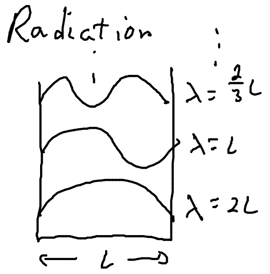
Black body Radiation

Cavity

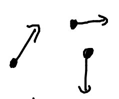


Light is a combination of these standing waves with different amplitudes → different energies

state of light is given by list of amplitudes (1, 3, 2.4, 9.6, ...)

these #s are degrees of freedom (d.o.f.)

e.g.



3 particles in a gas

a d.o.f. is a coordinate required to specify state of the system.

$\vec{x}_1\ \vec{x}_2\ \vec{x}_3$

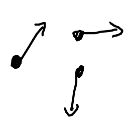
$\vec{v}_1\ \vec{v}_2\ \vec{v}_3$

in 2D $(x_1, y_1, x_2, y_2, x_3, y_3, V_{x1}, V_{y1}, V_{x2}, V_{y2}, V_{x3}, V_{y3})$

Equipartition Theorem

In a system in equilibrium, every degree of freedom which contributes to the total energy quadratically carries an average energy of $\frac{1}{2} k_B T$

$k_B = 1.38 \times 10^{-23}$ J/K Boltzmann's constant



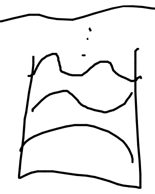
$$E = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + \frac{1}{2} m v_3^2$$

$$= \frac{1}{2} m (v_{1x}^2 + v_{1y}^2) + \frac{1}{2} m (v_{2x}^2 + v_{2y}^2) + \frac{1}{2} m (v_{3x}^2 + v_{3y}^2)$$

average over time $\left\langle \frac{1}{2} m v_{1x}^2 \right\rangle = \frac{1}{2} k_B T = \left\langle \frac{1}{2} m v_{3y}^2 \right\rangle$

total $E = \frac{1}{2} kT \times$ # of d.o.f.

Black body again



Each standing wave mode is a d.o.f.

Light reaches equilibrium,

total energy $= \frac{1}{2} kT \times \infty$

$= \infty$! Uh oh.

Ultraviolet catastrophe b/c too many low-λ, high-f modes.

