Coefficient of Volume Expansion

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P,N} \qquad \% \text{ increase} / C^\circ$$

Maxwell relation $\frac{\partial^2 G}{\partial P \partial T}$

$$\text{or} \quad \beta V = \underbrace{\left(\frac{\partial V}{\partial T}\right)_{P,N}}_{\text{natural variables, matches } G} = -\left(\frac{\partial S}{\partial P}\right)_{T,N}$$

$dU = TdS - PdV + \mu dN$
$dH = TdS + VdP + \mu dN$
$dF = -SdT - PdV + \mu dN$
$dG = -SdT + VdP + \mu dN$

$$\beta = -\frac{1}{V}\left(\frac{\partial S}{\partial P}\right)_{T,N}$$

isothermal change in entropy with pressure

T, N constant

$$dS = -\beta V dP$$

$Q = TdS$

as P increases at constant T, heat flows out $(dS < 0)$

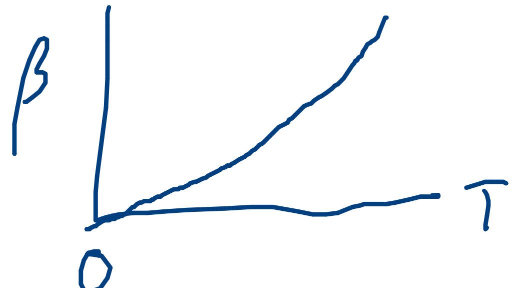
$$Q = -\beta V T dP$$

At $T = 0$, $S = 0$ and doesn't change

$$dS = 0 = -\beta V dP \quad \text{at} \quad T = 0$$

either $\boxed{\beta = 0 \quad \text{solid}}$ or $V = 0$ at $T = 0$

ideal gas
$PV = NkT$
not realistic
(singularity)



coefficient of volume expansion goes to zero as $T \to 0$.

$$\left(\frac{\partial T}{\partial P}\right)_{S,N} = +\left(\frac{\partial V}{\partial S}\right)_{P,N} \quad (H)$$

$dU = TdS - PdV + \mu dN$
$dH = TdS + VdP + \mu dN$
$dF = -SdT - PdV + \mu dN$
$dG = -SdT + VdP + \mu dN$

$$\left(\frac{\partial T}{\partial V}\right)_{P,N} = -\left(\frac{\partial P}{\partial S}\right)_{T,N}$$

$$\underbrace{\left(\frac{\partial V}{\partial T}\right)_{P,N}}_{dG} = -\left(\frac{\partial S}{\partial P}\right)_{T,N}$$

Another neat trick

$$\left(\frac{\partial a}{\partial b}\right)_c \left(\frac{\partial b}{\partial c}\right)_a \left(\frac{\partial c}{\partial a}\right)_b = -1$$

Const N

$$\left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = -1$$

$$\left(\frac{\partial V}{\partial P}\right)_T = -\kappa V \qquad \left(\frac{\partial V}{\partial T}\right)_P = \beta V$$

isothermal compressibility

coefficient of volume expansion

$$\left(\frac{\partial T}{\partial P}\right)_V \left(-\frac{1}{\kappa V}\right)(\beta V) = -1$$

$$\left(\frac{\partial T}{\partial P}\right)_V = \frac{\kappa}{\beta}$$