Probability

if there are Ω states of a system and every state is equally likely then probability of being in one state is $P = 1/\Omega$

"accessible states": states that system could be in

microstate: complete description of state of the system

e.g. 3 coins,

HHH   HTH   THH   TTH $\Omega = 8$ microstates

HHT   HTT   THT   TTT $P = 1/8$

in many cases in physics, the accessible microstates of a system are equally likely

Macrostate: a partial description

e.g. "exactly one head",

"first coin is tails"

multiplicity of a macrostate is # of microstates which satisfy the macrostate condition

eg. 3 coins

"exactly one head" $\Omega = 3$ HTT, THT, TTH

"first coin is tails" $\Omega = 4$ THH, THT, TTH, TTT

probability of a macrostate, if all microstates are equally likely,

$$P(\text{macro}) = \frac{\Omega(\text{macro})}{\Omega_{all}}$$

Ω_{all}: total # of accessible microstates

e.g. P of getting 2H after flipping 10 coins?

$$\Omega_{all} = 2^{10} = 1024 \qquad \Omega = \binom{10}{2} = \frac{10!}{8!\,2!} = \frac{10 \cdot 9}{2} = 45$$

$$P = \frac{45}{1024} = 4.4\%$$

Useful Models for Statistical Mechanics

- "toy" models
- easy mathematically
- demonstrate interesting physical properties

1) Paramagnet

- small magnetic dipoles $\vec{\mu}$, independent

classical: ↑ ↓ ↙ → ↖ ↑ ↙ ↘ ↙

in quantum mechanics: ↑↓↓↑↑↑↓↓↑↓

$\vec{\mu} \cdot \hat{z} = \mu_z = \pm \mu_0$ for spin-$\frac{1}{2}$ particle

if N dipoles, $\Omega_{all} = 2^N$

$N_\uparrow$ point up, $N_\downarrow$ point down

$$N = N_\uparrow + N_\downarrow$$

Now place dipoles in an external magnetic field $\vec{B} = -B\hat{z}$ ⇓

dipoles want to align with the field

$\vec{B}$ ⇓   ↑ $\vec{\mu}$ high PE   ↓ $\vec{\mu}$ low PE $U = -\vec{\mu} \cdot \vec{B}$

$U_\uparrow = +\mu_0 B$ $U_\downarrow = -\mu_0 B$

total PE

$$U = N_\uparrow(+\mu_0 B) + N_\downarrow(-\mu_0 B)$$
$$= \mu_0 B(N_\uparrow - N_\downarrow)$$
$$= \mu_0 B(2N_\uparrow - N)$$

$N_\downarrow = N - N_\uparrow$

$$U = \underbrace{2\mu_0 B}_{\substack{\text{choose units of}\\ \text{energy where}\\ 2\mu_0 B = 1}} N_\uparrow - \mu_0 B N$$

baseline, can ignore

$$\rightarrow \quad U = N_\uparrow$$

How many values can U take?

$U = 0, 1, \ldots N$ $N+1$ possibilities

$N+1$ energy macrostates

e.g. $U=2$, $N=4$ ↑↑↓↓   ↑↓↑↓ ← two microstates in the $U=2$ macrostate

multiplicity? $\quad \Omega(U) = \binom{N}{U}$

$$\Omega(N_\uparrow) = \frac{N!}{N_\uparrow!(N-N_\uparrow)!} = \frac{N!}{N_\uparrow! N_\downarrow!}$$

isolated paramagnet (no energy exchange with environment)

U will be constant (conservation of energy)

accessible microstates are $N_\uparrow = U$

Paramagnet jumps from one microstate to the next as energy sloshes around inside

at any moment   probability of any one accessible microstate

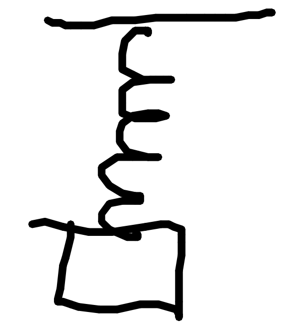
$$P = \frac{1}{\Omega(U)}$$

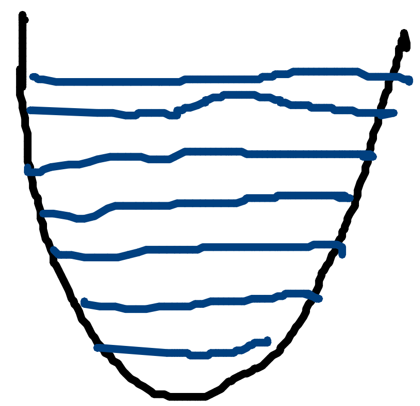
if $N = 10$, $U = 2$ $\Omega(U) = \binom{10}{2} = 45$

P of ↑↑↓↓↓↓↓↓↓↓ is $\frac{1}{45}$

Einstein Solid

quantum harmonic oscillator





$$E = \left(q + \frac{1}{2}\right) hf$$ ← frequency of oscillator (fixed)

$q: 0, 1, 2, 3, \ldots$

consider N harmonic oscillators

quanta of energy

$$U = q = q_1 + q_2 + q_3 + q_4 + q_5$$

2 1 3 0 1

$$U = \sum_{i=1}^{N} E_i$$

$$= \sum_{i=1}^{N} \left(q_i + \frac{1}{2}\right) hf$$

$$= \underbrace{hf}_{=\,1} \sum_{i=1}^{N} q_i + \underbrace{\frac{1}{2}\cancel{hf}N}_{\text{baseline}}$$

$hf = 1$ in same units