Antenna and propagation

Antenna & propagation

* Static charge produce Electric field
* when charge particle moving with constant speed produce magnostatic

Sol^n for pth fxn

$\nabla^2 A - \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu J$ ↗ mgn

$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\rho/\varepsilon$

↳ Electric

$\frac{\partial}{\partial t^2} = -\omega^2$

$\nabla^2 A + \omega^2 \mu\varepsilon A = -\mu J$

$\nabla^2 V + \omega^2 \mu\varepsilon V = -\rho/\varepsilon$

$\nabla^2 G + \beta^2 G = \delta(\text{space})$ —

Spherical coordinate system

$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial G}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial r}\sin\theta\frac{\partial G}{\partial\theta}$

$+ \frac{1}{r^2\sin^2\theta}\frac{\partial^2 G}{\partial\phi^2} + \beta^2 G = \delta(\text{space})$

origin symmetric

$rG = \psi$

$\frac{d^2\psi}{dr^2} + \beta^2\psi = \delta(r)$

Solving $\psi = Ce^{-j\beta r} + De^{j\beta r}$

$\psi = Ce^{-j\beta r}$

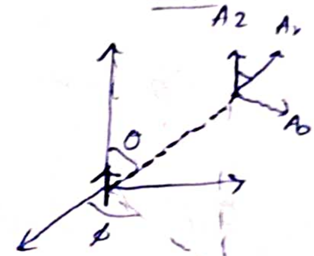
↳ sobtitu can be $\quad C = -\frac{1}{4\pi}$

$G = -\frac{1}{4\pi r} e^{-j\beta r}$

In Real general:

$A = \int_V \frac{\mu J \, e^{-j\beta|r-r'|}}{4\pi|r-r'|} dv$

The Hertzian Diapole - Infinite small charge



$A_r = A_z\cos\theta$

$A_\theta = -A_z\sin\theta$

$A_\phi = 0$

$H = \frac{1}{\mu}\nabla\times A$

$\frac{1}{\mu r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \cancel{\partial/\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$

$H_r = 0$

$H_\theta = 0$

$H_\phi = \frac{I dl\, e^{-j\beta r}\sin\theta}{4\pi r}\left\{ j\beta + \frac{1}{r}\right\} e^{j\omega t}$

$E = \frac{1}{j\omega\varepsilon}\nabla\times H$

$E_r = \frac{2 I_0 dl\, e^{j\omega t} e^{-j\beta r}}{4\pi\varepsilon}\left\{\frac{\beta}{\omega r^2} - \frac{j}{\omega r^3}\right\}$

$E_\theta = \frac{I_0 dl\, e^{j\omega t} e^{-j\beta r}\sin\theta}{4\pi\varepsilon}\left\{\frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3}\right\}$

$E_\phi = 0$

~~# near field~~

↳ ~~electric~~ field

$1/r$ (radition) $\propto \omega$

$1/r^2$ (Inductive) $\cancel{\propto} \omega$

$1/r^3$ $\propto \frac{1}{\omega}$ (Electric)

three field Equal at

$$r = \frac{\lambda}{2\pi} = \frac{1}{\beta}$$

$r \ll \lambda/6$ {dominant field Electric field} near field

$E_r =$

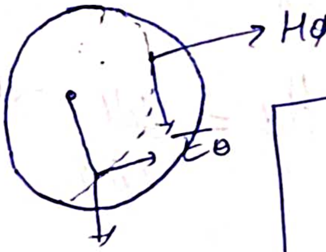
$E_\theta =$

Angular radiation of electric field (Hertz diapole)

$$|E| = \sqrt{E_r^2 + E_\theta^2}$$

$$|E| = \frac{I_0\, dl}{4\pi\varepsilon\omega r^3}\sqrt{1+3\cos^2\theta}$$

far field



$E_\theta =$

$H_\phi =$

$$\boxed{\frac{E_\theta}{H_\phi} = \boxed{\frac{\beta}{\omega\varepsilon} = \eta} = 120\pi}$$

Power Savg ?

$$\boxed{P_{avg} = \frac{1}{2}\eta|H_\phi|^2\, \hat{a}_r}$$

Total power

$$\boxed{W = 40\pi^2 I_0^2\left[\frac{dl}{\lambda}\right]^2 \cancel{I_0}}$$

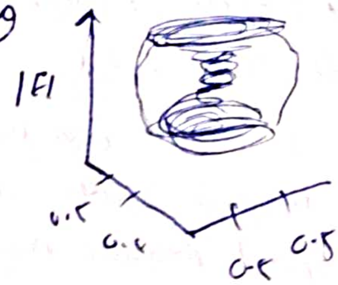
Radion Resistance

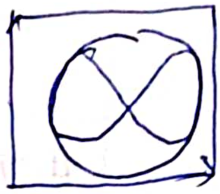
$$W = \frac{1}{2} I_0^2 R_{rad} \Rightarrow \boxed{R_{rad} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2}$$

$dl = 0.1\lambda$ $R = 8\Omega$

Radition pattern of Hertz diapole

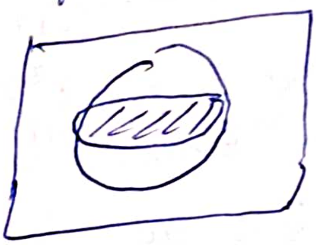
$$F(\theta,\phi) = \sin\theta$$

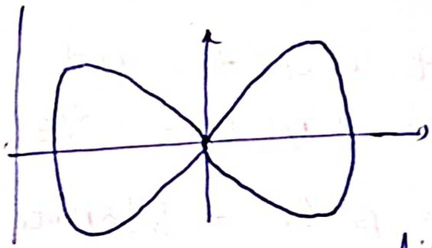




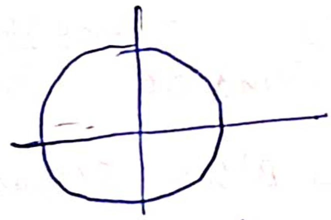
Vertical planee

Horizontal plu



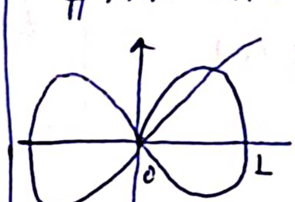


E plane Radition patter

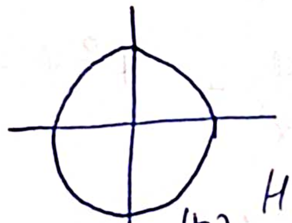


H plane Radition patter

Antenna pattern



(a) normalized E plan



(b) normal H plan

3D patter

Antenna and propagation

$$\nabla^2 A - \mu \varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu J$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