Max Planck in 1900

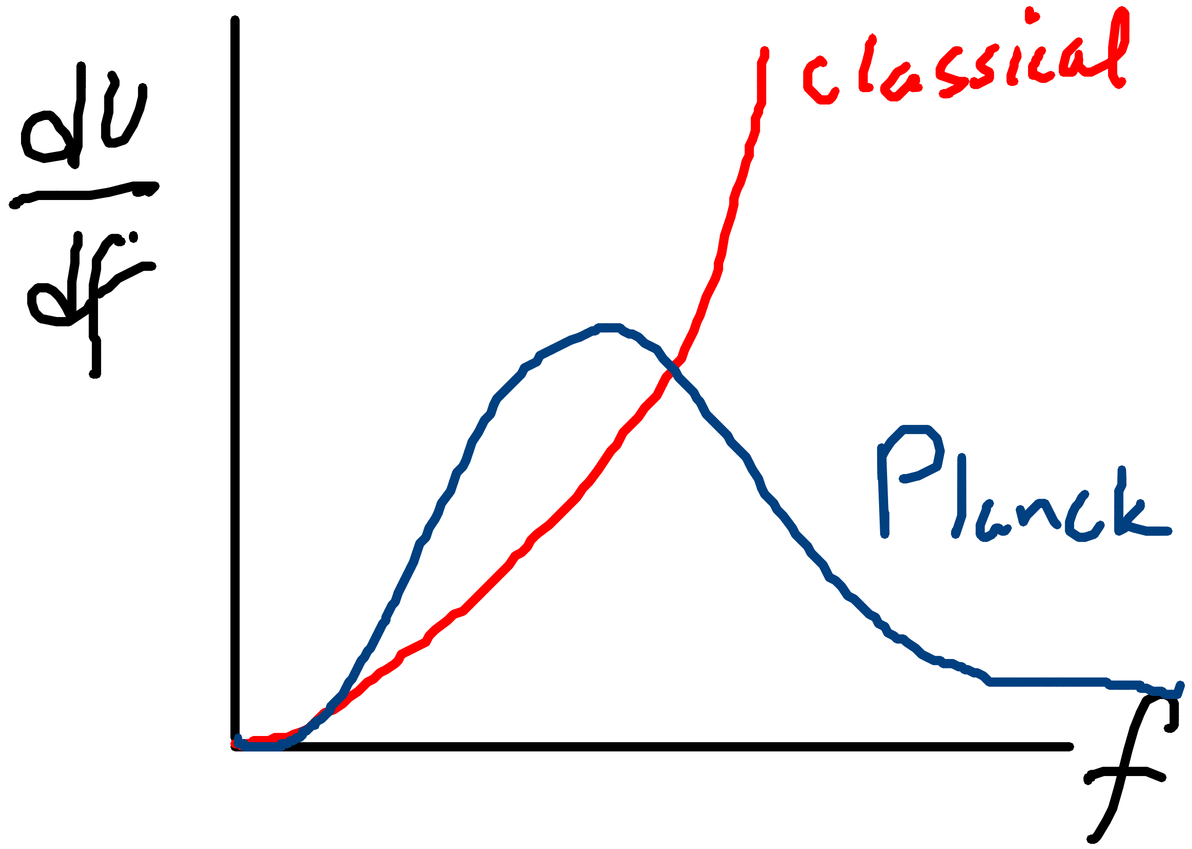
suppose energy in each mode can't take only value, but only an integer multiple of hf

f: frequency of mode h: constant

$$E_f = n h f \qquad n = 0, 1, 2, \ldots \infty$$

Each mode could contain large amounts of energy but high frequency modes can only increase in energy in large chunks hf.

Harder for energy to flow low f → high f than vice versa.



$\frac{dU}{df}$ = energy per frequency

tiny amount of energy stored in a tiny range of frequency

spectrum

Classical $$\frac{dU}{df} = \overbrace{kT}^{\text{energy per wave}} \times \underbrace{\frac{8\pi V}{c^3} f^2}_{\text{\# of waves with frequency } f \text{ in 3D}}$$

$$U = \int_0^\infty \frac{dU}{df}\, df \propto \int f^2\, df = \infty$$

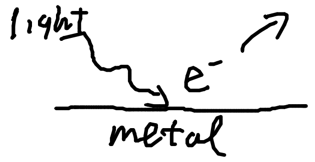
Planck $$\frac{dU}{df} = \frac{hf}{e^{hf/kT} - 1} \times \frac{8\pi V}{c^3} f^2$$

perfect fit

$h = 6.636 \times 10^{-34}\ \mathrm{J/Hz}$

Planck's constant

Photoelectric Effect



To kick an electron free requires a certain minimum amount of energy ϕ: work function

surplus energy goes into giving e^- kinetic energy,

Studied how rate of electrons depended on light

- weak light at 500nm ejects electrons from sodium with no time lag
- stronger light at 600nm – no electrons

Wave Model:

- energy proportional to intensity, not λ or f
- high intensity of light of any color should kick out electrons
- low intensity light will kick out e^- w/ a time lag as energy builds up

Einstein:

Light of frequency f is packaged in chunks (quanta) of energy hf. (particles!)

If one particle hits an electron

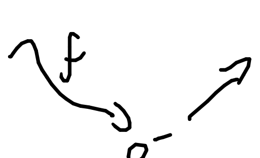
and if $hf > \phi$

then electron is kicked free

regardless of intensity.

Don't have multiple particles hitting same electron (or very rare)

photons



$KE_{max} = hf - \phi$