N_A	N_B
q_A	q_B

$N = N_A + N_B$

$q = q_A + q_B$

↑ varies

HW 4, #3

$q >> N >> 1$

What value does q_A have at equilibrium?

$$\frac{\partial S}{\partial q_A} = 0$$

$$\Omega_A = \left(\frac{e q_A}{N_A}\right)^{N_A} \qquad S_A = k N_A \left[1 + \ln q_A - \ln N_A\right]$$

$$S(q_A) = k N_A \left[1 + \ln q_A - \ln N_A\right] + k N_B \left[1 + \ln(q - q_A) - \ln N_B\right]$$

$$\frac{\partial S}{\partial q_A} = \frac{k N_A}{q_A} - \frac{k N_B}{q_B} = 0$$

$$\frac{N_A}{q_A} = \frac{N_B}{q_B} \qquad \frac{q_A}{q_B} = \frac{N_A}{N_B} \qquad \frac{q_A}{N_A} = \frac{q_B}{N_B}$$

$$dU = T\,dS - P\,dV + \mu\,dN$$

$$\left(\frac{\partial U}{\partial N}\right)_{S,V} = \mu$$

Add a particle ($dN = 1$) at constant S & V,

$dU = \mu$.

Normally increasing N would increase S, unless you reduce U at the same time.

$\mu < 0$ normally.

e.g. Einstein solid $N = q = 3$ $\qquad \Omega = \binom{N+q-1}{q}$

$$\Omega = \binom{3+3-1}{3} = \binom{5}{3} = 10$$

Add oscillator : $N = 4,\ q = 3$

$$\Omega = \binom{6}{3} = 20$$

To keep Ω constant, remove energy

$N = 4,\ q = 2$

$$\Omega = \binom{4+2-1}{2} = \binom{5}{2} = 10$$

$\therefore\ \mu = -1$ units of energy

$$[\mu] = \left[\frac{\partial U}{\partial N}\right] = \text{Joules or eV}$$

Paramagnet

N spins $\qquad U = N_\uparrow$

$\Omega(N_\uparrow = 0) = 1 \qquad S(U=0) = 0.$

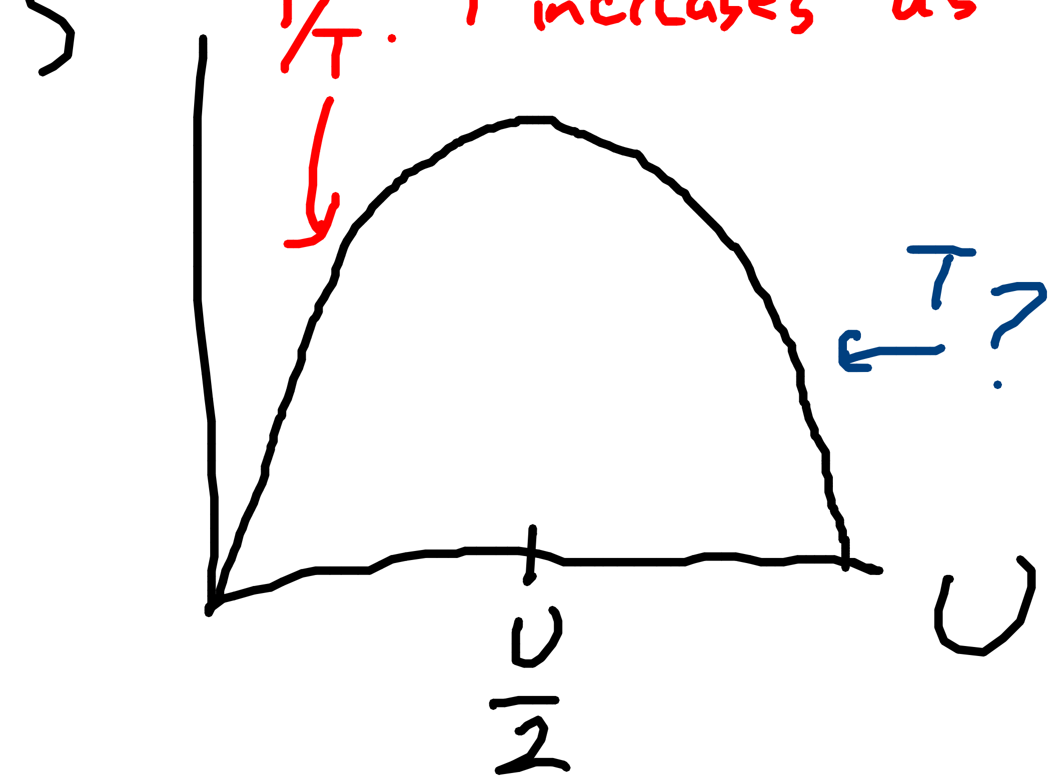
$\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow$

