Ideal Gas: set of non-interacting particles

reasonable approximation for dilute gases

physics

$$PV = N k_B T$$

or

chemistry

$$PV = nRT$$

Standard Temperature & Pressure (STP)

$T = 300\,K$

$1\ atm = 1.013 \times 10^5\ Pa$

$1\ bar = 10^5\ Pa$

P: pressure in Pascals ($N/m^2$)

V: volume in $m^3$

T: temperature in Kelvin

N: # of particles

n: # of moles

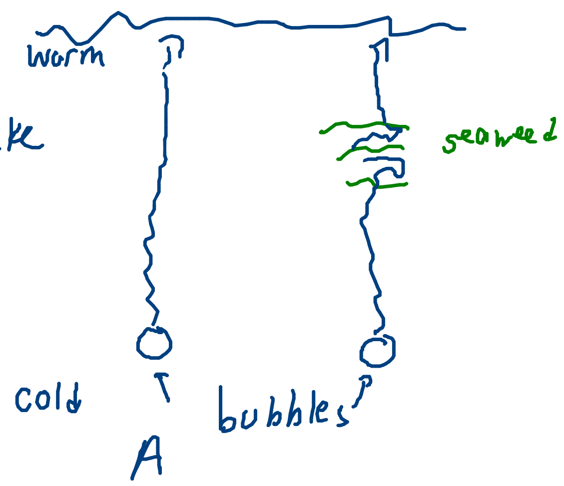
$k_B = 1.38 \times 10^{-23}\ J/K$

$R = k_B N_A = 8.31\ J/K\,mol$

"gas constant"

$N_A = 6.02 \times 10^{23}\ /mol$

e.g.




$PV = NkT$

What happens to the bubble's size as it rises?

$P$ decreases

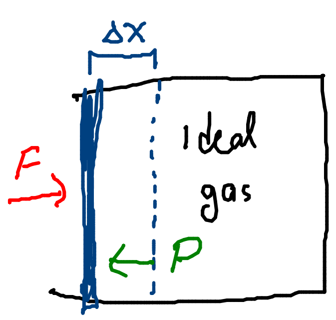
$T$ increases

$\downarrow P V \uparrow = N k T \uparrow$

expands

Which is bigger at the top?

B, because it is warmer (more time to reach thermal equilibrium)




Ideal piston

massless

frictionless

perfect seal

Compression Work

$W_{\substack{on \\ gas}} = F \Delta x$

for piston $F_{net} = m\overset{0}{a} = 0.$

so $F = PA$ ← cross-sectional area of piston

$W = PA\Delta x$

but $A\Delta x = -\Delta V$ volume that gas loses

$$\boxed{W = -P\Delta V}$$

if gas expands, $\Delta V > 0$, and $W < 0$

work flows out into the environment

Caveat: P is well-defined or this isn't true.

If you exert force too quickly

shock waves (pressure waves) form

& P is no longer single-valued.

Some of the energy you put into the gas is dissipated as turbulence

We'll work in the quasistatic limit

- allow system to maintain "pressure equilibrium" if $P(\bar{V})$ changes with volume

$$W = -\int_{V_i}^{V_f} P(\bar{V})\, d\bar{V}$$

$$- \quad - \quad - \quad -$$

# Heat Capacities of an Ideal Gas

$$C = \frac{Q}{\Delta T} = \frac{\Delta U}{\Delta T} - \frac{W}{\Delta T}$$

if $W=0$, then $C = \frac{\Delta U}{\Delta T}$

for ideal gas, $W=0$ means $\Delta V$ constant

h.c. at constant volume $\quad C_V = \left(\frac{\partial U}{\partial T}\right)_V$ constant volume

Aside: $f(x, y, z)$ $\quad \frac{\partial f}{\partial x}$ assumes $y$ & $z$ are constant or independent of $x$

Full derivative $\quad \frac{df}{dx}$ assumes $y$ & $z$ do depend on $x$

$$\left(\frac{\partial f}{\partial x}\right)_y \neq \left(\frac{\partial f}{\partial x}\right)_z$$

assume $y$ is constant but $z$ depends on $x$

vice versa

essential in thermodynamics where everything is interrelated

$C_p$ is heat capacity at constant pressure

e.g. a gas surrounded by & in equilibrium with air


$$C_p = \frac{\Delta U}{\Delta T} - \frac{W}{\Delta T} = \frac{\Delta U}{\Delta T} + \frac{P\Delta V}{\Delta T}$$

$$C_p = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P$$

e.g. ideal gas obeys equipartition theorem

$$U = N\frac{f}{2}kT \qquad \left(\frac{\partial U}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_P = N\frac{f}{2}k$$

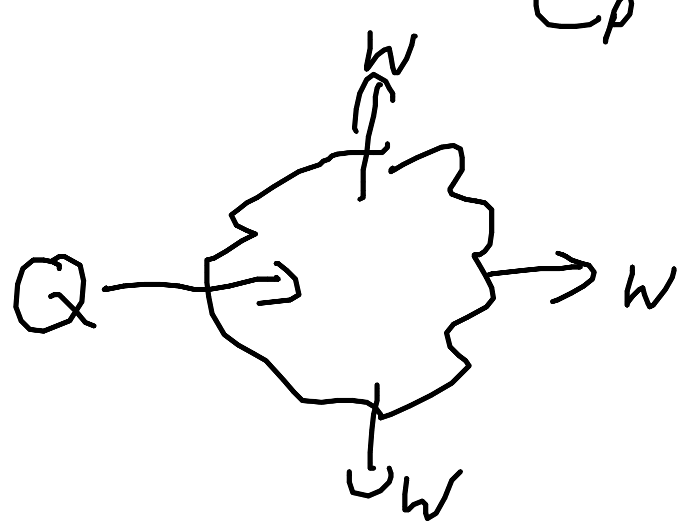
$$C_p = N\frac{f}{2}k + P\frac{Nk}{P}$$

$$\frac{\partial V}{\partial T} = \frac{\partial}{\partial T}\left(\frac{NkT}{P}\right) = \frac{Nk}{P}$$

$$C_p = C_V + Nk$$

$\uparrow$

$C_p$ is larger than $C_V$




less energy remains to increase T because some of Q goes into moving surrounding air out of the way.