in $|a_1\rangle$ $|a_2\rangle$ basis

$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ shorthand for $3|a_1\rangle + 4|a_2\rangle$

$\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ shorthand for $\left(1\langle a_1| + 2\langle a_2|\right)\left(3|a_1\rangle + 4|a_2\rangle\right)$

$$|x_1\rangle = a|y_1\rangle + b|y_2\rangle$$

$$|x_2\rangle = c|y_1\rangle + d|y_2\rangle$$

$$\Rightarrow \quad |y_1\rangle = a'|x_1\rangle + b'|x_2\rangle$$

$$|y_2\rangle = c'|x_1\rangle + d'|x_2\rangle$$

A spin in a magnetic field

$$\vec{B} = B_0 \hat{z} + B_1 \left[\cos \omega t \, \hat{x} + \sin \omega t \, \hat{y}\right]$$

$$H \doteq_{\uparrow\downarrow} \begin{pmatrix} \omega_0 & \omega_1 e^{-i\omega t} \\ \omega_1 e^{+i\omega t} & -\omega_0 \end{pmatrix} \qquad \omega_0 = \frac{eB_0}{m} \qquad \omega_1 = \frac{eB_1}{m}$$

Strength of fields

ω: frequency of spinning field

We need to solve $i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle$

In $\uparrow\downarrow$ basis

$$|\psi(t)\rangle = c_+(t)|\uparrow\rangle + c_-(t)|\downarrow\rangle = \begin{pmatrix} c_+ \\ c_- \end{pmatrix}$$

$$i\hbar \begin{pmatrix} \dot{c}_+ \\ \dot{c}_- \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} \omega_0 & \omega_1 e^{-i\omega t} \\ \omega_1 e^{i\omega t} & -\omega_0 \end{pmatrix} \begin{pmatrix} c_+ \\ c_- \end{pmatrix}$$

$$i\hbar \dot{c}_+ = \frac{\hbar}{2}\omega_0 c_+ + \frac{\hbar}{2}\omega_1 e^{-i\omega t} c_-$$

$$i\hbar \dot{c}_- = \omega_1 e^{i\omega t} c_+ - \omega_0 c_-$$

Let $c_\pm(t) = \alpha_\pm(t) e^{\mp i\omega t/2}$

$$i\hbar\left[\dot{\alpha}_+ \cancel{e^{-i\omega t/2}} + \alpha_+\left(-\frac{i\omega}{2}\right)\cancel{e^{-i\omega t/2}}\right] = \frac{\hbar}{2}\omega_0 \alpha_+ \cancel{e^{-i\omega t/2}} + \frac{\hbar\omega_1}{2}\alpha_- \cancel{e^{+i\omega t/2}} \cancel{e^{-i\omega t}}$$

$$i\hbar \dot{\alpha}_+ = i\hbar\left(+\frac{i\omega}{2}\right)\alpha_+ + \frac{\hbar\omega_0}{2}\alpha_+ + \frac{\hbar\omega_1}{2}\alpha_-$$

$\Delta\omega = \omega - \omega_0$

rotating freq. / strength of B_0; spacing between energy levels $\hbar\omega_0$

$$i\hbar \dot{\alpha}_+ = -\frac{\hbar\Delta\omega}{2}\alpha_+ + \frac{\hbar\omega_1}{2}\alpha_-$$

$$i\hbar \dot{\alpha}_- = \frac{\hbar\omega_1}{2}\alpha_+ + \frac{\hbar\Delta\omega}{2}\alpha_-$$

$$i\hbar \frac{d}{dt}\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} -\Delta\omega & \omega_1 \\ \omega_1 & \Delta\omega \end{pmatrix}\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$$

$$i\hbar \frac{d}{dt}|\alpha\rangle = H'|\alpha\rangle$$

This is just like spin precession problem before $H \doteq \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_1 \\ \omega_1 & -\omega_0 \end{pmatrix}$

if $|\alpha(0)\rangle = |\uparrow\rangle$

$$P_{\uparrow\to\downarrow}(t) = \frac{\omega_1^2}{(\Delta\omega)^2 + \omega_1^2} \sin^2\left(\frac{\sqrt{\Delta\omega^2 + \omega_1^2}}{2} t\right)$$

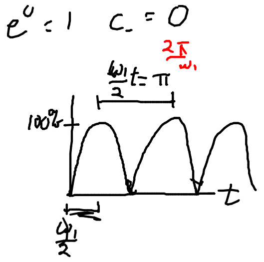
~~$|c\rangle = |\alpha\rangle$~~ $\quad c_\pm = \alpha_\pm e^{\mp i\omega t/2}$

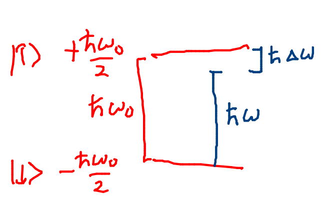
if $|\alpha(0)\rangle = |\uparrow\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \alpha_+ = 1 \quad \alpha_- = 0$

$c_+ = 1 e^0 = 1 \quad c_- = 0$

if $|c(0)\rangle = |\uparrow\rangle$

if $\Delta\omega \to 0$, then $P = \sin^2 \frac{\omega_1 t}{2}$





Choose rotation frequency to match energy jump required, & it will happen w/ 100% ~~freq~~ probability

Magnetic Resonance

Light-Matter Interactions similar
tune frequency of light to match difference in energy levels

Griffiths 3.2

Function Spaces

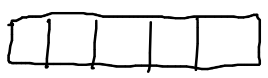
A vector space where the vectors are complex functions of x ($, y, z, \dots$)

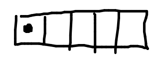
$|\psi\rangle \doteq \psi(x)$

In QM, we're particularly interested in square-integrable functions

$$\int |\psi|^2 dx < \infty$$

Suppose a particle can be in one of 5 positions



$|1\rangle$: definitely in 1st position 

~~In general~~ $|1\rangle, |2\rangle, |3\rangle, |4\rangle, |5\rangle$ form a basis

$P_{|1\rangle \text{ on square } 2} \cdot |\langle 2|1\rangle|^2 = 0$ orthogonal

In general

$$|\psi\rangle = \psi_1|1\rangle + \psi_2|2\rangle + \psi_3|3\rangle + \psi_4|4\rangle + \psi_5|5\rangle$$

$$\langle 1|\psi\rangle = \psi_1$$

$$|\psi\rangle \doteq \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \end{pmatrix} \qquad P(\psi \text{ is on square } 1) = |\langle 1|\psi\rangle|^2 = |\psi_1|^2$$

Now generalize to the number line

$|3.257.. m\rangle$ $|-5m\rangle$ etc

∞-dimensional vector space

$$\langle x_1|x_2\rangle = \delta_{x_1,x_2}$$

$$|\psi\rangle = \int \psi(x)|x\rangle\, dx$$

$$\psi(x) = \langle x|\psi\rangle$$

e.g. $\psi(3m)$ = coefficient of term with $|x=3m\rangle$ in it

$|x=3m\rangle$: state where particle is at position $x=3m$ w/certainty.

Position operator has eigenstates $|x\rangle$

& if I measure $|\psi\rangle$ with position operator

I get $|x\rangle$ with probability $P = |\langle x|\psi\rangle|^2 = |\psi(x)|^2$