Physics 4310 Homework #5

5 problems

Due by Monday, February 29

▷ **1.**

Evaluate $a_+\psi_1$ to find the $n = 2$ eigenstate of the harmonic oscillator. Give the correct normalization (which you can find in the text).

▷ **2.**

Considering the harmonic oscillator again.

(a) Show that the average position $\langle x \rangle$ and momentum $\langle p \rangle$ of every energy eigenstate ψ_n is zero. (Hint: Use Eq. 2.69, where the x and p operators are written in terms of the raising and lowering operators.)

(b) Find the average kinetic energy $\langle T \rangle$ of the nth eigenstate ψ_n.

▷ **3.**

A particle is in the ground state of the harmonic oscillator with frequency ω, when suddenly the spring constant quadruples, so that $\omega' = 2\omega$, without initially changing the wave function.

(a) What is the probability that a measurement of the energy would still return the value $\hbar\omega/2$?

(b) Show that the probability that it returns a value of $\hbar\omega$ is 0.943.

▷ **4.**

In the analytic derivation of the harmonic oscillator, use the recursion relation

$$a_{j+2} = \frac{-2(n-j)}{(j+1)(j+2)} a_j$$

to work out $H_5(\xi)$ and $H_6(\xi)$. To fix the overall constant, invoke the convention that the coefficient of the highest power of ξ is 2^n.

▷ **5.**

The Gaussian wave packet. A free particle has the initial wave function

$$\Psi(x, 0) = Ae^{-ax^2}$$

where A and a are constants (and a is real and positive).

(a) Find A to normalize $\Psi(x, 0)$

(b) Write an expression for $\Psi(x, t)$.

(c) Find a simple expression for $|\Psi(x, t)|^2$ (that is, do out all the integrals). You can use software to do out the integral, or see Griffiths problem 2.22 for a hint of how to do the integral by hand. You may find it helpful to use the change of variables $\xi = \sqrt{a}x$ and $\tau = \frac{2\hbar a}{m}t$, or something similar, to get all the constants out of the way as you do the math.

(d) Describe what $|\Psi(x, t)|^2$ does over time, qualitatively. You can figure this out by graphing the function at a few different times. You needn't include the graphs with your homework, though.