Section A

1. $f(x) = c\sqrt{x}$ $\qquad f(x) = \begin{cases} c\sqrt{x}, & x \in [0,4] \\ 0, & \text{otherwise} \end{cases}$

$$\underbrace{\int_{-\infty}^{0} f(x)dx}_{0} + \int_{0}^{4} f(x)\,dx + \underbrace{\int_{4}^{\infty} f(x)\,dx}_{0}$$

$$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

$$c\int_{0}^{4} \sqrt{x}\,dx = 1$$

$$\int \frac{x^{\frac{1}{2}+1}}{n+1} = \frac{2x^{\frac{3}{2}}}{3}\Big|_0^4$$

$$= \frac{2\cdot 0^{\frac{3}{2}}}{3} + \frac{2\cdot 4^{\frac{3}{2}}}{3} = 5.33 = \frac{16}{3}$$

$$c \cdot \frac{16}{3} = 1$$

$$\frac{}{\frac{16}{3}} \quad \frac{}{\frac{16}{3}}$$

$$\boxed{c = \frac{3}{16}}$$

2. a) $\frac{3+3+2+1+4+45+5}{7} = \boxed{9}$

b) $[\cancel{1}, \cancel{2}, \cancel{3}, \underline{3}, \cancel{4}, \cancel{5}, \cancel{45}]$ $\quad \boxed{3}$

c) The median is preffered to summarize the above data set because the median is not greatly affected (if at all) by the outlier 45. The mean however is greatly affected and causes the mean to not be highly representative of the entire data set.

3. $P(\frac{1}{3} \le x \le \frac{2}{3}) = \int_{\frac{1}{3}}^{\frac{2}{3}} f(x)dx = \int_{\frac{1}{3}}^{\frac{2}{3}} 2x - x^2$

$$= \int_{\frac{1}{3}}^{\frac{2}{3}} 2\int x - \int_{\frac{1}{3}}^{\frac{2}{3}} x^2 = 2\cdot\frac{x^2}{2}\Big|_{\frac{1}{3}}^{\frac{2}{3}} - \frac{x^3}{3}\Big|_{\frac{1}{3}}^{\frac{2}{3}} = x^2 - \frac{x^3}{3}\Big|_{\frac{1}{3}}^{\frac{2}{3}}$$

$$\Rightarrow x^2 - \frac{x^3}{3}\Big|_{\frac{1}{3}}^{\frac{2}{3}}$$

$$= \frac{20}{81}$$

$$\boxed{= \frac{20}{81}}$$

4. $E(3x^2+2) = 3E[x^2]+2$

$= 3(\frac{7}{3})+2$

$= 7+2$

$\boxed{= 9}$

$Var(x) = E[3x^2-2\mu]$
$= 3E[x^2]-2\mu$
$3 = 3E[x^2]-2(2)$
$7 = 3E[x^2]$
$\frac{7}{3} = E[x^2]$

5. $f(x) = \lambda e^{-\lambda x}$ $\quad x \sim Exp(0.5)$ $\quad P(x \geq 3)$
$f(x) = 0.5e^{-0.5x}$
$\int_3^{\infty} 0.5e^{-0.5x}dx = -e^{-0.5x}\Big|_3^{\infty} = 0-(-e^{-0.5 \cdot 3}) \approx .223$

$\boxed{\approx 0.223}$

6. $P(B \cap T) = 0.5$ $\quad P(B) = 0.6$ $\quad P(B \cup T) = 0.7$ $\quad P(T) = ?$
$P(B \cap T) = P(B) + P(T) - P(B \cup T)$
$P(B \cap T) - P(B) + P(B \cup T) = P(T)$
$0.5 - 0.6 + 0.7 = P(T)$
$\boxed{P(T) = 0.6}$

7. 36 possible rolls
6 can be made by: (1,5), (2,4), (3,3), (4,2), (5,1) = 5 ways

36 - 5 = 31. On average, it would take us 32 rolls to get a sum of rolls that equal exactly six.

8. $T = X + Y + Z$
$X = \frac{9+6}{2} = 12$
$Y = \frac{7+3}{2} = 5$
$Z = \frac{15+10}{2} = 12.5$

$T = 12 + 5 + 12.5 = 29.5$

$\boxed{T = 29.5}$

9. Standard Deviation $= \sqrt{12}$
average $= 65$
4.33 standard deviations to 80

There is a less than 1% (close to 0%) chance of a random classmate getting a score more than 80%

10. $56000 - 50{,}000 = 6{,}000$
$\frac{6{,}000}{12{,}000} = .5$ standard deviations away.

There is a 30.85% probability that your neiblarhood has an average income more than \$56,000.

