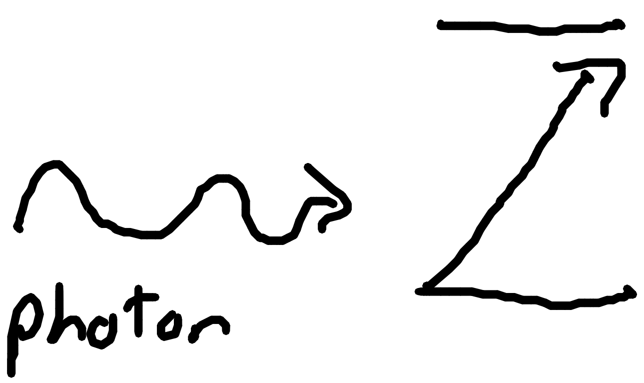
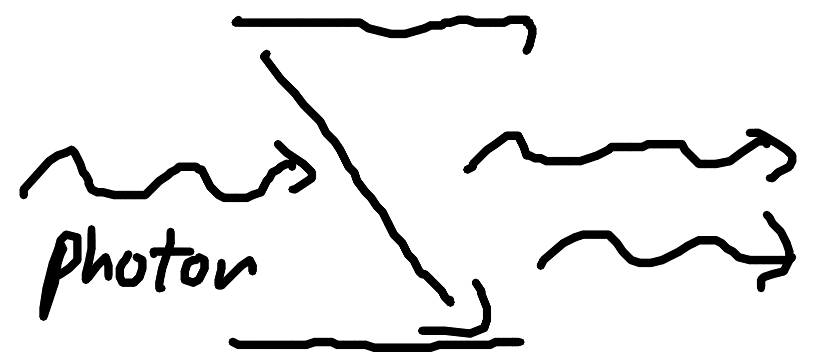
Only way to kick a particle out of an energy eigenstate is with a time-dependent Hamiltonian

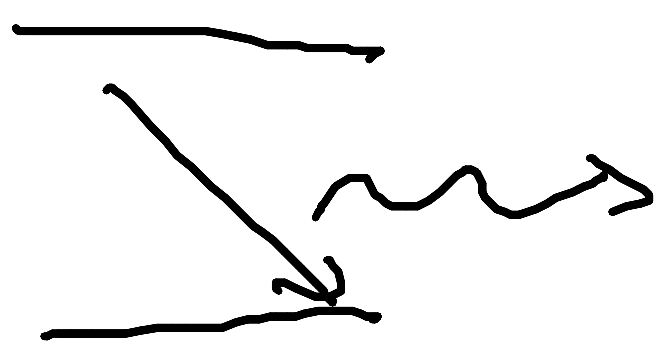


absorption



stimulated emission

Spontaneous emission is weird- how ~~did~~ it was it perturbed?



