Practice Questions Solutions (1-10)

CSCI 3022

June 18, 2020

1. $P(C^c \cap D) = P(D) - P(C \cap D) = 0.4 - 0.2 = 0.2$

2. (a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{1}{3} + \frac{1}{2} - \frac{3}{4} = \frac{1}{12}$

   (b) $P(A^c \cup B^c) = P(A \cup B) - P(A \cap B) = \frac{3}{4} - \frac{1}{12} = \frac{2}{3}$

3. Since $P(1) = p, P(2) = p^2, P(1) + P(2) = 1$. Therefore, $p + p^2 = 1, p \approx 0.618$

4. $P(E) = P(E \cap G) + P(E \cap F) = \frac{2}{3}$

5. The sample space will be the set of all non-negative integers since we are interested in the number of people in the two queues, so the all the possible outcome will be the number of people, which is all non-negative intergers.

6. (a) The sample space $S = \{2, 3, 4, ...\}$ Since we are looking for the number of tosses it takes until a head occur for the second time, so it will start with 2.

   (b) $P(X = 5) = \binom{5-1}{1} p^1 (1-p)^{5-2} p = 4p^2 (1-p)^3$

7. $P(B_2) = P(B_2|C_1) \cdot P(C_1) + P(B_2|D_1) \cdot P(D_1) = \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{3} = \frac{7}{36}$

8. (a) $P(S_1) = \frac{13}{52}, P(S_2|S_1) = \frac{12}{51}, P(S_2|S_1^c) = \frac{13}{51}$

   (b) $P(S_2) = P(S_2|S_1) \cdot P(S_1) + P(S_2|S_1^c) \cdot P(S_1^c) = \frac{12}{51} \cdot \frac{13}{52} + \frac{13}{51} \cdot \frac{52-13}{52} = \frac{1}{4}$

9. $P(R_2) = P(R_2|R_1) \cdot P(R_1) = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$

10. $P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{3}{4} + \frac{2}{5} - \frac{4}{5} = \frac{7}{20}, P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{7}{15}$