Module - 6

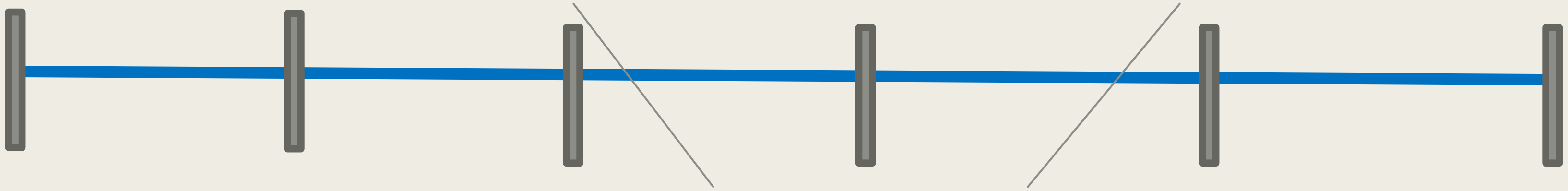
Module 5	**Time Value of Money**	5 hours
Time preference for money, Future value, Annuity, Perpetuity, Sinking fund factor, Present value, Annuity, Perpetuity, capital recovery factor, Multiple period Compounding.		

Time value of money

- A rupee expected soon is worth more than a rupee expected in the distant future
- Therefore, it is essential for financial managers to have a clear understanding of the time value of money and its impact on stock prices.

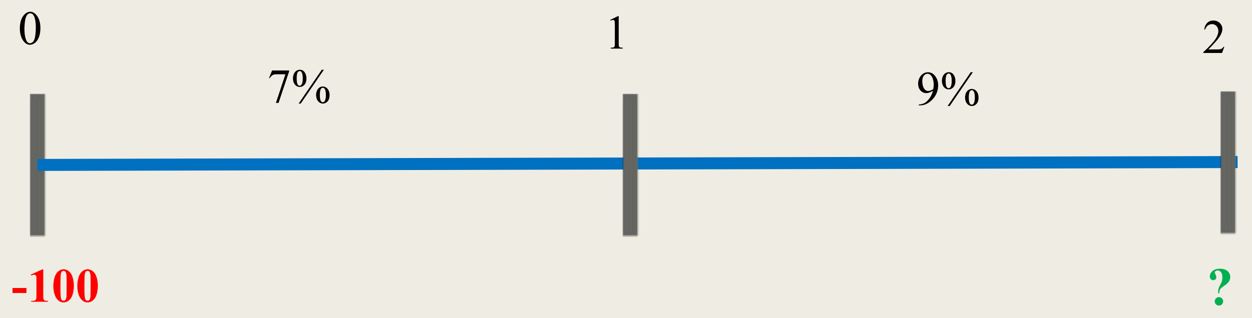
Time line

- One of the most important tools in time value analysis is the **time line**
- 0 **7%** 1 2 3 4 5
- Time:



cash flows: **-100** End of period values **?**

Time line



Future value of money

- The process of going from Present value (PV) to Future value (FV) is called **compounding**

- *Suppose you deposit $100 in a bank that pays 5 percent interest each year. How much would you have at the end of one year?*

PV	= the beginning amount in your account	($100)
i	= interest rate per year	(5% or 0.05)
INT	= interest amount	[$100 x(0.05)=$5]
n	= number of period	(5)
FV_n	= future value, ending amount	**?**

Future value of money

- $FV_n = FV_1 \ = \boldsymbol{PV + INT}$
- = PV+PV(i)
- =PV(1+i)
- \$100 (1+0.05)= \$100 (1.05)= \$105

- What would you end up with if you left your $100 in the account for **five** years?