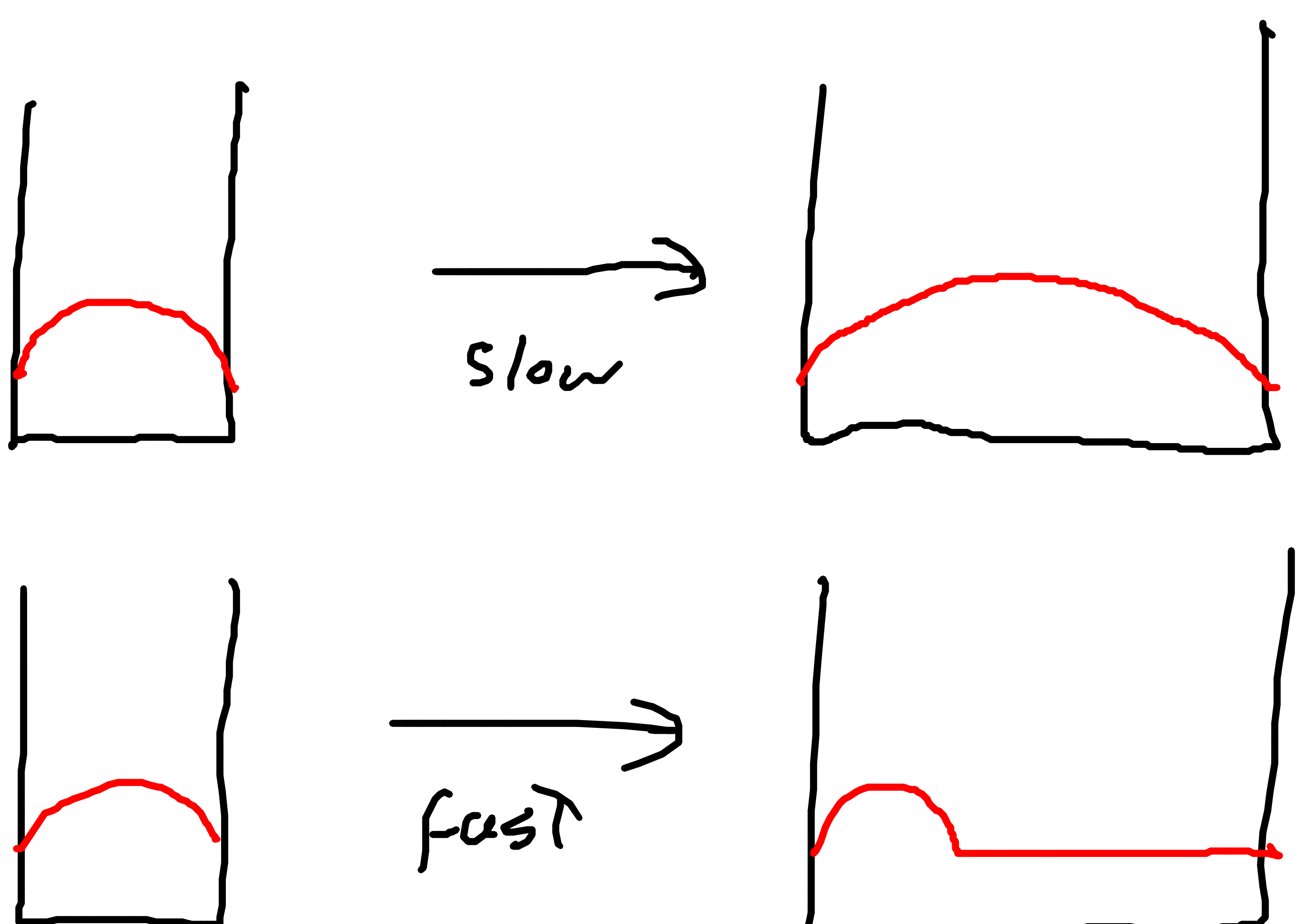
zero-point energy

Chapter 10:

Adiabatic Approximation

"adiabatic" : isentropic, reversible

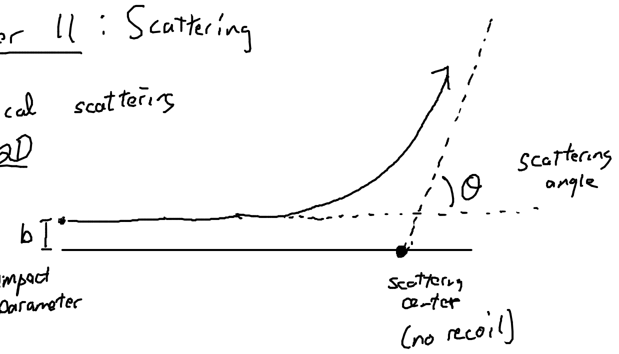
If a Hamiltonian changes slowly compared to motion of the object, the object will remain in the corresponding eigenstate in the final Hamiltonian



