Fermi gas at $T=0$

$$U = 2\iiint \varepsilon(n)\, dn_x\, dn_y\, dn_z$$

$$= \frac{\pi h^2}{8mL^2}\int_0^{n_{max}} n^4\, dn$$

$$\varepsilon = \frac{h^2}{8mL^2}(n_x^2+n_y^2+n_z^2) = \frac{h^2 n^2}{8mL^2}$$

$$n = \sqrt{\frac{8mL^2}{h^2}}\sqrt{\varepsilon}$$

$$dn = \sqrt{\frac{8mL^2}{h^2}}\frac{1}{2\sqrt{\varepsilon}}\, d\varepsilon$$

$$U = \pi\left(\frac{h^2}{8mL^2}\right)\int_0^{\varepsilon_F}\left(\frac{8mL^2}{h^2}\right)^{5/2}\frac{1}{2}\,\varepsilon^{3/2}\, d\varepsilon$$

$$U = \int_0^{\varepsilon_F}\varepsilon\left[\frac{\pi}{2}\left(\frac{8mL^2}{h^2}\right)^{3/2}\sqrt{\varepsilon}\right] d\varepsilon$$

$$= \sum_{\substack{\text{energies}\\ \varepsilon}}\begin{pmatrix}\text{energy of}\\ \text{one state}\end{pmatrix}\times\begin{pmatrix}\#\text{ of states with}\\ \text{energy } \varepsilon\end{pmatrix}$$

$$g(\varepsilon) = \frac{\pi}{2}\left(\frac{8mL^2}{h^2}\right)^{3/2}\sqrt{\varepsilon}$$

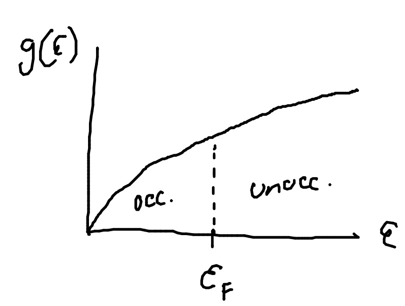
"density of states"

\# of single-particle states per unit energy

it's a distribution

$$\int_{\varepsilon_1}^{\varepsilon_2} g(\varepsilon)\, d\varepsilon = \#\text{ of states with } \varepsilon_1<\varepsilon<\varepsilon_2$$

At $T=0$, $N = \int_0^{\varepsilon_F} g(\varepsilon)\, d\varepsilon$ (every state from $0<\varepsilon<\varepsilon_F$ is occupied by 1 electron)



If $T\neq 0$,

average occupancy Fermi-Dirac distribution

$$N = \int_0^\infty g(\varepsilon)\,\bar{n}(\varepsilon)\, d\varepsilon$$

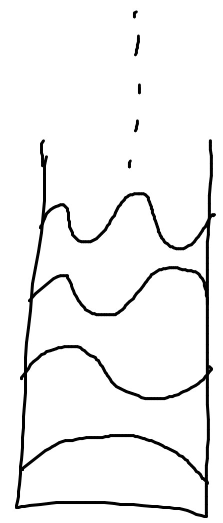
avg # per microstate

$$U = \int_0^\infty \underbrace{g(\varepsilon)\,\bar{n}(\varepsilon)}_{\text{average number of particles per energy macrostate}}\,\varepsilon\, d\varepsilon$$

Blackbody Radiation

- Ultraviolet catastrophe

if light is a wave,
then energy proportional to
intensity, or amplitude squared



put a light wave in a box
light could be in a standing
wave of any number of bumps n.

Equipartition Theorem: every state
has average energy $\frac{1}{2}kT$

∞ # of states $\rightarrow$ energy $= \infty$! Uh oh.

Planck:
energy is related to frequency, not intensity alone
every standing wave state can have energy
$E = 0, hf, 2hf, 3hf, \ldots$
where f is frequency of that standing wave
energy is quantized

for one standing wave

$$Z = 1 + e^{-\beta hf} + e^{-2\beta hf} + e^{-3\beta hf} + \ldots \quad \frac{1}{1-e^{-\beta hf}}$$

$$\langle E \rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = \frac{hf}{e^{hf/kT}-1} \qquad \text{not } \frac{1}{2}kT$$

hf: energy per quantum

$$\frac{\langle E \rangle}{hf} = \text{avg \# of quanta} = \frac{1}{e^{hf/kT}-1}$$ Bose-Einstein dist.

these "quanta" are photons

they are bosons
with $\mu = 0$.

$$0 = \mu = \left(\frac{\partial F}{\partial N}\right)_{T,V} \rightarrow$$ N will adjust itself in equilibrium so that F is minimized.

$\rightarrow$ photons can be created or destroyed freely
$\rightarrow$ massless