(4) Otherwise consider the selected vertex in the maximal independent set and remove all its neighbours from it. Proceed to find the maximal independence set possible excluding its neighbour.

(5) Repeat this process for all vertices and print the maximal independence set of them.

Time Complexity of Maximum Independence set algorithm:—

$= O(2^n)$ [exponential time]

~~d~~ [algo, ~~time~~ procedure, time complexity]

Short note

BIPARTITE GRAPH :-

$G = (V, E)$

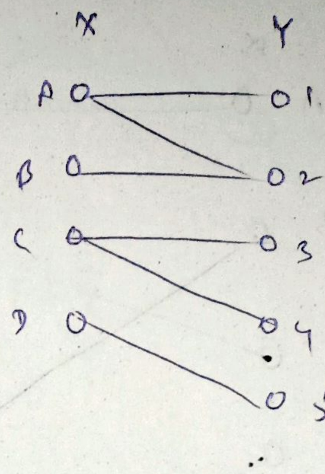
$V = \{A, B, C, D, 1, 2, 3, 4, 5\}$

$X = \{A, B, C, D\}$

$Y = \{1, 2, 3, 4, 5\}$

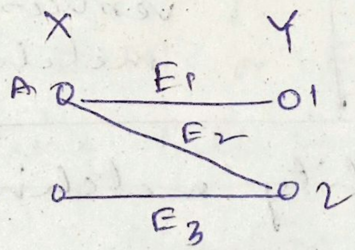
$X \cup Y = V$

$X \cap Y = \phi$



Matching in Bipartite Graph :-

Matching in a graph is a subset of edges that ~~know~~ no two edges share a vertex. (2 colorable graph)

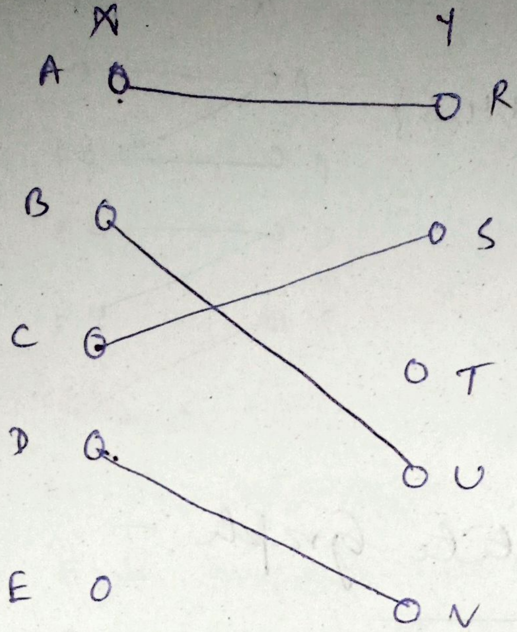


$M = \{E_1, E_3\}$

Chromatic number of Bipartite graph $= 2$

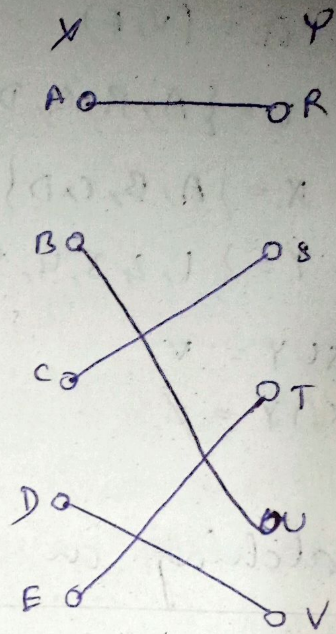
Complete Matching :-

A complete matching is a matching when every member of X is paired with 1 member of Y



Incomplete

Cause here E has no match

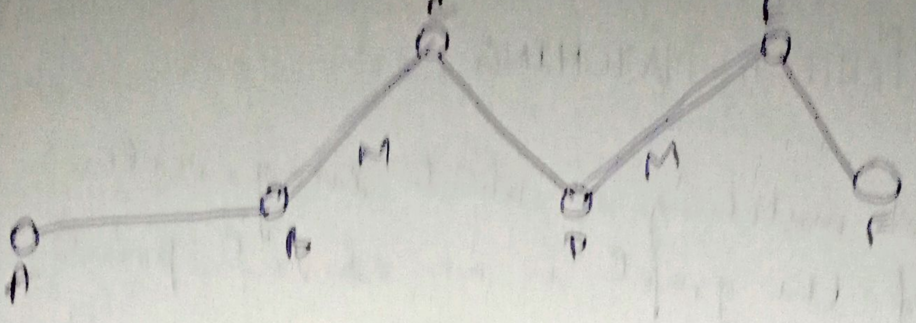


Complete

E. Here all vertices have match.

→ Maximum & Cardinality matching:—

It is a matching which contains ~~which~~ a maximum no. of edges.
Consider a general graph if M is the proper subset of E
$M \subseteq E$ is a matching for G then any vertex V is called a free vertex if it is not an end point of any element of M