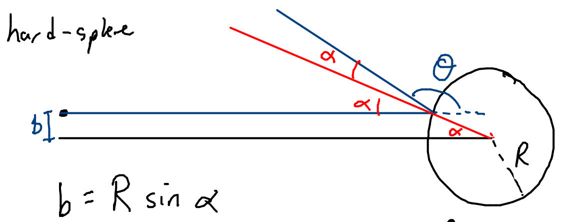
Chapter 11 : Scattering

Classical scattering

2D



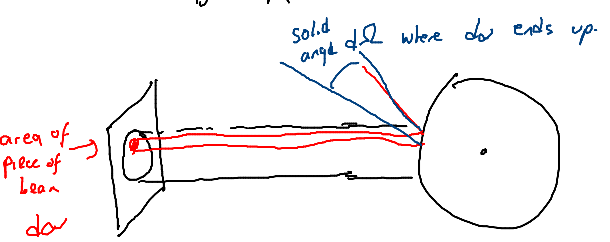
e.g. hard-sphere



$b = R \sin\alpha$

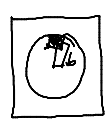
$\theta = \pi - 2\alpha \rightarrow \alpha = \frac{\pi}{2} - \frac{\theta}{2}$

$b = R \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = R\cos\frac{\theta}{2}$



$\frac{d\sigma}{d\Omega}(\theta, \phi)$ differential/scattering cross-section

azimuthal symmetry, $d\sigma = b\, db\, d\phi$



$d\Omega = \sin\theta \; d\theta \; d\phi$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Hard sphere

$\frac{db}{d\theta} = \frac{d}{d\theta}\left(R\cos\frac{\theta}{2}\right)$

$= -\frac{R}{2}\sin\frac{\theta}{2}$

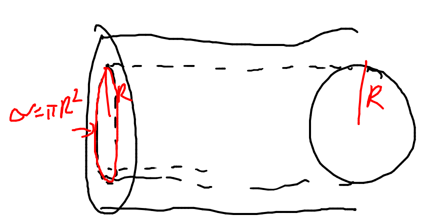
$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left(\frac{R}{2}\sin\frac{\theta}{2}\right)$

$= \frac{R^2}{4}$ independent of θ

Total cross-section

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega =$$

hard sphere $\sigma = \int \frac{R^2}{4}\, d\Omega = \frac{R^2}{4}\, 4\pi = \pi R^2$



Quantum Scattering Theory

imagine beam $\psi(z) = Ae^{ikz}$

hits scattering potential $V(\vec{r})$

outgoing spherical wave $f(\theta,\phi)\frac{e^{ikr}}{r}$

$$\psi(\vec{r}) \approx A\left[e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right]$$ for large r

$k = \frac{\sqrt{2mE}}{\hbar}$

$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$

$$\psi(\vec{r}) = A\left[e^{ikz} + f(\theta,\phi)\,\frac{e^{ikr}}{r}\right]$$

must obey $\quad -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$

Born approximation

suppose $V(\vec{r})$ dies away far from center

& we're only interested in region where $V \approx 0$

suppose k is small (low-energy scattering)

$$\boxed{f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\int V(\vec{r})\,d^3\vec{r}}$$

~~e.g. Rutherford scattering~~ $\cancel{V(r) = \frac{q_1 q_2}{4\pi\epsilon_0 r}}$

$$V(\vec{r}) = \begin{cases} V_0, & r \le a \\ 0, & r \ge a \end{cases} \qquad V_0 \text{ small}$$

$$f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\int_0^a V_0\,d^3r$$

$$= -\frac{m}{2\pi\hbar^2}V_0\,\frac{4}{3}\pi a^3$$

$$\frac{d\sigma}{d\Omega} = |f(\theta\phi)|^2 = \left(\frac{2mV_0 a^3}{3\hbar^2}\right)^2$$

$$\sigma \approx 4\pi\left(\frac{2mV_0 a^3}{3\hbar^2}\right)^2$$