$\boxed{A = \frac{\mu J e^{-j\beta r}}{4\pi r}}$

$$H = \frac{1}{\mu} \nabla \times A$$

Hertzian diapole

$$H_\phi = \frac{I_o dl\, e^{j\omega t} \sin\theta\, e^{-j\beta r}}{4\pi r} \left[j\beta + \frac{1}{r} \right]$$

$$E = \frac{1}{j\mu\varepsilon} \nabla \times H$$

$$E_r = \frac{2 I_o dl\, e^{j\omega t} \cos\theta\, e^{-j\beta r}}{4\pi\varepsilon} \left[\frac{\beta}{\omega r^2} - \frac{j}{\omega r^3} \right]$$

$$E_\theta = \frac{I_o dl\, e^{j\omega t} \sin\theta\, e^{-j\beta r}}{4\pi\varepsilon} \left[\frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3} \right]$$

near field → Electric field $[1/r^3] \to E_r\ E_\theta$

far field → H_ϕ and E_θ

$P_{avg} = \frac{1}{2} \left(\frac{I_o dl \sin\theta}{4\pi r} \right)^2 \frac{\beta^3}{\omega\varepsilon}$

Total power $\boxed{W = 40\pi^2 \left(\frac{dl}{\lambda} \right)^2 I_o}$

$$W = \frac{1}{2} I_o^2 R_{rad}$$

Half wave diapole Antenna

$$E_\theta = \frac{j 60 I_o e^{-j\beta r}}{r} F(\theta)$$

$$F(\theta) = \frac{\cos(\beta H \cos\theta) - \cos(\beta H)}{\sin\theta}$$

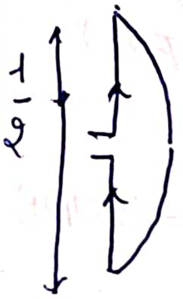
$$\boxed{H_\phi = \frac{E_\theta}{\eta}}$$

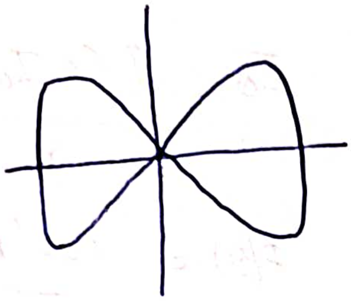
$$P_{rad} = 36.5 I_o^2$$

$$R_{rad} \simeq 73\ \Omega$$

Current distribution and E-plane Radition pattern

1) Half wave $\lambda/2$

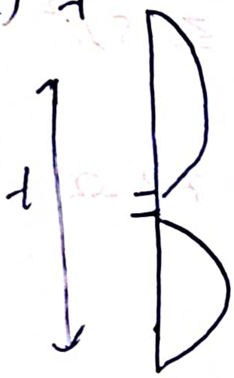


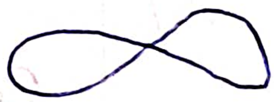


current

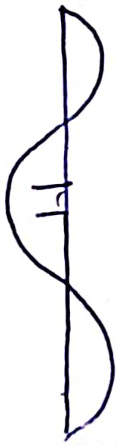
E plane rad^n pattern

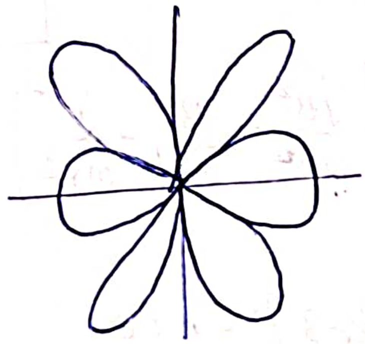
2) λ



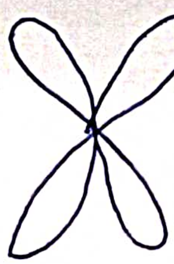


3) $3\lambda/2$ $\lambda + \lambda/2$



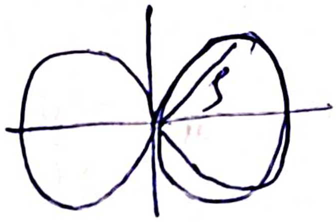


4) 2λ



Hertzian Diapole

H plane



E plane

Half wave dipole antenna Rad^n Resistance

$$E_\theta = \frac{j 60 I_0 e^{-jBR}}{R} F(\theta) \Rightarrow \frac{\cos(BH\cos\theta) - \cos\theta}{\sin\theta}$$

$\theta = \pi/2$

$\frac{2\pi}{\lambda}\frac{\lambda}{4}$

$$E_\theta = \frac{j 60 I_0 e^{-jB.R}}{R} \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$$

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta}$$

$\frac{E}{H} = \eta$

$$\Rightarrow \frac{1}{2 \times 120\pi}\left[\left(\frac{60 I_0 \cos(\pi/2 \cos\theta)}{R\sin\theta}\right)^2\right]$$

$\frac{\cancel{60} \times \cancel{60}}{2 \times \cancel{120}\pi}$ (30, 15)

$$W = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_r\, r^2 \sin\theta\, d\theta\, d\phi$$

$$\Rightarrow \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{15}{\pi}\left(\frac{I_0 \cos(\pi/2 \cos\theta)}{r \sin\theta}\right)^2 r^2 \sin\theta\, d\theta\, d\phi$$

$$\Rightarrow 2\pi \int \frac{15}{\pi} I_0^2 \left(\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right)^2 \sin\theta\, d\theta$$

$$= 30 I_0^2 \int_0^{\pi} \left(\frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta}\right) d\theta$$

$\underbrace{\quad}_{1.218}$

$$W = 36.54 I_0^2 \qquad R = \frac{36.54 I_0^2 \times 2}{I_0} = 73\,\Omega$$

Radition Resistance Derivation of Hertz diapole

$$W = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_{avg}\, r^2 \sin\theta \, d\theta \, d\phi$$

$$= \iint \frac{1}{2}\left(\frac{I\,dl \sin\theta}{4\pi r}\right)^2 \frac{\beta^3}{\omega\varepsilon} r^2 \sin\theta \, d\theta \, d\phi$$

$$\Rightarrow \frac{1}{2}\left(\frac{I\,dl}{4\pi}\right)^2 \frac{\beta^3}{\omega\varepsilon} \iint \sin^3\theta \, d\theta \, d\phi$$

