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Experiment No. -2

Generation of pulse code modulation and its Reconstruction.

Aim :-

To study Sampling and Simulation study of PCM and reconstruction of analog signals. Observe the effect of number of quantization level on signal to quantization ratio (SQNR) and validate with the theoretical value.

Further study on nonlinear quantization in PCM system.

Tools Used :-

Anaconda's Spyder Python Simulation Software.

Theory :-

Sampling is the conversion of a continuous-time signal into a discrete-time signal obtained by taking "samples" of the continuous time signal at discrete time instants. One of the sampling techniques to sample an analog signal is periodic or uniform sampling described by the relation,

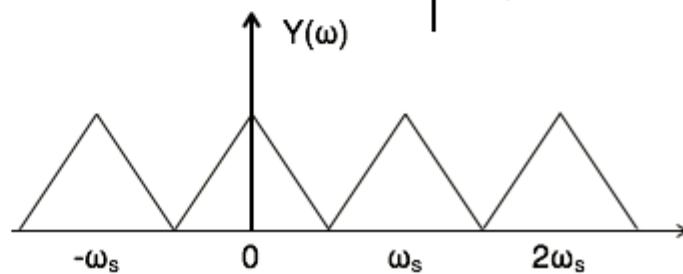
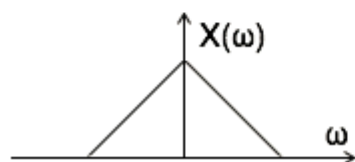
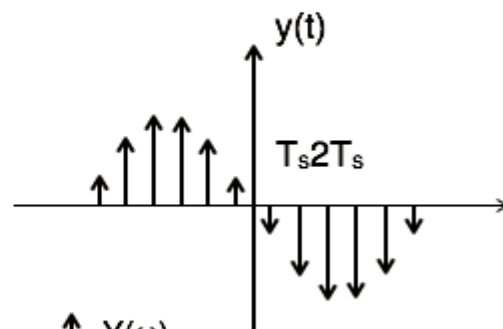
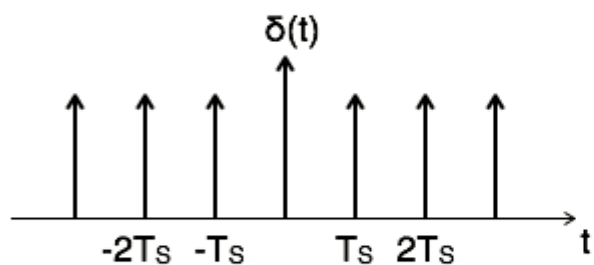
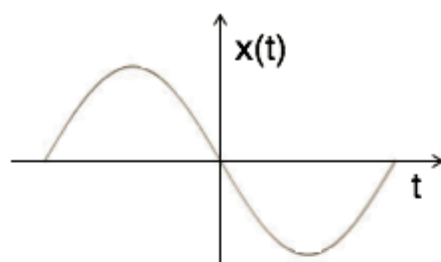
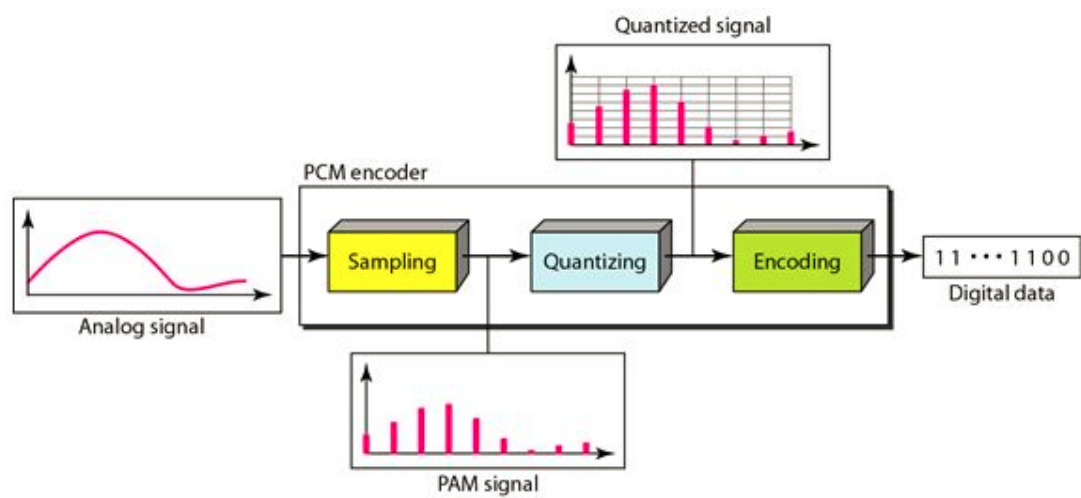
$$x(n) = x(nT), \quad -\infty < n < \infty$$

