Full Name (As It Appears on Gradescope):

Preferred Name:

ID:

CSCI 3104, Algorithms **Profs. Chen & Grochow**

Quiz 5A **Spring 2020, CU-Boulder**

Instructions: This quiz is closed book and an individual effort. Electronic devices are NOT allowed. Possession of such electronics is grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

Please provide these:

Left neighbor name :

Right neighbor name :

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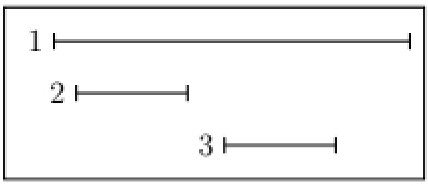
Quiz 5A **Spring 2020, CU-Boulder**

1. **Standard 11.** *Consider the interval scheduling problem from class. You are given a list of intervals, each with a start and end time* (s_i, e_i). *Your goal is to select a subset* S *of the given intervals such that (a) no two intervals in* S *overlap, and (b)* S *contains as many intervals as possible subject to condition (a).*

   Consider the greedy algorithm which selects the interval with the earliest starting time s_i *first. (Then it removes all intervals that conflict with the chosen interval, then selects the* remaining *interval with the earliest starting time, and so on.) Draw an example to show that this algorithm is not optimal. Show what this algorithm does on your example, what subset it outputs, and show a strictly larger set of non-overlapping intervals.*

   Solution: Consider the following intervals.

   Interval 1 has the earliest starting time, so the algorithm picks it. But then since interval 1 conflicts with intervals 2 and 3, the algorithm is done and simply outputs 1. The set of intervals $\{2, 3\}$ is a nonoverlapping set of intervals with more than the number output by the algorithm, so the algorithm is not optimal.

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2. **Standard 11.** *Suppose you want to drive from Town A to Town B along some fixed route of distance d, and you have a gas tank whose capacity will take you at most m miles along the route. Let $0 < d_1 < d_2 < \ldots < d_n < d$ be the distances of each gas station along the route from Town A. (So the distance from A to gas station 2 is d_2; the distance between the first two gas stations is $d_2 - d_1$.) Your goal is to get from A to B (equivalently, distance 0 to distance d) (a) without running out of gas, and (b) stopping as few times as possible.*

   The natural strategy most people use is a greedy one: go as far as you can, but refuel at gas station i if you don't have enough to get you to gas station $i+1$. This strategy indeed minimizes the number of stops you need to make.

   Consider a different greedy strategy, in which you stop at the nearest available gas station. Give an example (specify d, m, and the distances between the gas stations) showing that this strategy is not optimal. Show what this greedy algorithm does on your example, which subset of gas stations it outputs, and exhibit a strictly smaller set of gas stations that would still allow you to successfully complete the trip.

   Solution: This greedy strategy is ridiculous—it just stops at every gas station! Consider gas stations at positions 1, and 2, with $d = 3$, $m = 3$. The greedy strategy here stops at both gas stations. But a single tank has enough to get from the starting point at 0 all the way to the destination $d = 3$ without stopping at *any* gas stations.