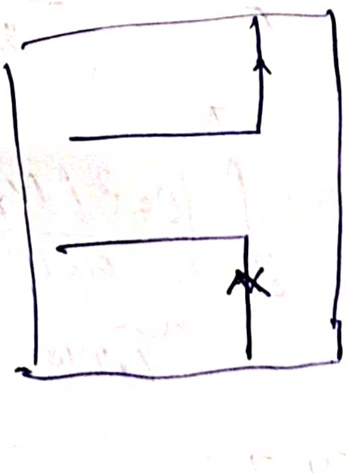
$$\Rightarrow \frac{1}{2}\left(\frac{I\,dl}{4\pi}\right)^2 \frac{\beta^3}{\omega\varepsilon} \int_0^{2\pi} d\phi \underbrace{\int_0^{\pi} \sin^3\theta \, d\theta}_{I_1} \qquad I_1 = 4/3$$

$$\Rightarrow \frac{1}{2}\left(\frac{I\,dl}{4\pi}\right)^2 \frac{\beta^3}{\omega\varepsilon}\, 2\pi\, (4/3) \qquad \because \frac{\beta^3}{\omega\varepsilon} = \frac{4\pi^2 \eta_0}{\lambda^2}$$

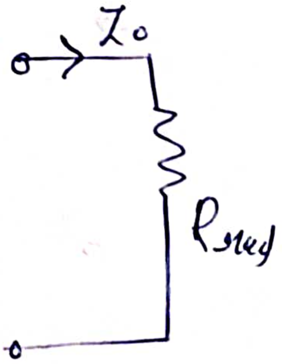
$$W \Rightarrow 40\pi^2 I_0^2 \left(\frac{dl}{\lambda}\right)^2$$

$$W = \frac{1}{2} I_0^2 R_{rad} \qquad \frac{1}{2} I_{rms}^2 R_{rad}$$

$$40\pi^2 I_0^2 \left(\frac{dl}{\lambda}\right)^2 = \frac{1}{2} I_0^2 R_{rad}$$



$$\boxed{R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2}$$



Equivalent Resistance looking from I/p

$dl = 0.1\lambda$ $\quad R_{rad} \approx 8\,\Omega$

Small for Hertz diapole

Half wave dipole antenna Rad^n Resistance

$$E_\theta = \frac{j 60 I_0 e^{-j\beta R}}{R} F(\theta) \Rightarrow \frac{\cos(\beta H \cos\theta) - \cos\theta}{\sin\theta}$$

$\theta = \pi/2$

$\frac{2\pi}{\lambda} \frac{\lambda}{4}$

$$E_\theta = \frac{j 60 I_0 e^{-j\beta R}}{R} \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$$

$$P_r = \frac{1}{2} \frac{|E_\theta|^2}{\eta}$$

$\frac{E}{H} = \eta$

$$\Rightarrow \frac{1}{2 \times 120\pi}\left[\left(\frac{60 I_0 \cos(\pi/2 \cos\theta)}{R\sin\theta}\right)^2\right]$$

$\frac{60 \times 60}{2 \times 120\pi}$ 30 15

$$W = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_r r^2 \sin\theta \, d\theta \, d\phi$$

$$\Rightarrow \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{15}{\pi}\left(\frac{I_0 \cos(\pi/2 \cos\theta)}{r \sin\theta}\right)^2 r^2 \sin\theta \, d\theta \, d\phi$$

$\sin\theta$

$$\Rightarrow 2\pi \int \frac{15}{\pi} I_0^2 \left(\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right)^2 d\theta$$

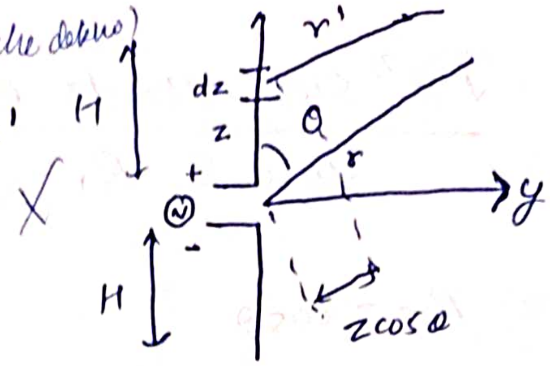
$$\Rightarrow 30 I_0^2 \int_0^{\pi} \left(\frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta}\right) d\theta$$

1.218

$W \Rightarrow 36.54 I_0^2$ $R = \frac{36.54 I_0^2 \times 2}{I_0} \simeq 73\,\Omega$

Half wave Dipole Antenna (piche dekho)

$$dA_z = \frac{\mu_0 I_0 \cos\beta z\, dz\; e^{-j\beta r'}}{4\pi r'}$$



$r >> l$

$r' - r' = z\cos\theta$

$r' = r - z\cos\theta$

$$A_{zs} = \frac{\mu_0 I_0}{4\pi r} \int_{-\lambda/4}^{\lambda/4} e^{-j\beta(r - z\cos\theta)} \cos(\beta z)\, dz$$

$$= \frac{\mu_0 I_0 e^{-j\beta r}}{4\pi r} \int_{-\lambda/4}^{\lambda/4} e^{+j\beta z\cos\theta} \cos(\beta z)\, dz$$

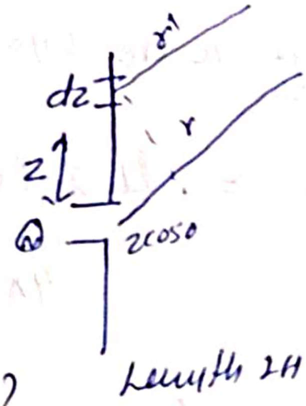
$$\Rightarrow \frac{\mu_0 I_0 e^{-j\beta r}\; e^{j\beta z\cos\theta}}{4\pi r(-\beta^2\cos^2\theta + \beta^2)} \Big[j\beta\cos\theta \cos(\beta z) + \beta \sin\beta z \Big]_{-\lambda/4}^{\lambda/4}$$

