Practice Questions

CSCI 3022

June 18, 2020

1. Let C and D be two events for which one knows that $P(C) = 0.3$, $P(D) = 0.4$, and $P(C \cap D) = 0.2$. What is $P(C^c \cap D)$?

2. When $P(A) = 1/3$, $P(B) = 1/2$, and $P(A \cup B) = 3/4$, what is
   (a) $P(A \cap B)$?
   (b) $P(A^c \cup B^c)$?

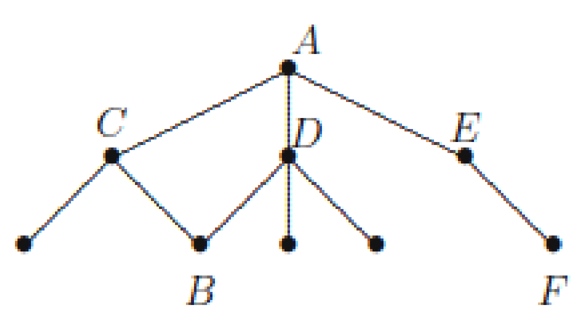
3. An experiment has only two outcomes. The first has probability p to occur, the second probability p^2. What is p?

4. Three events E, F, and G cannot occur simultaneously. Further it is known that $P(E \cap F) = P(F \cap G) = P(E \cap G) = 1/3$. Can you determine $P(E)$?

5. A post office has two counters where customers can buy stamps, etc.If you are interested in the number of customers in the two queues that will form for the counters, what would you take as sample space?

6. We repeatedly toss a coin. A head has probability p, and a tail probability $1-p$ to occur, where $0 < p < 1$.The outcome of the experiment we are interested in is the number of tosses it takes until a head occurs for the *second* time.
   (a) What would you choose as the sample space?
   (b) What is the probability that it takes 5 tosses?

7. Your lecturer wants to walk from A to B (see the map). To do so, he first randomly selects one of the paths to C, D,or E. Next he selects randomly one of the possible paths at that moment (so if he first selected the path to E, he can either select the path to A or the path to F), etc. What is the probability that he will reach B after two selections?



8. We draw two cards from a regular deck of 52. Let S_1 be the event "the first one is a spade," and S_2 "the second one is a spade."
   (a) Compute $P(S_1)$, $P(S_2|S_1)$, and $P(S_2|S_1^c)$
   (b) Compute $P(S_2)$ by conditioning on whether the first card is a spade.

9. A ball is drawn at random from an urn containing one red and one white ball. If the white ball is drawn, it is put back into the urn. If the red ball is drawn, it is returned to the urn together with two more red balls. Then a second draw is made. What is the probability a red ball was drawn on *both* the first and the second draws?

10. A certain grapefruit variety is grown in two regions in southern Spain. Both areas get infested from time to time with parasites that damage the crop. Let A be the event that region R_1 is infested with parasites and B that region R_2 is infested. Suppose $P(A) = 3/4$, $P(B) = 2/5$ and $P(A \cup B) = 4/5$. If the food inspection detects the parasite in a ship carrying grapefruits from R_1, what is the probability region R_2 is infested as well?

11. A student takes a multiple-choice exam. Suppose for each question he either knows the answer or gambles and chooses an option at random. Further suppose that if he knows the answer, the probability of a correct answer is 1, and if he gambles this probability is 1/4. To pass, students need to answer atleast 60% of the questions correctly. The student has "studied for a minimal pass," i.e., with probability 0.6 he knows the answer to a question. Given that he answers a question correctly, what is the probability that he actually *knows* the answer?

12. A breath analyzer, used by the police to test whether drivers exceed the legal limit set for the blood alcohol percentage while driving, is known to satisfy

$$P(A|B) = P(A^c|B^c) = p,$$

where A is the event "breath analyzer indicates that legal limit is exceeded" and B "driver's blood alcohol percentage exceeds legal limit." On Saturday night about 5% of the drivers are known to exceed the limit.

    (a) Describe in words the meaning of $P(B^c|A)$.
    (b) Determine $P(B^c|A)$ if $p = 0.95$.
    (c) How big should p be so that $P(B|A) = 0.9$?

