Chapter 5

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = i\hbar\frac{\partial\Psi}{\partial t}$$

$\psi = Ae^{i(kx-\omega t)}$

$$\frac{p^2}{2m}\Psi = E\Psi$$

$$(KE)\Psi = E\Psi$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}}$$

(time-dependent) Schrödinger equation

$$\Psi(x,t) = \psi(x)\phi(t)$$

$$-\frac{\hbar^2}{2m}\psi''\phi + U(x)\psi\phi = i\hbar\psi\phi' \quad \times\frac{1}{\psi\phi}$$

$$-\frac{\hbar^2}{2m}\frac{\psi''}{\psi} + U(x) = i\hbar\frac{\phi'}{\phi} = E$$

$$i\hbar\overset{\frac{d\phi}{dt}}{\phi'} = E\phi$$

$$\int\frac{d\phi}{\phi} = \int\frac{E}{i\hbar}dt$$

$$\ln\phi = \frac{E}{i\hbar}t + C$$

$$\phi = Ce^{-i\frac{E}{\hbar}t} \qquad E = \hbar\omega$$

$$\phi(t) = Ce^{-i\omega t}$$

$$\Psi(x,t) = \psi(x)e^{-i\omega t}$$

$$\text{Prob} = |\Psi(x,t)|^2 = |\psi(x)|^2\,\underbrace{|e^{-i\omega t}|^2}_{1}$$

$$= |\psi(x)|^2$$

probability is time independent

Wait ... what?

$\Psi(x,t) = \psi(x)\phi(t)$ are <u>stationary</u> <u>states</u>.

Schrodinger equation is linear

$\therefore$ if $\psi_1(x)e^{-i\omega_1 t}$ & $\psi_2(x)e^{-i\omega_2 t}$

are solutions, then

$$A\psi_1(x)e^{-i\omega_1 t} + B\psi_2(x)e^{-i\omega_2 t}$$

is a solution too.

$$|\psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t}|^2 =$$

$$(\psi_1^* e^{+i\omega_1 t} + \psi_2^* e^{+i\omega_2 t})(\psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t})$$

$$= |\psi_1|^2 + \psi_2^*\psi_1 e^{i(\omega_2-\omega_1)t} + \psi_1^*\psi_2 e^{i(\omega_1-\omega_2)t} + |\psi_2|^2$$

complex conjugates

$$-\frac{\hbar^2}{2m}\psi'' + U\psi = E\psi$$

"time-independent Schrodinger equation"

we need $U(x)$ to know more

When does a $\psi(x)$ describe a physical object.

1) it should be normalizable

$$\int_{-\infty}^{\infty} |\psi(x)|^2\,dx = 1$$

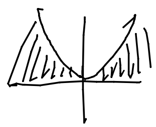
unless you want probability

it's ok to not normalize ψ

but we have to be able to do it

$\psi(x) = x$ not normalizable

$$\int_{-\infty}^{\infty} |x|^2\,dx = \infty$$



$\psi(x) = x^{3/2}$

$$\int_{-\infty}^{\infty} |x^3|\,dx = \infty$$

To be normalizable, $\psi(x) \to 0$ @ $\pm\infty$

e.g. $\psi(x) = \frac{1}{x}$ $\quad \int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$

$$= 2\int_0^{\infty} \frac{1}{x^2}\,dx = 2\left[-\frac{1}{x}\right]_0^{\infty}$$

$$= 2\left[-\frac{1}{\infty} + \frac{1}{0}\right]$$

$= \infty$. NOPE.

∞ at 0.

$\psi(x) = e^{-x^2}$ is normalizable

$$\frac{1}{x^2+1}$$

2) Must be continuous.

3) 1st derivative is continuous when $U(x)$ is finite.