Class Name: Assignment 0

Due on February 14, 2037 at 11:59pm

Instructor Name

Your Name

Problem 1

Compute the derivative of

$$x^3. \tag{1}$$

Solution

Using the power rule,

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^3) = 3x^2. \tag{2}$$

Problem 2

Give the pseudocode for QuickSort.

QUICKSORT

```
1: procedure QUICKSORT(A, p, r)
2:     if p < r then
3:         q = Partition(A, p, r)
4:         QUICKSORT(A, p, q − 1)
5:         QUICKSORT(A, q + 1, r)
6:     end if
7: end procedure
```

Problem 3

Answer the following questions:

(a) Find the partial derivative of $f(x, y) = x^2y + \sin(xy)$ with respect to x.

The partial derivative is:

$$\frac{\partial}{\partial x}(x^2y + \sin(xy)) = 2xy + y\cos(xy) \tag{3}$$

(b) Evaluate the integral:

$$\int_0^1 3x^2\mathrm{d}x \tag{4}$$

Using the power rule for integration:

$$\int_0^1 3x^2\mathrm{d}x = \left[x^3\right]_0^1 = 1 - 0 = 1 \tag{5}$$

Problem 3 continued on next page...

(c) Determine the expectation $\mathrm{E}[X]$ for a random variable X with probability density function $f(x) = 2x$ for $0 \leq x \leq 1$.

The expectation is:

$$\mathrm{E}[X] = \int_0^1 x \cdot 2x \mathrm{d}x = \int_0^1 2x^2 \mathrm{d}x = \left[frac2x^3 3\right]_0^1 = \frac{2}{3} \tag{6}$$

Problem 4

Consider random variables X and Y with the following properties:

$$\mathrm{E}[X] = 3 \tag{7}$$
$$\mathrm{E}[Y] = 5 \tag{8}$$
$$\mathrm{Var}[X] = 2 \tag{9}$$
$$\mathrm{Var}[Y] = 4 \tag{10}$$
$$\mathrm{Cov}[X, Y] = 1 \tag{11}$$

Let $\hat{\theta}$ be an estimator for parameter θ with $\mathrm{Bias}[\hat{\theta}] = 0.5$.

Solution

Given the information above, we can determine:

1. The variance of the sum $X + Y$:

$$\mathrm{Var}[X + Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] + 2\mathrm{Cov}[X, Y] = 2 + 4 + 2(1) = 8 \tag{12}$$

2. The covariance between $2X$ and $3Y$:

$$\mathrm{Cov}[2X, 3Y] = 2 \cdot 3 \cdot \mathrm{Cov}[X, Y] = 6 \cdot 1 = 6 \tag{13}$$

3. Since $\mathrm{Bias}[\hat{\theta}] = 0.5$, the estimator $\hat{\theta}$ is biased. To create an unbiased estimator, we can define $\hat{\theta}_{unbiased} = \hat{\theta} - 0.5$, which would have $\mathrm{Bias}[\hat{\theta}_{unbiased}] = 0$.

Problem 5

What Python function is commonly used to print text to the console?

Solution

The Python function commonly used to print text to the console is `print()`.