Making sense of the lecture question :

Total 5 questions with 4 options each. What is the probability of 3 correct answers.

Soln :

Let A_i be the event of correctly answering the question i.

Thus;

Final event : $(A_1 \cap A_2 \cap A_3 \cap A_4^c \cap A_5^c)$ C_1

$\cup \ (A_1 \cap A_2 \cap A_3^c \cap A_4^c \cap A_5)$ C_2

$\cup \ (A_1^c \cap A_2^c \cap A_3 \cap A_4 \cap A_5)$ C_3

$\vdots$

where C_i : ith configuration. Total configurations are $\boxed{5C_3}$

for, C_1 ;

$$P(C_1) = P(A_1 \cap A_2 \cap A_3 \cap A_4^c \cap A_5^c)$$

since. C_i s are independent events.

$$= P(A_1)\,P(A_2)\,P(A_3)\,P(A_4^c)\,P(A_5^c)$$

$$= p \cdot p \cdot p \cdot (1-p) \cdot (1-p)$$

$$= p^3 \cdot (1-p)^2 = (0.25)^3 (0.75)^2$$

Similarly; $P(C_2) = P(C_3) = \cdots P(C_{10})$ $\left\{ \binom{5}{3} = 10 \right\}$

Since, $C_1, C_2, \ldots C_{10}$ are disjoint events.

$$P(\text{Required}) = \sum_{i=1}^{10} P(C_i)$$

$$= \boxed{\binom{5}{3} \cdot (0.25)^3 (0.75)^2}$$