$\boxed{\begin{matrix} N_A \\ q_A \end{matrix}}\ \boxed{\begin{matrix} N_B \\ q_B \end{matrix}}$

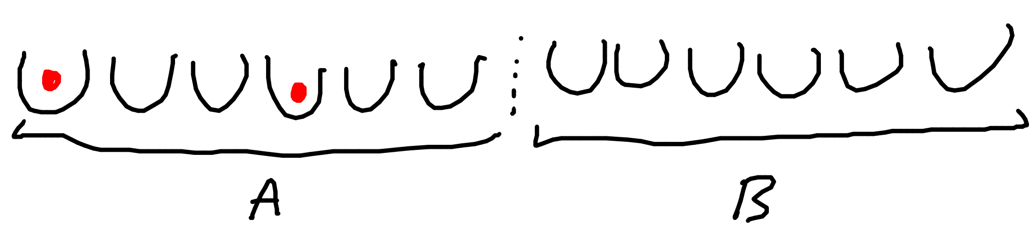
$\underline{N} = \underline{N_A} + \underline{N_B}$ constant

$\underline{q} = q_A + q_B \leftarrow q_B = q - q_A$

↑ variable: parametrizes "energy macrostates" of this system

$$P(q_A) = \frac{\binom{N_A + q_A - 1}{q_A}\binom{N_B + q_B - 1}{q_B}}{\binom{N + q - 1}{q}} = \frac{\Omega_A \Omega_B}{\Omega_{all}}$$

e.g. $N_A = N_B = 6 \qquad q = 2$



$q_A = 2$
$q_B = 0$

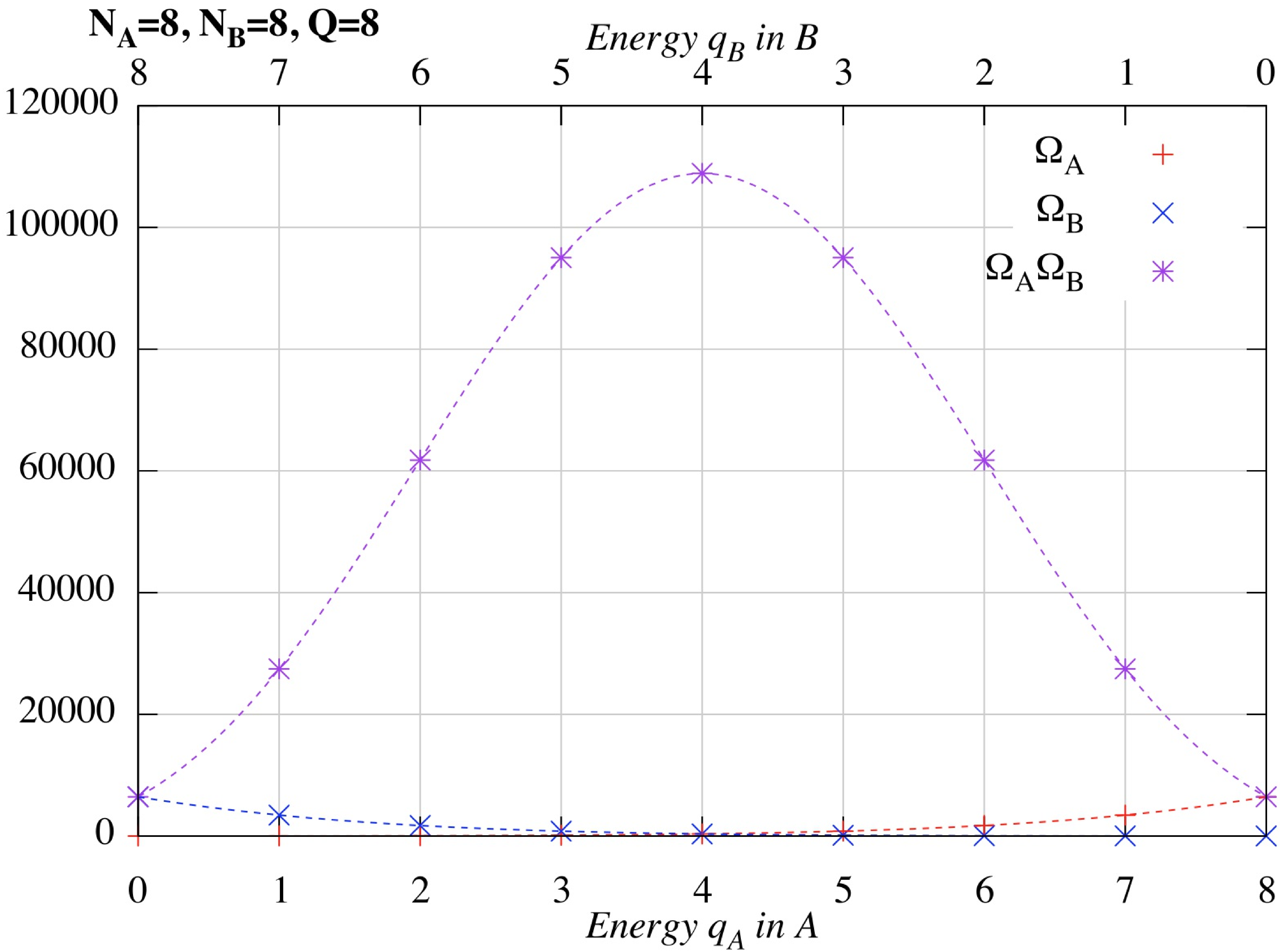
$$\Omega(q_A = 2) = \Omega_A \Omega_B = \binom{6+2-1}{2}\binom{6+0-1}{0}$$

