Practice Questions Solutions (21-24)

CSCI 3022

June 18, 2020

21. (a) $P(\frac{1}{4} \leq X \leq \frac{3}{4}) = \int_{\frac{1}{4}}^{\frac{3}{4}} f(x)dx = F(x)|_{\frac{1}{4}}^{\frac{3}{4}} = (2x^2 - x^4)|_{\frac{1}{4}}^{\frac{3}{4}} = 2 \cdot \frac{3}{4}^2 - \frac{3}{4}^4 - (2 \cdot \frac{1}{4}^2 - \frac{1}{4}^4) = \frac{11}{16}$ ≈ 0.6875

    (b) $f(x) = \frac{dF(x)}{dx} = 4x - 4x^3, 0 \leq x \leq 1$

22. $f(x) = \lambda e^{-\lambda x}$ Since X have an $Exp(0.2), f(x) = 0.2e^{-0.2x}, x > 0$
    Therefore, for $P(X > 5) = 1 - P(X \leq 5) = 1 - \int_0^5 0.2e^{-0.2x}dx = 1 - (-e^{-0.2x}|_0^5) = 1 - (1 - e^{-1}) = e^-1 \approx 0.3679$

23. (a) $P(x < 0.55) = \int_0^{0.55} f(x)\ dx = \int_0^{\frac{1}{2}} 4x\ dx + \int_{\frac{1}{2}}^{0.55}(4 - 4x)\ dx = 2x^2|_0^{\frac{1}{2}} + (4x - 2x^2|_{\frac{1}{2}}^{0.55}) = 2 \cdot \frac{1}{2}^2 + 4 \cdot 0.55 - 2 \cdot (0.55)^2 - 4 \cdot \frac{1}{2} + 2(\frac{1}{2})^2 = 0.595$

    (b) $S = \int_0^{\frac{1}{2}} f(x)\ dx = \int_0^{\frac{1}{2}} 4x\ dx = 2x^2|_0^{\frac{1}{2}} = 2 \cdot (\frac{1}{2})^2 = 0.5$
    The 50th percentile of the score distribution is 0.5

24. For the median of $Exp(\lambda)$, $F(x) = \int_0^t f(x)\ dx = \int_0^t \lambda e^{-\lambda x}\ dx = 0.5$
    $\int_0^t \lambda e^{-\lambda x}\ dx = -e^{-\lambda x}|_0^t = -e^{-\lambda t} + 1 = 0.5$
    $t = \frac{ln(2)}{\lambda}$
    The median of an $Exp(\lambda)$ distribution is $\frac{ln(2)}{\lambda}$