Eigenfunction

$$\hat{Q}\,\psi(x) = \lambda\psi(x)$$

e.g. $\hat{p} = -i\hbar\frac{\partial}{\partial x}$

$$\hat{p}\,Ae^{ikx} = -i\hbar A\frac{\partial}{\partial x}e^{ikx}$$
$$= (-i\hbar)(ik)\,Ae^{ikx}$$
$$= \hbar k\, Ae^{ikx}$$

$\psi(x) = Ae^{ikx}$

$$\hat{p}\,\psi(x) = \hbar k\,\psi(x)$$

$\hbar k = p$ is eigenvalue associated with e^{ikx} & $\hat{p}$.

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if ψ is eigenfunction of $\hat{Q}$ with eigenvalue λ.

$$\langle Q\rangle = \int \psi^*\hat{Q}\,\psi\,dx$$
$$= \int \psi^*\lambda\psi\,dx$$
$$= \lambda\int\psi^*\psi\,dx$$
$$= \lambda$$

$\hat{Q}$ is linear operator
$\hat{Q}(c\psi) = c\hat{Q}\psi$
$\hat{Q}(\psi+\phi) = \hat{Q}\psi + \hat{Q}\phi$

$$\langle Q^2\rangle = \int \psi^*\hat{Q}\hat{Q}\psi\,dx$$
$$= \lambda\int\psi^*\hat{Q}\psi\,dx$$
$$= \lambda^2\int\psi^*\psi\,dx$$
$$= \lambda^2$$

$\hat{Q}\hat{Q}\psi$
$= \hat{Q}(\lambda\psi)$
$= \lambda\hat{Q}\psi$
$= \lambda^2\psi$

$$\Delta Q = \sqrt{\langle Q^2\rangle - \langle Q\rangle^2} = \sqrt{\lambda^2-\lambda^2} = 0$$

$\psi(x)$ has a definite value of Q.

Eigenfunction of position operator?

$$x\,\psi(x) = \lambda\psi(x)$$

$$x\,\delta(x-x_0) = \begin{cases} 0 & \text{if } x\neq x_0 \\ \infty & \text{if } x = x_0\end{cases}$$

$$x_0\,\delta(x-x_0) = \begin{cases} 0 & \text{if } x\neq x_0 \\ \infty & \text{if } x = x_0\end{cases}$$

$\delta(x-x_0)$ is eigenf. of $\hat{X}$ w/ eigenvalue x_0.

Stationary State

$$\Psi(x,t) = \psi(x)\,e^{-i\omega t}$$

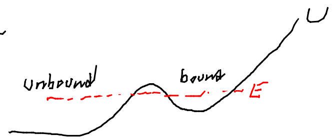
$$\hat{E}\Psi = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$
$$= i\hbar\,\psi(x)\,(-i\omega e^{-i\omega t})$$
$$= \hbar\omega\,\Psi(x,t)$$

$$\hat{E}\Psi = E\Psi$$

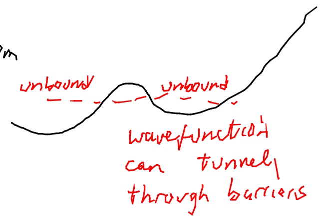
Ch 6: Unbound States
(Scattering States)

not restricted to a particular region

Classical



Quantum



unbound: $E > U(\infty)$ or $U(-\infty)$

bound: $E < U(\infty)$ & $U(-\infty)$

unbound: energy <u>not quantized</u>

e.g. Free particle

$$\Psi(x,t) = Ae^{ikx}e^{-i\omega t}$$

if I follow a crest as time increases,

$\Psi(x,t)$ stays same at crest

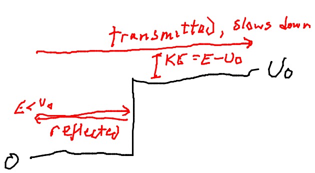
as t increases, x increases

$f(x-vt)$ describes wave moving to the right

$\therefore$ $Ae^{ikx}e^{-i\omega t}$ moves to the right

$Ae^{-ikx}e^{-i\omega t}$ moves to left

$$\boxed{\begin{array}{ll} \psi(x) = Ae^{ikx} & \text{wave moving to right} \\ \quad\quad\;\; Ae^{-ikx} & \text{wave moving to left} \end{array}}$$



$$U(x) = \begin{cases} 0, & x < 0 \\ U_0, & x > 0 \end{cases}$$

$F(x) = -\frac{dU}{dx}$

incident Ae^{ikx} $\longrightarrow$ $\quad$ $Ce^{ik'x}$ $\longrightarrow$ transmitted

$\longleftarrow$ Be^{-ikx} reflected

$\psi_L = Ae^{ikx} + Be^{-ikx}$ $\quad\quad$ $\psi_R = Ce^{ik'x}$