9. Maximize : $Z = -3x_1 - 2x_2$

subject to :

$$x_1 + x_2 \geq 1,$$
$$x_1 + x_2 \leq 7,$$
$$x_1 + 2x_2 \geq 10,$$
$$x_2 \leq 3,$$
$$x_1, x_2 \geq 0$$

by using dual simplex method.





**(Graph Paper)**

**B. Tech. 4th Semester (BME) Examination,**

**May–2016**

**MATHEMATICS–III**

**Paper–Math–201–F**

*Time allowed : 3 hours]* *[Maximum marks : 100*

***Note :*** *Attempt **five** questions in total, selecting at least **one** question from each section. **Q. No. 1 is compulsory.***

1. (a) Find $a_n$ for $f(x) = x - x^2$, $-1 < x < 1$.

(b) Examine sin z is analytic or not analytic.

(c) Define residue at a pole and state Cauchy's residue theorem.

(d) Minimize : $Z = 20x_1 + 10x_2$

subject to :

$$x_1 + 2x_2 \leq 40,$$
$$3x_1 + x_2 \geq 30,$$
$$4x_1 + 3x_2 \geq 60,$$
$$x_1, x_2 \geq 0$$

by graphical method.

**Section–A**

2. (a) Find Fourier series for

$$f(x) = \begin{cases} \pi + x & \text{for } -\pi < x < 0 \\ 0 & \text{for } 0 \leq x < \pi \end{cases}$$

(b) Obtain half range cosine series for

$$f(x) = x, \qquad 0 < x < \frac{\pi}{2},$$

$$= \pi - x, \qquad \frac{\pi}{2} < x < \pi.$$

**3.** Solve :

$\frac{\partial V}{\partial t} = k \frac{\partial^2 v}{\partial x^2}$ for $x > 0, t > 0$ under the boundary conditions $V = V_0$ when $x = 0, t > 0$ and the initial condition $V = 0$ when $t = 0, x > 0$.

## Section–B

**4.** (a) If $\tan(\theta + i\phi) = \cos\alpha + i \sin\alpha = e^{i\alpha}$, prove that :

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2} \log \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right).$$

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, even though C. R. equations are satisfied there at.

**5.** (a) State and prove Cauchy's integral theorem.

(b) Evaluate :

$$\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz \text{ where C is the circle } |z| = 3.$$





## Section–C

**6.** (a) Find the series expansion of

$f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ about $z = 0$ in the region

(i) $|z| < 2$

(ii) $2 < |z| < 3$.

(b) Evaluate :

$\oint_C \tan z \, dz$, where C is the circle $|z| = 2$.

**7.** (a) Prove that the total area under normal probability curve is unity.

(b) A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A were defective and 1% produced by machine B were defective. If a defective item is drawn at random, what is the probability that it was produced by machine A ?

## Section–D

**8.** Verify whether binomial distribution can be assumed from the data given below :

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| f | 13 | 25 | 52 | 58 | 32 | 16 | 4 |