Physics 370 Homework #8

7 problems

Due by Wednesday, October 26

▷ **1.**

(Harris 5.31) A 50 eV is trapped between electrostatic walls 200 eV high. How far does its wave function extend beyond the walls?

▷ **2.**

(Harris 5.52) To a good approximation, the hydrogen chloride molecule HCl behaves vibrationally as a quantum harmonic oscillator of spring constant 480 N/m and with effective oscillating mass just that of the lighter atom, hydrogen. If it were in its ground vibrational state, wht wavelength photon would be just right to bump this molecule up to its next-higher vibrational energy state?

▷ **3.**

(Harris 5.56-58) Consider a particle in an infinite square well with the wavefunction $\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$.

(a) Show that the uncertainty in a particle's position is given by

$$\Delta x = L\sqrt{\frac{1}{12} - \frac{1}{2n^2\pi^2}}$$

(b) Show that the uncertainty in a particle's momentum is $\Delta p = \frac{n\pi\hbar}{L}$.

(c) What is the product of these uncertainties? Discuss the result.

▷ **4.**

Prove that $\psi(x) = xe^{-\frac{1}{2}b^2x^2}$ is a solution to the Schrodinger equation with a harmonic-well potential

$$-\frac{\hbar^2}{2m}\psi''(x) + \frac{1}{2}\kappa x^2\psi(x) = E\psi(x)$$

and find b and E in terms of the spring constant κ and the mass m. (Hint: Because derivatives of complex exponentials makes me nervous, I would write $\psi(x) = xf(x)$ where $f(x)$ is the exponential, and I would calculate $f'(x)$ and $f''(x)$ separately.)

▷ **5.**

Prove that the variance $(\Delta x)^2 = \langle (x - \langle x \rangle)^2 \rangle$, can also be written as $\langle x^2 \rangle - \langle x \rangle^2$.

▷ **6.**

Show that the vector $v = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is an eigenvector of $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$, and find its eigenvalue.

▷ **7.**

Use integration by parts to show that the expectation value of the momentum of a bound state $\psi(x)$ is zero. (Hints: you can assume that $\psi(x)$ is real with no loss of generality. Your solution must not hold for a free state like e^{ikx}.)