$p = \gamma_u m u = \dfrac{mu}{\sqrt{1-u^2}}$ $\quad c=1$

total energy $\quad E = \gamma_u m = \underset{\text{rest energy}}{m} + \underset{\text{kinetic energy}}{(\gamma_u - 1) m}$

for massless particles,

$E = p$

Collision

$9m_0$ ① $\xrightarrow{0.8c}$ $\quad \xleftarrow{0.6c}$ ② $12m_0$ $\qquad m_0$: unit mass

hit & stick together

Find mass & velocity of final object.

Classical: $21m_0$ $\quad 0$

$\gamma_{0.8} = \frac{5}{3}$

$\gamma_{0.6} = \frac{5}{4}$

Momentum conserved: $\quad \gamma_{u_1} m_1 u_1 + \gamma_{u_2} m_2 u_2 = \gamma_{u_f} m_f u_f$

$\frac{5}{3}(9m_0)(0.8) - \frac{5}{4}(12m_0)(0.6) = \gamma_{u_f} m u_f$

$12m_0 - 9m_0$

$3m_0 = \underline{\gamma_{u_f} m_f} u_f$

Total Energy Conserved (!) : $\quad \gamma_{u_1} m_1 + \gamma_{u_2} m_2 = \gamma_{u_f} m_f$

$\frac{5}{3}(9m_0) + \frac{5}{4}(12m_0)$

$15m_0 + 15m_0$

$30m_0 = \underline{\gamma_{u_f} m_f}$

$3m_0 = (30m_0) u_f \rightarrow u_f = 0.1c$

$\gamma_{u_f} = \dfrac{1}{\sqrt{1-u_f^2}} = \dfrac{1}{\sqrt{1-0.1^2}} = 1.005$

$\therefore m_f = \dfrac{30m_0}{1.005} = 29.9m_0 > 21m_0$

$\Delta KE = -8.9m_0$ $\qquad KE = (\gamma - 1)m$

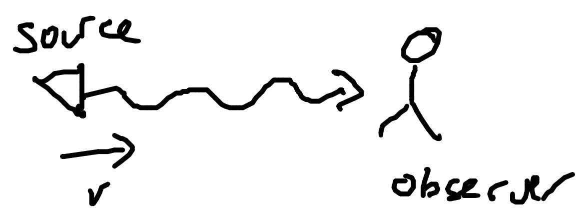
When particles collide inelastically,

KE decreases & mass increases

this is what particle accelerators do

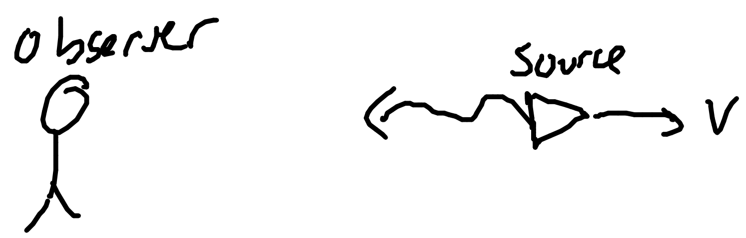
to create massive particles. (e.g. Higgs)

Doppler Effect for Light



$$f' = f \frac{v}{v - v_s}$$

For light, math is a little different



Let Δt_p be time between wavefronts produced by source, as seen in observer's frame

Let Δt_a be time between wavefronts arriving to the observer, in her frame.

$$\Delta t_a = \Delta t_p + \frac{v \Delta t_p}{c}$$

second wavefront has to travel additional distance $v \Delta t_p$

period of produced light is $T = \Delta t_p'$ in source's frame.

$$\Delta t_p = \gamma \Delta t_p' = \gamma T.$$

$c=1$

$$\Delta t_a = \Delta t_p (1+v) = \Delta t_p' \gamma (1+v)$$

$$\Delta t_a = \frac{1+v}{\sqrt{1-v^2}} T$$