Section B

1. a) $X_1 = \{1, 3, 4, 5, 6, 6, 7, 7, 9\}$ = $\boxed{\text{I for } X_1}$

median = 6 $Q_2 = 4$ $Q_3 = 7$

X_2 median = 6 $Q_2 = 5$ $Q_3 = 7$ = $\boxed{\text{H for } X_2}$

X_3 median = 5 $Q_2 = 2$ $Q_3 = 6$ = $\boxed{\text{A for } X_3}$

X_4 median = 5 $Q_2 = 4$ $Q_3 = 6$ = $\boxed{\text{F for } X_4}$

X_5 median = 6 $Q_2 = 3$ $Q_3 = 7$ = $\boxed{\text{E for } X_5}$

X_6 median = 5 $Q_2 = 3$ $Q_3 = 6$ = $\boxed{\text{C for } X_6}$

b) ~~1~~, ~~3~~, 3, ~~5~~, ~~5~~, 6, ~~6~~, ~~7~~, ~~8~~, ~~10~~, ~~11~~

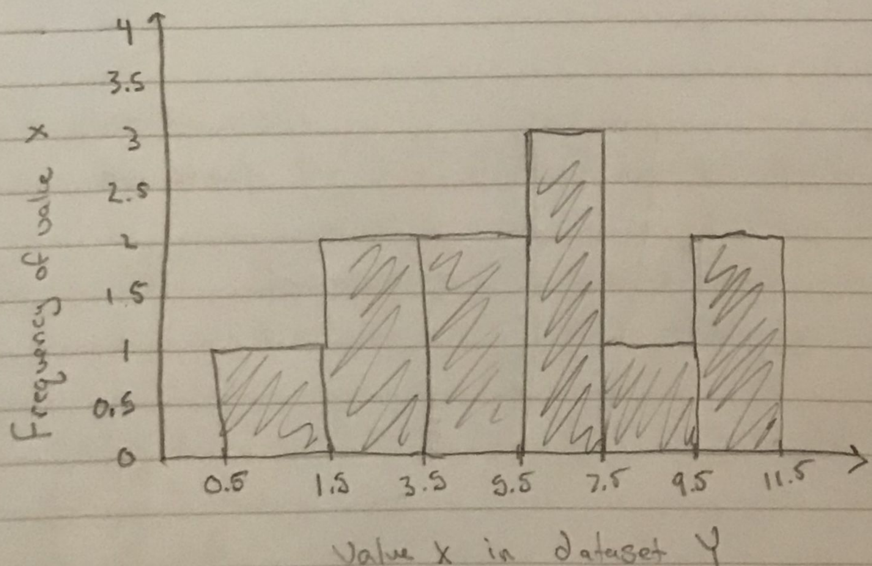
median = 6 $Q_3 = 8$

Q_2 Q_3 Q_4

1, 3, 3, 5, 5, 6 – 6, 6, 7 8, 10, 11

$\boxed{\text{median} = 6,\ Q_3 = 8}$

c)



Bin width 2
Start at -0.5

10

$\underline{H}$ $\underline{H}$ $\underline{HHH}$ $\underline{HHH}$ $\underline{H}$ $\underline{H}$

$\underline{T}$ $\underline{T}$ $\underline{TT}$ $\underline{TTT}$ $\underline{TTT}$

10

2. a) 2 fair coins.
8 coins total.
$\frac{2}{8} \rightarrow \boxed{\frac{1}{4} \text{ chance of fair coin.}}$

b) There is a $\frac{1}{2}$ chance the coin comes up heads.

c) P(F|R) - fair and TTH   P
P(UT|R) - unfair tails favored & TTH
P(UH|R) unfair heads favored & TTH
P(F) - fair die chosen
P(UT) - unfair tails favored die chosen
P(UH) - unfair heads favored die chosen
P(R) - roll was TTH

$P(R) = P(R|F) \cdot P(F) + P(R|UT) \cdot P(UT) + P(R|UH) \cdot P(UH)$

$= (.25 \cdot .25) + (.140625 \cdot .375) + (.046875 \cdot .375)$

$= .1328125$

$P(F|R) = P(R|F) \cdot \frac{P(F)}{P(R)} = .25 \cdot \frac{(1/4)}{.1328125} = .471$

The probability that the coin was fair given the flips TTH is .47%.

d) P(H) - probability of heads
P(F) - probability of fair
$P(H) = \frac{1}{2}$
$P(H|F) = \frac{1}{2}$

Since P(H) and P(H|F) are equal, the events are independent.

3. a) $E[2^x] = 2^0 \cdot (1-p) + 2^1 \cdot p$

$= 1(1-p) + 2p$

$= 1 - p + 2p$

$\boxed{= 1 + p}$

b) $E[X] = \frac{2}{2} + \frac{5}{2} = \boxed{3.5}$

c) $Y = X^4 \rightarrow -1^4 \cdot (\frac{1}{5}) + 0^4 \cdot (\frac{2}{5}) + 1^4 (\frac{2}{5})$

$\boxed{\begin{array}{l} y = 0, \ P(Y) = 2/5 \\ y = 1, \ P(Y) = 3/5 \end{array}}$

d) $E[Y] = E[X^4] = -1^4 \cdot (\frac{1}{5}) + 0^4 \cdot (\frac{2}{5}) + 1^4 (\frac{2}{5})$

$1 \cdot (\frac{1}{5}) + 0 + (\frac{2}{5})$

$\boxed{= \frac{3}{5}}$