Physics 4310 Homework #8
3 problems
Due by March 28

▷ **1.**

Using the recursion relation for c_j, work out R_{30} and R_{31} for the hydrogen atom.

▷ **2.**

For the following problems, feel free to use the tables in Griffiths; you don't need to work out ψ_{nlm}. If you want to use Eq. 4.89 and are using Mathematica, note that Mathematica uses a different normalization convention, so

$$L_a^b(x) = (a+b)!\, \texttt{LaGuerreL(a, b, x)}$$

(a) Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of hydrogen. Express your answers in terms of the Bohr radius.

(b) Find $\langle z \rangle$ and $\langle z^2 \rangle$ for an electron in the ground state of hydrogen. Hint: This requires no new integration— note that $r^2 = x^2 + y^2 + z^2$, and exploit the fact that the ground state is spherically symmetric.

(c) Find $\langle z^2 \rangle$ for an electron in the state $n = 2$, $l = 1$, and $m = 0$. Note that this state is *not* spherically symmetric, so we do need to integrate. Use $z = r \cos\theta$.

▷ **3.**

Consider the equation

$$x^2 v'''(x) + x^2 v''(x) + v'(x) + \lambda v(x) = 0$$

(a) Using the power-law method we used for the hydrogen atom, write $v(x) = \sum_{j=0}^{\infty} c_j x^j$, and find a recursion relation for the coefficients c_j.

(b) What is the recursion relation when $j \gg 1$? Prove that $v(x) \approx e^x$ unless the series terminates.

(c) What has to be true about λ if the series terminates?