$$= \binom{7}{2} 1 = \frac{7 \cdot 6}{2} = 21$$

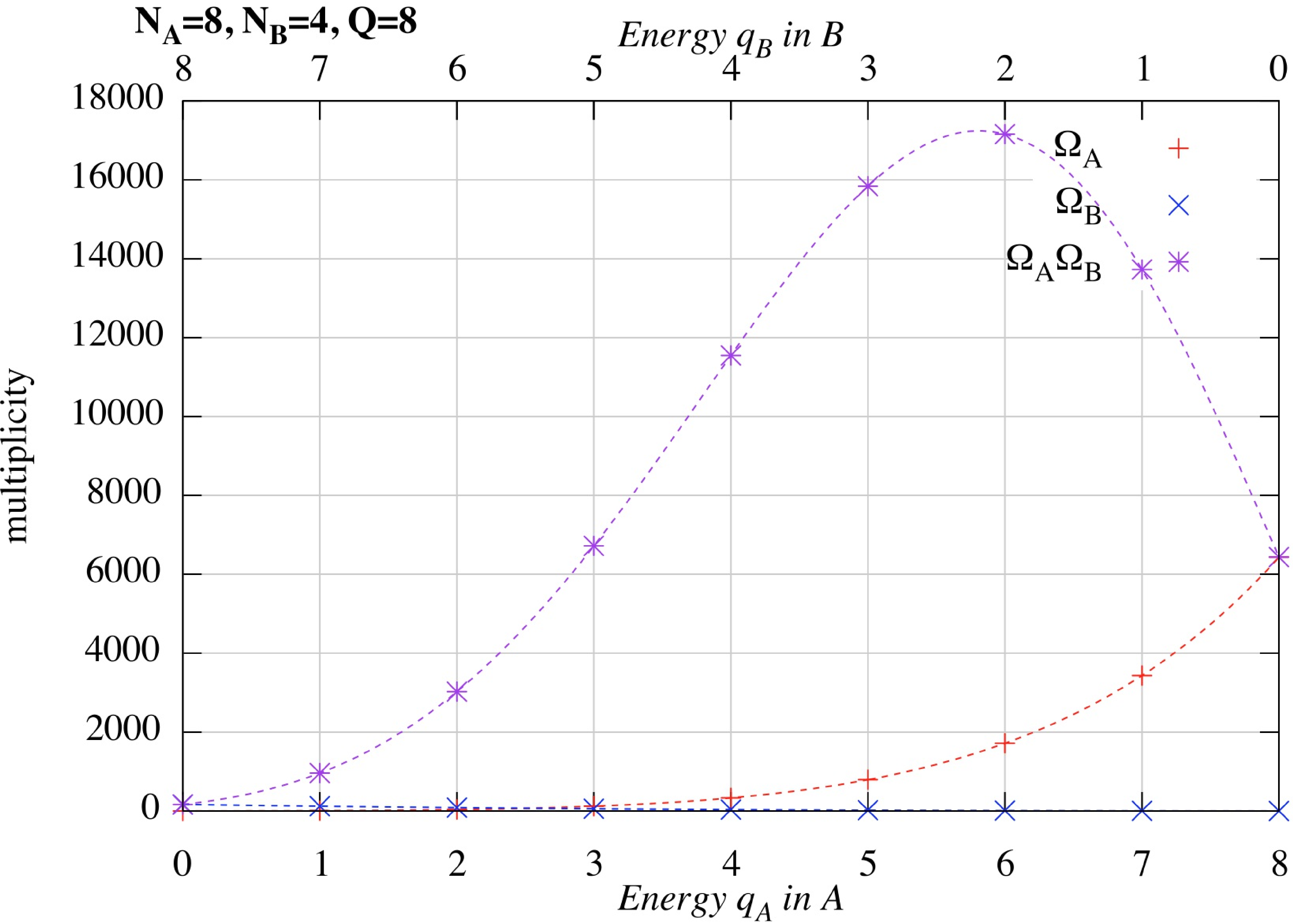
$$\Omega(q_A = 0) = 1\binom{7}{2} = 21$$

