

Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit (30 min for 1x, 37.5 min for 1.5x, 45 min for 2x). Questions posted to Piazza **must be posted as PRIVATE QUESTIONS**. Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences**. **Show and justify all work to receive full credit.**

Standard 19. We define the Multiplicative Rod Cutting problem as follows.

Input: A list of weights $w_1, \dots, w_n \geq 1$ for rods of length $1, \dots, n$, respectively.

Goal: Divide a rod of length n into pieces of lengths ℓ_1, \dots, ℓ_k (k can vary) to maximize the total value $\prod_{i=1}^k w_{\ell_i}$. You may assume that a rod of length 0 has weight 1.

Write down the recurrence for the optimal solution. Justify your answer.

Solution: Denote r_n to be the maximum revenue from a rod of length n . We note that r_n satisfies the recurrence:

$$r_n = \max \left(p_n, r_1 r_{n-1}, r_2 r_{n-2}, \dots, r_{n-1} r_1 \right),$$

where $r_1 = p_1$.