Physics 370 Homework #1

5 problems

Solutions

▷ **1.**

You are gliding over Earth's surface at a high speed, carrying your high-precision clock C. At points X and Y on the ground are similar clocks, synchronized in the ground frame of reference. As you pass over clock X, it and your clock both read 0.

(a) According to you, do clocks X and Y advance slower or faster than clock C?

(b) When you pass over clock Y, does it read the same time, an earlier time, or a later time than C, according to you?

(c) Answer the same question (b) from the point of view of an observer on the ground.

(d) Reconcile any seeming contradictions between your answers to parts (a) and (b).

Answer:

(a) X and Y are moving relative to you, so they will advance more **slowly** than clock C, which is in your frame.

(b) When you pass over clock Y, the time it reads is what a ground observer would see. To a ground observer, clock C is running more slowly than clock Y, so clock Y will read a **later** time than clock C.

(c) The lining up of clocks C and Y is an event, a single point in spacetime. (Granted, they have different values of z, but as the z axis is perpendicular to the motion it does not matter.) At that specific event, any observer—ground, rocket, or someone else entirely–would see clock Y reading a later time than clock C.

(d) So how can clock Y show a later time, when it appears to be running more slowly than clock C? Because when I pass clock X, clock Y does not show $t = 0$ but some later time. (X and Y are synchronized in the ground frame, but they are not synchronized in mine.) For example, when I pass clock X, clock C will read $0\,\mathrm{s}$ but clock Y might read $3\,\mathrm{s}$. Then, when I pass clock Y, it could be that clock C reads $4\,\mathrm{s}$ but clock Y reads $5\,\mathrm{s}$: clock Y shows a later time, but it has only advanced $2\,\mathrm{s}$ when my clock has advanced $4\,\mathrm{s}$.

(This was an awful puzzler to start with!)

▷ **2.**

Through a window in Carl's spaceship, passing at $v = 0.5c$, you watch Carl doing an important physics calculation. By your watch it takes him 1 minute. How much time did Carl spend on his calculation?

Answer:____________________________________

Carl is moving relative to you, so he appears to be moving more slowly. Let $\Delta t = 1\,\mathrm{min}$ be the time it takes from your perspective, and $\Delta t'$ be the time it takes from Carl's; then $\Delta t = \gamma \Delta t'$, or $\Delta t' = \frac{1}{\gamma}\Delta t$. Now $\gamma = (1-v^2)^{-1/2}$, so

$$\Delta t' = \sqrt{1-v^2}\Delta t = \sqrt{1-(0.5)^2}(1\,\mathrm{min}) = 0.87\,\mathrm{min}$$

or $52\,\mathrm{s}$. (I used the $c=1$ units, so that $v = 0.5$.)

▷ **3.**
According to an observer on Earth, a spacecraft whizzing by at $0.6c$ is $35\,\mathrm{m}$ long. What is the length of the spacecraft according to passengers on board?

Answer:____________________________________
In this case,

$$\gamma = \frac{1}{\sqrt{1-(0.6)^2}} = 1.25$$

The spacecraft looks contracted to the observer, and so its length in the passengers' frame must be longer, specifically

$$L = (1.25)(35\,\mathrm{m}) = 44\,\mathrm{m}$$

▷ **4.**
The variable γ_v is a reasonable measure of the size of relativistic effects. Roughly speaking, at what speed would observations deviate from classical expectations by 1%?

Answer:____________________________________
It's useful to calculate

$$\frac{1}{\sqrt{1-v^2}} = \gamma \implies \sqrt{1-v^2} = \frac{1}{\gamma} \implies 1 - v^2 = \frac{1}{\gamma^2}$$
$$\implies v = \sqrt{1-\frac{1}{\gamma^2}}$$

The relativistic effects will deviate by about 1% when $\gamma = 1.01$, or when

$$v = \sqrt{1-\frac{1}{(1.01)^2}} = 0.14$$

or, in SI units, $v = 0.14c = 4 \times 10^7\,\mathrm{m/s}$. In other words, even a rocket travelling at 10% the speed of light will show a very small relativistic effect.

▷ **5.**

A pole-vaulter holds a 16 ft pole. A barn has doors at both ends, 10 ft apart. The pole-vaulter on the outside of the barn begins running toward one of the open barn doors, holding the pole level in the direction he's running. When passing through the barn, the pole fits (barely) entirely within the barn all at once.
(a) How fast is the pole-vaulter running?
(b) According to whom—the pole-vaulter or an observer stationary in the barn—does the pole fit in all at once?
(c) According to the other person, which occurs first: the front end of the pole leaving the barn or the back end entering, and
(d) what is the time interval between these two events?

Answer:

(a) If the pole barely fits inside the barn, that means the pole has become contracted to $10\,\mathrm{ft}$. Because $\gamma \geq 1$, we must have

$$16 = \gamma(10) \implies \gamma = 1.6 \implies v = \sqrt{1 - \frac{1}{\gamma^2}} = 0.78$$

so the pole-vaulter is running at $0.78\,\mathrm{c}$.

(b) The pole only contracts from the barn's point of view, so only the stationary observer will see the pole fitting in the barn.

(c) The vaulter will see the barn as much shorter than the pole, so the front end of the pole will leave the barn before the back end enters.

(d) Let S be the stationary frame and S' the vaulter's frame. Let x_1 and x_2 be the positions of the doors of the barn, and x_B and x_F are the positions of the ends of the pole, as shown. Let t_L be the time the front end of the pole leaves the barn, and t_E be the time the back end of the pole enters the barn; all these quantities are in the barn's frame unless primed. We would like to know what $t'_E - t'_L$ is. We know that $t_E - t_L = 0$ because the pole just barely fits in the barn in S, so the two events are simultaneous in that frame. The formula is

$$(t'_E - t'_L) = \gamma[(t_E - t_L) - \frac{v}{c^2}(x_E - x_L)]$$

In this case, x_E is the position where the back end of the pole enters the barn, which is x_1, and x_L is the position where the front end of the pole leaves the barn, which is x_2. The difference

$x_2 - x_1$ is the length of the barn, which in meters is

$$x_2 - x_1 = 16\,\mathrm{ft} \times \frac{0.3048\,\mathrm{m}}{1\,\mathrm{ft}} = 4.9\,\mathrm{m}$$

so

$$t'_E - t'_L = \gamma \frac{v}{c^2}(x_2 - x_1) = (1.6)\left(\frac{0.78c}{c^2}\right)(4.9\,\mathrm{m}) = 1.25 \frac{4.9\,\mathrm{m}}{3 \times 10^8\,\mathrm{m/s}} = 2 \times 10^{-8}\,\mathrm{s}$$

or $\boxed{20\,\mathrm{ns}}$.