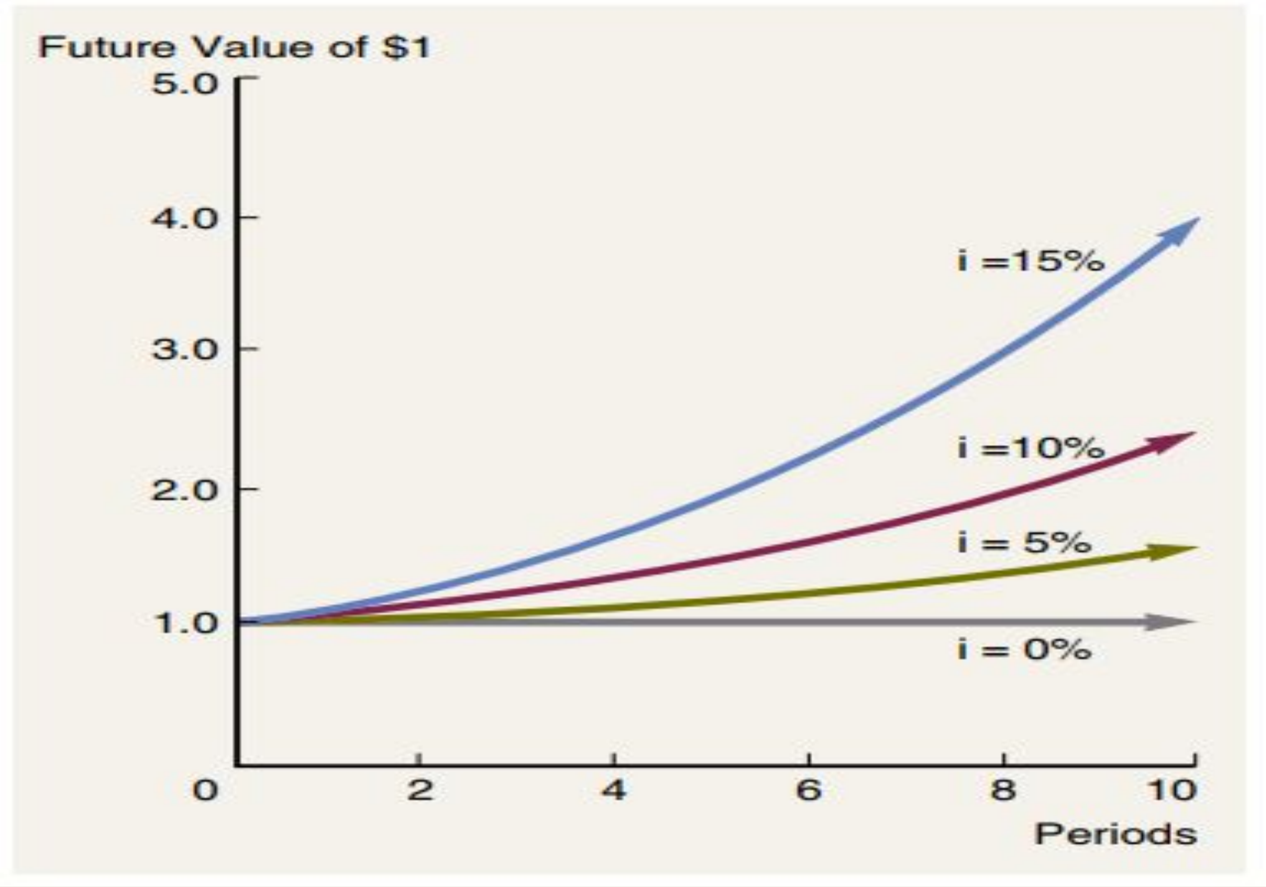
	0	5% 1	2	3	4	5
Initial deposit:	−100	FV_1 = ?	FV_2 = ?	FV_3 = ?	FV_4 = ?	FV_5 = ?
Interest earned:		5.00	5.25	5.51	5.79	6.08
Amount at the end of each period = FV_n:		105.00	110.25	115.76	121.55	**127.63**

- $FV_1 = PV(1+i)$
- $FV_2 = FV_1(1+i)$
- $= PV(1+i)(1+i)$
- $= PV(1+i)^2$
- $FV_3 = PV(1+i)^3$
- $\boldsymbol{FV_n = PV(1+i)^n}$

$FV_5 = 100(1.05)^5 = \$127.63$

- $(1+i)^n$ = Future value interest factor
- OR
- The *Compound value factor*

Relationships among FV, Growth,I, & time



Present Value

- Opportunity cost rate: The rate of return on the best available alternative investment of equal risk
- Let's compare the rate of return from a security and an FD that offers 5% rate of interest
- For eg. How much would you willing to pay for the security if that pays you 127.63 at the end of five years?

$$\frac{127.63}{(1+0.05)^5} = 100$$

- **Present value**
- *the present value of a cash flow due n years in the future is the amount which, if it were on hand today, would grow to equal the future amount*
- *Discounting: finding the present value*

- $FV_n = \mathbf{PV}(1+i)^n$

$$PV = \frac{FV_n}{(1+i)^n} = FV_n\left(\frac{1}{1+i}\right)^n =$$

- Can solve it for ‘i’ & ‘n’ also

deposit, invest, ... give → PV calculate hogi

available, ... recive → FV calculate hogi

- Rudy will retire in 20 years. This year he wants to fund an amount of €15,000 to become available in 20 years. How much does he have to deposit into a pension plan earning 7% annually?

