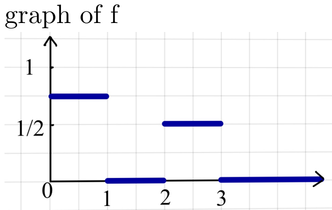
Practice Questions Solutions (16-20)

CSCI 3022

June 18, 2020

16. (a) It is a binomial distribution, $X \sim Bin(n,p) n = 1000, p = 0.1\%$
    (b) The probability that the batch contains no defective lamps is
    $P(X = 0) = \binom{0}{1000} \cdot 0.001^0 \cdot (1 - 0.001)^1 000 \approx 0.3677$
    $P(X = 1) = \binom{1}{1000} \cdot 0.001^1 \cdot (1 - 0.001)^1 000 - 1 \approx 0.3681$
    $P(X > 2) = 1 - P(X = 0) - P(X = 1) - P(X = 2) \approx 0.802$
17. It is a geometric distribution $Geo(p)$ with parameter $p = 1 - (1 - p_1)(1 - p_2)$
18. You would not agree to this arrangement.
    Let X be the number of tossing until a head appears, X has a geometric distribution $Geo(p)$ the pmf will be given as $p(k) = P(X = k) = (1 - p)^{k-1}p, k = 1, 2, 3...$
    $P(\text{you friend}) = \sum_{m=0}^{\infty} p_x(k = 2m + 1) = \sum_{m=0}^{\infty}(1 - p)^{2m}p = \frac{p}{1-(1-p)^2}$
    $P(\text{you}) = \sum_{m=0}^{\infty} p_x(k = 2m) = \sum_{m=0}^{\infty}(1 - p)^{2m-1}p = \frac{p(1-p)}{1-(1-p)^2}$
    Since $0 < p < 1$, so $P(\text{you friend}) > P(\text{you})$, so it is not a fair arrangement.
19. (a) $P(X = 2) = p^2, P(X = 3) = C_2^1 p^1(1 - p)^{2-1}p = 2p^2(1 - p)$,
    $P(X = 4) = C_1^3 p(1 - p)^{3-1}p = 3p^2(1 - p)^2$
    (b) for X=n, $P(X = n) = C_{n-1}^1 p(1 - p)^{n-1}p = (n - 1)p^2(1 - p)^{n-2}$
20. (a) graph of f



    (b) the distribution function $F = \begin{cases} 0 & x < 0 \\ \frac{3}{4}x & 0 \le x \le 1 \\ 0.75 & 1 < x < 2 \\ \frac{3}{4} + \frac{x-2}{4} & 2 \le x \le 3 \\ 1 & x > 3 \end{cases}$