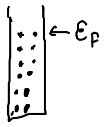
Fermi gas at $T=0$.

In 1D



in 3D,

$$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

Total energy,

2 spins per micro state

energy of microstate $\vec{n}$

$\vec{n} = n_x\hat{x} + n_y\hat{y} + n_z\hat{z}$

$$U = 2\iiint \epsilon(\vec{n})\, dn_x\, dn_y\, dn_z \qquad \epsilon(\vec{n}) = \frac{h^2}{8mL^2}|\vec{n}|^2$$

use spherical coordinates (n,θ,ϕ)

$$= 2\int_0^{\pi/2}\int_0^{\pi/2}\int \frac{h^2}{8mL^2}\, n^2 \quad n^2 \sin\theta\, dn\, d\theta\, d\phi$$

$$= \left(\int_0^{\pi/2}\int_0^{\pi/2} \sin\theta\, d\theta\, d\phi\right)\frac{h^2}{8mL^2}2\int_0^R n^4\, dn$$

$\frac{4\pi}{8}$

$$\frac{\pi}{2}\, 2\, \frac{h^2}{8mL^2}\left[\frac{1}{5}n^5\right]_0^R$$

$$U = \frac{h^2\pi}{40mL^2}R^5 \qquad R^3 = \frac{3}{\pi}N$$

$$= \left(\frac{h^2}{8mL^2}R^2\right)\frac{\pi}{5}R^3$$

$$= \epsilon_F \quad \frac{\pi}{5}\frac{3}{\pi}N$$

$$U = \frac{3}{5}N\epsilon_F$$

(if all electrons were at surface then U would be $N\epsilon_F$).

For conduction electrons in a metal, $\epsilon_F \approx 1\text{–}2\ \text{eV}$

$kT = \frac{1}{40}\ \text{eV}$ at room temperature

$\rightarrow kT \ll \epsilon_F \rightarrow$ "low T"

$\epsilon_F \propto \left(\frac{1}{V}\right)^{2/3}$ so compressing a metal (Fermi gas) will increase ϵ_F & U.

$\rightarrow$ positive work is required to compress

$\rightarrow$ Fermi gas is fighting back $\rightarrow$ pressure

"degeneracy pressure" due to Pauli exclusion principle

$$P = -\left(\frac{\partial U}{\partial V}\right)_{S,N} = -\frac{\partial}{\partial V}\left(\frac{3}{5}N\epsilon_F\right) = -\frac{3}{5}N\frac{\partial}{\partial V}\left(\frac{h^2}{8m}\left(\frac{3N}{\pi}\right)^{2/3}V^{-2/3}\right)$$

3rd Law: $S=0$ @ $T=0$

$$= -\frac{3N}{5}\frac{h^2}{8m}\left(\frac{3N}{\pi}\right)^{2/3}\left(-\frac{2}{3}V^{-5/3}\right)$$

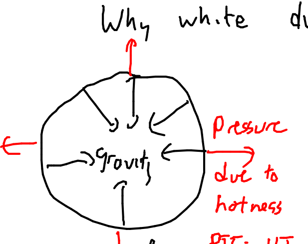
$$= +\frac{2}{3}\left(\frac{3N}{5}\frac{h^2}{8m}\left(\frac{3N}{\pi}\right)^{2/3}\right)\frac{V^{-2/3}}{V} \qquad = U$$

$$P = \frac{2}{3}\frac{U}{V}$$

Why atoms don't collapse

Why solids are solid

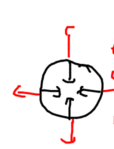
Why white dwarf stars aren't black holes



Pressure due to hotness

$PV = NkT$

normal star



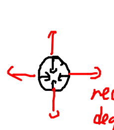
electron degeneracy pressure

white dwarf

If gravity too strong

$p^+ + e^- \rightarrow n^0$

neutrons



neutron degeneracy pressure

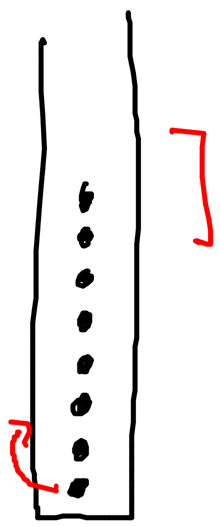
neutron star

if gravity is too strong for that

black hole

If $T > 0$ but small

- in normal gas, every particle would gain energy kT
- in Fermi gas, that won't work



only e^- at surface can gain energy

$\hookrightarrow$ # of e^- that can gain energy is prop. to N & kT (higher T → bigger jumps → more particles can reach empty slot)

wants to gain kT of energy, it can't — it's blocked

$$\Delta U \propto \underset{\substack{\text{\# of } e^- \\ \text{that jump}}}{(NkT)} \underset{\substack{\text{energy} \\ \text{per jump}}}{(kT)}$$

$$U = \frac{3}{5} N \varepsilon_F + A N (kT)^2$$

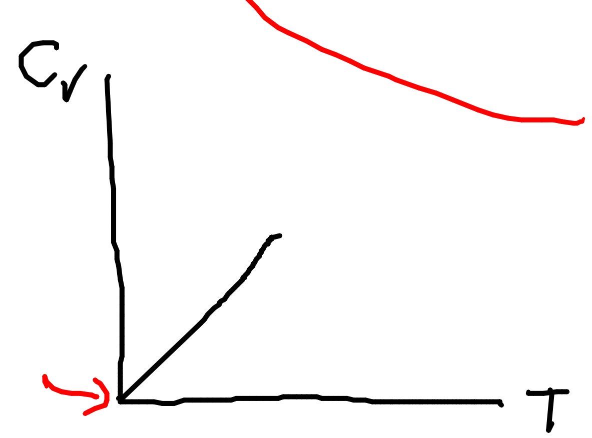
$A \propto \frac{1}{\varepsilon_F}$ to get dimensions right

$$= \frac{3}{5} N \varepsilon_F + \frac{\pi^2}{4} \frac{1}{\varepsilon_F} N (kT)^2$$

$$C_V = \frac{\partial U}{\partial T} = \frac{\pi^2}{2} \frac{N}{\varepsilon_F} k^2 T$$

→ heat capacity is linear with temperature

actually seen in metals at low T (2-4K)



goes to 0 at $T=0$ (3rd law)

at room T, $C_V \propto T^3$ (phonons)