- $PV = \frac{FV_n}{(1+i)^n}$ $= \frac{15{,}000}{(1+0.07)^{20}} = 3877.$

- Assume that an individual invests \$10,000 in a four-year certificate of deposit account that pays 10% interest at the end of each year. Calculate the amount of money available in his account the maturity

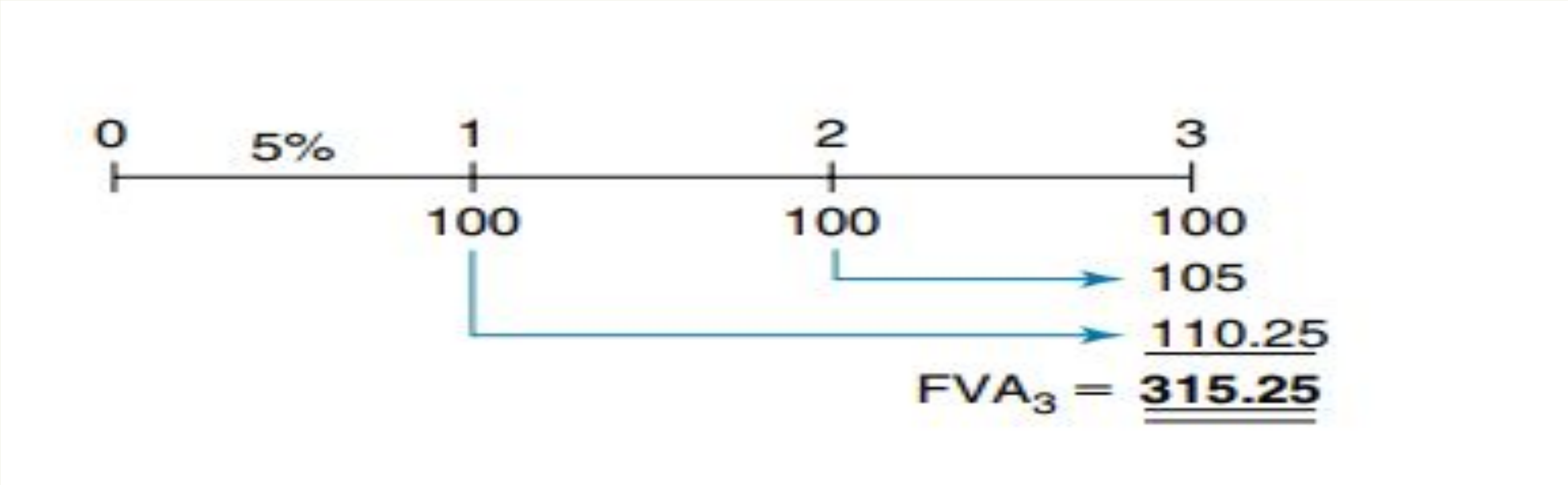
$$10{,}000(1+0.10)^4 = 1464$$

Future value of Annuity

- An **annuity** is a series of equal payments made at fixed intervals for a specified number of periods.
- Ordinary annuity: payments occur at the end of each period A N
- Annuity due: payments occur at the beginning of each period Adue
-

Ordinary annuity

Payment at End of month.

