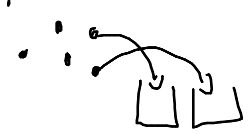
Choose c items from a collection of N items

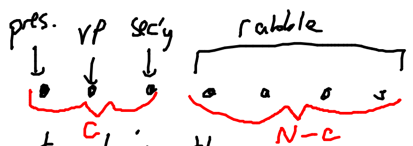
If order matters, "permutation"

(e.g. president, vice-president, secretary)

(e.g.



- put all items in a line, & take first c of them



$N!$ ways to line things up

can rearrange the rabble any way without changing result, so $N!$ overcounts by a factor of $(N-c)!$

$$\Omega = \frac{N!}{(N-c)!} \quad \text{Permutation}$$

If order doesn't matter, you can rearrange the rabble and the chosen ones, so $N!$ overcounts by $(N-c)!$ and $c!$

$$\Omega = \frac{N!}{(N-c)!\,c!} \quad \text{Combination}$$

Which?	P	C
Order?	Y	N
# in denom?	1	2

"N choose c": $\binom{N}{c} \equiv \frac{N!}{c!(N-c)!}$ binomial coefficients

$$(x+y)^3 = (x+y)(x+y)(x+y)$$
$$= \binom{3}{3}x^3 + \binom{3}{2}x^2y + \binom{3}{1}xy^2 + \binom{3}{0}y^3$$
$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

choose 2 objects out of 5

ABCDE

$$\binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5\cdot4\cdot\cancel{3\cdot2\cdot1}}{(2\cdot1)(\cancel{3\cdot2\cdot1})} = \frac{20}{2} = \boxed{10}$$

AB CD
AC CE
AD DE
AE
BC
BD
BE

$0! = 1$

$$\binom{N}{0} = \frac{N!}{N!\,0!} = \frac{N!}{N!} = 1$$

$$\binom{N}{N} = 1$$

$$\binom{N}{1} = \frac{N!}{1!(N-1)!} = N$$

$$\binom{N}{c} = \binom{N}{N-c}$$

Very Large Numbers

if $N >> 1$, N is "large"

$N+1 \approx N$ $\qquad 10^{23}+5 \not\approx 10^{23}$

$2N \not\approx N$

if $N >> 1$, then $N!$, N^N, 2^N, etc

are very large numbers

VLN $\times$ LN

e.g. $10^{10^{23}} \times 10^{23} = 10^{10^{23}+23} \approx 10^{10^{23}}$

e.g. $N 2^N \approx 2^N$ $\qquad N N! \not\approx N!$

$\ln(\text{VLN}) = \text{LN}$ so logarithms are handy when working with VLNs.

$$\text{VLN} \rightarrow \ln N^N = N \ln N \leftarrow \text{LN}$$

$$\ln 2^N = N \ln 2 \leftarrow \text{LN}$$

$$\boxed{\ln N! = N \ln N - N}$$ Stirling's approximation

if $N >> 1$

e.g. $N = 10$ $\qquad \ln(10!) = \ln(3628800) = 15.10$

$10 \ln 10 - 10 = 13.0$ $\quad$ 14% error.

for $N=100$, Stirling's approximation has a 0.8% error

e.g. how many ways can I rearrange 50 red balls & 50 blue balls?

$$\Omega = \frac{100!}{50!\,50!} =$$

$$\ln \Omega = \ln 100! - \ln 50! - \ln 50!$$

$$\approx (100 \ln 100 - 100) - (50 \ln 50 - 50) - (50 \ln 50 - 50)$$

$$= (100 \ln 100 - 50 \ln 50 - 50 \ln 50) + \cancel{(-100 + 50 + 50)} \rightarrow 0$$

$$= 100 \ln 100 - 100 \ln 50$$

$$\ln \Omega = 100 \ln \frac{100}{50} = 100 \ln 2 = \ln 2^{100}$$ $\qquad a \ln b = \ln b^a$

$$\Omega \approx 2^{100}$$

$N >> 1$ $\quad N! \approx \underbrace{N^N e^{-N}}_{\text{VLNs}} \underbrace{\sqrt{2\pi N}}_{\text{LN}} \not\approx \left(\frac{N}{e}\right)^N.$

2nd order approximations