The most likely macrostate is the one with the largest number of accessible microstates — largest multiplicity. Ω

System in equilibrium is in this most likely macrostate or near it (given fluctuations)

$$\frac{\partial \Omega}{\partial U} = 0 \quad \text{or} \quad \frac{\partial \Omega}{\partial V} = 0$$

or whatever parameters determine what macrostate we're in.

if $\frac{\partial \Omega}{\partial U} = 0$, $\frac{\partial \ln \Omega}{\partial U} = 0$

$\ln \Omega$ is easier to deal with since Ω is a VLN

$$S = k_B \ln \Omega \qquad \text{"entropy"}$$

2nd Law of Thermodynamics: Entropy is maximized at equilibrium

Some properties of entropy

- generally increases with N, U
- for two isolated, noninteracting systems

$$\Omega = \Omega_A \Omega_B$$

$$\ln \Omega = \ln \Omega_A + \ln \Omega_B \rightarrow S = S_A + S_B$$

- measure of disorder or unpredictability, but only in a sense

lower entropy

higher entropy

but if I measure temperature of water at random, warm water is more "predictable".

- arrow of time
  systems tend to approach equilibrium, so "forward in time" = "increasing entropy"

if $\Omega = 1$, $S = 0$.

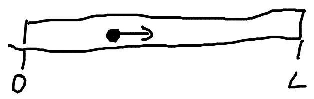
e.g. shuffling a 52-card deck

$$S = k \ln \Omega = k \ln 52! \approx k(52 \ln 52 - 52)$$
$$= 156 k$$

stop shuffling: $S = 0$
because stuck in one state and can't reach the others,

Ideal Gas

1 point particle in a 1D box, length L

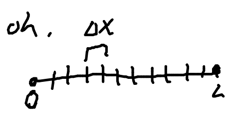


"microstate" depends on its momentum (or velocity) and its position

x & p_x

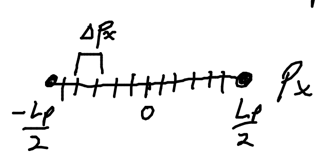
How many microstates are there? ∞ uh oh.

divide line up into bins of size Δx



How many values can x have? $\frac{L}{\Delta x}$

Also divide "momentum space" into bins Δp_x

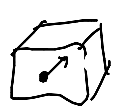


$$\Omega = \Omega_x \Omega_p = \frac{L}{\Delta x} \frac{L_p}{\Delta p_x} = \frac{L L_p}{\Delta x \Delta p_x}$$

Quantum mechanics says: $\Delta x \Delta p \geq \frac{\hbar}{2} = \frac{h}{4\pi}$ $\quad \Delta x \Delta p \approx h$

$$\Omega = \frac{L L_p}{h}$$

in 3D



$$\Omega = \Omega_{pos} \Omega_{mom}$$

$$\Omega_{pos} = \Omega_x \Omega_y \Omega_z = \frac{V}{(\Delta x)^3}$$

$$\Omega_{mom} = \frac{V_p}{(\Delta p)^3}$$

$$\Omega = \frac{V V_p}{h^3}$$

V: volume of gas

What is V_p?

Well, it depends on energy U of the system

In momentum space, particle's momentum is given by (p_x, p_y, p_z)

$\frac{1}{2} m \vec{v}^2 = U$ $\qquad \frac{1}{2} m \vec{v}^2 = \frac{\vec{p}^2}{2m} = U$

$$p_x^2 + p_y^2 + p_z^2 = 2mU$$

Shell of a sphere with radius $\sqrt{2mU}$ in ~~position~~ momentum space

V_p = surface area of this sphere

$$= 4\pi R^2 = 4\pi (\sqrt{2mU})^2 = 8\pi m U$$

1 particle in 3D, $\quad \Omega = \frac{V \, 8\pi m U}{h^3}$

Two particles, total energy U

$$\Omega_{pos} \propto V^2$$

positions are independent

but momenta are not

$$p_{1x}^2 + p_{1y}^2 + p_{1z}^2 + p_{2x}^2 + p_{2y}^2 + p_{2z}^2 = 2mU$$

6-dimensional sphere!