Know the wavefunction $\psi(x)$ of object

what is $\langle x^2 \rangle$?

$\langle x \rangle = \int x \, |\psi|^2 \, dx$

$$\langle x^2 \rangle = \int \cancel{x}^2 \, |\psi(x)|^2 \, dx$$

$\int x^2 |\psi|^2 \, dx$

$\int x^2 |\psi|^4 \, dx$

$$(\Delta x)^2 = \langle (x - \bar{x})^2 \rangle$$
$$= \int (x - \bar{x})^2 \, |\psi(x)|^2 \, dx$$
$$= \int |(x - \bar{x}) \psi(x)|^2 \, dx$$

This can only be zero

if $\psi(x) = 0$ everywhere

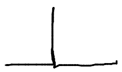
except $x = \bar{x}$

area under $\psi(x)$ is 1 (normalized)

$\therefore \ \psi(\bar{x}) = \infty$

Dirac delta function $\delta(x)$

$$\int_{-\infty}^{\infty} \delta(x) = 1$$



$\delta(x) = 0$ unless $x = 0$,

$$(\Delta x)^2 = \langle (x - \bar{x})^2 \rangle$$
$$= \langle x^2 \rangle - \langle x \rangle^2$$
$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

e.g. $\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$ Ground state ∞ ⊔ well

$$\langle x \rangle = \int_0^L x \, |\psi(x)|^2 \, dx = \frac{2}{L} \int_0^L x \sin^2 \frac{\pi x}{L} \, dx$$
$$= \frac{2}{L} \left(\frac{L^2}{4} \right) = \frac{L}{2}$$

$$\langle x^2 \rangle = \frac{1}{3} L^2 - \frac{1}{2\pi^2} L^2$$

$$\Delta x = \sqrt{\left(\frac{1}{3} L^2 - \frac{1}{2\pi^2} L^2 \right) - \left(\frac{L}{2} \right)^2}$$
$$= L \sqrt{\frac{1}{12} - \frac{1}{2\pi^2}} = 0.18 L$$

Gaussian $\psi(x) = A e^{-[(x-b)/2\sigma]^2}$

$$\langle x \rangle = b$$
$$\Delta x = \sigma$$

Any property of an object that can be measured is an <u>observable</u>

position, momentum, E, L, ...

For observable Q

$$\langle Q \rangle = \int \Psi^*(x,t)\, \hat{Q}\, \Psi(x,t)\, dx$$

$\hat{Q}$: operator of Q

e.g. $\hat{x} = x$ $\qquad \widehat{x^2} = \hat{x}^2 = x^2$

$\widehat{f(x)} = f(\hat{x})$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\langle p \rangle = \int \Psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \Psi \, dx$$

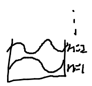
$$= -i\hbar \int \Psi^* \frac{\partial \Psi}{\partial x}\, dx$$

e.g. $\Psi(x) = A \sin \frac{n\pi x}{L}$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$$\langle p \rangle = -A^2 i\hbar \int_0^L \sin\frac{n\pi x}{L} \frac{n\pi}{L} \cos\frac{n\pi x}{L}\, dx$$

$$= -A^2 i\hbar \frac{n\pi}{L} \int_0^L \frac{1}{2} \sin\frac{2n\pi x}{L}\, dx$$

$$= 0$$



moves to right just as often as to the left

$\langle KE \rangle$

$$\widehat{KE} = \frac{1}{2m}\hat{p}^2 = \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial x}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi(x) = i\hbar\frac{\partial}{\partial t}\Psi(x)$$

$$\widehat{KE}\,\Psi + \hat{U}\,\Psi = \hat{E}\,\Psi$$

$\hat{E} = i\hbar\frac{\partial}{\partial t}$ or if Ψ is constant $\hat{E} = E$

$\hat{H} \equiv \widehat{KE} + \hat{U}$ Hamiltonian operator

$$\hat{H}\Psi = \hat{E}\Psi$$

Eigenvalues & eigenfunctions of operators

matrix: $A\vec{v} = \lambda\vec{v}$

$\vec{v}$ is eigenvector of A
& λ corresponding eigenvalue

$$\hat{Q} f(x) = \lambda f(x)$$

$f(x)$ is eigenfunction of $\hat{Q}$
& λ is eigenvalue