Name: $\boxed{\text{Sahib Bajwa}}$
ID: $\boxed{107553096}$

**CSCI 3104, Algorithms** | **Profs. Chen & Grochow**
**Final Exam S6–S8** | **Spring 2020, CU-Boulder**

**Instructions:** This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit. Questions posted to Piazza **must be posted as PRIVATE QUESTIONS.** Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these are grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

**TIMING:** If you are not attempting all the standards in a given quiz, please only use the ordinary amount of time for the number of standards you attempt. For example, if you are only attempting one standard on a 4-standard quiz, please only use 30 min (or 38 for 1.5x, 45 for 2x).

**YOU MUST SIGN THE HONOR PLEDGE. Your quiz will otherwise not be graded.**
**Honor Pledge:** On my honor, I have not used any outside resources (other than my notes and book), nor have I given any help to anyone completing this assignment.

**Your Name:** Sahib Bajwa

Quicklinks: 6 7 8

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6. **Standard 6.** *Use the* **substitution/unrolling** *method to determine a good asymptotic upper bound on the recurrence*

$$T(n) = \begin{cases} 5, & n < 4 \\ 7T(n-4) + 5, & n \geq 4. \end{cases} \tag{1}$$

(BEGIN YOUR ANSWER ON THE NEXT PAGE.)

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6 $T(n) = 7T(n-4)+5$

$= 7(7T(n-8)+5)+5$

$= 7^2 T(n-8) + (1+7)\cdot 5$

$= 7(7^2 T(n-12)+5) + (1+7)\cdot 5.$

$= 7^3 T(n-12) + (1+7+7^2)\cdot 5$

$\hookrightarrow T(n) = 7^k T(n-4k) + 5 \sum_{i=0}^{k-1} 7^i$

Solve for k:

$n-4k < 4$ (base case)
$+4k \quad +4k$

$= n < 4+4k \quad = n-4 < 4k = \frac{n-4}{4} < k$
$-4 \quad -4 \qquad\quad /4$

$T(n-4k) = 5$ (base case)

Plug back in:

$T(n) = 7^{((n-4)/4)-1} \cdot 5 + 5 \sum_{i=0}^{k-1} 7^i$

$\sum_{i=0}^{k-1} 7^i$ is a geometric series

$\sum_{i=0}^{k-1} 7^i = \frac{1-7^k}{1-7}$

Plug back into reccurance.

$T(n) = 7^{((n-4)/4)-1} \cdot 5 + 5 \cdot \frac{1-7^k}{1-7}$

$= 7^{((n-4)/4)\cancel{-1}} \cancel{\cdot 5} - \cancel{5} \cdot (\cancel{1} - 7^{(n-4)/4})$

$= \Theta(7^{(n-4)/4})$

or

$= \Theta(7^{n/4})$

$\boxed{\Theta(7^{n/4})}$

7. **Standard 7.** *Use* **recursion tree method** *to determine a good asymptotic upper bound on the following recurrence.*

$$T(n) = \begin{cases} 1, & n < 2 \\ 8T(n/4) + n^2, & \text{otherwise.} \end{cases}$$

(BEGIN YOUR ANSWER ON THE NEXT PAGE.)

YOUR ANSWER HERE FOR STANDARD 7. (YOU CAN DELETE ALL THIS TEXT IN CAPS.)

IF YOU ARE HANDWRITING AND INSERTING AN IMAGE, SEE THE COMMENTED CODE BELOW IN THE .TEX FILE. PLEASE BE SURE TO ROTATE YOUR IMAGE TO THE CORRECT ORIENTATION (CAN BE DONE IN THE LATEX DIRECTLY; SEE COMMENTS.)

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8. **Standard 8.** *Given an array $A[1, ..., n]$, we say it contains a zero-sum triple if there are three distinct indices $i \neq j \neq k$ such that $A[i] + A[j] + A[k] = 0$. Consider the following divide and conquer algorithm for counting the number of zero-sum triples.*

```
countZeroSumTriples(A, p, q):
        // base cases
        if q < p+2 { return 0; } // fewer than 3 elements

        if q == p+2 { // exactly 3 elements
          if A[p] + A[p+1] + A[p+2] == 0 {
            return 1;
          } else {
            return 0;
          }
        }

        // recursive case: more than 3 elements
        if q > p+2 {
            r = floor(p + (q-p)/3)
            s = floor(p + 2(q-p)/3)
            L = countZeroSumTriples(A, p, r):
            M = countZeroSumTriples(A, r+1, s):
            R = countZeroSumTriples(A, s+1, q)
            return L + M + R
        }
```

*Will the above algorithm return the correct number of zero-sum triples?* **Explain and justify** *your answer. If the algorithm does not perform correctly, give an example to illustrate that the algorithm fails.*

(BEGIN YOUR ANSWER ON THE NEXT PAGE.)

YOUR ANSWER HERE FOR STANDARD 8. (YOU CAN DELETE ALL THIS TEXT IN CAPS.)

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