$E = \{ AB, BC, CD, DE, EF \}$

$M = \{ BC, DE \}$

[If $M = \{AB, EF\}$ then there is no free vertex.]

Alternative Path ~~Example~~ :-

An alternative path is a simple path in G when the edges alternatively belongs to M and E-M or E-M and M. BCDEF

Augmenting Path:

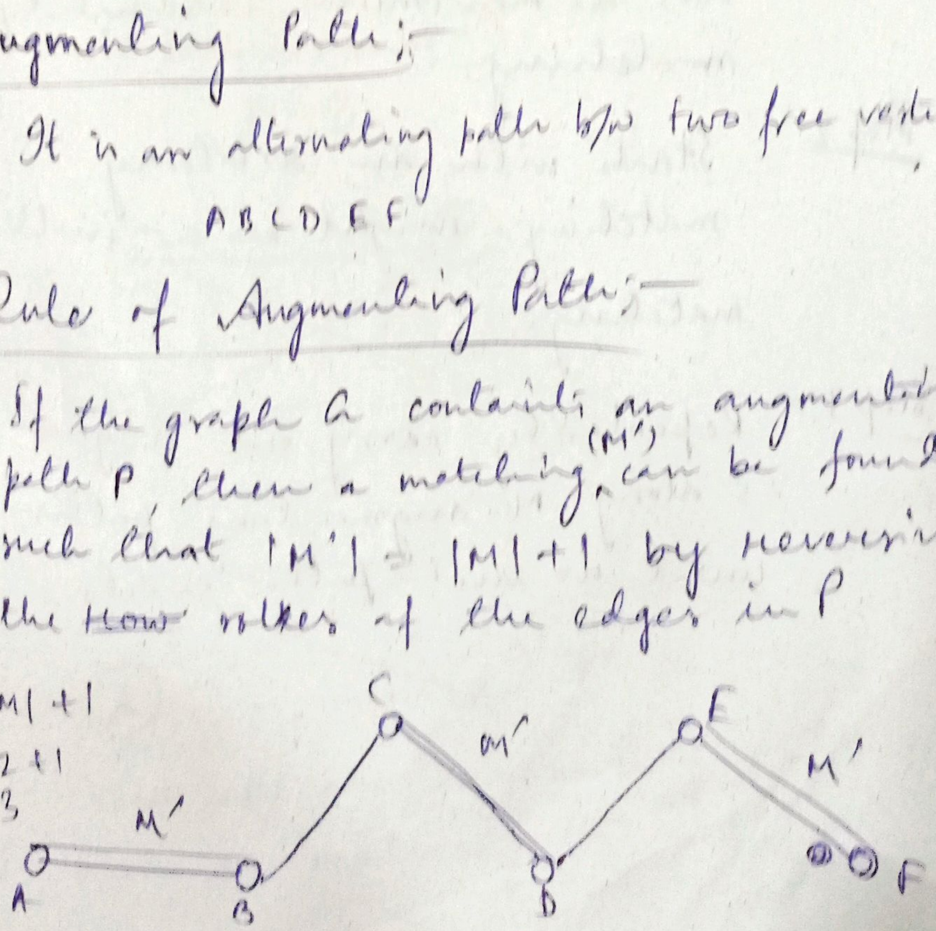
It is an alternating path b/w two free vertices.

ABCDEF

Rule of Augmenting Path:-

If the graph G contains an augmenting path P, then a matching (M') can be found such that $|M'| = |M| + 1$ by reversing the ~~How~~ roles of the edges in P

$|M'| = |M| + 1$
$= 2 + 1$
$= 3$



PERFECT MATCHING

A matching in which every vertex of the graph is an ~~ed~~ end point of an edge in matching. Every graph may containt a perfect matching. If a graph contents a perfect matching M then M is a maximum cardina-lity matching.

