Name:

ID:

CSCI 3104, Algorithms **Profs. Chen & Grochow**
Problem Set 4 – Due Thurs Feb 13 11:55pm **Spring 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Informal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solutions:

- All submissions must be easily legible.
- You should submit your work through the **class Canvas page** only.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please allot at least as many pages per problem (or subproblem) as are allotted in this template.
- For Problem 4 **you must submit your code in a separate file on Canvas** (*not* in a zip file). **Solutions must have both code in a separate file and answers in the PDF, and your plots must match up with your code; solutions which don't will receive a 0 for Problem 4.**

Quicklinks: 1 2 3a 3b 3c 3d 4 4a 4b 4c 4d

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1. Provide a one-sentence description of each of the components of a divide and conquer algorithm.

   - **divide**: break a problem instance into several smaller instances of the same problem
   - **conquer**: if a smaller instance is trivial, solve it directly; otherwise, divide again
   - **combine**: combine the results of smaller instances together into a solution to a larger instance

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2. For the following problems, **you must** use the pseudocode for QuickSort and Partition in Section 1.1 of the course note "Week 4: QuickSort" on canvas and the array $A = [2, 4, 7, 5, 1, 9, 6]$.

   (a) What is the value of the pivot in the call $partition(A, 1, 7)$?
   In the pseudocode, we take A[right] as pivot, so the pivot is A[7] = **6**.

   (b) What is the index of that pivot value at the end of that call to $partition()$?

   5
   We entered the if statement 4 times (for A[g] equals 2, 4, 5, 1), so l increased 4 times. l is initialized as 0, so after the loop l is 4. After the exchange, A[g] (the pivot) was exchanged to A[l+1], whose index is $l + 1 = 5$.

   (c) On the next recursive call to Quicksort, what sub-array does $partition()$ evaluate? (Give the indices specifying the subarray.)

   The return value for partition is 5 (l+1 in Partition, p in Quicksort). Then Patition will be called for A[left, l+1-1] and A[l+1+1, right], which are **A[1:4] and A[6:7] (both sides are included)**.

3. Suppose that we modify the *Partition* algorithm in QuickSort in such a way that on alternating levels of the recursion tree, *Partition* either chooses the best possible pivot or the worst possible pivot.

   (a) Given an array, what choice of pivot will result in the best partitioning, and which one will result in the worst partitioning?

   Solution: Picking the best-possible pivot means choosing the median, so that when Partition returns, it has divided the problem into two sub-problems of size $(n/2)$ and $(n/2)$ which is approximately half. Picking the worst-possible pivot means choosing the most minimum or most maximum, so that when Partition returns, it has divided the problem into one part of size 0 and one part of size $n-1$.