13. Two independent events A and B are given, and $P(B|A \cup B) = 2/3$, $P(A|B) = 1/2$. What is $P(B)$?

14. Let X be a discrete random variable with probability mass function p given by:

a	-1	0	1	2
$p(a)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

and $p(a) = 0$ for all other a.

    (a) Let the random variable Y be defined by $Y = X^2$, i.e., if $X = 2$,then $Y = 4$. Calculate the probability mass function of Y.
    (b) Calculate the value of the distribution functions of X and Y in $a = 1$, $a = 3/4$, and $a = \pi - 3$.

15. Three times we randomly draw a number from the following numbers:

$$1 \; 2 \; 3$$

If X_i represents the ith draw, $i = 1, 2, 3$, then the probability mass function of X_i is given by

a	1	2	3
$\mathrm{P}(X_i = a)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

and $P(X_i = a) = 0$ for all other a. We assume that each draw is independent of the previous draws. Let $\bar{X}$ be the average of X_1, X_2, and X_3, i.e.,

$$\bar{X} = \frac{X_1 + X_2 + X_3}{3}$$

(a) Determine the probability mass function $p_{\bar{X}}$ of $\bar{X}$.

(b) Compute the probability that exactly two draws are equal to 1.

16. A shop receives a batch of 1000 cheap lamps. The odds that a lamp is defective are 0.1%. Let X be the number of defective lamps in the batch.

    (a) What kind of distribution does X have? What is/are the value(s) of parameter(s) of this distribution?

    (b) What is the probability that the batch contains no defective lamps? One defective lamp? More than two defective ones?

17. You decide to play monthly in two different lotteries, and you stop playing as soon as you win a prize in one (or both) lotteries of at least one million euros. Suppose that every time you participate in these lotteries, the probability to win one million (or more) euros is p_1 for one of the lotteries and p_2 for the other. Let M be the number of times you participate in these lotteries until winning at least one prize. What kind of distribution does M have, and what is its parameter?

18. You and a friend want to go to a concert, but unfortunately only one ticket is still available. The man who sells the tickets decides to toss a coin until heads appears. In each toss heads appears with probability p, where $0 < p < 1$, independent of each of the previous tosses. If the number of tosses needed is odd, your friend is allowed to buy the ticket; otherwise you can buy it. Would you agree to this arrangement?

19. We throw a coin until a head turns up for the second time, where p is the probability that a throw results in a head and we assume that the outcome of each throw is independent of the previous outcomes. Let X be the number of times we have thrown the coin.

    (a) Determine $P(X = 2)$, $P(X = 3)$, and $P(X = 4)$.

    (b) Show that $P(X = n) = (n-1)p^2(1-p)^{n-2}$ for $n \geq 2$.

20. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{4} & for\ \ 0 \leq x \leq 1 \\ \frac{1}{4} & for\ \ 2 \leq x \leq 3 \\ 0 & elsewhere \end{cases}$$

    (a) Draw the graph of f.

    (b) Determine the distribution function F of X, and draw its graph.

21. Let a continuous random variable X be given that takes values in $[0, 1]$, and whose distribution function F satisfies

$$F(x) = 2x^2 - x^4 \quad for\ \ 0 \leq x \leq 1.$$

(a) Compute $P(\frac{1}{4} \leq X \leq \frac{3}{4})$.

(b) What is the probability density function of X?

22. Let X have an $Exp(0.2)$ distribution. Compute $P(X > 5)$.

23. The score of a student on a certain exam is represented by a number between 0 and 1. Suppose that the student passes the exam if this number is at least 0.55. Suppose we model this experiment by a continuous random variable S, the score, whose probability density function is given by

$$f(x) = \begin{cases} 4x \;\; for \;\; 0 \leq x \leq \frac{1}{2} \\ 4\text{–}4x \;\; for \;\; \frac{1}{2} \leq x \leq 1 \\ 0 \;\; elsewhere \end{cases}$$

(a) What is the probability that the student fails the exam?

(b) What is the score that he will obtain with a 50% chance, in other words, what is the $50th$ percentile of the score distribution?

24. Compute the median of an $Exp(\lambda)$ distribution.