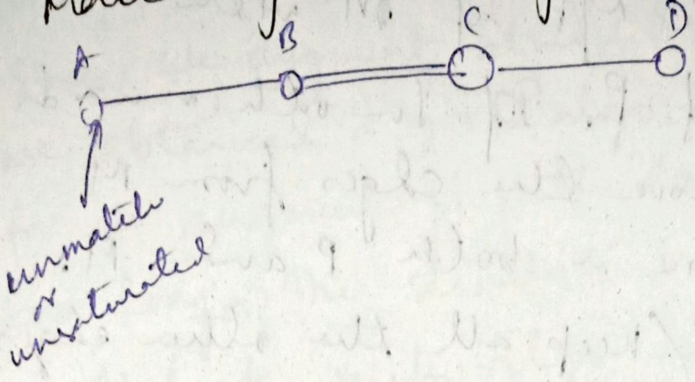
$\boxed{\text{Algo}}$

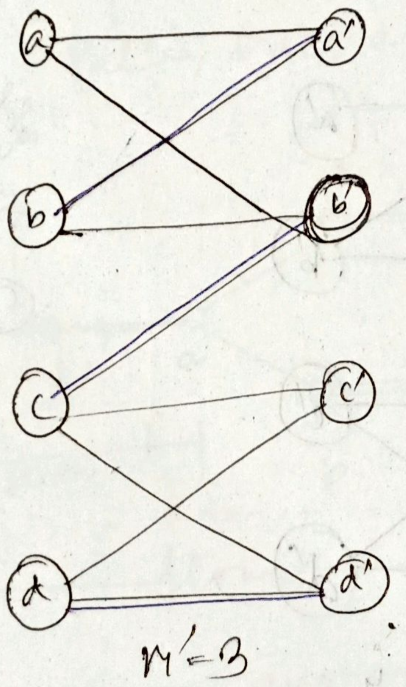
Step 1 :- There is an M augmenting path if and only if M is not a maximum cardinality matching.

Step 2 :- Starts with an arbitary matching, might be a null matching.

Step 3 :- Repeatedly carry out augmentation-s along M augmenting paths until no such path exists.

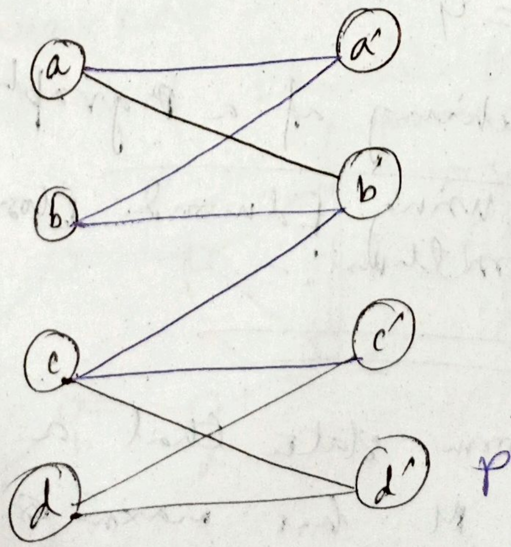
Roles of Augmenting Pattern Matching theory :—



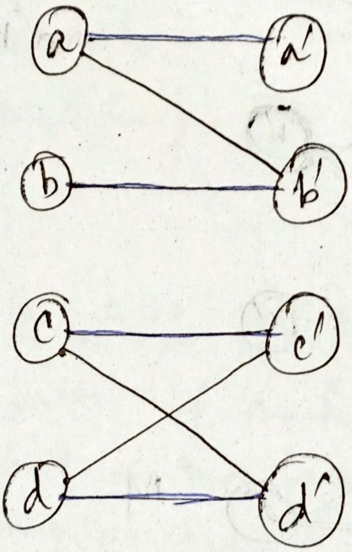


Augmenting path

a a' b b' c c'



We can use an augmenting path P to turn a matching M into a a larger matching taking the semmetric diff of M with the edges of P. ~~Df~~ In other words we remove the edges from M which are in both P and M. We add/keep all the other edges



