Name:

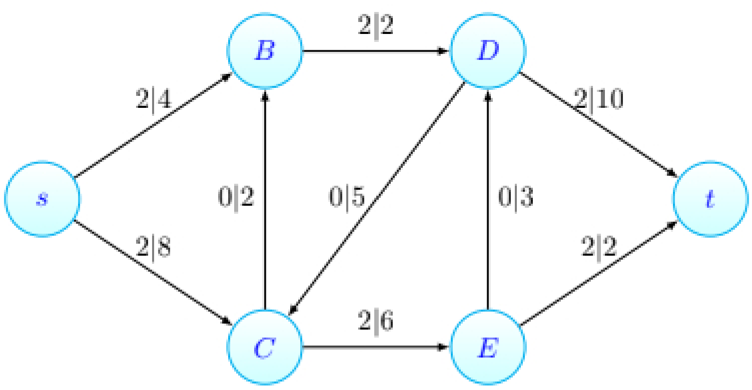
ID:

CSCI 3104, Algorithms | **Profs. Chen & Grochow**
Quiz 8 Q1 S16 | **Spring 2020, CU-Boulder**

Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit (30 min for 1x, 37.5 min for 1.5x, 45 min for 2x). Questions posted to Piazza **must be posted as PRIVATE QUESTIONS.** Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these are grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences. Show and justify all work to receive full credit.**

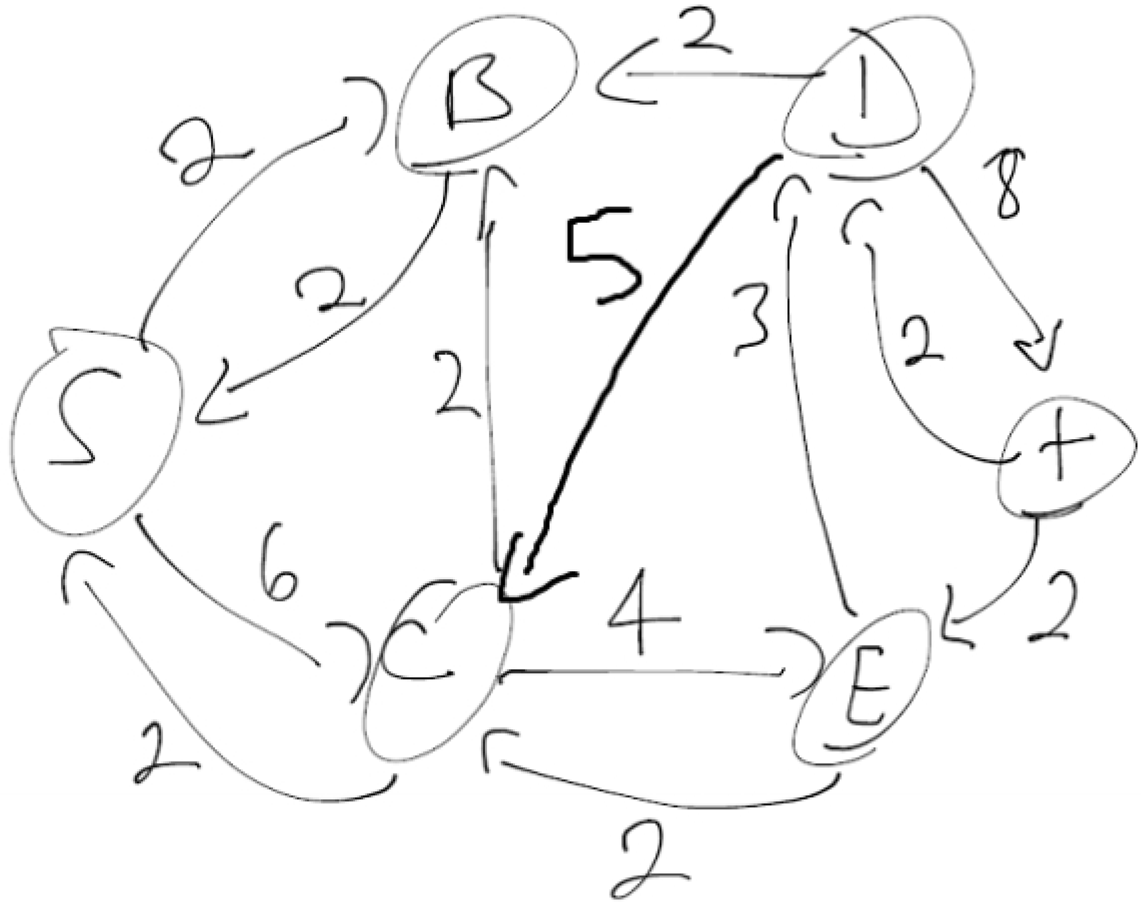
Standard 16. Consider the following for the flow network where $a|b$ on an edge denotes a flow of a and a total capacity of b (meaning, starting from this, you cannot add more than $b - a$ flow to that edge without exceeding its capacity). The node s is the source and t is the sink.

Several iterations of the Ford–Fulkerson algorithm have already been completed. In this problem you will complete the Ford–Fulkerson algorithm in parts (a) and (b) below.



a. Draw the residual graph of the graph above. (Because there are no 2-cycles in this graph, you do not have to worry about the gadget from class that deals with 2-cycles.)

Solution:

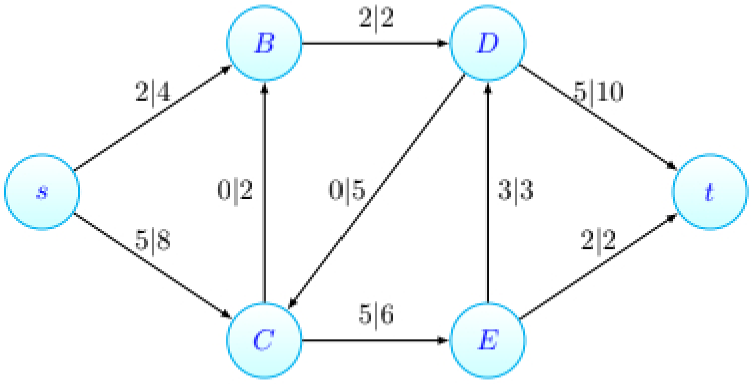


b. Complete the Ford–Fulkerson algorithm. Show your work. End your answer with your final flow, its value, and the corresponding min cut.

Solution: We have the following flow-augmenting path:

$$s \to C \to E \to D \to t.$$

We push 3 units of flow from $s \to t$ along this path, which yields the updated flow network.



We note that there are no more flow-augmenting paths. There are 7 units of flow that reach t. In order to find a minimum cut, we consider the vertices reachable from s as one side of our cut. So $S = \{s, B, C, E\}$, and $V - S = \{D, T\}$. The amount of flow we can push through our minimum cut is given by the capacities of the edges going from $S \to (V - S)$ (note the direction). So the cut edges are $\{(B, D), (E, D), (E, T)\}$, with a capacity of 7.