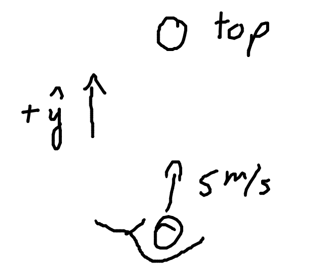
4) I throw a ball into the air at $5\,m/s$ (straight up)

How long until it reaches the top of its flight?



$\Delta y =$

$V_{iy} = +5\,m/s$

$V_{fy} = 0\,m/s$

$a_y = -9.8\,m/s^2$

$\Delta t =$ NEED

initial: right after ball leaves my hand

final: top of its flight

$V_f = V_i + a\Delta t$

$\Delta t = \dfrac{V_f - V_i}{a}$

$= \dfrac{0-5}{-9.8} = 0.51\,s$

• How long does it take to hit your hand again?

$\Delta y =$

$V_{iy} = 0\,m/s$

$V_{fy} =$

$a_y = -9.8\,m/s^2$

$\Delta t =$ NEED

initial: top of its flight

final: right before it hits my hand

Can't solve this yet, but I could go back & get Δy

Instead,

initial: when ball leaves my hand

$\Delta y = 0\,m$ ← back where it started

$V_{iy} = +5\,m/s$

$V_{fy} =$

$a_y = -9.8\,m/s^2$

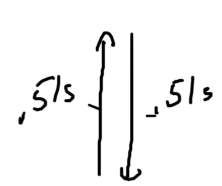
$\Delta t =$ NEED

$\Delta y = V_i \Delta t + \frac{1}{2}a(\Delta t)^2$

$0 = 5\Delta t - 4.9(\Delta t)^2$

$= \Delta t\,[5 - 4.9\Delta t]$

$\Rightarrow \Delta t = 0,\ \frac{5}{4.9} = 1.02\,s$



same time up as down

What is V_{fy}?

$V_{fy}^2 = V_{iy}^2 + 2a\Delta y$

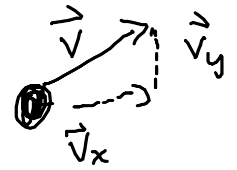
$V_{fy}^2 = 25$

$V_{fy} = \cancel{+}\!\!-\,5\,m/s$

$V_{fy} = -5\,m/s$

<u>speed</u> is same coming down as going up

Two-Dimensional Motion w/ Constant Acceleration



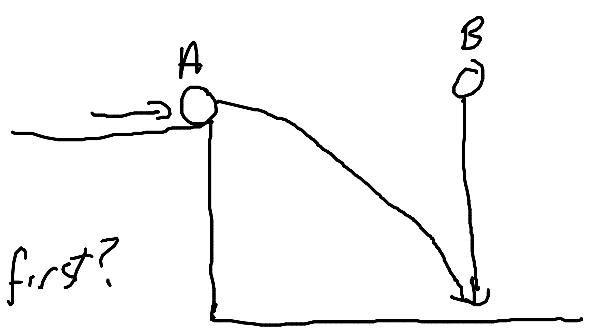
Δx	Δy
v_{ix}	v_{iy}
v_{fx}	v_{fy}
a_x	a_y
Δt	

- 9 unknowns
- − • 4 independent equations

need 5 of these values

Dimensions are independent

e.g.



When A leaves the table, B is dropped

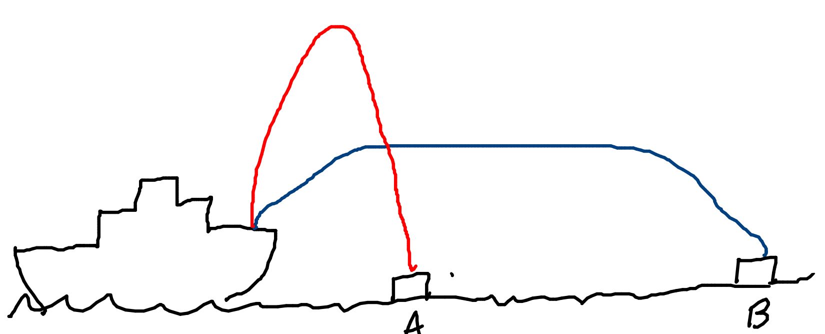
Which hits the ground first?

A) A B) B
C) Both at same time

Looking at vertical components

Δy is same for both
$v_{iy} = 0$ is same for both (but A has additional x component)
$a = 9.81$ for both.

so you can solve for Δt

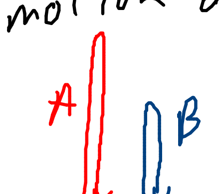


Cannon balls are not neccessarily launched with same speed

Which target is hit first?

