Name:

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CSCI 3104, Algorithms
Problem Set 11 – Due Wed April 29 11:55pm

Profs. Chen & Grochow
Spring 2020, CU-Boulder

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Informal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solutions:

- All submissions must be typed.
- You should submit your work through the **class Canvas page** only.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please allot at least as many pages per problem (or subproblem) as are allotted in this template.

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1. Indiana Jones is gathering n artifacts from a tomb, which is about to crumble and needs to fit them into 5 cases. Each case can carry up to W kilograms, where W is fixed. Suppose the weight of artifact i is the positive integer w_i. Indiana Jones needs to decide if he is able to pack all the artifacts. We formalize the Indiana Jones decision problem as follows.

   - Instance: The weights of our n items, $w_1, \ldots, w_n > 0$.
   - Decision: Is there a way to place the n items into different cases, such that each case is carrying weight at most W?

   Show that Indiana Jones $\in$ NP.

   Solution A *certificate* y for Indiana Jones takes the form of a partition of $w_1, \ldots, w_n$ into five sets, $C_1, \ldots, C_5$, such that for each C_i, $W \geq \sum_{x \in C_i} wt(x)$. We can verify y as follows:

   (a) Verify that all items are present in exactly one C_i and that no items appear more than once. This can be done in $\mathcal{O}(n)$ time.

   (b) For each C_i, check that $W \geq \sum_{x \in C_i} wt(x)$. This requires a simple sum over the items in each bin, thus taking $\mathcal{O}(n)$ time.

   If either (a) or (b) fail to hold, we reject y. Otherwise, we accept it as a valid solution. Since both steps can be computed in linear time, we conclude that Indiana Jones is polynomial-time verifiable and thus belongs to NP.

2. A student has a decision problem L, which they know belongs to NP. This student wishes to show that L is NP-Complete. They attempt to do so by constructing a polynomial time reduction from L to SAT, a known NP-Complete problem. That is, the student attempts to show that $L \leq_p \mathsf{SAT}$. Determine if this student's approach is correct and justify your answer.

Solution The student's approach is wrong. To prove L is NP-Complete, we have 2 steps: First, prove L belongs to NP, which is already known. Second, prove a known NP-Complete problem can be reduced to L in polynomial time.

Note that as SAT is NP-Complete, every language in NP reduces **to** SAT in polynomial time. So as $L \in \mathsf{NP}$, we have immediately that $L \leq_p \mathsf{SAT}$. This says nothing about whether L is NP-Complete. The student would instead need to show that $\mathsf{SAT} \leq_p L$.