Physics 4310 Homework #3

3 problems

Due by Monday, February 8

Note: Please feel free to use software to calculate eigenvectors and eigenvalues etc.

▷ **1.**

In the ↑↓ spin–1/2 basis, consider the two operators

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$$

(a) Find the commutator $[A, B]$.
(b) What does the result to (a) say about the eigenvectors of A and B? Confirm this.
(c) Suppose we measure a number of particles in state $|\uparrow\rangle$, using A and B. Find the average values $\langle A\rangle$ and $\langle B\rangle$ from these measurements.
(d) Use the uncertainty principle to find the lower bound on $\Delta A\,\Delta B$, for the same set of particles in state $|\uparrow\rangle$.
(e) What is the lower bound on $\Delta A\,\Delta B$ if the particles' state is one of the eigenvectors of A? You can either do the calculation, or make a clever argument for your answer.

▷ **2.**

Consider a two-state quantum system with a Hamiltonian $H \doteq \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$.

Another physical observable A is described by the operator $A \doteq \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix}$ where a is real and positive. Let the initial state of the system be $|\psi(0)\rangle = |a_1\rangle$, the eigenstate of A corresponding to the larger of the two eigenvalues of A.
(a) Find $|\psi(t)\rangle$.
(b) What is the frequency of oscillation (i.e. the Bohr frequency) of $\langle A\rangle$?

▷ **3.**

A quantum mechanical system starts out in the state

$$|\psi(0)\rangle = C(3|a_1\rangle + 4|a_2\rangle)$$

where $|a_i\rangle$ are the normalized eigenstates of the operator A corresponding to the eigenvalues a_i. In this $|a_i\rangle$ basis, the Hamiltonian of this system is represented by the matrix

$$H \doteq E_0 \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

(a) If you measure the energy of this system, what values are possible, and what are the probabilities of measuring those values?
(b) Calculate the expectation value $\langle A\rangle$ as a function of time.