Physics 370 Homework #4

7 problems

Due by Monday, September 19th

▷ **1.**

According to an observer at Earth's equator, by how much would his clock and one on a satellite in geosynchronous orbit differ in one day?

▷ **2.**

(a) Show, using the chain rule, that

$$\frac{dU}{d\lambda} = \frac{8\pi V hc}{e^{hc/\lambda k_B T} - 1} \frac{1}{\lambda^5}$$

(b) According to *Wien's Law*, the wavelength $\lambda_{\max}$ of maximum thermal emission of electromagnetic energy from a body of temperature T obeys

$$\lambda_{\max} T = 2.898 \times 10^{-3}\,\mathrm{m \cdot K}$$

Show this is true by obtaining an expression that, when solved, would yield the wavelength at which this function is maximized. Solving this equation is impossible, but show that the value of $\lambda_{\max} T$ solves it to a reasonable degree.

▷ **3.**

The electromagnetic intensity of all wavelengths thermally radiated by a body of temperature T is given by

$$I = \sigma\, T^4 \quad \text{where} \quad \sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2/K^4}$$

This is the *Stefan-Boltzmann law*. TO derive it, show that the total energy of the radiation in a volume V at temperature T is

$$U = \frac{8\pi^5 k_B^4 V T^4}{15 h^3 c^3}$$

by integrating Planck's spectral energy density over all frequencies. Note that

$$\int_0^\infty \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15}$$

Intensity, or power per unit area, is then the product of energy per unit volume and distance per unit time. But because intensity is a flow in a given direction away from the blackbody, c is not the correct speed. For radiation moving uniformly in all directions, the average *component* of velocity in a given direction is $\frac{1}{4}c$.

▷ **4.**
Light of 300 nm wavelength strikes a metal plate, and photoelectrons are produced moving as fast as $0.002c$.
(a) What is the work function of the metal?
(b) What is the threshold wavelength for this metal?

▷ **5.**
When a beam of monoenergetic electrons is directed at a tungsten target, X-rays are produced with wavelengths no shorter than 0.062 nm. How fast are the electrons in the beam moving?

▷ **6.**
A photon has the same momentum as an electron moving at 10^6 m/s.
(a) Determine the photon's wavelength.
(b) What is the ratio of the kinetic energies of the two? (Note: a photon is *all* kinetic energy.)

▷ **7.**
Show that the laws of momentum and energy conservation forbid the complete *absorption* of a photon by a free electron. (This isn't the photoelectric effect, because electrons aren't entirely free in a metal.)