Any complex number c

$$c = a + ib$$

$$c = |c| e^{i\phi}$$

$$x + iy$$
$$\overline{x - iy}$$

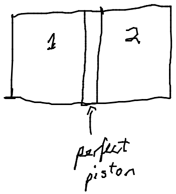
$$e^{i\phi} = \cos\phi + i\sin\phi$$

$$c = |c|(\cos\phi + i\sin\phi)$$

$$a = |c|\cos\phi$$

$$b = |c|\sin\phi$$

$$\tan^{-1}\frac{b}{a} = \phi$$



$$dV_1 = -dV_2$$

$$dS = dS_1 + dS_2$$

$$= \left(\frac{\partial S_1}{\partial V_1}\right) dV_1 + \left(\frac{\partial S_2}{\partial V_2}\right) dV_2$$

$$dS = \left(\frac{\partial S_1}{\partial V_1} - \frac{\partial S_2}{\partial V_2}\right) dV_1 > 0$$

if $dV_1 > 0$, then $\frac{\partial S_1}{\partial V_1} > \frac{\partial S_2}{\partial V_2}$

as system approaches equilibrium

$$P \equiv T\left(\frac{\partial S}{\partial V}\right)_U$$ pressure

If both change

$$dS = \left(\frac{\partial S}{\partial E}\right)_V dE + \left(\frac{\partial S}{\partial V}\right)_E dV$$

$$= \frac{1}{T} dE + \frac{P}{T} dV$$

$$dE = T\,dS - P\,dV + \mu\,dN$$

thermodynamic identity

$$\left(\frac{\partial S}{\partial E}\right)_{V,N}$$ $dV = 0$, $dN = 0$ $\quad dE = T\,dS \rightarrow \frac{dS}{dE} = \frac{1}{T}$

chemical potential

$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{E,V}$$

& $dE = dV = 0$

$$0 = T\,dS + \mu\,dN$$

particles flow from small to big

so " " from high μ to low μ

$\mu < 0$ almost always

$$\mu = -30\,eV \xleftarrow[\text{flow}]{\text{particles}} \mu = -20\,eV$$

$E_n = (n+\frac{1}{2})\hbar\omega_0 \qquad n = 0, 1, 2, \ldots$

What is prob. that in ground state $n=0$?

- if all states are equally likely
  $P = \frac{1}{\#\text{ states}} = \frac{1}{\infty} = 0$
- if energy is conserved (isolated)
  $P = 1$ if already in g.s.
  $P = 0$ if it has more energy
- if in contact with a thermal reservoir
  i.e. object with constant T
  and ∞ heat capacity
  (T stays constant even if heat flows in or out)

Suppose universe is oscillator & reservoir

$$\Omega_U = \Omega_0 \Omega_R$$

Consider case where oscillator is in ground state or any other state

$$\Omega_0 = 1$$

Consider two energy states of H.O. A & B

$$E(A) < E(B)$$

$$\rightarrow E_R(A) > E_R(B)$$

$$\rightarrow \Omega_R(A) > \Omega_R(B)$$

$S = k \ln \Omega$

$\Omega = e^{S/k}$

$$\rightarrow \Omega_U(A) > \Omega_U(B)$$

$$P(A) > P(B)$$

$$\frac{P(A)}{P(B)} = \frac{\Omega_R(A)}{\Omega_R(B)} = \frac{e^{S_R(A)/k}}{e^{S_R(B)/k}} = e^{\overbrace{[S_R(A) - S_R(B)]}^{\Delta S_R}/k}$$

$$dS_R = \frac{1}{T} dE_R = -\frac{1}{T} dE_0$$

$$\frac{P(A)}{P(B)} = e^{-[E_0(A) - E_0(B)]/kT} = \frac{e^{-E(A)/kT}}{e^{-E(B)/kT}}$$

$$\boxed{P(A) = \frac{1}{Z} e^{-E(A)/kT}}$$

for any system in contact with a thermal reservoir

$e^{-E/kT}$ Boltzmann factor

Z partition function

$$1 = \sum_s P(s) = \frac{1}{Z} \sum_s e^{-E_s/kT} \rightarrow Z = \sum_s e^{-E_s/kT}$$

if energy increases by kT, probability drops by e^{-1} 37%

kT: sets energy scale

at 300 K $\quad kT = \frac{1}{40}$ eV