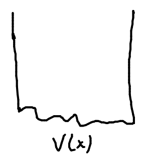
$$\psi(x) = \frac{C}{\sqrt{p(x)}} e^{\pm \frac{i}{\hbar}\int^x p(x)\,dx'}$$



$$p(x) = \sqrt{2m[E - V(x)]}$$

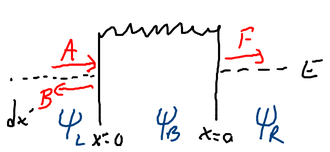
$E \gg V(x)$

Why? $\lambda \sim \frac{1}{p} = \frac{1}{\sqrt{2m(E-V(x))}}$ we need this to be small

If $E < V$,

$p(x)$ is imaginary

$$\psi(x) = \frac{C}{\sqrt{|p(x)|}} e^{\pm\frac{1}{\hbar}\int^x |p(x)|\,dx'}$$



$$\psi_L(x) = Ae^{ikx} + Be^{-ikx}$$

$$\psi_R(x) = Fe^{ikx}$$

$$\psi_B(x) = \frac{C}{\sqrt{|p(x)|}} e^{+\frac{1}{\hbar}\int^x |p(x')|dx'} + \frac{D}{\sqrt{|p(x)|}} e^{-\frac{1}{\hbar}\int^x |p(x')|dx'}$$

exponential increasing — exponentially decreasing

if ~~wall~~ barrier is high or wide & tunnelling is unlikely, then $C \approx 0$.

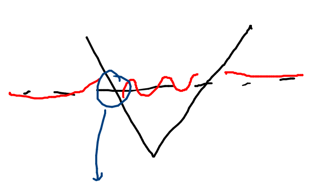
$$\cancel{\psi_L(0) = A + B = \frac{D}{\sqrt{|p(0)|}}}$$

$$\frac{|F|}{|A|} \sim \frac{e^{-\frac{1}{\hbar}\int_0^a |p(x')|dx'}}{\cancel{e^{-\frac{1}{\hbar}\int_0^0 |p(x')|dx'}}_{\,1}}$$

$$T = \frac{|F|^2}{|A|^2} \simeq e^{-2\gamma} \qquad \gamma = \frac{1}{\hbar}\int_0^a |p(x)|\,dx$$

if $T \ll 1$

If walls aren't vertical



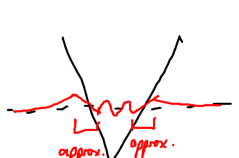
$E = V(x)$ at "turning point"

$$p(x) = \sqrt{2m(E - V(x))} = 0$$

$$\to \lambda \approx \frac{1}{p(x)} = \infty$$

WKB fails just where you need it most → @ boundaries

Solution: approximate wavefunction in boundary region by a straight line



Chapter 9: Time-Dependent Perturbation Theory.

Suppose a particle is in one energy eigenstate

~~But~~ How can I move that particle to a different energy eigenstate?

nonzero if particle transitions

$$\frac{d\langle H\rangle}{dt} = \frac{i}{\hbar}\langle \underbrace{[H,H]}_{0}\rangle + \left\langle \frac{dH}{dt}\right\rangle$$

... only if H has time-dependence.

Easier if time dependence is a perturbation

$$H(t) = H_0 + H'(t)$$

e.g. two-state Hamiltonian

$$H_0\psi_a = E_a\psi_a \qquad H_0\psi_b = E_b\psi_b \qquad \langle\psi_a|\psi_b\rangle = 0$$

$$\Psi(0) = c_a\psi_a + c_b\psi_b \qquad |c_a|^2+|c_b|^2 = 1$$

if $H'=0$: $$\Psi(t) = c_a\psi_a e^{-iE_at/\hbar} + c_b\psi_b e^{-iE_bt/\hbar}$$

if $H'\neq 0$. $$\Psi(t) = c_a(t)\psi_a e^{-iE_at/\hbar} + c_b(t)\psi_b e^{-iE_bt/\hbar}$$

$$(H_0+H')\Psi = H\Psi = i\hbar\frac{d\Psi}{dt}$$