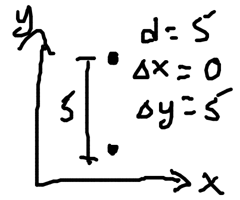
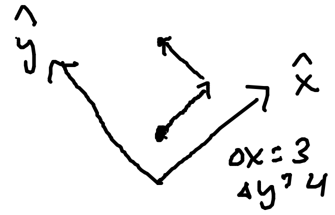
Time

Space

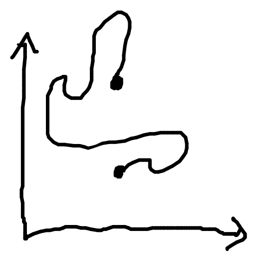




in relativity, Δx & Δy $\longleftrightarrow$ Δx & Δt

basis $\longleftrightarrow$ frame

space



invariant under basis change
- distance between them
- length of any particular path that you draw between them

Time Intervals

- coordinate time Δt
  as measured in one particular frame
  frame - dependent (cf Δy above)
- proper time $\Delta \tau$ between two events
  take a clock from 1st event to 2nd event
  clock will record proper time between them

  frame - independent (everyone reads same clock)
  BUT depends on clock's worldline

  clock isn't necessarily inertial

- spacetime interval Δs
  is proper time on a clock with an inertial worldline between events

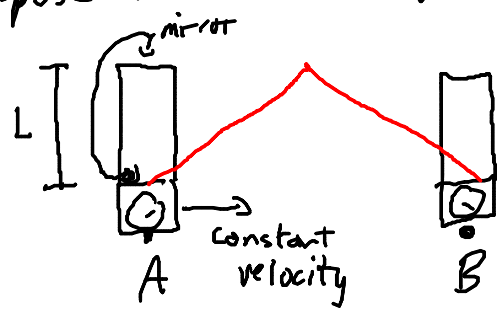
  universal - everyone agrees

  unique

also the coordinate time Δt in clock's inertial frame

Suppose I have two events, A & B

light clock



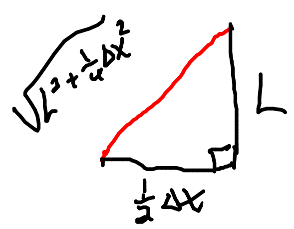
going from A to B is 1 "tic"

$\Delta s = 2L$ (= Δt in clock's frame)

in clock's frame, $\Delta s = \Delta t$

If A & B are a distance Δx apart in rest frame.

how far does the light travel in rest frame?



$\times 2$

light travels a distance

$$2\sqrt{L^2+\tfrac{1}{4}\Delta x^2}$$

In rest frame $\searrow$ $\Delta t = \sqrt{4L^2+\Delta x^2}$ $\leftarrow$ In rest frame

$$(\Delta t)^2 = (2L)^2 + (\Delta x)^2$$

$$\Delta t^2 = \Delta s^2 + \Delta x^2$$

$$\boxed{\Delta s^2 = \Delta t^2 - \Delta x^2}$$

invariant

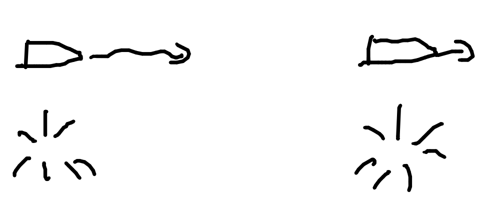
<u>metric equation</u>

In 3D $\Delta s^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$

$= \Delta t^2 - \Delta r^2$

Notice that $\Delta s \leq \Delta t$

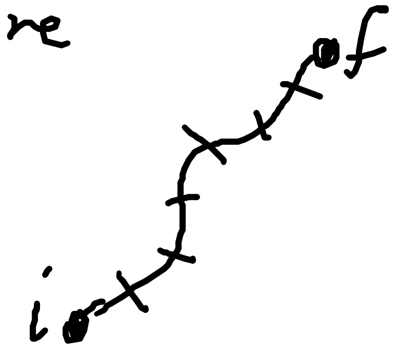
& only $\Delta s = \Delta t$ in frame where two events happen at the same place



in rocket's frame, $\Delta x = 0$

Proper Time for a Worldline

break into a bunch
of little inertial paths



$$\Delta\tau = \int_i^f ds = \int_i^f \sqrt{dt^2 - dx^2}$$

$$= \int_{t_i}^{t_f} dt \sqrt{1 - \left(\frac{dx}{dt}\right)^2}$$

$$= \int_{t_i}^{t_f} \sqrt{1 - v^2}\, dt$$

t & v are measured in some particular frame

if $|\vec{v}|$ is constant

$$\Delta\tau = \sqrt{1 - v^2}\,\Delta t = \frac{1}{\gamma}\Delta t$$

clock reads

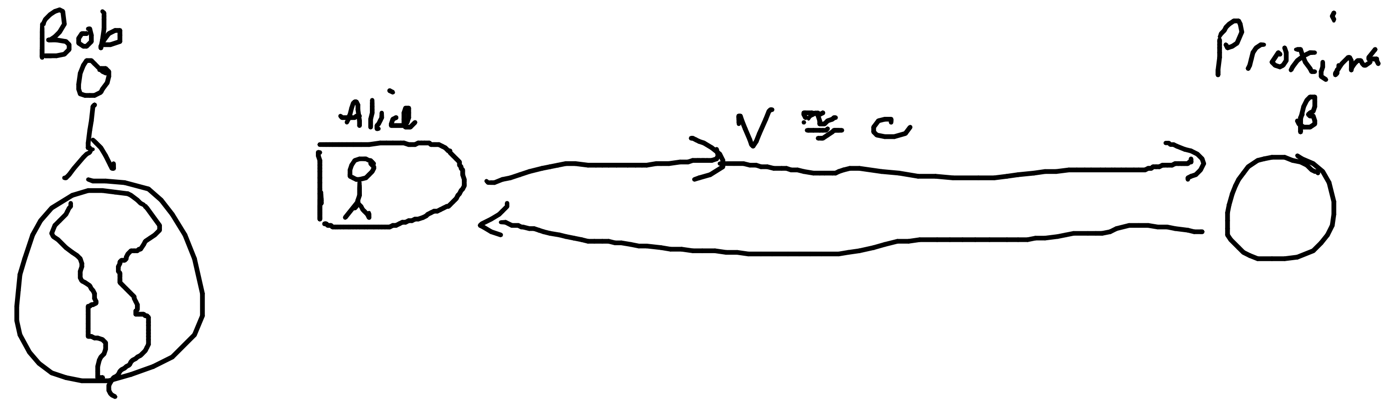
time dilation

less than what observer thinks it should

$$\Delta\tau \leq \Delta t$$

$$\Delta\tau \leq \Delta s \leq \Delta t$$

Twin Paradox



Alice & Bob are twins

- Bob says, "Alice was moving really fast, so she aged more slowly than me."
  She should be younger
- Alice says, "Bob was moving really fast, so Bob should be younger."

Alice accelerated, so travelled a noninertial worldline.

In Sun's frame,

Bob is not moving

& his age $\Delta \tau_B = \Delta t$ because Bob is not moving in Sun's frame.

Alice is moving relative to Sun,

$$\Delta \tau_A < \Delta t$$

Alice can't make the same claim because there is no Δt in a noninertial reference frame.

$\Rightarrow$ Alice will be younger.