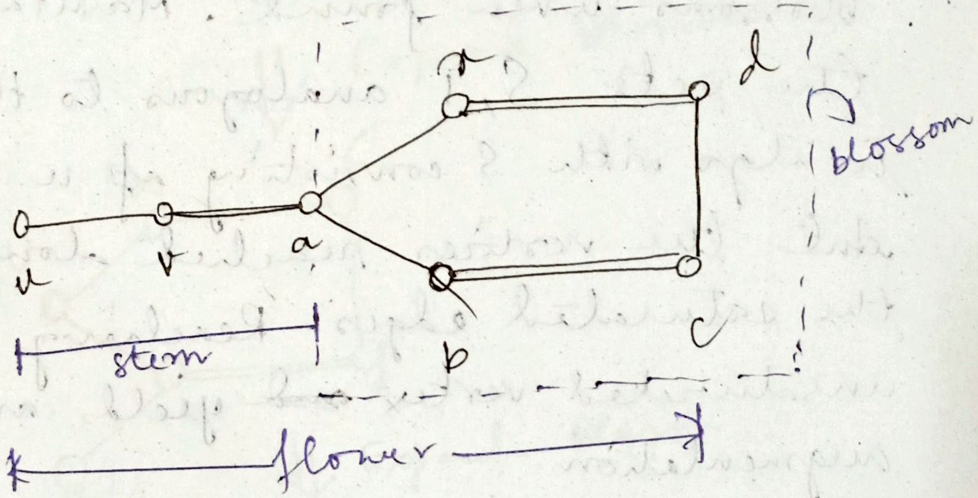
match = 4
M' = 4

Matching of a ~~B~~ graph using Edmond Blossom's Algorithm :-

Berges Theorm states that a matching M has maxm size iff ~~G~~ G has no ~~M~~ M-augmenting path.

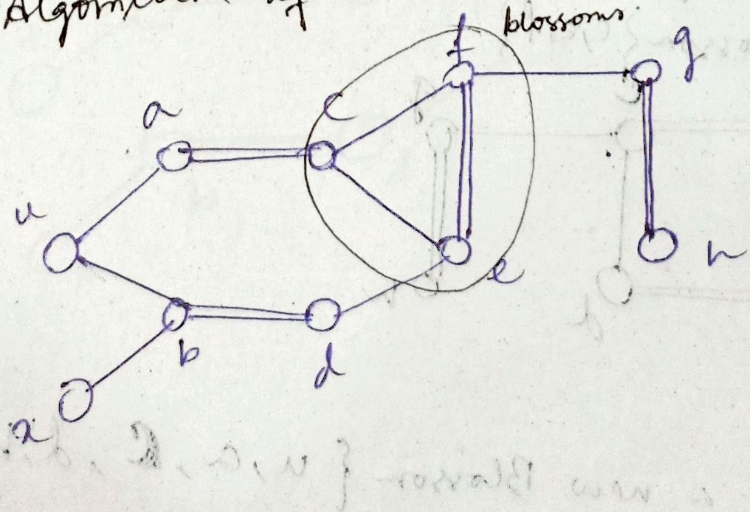
[illegible]

We can, thus find a max^m matching using successive augmenting path. Since, we augment atmost ~~2~~ $n/2$ times, we obtained a good algo if the search for an augmenting path does not take too long. The scientist Edmond present a ~~b~~ fast such algo in his famous paper "Paths, trees and flowers".



uvax — augmenting path
uvabcdx — "

Algorithm of Edmond Blossom:-



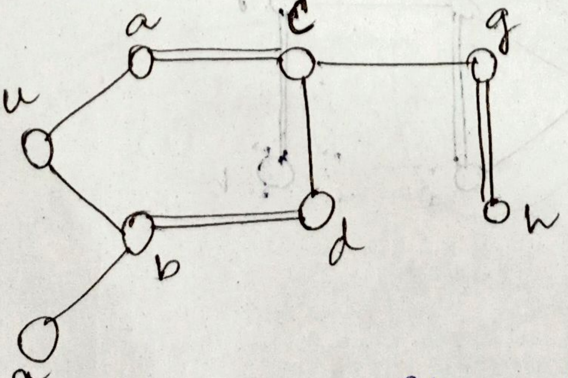
- <u>Input</u>:- A graph G, a matching M in G, and unsaturated vertex u.

