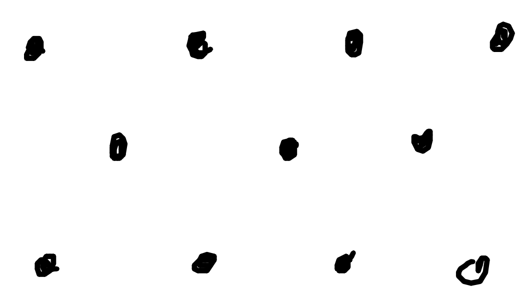
1927 Davisson & Germer

scattered electrons through a crystal of nickel

electrons preferred certain specific angles to emerge (reflected)

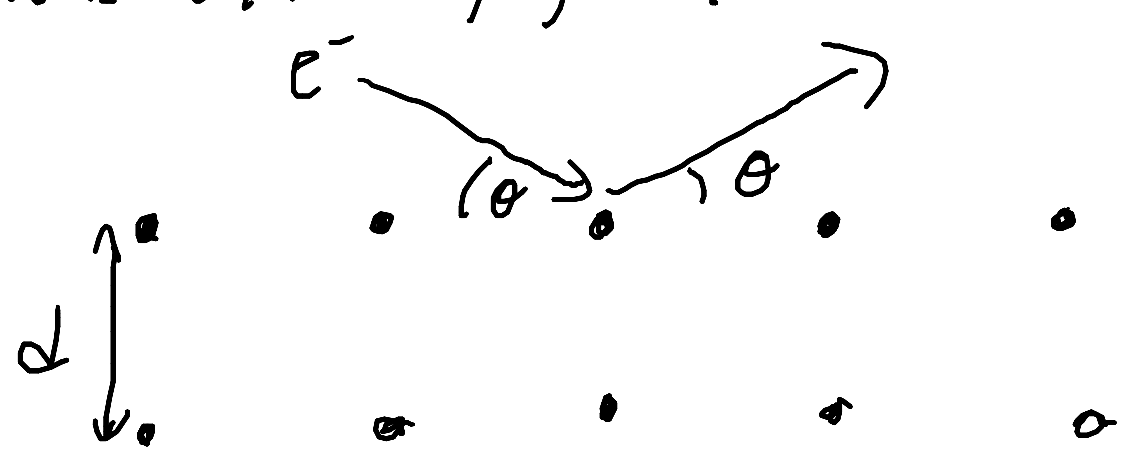
interference

Crystal



Each atom reflects electron wave in all directions, acting like a point source – interference just as with a diffraction grating

Bragg Law determines the angles that the electrons will emerge from.



Constructive interference occurs when

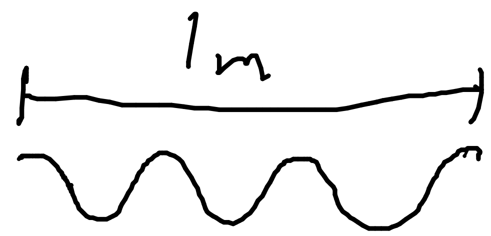
$$2d \sin\theta = n\lambda \quad n \in \mathbb{Z} \leftarrow \text{integers}$$

$\lambda = \frac{h}{p}$ & $f = \frac{E}{h}$ for light and particles alike

Define wavenumber $k \equiv \frac{2\pi}{\lambda}$

λ : length/crest

$1/\lambda$: crests/length



$1/\lambda = 3/m$

$\frac{1}{\lambda} \frac{\text{crests}}{\text{length}} \times \frac{2\pi \text{ radians}}{\text{crest}}$

$\therefore k = \frac{2\pi}{\lambda} \frac{\text{radians}}{\text{length}}$

$k : \frac{\text{radians}}{\text{length}}$

angular velocity $\omega = 2\pi f$

$\omega : \frac{\text{radians}}{\text{time}}$

$p = \frac{hk}{2\pi}$ $E = \frac{h\omega}{2\pi}$

$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ J/Hz

"h-bar"

$v = f\lambda = \frac{E}{h}\frac{h}{p} = \frac{E}{p}$

For light, $E = pc$, so $\frac{E}{p} = c$ ✓

if $E = \frac{1}{2}mv^2$ & $p = mv$ $\frac{E}{p} = \frac{\frac{1}{2}mv^2}{mv} = \frac{1}{2}v$.

phase velocity
velocity of wave

group velocity
velocity of particle

For a wave on a string,

$y(x,t)$ is displacement of string at (x,t)

$$v^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2} \rightarrow y = A \sin(kx - \omega t)$$

$$\frac{\omega}{k} = v$$

Matter waves $\Psi(x,t)$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial t^2} = i\hbar \frac{\partial \Psi}{\partial t}$$

if no external forces

Schrödinger's Equation

Ψ is complex!

$$\Psi = \Psi_R + i\,\Psi_I$$

has 2 parts just like EM waves

Remember $|\overbrace{a+ib}^{\Psi}|^2 = (a+ib)^*(a+ib)$

$|\Psi|^2 = \Psi^* \Psi$

$$= (a-ib)(a+ib)$$

$$= (a^2 \cancel{+iab} \cancel{-iab} - i^2 b^2)$$

$$= a^2 + b^2$$

real, non-negative

We can observe $|\Psi|^2$, but not Ψ

Solving the Schrodinger equation above

$$\Psi(x,t) = A e^{i(kx - \omega t)}$$

where $\frac{\hbar^2 k^2}{2m} = \hbar\omega$

$\hbar k = \frac{hk}{2\pi} = p$

$\hbar \omega = E$

$\frac{p^2}{2m} = E$

$\frac{p^2}{2m} = \frac{(mv)^2}{2m} = \frac{1}{2}mv^2$