Name: _____

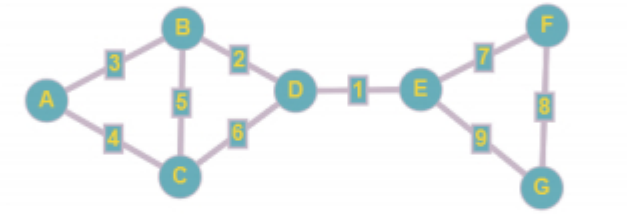
ID: _____

CSCI 3104, Algorithms **Profs. Chen & Grochow**

Quiz 7 Q1 S12 **Spring 2020, CU-Boulder**

Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit (30 min for 1x, 37.5 min for 1.5x, 45 min for 2x). Questions posted to Piazza **must be posted as PRIVATE QUESTIONS.** Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

Standard 12. Consider the following graph G.



(a) Determine all the edges of G which do not belong to any MST. Clearly justify your answer.

Solution: We note that the edges of G are distinct. So by the Cycle Property, if e is the heaviest edge on some cycle, then e does not belong to any MST of G. In light of the Cycle Property, we note that the following edges do not belong to any MST of G:

- (B, C), which lies on the cycle ABC.
- (C, D), which lies on the cycle BCD.
- (E, G), which lies on the cycle EFG.

(b) Determine all edges of G which belong to every MST. Clearly justify your answer.

Solution: We note that the edges of G are distinct. So by the Cut Property, if e is the smallest weight edge on some cut, then e belongs to every MST of G. In light of the Cut Property, we note that the following edges belong to every MST of G:

- (A, B), which belongs to the cut $\{(A, B), (A, C)\}$.
- (A, C), which belongs to the cut $\{(A, C), (C, B), (C, D)\}$.
- (B, D), which belongs to the cut $\{(B, D), (C, D)\}$.
- (D, E), which is itself a cut edge of G.
- (E, F), which belongs to the cut $\{(E, F), (F, G)\}$.
- (F, G), which belongs to the cut $\{(F, G), (E, G)\}$.