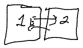
Consider two systems that can exchange energy with each other



$dE_2 = -dE_1$

$$dS_{total} = dS_1 + dS_2$$
$$= \left(\frac{\partial S_1}{\partial E_1}\right) dE_1 + \left(\frac{\partial S_2}{\partial E_2}\right) dE_2$$
$$dS_{tot} = \left(\frac{\partial S_1}{\partial E_1} - \frac{\partial S_2}{\partial E_2}\right) dE_1$$

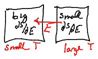
Equilibrium is reached when entropy is maximized

$$dS_{total} = 0$$
$$\frac{\partial S_1}{\partial E_1} = \frac{\partial S_2}{\partial E_2}$$

What happens if $\frac{\partial S_1}{\partial E_1} > \frac{\partial S_2}{\partial E_2}$?

$dS_{tot} > 0$

$\therefore dE_1 > 0$



$$\frac{1}{T} \equiv \frac{\partial S}{\partial E} \rightarrow T = \frac{\partial E}{\partial S}$$

$S = k \ln \Omega$

$k = 1.38 \times 10^{-23}$ J/K

$\left[\frac{\partial S}{\partial E}\right] = \frac{J/K}{J} = \frac{1}{K}$

heat

$$dS = \frac{1}{T} dE \rightarrow dS = \frac{Q}{T}$$

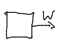
$Q > 0$: heat flowing in

$Q < 0$: heat out

If T is constant,

$$\Delta S = \frac{Q}{T}$$

e.g. heat engines

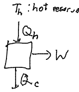


perpetual motion machine of the 1st kind (violates conservation of energy)

perpetual motion machine of the 2nd kind

will build up entropy & not be cyclic

T_h : hot reservoir



T_c : cold reservoir

$T_{in} < T_h$ is T of engine when heat flows in

$T_{out} > T_c$

$$\Delta S_{in} = \frac{Q_h}{T_{in}}$$
$$\Delta S_{out} = -\frac{Q_c}{T_{out}}$$
$$\Delta S = \frac{Q_h}{T_{in}} - \frac{Q_c}{T_{out}} = 0$$
$$\frac{Q_h}{Q_c} = \frac{T_{in}}{T_{out}}$$

efficiency $\eta = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$

$$\eta = 1 - \frac{T_{out}}{T_{in}} = \frac{T_{in} - T_{out}}{T_{in}}$$

$$\frac{T_{out}}{T_{in}} \geq \frac{T_c}{T_h} \rightarrow \boxed{\eta \leq 1 - \frac{T_c}{T_h}}$$

e.g. Diesel engine $T_c = 300K$ $T_h = 1000K$

$$\eta \leq 1 - \frac{300}{1000} = 70\%$$

Efficiency is maximized (for given T_c, T_h) when $T_{out} \simeq T_c$ & $T_{in} \simeq T_h$

Carnot engine

very slow because heat flow takes a long time when ΔT is very small