Name:

ID:

CSCI 3104, Algorithms **Profs. Chen & Grochow**

Problem Set 2 – Due Thurs Jan 30 11:55pm **Spring 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Informal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solutions:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this template of at least 9 pages (or Gradescope has issues with it).

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1. *Name (a) one advantage, (b) one disadvantage, and (c) one alternative to worst-case analysis. For (a) and (b) you should use full sentences.*

   There are many reasonable answers to all of these. Here are some good ones:

   (a) It is often easier to derive a worst-case upper bound than average-case or typical-case. Worst-case upper bounds give absolute guarantees.

   (b) The worst case may not arise in practice, so a worst-case bound can be far from runtimes in practice. It can be difficult to actually construct a worst-case example, and thus to show that a worst-case bound is tight.

   (c) Average-case, relative to various distributions. Typical-case. Best-case (although best-case isn't usually a good measure of performance). Worst-case analysis on restricted kinds of inputs.

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2. *For each part of this question, put the growth rates in order, from slowest-growing to fastest. That is, if your answer is $f_1(n), f_2(n), \ldots, f_k(n)$, then $f_i(n) \leq O(f_{i+1}(n))$ for all i. If two adjacent ones are asymptotically the same (that is, $f_i(n) = \Theta(f_{i+1}(n))$), you must specify this as well.*

   Justify your answer (show your work). You may assume transitivity: if $f(n) \leq O(g(n))$ and $g(n) \leq O(h(n))$, then $f(n) \leq O(h(n))$, and similarly for little-oh, etc.

   (a) *Polynomials.*

   $$n+1 \qquad n^4 \qquad 1/n \qquad 1 \qquad n^2+2n-4 \qquad n^2 \qquad \sqrt{n} \qquad 10^{100}$$

   Correct order (two functions that are Θ of one another are listed together in {braces}:

   $$1/n \qquad \{1, 10^{100}\} \qquad \sqrt{n} \qquad n+1 \qquad \{n^2+2n-4, n^2\} \qquad n^4$$

   Justification: In all these cases we can use the limit test.

   - $\lim_{n\to\infty} \frac{1/n}{1} = \lim_{n\to\infty} 1/n = 0$, so the limit test tells us that $1/n < o(1)$, and thus $1/n \leq O(1)$ but they are not Θ of one another.
   - $\lim_{n\to\infty} 10^{100}/1 = 10^{100}$. As this is a nonzero constant, the limit test tells us that $1 = \Theta(10^{100})$.
   - $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$, so $1 < o(\sqrt{n})$.
   - We note that:

   $$\begin{aligned}\lim_{n\to\infty} \frac{n+1}{\sqrt{n}} &= \lim_{n\to\infty} \left(\sqrt{n} + \frac{1}{\sqrt{n}}\right) \\ &= \infty + 0 \\ &= \infty.\end{aligned}$$

   So $n+1 > \omega(\sqrt{n})$ and thus $n+1 \geq \Omega(\sqrt{n})$. In particular, $\sqrt{n} \leq O(n+1)$; however, $\sqrt{n} \neq \Theta(n+1)$.

   - We note that:

   $$\begin{aligned}\lim_{n\to\infty} \frac{n^2+2n-4}{n^2} &= \lim_{n\to\infty} \left(1 + \frac{2}{n} - \frac{4}{n^2}\right) \\ &= 1+0+0 \\ &= 1,\end{aligned}$$

which is a nonzero constant. So we conclude $n^2 = \Theta(n^2 + 2n - 4)$.

- We have that:

$$\begin{aligned}\lim_{n\to\infty} \frac{n+1}{n^2} &= \lim_{n\to\infty} \left(\frac{1}{n} + \frac{1}{n^2}\right) \\ &= 0.\end{aligned}$$

  So we conclude $n + 1 < o(n^2)$.

- $\lim_{n\to\infty} n^2/n^4 = \lim_{n\to\infty} 1/n^2 = 0$, so we conclude $n^2 < o(n^4)$.