Practice Questions Solutions (11-15)

CSCI 3022

June 18, 2020

11. $P(C) = P(C|knows) \cdot P(knows) + P(C|gamble) \cdot P(gamble) = 1 \cdot 0.6 + \frac{1}{4} \cdot (1 - 0.6) = 0.7$
    $P(knows|C) = \frac{P(knows \cap C)}{P(C)} = \frac{0.6 \cdot 1}{0.7} = \frac{6}{7}$

12. (a) $P(B^c|A)$ means driver's blood alcohol percentage does not exceeds legal limit while the breath analyzer indicates that legal limit is exceeded.

    (b) $P(A) = P(B) \cdot P(A|B) + P(B^c) \cdot P(A|B^c) = 0.05 * 0.95 + (1 - 0.05) * (1 - 0.95) = 0.095$,
        $P(B^c|A) = \frac{P(B^c) \cdot P(A|B^c)}{P(A)} = \frac{(1-0.05)*(1-0.95)}{0.095} = 0.5$

    (c) $P(B|A) = \frac{P(B) \cdot P(B|A)}{P(A)}$, Therefore
        $0.9 = \frac{0.05 \cdot p}{0.05 \cdot p + (1-0.05) \cdot (1-p)}, p \approx 0.9942$

13. $P(A) = P(A|B) = \frac{1}{2}$
    $P(B|A \cup B) = \frac{P(B \cap A \cup B)}{P(A \cup B)} = \frac{P(B)}{P(A) + P(B) - P(A)P(B)}$
    $\frac{2}{3} = \frac{P(B)}{\frac{1}{2} + P(B) - \frac{1}{2}P(B)}, P(B) = \frac{1}{2}$

14. (a) $Y = X^2$, $P(Y = 0) = P(X = 0), P(Y = 1) = P(X = -1) + P(X = 1), P(Y = 4) = P(X = 2)$ so the probability mass function $p(Y)$ will be given below

Y=x^2	0	1	4
p(Y=x^2)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$

    (b) for X:
        $P(a = 1) = P(a = -1) + P(a = 0) + P(a = 1) = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$
        $P(a = \frac{3}{4}) = P(a = -1) + P(a = 0) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$
        $P(a = \pi - 3) = P(a = -1) + P(a = 0) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$
        for Y:
        $P(a = 1) = P(a = 0) + P(a = 1) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$
        $P(a = \frac{3}{4}) = P(a = 0) = \frac{1}{8}$
        $P(a = \pi - 3) = P(a = 0) = \frac{1}{8}$

15. (a)

$\hat{X}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
$p_{\hat{X}}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{7}{27}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{1}{27}$

    (b) $P(2draws) = \binom{2}{3} * \frac{1}{3} * \frac{1}{3} * \frac{2}{3} = \frac{2}{9}$