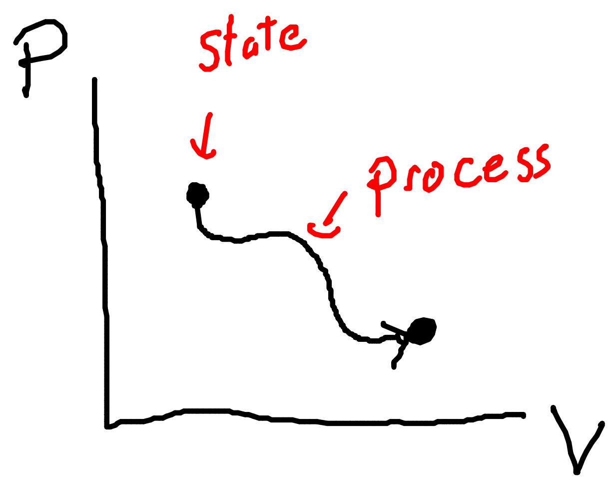
Chapter 4

Heat Engines & Refrigerators



ideal gas

N constant

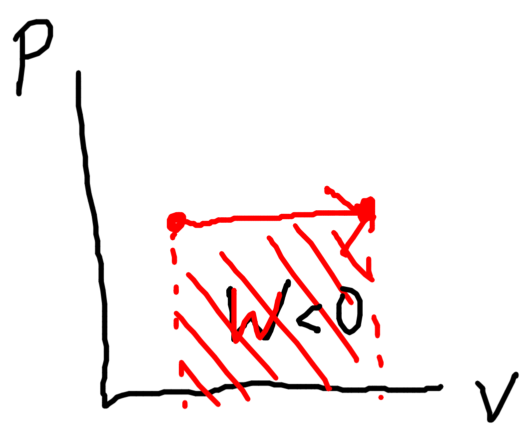
$PV = NkT$

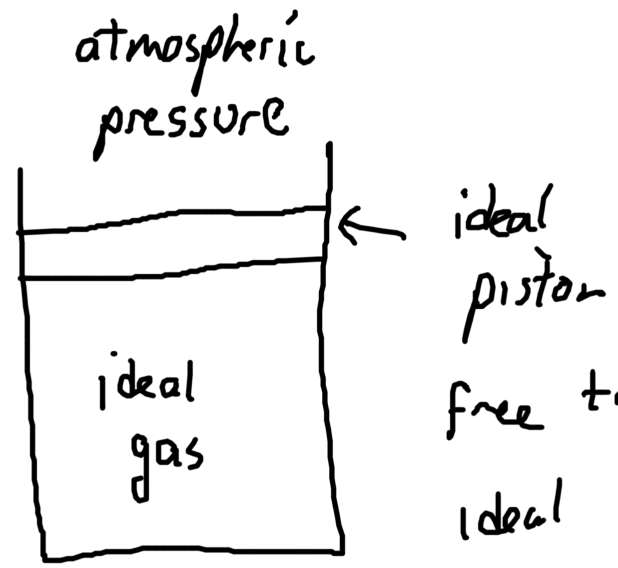
T is determined by P & V.

