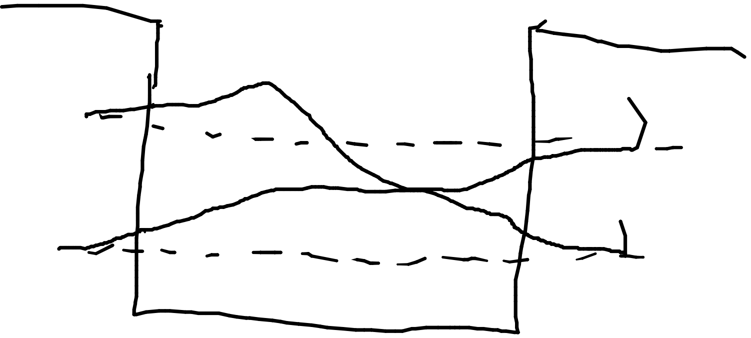
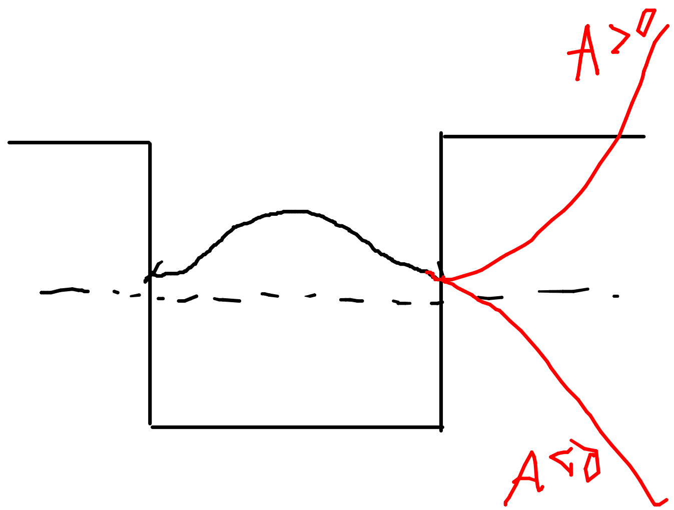
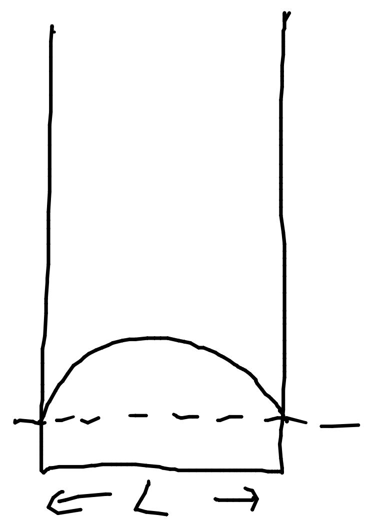




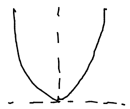
$$\underbrace{A}_{\text{needs to be zero to have a normalizable solution}} e^{\alpha x} + B e^{-\alpha x}$$

lower energy than

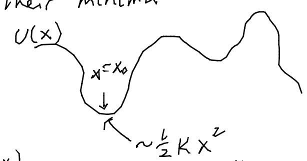


Harmonic Oscillator

$$U(x) = \frac{1}{2} K x^2$$



Most potential functions are like H.O. close to their minima

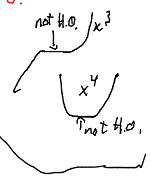


$$U(x) = \underbrace{U(x_0)}_{\text{constant}} + (x-x_0)\underbrace{U'(x_0)}_{\text{0 at min.}} + \underbrace{\frac{1}{2}(x-x_0)^2 U''(x_0)}_{\text{H.O.}} + \dots$$

This

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}Kx^2\psi = E\psi$$

can be solved explicitly.



Energies $E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0 \quad n = 0, 1, 2, 3, \dots$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

evenly spaced

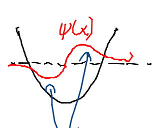
$$\psi_0(x) \propto e^{-\frac{1}{2}b^2x^2} \qquad b = \left(\frac{mK}{\hbar}\right)^{1/4}$$

$$\psi_1(x) \propto 2bx\, e^{-\frac{1}{2}b^2x^2}$$

$$\psi_2(x) \propto \left(4b^2x^2 - 2\right) e^{-\frac{1}{2}b^2x^2}$$

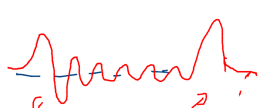
$$\psi_n(x) \propto H_n(bx)\, e^{-\frac{1}{2}b^2x^2}$$

Hermite polynomials



Classically, expect to find particle at extremes of its motion

most likely positions quantum mechanically



at higher energies, most likely positions move towards edges

Expectation value of a property of a wavefunction

- average value

e.g. 1d6 $\underset{\text{probability}}{\frac{1}{6}}\underset{\text{outcome}}{(1)} + \frac{1}{6}(2) + \frac{1}{6}(3) \ldots$

avg value is 3.5

$$\bar{v} = \langle v \rangle = \sum_i p_i v_i \quad \text{or} \quad \int p(x)\, v(x)\, dx$$

For wavefunctions,

$$\langle x \rangle = \int |\Psi(x)|^2\, x\, dx$$

not a time average

- Quantum objects don't have well-defined positions unless you measure them which collapses Ψ.

Ensemble average

- create an ensemble (group) of objects with same Ψ, & measure x of each

$\langle x \rangle$ is average result of these measurements!