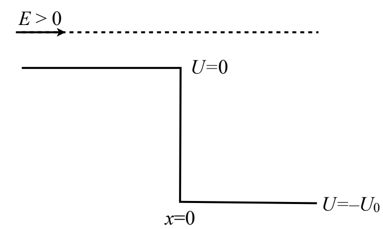
Physics 370 Homework #9

6 problems

Due by Wednesday, November 2

▷ **1.**

Suppose an incident wave Ae^{ikx} with energy $E > 0$ passes a potential which drops down to $U = -U_0$. (Notice that the constant U_0 is positive here.)



(a) Find the reflection and transmission coefficients as a function of $\mathcal{E} = \frac{E}{U_0}$.

(b) Graph R and T as a function of $\mathcal{E}$, and on the same graph the result of the step up,

$$R = \left(\frac{\sqrt{\mathcal{E}} - \sqrt{\mathcal{E}-1}}{\sqrt{\mathcal{E}} + \sqrt{\mathcal{E}-1}} \right)^2 \quad \text{and} \quad T = \frac{4\sqrt{\mathcal{E}(\mathcal{E}-1)}}{(\sqrt{\mathcal{E}} + \sqrt{\mathcal{E}-1})^2}$$

(c) Find the reflection probability for a 5 eV electron encountering a step in which the potential drops by 2 eV.

▷ **2.**

A beam of particles of energy E and incident upon a potential step of $U_0 = \frac{5}{4}E$ is described by a wave-function

$$\psi_{inc} = 1e^{ikx}$$

(a) Determine completely the reflected wave, and the wave inside the step, by enforcing the required continuity conditions to obtain their (possibly complex) amplitudes.

(b) Verify by explicit calculation that $R = 1$.

▷ **3.**

It is shown in Section 6.1 that for the $E < U_0$ potential step,

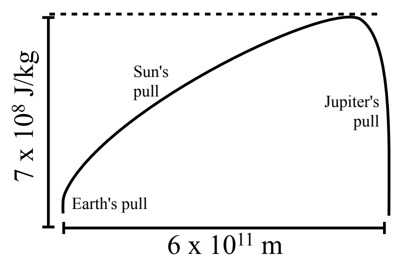
$$B = -\frac{\alpha + ik}{\alpha - ik} A$$

(a) Use this to calculate the probability density to the left of the step $P_L(x) = |\psi_L(x)|^2$. You should get a typical standing wave pattern.

(b) What is the largest value that $P_L(x)$ takes?

▷ **4.**

The gravitational potential energy of a 1 kg object is plotted versus position from Earth's surface to the surface of Jupiter. Mostly it is due to the Sun, but there are downturns at each end due to the attractions to the two planets. Make the crude approximation that this is a rectangular barrier with the same average height (you can estimate this by eye). If a 65 kg person jumps upwards at 4 m/s on Earth, what is the probability that they will tunnel to Jupiter? (See equation 6.18).

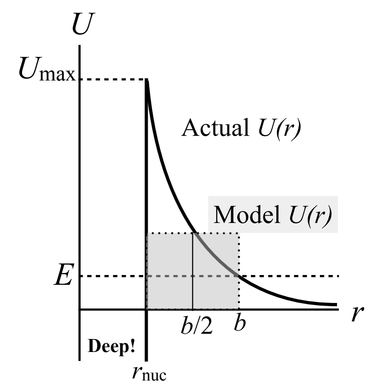


▷ **5.**

For wavelengths less than about 1 cm, the dispersion relation for waves on the surface of water is $\omega = \sqrt{(\gamma/\rho)k^3}$, where γ and ρ are the surface tension and density of water. Given $\gamma = 0.072\,\mathrm{N/m}$ and $\rho = 10^3\,\mathrm{kg/m^3}$, calculate the phase and group velocities for a wave of 5 mm wavelength.

▷ **6.**

Fusion in the Sun: Without tunneling, our Sun would fail us. The source of its energy is nuclear fusion, and a crucial step is the fusion of a hydrogen nucleus (a proton) and a deuterium nucleus (a protons and a nucleus). When these nuclei get close enough, their short-range attraction via the strong force overcomes their Coulomb repulsion. This allows them to stick together, resulting in a reduced total mass/internal energy and a consequent release of kinetic energy. However, the Sun's temperature is simply too low to ensure that nuclei move fast enough to overcome their repulsion.



(a) By equating the average thermal kinetic energy that the nuclei would have when distant, $\frac{3}{2}k_BT$, and the Coulomb potential energy they would have when 2 fm apart, roughly the separation at which they stick, show that a temperature of about a billion Kelvin would be needed.

(b) The Sun's core is only about 10^7 K. If nuclei can't make it "over the top", they must tunnel. Consider the following model, illustrated in the figure: One nucleus is fixed at the origin, while the other approaches from far away with energy E. As r decreases, the Coulomb potential energy increases, until the separation r is roughly the nuclear radius r_{nuc}, whereupon the potential energy is $U_{\max}$ and then quickly drops down a very deep "hole" as the strong-force attraction takes over. Given that $E \ll U_{\max}$, the point b where tunneling must begin will be very large compared with r_{nuc}, so we approximate the barrier's width L as simply b. Its height U_0, we approximate by the Coulomb potential evaluated at $b/2$. Finally, let the energy be $E = 4(\frac{3}{2}k_BT)$ which is a reasonable limit, given the natural range of speeds in a thermodynamic system. Combining these approximations, show that the exponential factor in the wide-barrier tunneling probability (see 6.18) is

$$\exp\left[\frac{-e^2}{(4\pi\epsilon_0)\hbar}\sqrt{\frac{4m}{3k_BT}}\right]$$

(The e above is the proton charge, not 2.781828.)

(c) Using the proton mass for m, evaluate this factor for a temperature of 10^7 K. Then evaluate it at 3000 K, about that of a hot flame. Discuss the consequences.