$U = N\frac{f}{2}kT$

same pressure

isobaric expansion





free to move

ideal gas is in equilibrium with atmosphere

$$\underset{\text{const}}{P}\ V = NkT$$

expansion: work is flowing out $\quad W < 0$

$$W = -P\Delta V = -\int P\,dV$$

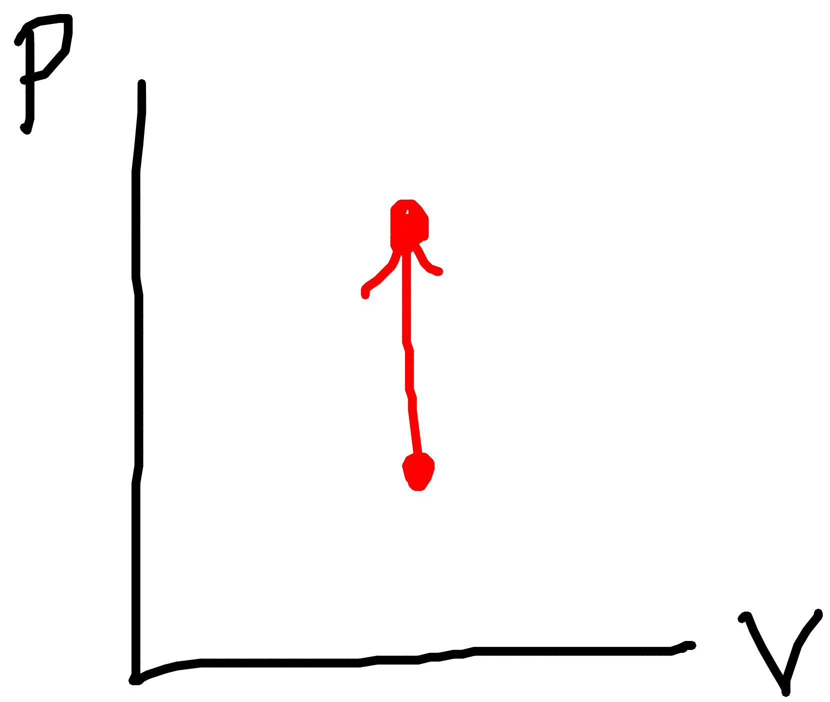
$W \propto$ area under a process

process $\rightarrow$ $\quad W < 0$

process $\leftarrow$ $\quad W > 0$

isometric / isochoric process

"constant volume" process



$$PV = NkT$$

const

$$W = 0$$

$$\Delta U = N\frac{f}{2}k\Delta T > 0$$

$Q > 0$ heat flows in

isothermal process

T = constant

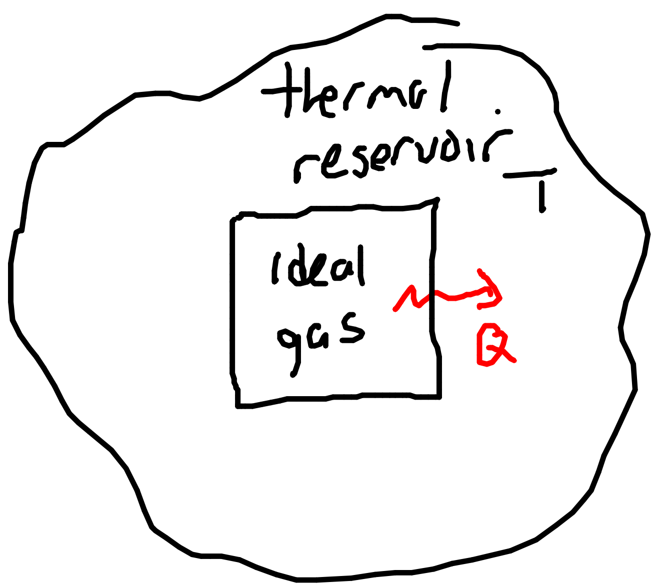
$PV = NkT$ = constant

$P = \frac{c}{V}$

$\Delta U = 0$

$W > 0$ because it's expanding

$\therefore\ Q < 0$ heat flows out



must stay in thermal equilibrium with reservoir

$\therefore$ isothermal process is relatively slow

$$W = -\int P\,dV = -\int \frac{NkT}{V}\,dV = -NkT\int \frac{dV}{V}$$

$$= -NkT \ln V \longrightarrow -NkT \ln \frac{V_f}{V_i}$$

$\Delta U = 0$

$$Q = -W = +NkT \ln \frac{V_f}{V_i}$$

Adiabatic Process : no heat flow, or no change in entropy

- rather quickly so no heat flows (insulation helps)
- not too fast or else you'll get turbulence & S increases
  quasistatic but fast