- <u>Idea</u>:- Explore M-alternating paths from u, recording for each vertex from which it was reached and contracting blossoms when found. Maintain the sets S, T analogous to those in algo with S consisting of u. And the vertices reached along the saturated edges. Reaching an unsaturated vertex ~~and~~ yields an augmentation

- <u>Initialization</u>:-

$S = \{u\}$ and $T = \phi$

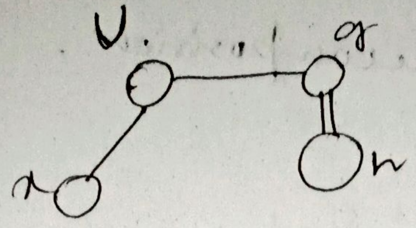
contract blossom $\{g, e, f\}$



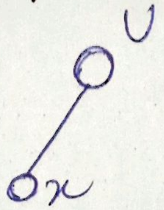
Find a new Blossom $\{u, a, c, d, b\}$

$\Downarrow$

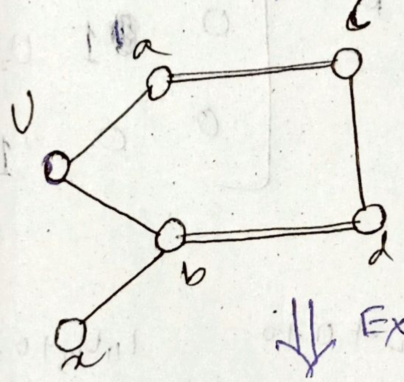
contract blossom



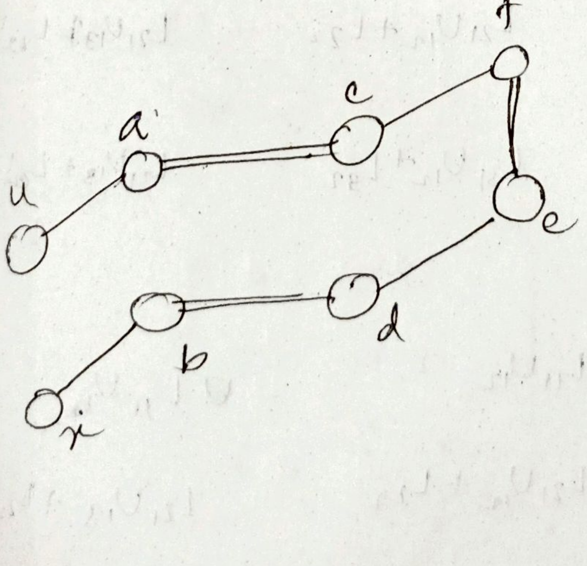
new augmenting path



Expand Blossoms U



Expand Blossom C



Find out the inverse matrix using a new decomposision.

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 2 & 9 & 8 \\ 6 & 5 & 2 \end{bmatrix}$$

$A = LU$

$$L = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \quad U = \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} L_{11}+0+0 & L_{11}U_{12}+0+0 & L_{11}U_{13}+0+0 \\ L_{21} & L_{21}U_{12}+L_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31} & L_{31}U_{12}+L_{32} & L_{31}U_{13}+L_{32}U_{23}+L_{33} \end{bmatrix}$$

$$= \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ L_{21} & L_{21}U_{12}+L_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31} & L_{31}U_{12}+L_{32} & L_{31}U_{13}+L_{32}U_{23}+L_{33} \end{bmatrix}$$

$L_{11} = 1$

$L_{11}U_{12} = -1$
$1 \cdot U_{12} = -1$
$U_{12} = -1$

$L_{11}U_{13} = 4$
$U_{13} = 4$

$L_{21} = 2$

$L_{21}U_{12} + L_{22} = 9$
$2(-1) + L_{22} = 9$
$L_{22} = 9 + 2$
$= 11$

$L_{21}U_{13} + L_{22}U_{23} = 8$
$(2)(4) + (11)U_{23} = 8$
$8 + 11U_{23} = 8$
$11U_{23} = 8 - 8 = 0$
$U_{23} = 0$

$L_{31} = 6$

$L_{31}U_{12} + L_{32} = 5$
$-6 + L_{32} = 5$
$L_{32} = 5 + 6 = 11$

$L_{31}U_{13} + L_{32}U_{23}$
$+ L_{33} = 2$
$(6)(4) + (11)(0)$
$+ L_{33} = 2$
$24 + 0 + L_{33}$
$= 2$
$L_{33} = 2 - 24$
$= -22$

