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CSCI 3104, Algorithms **Profs. Chen & Grochow**

Exam 2 – S19 **Spring 2020, CU-Boulder**

Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit (30 min for 1x, 37.5 min for 1.5x, 45 min for 2x). Questions posted to Piazza **must be posted as PRIVATE QUESTIONS.** Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these are grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

YOU MUST SIGN THE HONOR PLEDGE. Your quiz will otherwise not be graded. Honor Pledge: On my honor, I have not used any outside resources (other than my notes and book), nor have I given any help to anyone completing this assignment.

Your Name: Sahib Bajwa

Standard 19. Given a directed graph $G = (V, E)$ with positive weight $c_e > 0$ for each edge $e \in E$, you are asked to find the shortest path from a source $s \in V$ to a destination $t \in V$. For each node $v \in V$, denote by d_v the cost of shortest path from v to the destination t. Write down the optimal substructure property (equivalently, the recurrence for the optimal solution) in terms of d_v for this single-source-single-destination shortest path problem. Justify adequately your answer,

Case 1: edge e is in the shortest path.
If this is true, then we add edge e to d_v and add its weight c_e. Remove vertices that we can no longer use {p(e) + 1, p(e) + 2, ... , e - 1)}.
Must include optimal solution to problem consisting of remaining edges 1, 2, ..., p(e).

Case 2: edge e is not in the shortest path.
If this is true, then we discard edge e.

d_v(e) =
0 if e = 0
min $\{c_e + d_v(\text{p(e)}), d_v(\text{e - 1})\}$ otherwise