Physics 4310 Homework #6

4 problems

Due by March 14 (after break)

▷ **1.**

Evaluate the following integrals. (These are easy, but you need to be a *little* careful.)

(a) $\int_{-3}^{+1} (x^3 - 3x^2 + 2x - 1)\delta(x+2)\, dx$

(b) $\int_{0}^{\infty} [\cos(3x) + 2]\, \delta(x - \pi)\, dx$

(c) $\int_{-1}^{+1} \exp(|x| + 3)\delta(x - 2)\, dx$

▷ **2.**

Delta functions are actually "distributions" because they live under integral signs. Two distributions ($D_1(x)$ and $D_2(x)$) are said to be equal if

$$\int_{-\infty}^{\infty} D_1(x) f(x)\, dx = \int_{-\infty}^{\infty} D_2(x) f(x)\, dx$$

for every (ordinary) function $f(x)$.

(a) Show that

$$\delta(cx) = \frac{1}{|c|}\delta(x)$$

where c is a real constant. (Be sure to check the case where c is negative.)

(b) Let $\theta(x)$ be the *step function*:

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

Show that $\frac{d\theta}{dx} = \delta(x)$. Hint: Use integration by parts, along with the definition of "equal distributions" above.

▷ **3.**

In class we discussed the *even* (symmetric) bound state wave functions for the finite square well. I want you to analyze the *odd* (i.e. antisymmetric) bound state wave functions now. Derive the transcendental equation for the allowed energies, and solve it graphically. Is there always at least one even bound antisymmetric state?

▷ **4.**

Consider the "step" potential:

$$V(x) = \begin{cases} 0, & x \le 0 \\ V_0, & x > 0 \end{cases}$$

(a) Calculate the reflection coefficient for the case $E < V_0$, assuming the incident wave comes in from the left (as in the finite square well).

(b) Calculate the reflection coefficient for the case $E > V_0$.