Physics 4310 Homework #4

5 problems

Due by Monday, February 15

▷ **1.**

Suppose a spin–1/2 particle starts with the initial state $|\psi(0)\rangle = |\odot\rangle$. It is placed in a magnetic field $\vec{B} = B_0\hat{z}$.

(a) What is $|\psi(t)\rangle$?

(b) Find $\langle S_x\rangle$ as a function of time.

▷ **2.**

Consider an electron that starts in the state $|\uparrow\rangle$, in a magnetic field of $B_0 = 0.1\,\text{T}$ pointing upward. A secondary magnetic field of $B_1 = 0.001\,\text{T}$ is applied to the electron, perpendicular to the initial field, that rotates around the z axis with angular frequency. That is,

$$\vec{B} = B_0\hat{z} + B_1(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$$

when the field is turned on.

(a) What should the frequency of rotation ω be to maximize the probability that the spin will flip to $|\downarrow\rangle$?

(b) Given the answer in part (a), what is the shortest amount of time that B_1 should be turned on, to guarantee that the spin flips?

▷ **3.**

Define the wavefunctions

$$\psi_n(x) = \begin{cases} C_n(1 - x^n) & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Consider the two functions $|\psi_2\rangle$ and $|\psi_4\rangle$, specifically.

(a) Find C_2 and C_4 so that the two functions are normalized.

(b) Find $\langle\psi_2|\psi_4\rangle$. Are the functions orthogonal?

(c) Find the average value $\langle x\rangle$ and $\langle p\rangle$ for both functions.

▷ **4.**

In the time-independent Schrodinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

Show that the eigenvalues E must exceed the minimum value of $V(x)$, for every normalizable solution to the time-independent Schrödinger equation. Hint: Isolate $\frac{d^2\psi}{dx^2}$, and show that, if $E < V_{\min}$, then ψ and ψ'' have the same sign. Argue that such a function cannot be normalized.

▷ **5.**

A particle in the infinite square well has as its initial wave function an even mixture of the first two stationary states:

$$\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)]$$

(a) Normalize $\Psi(x, 0)$; that is, find A.
(b) Find $\Psi(x, t)$ and $|\Psi(x, t)|^2$. Express the latter as a sinusoidal function of time. To simplify, let $\omega = \pi^2\hbar/2ma^2$.
(c) Compute $\langle x\rangle$. Notice that it oscillates in time. What is the angular frequency of the oscillation? The amplitude? (If your amplitude is greater than $a/2$, then the particle has left the well which is impossible; try again.)
(d) Compute $\langle p\rangle$.
(e) If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? Find the expectation value of H.