$L_{11} =$ U_{12}
L_{21} L_{22}
$L_{31} =$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 11 & 0 \\ 6 & 11 & -22 \end{bmatrix} \qquad U = \begin{bmatrix} \cancel{0}\ 1 & -1 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 11 & 0 \\ 6 & 11 & -22 \end{bmatrix}$$

$$L^{-1} = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$$

$$LL^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 11 & 0 \\ 6 & 11 & -22 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$$

$$= \begin{bmatrix} a & 0 & 0 \\ 2a+11b & 11c & 0 \\ 6a+11b-22d & 11c-22e & -22f \end{bmatrix}$$

$$LL^{-1} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $a = 1$

$\cancel{2a + 11b = 0}$

$\cancel{2 + 11b = 0}$

$\cancel{11b = -2}$

$\cancel{b = -\frac{2}{11}}$

$b = -\frac{2}{11}$

$11c = 1$

$c = \frac{1}{11}$

$2a + 11b = 0$

$2 + 11b = 0$

$11b = -2$

$b = -\frac{2}{11}$

$11c - 22e = 0$ $\qquad$ $-22f = 1$

$11 \cdot \frac{1}{11} - 22e = 0$ $\qquad$ $f = -\frac{1}{22}$

$1 - 22e = 0$

$22e = 1$

$e = \frac{1}{22}$

$6a + 11b - 22d = 0$

$6 + 11\left(-\frac{2}{11}\right) - 22d = 0$

$6 - 2 - 22d = 0$

$4 - 22d = 0$

~~or~~ $4 = 22d$

$d = \frac{\cancel{4}\,2}{\cancel{22}\,11} = \frac{2}{11}$

$a = 1$, $b = -\frac{2}{11}$, $c = \frac{1}{11}$, $d = \frac{2}{11}$,

$e = \frac{1}{22}$, $f = -\frac{1}{22}$

$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{11} & \frac{1}{11} & 0 \\ \frac{2}{11} & \frac{1}{22} & -\frac{1}{22} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad U^{-1} = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

$$UU^{-1} = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & a-1 & b-c+4 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & a-1 & b-c+4 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore\ a-1=0 \qquad \cancel{b}\ c=0 \qquad b-c+4=0$

$a=1 \qquad\qquad\qquad\qquad b+4=0$

$\qquad\qquad\qquad\qquad\qquad b=-4$

$$U^{-1} = \begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A = LU$

$A^{-1} = (LU)^{-1}$

$A^{-1} = U^{-1} L^{-1}$

$$= \begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{11} & \frac{1}{11} & 0 \\ \frac{2}{11} & \frac{1}{22} & -\frac{1}{22} \end{bmatrix}$$

$$= \begin{pmatrix} 1 - \frac{2}{11} - \frac{8}{11} & \frac{1}{11} - \frac{4}{22} & \frac{4}{22} \\ -\frac{2}{11} & \frac{1}{11} & 0 \\ \frac{2}{11} & \frac{1}{22} & -\frac{1}{22} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{11-2-8}{11} & \frac{1-2}{11} & \frac{2}{11} \\ -\frac{2}{11} & \frac{1}{11} & 0 \\ \frac{2}{11} & \frac{1}{22} & -\frac{1}{22} \end{pmatrix}$$

$$= \begin{bmatrix} \frac{1}{11} & -\frac{1}{11} & \frac{2}{11} \\ -\frac{2}{11} & \frac{1}{11} & 0 \\ \frac{2}{11} & \frac{1}{22} & -\frac{1}{22} \end{bmatrix}$$

~~Shortest Path~~

Chinese Remainder Theorem :-

Congruence Modulo m :-

Two integers a and b are congruence modulo m if they have the same ~~numerical~~ remainders when divided by m.

It is denoted by

$$a \equiv b \pmod{m}$$

Statement of Chinese Remainder Theorem :-

Let $m_1, m_2 \ldots m_r$ be the relatively prime numbers such that GCD of $m_i, m_j = 1$; $i \neq j$ then the linear congruences

$$x \equiv a_1 \pmod{m_1}$$

$$x \equiv a_2 \pmod{m_2}$$

$$\vdots$$

$$x \equiv a_r \pmod{m_r}$$

has a simultaneous solution

which is ~~unit mo~~ unique modulo $(m_1, m_2 \cdots m_r)$.

General form :-

$$x \equiv a_1 \pmod{m_1}$$
$$x \equiv a_2 \pmod{m_2}$$
$$\vdots$$
$$x \equiv a_n \pmod{m_n}$$

Our main task is to find out the value of x such that these conditions are satisfied. To do this, we have to perform the following steps ——

Step-1 :-

Find out the common modulus (M).

$$M = m_1 \times m_2 \times m_3 \times \cdots \times m_n$$

Step2 :-

Find $M_1 = \frac{M}{m_1}$, $M_2 = \frac{M}{m_2}$, ... $M_n = \frac{M}{m_n}$.

