Name: Sahib Bajwa

ID: 107553096

CSCI 3104, Algorithms **Profs. Chen & Grochow**

Problem Set 11 – Due Wed April 29 11:55pm **Spring 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Informal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solutions:

- All submissions must be typed.
- You should submit your work through the **class Canvas page** only.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please allot at least as many pages per problem (or subproblem) as are allotted in this template.

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1. Indiana Jones is gathering n artifacts from a tomb, which is about to crumble and needs to fit them into 5 cases. Each case can carry up to W kilograms, where W is fixed. Suppose the weight of artifact i is the positive integer w_i. Indiana Jones needs to decide if he is able to pack all the artifacts. We formalize the Indiana Jones decision problem as follows.

   - Instance: The weights of our n items, $w_1, \ldots, w_n > 0$.
   - Decision: Is there a way to place the n items into different cases, such that each case is carrying weight at most W?

   Show that Indiana Jones $\in$ NP.

   The Indiana jones problem is np because we can check the solution to the problem in polynomial time. We accomplish this by doing the following check:

   If the weight of any case, W, is greater than or equal to the weight of all items contained within it, then the solution is correct. If the weight of any case, W, is less than the weight of all items contained within it, then the solution is incorrect.

   Since we are going through each item and summing them (then checking the sum to the weight of the case they are in), we are checking the solution in polynomial time. Thus, Indiana Jones $\in$ NP.

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2. A student has a decision problem L, which they know belongs to NP. This student wishes to show that L is NP-Complete. They attempt to do so by constructing a polynomial time reduction from L to SAT, a known NP-Complete problem. That is, the student attempts to show that $L \leq_p$ SAT. Determine if this student's approach is correct and justify your answer.

   The student's approach is correct. This is because if the student is able to reduce the problem to SAT, then it is as hard as SAT. if we have an algorithm that solves SAT, and L is a special case of SAT, then our algorithm can solve the hardest cases in L. Thus, if $L \leq_p$ SAT, then L is NP-Complete.