Length only contracted along direction of motion, not $\perp$ to it

e.g. $\otimes$ (moving away from us)

~~fall of~~

if train got narrower, would fall off tracks & wheels would be between tracks

train's frame:

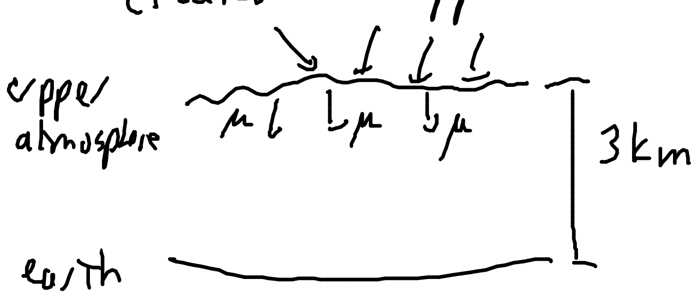
rails are moving fast & would contract, & wheels would be outside the tracks

impossible!

$\therefore$ no $\perp$ contraction - wheels stay on the tracks

Muons

created in upper atmosphere by cosmic rays



$\mu^- \to e^- + \bar{\nu} + \nu$

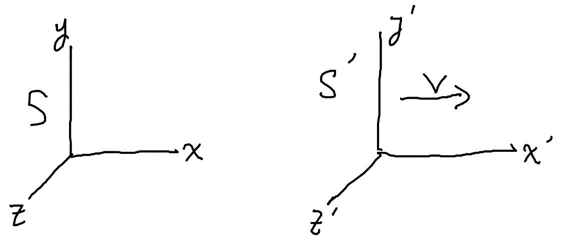
average life span of $2.2\mu s$

Would need a speed $\frac{3km}{2.2\mu s} \approx 1.5 \frac{10^3}{10^{-6}} = 1.5 \times 10^9 \, m/s = 5c$

so muons shouldn't be able to reach surface — but they do!

Muon frame: distance to ground contracts & they have enough time to get there

Earth's frame: muons are time-dilated & last longer than $2.2\mu s$



When origins of S & S' coincide, $t=0$.

Classical physics (Galilean relativity)

if event in S' has coordinates (x', y', z', t') what are coordinates in S?

$t = t' \quad y = y' \quad z = z' \quad x = x' + vt'$

$x' = x - vt$

Add relativity

. Suppose an object moves at constant velocity to the right in S. u

$x = ut \cancel{+x_0} \qquad x' = u't'$

We want a relationship between (x,t) & (x',t') such that, if $\frac{x}{t}$ is constant, $\frac{x'}{t'}$ is constant.

This requires that

$x' = Ax + Bt \qquad t' = Cx + Dt$

1) if object is fixed at origin in S'

$x' = 0, \quad u = v, \; u' = 0, \; x = vt$

$0 = Avt + Bt \rightarrow B = -Av$

2) If object is fixed at S's origin

$x = 0 \quad u' = -v \quad x' = -vt'$

$D = A$

3) if object is light,

$u = u' = c$

$x' = ct' \quad \& \quad x = ct$

$C = -A\frac{v}{c^2}$

$x' = A(x - vt) \quad t' = A(-\frac{v}{c^2}x + t)$

$c=1$

$x' = A(x - vt) \quad t' = A(t - vx)$

solve for x $\quad x = A(x' + vt')$ (common sense, switch ' to other frame)

$x = \frac{1}{A(1-v^2)}(x' + vt')$

$\Rightarrow A = \frac{1}{\sqrt{1-v^2}} \equiv \gamma_v \quad (c=1)$

$\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$

$x' = \gamma(x - vt)$

$t' = \gamma(t - \frac{v}{c^2}x)$

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

$$\Delta x' = x_f' - x_i' = \gamma(x_f - vt_f) - \gamma(x_i - vt_i)$$
$$= \gamma\left[(x_f - x_i) - v(t_f - t_i)\right]$$
$$= \gamma\left[\Delta x - v\Delta t\right]$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \geq 1$$

$\gamma = 1$ $\quad v = 0$

$\gamma \approx 1$ $\quad$ nonrelativistic speeds

$\gamma \to \infty$ as $v \to c$

Lorentz transformations

time dilation

- two events at same location in S'

$$\Delta x' = 0$$

$$\Delta t = \gamma(\Delta t' + v\cancel{\Delta x'}) \Rightarrow \Delta t = \gamma \Delta t' \geq \Delta t'$$

length contraction

observe 2 points on object at same time in S

$$\Delta t = 0$$

$$\Delta x' = \gamma(\Delta x - \cancel{v\Delta t}_{0}) \qquad \Delta x' = \gamma \Delta x$$

$$\Delta x = \frac{1}{\gamma}\Delta x' \leq \Delta x'$$