

Introduction to Data Science With Probability and Statistics

Quiz 2 Solution

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Question 1

Your friend rolls a standard 6-sided die and he only tells you that the result was an odd number. What is the probability that the result was actually 3?

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

Let A : Event when the result is an odd no. $\{1, 3, 5\}$

B : Event of getting a 3. $\{3\}$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}$$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/6}{1/2} = \frac{1}{3} = \boxed{0.33}$$

Question 2

Ans = 0.625

There are 2 bags with green and purple socks. The configuration is:

- Bag 1 has 4 green and 4 purple socks. $\mathbb{P}(P_1 | B_1) = 4/(4+4)$
- Bag 2 has 4 green and 12 purple socks. $\mathbb{P}(P_2 | B_2) = 12/(12+4)$

You randomly pick one bag with equal probability and select one sock out of that bag. What is the probability that the sock is a purple one?

Let P : Total prob. that finally you pick up a purple sock

B_i : event of choosing Bag i

P_i : event of choosing purple sock from Bag i .

$$\begin{aligned}\therefore \mathbb{P}(P) &= \mathbb{P}(B_1 \cap P_1) + \mathbb{P}(B_2 \cap P_2) \\ &= \mathbb{P}(B_1) \mathbb{P}(P_1 | B_1) + \mathbb{P}(B_2) \mathbb{P}(P_2 | B_2) \\ &= (0.5)(4/8) + (0.5)(12/16)\end{aligned}$$

Question 3

For the above setting of socks and bags, you observe that the sock is indeed a purple one but you don't remember which bag you chose initially. What is the probability that it came from bag 1?

Priors, $\mathbb{P}(B_1) = \mathbb{P}(B_2) = 0.5$, Evidence = purple sock.

$$\therefore \mathbb{P}(B_1 | P) = \frac{\mathbb{P}(P \cap B_1)}{\mathbb{P}(P)} = \frac{\mathbb{P}(B_1) \mathbb{P}(P | B_1)}{\mathbb{P}(P)} = \boxed{0.40}$$

total prob. comp.
last slide.

Question 4

You toss a fair coin twice. X is a random variable which is equal to the number of heads. Write the probability mass function of X .

$$\Omega = \{ HH, HT, TH, TT \}$$

Handwritten arrows indicate the mapping from outcomes to the number of heads:

- $HH \rightarrow 2$
- $HT, TH \rightarrow 1$
- $TT \rightarrow 0$

$$X = \{ 2, 1, 0 \}$$

x	$P(X=x)$
0	$1/4$
1	$1/2$
2	$1/4$

$$P(HH) = 1/4$$

$$P(\{HT, TH\}) = 1/2$$

$$P(TT) = 1/4.$$