Name:_________________________

Physics 4310

Exam 1

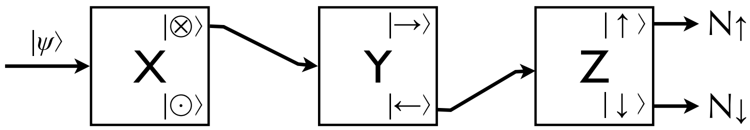
February 24, 2016

- Turn off your cell phone NOW, if you have one, and put it away. Avoid the appearance of impropriety.
- This test contains 18 questions and 67 points. The point value of each question may be found in a little box, like so: $\boxed{3}$.
- If any question seems ambiguous, ask me about it. Raise your hand (and maybe clear your throat if I'm not looking) and I will come to you; please remain seated.
- Partial credit is available *everywhere*; when in doubt, explain your reasoning. If you need more room to write, use the back of a sheet, but tell me that you are continuing on the back.
  Show your work.
- In the event that I have to make a correction or clarification to the exam, I will announce it and write it on the board; if I do so, you are responsible for taking these corrections into account.
- Look out for *emphasized* and **bolded** words; they are usually important.
- Please use the little blank (______) for your answers, where provided. If there is no blank, please $\boxed{\text{box}}$ or circle your final answer.
- When you're done, place the exam in the appropriate pile, and leave quietly; please do not stand outside the doors talking about the exam.

Good luck!

$\boxed{3}$ 1. ______ A large number of particles with arbitrary state $|\psi\rangle$ are run through three Stern-Gerlach apparatuses, as shown. $N_\uparrow$ particles leave the final S_z apparatus in the state $|\uparrow\rangle$ and $N_\downarrow$ particles leave in the state $|\downarrow\rangle$. Which of the following is true?

A) $N_\uparrow = 0$ **B)** $N_\uparrow \approx N_\downarrow$ **C)** $N_\downarrow = 0$



2. In the spin–1/2 system,

$\boxed{2}$ (a) ______ What is $\left|\langle\leftarrow | \rightarrow\rangle\right|^2$?

A) 0 **B)** $\frac{1}{2}$ **C)** $\frac{1}{\sqrt{2}}$ **D)** 1

$\boxed{2}$ (b) ______ What is $\left|\langle\downarrow | \leftarrow\rangle\right|^2$?

A) 0 **B)** $\frac{1}{2}$ **C)** $\frac{1}{\sqrt{2}}$ **D)** 1

$\boxed{3}$ 3. If $P_\rightarrow$ is the projection operator for $|\rightarrow\rangle$, what is $P_\rightarrow\Big(3|\rightarrow\rangle - 2|\leftarrow\rangle\Big)$?

$\boxed{3}$ 4. If $|\psi\rangle = |\uparrow\rangle - i|\downarrow\rangle$ and $|\phi\rangle = 2i|\uparrow\rangle + 3|\downarrow\rangle$, find $\langle\psi|\phi\rangle$.

5. Consider the vector $|\psi\rangle = 3i|\uparrow\rangle - |\downarrow\rangle$.

$\boxed{3}$ (a) Normalize $|\psi\rangle$.

$\boxed{2}$ (b) Write $|\psi\rangle$ as a matrix in the S_z basis (normalized or not, as you like).

$\boxed{2}$ (c) Write $\langle\psi|$ as a matrix in the S_z basis (normalized or not, as you like).

6. A system's Hamiltonian has three energy states: $E_1 = 1\,\mathrm{J}$, $E_2 = 2\,\mathrm{J}$, and $E_3 = 3\,\mathrm{J}$. The system is in the state $|\psi\rangle = \frac{1}{2}|E_1\rangle - |E_2\rangle$, and its energy is then measured.

$\boxed{3}$ (a) _______ What is the probability that the measurement will return the value $1\,\mathrm{J}$?

A) $\frac{1}{5} = 20\%$ **B)** $\frac{1}{4} = 25\%$ **C)** $\frac{1}{\sqrt{5}} = 45\%$ **D)** $\frac{1}{2} = 50\%$

$\boxed{3}$ (b) What is the probability that the measurement will return the value $3\,\mathrm{J}$?

$\boxed{3}$ 7. _______ $S_x|\uparrow\rangle$ is equal to

A) $\frac{\hbar}{2}|\uparrow\rangle$ **B)** $\frac{\hbar}{2}|\downarrow\rangle$ **C)** $\frac{\hbar}{2}|\odot\rangle$ **D)** $\frac{\hbar}{2}|\otimes\rangle$ **E)** $\frac{\hbar}{2}$ **F)** 0 **G)** $-\frac{\hbar}{2}$

$\boxed{3}$ 8. _______ Which of the following matrices could represent a measurement?

