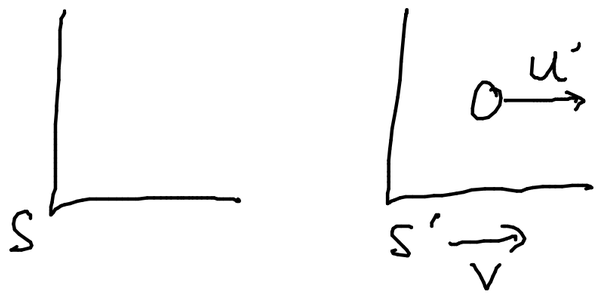
Lorentz equations if S' has speed v in S

(c=1) $\Delta x = \gamma(\Delta x' + v\Delta t')$

$\Delta t = \gamma(\Delta t' + v\Delta x')$

$\gamma = \frac{1}{\sqrt{1-v^2}}$



$u' = \frac{\Delta x'}{\Delta t'}$

$\Delta x' = u'\Delta t'$

$u = \frac{\Delta x}{\Delta t} = \frac{\gamma(\Delta x' + v\Delta t')}{\gamma(\Delta t' + v\Delta x')}$

$= \frac{u'\Delta t' + v\Delta t'}{\Delta t' + vu'\Delta t'}$

$= \frac{u' + v}{1 + vu'}$

For velocities parallel to S''s motion

$u_x = \frac{u_x' + v}{1 + vu_x'}$

if $u', v \ll 1$ then $u \approx u' + v$

if $u' = 1$, then $u = 1$

How about u_y?

$u_y' = \frac{\Delta y'}{\Delta t'}$

$u_y = \frac{\Delta y}{\Delta t} = \frac{\Delta y'}{\gamma(\Delta t' + v\Delta x')} = \frac{\Delta y'}{\gamma(\Delta t' + vu_x'\Delta t)}$

$u_y = u_y' \frac{1}{\gamma(1 + vu_x')}$

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$
$$\Delta t' = \gamma(\Delta t - v\Delta x)$$

Suppose event B cannot be reached from event A without moving faster than light

e.g. $\Delta x = 2$ yr $\quad \Delta t = 1$ yr

B is 1 yr after A

$\Delta t' = \gamma(1 - 2v)$

if $v = \frac{3}{4}$, $\Delta t' = \gamma(1 - \frac{3}{2}) = -\frac{1}{2}\gamma < 0$

A happens after B.

Which is fine!

BUT if I can go from A to B

I can go from effect to cause. Violate causality,

if $\Delta x > \Delta t$, spacelike relationship

(light can't travel between events,

no causal relationship between events.)

if $\Delta x < \Delta t$, timelike relationship

- causal relationships

$\Delta x = \Delta t$ lightlike relationship