Name: Sahib Bajwa

ID: 107553096

CSCI 3104, Algorithms **Profs. Chen & Grochow**

Final Exam S16–S17 **Spring 2020, CU-Boulder**

Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit. Questions posted to Piazza **must be posted as PRIVATE QUESTIONS.** Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these are grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

TIMING: If you are not attempting all the standards in a given quiz, please only use the ordinary amount of time for the number of standards you attempt. For example, if you are only attempting one standard on a 4-standard quiz, please only use 30 min (or 38 for 1.5x, 45 for 2x).

YOU MUST SIGN THE HONOR PLEDGE. Your quiz will otherwise not be graded.
Honor Pledge: On my honor, I have not used any outside resources (other than my notes and book), nor have I given any help to anyone completing this assignment.

Your Name: Sahib Bajwa

Quicklinks: 16 16a 16b 16c 17

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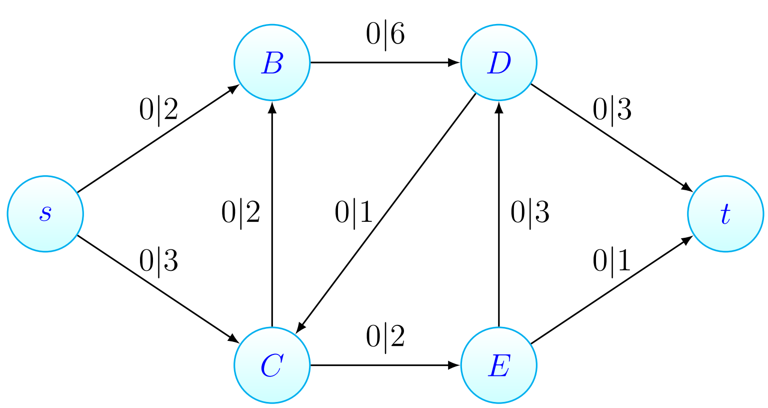
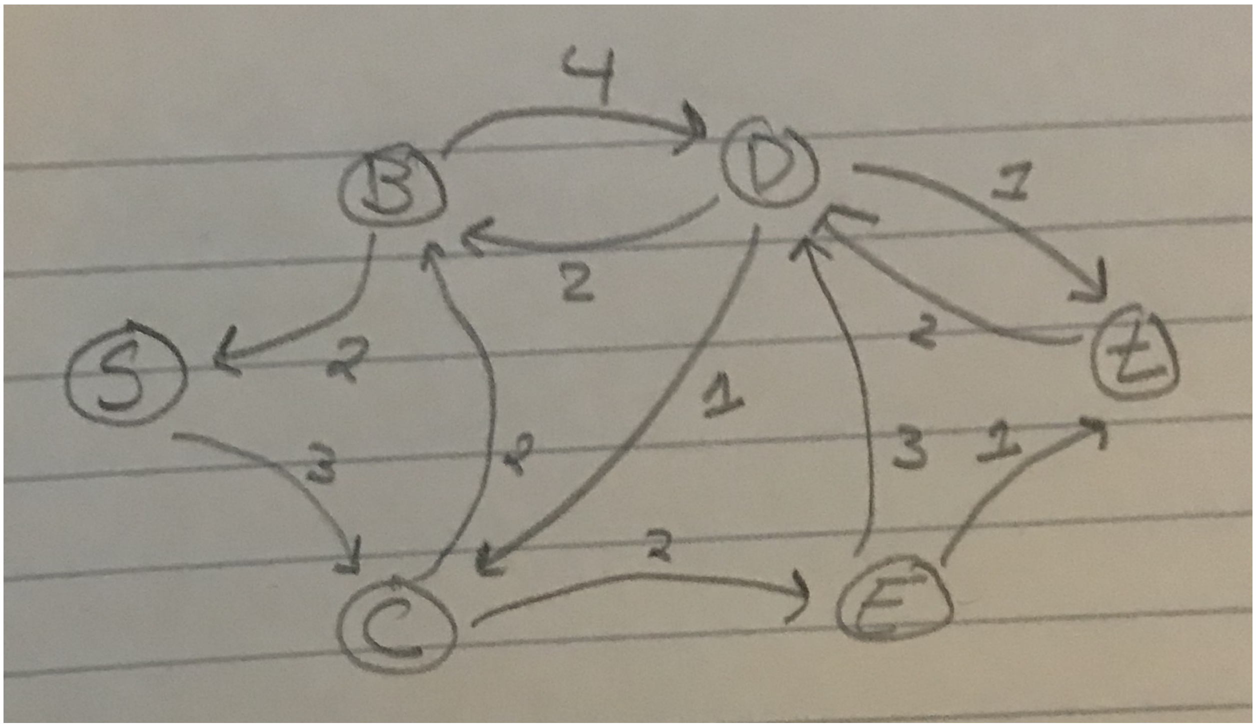
16. **Standard 16.** Given the following network G with edge capacities and an initial flow of 0 ($f|c$ on an edge denotes that there is currently flow of f on the edge, and the edge has total capacity c). For the first iteration of the Ford-Fulkerson algorithm, we **select the path** $s \to B \to D \to t$ **(Do not choose the first s-t path on your own—use this one!).**



THIS QUESTION HAS THREE PARTS, (a)–(c).