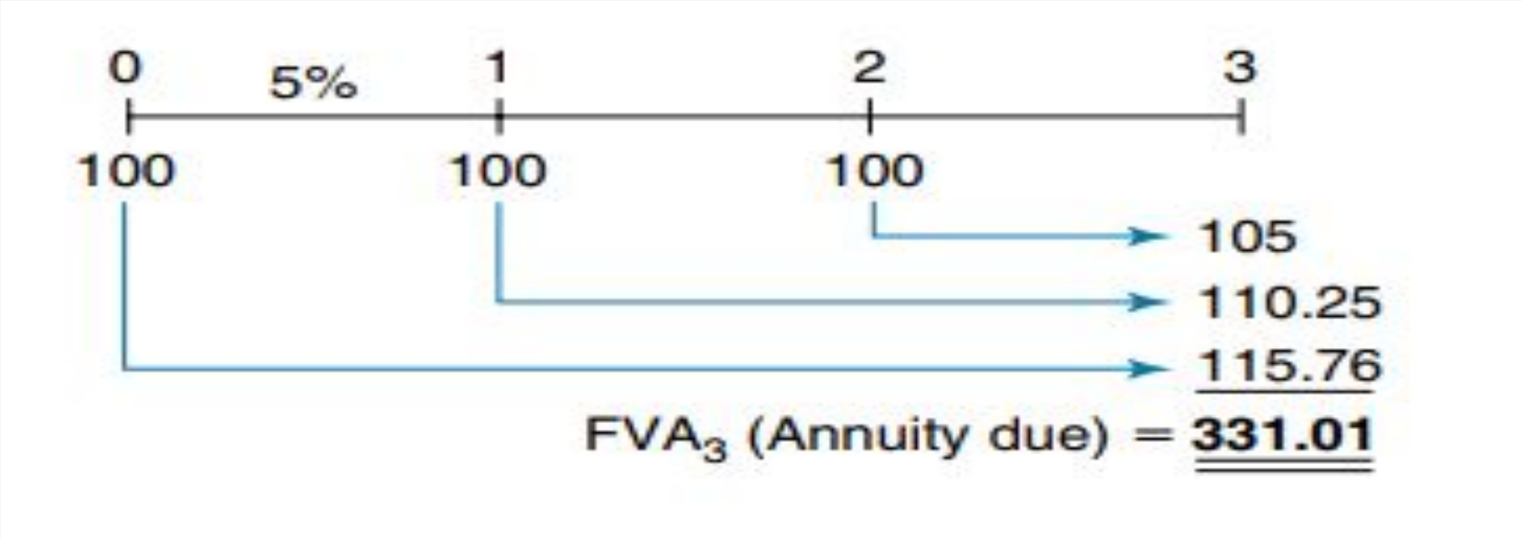


$$\begin{aligned}
FVA_n &= PMT(1+i)^{n-1} + PMT(1+i)^{n-2} + PMT(1+i)^{n-3} + \cdots \\
&\quad + PMT(1+i)^0 \\
&= PMT \sum_{t=1}^{n} (1+i)^{n-t} \\
&= PMT(FVIFA_{i,n}).
\end{aligned}$$

$$FVIFA_{i,n} = \frac{(1+i)^n - 1}{i}.$$

Annuity dues

- Had the three $100 payments in the previous example been made at the *beginning* of each year, the annuity would have been an *annuity due*



advance payment

- $FV_n = PV(1+i)^n$

$$PV = \frac{FV_n}{(1+i)^n} = FV_n\left(\frac{1}{1+i}\right)^n =$$

Payment at fixed Interval of time

- $FVA_N = PMT\left[\frac{(1+i)^n-1}{i}\right]$ $= 100\left[\frac{(1+0.05)^3-1}{0.05}\right] = 315.25$

- $FVA_{due} = FVA_N(1+i)$ $\Rightarrow 315.25 * [1+0.05]$

$\Rightarrow 331.0125$

- You would like to buy a house that is currently on the market at $85,000, but you cannot afford it right now. However, you think that you would be able to buy it after 4 years. If the expected inflation rate as applied to the price of this house is 6% per year, what is its expected price after four years?

$$FV = 85{,}000(1+0.06)^4$$
$$= 1{,}07{,}310$$

- You expect to receive $10,000 as a bonus after 5 years on the job. You have calculated the present value of this bonus at an opportunity cost rate of 4.56. ??

$$PV = \frac{10000}{(1+0.0456)^5} = 8001$$

- If you deposit Rs 5,000 at the end of every year in a bank for 5 years. The bank is paying 10% rate of interest. Calculate the future value

$$AN_N = 5000\left[\frac{(1+0.1)^5-1}{0.1}\right] = 30{,}525$$

$$AN_{Due} = 30{,}525 * 1.1 = 33577.5$$

(1) $FV = PV(1+i)^n = 85k(1.06)^4 = 107,331$

(2) $PV = \dfrac{FV}{(1+i)^n} = \dfrac{10000}{\cancel{1.456}\ (1.046)^5} = 8000$

(3) $FVA = 5000\left[\dfrac{(1.10)^5 - 1}{.10}\right] =$

$= 5000 \times \dfrac{1.6105 - 1}{.1} \Rightarrow 5K \times \left(\dfrac{.6105}{.1}\right)$

$\Rightarrow 5K \times 6.105$

$= \underline{30,525/-}$

(4) $FVA_D = 30,525 \times (1.1) = \underline{33577.5}$

Perpetuities Pension Scheme, Unemployment Wages

- An annuity that goes indefinitely or a stream of equal payments expected to continue forever

$$PV(\text{Perpetuity}) = \frac{\text{Payment}}{\text{Interest rate}} = \frac{PMT}{i}.$$