where $x(n)$ is the discrete time signal obtained by "taking samples" of the analog signal x_a every T seconds. The time interval T is called the sampling interval and its reciprocal $1/T = F$ is called the sampling rate. The desired relationship between the spectrum $X(F)$ of the discrete-time signal and the spectrum X_a is given by,

(1)

its reciprocal $1/T = F$ is called the sampling rate. The desired relationship between the spectrum $X(F)$ of the discrete-time signal and the spectrum X_a is given by,

$$X(F) = F_s \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$



Derivation :

The mathematical expression of sampled signal can be given as,

$$x(t) = x_a(t) \cdot \delta(t) \dots \dots \dots (1)$$

The unit impulse function $\delta(t)$ can be represented in terms of Fourier Series as

$$\delta(t) = a_0 + \sum_{n=-\infty}^{\infty} a_n \cos(n\omega_s t) + \sum_{n=-\infty}^{\infty} b_n \sin(n\omega_s t) \dots \dots (2)$$

$$\text{Where } a_0 = \frac{1}{T_s} \int_{-T/2}^{T/2} \delta(t) dt = \frac{1}{T_s} \delta(0) = \frac{1}{T_s}$$

$$a_n = \frac{2}{T_s} \int_{-T/2}^{T/2} \delta(t) \cos(n\omega_s t) dt = \frac{2}{T_s} \delta(0) \cos(n\omega_s 0) = \frac{2}{T_s}$$

$$b_n = \frac{2}{T_s} \int_{-T/2}^{T/2} \delta(t) \sin(n\omega_s t) dt = \frac{2}{T_s} \delta(0) \sin(n\omega_s 0) = 0$$

Substitute above values in equation 2.

$$\therefore \delta(t) = \frac{1}{T_s} + \sum_{n=-\infty}^{\infty} \frac{2}{T_s} \cos(n\omega_s t) + 0$$

Substitute $\delta(t)$ in equation 1.

$$\begin{aligned} \rightarrow y(t) &= x(t) \cdot \delta(t) \\ &= x(t) \left[\frac{1}{T_s} + \sum_{n=-\infty}^{\infty} \frac{2}{T_s} \cos(n\omega_s t) \right] \\ &= \frac{1}{T_s} x(t) + \frac{2}{T_s} \sum_{n=-\infty}^{\infty} \cos(n\omega_s t) x(t) \end{aligned}$$