Since $\beta = \frac{2\pi}{\lambda}$ or $\frac{\beta\lambda}{4} = \frac{\pi}{2}$ and $\beta^2(1-\cos^2\theta) = \beta^2\sin^2\theta$

$$A_{zs} = \frac{\mu I_0 e^{-j\beta r}}{4\pi r \beta^2 \sin^2\theta} \Big[e^{j\frac{\pi}{2}\cos\theta}(0+\beta) - e^{-j\frac{\pi}{2}\cos\theta}(0-\beta) \Big]$$

$$\Rightarrow \frac{\mu I_0 e^{-j\beta r}\, \cancel{2\beta}\big[2\cos(\tfrac{\pi}{2}\cos\theta)\big]}{4\pi r \beta^{\cancel{2}} \sin^2\theta}$$

$$A_{zs} = \frac{\mu I_0 e^{-j\beta r}\big[\cos \tfrac{\pi}{2}\cos\theta\big]}{2\pi r \beta \sin^2\theta}$$



$$dE_\theta = \frac{j\beta^2 \sin\theta \, I(z)dz \, e^{-j\beta R_1}}{4\pi\omega\varepsilon R_1}$$

$$R_1 = R - z\cos\theta$$

$$dE_\theta = \frac{j\beta^2 \sin\theta \, I(z)dz}{4\pi\omega\varepsilon R} e^{-j\beta(R - z\cos\theta)}$$

$$\Rightarrow \frac{j\beta^2 \sin\theta \, e^{-j\beta R}}{4\pi\omega\varepsilon R} \int_{-H}^{H} I(z) e^{j\beta z\cos\theta} dz$$

$$\Rightarrow \frac{j\beta^2 \sin\theta \, e^{-j\beta R}}{4\pi\omega\varepsilon R} \int_{-H}^{0} I_1(z)dz + \int_{0}^{H} I_2(z)dz$$

where ~~$I_1(z) = I_m \beta(H+z)$~~

~~$I_2(z) = I_m \beta(H-z)$~~

$$\Rightarrow \frac{j60 I_m e^{-j\beta R}}{R} F(\theta)$$

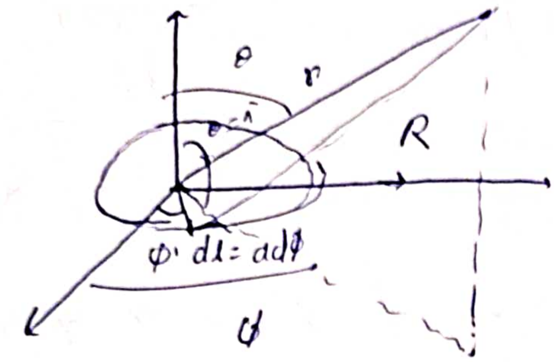
$$F(\theta) = \frac{\cos(\beta H \cos\theta) - \cos(\beta H)}{\sin\theta}$$

where

$$I_1(z) = I_m \sin(\beta)(H+z) e^{j\beta z\cos\theta}$$

$$I_2(z) = I_m \sin[\beta(H-z)] e^{j\beta z\cos\theta}$$

Small loop antenna



Assuming antenna is located symmetrically on the xy plane z=0

$I_\phi = I_0$

$$A(x,y,z) = \frac{\mu}{4\pi} \int_C \frac{I_e(x',y',z') e^{-j\beta R}}{R} dl'$$

$$I_e(x',y',z') = I_x(x',y',z') a_x + I_y(x',y',z') a_y + I_z(x',y',z') \quad (1)$$

cylindrical components using I_{ρ}, I_ϕ, $I_z(x',y',z')$

$$I_x = I_\rho \cos\phi - I_\phi \sin\phi$$

$$I_y = I_\rho \sin\phi + I_\phi \cos\phi$$

$$I_z = I_z \quad (2)$$

$$\begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_\rho \\ I_\phi \\ I_z \end{bmatrix}$$

Similarly unit vectors in spherical coordinate system

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix}$$

Substituting in eq^n (1) a_x, a_y, a_z & I_x, I_y, I_z

$$I_e = a_r \left[I_\rho \sin\theta \cos(\phi-\phi') + I_\phi \sin\theta \sin(\phi-\phi') + I_z \cos\theta \right]$$

$$+ \; a_\theta \left[I_\rho \cos\theta \cos(\phi-\phi') + I_\phi \cos\theta \sin(\phi-\phi') - I_z \sin\theta \right.$$

$$\left. a_\phi \left[I_\rho (-\sin(\phi-\phi')) + I_\phi \cos(\phi-\phi') \right) \right.$$

$$I_e = a_r I_\phi \sin\theta \sin(\phi-\phi') + a_\theta I_\phi \cos\theta \sin(\phi-\phi') + I_\phi \cos(\phi-\phi')$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$R = \sqrt{r^2 + a^2 - 2ar\sin\theta\cos(\phi-\phi')}$$

$dl = a\,d\phi'$

$$A_\phi = \frac{a\mu}{4\pi}\int_0^{2\pi} I_\phi \cos(\phi-\phi')\, \frac{e^{-j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}}\, d\phi$$

I_ϕ is constant and Not fxn of ϕ $\quad I_\phi = I_0$ & $\phi' = 0$ for simplici

$$A_\phi = \frac{a\mu I_0}{4\pi}\int_0^{2\pi} \cos\phi\, \frac{e^{-j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos\phi}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi}}\, d\phi$$

$$f = \frac{e^{-j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos\phi}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi}}\, d\phi$$

maclaurin series

$$f(0) = f(0) + f'(0)\,a + \frac{1}{2!}f''(0)\,a^2$$

$$f'(0) = \left.\frac{\partial f}{da}\right|_{a=0} \qquad f(0) = \frac{e^{-j\beta r}}{r}$$

$$f'(0) = \quad \log f = j\beta\sqrt{r^2+a^2-2ar\sin\theta} - \frac{1}{2}\log\sqrt{}$$

$$\frac{1}{f}\frac{df}{da} = \frac{j\beta(2a-2r\sin\theta\cos\phi)}{2\sqrt{r^2+a^2-2ar\sin\theta\cos\phi}} - \frac{1}{2}\,\underline{\qquad}$$

$$f'(0) = \left(\frac{j\beta}{r} + \frac{1}{r^2}\right) e^{-j\beta r}\sin\theta\cos\phi'$$

$$f = \left[\frac{1}{r} + a\left(\frac{j\beta}{r}+\frac{1}{r^2}\right)\sin\theta\cos\phi'\right] e^{-j\beta r}$$

$$A_\phi \simeq \frac{a\mu I_0}{4\pi}\int_0^{2\pi} \cos\phi\left[\frac{1}{r} + a\left(\frac{j\beta}{r}+\frac{1}{r^2}\right)\sin\theta\cos\phi'\right]e^{-j\beta r}\, d\phi$$

$$A_\phi = \frac{a \mu_0 I_0}{4\pi}\left[a\left(\frac{jB}{r} + \frac{1}{r^2}\right)\sin\theta\right]e^{-jBr}$$

$$A = A_\phi \hat{a}_\phi = \frac{a^2 \mu I_0}{4} \sin\theta \left[\frac{jB}{r} + \frac{1}{r^2}\right] e^{-jBr}$$

$$A = \frac{a^2 \mu I_0}{4} e^{-jBr} \sin\theta \left(\frac{jB}{r} + \frac{1}{r^2}\right)$$

$$H_r = \frac{Ba^2 I_0 \cos\theta}{2r^2}\left[1 + \frac{1}{jBr}\right]e^{-jBr}$$

$$H_\theta = \frac{-(Ba)^2 I_0 \sin\theta}{4r}\left[1 + \frac{1}{jBr} - \frac{1}{Br^2}\right]e^{-jBr}$$

$$H_\phi = 0$$

$$E_r = E_\theta = 0 \qquad E_\phi = \frac{\eta (Ba)^2 I_0 \sin\theta}{4r}\left[1 + \frac{1}{jBr}\right]e^{-jBr}$$

