Introduction to Data Science With Probability and Statistics

Lecture 5: Conditional Probability & Notebooks Review

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Birthday Problem

Your friend tells you that he was born on a long month, i.e month with 31 days, now you wonder what is the probability that his birth month has an ‘r’ in its name?

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$\Omega = \{Jan, Feb, Mar, \dots, Dec\}$; $|\Omega| = 12$

$L = \{\underline{Jan}, \underline{Mar}, May, July, Aug, \underline{oct}, \underline{Dec}\}$, $|L| = 7$

$R = \{\underline{Jan}, Feb, \underline{Mar}, Apr, Sep, \underline{Oct}, Nov, \underline{Dec}\}$, $|R| = 8$

$P(L) = \frac{|L|}{|\Omega|} = \frac{7}{12}$, $P(R \cap L) = \frac{|R \cap L|}{|\Omega|} = \frac{4}{12}$

$\therefore P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{4}{7}$ //

Die roll problem

You roll a standard die twice, but you can't see the results. Your friend who can see the results, tells you that the minimum value of the three rolls was 2. Now, you wonder what is the probability that the maximum of the three rolls was 4? Assuming each outcome being equally likely.

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Let; (i) The results of the first die be x and second be y

(ii) Given event be $A = \{\min(x,y) = 2\}$

(iii) Event $B = \{\max(x,y) = 4\}$

$\therefore$ Required probability is $P(B|A)$

Die roll problem

$\Omega = \{(1,1), (1,2), \ldots (2,1), (2,2) \ldots (6,6)\}$; $|\Omega| = 6^2 = 36$

- A = outcomes where $x = 2, y = \{2,3,4,5,6\}$ or $x = \{2,3,4,5,6\}, y = 2$

  $|A| = 9$.

- $A \cap B$ = $x=2, y=4$ or $x=4, y=2$. $|A \cap B| = 2$

$\therefore$ $P(A) = |A|/|\Omega| = 9/36$; $P(A \cap B) = |A \cap B|/|\Omega| = 2/36$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2}{9}$$

Multiplication Rule

While modelling situations or experiments which have a sequential character, it's often convenient to find the conditional probabilities first and use them to find the unconditional probabilities by restating the definition of conditional probability in the below format.

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

Multiplication Rule

The general form of the multiplication rule is stated below for the sequential events $A_1, A_2, .., A_n$

$$P(\bigcap_{i=1}^{n} A_i) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)...P(A_n|\bigcap_{i=1}^{n-1} A_i)$$

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Let; $A = \{$ first draw result is a red sock $\}$

$B = \{$ second draw result is a red sock $\}$

$\Omega = \{b, b, b, r, r, r, r, r\}$

$|\Omega| = 8$

We want : $P(A \cap B) = P(A) P(B|A)$.

$$= \frac{5}{8} \cdot \frac{4}{7} = \frac{5}{14}$$

Independence

An event A is said to be independent of event B if $P(A \mid B) = P(A)$

This definition, combined with the product rule and the definition of conditional probability, give us a few equivalent tests for independence of two events:

1. $P(A \mid B) = P(A)$
2. $P(B \mid A) = P(B)$
3. $P(A \cap B) = P(A)P(B)$

Events $A_1, A_2, \ldots, A_m$ are **independent** if $P(A_1 \cap A_2 \cap \ldots \cap A_m) = P(A_1)P(A_2)\ldots P(A_m)$

Independence

Example: Suppose you flip a fair coin twice. Let A= "Heads on flip 1", B = "Heads on flip 2", and C = "Same outcome on both flips". Are these events independent or dependent?

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We need to check wether;

$$\underset{1/2}{P(A)}\,\underset{1/2}{P(B|A)}\,\underset{1}{P(C|A\cap B)} \stackrel{?}{=} \underset{1/2}{P(A)}\,\underset{1/2}{P(B)}\,\underset{1/2}{P(C)}$$

For; $P(C)$: $|\Omega| = 4$; $\Omega = \{\underline{HH}, HT, TH, \underline{TT}\}$; $\therefore P(C) = 2/4 = 1/2$

For, $P(C|A\cap B)$ or $P(C|A,B)$

$A\cap B = \{HH\}$

$\therefore P(C|A,B) = 1.$