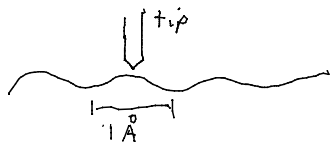


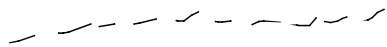
Scanning Tunneling Microscope



electrons tunnel between
surface & tip

electrons is very sensitive
to size of gap

build up a topological map of surface
down to 0.1 Å — see atoms



Real unbound objects

described as wave packets — sums of
plane waves $\int A(k) e^{ikx} dx$

Consider sum of 2 plane waves

with $k = k_0 \pm dk$ $\omega = \omega_0 \pm d\omega$

$$\Psi(x, t) = A \left[e^{i(k_0 + dk)x} e^{-i(\omega_0 + d\omega)t} + e^{i(k_0 - dk)x} e^{-i(\omega_0 - d\omega)t} \right]$$

$$\begin{aligned} |e^{ia}| &= 1 \\ \text{if } a \in \mathbb{R} \end{aligned} \quad \begin{aligned} &= A e^{ik_0 x - i\omega_0 t} \left[e^{i(dk x - d\omega t)} + e^{-i(dk x - d\omega t)} \right] \\ &= A e^{i(k_0 x - \omega_0 t)} 2 \cos(dk x - d\omega t) \end{aligned}$$

$$f(x - vt)$$

↑ plane wave
speed $\frac{\omega_0}{k_0}$
↑ speed $\frac{d\omega}{dk}$

$|\Psi|^2$ depends on group velocity

$$= |A|^2 e^{-i(k_0 x - \omega_0 t)} e^{i(k_0 x - \omega_0 t)} 4 \cos^2(dk x - d\omega t)$$

$$\begin{aligned} v_{\text{phase}} &= \frac{\omega_0}{k_0} \\ &= \lambda_0 f_0 \end{aligned} \quad v_{\text{group}} = \frac{d\omega}{dk}$$

$\omega(k)$ is dispersion relation

EM wave $\omega = ck$ $\frac{d\omega}{dk} = c = \frac{\omega}{k}$

Matter waves $\omega = \frac{\hbar k^2}{2m}$

$v_{\text{phase}} = \frac{\omega}{k} = \frac{\hbar k}{2m}$ $v_{\text{group}} = \frac{d\omega}{dk} = \frac{\hbar k}{m} = 2v_{\text{phase}}$

Light in a material will have a different dispersion relation

$v_{\text{light}} = v(k)$

speed depends on λ

& angle of refraction \rightarrow

RAINBOWS

e.g. ionosphere $n(\omega) = \sqrt{1 - \frac{b}{\omega^2}}$

$\omega(k)$

$n = \frac{c}{v} \leftarrow \text{phase}$

$v = \frac{\omega}{k}$

$n = \frac{ck}{\omega} = \sqrt{1 - \frac{b}{\omega^2}}$

algebra happens

$\omega = \sqrt{(kc)^2 + b}$

$v_{\text{phase}} > c$ but that's OK -

no matter/info is travelling

at v_{phase}

$v_{\text{group}} = \frac{d\omega}{dk} = \frac{kc^2}{\sqrt{(kc)^2 + b}} = c \sqrt{1 - \frac{b}{\omega^2}} < c$