---

① Hertzian diapole

$$A = \frac{\mu I e^{-jBr}}{4\pi r}$$

$$H_r = H_\theta = 0$$

$$H_\phi = \frac{I_0 dl\, e^{j\omega t} e^{-jBr} \sin\theta}{4\pi r}\left[jB + \frac{1}{r}\right]$$

$$E_r = \frac{2 I_0 dl\, e^{j\omega t} e^{-jBr} \cos\theta}{4\pi \epsilon}\left[\frac{B}{\omega r^2} - \frac{j}{\omega r^3}\right]$$

$$E_\theta = \frac{I_0 dl\, e^{j\omega t} e^{-jBr} \sin\theta}{4\pi \epsilon}\left[\frac{jB^2}{\omega r} + \frac{B}{\omega r^2} - \frac{j}{\omega r^3}\right]$$

$$E_\phi = 0$$

② Half wave

$$A_{zs} = \frac{\mu I_0 e^{-jBr}}{2\pi r} F(\theta)$$

$$E_{\theta s} = \frac{j60 I_0 e^{-jBr}}{r} F(\theta)$$

$$F(\theta) = \frac{\cos(BH\cos\theta) - \cos BH}{\sin\theta}$$

\# Small loop antenna

$$A_\phi = \frac{a^2 \mu I_0 e^{-jBr}}{4} \sin\theta \left[\frac{jB}{r} + \frac{1}{r^2}\right]$$

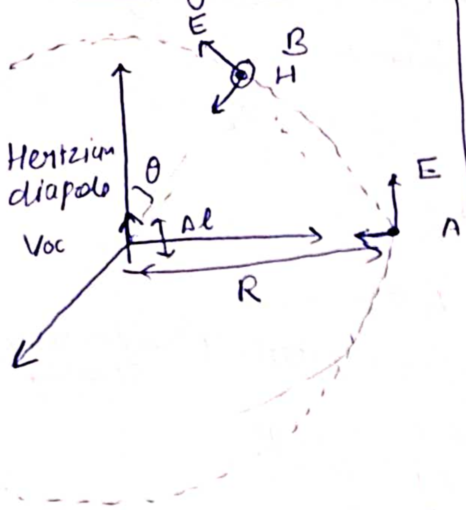
$$H_r = \frac{Ba^2 I_0 \cos\theta}{2r}\left[1 + \frac{1}{jBr}\right]e^{-jBr}$$

$$H_\theta = -\frac{(Ba)^2 I_0 \sin\theta}{4r}\left[1 + \frac{1}{jBr} - \frac{1}{Br}\right]$$

$$E_r = E_\theta = 0$$

$$E_\phi = \frac{\eta (Ba)^2 I_0 \sin\theta}{4r}\left(1 + \frac{1}{jBr}\right) e^{-jBr}$$

Reciving Antenna



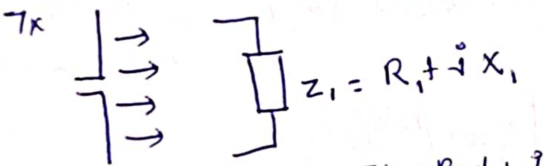
Let us consider source at A

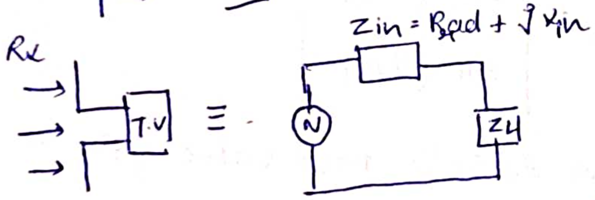
$Voc = E.\Delta l = V_A$

Now source at B

$Voc = V_B = E\Delta l \cos(90-\theta)$

$= E\Delta l \sin\theta$





Power max when $Z_L = R_{rad} - jX_{in}$

Power delived to matched load $P_r = \frac{|V_{oc}|^2}{8R} \quad \frac{1}{2}\left|\frac{V_{oc}}{2R_{rad}}\right|^2 R_{rad}$

$P_r = \frac{|V_{oc}|^2}{8R_{rad}}$

→ When EM wave is normal to surface, recieved power is

$P_r = \int P_{avg}.ds = P_{avg}\, dS$
↳ avg power density

$$\boxed{A_e = \frac{P_r}{P_{avg}}}$$

For Herz diapole

$G = D = 1.5 \quad D = 1.64$ small

$A_e = \frac{D\lambda^2}{4\pi} \Rightarrow$

Effective Area derivation

$P_r = \cancel{P_{rad}}\ \cancel{G_d}\ \frac{P_{rad} A_e}{4\pi r^2}$

$P_{avg} = \frac{G_d P_{rad}}{4\pi r^2}$

$P_{avg} = \frac{P_{rad}}{4\pi r^2}$

$P_r = P_{avg}.A_e$ ① step

$$\boxed{A_e = \frac{P_r}{P_{avg}}}$$

#

$P_r = \frac{|V_{oc}|^2}{8R_{rad}}$

for Hertz diapole

$R_{rad} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$

& $V_{oc} = Edl$

$\Rightarrow P_r = \frac{|E|^2 dl^2 \lambda^2}{8\times 80\pi^2 dl^2}$

$P_r = \frac{E^2\lambda^2}{640\pi^2}$

We know avg power at antenna

$P_{avg} = \frac{E^2}{2\eta} = \frac{E^2}{240\pi}$

$A_e = \frac{\frac{E^2\lambda^2}{8\ 640\pi^2}}{\frac{E^2}{3\ 240\pi}}$

$= \frac{3}{8}\frac{\lambda^2}{\pi}$

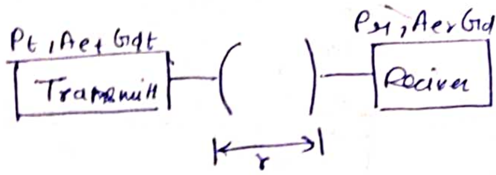
$A_e = \frac{1.5}{G}\frac{\lambda^2}{4\pi}$

$$\boxed{A_e = G_d(\theta,\phi)\frac{\lambda^2}{4\pi}}$$

Friis Transmission Formula

recived power

$P_{rad} = G_d$



At reciving antenna

$P_r = \underbrace{\frac{\lambda^2}{4\pi} G_{dr}}_{A_e} P_{avg}$ — A

At the transmitter antenna

$G_{dt} = \frac{4\pi U}{P_t} = \frac{4\pi r^2 P_{avg}}{P_t}$ $r^2 P_{avg}$

$P_{avg} = \frac{P_t G_{dt}}{4\pi r^2}$ — (B)