$\Omega(N_\uparrow = N) = 1 \qquad S(U = \overset{N}{U_{max}}) = 0.$

$\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$

slope $= \dfrac{\partial S}{\partial U} = \dfrac{1}{T}$

$1/T$: T increases as slope decreases

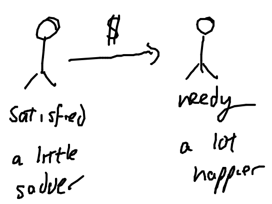


negative temperature?

What does that mean?

Analogy

- People exchange money to increase total happiness of their society
- Most people become happier when they get money
- People who are "needy" become *much* happier when given money.
- People who are "satisfied" or "comfortable" become only a little happier



Society's happiness increases

$$\text{need} = \frac{\partial ☺}{\partial \$} = \frac{1}{\text{comfort}}$$

money ⟷ energy
happiness ⟷ entropy
comfort ⟷ temperature

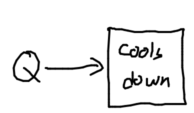
What happens to a normal person as they get richer?

become more comfortable
less needy
$\text{need} = \frac{\partial ☺}{\partial \$}$ will decrease

A *miser* becomes needier (less satisfied) as they get richer

slope increases
$\frac{\partial ☺}{\partial \$}$ increases

"Miserly systems" as U increases, $\frac{1}{T}$ increases
$\therefore$ T decreases



Miserly systems usually have an associated potential energy

e.g. planet in orbit

$F = G\frac{Mm}{r^2}$ $\quad PE = -G\frac{Mm}{r}$

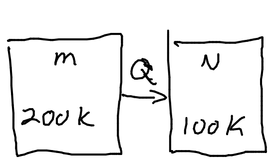
$\rightarrow PE = -rF$

$KE = \frac{1}{2}mv^2$ $\quad F = \frac{mv^2}{r}$

$KE = \frac{1}{2}rF$

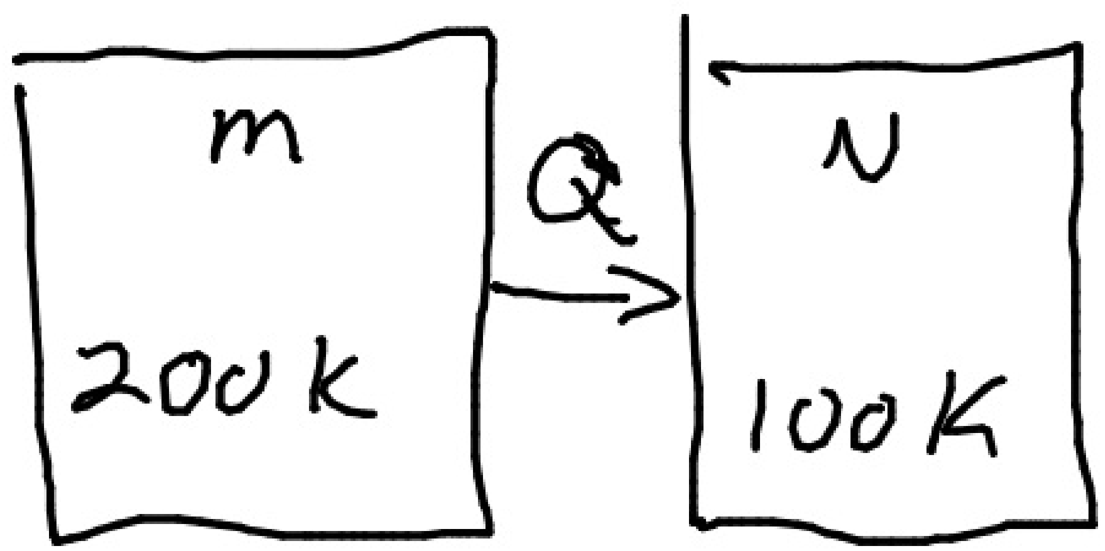
$U = KE + PE = -\frac{1}{2}rF = -KE$

Speed up planet (~ increase T)
KE rises, U falls.



Heat still flows from high T to low T

N gets warmer
m gets warmer too.



Heat still flows from high T to low T

N gets warmer
M gets warmer too.

If M gets warmer faster than N does, then they might never reach equilibrium

unless M system becomes Normal at high temperatures