = -l(l+1)\sin^2\theta\, Y$$

▷ **5.**

For the infinite spherical well of radius $a = 10^{-10}\,\text{m}$, find the energy eigenvalue (in eV) corresponding to $n = 3$, $l = 2$, and $m = 1$.