Substitute (B) in A

$P_r = \frac{\lambda^2}{4\pi} G_{dr}\left[\frac{P_t G_{dt}}{4\pi r^2}\right]$

$$\boxed{P_r = G_{dr} G_{dt}\left(\frac{\lambda}{4\pi r}\right)^2 P_t}$$

(Friis transmission formula

$r \geq \frac{2d^2}{\lambda}$

At reciver

$P_r = P_{avg} A_e$

$= P_{avg}\, G_{dr}\frac{\lambda^2}{4\pi}$

At transmitter

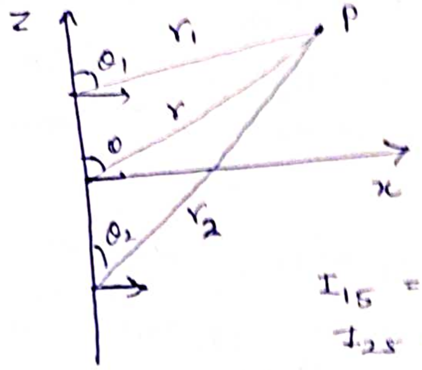
$P_{avg} = G_{dt}\frac{P_t}{4\pi r^2}$

$P_r = G_{dt} G_{dr}\left(\frac{\lambda}{4\pi r}\right)^2 P_t$

$\frac{P_r}{P_t} = G_{dt} G_{dr}\left(\frac{\lambda}{4\pi r}\right)^2$

Antenna Array

→ Two Element array



$I_{1S} = I_0 \angle \alpha$

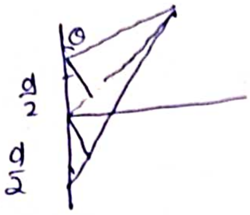
$I_{2S} = I_0 \angle 0$

$E_S = E_{1S} + E_{2S}$

$$\rightarrow \frac{j\eta\beta I_0 dl \cos\theta_1 \, e^{-j\beta r_1} e^{j\alpha}}{4\pi r_1} + \frac{j\eta\beta I_0 dl \cos\theta_2 \, e^{-j\beta r_2}}{4\pi r_2}$$

$\theta_1 \simeq \theta_2 \simeq \theta$
$r_1 \simeq r_2 \simeq r$] For amplitude

$$E_S = \frac{j\eta\beta I_0 dl \cos\theta}{4\pi r}\left[e^{-j\beta r_1} e^{j\alpha} + e^{-j\beta r_2}\right]$$



$r_1 = r - \frac{d}{2}\cos\theta$

$r_2 = r + \frac{d}{2}\cos\theta$

$$E_S = \frac{j\eta\beta I_0 dl \cos\theta}{4\pi r}\left[e^{j\alpha} e^{-j\beta(r - \frac{d}{2}\cos\theta)} + e^{-j\beta(r + \frac{d}{2}\cos\theta)}\right]$$

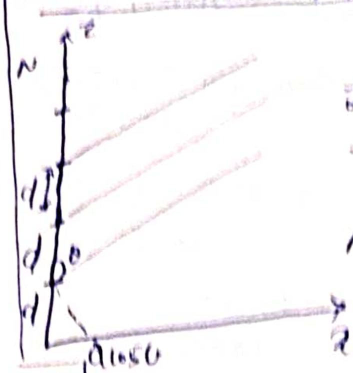
$$= j\eta\beta I_0 dl \cos\theta \, e^{\frac{j\alpha}{2}} e^{-j\beta r}\left[e^{-j\frac{\beta d\cos\theta + \alpha}{2}} + e^{j\frac{\beta d \cos\theta + \alpha}{2}}\right]$$

$$= \underbrace{\frac{j\eta\beta I_0 dl \cos\theta \, e^{-j\beta r}}{4\pi r}}_{E \text{ due to single}} \underbrace{e^{\frac{j\alpha}{2}} 2 \cos\left(\frac{\beta d\cos\theta + \alpha}{2}\right)}_{AF}$$

$$AF = 2\cos\left[\frac{\beta d \cos\theta + \alpha}{2}\right] e^{j\alpha/2}$$

$$(AF)_n = \cos\left(\frac{\beta d\cos\theta + \alpha}{2}\right)$$

N-Element Uniform Array



→ current distribution of isotropic element

$I_S = I_0\angle 0 + I_0\angle\alpha + I_0\angle 2\alpha \dots$

$AF = 1 + e^{j\psi} + \dots + e^{j(N-1)\psi}$ —(1)

$\psi = \beta d\cos\theta + \alpha$

α → progressive phase, d → interelement spacing

$$AF = \frac{1 - e^{jN\psi}}{1 - e^{j\psi}}$$

$$AF = \frac{e^{jN\psi/2}}{e^{j\psi/2}} \frac{e^{jN\psi/2} - e^{-jN\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}}$$

$$AF = e^{j(N-1)\psi/2}\left[\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right]$$

→ If array center about origin $e^{j(N-1)\psi/2}$ not present

$$|AF| = \left[\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right]$$

From Eqⁿ (1) max value of AF = N

$$(AF)_n = \frac{1}{N}\frac{\sin(N\psi/2)}{\sin\psi/2}$$

→ maximum value of AF at

$$\frac{\psi}{2} = \frac{\beta d\cos\theta + \alpha}{2} = \pm m\pi$$

$$\boxed{\theta_m = \cos^{-1}\frac{\lambda(2\pi m - \alpha)}{2\pi d}}$$

AF Only one global maxima $m = 0$

$\Rightarrow \theta_m = \cos^{-1}(-\alpha\lambda/2\pi d)$

→ maximum radiation along axis — End Fire Array

→ maximum radⁿ ⊥ to axis → broad side array

Herzian diapole

$$\nabla^2 A - \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = -\rho/\varepsilon$$

$$A = \frac{\mu J e^{-j\beta r}}{4\pi r}$$

$$H = \frac{1}{\mu}\nabla \times A$$

$$H_r = H_\theta = 0$$

$$H_\phi = \frac{I_0 dl \sin\theta\, e^{-j\beta r} e^{j\omega t}}{4\pi r}\left[j\beta + \frac{1}{r}\right]$$

