Physics 4310 Homework #1

4 problems

Due by January 22

▷ **1.**

The possible outcomes of a S_y analyzer are

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle) \quad \text{and} \quad |\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle)$$

Prove the following:

(a) Prove these vectors are normalized. (That is: $\langle\leftarrow \mid \leftarrow\rangle = \langle\rightarrow \mid \rightarrow\rangle = 1$.)

(b) Prove they are orthogonal: $\langle\leftarrow \mid \rightarrow\rangle = 0$

(c) Prove that, if passed through a S_z analyzer, they will come out as $|\uparrow\rangle$ with probability 1/2.

(d) Prove that, if passed through a S_x analyzer, they will come out as $|\odot\rangle$ with probability 1/2.

▷ **2.**

In the spin–1/2 quantum system, consider the ket $|\psi\rangle = -3|\uparrow\rangle + 4i|\downarrow\rangle$.

(a) Normalize the ket.

(b) Find the probability that, if the ket were fed into an S_z analyzer, it would give a result of $S_z = +\frac{\hbar}{2}$.

(c) Find the probability that, if the ket were fed into an S_x analyzer, it would come out as $|\otimes\rangle$.

(d) Write the normalized ket as a column vector in the S_z basis.

(e) Write the normalized ket as a column vector in the S_x basis.

▷ **3.**

Show that a change in the overall phase of a quantum state vector does not change the probability of obtaining a particular result in a measurement. To do this, consider how the probability is affected by changing the state $|\psi\rangle$ to the state $e^{i\delta}|\psi\rangle$.

▷ **4.**

Prove the *Schwarz inequality*:

$$|\langle\alpha|\beta\rangle|^2 \leq \langle\alpha|\alpha\rangle\langle\beta|\beta\rangle$$

Hint: Consider the vector

$$|\gamma\rangle = |\beta\rangle - \frac{\langle\alpha|\beta\rangle}{\langle\alpha|\alpha\rangle}|\alpha\rangle$$

and the fact that $\langle\gamma|\gamma\rangle \geq 0$.