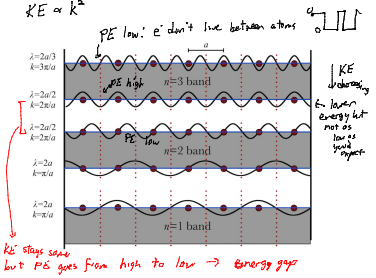
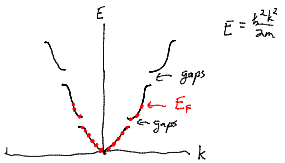
$KE \propto k^2$





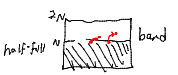
$E = \frac{\hbar^2 k^2}{2m}$

At $T=0$, electrons fill states from the bottom up

Each band has N spatial states $\times 2$ per spin
$2N$ states per band
(N = # of atoms)

Naïve:

- Suppose each atom has 1 valence electron
  N electrons.



electrons at top easily move around and gain energy

Conductor

- Suppose each atom has 2 valence electrons
  $2N$ atoms

electrons at top can only move by jumping gap

Insulator

In practice, bands can overlap
e.g. 2s & 2p in Beryllium
group these into 1 band

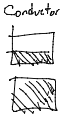
Conduction band

valence band - completely full at $T=0$

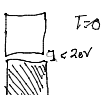
Insulator

empty conduction band

Conductor

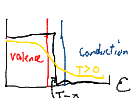


non-empty conduction band

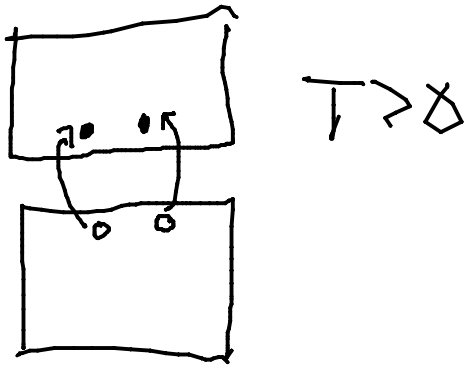


empty conduction band, small gap

$\bar{n}$

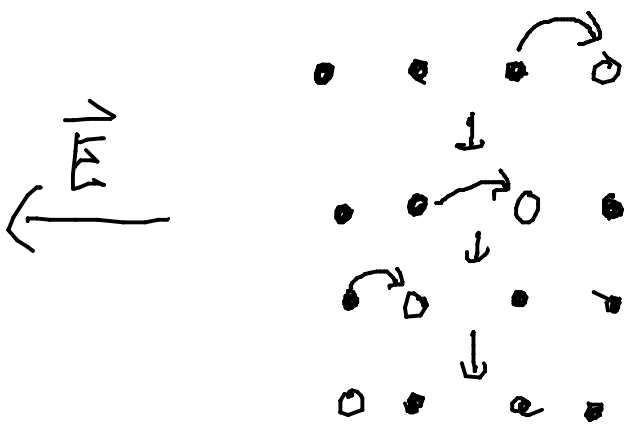


$\bar{n} = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$



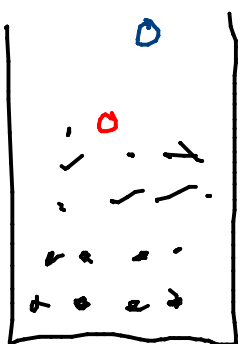
T > 0

electrons leave holes in valence band.



$\vec{E}$

hole moves with electric field - positively-charged



electrons at top have higher energy

holes have lower energy at top

Effective mass of electrons

$$m = \frac{F_{net}}{a}$$

$$m_{eff} = \frac{F_{ext}}{a} \leftarrow \text{electric field, e.g.}$$

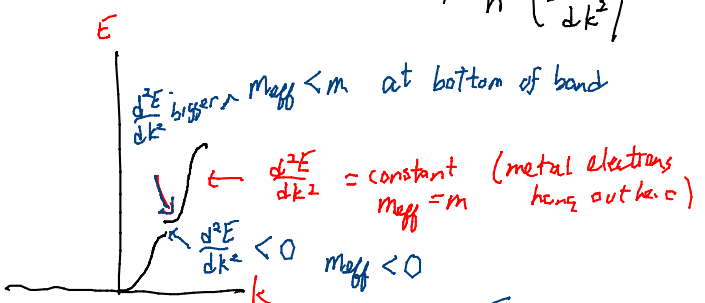
$$a = \frac{dv_g}{dt} = \frac{d}{dt}\frac{d\omega}{dk} = \frac{d^2\omega}{dk^2}\frac{dk}{dt}$$

$$F_{ext} v_g = \frac{dE}{dt} = \hbar \frac{d\omega}{dt} = \hbar \frac{d\omega}{dk}\frac{dk}{dt} = \hbar v_g \frac{dk}{dt}$$

$$F_{ext} = \hbar \frac{dk}{dt}$$

$$m_{eff} = \frac{F_{ext}}{a} = \frac{\hbar \frac{dk}{dt}}{\frac{d^2\omega}{dk^2}\frac{dk}{dt}} = \hbar \left(\frac{d^2\omega}{dk^2}\right)^{-1}$$

$$= \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$



$\frac{d^2E}{dk^2}$ bigger, $m_{eff} < m$ at bottom of band

$\leftarrow \frac{d^2E}{dk^2}$ = constant ($m_{eff} = m$) (metal electrons having out here)

$\frac{d^2E}{dk^2} < 0$ $m_{eff} < 0$

$$F_{ext} = m_{eff} a$$

If $m_{eff} < 0$, object moves opposite external force

Holes have "negative mass" normally, at top of band, they have positive effective mass.