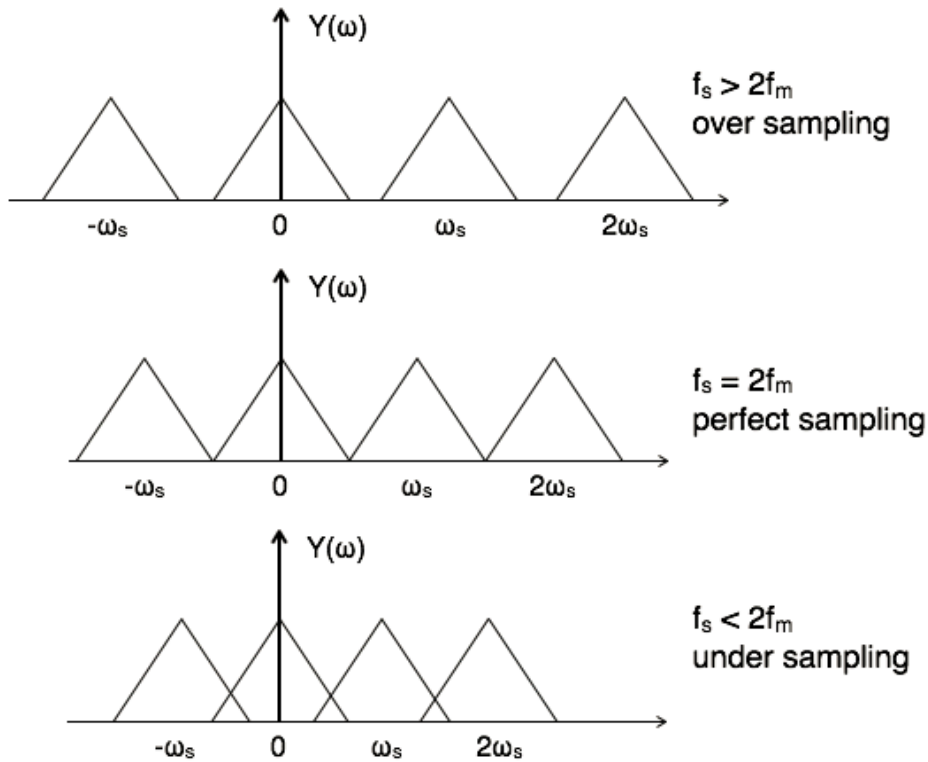
$$y(t) = \frac{1}{T_s} [x(t) + 2\cos\omega_s t \cdot x(t) + 2\cos 2\omega_s t \cdot x(t) + 2\cos 3\omega_s t \cdot x(t) \dots \dots \dots]$$

Take Fourier transform on both sides

$$Y(\omega) = \frac{1}{T_s} [X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) + \dots]$$

$$\therefore Y(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s) \text{ where } n = 0, \pm 1, \pm 2, \dots$$

Possibility of sampled frequency spectrum with different conditions is given by the following diagrams:

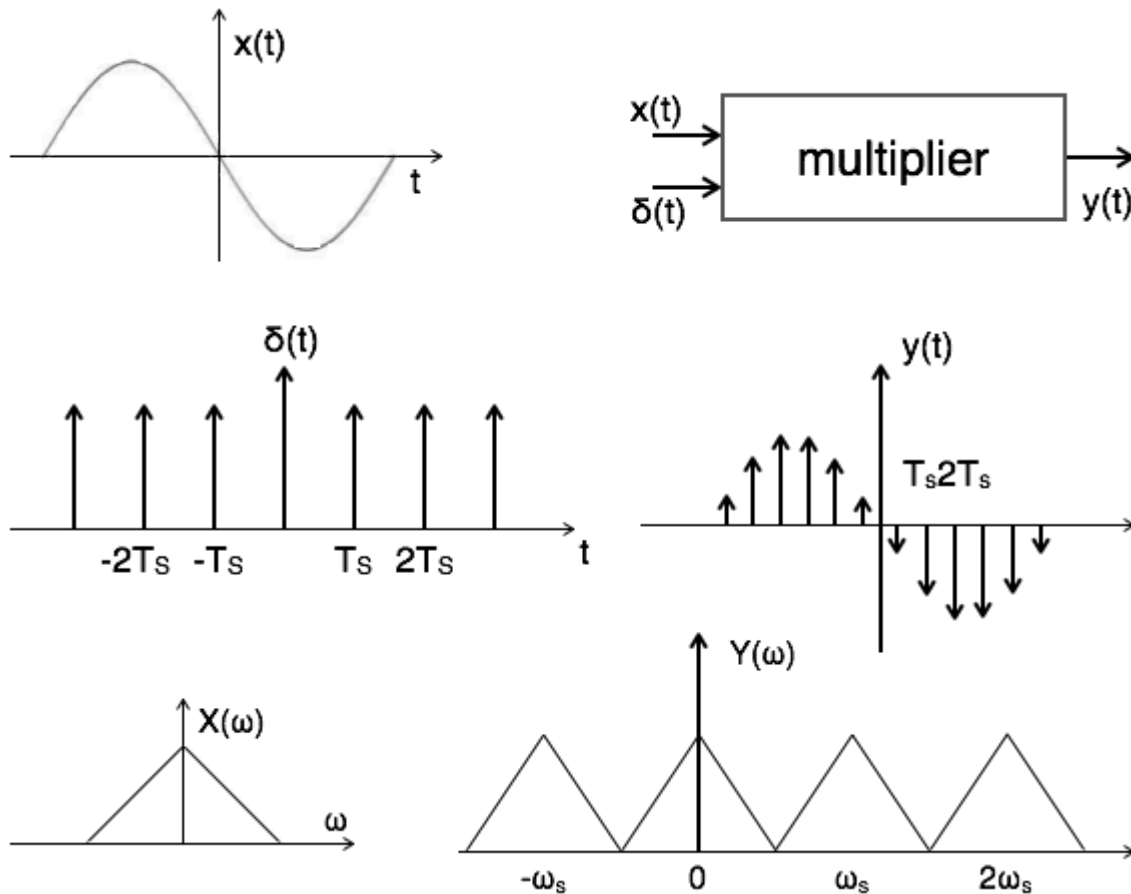


Sampling Theorem: A bandlimited continuous-time signal, with highest frequency B Hz, can be uniquely recovered from its samples provided that the sampling rate ≥ 2 samples per second. If this condition is met, then () can be exactly

reconstructed from,

If the sampling frequency is less than two times the highest frequency component in the sampled signal, then it leads to an aliasing effect and the signal cannot be reconstructed perfectly.

Bandpass Signal: A continuous time bandpass signal with bandwidth B and center frequency has its frequency content in the two frequency bands defined by



Aliasing Effect

The overlapped region in case of under sampling represents aliasing effect, which can be removed by

- considering $f_s > 2f_m$
- By using anti aliasing filters.

Signals Sampling Techniques

There are three types of sampling techniques:

- Impulse sampling.
- Natural sampling.
- Flat Top sampling.

Procedure :-

1. Generate a band limited signal $x(t)$ of your choice with frequency f_m .
2. Choose sampling frequency f_s (more than Nyquist rate) and sample the signal $x(t)$.
Let $x_s(n)$ is the sampled signal
3. Determine the DFT $X_s(K)$ of the sampled signal $x_s(n)$.
4. Pass the sampled signal through a low pass filter of cut-off frequency $f_s/2$.
5. Plot all the signals and spectrums with proper labelling.
6. Vary the sampling frequency to observe under sampling.
7. Repeat the same for the passband signal.

Code :-

```
import numpy as np
import matplotlib.pyplot as plt

# Function to generate a sinusoidal signal
def generate_sinusoidal_signal(amplitude, frequency, duration, sampling_rate):
    t = np.arange(0, duration, 1/sampling_rate)
    signal = amplitude * np.sin(2 * np.pi * frequency * t)
    return t, signal

# Function to perform uniform quantization
def uniform_quantization(signal, num_levels):
    max_amplitude = max(abs(signal))
    step_size = 2 * max_amplitude / num_levels
    quantized_signal = np.round(signal / step_size) * step_size
    return quantized_signal
```

```

# Function to calculate Signal-to-Quantization Noise Ratio (SQNR)
def calculate_sqnr(original_signal, quantized_signal):
    noise = original_signal - quantized_signal
    signal_power = np.sum(original_signal**2)
    noise_power = np.sum(noise**2)
    sqnr = 10 * np.log10(signal_power / noise_power)
    return sqnr

# Parameters
amplitude = 2
frequency = 10
duration = 2 # Two cycles
sampling_rate = 1000
num_levels_8 = 8
num_levels_16 = 16

# Generate sinusoidal signal
t, original_signal = generate_sinusoidal_signal(amplitude, frequency, duration,
sampling_rate)

# Uniform quantization with 8 levels
quantized_signal_8 = uniform_quantization(original_signal, num_levels_8)
sqnr_8 = calculate_sqnr(original_signal, quantized_signal_8)

