If N particles, each can be in state $A, B, C, D, \ldots$

"state of all particles combined" – if particles are indistinguishable – is given by indicating how many particles are in each state.

occupancy n

bosons

$$\bar{n}(E) = \frac{1}{e^{(E-\mu)/kT}-1} \quad \text{Bose-Einstein distribution}$$

μ: chemical potential
(we'll talk about this soon)

fermions

$$\bar{n}(E) = \frac{1}{e^{(E-\mu)/kT}+1} \quad \text{Fermi-Dirac distribution}$$

• $E \gg \mu$: $e^{(E-\mu)/kT} \gg 1$

both $\bar{n}(E) \approx \frac{1}{e^{(E-\mu)/kT}}$ = tiny — high-energy states mostly empty

$$= e^{\mu/kT} e^{-E/kT} = \frac{N}{Z} e^{-E/kT}$$

Boltzmann distribution

$$e^{\mu/kT} = \frac{N}{Z}$$

$E \ll \mu$ $\quad e^{(E-\mu)/kT}$ tiny

$$\bar{n}_{BE}(E) = \frac{1}{e^{(E-\mu)/kT}-1} \approx -1 \quad \text{HUH???}$$

$$\bar{n}_{FD}(E) = \frac{1}{e^{(E-\mu)/kT}+1} \approx 1$$

guaranteed to be occupied by 1 fermion

$E = \mu$ $\quad e^{(E-\mu)/kT} = 1$

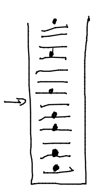
$$\bar{n}_{FD}(E) = \frac{1}{1+1} = \frac{1}{2}$$

for Fermions, μ: energy of a state occupied half the time

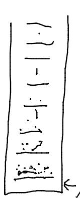
$$\bar{n}_{BE}(E) = \frac{1}{1-1} = \infty \quad \text{impossible!}$$

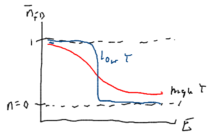
$E > \mu$

Fermions

μ 

Bosons

 μ



Harmonic Oscillator

at $T=0$

maximum energy level filled when $T=0$: Fermi energy E_F

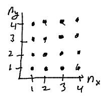
$\mu \gtrsim E_F$

Another fermion situation:

3D ∞ square well

$$E_{n_x,n_y,n_z} = (n_x^2+n_y^2+n_z^2)\frac{\pi^2\hbar^2}{2mL^2}$$

$$\vec{n} = n_x\hat{x}+n_y\hat{y}+n_z\hat{z} \qquad E = |\vec{n}|^2\frac{\pi^2\hbar^2}{2mL^2}$$



Fermions fill states one at a time, always picking lowest energy

we will get an $\frac{1}{8}$th sphere

$$\frac{1}{8}\,\frac{4}{3}\pi|\vec{n}|^3 = \frac{1}{6}\pi|\vec{n}|^3 = N$$

radius of sphere

$$\frac{2mL^2}{\pi^2\hbar^2}E_F = |n_{max}|^2$$

$$|n_{max}|^3 = (2mE_F)^{3/2}\left(\frac{L^3}{\pi^3\hbar^3}\right) \leftarrow V \text{ volume}$$

$$N = \frac{1}{6}\cancel{\pi}(2mE_F)^{3/2}\left(\frac{V}{\pi^{\cancel{3}2}\hbar^3}\right)$$

$$E_F = \frac{\pi^2\hbar^2}{m}\left(\frac{3}{\pi\sqrt{2}}\,\frac{N}{V}\right)^{2/3}$$

if $k_BT >> E_F$ then

fermion nature can be ignored

if $k_BT << E_F$ quantum effects

e.g. conduction electrons

conduction electrons $E_F = 1-10\,eV \qquad kT < \frac{1}{40}eV$

electrons in a metal are confined to a potential made up of all positive ions

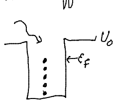
1 ion

bunch of ions

$\approx$

finite square well (sort of)

Photoelectric effect



Top Electron needs energy

$\phi = U_0 - \epsilon_F$ to be kicked out

work function