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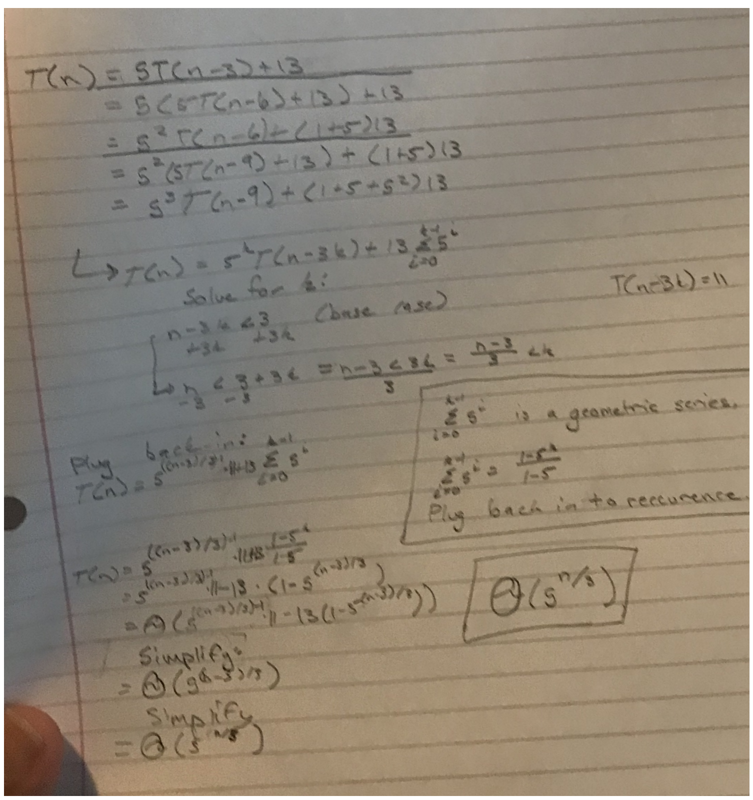
CSCI 3104, Algorithms
Requiz Standard 6, Version A

Profs. Chen & Grochow
Spring 2020, CU-Boulder

Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit (30 min for 1x, 37.5 min for 1.5x, 45 min for 2x). Questions posted to Piazza **must be posted as PRIVATE QUESTIONS.** Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited and will count as violations of the academic honor code. Such violations are, at a minimum, grounds to receive a 0. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

Standard 6. *Using the plug-in/unroll/substitution method, solve the following recurrence relation.*

$$T(n) = \begin{cases} 11 & : n < 3, \\ 5T(n-3) + 13 & : n \geq 3. \end{cases} \tag{1}$$



$T(n) = 5T(n-3) + 13$

$= 5(5T(n-6) + 13) + 13$

$= 5^2 T(n-6) + (1+5)13$

$= 5^2(5T(n-9) + 13) + (1+5)13$

$= 5^3 T(n-9) + (1+5+5^2)13$

$\to T(n) = 5^k T(n-3k) + 13 \sum_{i=0}^{k-1} 5^i$

Solve for k: (base case) $T(n-3k) = 11$

$n - 3k < 3$
$+3k \quad +3k$

$\to n < 3 + 3k = n - 3 < 3k = \frac{n-3}{3} < k$
$-3 \quad -3$

$\sum_{i=0}^{k-1} 5^i$ is a geometric series.

$\sum_{i=0}^{k-1} 5^i = \frac{1-5^k}{1-5}$

Plug back in to reccurence.

Plug back in: $T(n) = 5^{((n-3)/3)+1} \cdot 11 + 13 \sum_{i=0}^{k-1} 5^i$

$T(n) = 5^{((n-3)/3)+1} \cdot 11 + 13 \frac{1-5^k}{1-5}$

$= 5^{((n-3)/3)+1} \cdot 11 - 13 \cdot (1 - 5^{(n-3)/3})$

$= \Theta(5^{((n-3)/3)+1} \cdot 11 - 13(1 - 5^{(n-3)/3}))$

Simplify:

$= \Theta(5^{(n-3)/3})$

Simplify

$= \Theta(5^{n/3})$

$\boxed{\Theta(5^{n/3})}$