

Name:
ID:

CSCI 3104, Algorithms
Problem Set 10 – Due Wed April 22 11:55pm

Profs. Chen & Grochow
Spring 2020, CU-Boulder

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
 - You should submit your work through **Gradescope** only.
 - If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
 - Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
 - You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
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1. Consider the following DP table for the Knapsack problem for the list $A = [(4, 3), (1, 2), (3, 1), (5, 4), (6, 3)]$ of (weight, value) pairs. The weight threshold $W = 10$.

- Fill in the values of the table.
- Draw the backward path consisting of backward edges and do not draw (or erase them) the edges that are not part of the optimal backward paths.

- (a) Fill the table with the above requirements (You can also re-create this table in excel/sheet).

Solution.

Weight	Value	$i \backslash w$	0	1	2	3	4	5	6	7	8	9	10
4	3	$A[0..0]$	0	0	0	0	3	3	3	3	3	3	3
1	2	$A[0..1]$	0	2	2	2	3	5	5	5	5	5	5
3	1	$A[0..2]$	0	2	2	2	3	5	5	5	6	6	6
5	4	$A[0..3]$	0	2	2	2	3	5	6	6	6	7	9
6	3	$A[0..4]$	0	2	2	2	3	5	6	6	6	7	9

- (b) Which cell has the optimal value and what is the optimal value for the given problem?

Solution. Cell (5,10) has the best value if you start rows from 1 – index. The optimal value is 9.

- (c) List out the optimal subset and provide it's weight and value.

Solution. Optimal subset: $\{(5,4), (1,2), (4,3)\}$. Its weight is 10 and its value is 9.

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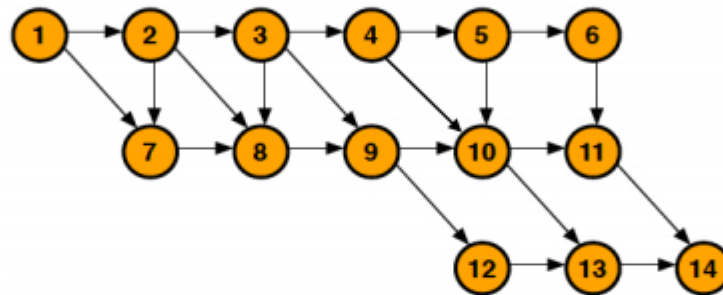
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2. Given the following directed acyclic graph. Use dynamic programming to fill in a table that counts number of paths from each node j to 14, for $j \geq 1$. Note that a single vertex is considered a path of length 0.



Solution. Note that for this question we want to find the value for the number of paths from i to 14 ($X_{i,14}$) where i goes from 14 to 1 in an iterative way. We start by computing $X_{14,14}$ and go to $X_{13,14}$to... $X_{1,14}$.

Also, note that this is **different** than going from $X_{1,1}$ to $X_{1,2}$to... $X_{1,14}$.

We fill the below table starting from the last row and going to the first.

Node	#Path
1	20
2	17
3	11
4	5
5	3
6	1
7	3
8	3
9	3
10	2
11	1
12	1
13	1
14	1