$$\Omega(q_A = 1) = \binom{6+1-1}{1}\binom{6+1-1}{1} = \left(\frac{6!}{5!\,1!}\right)^2 = (6)^2 = 36.$$

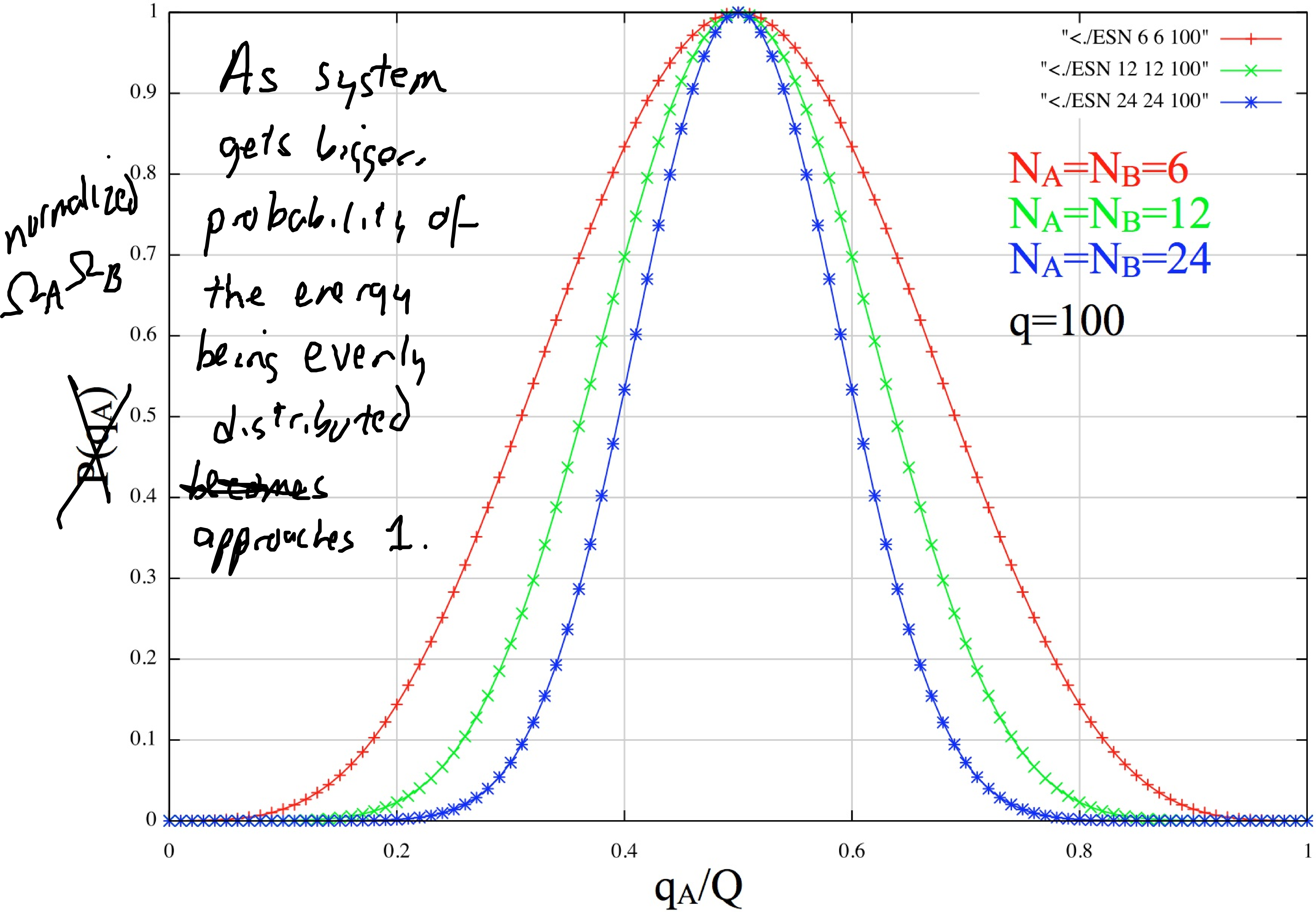
$$P(q_A = 1) = \frac{36}{21+36+21} = 46\% \leq 50\%$$



