Coordinate time Δt is in a particular inertial frame

Proper time $\Delta\tau$ is frame-independent and depends on particular clock

Spacetime interval Δs is unique

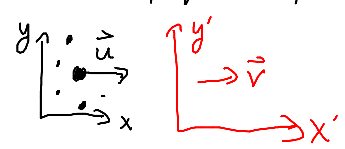
- proper time of an object that moves between events inertially.

$$\Delta s^2 = \Delta t^2 - \Delta x^2$$

Momentum

$$\vec{F}_{\substack{net\\ external}} = \frac{d\vec{p}_{tot}}{dt}$$

In classical physics, $\vec{p} = m\vec{u}$



$$\vec{P}'_{total} = \sum_i m_i \vec{u}_i'$$
$$= \sum_i m_i (\vec{u}_i - \vec{v})$$
$$= \sum_i m_i \vec{u}_i - \sum_i m_i \vec{v}$$
$$= \vec{P}_{total} - \vec{v} M$$

if M is conserved, If $\vec{P}_{total}$ is const, so is $\vec{P}'_{total}$

That's not how velocities add in relativity!

In relativity,

$$p = \gamma_u m u \quad \text{(in 1D)}$$

you can prove that $\frac{dp'}{dt} = \frac{dp}{dt}$ in any two frames (HW)

$$p = \frac{1}{\sqrt{1-u^2}} m u$$

for momentum to be conserved,

$\sum_i \gamma_{u_i} m_i$ must be conserved too.

total energy of system

SI units: $E = \gamma_u m c^2$

includes thermal energy, chemical bonding energy, etc

$\rightarrow$ heat an object up, its mass increases

if object is at rest, $\gamma_u = 1$

rest energy $E = mc^2$

$9 \times 10^{16}\,J = (1\,kg)(3 \times 10^8\, m/s)^2$

so ordinary thermal energy increases result in undetectable mass changes.

mass is not conserved

for particles with no internal structure, mass is invariant.

Kinetic energy = total energy − rest energy

$$KE = (\gamma_u - 1) m c^2$$

$p = \gamma m u$ & $E = \gamma m c^2$ (SI)

$p = \gamma m u$ $\quad E = \gamma m$ ($c = 1$ units)

$u \rightarrow 1$ or $u \rightarrow c$

$\Rightarrow \gamma \rightarrow \infty$

$\gamma_u = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$

$u \ll 1$ $\quad \simeq 1 + \frac{1}{2}\frac{u^2}{c^2}$

$KE = \frac{1}{2}\frac{u^2}{c^2} m c^2$

$= \frac{1}{2} m u^2$

To reach speed of light requires ∞ energy & momentum

Aside: Some call γm "relativistic mass"

Aside! some call γm "relativistic mass" which depends on frame

"moving objects become heavier"

most physicists do not use this quantity

$E = \gamma m \qquad p = \gamma m u$

if $m = 0$ then $E = 0$ unless $\gamma = \infty$

massless particles must move at speed of light

$E = pc$ or $E = p$