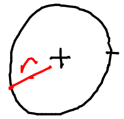
Bohr model of Hydrogen !

- semiclassical explanation

suppose electron is a particle



$$|F| = k\frac{e^2}{r^2} = m\frac{v^2}{r}$$

$$\rightarrow v^2 = \frac{ke^2}{mr}$$

Bohr says: suppose angular momentum is quantized

$$L = n\hbar \qquad n = 1, 2, 3, \ldots$$

$$mvr = n\hbar$$

$$\rightarrow v^2 = \left(\frac{n\hbar}{mr}\right)^2$$

$$\frac{ke^2}{mr} = \left(\frac{n\hbar}{mr}\right)^2 \longrightarrow r = \underbrace{\boxed{\frac{\hbar^2}{kme^2}}}_{\text{Bohr radius}}\overset{}{} n^2 \qquad a_0$$

Bohr radius

$a_0 = 0.053\,\text{nm}$

$\simeq \frac{1}{2}\,\text{Å}$

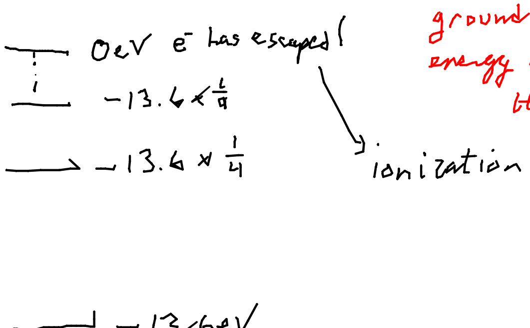
$$r = a_0 n^2$$

$$E = \frac{1}{2}mv^2 - k\frac{e^2}{r}$$

$$= \frac{1}{2}k\frac{e^2}{r} - k\frac{e^2}{r} = -\frac{1}{2}k\frac{e^2}{r}$$

$$E = -\frac{1}{2}k\frac{e^2}{a_0}\frac{1}{n^2} = (-13.6\,\text{eV})\frac{1}{n^2}$$

ground state energy of Hydrogen



$\Delta x \Delta p_x \geq \frac{\hbar}{2}$ $\Delta y \Delta p_y \geq \frac{\hbar}{2}$ $\Delta z \Delta p_z \geq \frac{\hbar}{2}$

but dimensions are independent

$\Delta x \Delta p_y =$ whatever

$\Delta E \Delta t \geq \frac{\hbar}{2}$

- if a state or particle has a finite lifetime, its energy is uncertain
- a state with a definite energy is permanent

"stationary state"

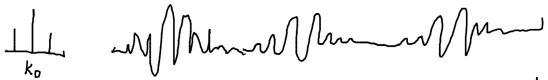
compare

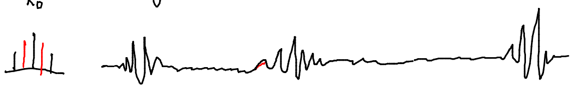
$\Delta \lambda \neq 0$

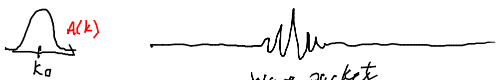
$\Delta p \neq 0$

Single sine wave with wavenumber k_0

$$A \sin k_0 x$$







wave packet

$$\psi(x) = \int_{-\infty}^{\infty} A(k)\, e^{ikx}\, dk$$

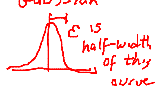
amount of e^{ikx} to add to mix

$$A(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(x)\, e^{-ikx}\, dx$$

Fourier transform of $\psi(x)$

e.g. $$\psi(x) = C e^{-(x/2\varepsilon)^2} e^{ik_0 x}$$

Gaussian — oscillatory

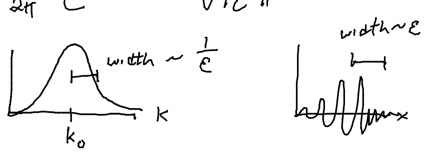


Re $\psi(x)$ =

Gaussian wavepacket

$$A(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{C e^{-(x/2\varepsilon)^2} e^{ik_0 x}}_{\psi(x)} e^{-ikx}\, dx$$

$$= \frac{C}{2\pi} e^{-\varepsilon^2(k-k_0)^2} \sqrt{4\varepsilon^2\pi}$$



$\Delta k \sim \frac{1}{\varepsilon}$ $\Delta x \sim \varepsilon$

$$\Delta x \Delta k \sim 1$$

$\Delta k \sim \Delta p \rightarrow \Delta x \Delta p \sim 1$