Physics 3410 Homework #2

6 problems

Solutions

▷ **1.**

How many molecules are in a cubic centimeter of air at standard temperature and pressure? (You needn't worry about the different types of molecules.)

Answer:

Standard temperature and pressure are $P = 10^5\,\mathrm{Pa}$ and $T = 300\,\mathrm{K}$. The volume is $(1\,\mathrm{cm})^3 = (10^{-2}\,\mathrm{m})^2 = 10^{-6}\,\mathrm{m}^3$. Using the ideal gas law, we have that

$$PV = NkT \implies N = \frac{PV}{kT} = \frac{(10^5\,\mathrm{Pa})(10^{-6}\,\mathrm{m}^3)}{(1.38\times10^{-23}\,\mathrm{J/K})(300\,\mathrm{K})} = \boxed{2.4\times10^{19}}$$

▷ **2.**

What is the internal energy U of $0.05\,\mathrm{m}^3$ of an ideal gas of point particles with pressure $P = 8\times10^4\,\mathrm{Pa}$?

Answer:

The internal energy is $U = \frac{f}{2}NkT$ and the ideal gas law is $PV = NkT$; combining the two gives us $U = \frac{f}{2}PV$. For point particles $f = 3$ (translational dof only), so

$$U = \frac{3}{2}(8\times10^4\,\mathrm{Pa})(0.05\,\mathrm{m}^3) = \boxed{4.2\,\mathrm{J}}$$

▷ **3.**

An ideal gas is compressed from some volume V_i to half that volume ($V_f = \frac{1}{2}V_i$) at constant temperature $T = 290\,\mathrm{K}$.

(a) How much work is done on the gas?

(b) How much heat flows into or out of the gas? (Hint: think about ΔU, how the internal energy changes.)

Answer:

(a) The work done on the gas is $W = -P\,\Delta V$, but the pressure changes as the volume does so we need to use the more general form:

$$W = -\int_{V_i}^{\frac{1}{2}V_i} P\,dV$$

$$= -\int_{V_i}^{\frac{1}{2}V_i} \frac{NkT}{V}\, dV$$

$$= -NkT \int_{V_i}^{\frac{1}{2}V_i} \frac{1}{V}\, dV \qquad NkT \text{ is constant}$$

$$= -NkT \left[\ln V\right]_{V_i}^{\frac{1}{2}V_i}$$

$$= -NkT \ln \frac{V_i/2}{V_i} = -NkT \ln \frac{1}{2} = \boxed{+NkT \ln 2}$$

It would have been nice if I'd given you N, but I didn't so I might as well leave it in this form.

(b) The temperature is constant, so the internal energy $U = \frac{f}{2}NkT$ is constant as well, so $\Delta U = 0$. According to the first law of thermodynamics, $\Delta U = Q + W$, so $\boxed{Q = -NkT \ln 2}$.

▷ **4.**
Two hundred joules of heat flows into an ideal gas of $N = 10^{23}$ point particles which maintains a constant pressure of $P = 3 \times 10^5$ Pa throughout the flow of heat.
(a) Is the volume of the gas increasing, staying the same, or decreasing?
(b) What is the heat capacity at constant pressure C_P?
(c) How much does the gas's temperature increase?

Answer:______________________________

(a) Look at the ideal gas law: $PV = NkT$. Heat is flowing into the gas, so the temperature is rising (because $\Delta T = Q/C$). Since N and P are constant, V must be $\boxed{\text{increasing}}$ as well.
(b) The heat capacity at constant pressure, as indicated in class, is

$$C_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P$$

The ideal gas obeys the equipartition theorem with $f = 3$ (point particles), so $U = \frac{3}{2}NkT$, and $\left(\frac{\partial U}{\partial T}\right)_P = \frac{3}{2}Nk$. From the ideal gas law, $V = \frac{NkT}{P}$, so

$$C_P = \frac{3}{2}Nk + P\left(\frac{\partial(NkT/P)}{\partial T}\right)_P = \frac{3}{2}Nk + Nk = \frac{5}{2}Nk = \frac{5}{2}(10^{23})(1.38 \times 10^{-23}) = \boxed{3.45\,\mathrm{J/K}}$$

(c) Because the pressure is constant, the relationship between heat and temperature change is

$$Q = C_P \Delta T \implies \Delta T = \frac{Q}{C_P} = \frac{200\,\mathrm{J}}{3.45\,\mathrm{J/K}} = \boxed{58\,\mathrm{K}}$$

▷ **5.**

How many different letter sequences can I make with the letters in the word "RACECAR"?

Answer:____________________________________

There are seven letters, but three sets of duplicates: 2 R's, 2 A's, and 2 C's. Thus the number of ways to rearrange the letters is

$$\Omega = \frac{7!}{2!2!2!} = \boxed{630}$$

▷ **6.**

Twenty people enter a raffle.

(a) How many different ways can I hand out identical prizes to three different people in the raffle? Give me a number, please, not just an expression.

(b) What if the prizes are different?

Answer:____________________________________

(a) I choose 3 people out of 20 to give prizes, so

$$\Omega = \binom{20}{3} = \frac{20!}{17!3!} = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = \boxed{1140}$$

(b) Once I choose the prize winners, I can rearrange the prizes $3!$ different ways, so

$$\Omega = (1140)(3!) = \boxed{6840}$$

Or you can think of it as a permutation of 3 out of 20, so $\Omega = \dfrac{20!}{(20-3)!} = 6840$.