$$E_r = \frac{2I_0\, dl \cos\theta\, e^{-j\beta r} e^{j\omega t}}{4\pi\varepsilon}\left[\frac{\beta}{\omega r^2} - \frac{j}{\omega r^3}\right]$$

$$E_\theta = \frac{I_0 dl \sin\theta\, e^{-j\beta r} e^{j\omega t}}{4\pi\varepsilon}\left[\frac{j\beta^2}{\omega r} + \frac{\beta^2}{\omega r^2} - \frac{j}{\omega r^3}\right]$$

$$P_{avg} = \frac{1}{2}\eta|H_\phi|^2 = \frac{1}{2\eta}|E_\phi|^2$$

$$= \frac{1}{2}\mathrm{Re}\left[E_\phi^* H_\theta^*\right]$$

$$W = \int_0^{\pi}\int_{\phi=0}^{2\pi} P_{avg}\, r^2 \sin\theta\, d\theta\, d\phi$$

$$W = \cancel{40\pi I_0^2}\ 40\pi^2\left(\frac{dl}{\lambda}\right)^2 I_0^2$$

$$R = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

Half wave mono pole

$$A = \frac{\mu I_0 dl\, e^{-j\beta r}}{2\pi r}F(\theta)$$

$$E = \frac{j60 I_m e^{-j\beta r}}{r}F(\theta)$$

$$W = 36.54\, I_0^2$$

$$R \approx 73\Omega \approx 73 + 42.5j$$

~~Small loop~~ Quater pole

$$W = 18.28\, I_0^2 \ (\text{Half}$$

$$R = 36.5\Omega$$

Small loop Antenna

$$A = \frac{a^2 \mu I_0}{4r} e^{-j\beta r}\left[\frac{j\beta}{} + \frac{1}{r}\right]$$

$$H_r = \frac{j\beta a^2 I_0 \cos\theta\, e^{-j\beta r}}{2r}\left[1 + \frac{1}{j\beta r}\right]$$

$$H_\theta = \frac{j(\beta a)^2 I_0 \sin\theta\, e^{-j\beta r}}{4r}\left[1 + \frac{1}{j\beta r} - \frac{1}{\beta r}\right]$$

$$H_\phi = 0$$

$$E_r = E_\theta = 0$$

$$E_\phi = \eta H_\phi = \frac{j\eta(\beta a)^2 I_0 \sin\theta\, e^{-j\beta r}}{4r}\left[1 + \frac{1}{j\beta r}\right]$$

2-element Antenna array

$$E = (\qquad) \times e^{j\alpha/2}\cos\left(\frac{\beta d\cos\theta + \alpha}{2}\right)$$

$$AF)_N = \cos\left(\frac{\beta d\cos\theta + \alpha}{2}\right)$$

N-Element Uniform array

$$AF = e^{j(N-1)\psi/2}\frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

$$(AF)_N = \frac{1}{N}\frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

→ End fire $\psi = 0$ $\theta = 0$ or π

$\alpha = -\beta d$ and $\alpha = \beta d$

→ Broad side

$\psi = 0$ $\theta = \pi/2$

$\alpha = 0$

for maximum

$$\frac{\psi}{2} = \cos\left(\frac{\beta d\cos\theta + \alpha}{2}\right) = \pm m\pi$$

$$\theta = \cos^{-1}\frac{1}{2\pi d}(-\alpha \pm 2\pi m)$$

for A.fonk

$$\theta_m = \cos^{-1}\frac{-\alpha\lambda}{2\pi d} = \cos^{-1}\frac{-\alpha}{\beta d}$$

Direction of nulls

Numerator $= 0$

$$\sin\frac{N\psi}{2} = 0 = \sin\cos N\left(\frac{\beta d\cos\theta + \alpha}{2}\right) = \pm n\pi$$

$$N(\beta d\cos\theta + \alpha)/2 = \pm n\pi$$

$$\theta = \cos^{-1}\left(-\alpha \pm 2n\pi/N\right)$$

Direction of side lobe

$$\sin\left(\frac{N\psi}{2}\right) = 1 =$$

$$N\psi/2 = \pm\left(\frac{2l+1}{2}\right)\pi$$

(2)
Directive Gain G_d

$$\boxed{G_d = \frac{U(\theta,\phi)}{U_{avg}} = \frac{4\pi\, U(\theta,\phi)}{P_{rad}}}$$

$$\boxed{P_{avg} = G_d \frac{P_{rad}}{4\pi r^2}}$$

for isotropic antenna

$G_d = 1$

Directivity

$$D = \frac{U_{max}}{U_{avg}} = G_{d\,max}$$

$$\boxed{D = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi\, U_{max}}{\int\int U(\theta,\phi)\, d\Omega}}$$

$$P_{avg} = \frac{1}{2\eta} |E_\phi|^2 \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

$$D = \frac{4\pi\, E_{max}^2}{\int\int |E(\theta,\phi)|^2 \sin\theta\, d\theta\, d\phi}$$

For Hertzian diapole

$\boxed{D = 1.5 \qquad G_d = 1.5 \sin^2\theta}$

For Half wave $D = 1.64$

(1)
Radition Intensity

$$U(\theta,\phi) = r^2 P_{avg}$$

$$U_{avg} = \frac{P_{rad}}{4\pi}$$

Power Gain

$$P_{in} = P_l + P_{rad}$$

$$P_{in} = \frac{1}{2} |I_{in}|^2 (R_l + R_{rad})$$

$$G_p(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_{in}}$$

$$\eta_r = \frac{G_p}{G_d} = \frac{P_{rad}}{P_{in}}$$

$$\eta_r = \frac{R_{rad}}{R_l + R_{rad}}$$

Recieiving Antenna

$$P_r = \frac{1}{2} \left|\frac{V_{oc}}{2R}\right|^2 R^2 = \frac{|V_{oc}|^2}{8 R_{rad}}$$

Effective area derivation

$$A_e = \frac{P_r}{P_{avg}} = \frac{\dfrac{|V_{oc}|^2}{8\left[80\pi^2 \left(\frac{dl}{\lambda}\right)^2\right]}}{\dfrac{1}{2\eta} |E|^2}$$

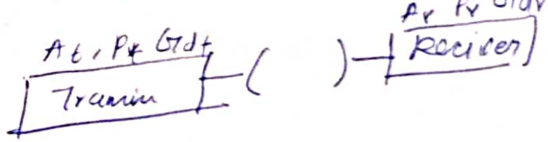
$$A_e = \frac{\dfrac{|E|^2 (dl)^2}{640\pi^2 \left(\frac{dl}{\lambda}\right)^2}}{2(120\pi)|E|^2}$$

$$A_e = \frac{3}{8}\frac{\lambda^2}{\pi} \Rightarrow A_e = 1.5 \frac{\lambda^2}{4\pi}$$