N_A=8, N_B=8, Q=8
Energy q_B in B
8 7 6 5 4 3 2 1 0
120000
100000
80000
60000
40000
20000
0
Ω_A
Ω_B
$\Omega_A\Omega_B$
0 1 2 3 4 5 6 7 8
Energy q_A in A



N_A=8, N_B=4, Q=8
Energy q_B in B
8
7
6
5
4
3
2
1
0
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
multiplicity
Ω_A
Ω_B
$\Omega_A\Omega_B$
0
1
2
3
4
5
6
7
8
Energy q_A in A



"<./ESN 6 6 100"
"<./ESN 12 12 100"
"<./ESN 24 24 100"
$N_A=N_B=6$
$N_A=N_B=12$
$N_A=N_B=24$
q=100
q_A/Q
~~$P(q_A)$~~
normalized $\Omega_A \Omega_B$
As system gets bigger probability of the energy being evenly distributed ~~becomes~~ approaches 1.

Suppose $N_A = N_B = \frac{1}{2}N \gg 1$

$$q_A = q\left(\tfrac{1}{2} + \delta\right) \qquad q_B = q\left(\tfrac{1}{2} - \delta\right)$$

$$q_A + q_B = q$$

δ: deviation from even distribution of energy.

$-\frac{1}{2} \leq \delta \leq \frac{1}{2}$

if $q \gg N$ high-temperature limit

$$\Omega(\delta) = \Omega_A \Omega_B$$

$$= \left(\frac{e q_A}{N_A}\right)^{N_A} \left(\frac{e q_B}{N_B}\right)^{N_B}$$

$N_A = N_B = \frac{1}{2}N$

$$= \left(\frac{e q_A}{\frac{1}{2}N}\right)^{\frac{1}{2}N} \left(\frac{e q_B}{\frac{1}{2}N}\right)^{\frac{1}{2}N} = \left(\frac{e}{N}\right)^N \left(4 q_A q_B\right)^{\frac{1}{2}N}$$

$$= \left(\frac{e}{N}\right)^N \left[4\, q\left(\tfrac{1}{2}+\delta\right) q\left(\tfrac{1}{2}-\delta\right)\right]^{\frac{1}{2}N}$$

$$= \left(\frac{eq}{N}\right)^N \left[4\left(\tfrac{1}{4} - \delta^2\right)\right]^{\frac{1}{2}N}$$

$$= \left(\frac{eq}{N}\right)^N \left[1 - 4\delta^2\right]^{\frac{1}{2}N}$$

$(1+\epsilon)^a \approx 1 + a\epsilon$

$\delta \ll 1$

$$\approx \left(\frac{eq}{N}\right)^N \left[1 - 2N\delta^2\right]$$

$e^{-\epsilon} \approx 1 - \epsilon$

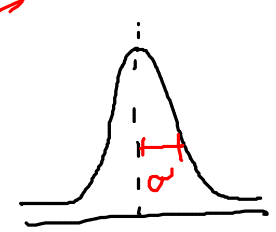
$N\delta^2 \ll 1$

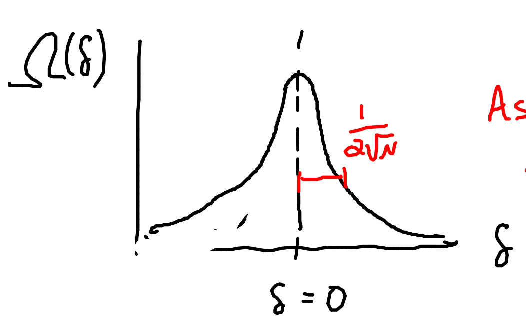
$\delta \ll \frac{1}{\sqrt{N}}$

$$\Omega(\delta) \approx \left(\frac{eq}{N}\right)^N e^{-2N\delta^2}$$

$\sigma = \frac{1}{2\sqrt{N}}$

$e^{-x^2/2\sigma^2}$





As $N \to \infty$,
$\sigma \to 0$.

$$q_A = q\left(\tfrac{1}{2} \pm \frac{1}{2\sqrt{N}}\right)$$

As $N \to \infty$
$q_A \to \frac{1}{2}q$

$$P(\delta = 0) = \frac{\Omega(\delta=0)}{\Omega_{all}} = \frac{\left(\frac{eq}{N}\right)^N e^0}{\left(\frac{eq}{N}\right)^N} = 1$$