```

```

# Uniform quantization with 16 levels
quantized_signal_16 = uniform_quantization(original_signal, num_levels_16)
sqnr_16 = calculate_sqnr(original_signal, quantized_signal_16)

# Plotting
plt.figure(figsize=(10, 6))

plt.subplot(3, 1, 1)
plt.plot(t, original_signal, label='Original Signal')
plt.title('Original Signal')
plt.xlabel('Time')
plt.ylabel('Amplitude')
plt.legend()

plt.subplot(3, 1, 2)
plt.plot(t, quantized_signal_8, label='Quantized (8 levels)')
plt.title('Uniform Quantization (8 levels)')
plt.xlabel('Time')
plt.ylabel('Amplitude')
plt.legend()

```

```

plt.subplot(3, 1, 2)
plt.plot(t, quantized_signal_8, label='Quantized (8 levels)')
plt.title('Uniform Quantization (8 levels)')
plt.xlabel('Time')
plt.ylabel('Amplitude')
plt.legend()

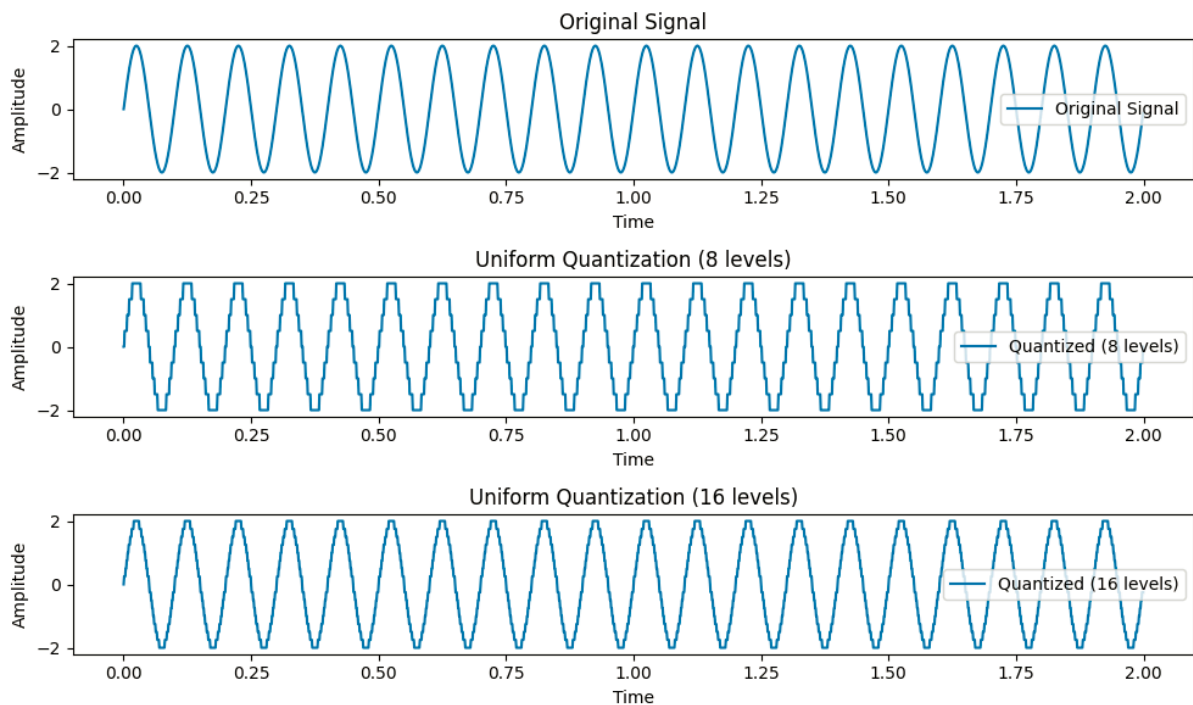
plt.subplot(3, 1, 3)
plt.plot(t, quantized_signal_16, label='Quantized (16 levels)')
plt.title('Uniform Quantization (16 levels)')
plt.xlabel('Time')
plt.ylabel('Amplitude')
plt.legend()

plt.tight_layout()
plt.show()

# Print SQNR values
print(f'SQNR (8 levels): {sqnr_8} dB')
print(f'SQNR (16 levels): {sqnr_16} dB')