Step3 :-

Find out the inverse of M_1^{-1} M_2^{-1} M_3^{-1}, ... M_n^{-1} with respect to ~~on~~ $m_1, m_2, \cdots m_n$

Step 4:-

$$x = \Big((a_1 * M_1 * M_1^{-1}) + (a_2 * M_2 * M_2^{-1}) + \cdots + (a_n * M_n * M_n^{-1}) \Big) \bmod M$$

Example:-

Pr 1

$x \equiv 4 \pmod{11}$ $\quad m_1 = 11 \quad a_1 = 4$

$x \equiv 5 \pmod{7}$ $\quad m_2 = 7 \quad a_2 = 5$

$x \equiv 6 \pmod{13}$ $\quad m_3 = 13 \quad a_3 = 6$

Step 1

$$M = m_1 * m_2 * m_3 = 11 \times 7 \times 13$$
$$= 1001$$

Step 2

Now, $M_1 = \dfrac{M}{m_1} = \dfrac{1001}{11} = 91$

$M_2 = \dfrac{M}{m_2} = \dfrac{1001}{7} = 143$

$M_3 = \dfrac{M}{m_3} = \dfrac{1001}{13} = 77$

Step 3

$M_1^{-1} \equiv 91^{-1} \pmod{11}$

$\Rightarrow (x * 91) \bmod 11 = 1$

$x = 4$ then $(4 * 91) \bmod 11 = 1$

$$\begin{array}{r|l|l} 11 & 364 & 33 \\ & 33 & \\ \hline & 34 & \\ & 33 & \\ \hline & 1 & \end{array}$$

$\therefore M^{-1} = 4$

$M_2^{-1} = 143^{-1} \pmod 7$

$\Rightarrow (x * 143) \bmod 7 = 1$

$\therefore M_2^{-1} = 5$

$M_3^{-1} = 77^{-1} \pmod{13}$

$\Rightarrow (x * 77) \bmod 13 = 1$

$M_3^{-1} = 12$

Step 4

$x = ((4 \times 91 \times 4) +$
$(5 \times 143 \times 5) +$
$(6 \times 77 \times 12)) \bmod 1001$

$= (1456 + 3575 + 5544) \bmod 1001$

$x = 10575 \pmod{1001}$

$\therefore x = 565$ (Ans)

for check $\rightarrow$ $x / m_1 = a_1$

$7 \,|\, 143 \,|\, 2$
$\quad 14$
$\quad 3$

$x = 5$

$7 \,|\, 715 \,|\, 102$
$\quad 7$
$\quad 15$
$\quad 14$
$\quad 1$

$77 \times 12 = 154$, $77 \times = 924$

$13 \,|\, 154 \,|\, 1$
$\quad 13$
$\quad 24$

$13 \,|\, 924 \,|\, 71$
$\quad 91$
$\quad 14$
$\quad 13$
$\quad 1$

$13 \,|\, 231 \,|\, 1$
$\quad 13$
$\quad 101$

$13 \,|\, 308 \,|\, 2$
$\quad 26$
$\quad 48$

$1001 \,|\, 10575 \,|\, 1$
$\quad 1001$
$\quad 565$

Prob: Find out the solution of the following problem using chinease remainder theorem:—

$x \equiv 3 \pmod 5$ $m_1 = 5$ $a_1 = 3$

$x \equiv 1 \pmod 7$ $m_2 = 7$ $a_2 = 1$

$x \equiv 6 \pmod 8$ $m_3 = 8$ $a_3 = 6$

→ (1) $M = 5 \times 7 \times 8 = 280$

(2) $M_1 = \frac{M}{m_1} = \frac{280}{5} = 56$

$M_2 = \frac{M}{m_2} = \frac{280}{7} = 40$

$M_3 = \frac{M}{m_3} = \frac{280}{8} = 35$

(3) $M_1^{-1} = 56^{-1} \pmod 5$

$(x * 56) \bmod 5 = 1$

$M_1^{-1} = 1$

$5 \overline{) 56 } 11$
5
$\overline{6}$
5
$\overline{1}$

$M_2^{-1} \equiv 40^{-1} \pmod 7$

$(x * 40) \bmod 7 = 1$

$M_2^{-1} = 3$

$x = 3$

$7 \,|\, 120 \,|\, 17$
7
$\overline{50}$
49
$\overline{1}$

$M_3^{-1} \equiv 35^{-1} \pmod 8$

$(x * 35) \bmod 8 = 1$

$M_3^{-1} = 3$

$8 \,|\, 70 \,|\, 8$
64
$\overline{6}$

$8 \,|\, 105 \,|\, 13$
8
$\overline{25}$

(4)

