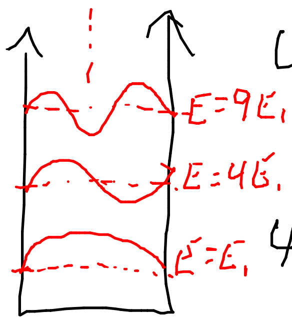
Infinite Well



$$U(x) = \begin{cases} 0, & 0<x<L \\ \infty, & \text{otherwise} \end{cases}$$

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} & 0<x<L \\ 0 & \text{otherwise} \end{cases}$$

$$E = n^2 \frac{\pi^2 \hbar^2}{2mL^2} = n^2 E_1$$

$n = 1, 2, 3, \ldots$

$n \neq 0$

ground-state energy is > 0

& $KE > 0$

because $\Delta x = L/2$

so $\Delta p \neq 0$

$\rightarrow KE \neq 0$

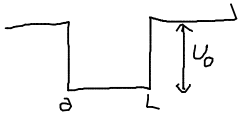
What is the probability that ground state particle is in the leftmost quarter of box?

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}$$

$$P = \int_0^{L/4} \underbrace{|\psi(x)|^2 dx}_{\text{prob. it is within } dx \text{ of } x}$$

$$= \int_0^{L/4} \frac{2}{L} \sin^2\frac{\pi x}{L} dx = \frac{1}{4} - \frac{1}{2\pi} = 9\%$$

Finite Wall



$$U(x) = \begin{cases} 0, & 0 < x < L \\ U_0, & \text{otherwise} \end{cases}$$

Bound states $\quad E < U_0$

$$-\frac{\hbar^2}{2m}\psi'' + U(x)\psi = E\psi$$

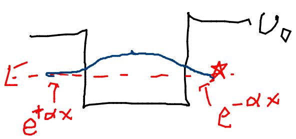
$$\psi'' = \frac{2m[U(x) - E]}{\hbar^2}\psi \equiv \alpha^2 \psi$$

- if α is real, $e^{\pm\alpha x}$ are solutions
- if α is imaginary
  $\alpha = ik \qquad \sin kx \;\&\; \cos kx$

inside well, $\quad \alpha = \sqrt{\frac{2m(\;-E)}{\hbar^2}} = ik$
$U(x) = 0 \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$
$\sin kx \;\&\; \cos kx$

outside well,
$U(x) = U_0 \qquad \alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$

$$e^{I\alpha x}$$



$\psi(x) \neq 0$ outside well
probability that particle is outside well
$E < U$ but $E = U + KE$
$KE < 0$ outside well

$e^{-x/\delta} \qquad \delta = \frac{1}{\alpha}$: penetration depth

"particle's outside" $\quad \Delta x < \delta$

$$\Delta p > \frac{\hbar}{2\delta} =$$

$$\Delta KE = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{8m\delta^2} = \frac{\hbar^2}{8m}\alpha^2$$

$$\frac{\cancel{\hbar^2}}{\cancel{8m}_{4}} \frac{\cancel{2m}(U_0 - E)}{\cancel{\hbar^2}} =$$

$$\Delta KE \sim U_0 - E$$