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CSCI 3104, Algorithms **Profs. Chen & Grochow**
Exam 2 – S17 **Spring 2020, CU-Boulder**

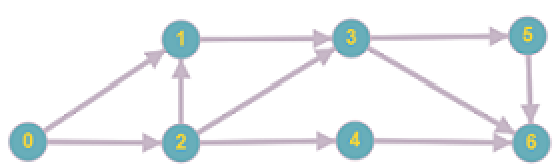
Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit (30 min for 1x, 37.5 min for 1.5x, 45 min for 2x). Questions posted to Piazza **must be posted as PRIVATE QUESTIONS.** Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these are grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

YOU MUST SIGN THE HONOR PLEDGE. Your quiz will otherwise not be graded. Honor Pledge: On my honor, I have not used any outside resources (other than my notes and book), nor have I given any help to anyone completing this assignment.

Your Name: Sahib Bajwa

Standard 17. We say that two $i \rightsquigarrow j$ paths are *edge-disjoint* if they do not share any common edges. Note however, these paths can (and in fact, often do) share common vertices (aside from i and j). As an example, consider the following graph.

- Observe that $0 \to 1 \to 3 \to 5 \to 6$ and $0 \to 2 \to 3 \to 6$ are edge-disjoint paths, as they do not share any directed edges. It is fine that they share the common vertex 3.
- Note however that $0 \to 2 \to 4 \to 6$ and $0 \to 2 \to 3 \to 6$ are **not** edge-disjoint paths, as they both share the $(0, 2)$ edge.



Consider the following problem.

- Input: A directed graph $G(V, E)$, as well as a start node i and an end node j.
- Solution: We seek to find a set $\mathcal{P}$ of $i \to j$ paths such that any two distinct paths $P_1, P_2 \in \mathcal{P}$ are edge-disjoint, and $|\mathcal{P}|$ is maximum. That is, we seek to find a maximum set of edge-disjoint $i \to j$ paths.

(a) Describe how to reduce the above problem to the (one-source, one-sink) max-flow problem from class. Your description should be **general**, and not tied to a specific example. (You may illustrate with an example for expository purposes, but an example alone is not sufficient. E.g., "This is how my construction is performed in general. Then for example, this is how we apply the construction to the graph I selected.")

To reduce this problem to a one-source, one-sink problem, we need to create two new vertexes. The first vertex s we create will have a directed edge with infinite capacity toward whatever vertex we are starting at (i). The second vertex t we create will have a directed edge towards t from whatever vertex we are ending at (j). We will then set every edge within the directed graph G to have a capacity of 1.
Now to find a maximum set of edge-disjoint $i \to j$ paths, we simply push 1 flow from $s \to t$ and record each path we get after pushing 1 flow. Doing this, we fill any edge we go through when going from $i \to t$, so the next time we push any flow, we do not use the same edge again. Once we can no longer push any flow, we are done and all paths we recorded after each push of 1 flow are edge-disjoint. The amount of edge-disjoint paths we have total will be equal to the flow from $s \to i$ or the flow from $j \to t$.

(b) Using your reduction, find a maximum set of edge-disjoint paths from $0 \rightsquigarrow 6$ in the graph above. Show your work, as well as your final answer. Note that there may be multiple maximum-size sets $\mathcal{P}$ in the graph above; you need only find one such set $\mathcal{P}$ of edge-disjoint paths, as long as it has the largest number of paths possible.

First we will push 1 flow from $s \to t$ and it will take the path of: $s \to 0 \to 1 \to 3 \to 5 \to 6 \to t$.
Add this path excluding s and t to $\mathcal{P}$.
Second we will push 1 flow from $s \to t$ and it will take the path of: $s \to 0 \to 2 \to 4 \to 6 \to t$.
Add this path excluding s and t to $\mathcal{P}$.
Since there are no more possible paths to push 1 more flow out of vertex 0, we have found the maximum set of edge-disjoint paths from $0 \to 6$ and a set $\mathcal{P}$ of edge-disjoint paths.
We have a maximum number of 2 edge-disjoint paths from $0 \to 6$ and 2 of those possible paths are in $\mathcal{P}$.