$x = \big((3 \times 56 \times 1) +$
$(1 \times 40 \times 3) +$
$(6 \times 35 \times 3) \big) \bmod 280$

280×3
$\overline{840}$

$= (168 + 120 + 630) \bmod 280$

$= 918 \pmod{280}$

$\therefore \boxed{x = 78}$

$280 \,|\, 918 \,|\, 3$
840
$\overline{78}$

Polynomial Interpolation :-

Polynomial Interpolation is a method of estimating values b/w known data points. When the graphical data contains a gap but data is available on either side of the gap or at a few specific points within the gap, and estimate of values within the gap can be made by interpolation. The simplest method of interpolation is to draw straight lines b/w the known data points, and consider the function as the combination of those straight lines. This method is called linear interpolation, usually introduce an error. A ~~kno m~~ more presize approach uses a polynomial func to connect the points. This interpolation is called polynomial interpolation.

Q. Find the interpolating polynomial $f(x)$ satisfying $f(0) = 0$, $f(2) = 4$, $f(4) = 56$, $f(6) = 204$, $f(8) = 496$, $f(10) = 980$ and also find $f(3)$ and $f(5)$.

Ans) ~~x f(x)~~

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
0	0					
		4				
			48			
2	4			48		
		52			0	
			96			0
4	56					
		148		48		
6	204		144		0	
		292		48		
8	496		192			
		484				
10	980					

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0.$$

$$r = \frac{x - x_0}{h} = \frac{x-0}{2} = \frac{x}{2}$$

~~where x = 3~~ ~~$r = \frac{3}{2}$~~

$$f(3) = 0 + \frac{3}{2}(4) + \frac{\frac{3}{2}\left(\frac{3}{2}-1\right)}{2}(48) +$$

$$\frac{\frac{3}{2}\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)}{6} 48$$

$$= 0 + 6 + \frac{\frac{3}{2} \times \frac{3-2}{2}}{\cancel{2}} \times \overset{24}{\cancel{48}} +$$

$$\frac{\frac{3}{2} \times \frac{3-2}{2} \times \frac{3-4}{\cancel{2}}}{\cancel{6}} \times \overset{8}{\cancel{48}}$$

$$= 0 + 6 + \frac{3}{\cancel{2}} \times \frac{1}{\cancel{2}} \times \overset{6}{\cancel{\overset{12}{\cancel{24}}}} - \frac{3}{\cancel{2}} \times \frac{1}{\cancel{2}} \times \frac{1}{\cancel{2}} \times \overset{2}{\cancel{\overset{4}{\cancel{8}}}}$$

$$= 0 + 6 + 18 - 3$$

$$f(3) = 21$$

$$f(x) = 0 + \frac{4x}{2} + \frac{\frac{x}{2}\left(\frac{x}{2}-1\right)}{2} 48$$

$$+ \frac{3}{2} \frac{\frac{x}{2}\left(\frac{x}{2}-1\right)\left(\frac{x}{2}-2\right)}{6} \times 48$$

$$= 0 + 2x + \frac{x}{\cancel{2}} \times \frac{x-2}{\cancel{2}} \times \overset{6}{\cancel{\overset{12}{\cancel{24}}}}$$

$$+ \frac{x}{\cancel{2}}\left(\frac{x-2}{\cancel{2}}\right)\left(\frac{x-4}{\cancel{2}}\right) \times \overset{1}{\cancel{\overset{4}{\cancel{8}}}}$$

$2x + 6x(x-2) + x(x-2)(x-4)$

$= 2x + 6x^2 - 12x + x(x^2 - 4x - 2x + 8)$

$= 2x + 6x^2 - 12x + x(x^2 - 6x + 8)$

$= 2x + \cancel{6x^2} - 12x + x^3 - \cancel{6x^2} + 8x$

$= x^3 - 2x$

$x=3 \quad f(3) = (3)^3 - 2(3) = 27 - 6 = 21$

$x=5 \quad f(5) = (5)^3 - 2(5) = 125 - 10 = 115$

H.W

The following data gives the melting points of an alloy of lead and zinc.

Percentage of lead in the alloy (P)	50	60	70	80
Temperature (°C)	205	225	248	274

Find the melting point of the alloy containing 54% of lead using appropriate interpolation formula.