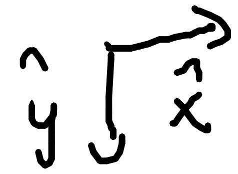
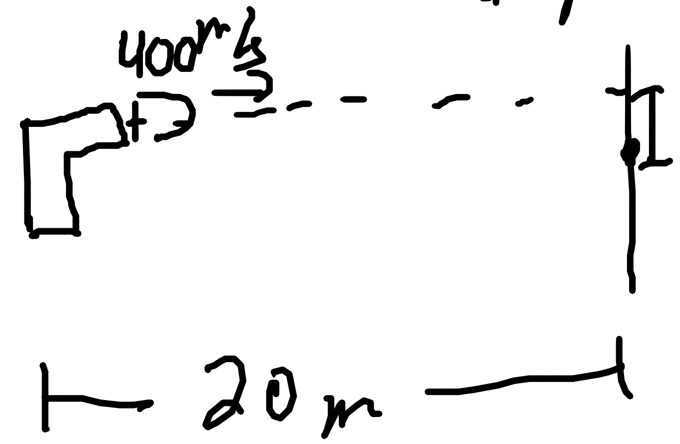
Fire a bullet horizontally at 400 m/s at target 20 m away. How far below horizontal does bullet drop?





$\Delta x: 20\,m$ $\quad \Delta y = \text{NEED}$

$V_{ix} = 400\,m/s$ $\quad V_{iy} = 0\,m/s$

$V_{fx}:$ $\quad V_{fy}:$

$a_x = 0$ $\quad a_y: 9.8\,m/s^2$

$\Delta t:$

Can't solve y right away, (only 2 given)

So solve x column for Δt first.

Δt: $\Delta x = V_{ix}\Delta t + \frac{1}{2} a_x (\Delta t)^2$

$20 = 400(\Delta t) + 0$

$= \Delta t = \frac{20}{400} = 0.05\,s$

$\Delta y = V_{iy}\Delta t + \frac{1}{2} a_y (\Delta t)^2$

$= 0 + \frac{1}{2}(9.8)(0.05)^2 = 0.012\,m$

or 12cm

Cannon on ground fires a ball at angle θ from horizontal, with speed v_0.

How far away does the ball land?

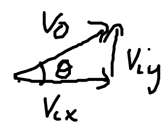
$\Delta x =$ NEED $\quad \Delta y = 0$

$v_{ix} = +v_0 \cos\theta \quad v_{iy} = +v_0 \sin\theta$

$v_{fx} = \quad v_{fy} =$

$a_x = 0 \quad a_y = -g$

$\Delta t =$



$\cos\theta = \frac{v_{ix}}{v_0} \rightarrow v_{ix} = v_0 \cos\theta$

x only has 2 "givens" but y has 3. (v_0 & θ are given here.)

$0 = v_0 \sin\theta (\Delta t) - \frac{1}{2} g (\Delta t)^2 \qquad v_0 \sin\theta - \frac{1}{2} g \Delta t = 0$

$0 = \Delta t \left[v_0 \sin\theta - \frac{1}{2} g \Delta t \right]$ or this is zero

Either $\Delta t = 0$ or $v_0 \sin\theta = \frac{1}{2} g \Delta t$

$\frac{2 v_0 \sin\theta}{g} = \Delta t$

$\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$

$\Delta x = (v_0 \cos\theta)\left(\frac{2 v_0 \sin\theta}{g}\right) + 0$

$\Delta x = \frac{v_0^2}{g} \underbrace{2 \sin\theta \cos\theta}_{\sin 2\theta}$

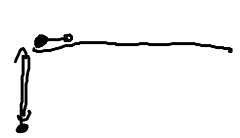
$\boxed{\Delta x = \frac{v_0^2}{g} \sin 2\theta}$ range of cannon

$\Delta x \propto v_0^2$ so as v_0 increases, Δx increases a lot

$\Delta x \propto \frac{1}{g}$ so for smaller g (e.g. the Moon) Δx increases

At $\theta = 0$, $\Delta x = \frac{v_0^2}{g} \sin 0 = \underline{0}$

At $\theta = 90^\circ$, $\Delta x = \frac{v_0^2}{g} \sin 180^\circ = 0$



Max range when $\sin 2\theta = 1$

$\sin 90^\circ = \underline{1}$

$\rightarrow 2\theta = 90^\circ \rightarrow \theta = 45^\circ$