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CSCI 3104, Algorithms — **Profs. Chen & Grochow**
Final Exam S2–S5 — **Spring 2020, CU-Boulder**

Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit. Questions posted to Piazza **must be posted as PRIVATE QUESTIONS.** Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these are grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

TIMING: If you are not attempting all the standards in a given quiz, please only use the ordinary amount of time for the number of standards you attempt. For example, if you are only attempting one standard on a 4-standard quiz, please only use 30 min (or 38 for 1.5x, 45 for 2x).

YOU MUST SIGN THE HONOR PLEDGE. Your quiz will otherwise not be graded.
Honor Pledge: On my honor, I have not used any outside resources (other than my notes and book), nor have I given any help to anyone completing this assignment.

Your Name: Sahib Bajwa

Quicklinks: 2 3 4 5

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2. **Standard 2.** *Let* $f(n) = 3(n+10)^2$ *and let* $g(n) = n^2$*. Determine which relation* ***best*** *applies:* $f(n) = \mathcal{O}(g(n))$*,* $f(n) = \Omega(g(n))$*, or* $f(n) = \Theta(g(n))$*.* **Show all work to receive full credit. This is the only problem on which you (may) need to specify details of calculus problems, such as limits.**

   (BEGIN YOUR ANSWER ON THE NEXT PAGE.)

2. ①

$$\lim_{n\to\infty} \frac{3(n+10)^2}{n^2} = \lim_{n\to\infty} \frac{3n^2+60n+300}{n^2} \quad \frac{\infty}{\infty}$$

$(n+10)(n+10)$
$n^2+10n+10n+100$
$n^2+20n+100$

②

$$= \lim_{n\to\infty} \frac{6n+60}{2n} \quad \frac{\infty}{\infty}$$

③

$$= \lim_{n\to\infty} \frac{6}{2} = 3$$

① Simplify by distributing 3
② Apply L'Hopital's rule due to the limit $= \frac{\infty}{\infty}$
③ Apply L'Hopital's rule due to the limit $= \frac{\infty}{\infty}$

3 is a zero constant. Thus, $\boxed{f(n) = \Theta(g(n)).}$

or

$3(n+10)^2 = \Theta(n^2)$

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3. **Standard 3.** *Write down a closed form expression in Θ notation for the runtime of the following algorithm.* **Show all of your work**.

```
// Assume A is an n x n matrix of real numbers
function DiagSumSquare(A):
    n = length(A)
    sum = 0.0
    for i = 1 to n {
        sum += A[i][i] * A[i][i]
    }
    return sum
```

(BEGIN YOUR ANSWER ON THE NEXT PAGE.)

YOUR ANSWER HERE FOR STANDARD 3. (YOU CAN DELETE ALL THIS TEXT IN CAPS.)

IF YOU ARE HANDWRITING AND INSERTING AN IMAGE, SEE THE COMMENTED CODE BELOW IN THE .TEX FILE. PLEASE BE SURE TO ROTATE YOUR IMAGE TO THE CORRECT ORIENTATION (CAN BE DONE IN THE LATEX DIRECTLY; SEE COMMENTS.)

4. **Standard 4.** *Write down a closed form expression in* Θ *notation for the the runtime of the following algorithm.* **Show all of your work**.

```
// Assume A is not empty, and length(A) = n
function MaxGap(A):
    maxGapSoFar = 0
    for i = 1 to length(A) {
       for j = i+1 to length(A) {
           gap = A[j] - A[i]
           if (gap > maxGapSoFar) { maxGapSoFar = gap }
    }}
    return maxGapSoFar
```

(BEGIN YOUR ANSWER ON THE NEXT PAGE.)

4.

Line	Cost	# times run
1		
2		
3		
4	1	n
5	1	n-i-1
6		↓
7		
8		
9	1	1

$$T(n) = O(1) + \sum_{i=1}^{n} c(n-i-1)$$
$$= O(1) + \sum_{i=1}^{n} cn - \sum_{i=1}^{n} ci - \sum_{i=1}^{n} 1$$
$$= O(1) + cn^2 - \frac{cn(n+1)}{2} - n$$
$$= O(1) + cn^2 - \frac{cn^2 + cn}{2} - n$$
$$= O(1) + c\left(\frac{2n^2 - n^2 - n}{2}\right) - n$$
$$= O(1) + c\left(\frac{n^2 - \cancel{n}}{\cancel{2}}\right) - \cancel{n}$$
$$\boxed{= \Theta(n^2)}$$

5. **Standard 5.** *Consider the following algorithm.*

```
def Fun(A, lo, hi):
       if hi > lo {
           mid1 = floor (lo + (hi-lo)/4)
           mid2 = floor (lo + (hi-lo)/2)
           mid3 = floor (lo + 3(hi-lo)/4)
           for i = lo to mid2 {
                A[i] += 1
                A[lo+hi-i] -= 1
           }
           Fun(A, lo, mid1)
           Fun(A, mid1+1, mid2)
           Fun(A, mid3+1, hi)
       }
       else {
             return 1
       }
```

*Write down a recurrence relation describing the number of steps the above algorithm performs on an input of size $n = length(A)$. You do **not** need to solve the recurrence you obtain.*

(BEGIN YOUR ANSWER ON THE NEXT PAGE.)

6. Our non-recursive work will be from lo to mid2. This will b be for $\frac{n}{2} - lo$ iterations. So the non-recursive work is $\Theta(n)$

There are 3 recursive calls. The first call iterates over $n/4$ of the array. The second call iterates over $(\frac{n}{2} - \frac{n}{4}) = \frac{n}{4}$ of the array. The third call iterates over $(\frac{n}{1} - \frac{3n}{4}) = \frac{n}{4}$ of the array. So our reccurence is:

$$T(n) = \begin{cases} T(\frac{n}{4}) + T(n/4) + T(n/4) + \Theta(n) & : hi > lo \\ 1 & : \text{otherwise} \end{cases}$$