

Exam 2 Outline

Spring 2016

Chapter 2

One-Dimensional Schrodinger Equation

- What is the standard Hamiltonian (kinetic energy plus potential energy)?
- What is a “stationary state”?
- The energy of a stationary state has to be larger than the potential *somewhere*.
- The wavefunction is always continuous.
- The wavefunction’s derivative is continuous except if the potential is infinite.

Infinite Square Well

- What are the stationary states in the infinite square well? The corresponding energy eigenvalues E_n ?
- The stationary states are orthogonal and complete.
- Given a general wavefunction ψ , how can you write it in terms of the stationary states?

Harmonic Oscillator

- What is the potential $V(x)$ of a harmonic oscillator? Why is it so important an example?
- What are the raising and lowering operators a_{\pm} , and what do they do to a stationary state?
- How do we find the wavefunction of the ground state?
- How can you find the harmonic oscillator wavefunction using a power-law expansion?
- Why must the recursion relation terminate? What consequence does this have?

Free Particle

- What is the stationary state when the potential is $V(x) = 0$?
- Why can’t a free particle exist in a pure stationary state?
- What is the difference between e^{+ikx} and e^{-ikx} ?
- How can you write a wavepacket of a free particle? How can you find the coefficients $\phi(k)$?
- What is the difference between the phase velocity and group velocity of a wavepacket?

Delta Potential

- What is the difference between a bound state and a scattering state? When do each occur?
- What is “tunneling”?
- What is the Dirac delta function $\delta(x)$ and how does it work?
- What is the boundary condition of a wavefunction’s derivative when the potential is infinite, such as at the delta function?
- What are the bound states of the delta well?
- Understand how we deal with the scattering states: the incident, reflected, and transmitted waves.
- How do we find the reflection and transmission coefficients?
- How do the solutions change when we use a delta *barrier* instead?

Finite Square Well

- Understand the basic solution method for the finite square well.

Chapter 3

- What are the eigenfunctions of the operators x and p ?
- Understand how to use the equation

$$\frac{d\langle Q \rangle}{dt} = \frac{i}{\hbar} \langle [H, Q] \rangle + \left\langle \frac{dQ}{dt} \right\rangle$$

- In the expression $\Delta E \Delta t \geq \frac{\hbar}{2}$, what does Δt mean?

Chapter 4

- What is the standard Hamiltonian in three dimensions?
- Legendre polynomials and associated Legendre functions
- For a radial potential, the wavefunction’s angular components are always the spherical harmonics $Y_{lm}(\theta, \phi)$. Know how they work.
- What are the restrictions on l and m ?
- What is the equation for the radial portion $R(r)$ of the wavefunction?
- What is the centrifugal term and what effect does it have?
- What are the radial energy eigenstates and eigenvalues for the infinite square well?

Hydrogen Atom

- What is the potential for an electron of a hydrogen atom?
- Understand how we built up the solution.
- Be able to find the energy eigenstates for a given set of values n , l , and m . What does the energy depend on?
- What are the restrictions on n , l , and m ?

Angular Momentum

- What are the angular momentum operators L_x , L_y , and L_z ?
- How do they commute?
- What are the angular momentum eigenstates?
- What are L_{\pm} and what do they do to an angular momentum eigenstate?

Spin

- How are the spin numbers s different from the angular momentum numbers l ? (There are two differences.)
- How do the spin raising and lowering operators act on up and down spins of a spin-1/2 operator.

Addition of Angular Momenta

- For two spin-1/2 particles, what are the three triplet states and the singlet state?
- Given s_1 and s_2 , what are the possible values of s ?
- Given m , s_1 , and s_2 , what are the possible values of m_1 and m_2 .
- How do Clebsch-Gordon coefficients work (to calculate probabilities, like in the homework).
- Be comfortable with the difference between one-particle and two-particle operators (e.g. S_{1z} and S_{2z} vs. $S_z = S_{1z} + S_{2z}$ and how it works: What is $S_{1z}|\uparrow\downarrow\rangle$?)

Chapter 5 (partial)

Two-Particle Systems

- What is the Hamiltonian for two particles?
- What property must the wavefunction $\psi(\vec{r}_1, \vec{r}_2)$ have for bosons? fermions?
- What is the exchange “force”? What effect does this have on the hydrogen molecule?
- How can two electrons have a symmetric spatial wavefunction $\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1)$, if they’re fermions?
- To a first approximation, each helium electron has the same energy eigenstates as a hydrogen electron, except for what changes? Why is this only an approximation—what’s missing?
- What’s the difference between parahelium and orthohelium?

Atoms with $Z > 2$

- What do *orbitals* correspond to?
- Why does the energy of an orbital depend on l , even though it only depends on n for hydrogen?