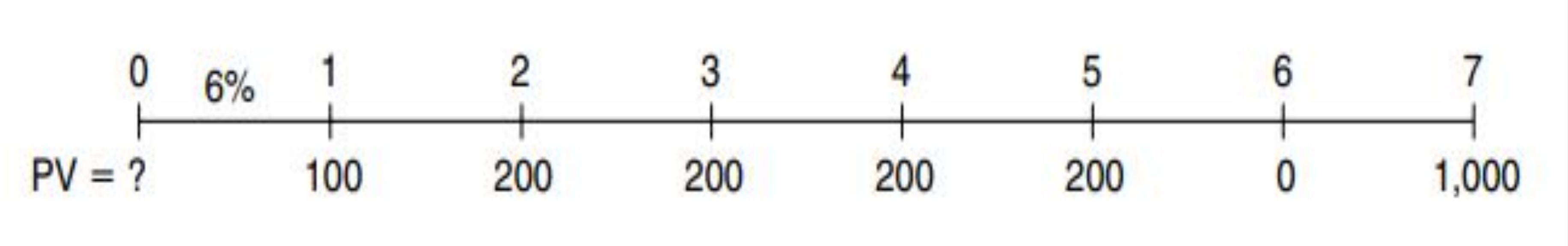
- Suppose a perpetual bond issued by the Government of India promised to pay ₹100 per year forever. What would be the worth of the bond if the opportunity cost rate is 5%.
- PV (perpetuity) = $\frac{₹100}{0.05}$ = ₹2000.

Uneven cash flow streams

- Till now we have considered ‘constant payments’
- Let’s extend our time value decision to non constant cash flows
- A series of cash flows in which the amount varies from one period to next.
- Lets use cash flow (CF) to denote the uneven cash flows

PV of uneven CFs

- It is the sum of the PVs of the individual CFs of a stream



- The PV will be found by applying the following formula:

$$PV = CF_1\left(\frac{1}{1+i}\right)^1 + CF_2\left(\frac{1}{1+i}\right)^2 + \cdots + CF_n\left(\frac{1}{1+i}\right)^n$$

$$= \sum_{t=1}^{n} CF_t\left(\frac{1}{1+i}\right)^t = \sum_{t=1}^{n} CF_t(PVIF_{i,t}).$$

FV of uneven CFs

- It is found by compounding each payment to the end of the stream and then summing the future values
- $$\begin{aligned} FV_n &= CF_1(1+i)^{n-1} + CF_2(1+i)^{n-2} + \cdots + CF_n \\ &= \sum_{t=1}^{n} CF_t(1+i)^{n-t} = \sum_{t=1}^{n} CF_t(FVIF_{i,n-t}). \end{aligned}$$

Multiple period compounding → Interest added 2 time a Year

Interest Rate → half ho Jayega
time period double

- Semi annual compounding:
- The arithmetic process of determining the final value of a cash flow or series of cash flows when interest is added *twice* a year
- To illustrate semiannual compounding, assume that ₹100 is placed into an account at an interest rate of 6 percent and left there for three years. Find the FV through annual and semi-annual compounding.
- Annual compounding: $FV_3 = 100(1.06)^3 = ₹119.1$
- Semi-annual compounding: $FV_6 = 100(1.03)^6 = 100(1.194) =$ ₹119.4

- Two thing to remember here
- 1. convert the stated interest rate to a 'periodic rate'
- 2. convert the number of years to 'number of periods'
- In our example:
  - ***Periodic rate = stated rate/number of payments per year***

    *= 6%/2 = **3%***
  - ***Number of periods (N) = number of years x periods per year***

    *= 3 x 2 = **6***
- Semi-annual compounding: $FV_6 = 100(1.03)^6 = 100(1.194) = ₹119.4$

Nominal and effective Interest rate

- Nominal interest rate is the stated or quoted interest rate
- Effective Interest rate (EFF%) is the annual rate of interest actually being earned
- Given the nominal rate , we can estimate the EFF% as:

$$\text{Effective annual rate} = \text{EAR (or EFF\%)} = \left(1 + \frac{i_{Nom}}{m}\right)^m - 1.0.$$

- The EFF% of 6% nominal interest rate which is compounding semi-annually is :
- EFF%=$(1+\frac{0.06}{2})^2-1$
- $=(1.03)^2-1 = 1.0609 - 1 = 0.0609 = 6.09\%$

EMI calculation

$$E = P \times r \times \frac{(1 + r)^n}{(1 + r)^n - 1}$$

Where,

E is the EMI

P is the principal amount

r is the monthly rate of interest

n is the number of months

Loan amortization

- A loan that is re-paid in equal payments over its life
- Interest is calculated by multiplying the loan balance at the beginning of the year by the interest rate

Year	Beginning Amount (1)	PMT (2)	Interest (I) (3)	Repayment of Principal 4 (2-3)	Remaining balance (1-4)