A) A $\boxed{\text{B) B}}$ c) Both the same

Look at vertical motion alone



Fire a bullet horizontally at $400 \, m/s$ at target 20 m away. How far below horizontal does bullet drop?

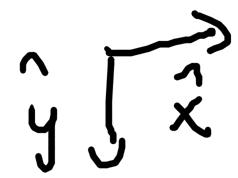
$\Delta x : 20m$ $\quad \Delta y = NEED$

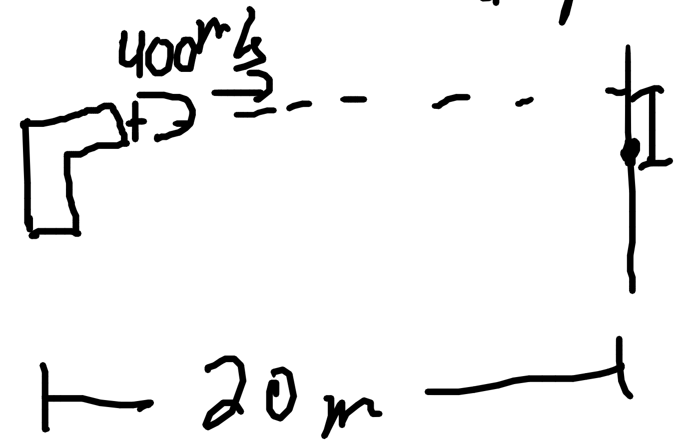
$V_{ix} = 400 \, m/s$ $\quad V_{iy} = 0 \, m/s$

$V_{fx}:$ $\quad V_{fy}:$

$a_x = 0$ $\quad a_y = 9.8 \, m/s^2$

$\Delta t =$





Can't solve y right away, (only 2 given)

So solve x column for Δt first.

Δt: $\Delta x = V_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$

$20 = 400(\Delta t) + 0$

$= \Delta t = \frac{20}{400} = 0.05 s$

$\Delta y = V_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$

$= 0 + \frac{1}{2}(9.8)(0.05)^2 = 0.012 m$

or 12 cm

Cannon on ground fires a ball at angle θ from horizontal, with speed v_0.
How far away does the ball land?

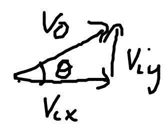
$\Delta x = $ NEED $\quad \Delta y = 0$

$v_{ix} = +v_0 \cos\theta \quad v_{iy} = +v_0 \sin\theta$

$v_{fx} = \quad v_{fy} =$

$a_x = 0 \quad a_y = -g$

$\Delta t =$



$\cos\theta = \frac{v_{ix}}{v_0} \rightarrow v_{ix} = v_0 \cos\theta$

x only has 2 "givens" but y has 3. (v_0 & θ are given here.)

$$0 = v_0 \sin\theta (\Delta t) - \frac{1}{2} g (\Delta t)^2$$

$$v_0 \sin\theta - \frac{1}{2} g \Delta t = 0$$

$$0 = \Delta t \left[v_0 \sin\theta - \frac{1}{2} g \Delta t \right]$$

or this is zero

Either $\Delta t = 0$ or $v_0 \sin\theta = \frac{1}{2} g \Delta t$

$$\frac{2 v_0 \sin\theta}{g} = \Delta t$$

$$\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

$$\Delta x = (v_0 \cos\theta)\left(\frac{2 v_0 \sin\theta}{g}\right) + 0$$

$$\Delta x = \frac{v_0^2}{g} \underbrace{2 \sin\theta \cos\theta}_{\sin 2\theta}$$

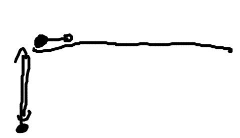
$$\boxed{\Delta x = \frac{v_0^2}{g} \sin 2\theta}$$ range of cannon

$\Delta x \propto v_0^2$ so as v_0 increases, Δx increases a lot

$\Delta x \propto \frac{1}{g}$ so for smaller g (e.g. the Moon) Δx increases

At $\theta = 0$, $\Delta x = \frac{v_0^2}{g} \sin 0 = \underline{0}$

At $\theta = 90^\circ$, $\Delta x = \frac{v_0^2}{g} \sin 180^\circ = 0$



Max range when $\sin 2\theta = 1$

$\sin 90^\circ = \underline{1}$

$\rightarrow 2\theta = 90^\circ \rightarrow \theta = 45^\circ$