# Business

$$(\Delta A)^2 (\Delta B)^2 \geq \left[\frac{1}{2i}\langle [A,B] \rangle\right]^2 \quad \text{true}$$

$$\Delta A \, \Delta B \geq \left| \frac{1}{2i}\langle [A,B] \rangle \right|$$

For Wednesday, read § 2.1 & 2.2.
prepare derivation of eigenstates
of infinite square well.

Last time $\qquad \vec{B} = B_0 \hat{z} \qquad H = \omega_0 S_z$

$\cancel{|\uparrow\rangle \;\&\; |\downarrow\rangle}$

If I measure a general state $|\psi(t)\rangle$

with $S_z$, $\quad \langle S_z \rangle$ is constant with time.

but, with $S_x$, $\quad \langle S_x \rangle \propto \cos(\omega_0 t)$

$$\omega_0 = \frac{eB}{m_e}$$ Larmor frequency

$\vec{B} = B_0 \hat{z} + B_1 \hat{x}$

$B_0$: set up energy levels — +, — -
$B_1$: precession at an angle which might allow system to flip from one energy level to the other

$\omega_0 = \frac{eB_0}{m}$ aka $\omega_z$ $\quad \omega_1 = \frac{eB_1}{m}$ aka $\omega_x$

$$H = -\vec{\mu}\cdot\vec{B} = \omega_0 S_z + \omega_1 S_x$$

↑↓ basis $\quad H \doteq \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_1 \\ \omega_1 & -\omega_0 \end{pmatrix}$

Eigen values

$$\left(\frac{\hbar}{2}\omega_0 - \lambda\right)\left(-\frac{\hbar}{2}\omega_0 - \lambda\right) - \left(\frac{\hbar}{2}\right)^2 \omega_1^2 = 0.$$

$$\lambda^2 - \frac{\hbar^2}{4}\omega_0^2 - \frac{\hbar^2}{4}\omega_1^2 = 0$$

$$E_\pm = \lambda = \pm\frac{\hbar}{2}\sqrt{\omega_0^2 + \omega_1^2}$$

$$|+\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}|\downarrow\rangle \qquad \tan\theta = \frac{\omega_x}{\omega_z} = \frac{B_x}{B_z}$$

$$|-\rangle = \sin\frac{\theta}{2}|\uparrow\rangle - \cos\frac{\theta}{2}|\downarrow\rangle$$

$B_x$, $\vec{B}$, $B_z$, $\theta$, $B_x$

Suppose $|\psi(0)\rangle = |\uparrow\rangle$. What is $|\psi(t)\rangle$?

$$|\psi(0)\rangle = \langle +|\uparrow\rangle |+\rangle + \langle -|\uparrow\rangle |-\rangle \qquad |+\rangle\langle +|\uparrow\rangle + |-\rangle\langle -|\uparrow\rangle$$

$\langle\uparrow|+\rangle = \cos\frac{\theta}{2}$
$\langle +|\uparrow\rangle = (\cos\frac{\theta}{2})^*$

$$= \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle$$

$$|\psi(t)\rangle = \cos\frac{\theta}{2} e^{-iE_+t/\hbar}|+\rangle + \sin\frac{\theta}{2} e^{-iE_-t/\hbar}|-\rangle$$

What is probability that $|\psi(t)\rangle$ points ↓?

spin flip $\quad \mathcal{P}_{\uparrow\to\downarrow} = |\langle\downarrow|\psi(t)\rangle|^2 \qquad$ $\psi(t)$ is normalized - Good!

$$= \left|\langle\downarrow|\cos\frac{\theta}{2}e^{-iE_+t/\hbar}|+\rangle + \langle\downarrow|\sin\frac{\theta}{2}e^{-iE_-t/\hbar}|-\rangle\right|^2$$

$$= \left|\cos\frac{\theta}{2}e^{-iE_+t/\hbar}\langle\downarrow|+\rangle + \sin\frac{\theta}{2}e^{-iE_-t/\hbar}\langle\downarrow|-\rangle\right|^2$$

$$= \left|\cos\frac{\theta}{2}e^{-iE_+t/\hbar}\sin\frac{\theta}{2} + \sin\frac{\theta}{2}e^{-iE_-t/\hbar}\left(-\cos\frac{\theta}{2}\right)\right|^2$$

$$= \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\left|e^{-iE_+t/\hbar} - e^{-iE_-t/\hbar}\right|^2 \qquad E_\pm = \pm\underbrace{\frac{\hbar}{2}\sqrt{\omega_0^2+\omega_1^2}}_{g}$$

$$= \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\left|e^{-igt/\hbar} - e^{+igt/\hbar}\right|^2$$

$$= \sin^2\theta\,\sin^2\left(\frac{gt}{\hbar}\right) = \sin^2\theta\,\sin^2\left(\frac{\sqrt{\omega_0^2+\omega_1^2}}{2}t\right)$$

use definition of $\theta$ from before

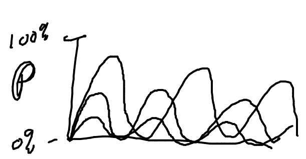
$$\mathcal{P}_{\uparrow\to\downarrow} = \frac{\omega_1^2}{\omega_0^2+\omega_1^2}\sin^2\left(\frac{\sqrt{\omega_0^2+\omega_1^2}}{2}t\right) \qquad \text{Rabi's formula}$$

If $\omega_1 = 0$, $\mathcal{P} = 0$ because $B_x = 0$, $\vec{B} = B\hat{z}$

If $\omega_0 = 0$,

$\mathcal{P}$ oscillates between 0% & 100% w/ frequency $\frac{\sqrt{\omega_0^2+\omega_1^2}}{2}$

bigger $\omega_1$ is compared to $\omega_0$, larger $\mathcal{P}$ is at at least part of the cycle




↑ As $\omega_1$ increases

· Suppose you start with $B_0$ $\quad \vec{B} = B_0\hat{z}$
$|\uparrow\rangle$ is the "excited" state

· Apply $B_1$ for a certain amount of time to flip spin to $|\downarrow\rangle$

- Suppose you start with $B_0$ $\vec{B} = B_0 \hat{z}$

  $|\uparrow\rangle$ is the "excited" state

- Apply $B_1$ for a certain amount of time

  to flip spin to $|\downarrow\rangle$

- Turn $B_1$ off, & $|\downarrow\rangle$ is the ground state.

# Time-Dependent Hamiltonian

$$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle$$

Schrodinger factors don't work
solve full Schrodinger equation
(or find a trick)

e.g. Magnetic Resonance

$$\vec{B} = B_0 \hat{z} + B_1 \left[\cos\omega t\, \hat{x} + \sin\omega t\, \hat{y}\right]$$

$\omega$: frequency of "flipping field"

$\omega_0 = \frac{eB_0}{m}$ : sets up energy levels

$\omega_1 = \frac{eB_1}{m}$ : flipping field strength

$$H = -\vec{\mu}\cdot\vec{B} = \omega_0 S_z + \omega_1\left[\cos\omega t\, S_x + \sin\omega t\, S_y\right]$$

$$H \doteq \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_1 e^{-i\omega t} \\ \omega_1 e^{i\omega t} & -\omega_0 \end{pmatrix}$$