Name:

ID:

CSCI 3104, Algorithms **Profs. Chen & Grochow**

Problem Set 8 – Due Thurs Apr 2 11:55pm **Spring 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Informal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solutions:

- All submissions must be typed.
- You should submit your work through the **class Canvas page** only.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please allot at least as many pages per problem (or subproblem) as are allotted in this template.

Quicklinks: 1a 1b 2a 2b

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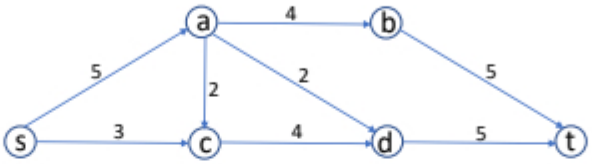
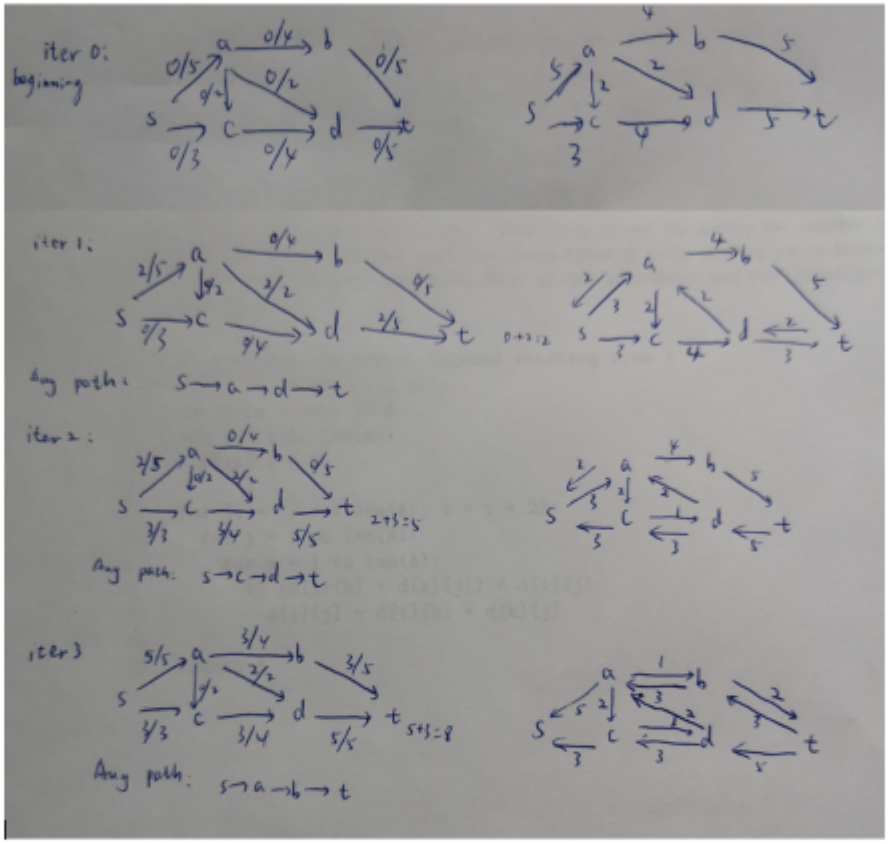
1. *Consider the following flow network, with each edge labeled by its capacity:*



   (a) Using the Ford-Fulkerson algorithm, compute the maximum flow that can be pushed from $s \to t$. **You must use $s \to a \to d \to t$ as your first flow-augmenting path.**

   In order to be eligible for full credit, you must include the following:

   - The residual network for each iteration, including the residual capacity of each edge.
   - The flow augmenting path for each iteration, including the amount of flow that is pushed through this path from $s \to t$.
   - The updated flow network **after each iteration**, with flows for each directed edge clearly labeled.
   - The maximum flow being pushed from $s \to t$ after the termination of the Ford-Fulkerson algorithm.



Flow graph is on the left, residual graph is on the right.

(b) The Ford-Fulkerson algorithm will terminate when there is no longer an augmenting path on the residual network. At this point, you can find a minimum capacity cut. Indicate this cut and its capacity, and verify if max flow min cut theorem holds.

Solution: The minimum cut includes the edge sa, sc. The capacity is $5+3=8$, which equals the max flow in (a).