$$Q = 0 \rightarrow \quad dU = W = -P\,dV$$

$$\frac{f}{2} Nk\,dT = -P\,dV$$

$$\frac{f}{2} \cancel{Nk}\,dT = -\frac{\cancel{NkT}}{V}\,dV$$

$$\int_{T_i}^{T_f} \frac{f}{2}\frac{dT}{T} = -\int_{V_i}^{V_f} \frac{dV}{V}$$

$$\frac{f}{2} \ln \frac{T_f}{T_i} = -\ln \frac{V_f}{V_i}$$

$$\left(\frac{T_f}{T_i}\right)^{f/2} = \left(\frac{V_f}{V_i}\right)^{-1}$$

$$\rightarrow \quad V_f T_f^{f/2} = V_i T_i^{f/2}$$

$$\rightarrow \quad V T^{f/2} = \text{constant}$$

$$\rightarrow \left[V (PV)^{f/2}\right]^{2/f} = \left[\text{constant}\right]^{2/f}$$

$$\rightarrow \quad P V^{1+\frac{2}{f}} = \text{constant}$$

$$\boxed{P V^{\frac{f+2}{f}} = \text{constant}}$$

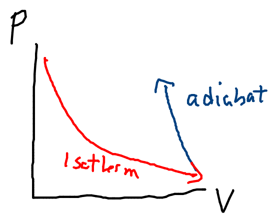
$\gamma = \frac{f+2}{f}$ adiabatic exponent

$f = 1 \quad \gamma = 3$

$f = 3, \quad \gamma = \frac{5}{3}$

$\vdots$

$f \to \infty \quad \gamma = 1$



e.g. Diesel engine quickly compresses air to $\frac{1}{20}$th of initial value.
What is T after compression?

$$T_f^{\frac{f}{2}} V_f = T_i^{f/2} V_i$$

$$T_f V_f^{2/f} = T_i V_i^{2/f}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{2/f}$$

$T_i = 300K$

$V_f = V_i \frac{1}{20} \quad \frac{V_i}{V_f} = 20$

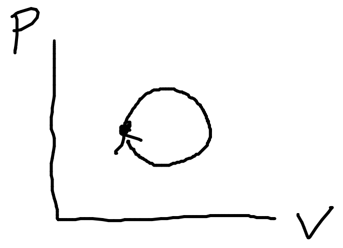
$f = 5$

$$T_f = (300K)(20)^{2/5}$$

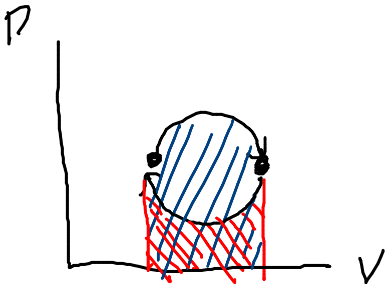
$$\approx 1000K \text{ or } 700°C$$

Diesel engines can ignite gas without sparks

Cyclic Process



After each cycle, all state variables must return to original variables especially the <u>entropy</u>



work done on bottom half

$W_b > 0$

work done on top half

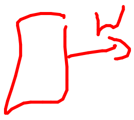
$W_t < 0$

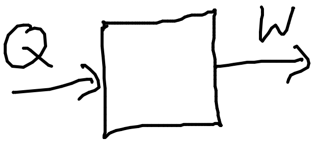
$|W_t| > |W_b|$

$W = W_t + W_b < 0$

U is a state variable, so if net work flows out of system after one cycle, heat must flow in

Perpetual motion 1st kind





heat is converted to work

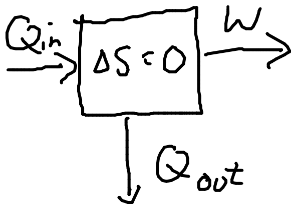
Perpetual motion machine of the 2nd kind - impossible