```

Graphs :-



Observations :-

In this simulation experiment the output graphs that were obtained, the following observations on the reconstruction and sampling of Baseband/Bandlimited signals and Passband signals may be made:

1. Oversampling (More Quantization Levels)

When using a higher number of quantization levels (e.g., 16 levels compared to 8 levels), it can be considered as oversampling in the context of PCM. Oversampling provides a more accurate representation of the original analog signal after quantization.

The higher number of quantization levels allows for finer granularity in representing the signal amplitudes, reducing quantization errors.

- In the experiment, oversampling can be observed by comparing the reconstructed signals with 8 and 16 quantization levels.

1. Perfect Sampling (Ideal Uniform Quantization)

In an ideal scenario, perfect sampling would occur when the quantization levels exactly match the amplitude levels of the original signal.

Theoretically, perfect sampling would result in minimal quantization error, and the reconstructed signal would closely resemble the original analog signal.

Achieving perfect sampling might be challenging, especially as the number of quantization levels increases, due to practical limitations and finite precision.

1. Undersampling (Insufficient Quantization Levels):

If the number of quantization levels is too low compared to the signal's amplitude range, it can be considered as undersampling.

- Undersampling leads to a coarser representation of the signal, introducing larger quantization errors and distortion in the reconstructed signal.
- In the experiment, undersampling can be observed by comparing the reconstructed signals with 8 quantization levels, where the signal might show more distortion compared to the 16 levels case.

4. SQNR and Sampling Quality:

The Signal-to-Quantization Noise Ratio (SQNR) can be used as a quantitative measure of the quality of signal representation.

Higher SQNR values indicate better signal fidelity, while lower SQNR values suggest increased quantization noise and potential signal distortion.

Comparing the SQNR values for different quantization levels in the experiment allows for an assessment of the impact of sampling quality on signal reconstruction.

By analyzing the observations mentioned above, you can gain insights into how the choice of quantization levels affects the accuracy and fidelity of signal reconstruction in the PCM system.

Results :-

Experiment Results

1. **Original Signal:**
 - A sinusoidal signal with amplitude 2 and frequency 10 Hz was generated.
2. **Uniform Quantization (8 Levels):**
 - The original signal was quantized using a uniform quantization technique with 8 levels.
 - The quantized signal was obtained and plotted.
3. **Uniform Quantization (16 Levels):**
 - The original signal was quantized using a uniform quantization technique with 16 levels.
 - The quantized signal was obtained and plotted.
4. **Comparison of SQNR:**
 - The Signal-to-Quantization Noise Ratio (SQNR) was calculated for both cases (8 levels and 16 levels).
5. **Reconstruction:**
 - The quantized levels were encoded, and the corresponding bits were stored in a matrix (X).
 - Signal reconstruction was performed using the encoded levels.

Conclusion :-

The experiment demonstrated the impact of the number of quantization levels on signal reconstruction in PCM. Oversampling with more levels improved accuracy, while undersampling with fewer levels increased distortion. Achieving perfect sampling proved challenging in practice. The SQNR analysis provided quantitative insights, emphasizing the importance of appropriate quantization levels for accurate signal representation in PCM systems.