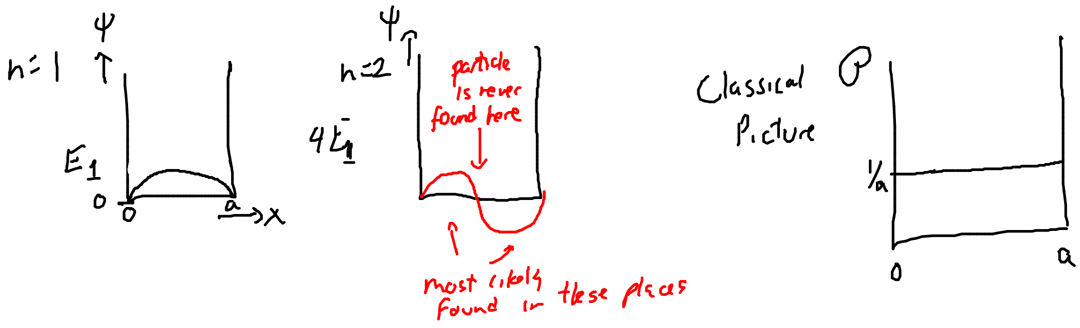
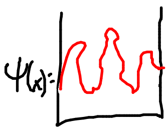
$$k = \frac{n\pi}{a} \qquad n = 1, 2, 3, \ldots$$

$$k = \frac{\sqrt{2mE}}{\hbar} \Rightarrow$$

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2ma^2} = n^2 E_1$$



Any other wavefunction $\psi(x)$ is perfectly valid description of particle state, but

e.g. 

$$\psi(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$$

energy eigenstates $|E_n\rangle$

We don't know what its energy is.

$$P(E_n) = |\langle E_n|\psi\rangle|^2 = \left|\langle E_n| \sum_{m=1}^{\infty} c_m |E_m\rangle\right|^2$$

$$= |c_n|^2$$

$$\psi(x) = \frac{1}{\sqrt{2}}\psi_0(x) + \frac{1}{\sqrt{2}}\psi_1(x)$$

$$P(E = E_1) = \frac{1}{2} \qquad P(E = 4E_1) = \frac{1}{2}$$

$$P(E = 9E_1) = 0.$$

in wavefunction notation $$P(E_n) = \left|\int \psi_n^*(x) \overbrace{\sum_{m=1}^{\infty} c_m \psi_m(x)}^{\psi(x)} dx\right|^2$$

$$\langle H\rangle = \langle\psi|H|\psi\rangle = \int \psi^*(x) H \psi(x)\, dx$$

$$\langle H\rangle = \sum_{m=1}^{\infty} |c_m|^2 E_m$$

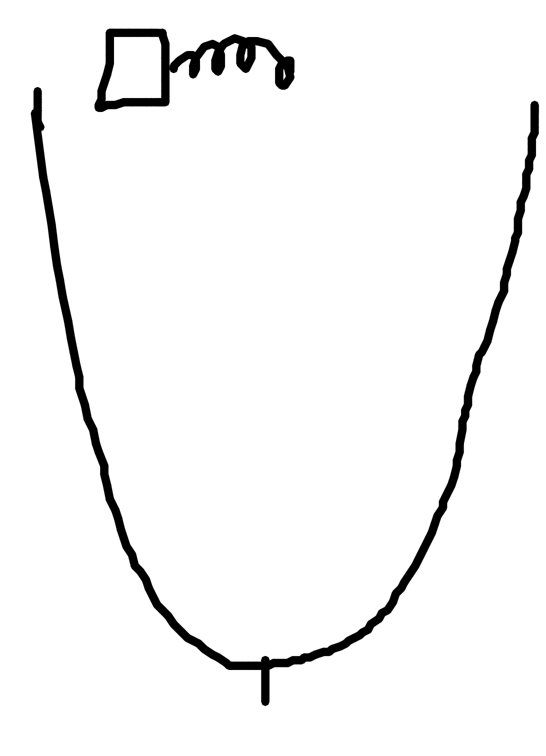
$$\Psi(x,0) = \sum_{m=1}^{\infty} c_m \psi_m(x)$$

$$\Psi(x,t) = \sum_{m=1}^{\infty} c_m e^{-iE_m t/\hbar} \psi_m(x)$$

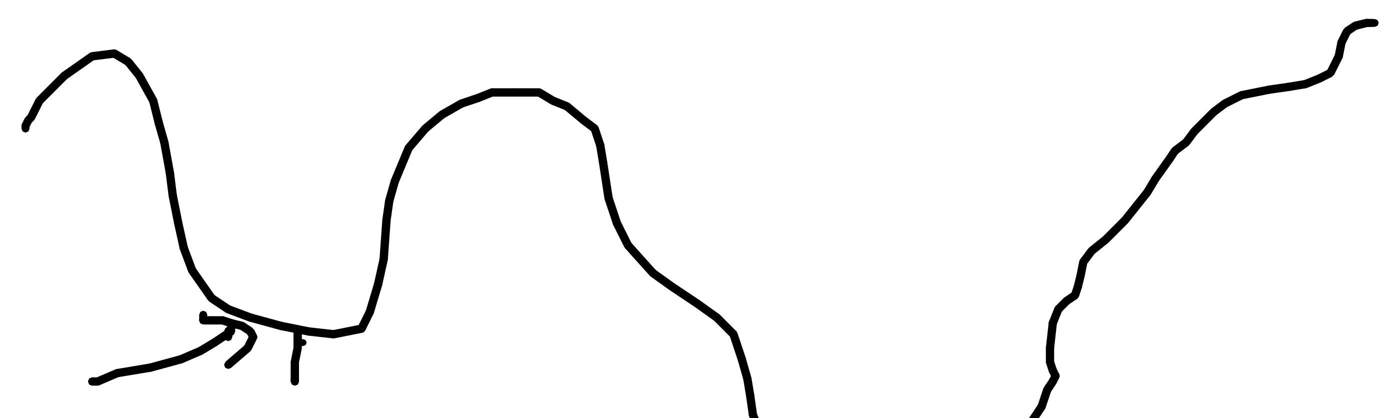
a general wavefunction changes with time but energy eigenstates don't.

Harmonic Oscillator

$$V(x) = \frac{1}{2} k x^2$$



This is a great approximation for local minimum of most potentials



$$V(x) = V(x_0) \leftarrow \text{baseline}$$
$$+ V'(x_0)(x-x_0) \leftarrow \text{zero}$$
minimum : $V' = 0$
$$+ \frac{1}{2} V''(x_0)(x-x_0)^2 \leftarrow$$
$$+ \ldots$$

if $x - x_0$ is small and $V''(x_0) \neq 0$, this is good approximation to $V(x)$