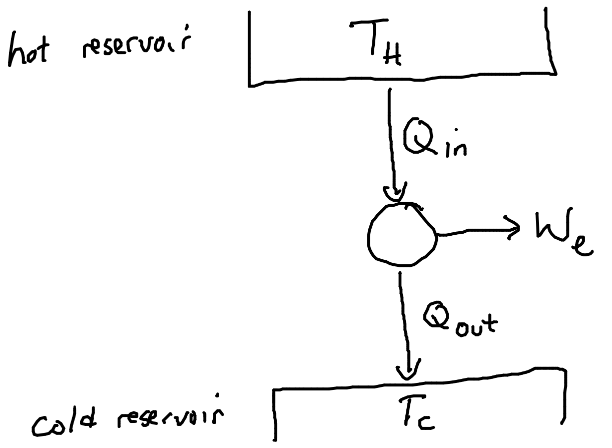
Why?

Q flows in, entropy increases

in a cyclic process, entropy has to return to its initial value, so excess entropy must be disposed of

Heat Engine:





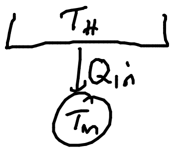
$$Q_{in} = W_e + Q_{out}$$

Efficiency of Engine

$$\eta = \frac{W_e}{Q_{in}} \leq 100\%$$

$$= \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Let T_{in} be temperature of engine when heat flows in

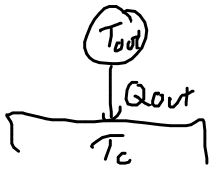


$$T_{in} < T_H$$

$$\Delta S = + \frac{Q_{in}}{T_{in}}$$

Let T_{out} be temperature of engine when heat flows out

$$\Delta S = - \frac{Q_{out}}{T_{out}}$$



$$T_{out} > T_c$$

$$\Delta S = \frac{Q_{in}}{T_{in}} - \frac{Q_{out}}{T_{out}} = 0$$

$$\frac{Q_{in}}{Q_{out}} = \frac{T_{in}}{T_{out}} \rightarrow \eta = 1 - \frac{T_{out}}{T_{in}}$$

make this small

To maximize η,

- make T_{out} small, but $T_{out} > T_c$
- make T_{in} big, but $T_{in} < T_H$

Best we can do

$$\eta < \underbrace{1 - \frac{T_c}{T_H}}_{\text{maximum possible efficiency}} = \frac{T_H - T_c}{T_H}$$

e.g. $T_c = 300K$ $T_H = 1000K \approx 700°C$

$$\eta < 70\%$$

$$\eta = 1 - \frac{T_{out}}{T_{in}}$$

Carnot engine is maximally efficient

bring $T_{out} \gtrsim T_c$

$T_{in} \lesssim T_H$

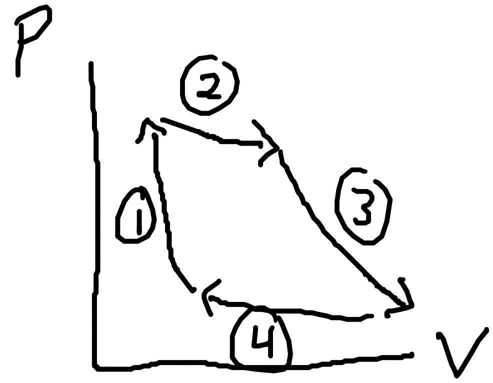
1) raise T of engine to almost T_H, adiabatically (no heat, fast)

2) let heat flow from hot reservoir into engine
almost isothermal

3) lower T to almost T_c, adiabatically

4) let heat flow out of engine (isothermal)

$$\eta_{carnot} = 1 - \frac{T_c}{T_H}$$



Steps 2 & 4 are isothermal - <u>slow</u>

When heat flows, <u>rate</u> of flow depends on ΔT

ΔT is very small, rate of flow is very slow

→ low-power, high-efficiency engine