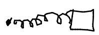
$\psi(x)$ describes a real system if

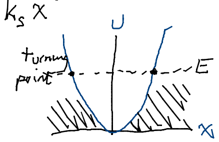
- normalizable $\int_{-\infty}^{\infty} |\psi|^2 dx < \infty$
- continuous
- $\psi'(x)$ is continuous except where potential energy is ∞

Bound State

- object is trapped in a finite region by an external force

e.g./ 

$U = \frac{1}{2} k_s x^2$

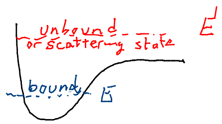


$E = U + KE$
conserved

turning point:
$KE = 0$

forbidden region: $KE < 0$

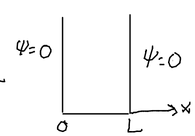
classically, any E is possible

eg. 

QM,
a bound wave is a standing wave

eg. Infinite Well

$U(x) = 0, \quad 0 < x < L$
$\infty, \quad$ otherwise



"particle in a box" 

$\psi = 0$ outside box

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \qquad 0 < x < L$$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

$k = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{p^2}{\hbar^2}}$

$\rightarrow p = \hbar k$

$$\psi'' = -k^2\psi$$

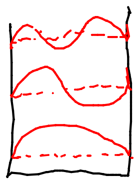
$$\psi(x) = A\sin kx + B\cos kx$$

$$\psi(0) = 0 = B \rightarrow \psi(x) = A\sin kx$$

$$\psi(L) = 0 = A\sin kL$$

$\sin kL = 0 \rightarrow kL = n\pi \quad n \in \mathbb{Z}$

$$k = \frac{n\pi}{L} = \sqrt{\frac{2mE}{\hbar^2}} \rightarrow \boxed{E = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

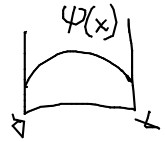


$n=3,\ E=9E_1$

$n=2,\ E=4E_1$

$n=1,\ E=E_1$

ground state $(n=1)$
object is most likely in middle



$n=2$:
P of being in center is 0
b/c $\psi(x)=0$

Ground state energy $E=\frac{\pi^2\hbar^2}{2mL^2}$
lowest KE possible
$\neq 0$: particle always moving!

if stopped, $\Delta p=0$ but $\Delta x=L$

$$\psi(x)=A\sin kx \qquad k=\frac{n\pi}{L}$$
$$=A\sin\frac{n\pi x}{L}$$

← normalization constant

$$1=\int_0^L |\psi(x)|^2\,dx = |A|^2\int_0^L \sin^2\frac{n\pi x}{L}\,dx$$

$$A=\sqrt{\frac{2}{L}}$$

$$\psi_n(x)=\begin{cases}\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} & 0<x<L\\ 0 & \text{otherwise}\end{cases}$$

$$E_n=n^2\frac{\pi^2\hbar^2}{2mL^2}$$