Magnetic Dipole in a magnetic field $\vec{B} = B_0 \hat{z}$

electron

$$H = -\vec{\mu}\cdot\vec{B} = \omega_0 S_z \qquad \omega_0 = \frac{eB_0}{m}$$

in $\uparrow\downarrow$ $\qquad H = \frac{\hbar\omega_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

Energy eigenstates : $\quad H|\uparrow\rangle = \frac{\hbar\omega_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{\hbar\omega_0}{2}\begin{pmatrix}1\\0\end{pmatrix}$

$\qquad H|\downarrow\rangle = \frac{\hbar\omega_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{\hbar\omega_0}{2}\begin{pmatrix}0\\-1\end{pmatrix}$

$|\uparrow\rangle \quad |\downarrow\rangle$

E: $+\frac{\hbar\omega_0}{2} \quad -\frac{\hbar\omega_0}{2}$

$+\frac{\hbar\omega_0}{2}$ $|\uparrow\rangle$

$\hbar\omega_0$

$-\frac{\hbar\omega_0}{2}$ $|\downarrow\rangle$

$\vec{B} = B_0\hat{z}$ $\Uparrow$

$|\uparrow\rangle$ has higher energy?

yes, $|\uparrow\rangle$ has $\vec{S}\uparrow$

$\vec{\mu} \propto q\vec{S}$ $\quad q<0$ for electron

$|\uparrow\rangle$ has $\vec{\mu}\downarrow$

Suppose $|\psi(0)\rangle = |\uparrow\rangle$

$$|\psi(t)\rangle = e^{-iEt/\hbar}|\uparrow\rangle = e^{-i\omega_0 t/2}|\uparrow\rangle$$

stationary state

if $|\psi(0)\rangle = |\theta\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}e^{-i\omega_0 t/2}|\uparrow\rangle + \frac{1}{\sqrt{2}}e^{+i\omega_0 t/2}|\downarrow\rangle$$

$E_\uparrow = +\hbar\omega_0/2$ $\qquad E_\downarrow = -\frac{\hbar\omega_0}{2}$

$$P_\uparrow(t) = |\langle\uparrow|\psi(t)\rangle|^2$$

$$= \left|\frac{1}{\sqrt{2}}e^{-i\omega_0 t/2}\right|^2 = \frac{1}{2}.$$

$P_\downarrow(t) = \frac{1}{2}$ $\qquad \langle S_z\rangle = 0$ independent of time because $[H, S_z] = 0$.

$$|\psi(0)\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}e^{i\phi}|\downarrow\rangle \qquad \theta, \phi \text{ constants}$$

$$|\psi(t)\rangle = \cos\frac{\theta}{2}e^{-i\omega_0 t/2}|\uparrow\rangle + \sin\frac{\theta}{2}e^{i\phi}e^{+i\omega_0 t/2}|\downarrow\rangle$$

$\uparrow\downarrow$ $$\doteq e^{-i\omega_0 t/2}\begin{pmatrix}\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i(\phi+\omega_0 t)}\end{pmatrix}$$

$$P_{\odot} = |\langle\odot|\psi(t)\rangle|^2$$

$$= \left|\frac{1}{\sqrt{2}}(1 \quad 1)e^{-i\omega_0 t/2}\begin{pmatrix}\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i(\phi+\omega_0 t)}\end{pmatrix}\right|^2$$

$$= \frac{1}{2}\left|\cos\frac{\theta}{2} + \sin\frac{\theta}{2}e^{i(\phi+\omega_0 t)}\right|^2$$

$$= \frac{1}{2}\left[\cos\frac{\theta}{2} + \sin\frac{\theta}{2}e^{i(\phi+\omega_0 t)}\right]\left[\cos\frac{\theta}{2} + \sin\frac{\theta}{2}e^{-i(\phi+\omega_0 t)}\right]$$

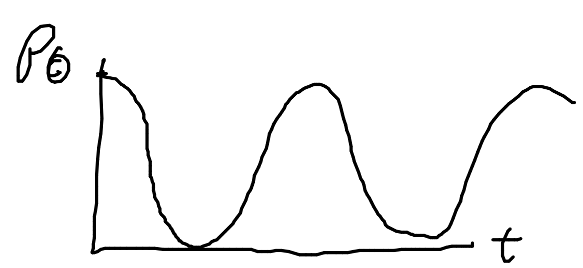
$$= \frac{1}{2}\left[\cos^2\frac{\theta}{2} + \cos\frac{\theta}{2}\sin\frac{\theta}{2}\left(e^{i(\phi+\omega_0 t)} + e^{-i(\phi+\omega_0 t)}\right) + \sin^2\frac{\theta}{2}\right]$$

$$= \frac{1}{2}\left[1 + \cos\frac{\theta}{2}\sin\frac{\theta}{2}\, 2\cos(\phi+\omega_0 t)\right]$$

$$P_{\odot} = \frac{1}{2}\left[1 + \sin\theta\cos(\phi+\omega_0 t)\right]$$

e.g. if $\theta = \frac{\pi}{2}$ $\phi = 0$ $\quad |\psi(0)\rangle = |\odot\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$

$$P_{\odot} = \frac{1}{2}\left[1 + \cos(\phi+\omega_0 t)\right]$$



Spin precesses around the z-axis, just like a top

$$\langle S_x\rangle = \frac{\hbar}{2}\sin\theta\cos(\phi+\omega_0 t)$$

Larmor precession $\quad \omega_0$: Larmor frequency

Why we think of spin as angular momentum.