A) $\begin{pmatrix} i & 3 \\ -3 & -i \end{pmatrix}$ **B)** $\begin{pmatrix} 2 & -i+2 \\ i+2 & 2 \end{pmatrix}$ **C)** $\begin{pmatrix} i & 1-2i \\ 1-2i & -i \end{pmatrix}$ **D)** None of these (Explain)

9. Consider the operator $A \doteq \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$ in the S_z basis. Note that $A^2 \doteq \begin{pmatrix} 10 & 9 \\ 9 & 13 \end{pmatrix}$.

$\boxed{3}$ (a) Find the expectation value $\langle A \rangle$ for the state $|\psi\rangle = |\uparrow\rangle$.

$\boxed{3}$ (b) Find the standard deviation ΔA for the state $|\psi\rangle = |\uparrow\rangle$.

$\boxed{2}$ 10. ______ Suppose the spin of a particle is $|\nearrow\rangle$, halfway between $\uparrow$ and $\rightarrow$. What is the probability that $|\nearrow\rangle$, when measured by S_z, will give a value of $+\frac{\hbar}{2}$?
A) 0% **B)** 15% **C)** 50% **D)** 85% **E)** 100%

$\boxed{3}$ 11. ______ I measured a particular state $|\psi\rangle$ using Stern-Gerlach equipment, and I got the following results:

$$\Delta S_y = \frac{\hbar}{2} \qquad \Delta S_z = \frac{2\hbar}{5} \qquad \langle S_x \rangle = \frac{2\hbar}{5}$$

Does this satisfy the uncertainty principle? (It's possible my results are wrong!) Prove your result.

A) yes **B)** yes, barely (= instead of >) **C)** no

$\boxed{3}$ 12. ______ If the Hamiltonian has an eigenstate $|E_1\rangle = \frac{3}{5}|\uparrow\rangle - \frac{4}{5}i|\downarrow\rangle$ with corresponding eigenvalue E_1, what is the probability that $|\uparrow\rangle$ has energy E_1 (when its energy is measured)?

A) 36% **B)** 40% **C)** 60% **D)** 64% **E)** 80%

13. If $A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ represents a measurement in a spin-1 system:

$\boxed{3}$ (a) _______ If a particle in an arbitrary state $|\psi\rangle$ is measured by this operator, which of the following states could the particle be after it is measured by A? You can ignore normalization.

A) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ **B)** $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ **C)** $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\boxed{3}$ (b) _______ If the particle comes out in the state given by your answer to part (a), what value will A return?

A) 1 **B)** 4 **C)** 5 **D)** 17 **E)** $\hbar/2$

$\boxed{3}$ 14. A particle in the state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$. It is placed in a magnetic field so that its Hamiltonian has the eigenvectors

$$H|\uparrow\rangle = E_0|\uparrow\rangle \quad \text{and} \quad H|\downarrow\rangle = 4E_0|\downarrow\rangle$$

. Write $|\psi(t)\rangle$.

$\boxed{3}$ 15. Given the following:

$$
\begin{aligned}
|\psi\rangle &= |\uparrow\rangle - 2|\downarrow\rangle \\
|a_1\rangle &= \frac{\sqrt{3}}{2}|\uparrow\rangle + \frac{i}{2}|\downarrow\rangle \\
|a_2\rangle &= \frac{1}{2}|\uparrow\rangle - \frac{i\sqrt{3}}{2}|\downarrow\rangle
\end{aligned}
$$

Write $|\psi\rangle$ in the a_1, a_2 basis. (That is, in the form $|\psi\rangle = c_1|a_1\rangle + c_2|a_2\rangle$.)

$\boxed{3}$ 16. Evaluate and fully simplify the commutator $[x^2, \frac{d}{dx}]$.

$\boxed{3}$ 17. If

$$\psi(x) = \begin{cases} x^2 - 1, & -1 \leq x \leq 1 \\ 0, & \text{otherwise,} \end{cases}$$

find the expectation value of the momentum $\langle p \rangle$. ($\hat{p} = \frac{\hbar}{i}\frac{d}{dx}$)

$\boxed{3}$ 18. Explain why $\psi(x) = e^{-x}$ is not a valid wavefunction—*i.e.* why it can't describe the state of a system—over the range $-\infty < x < \infty$.