(a) Draw the residual graph after we select the $s \to B \to D \to t$ flow-augmenting path.
(BEGIN YOUR ANSWER ON THE NEXT PAGE.)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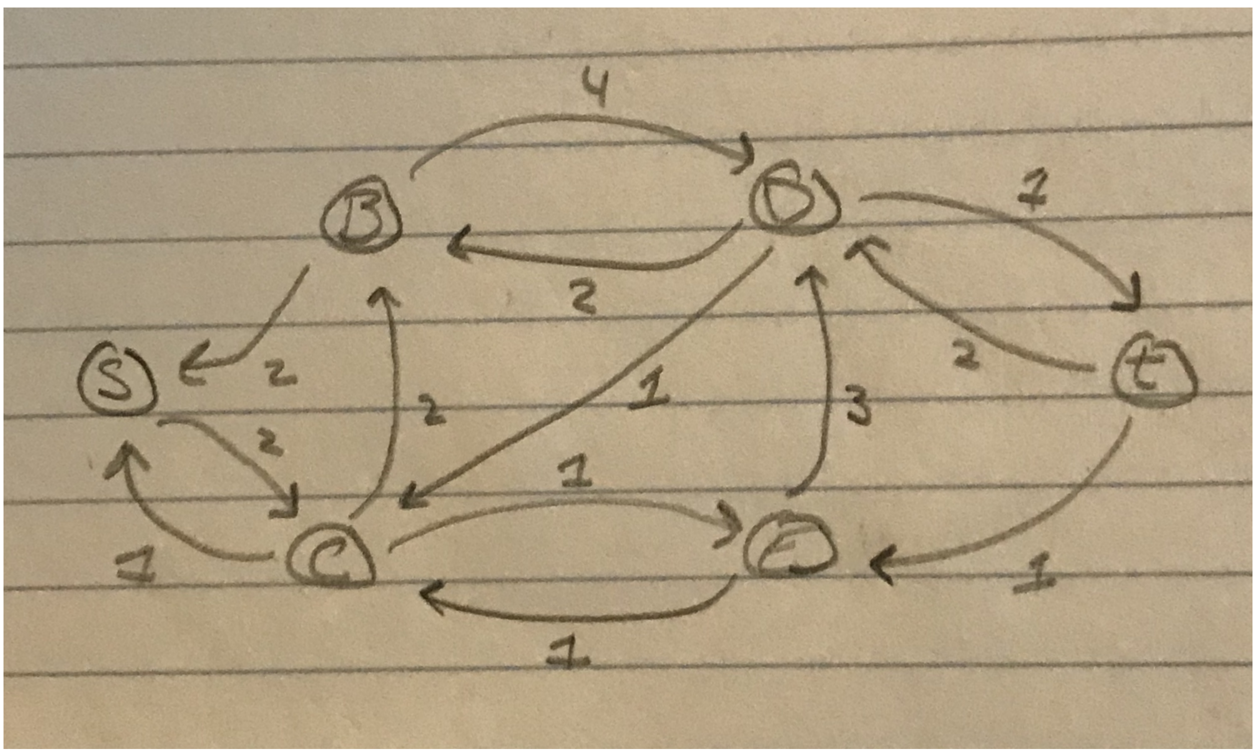
(b) Using $s \to C \to E \to t$ as the next flow-augmenting path, how much flow can you push along it? (Assuming you have already pushed flow along the $s \to B \to D \to t$ path as in (a).) Draw the residual graph after flow has been pushed along this path and the previous one.

(BEGIN YOUR ANSWER ON THE NEXT PAGE.)

You can push 1 flow along the path $s \to C \to E \to t$. This is because the edge from $E \to t$ has a capacity of 1. This capacity is the smallest of all edges from $s \to C \to E \to t$, so we can only push 1 flow through this path.



(c) Starting using the two paths from (a) and (b), now complete the Ford–Fulkerson Algorithm. Your answer should consist of the following:

- The flow-augmenting paths you selected (*after* those from (a) and (b)) and the amount of flow you pushed through each path.
- The maximum $s \to t$ flow.
- The corresponding minimum cut, as well as the capacity of the cut.

You do not need to draw the updated flow networks or residual graphs, though you are welcome to do so if you wish.

(BEGIN YOUR ANSWER ON THE NEXT PAGE.)

The next flow-augmenting path will be $s \to C \to E \to D \to t$. We can push 1 unit of flow through this path.

There are no more flow-augmetning paths due to there being no paths left to push flow from $s \to t$.

The maximum flow from $s \to t$ is 4.

The corresponding minimum cut includes the edge from $D \to t$ (Dt) and the edge from $E \to t$ (Et). The capacity of this cut is 4, which equals the max flow (which is 4).

17. **Standard 17.** Consider a WiFi network with a set P of access points (AP) and a set U of mobile users. Each AP i can only serve a subset $U_i \subset U$ of users, and each user j can only be served by a subset $P_j \subset P$ of APs. However, at any time, an AP can serve at most one user, and a user can be served by at most one AP.

    Design an algorithm to find the maximum number of users that can be served simultaneously, by describing how to reduce the preceding problem to the (one-source, one-sink) max-flow problem from class. Your description should be **general**, and not tied to a specific example. (You may illustrate with an example for expository purposes, but an example alone is not sufficient. E.g., "This is how my construction is performed in general. Then for example, this is how we apply the construction to a particular graph I selected.").

    (BEGIN YOUR ANSWER ON THE NEXT PAGE.)

YOUR ANSWER HERE FOR STANDARD 17. (YOU CAN DELETE ALL THIS TEXT IN CAPS.)

IF YOU ARE HANDWRITING AND INSERTING AN IMAGE, SEE THE COMMENTED CODE BELOW IN THE .TEX FILE. PLEASE BE SURE TO ROTATE YOUR IMAGE TO THE CORRECT ORIENTATION (CAN BE DONE IN THE LATEX DIRECTLY; SEE COMMENTS.)