Directivity

$$\boxed{A_e = D \frac{\lambda^2}{4\pi}} \qquad \boxed{A_e = G_d \frac{\lambda^2}{4\pi}}$$

Frii's Transmission Eqn



At reciver

$$P_r = P_{avg} A_e = P_{avg} \frac{\lambda^2 G_{dr}}{4\pi}$$

At transmitter

$$P_{avg} = \frac{G_{dt} P_t}{4\pi r^2}$$

$$\boxed{P_r = G_{dt} G_{dr} \left(\frac{\lambda}{4\pi r}\right)^2 P_t}$$

$r \geq \frac{2d^2}{\lambda}$

Radar: Radio detecting and Ranging

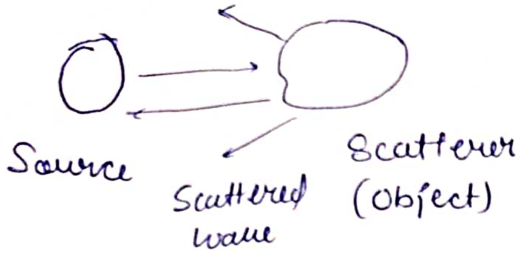
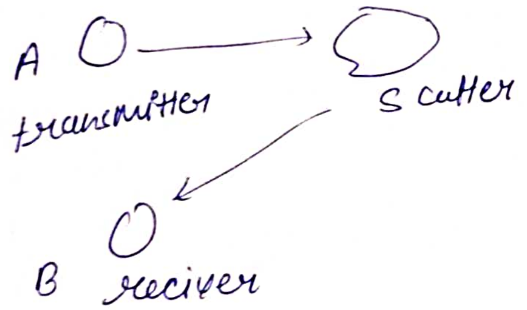


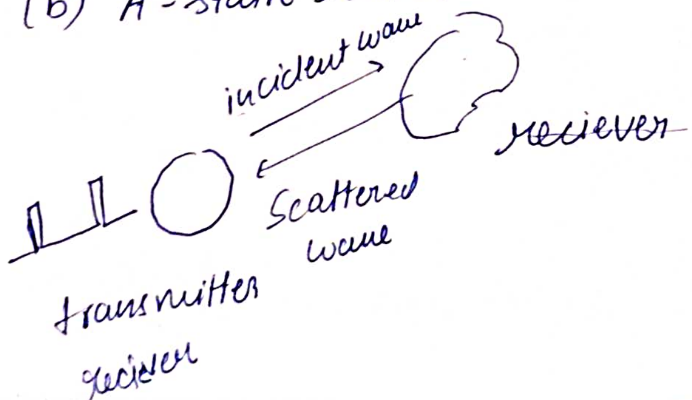
fig: Principle of radar operation reflection of wave from target

→ Two modes of Radar operation

(a) Bi-Static radar



(b) A-static radar



Transmitted power density

$P_i = \frac{P_{rad} G_{dt}}{4\pi r^2}$

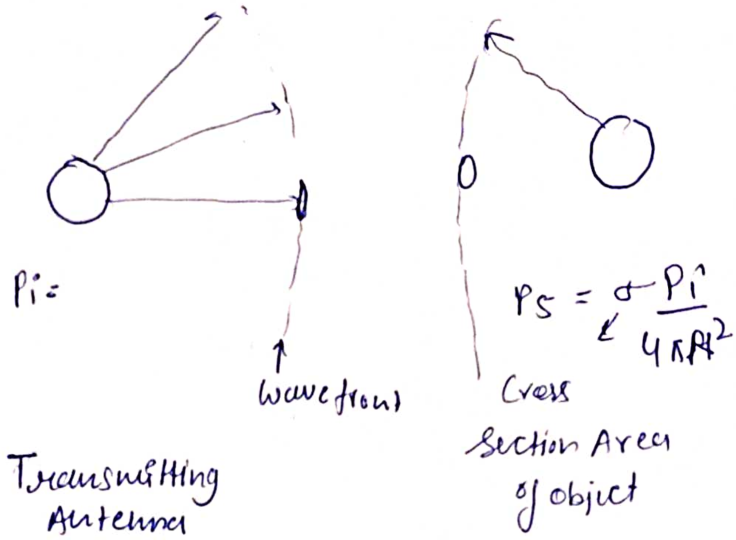
$P_s = \sigma P_i = \sigma \frac{P_{rad} G_{dt}}{4\pi r^2}$

↳ Scattered power

Scattered power den $= \frac{P_s}{4\pi r^2} = \frac{\sigma P_{rad} G_{dt}}{(4\pi r^2)^2}$

$P_{rec} = P_s A_e = \frac{\sigma P_{rad} G_{dt} G_{dr} \lambda^2}{4\pi (4\pi r^2)}$

↳ recived pow



$P_s = \sigma \frac{P_i}{4\pi R^2}$

Scattered power density

$P_i = P_{avg} = \frac{P_{rad}}{4\pi R^2} G_{dt}$

transmitted power

$P_s = \sigma \frac{P_{rad} G_{dt}}{(4\pi R^2)^2}$

Total recived power

$P_r = P_s A_{er} \quad G_{dr} \frac{\lambda^2}{4\pi}$

$P_r = \sigma \frac{G_{dt} G_{dr}}{4\pi (4\pi R^2)^2} P_{rad}$

$$\boxed{P_r = \sigma \frac{G_{dt} G_{dr} P_{rad}}{4\pi (4\pi R^2)^2}}$$

In case of same antenna

$G_{dt} = G_{dr} = G$

$$\boxed{P_r = \sigma \frac{P_{rad}}{4\pi} \left(\frac{G}{4\pi R^2}\right)^2}$$

Radar Eqn $R = (\quad)^{1/4}$

Radifion Intensity

$U(\theta,\phi) = r^2 P_{avg}$ (avg Power density)

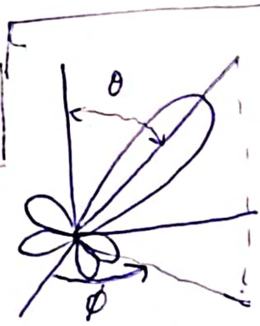
Total power $P_{rad} = \oint_s P_{avg} ds$

$= \int P_{avg}\, r^2 \sin\theta\, d\theta\, d\phi$

$= \int U(\theta,\phi) \sin\theta\, d\theta\, d\phi$

$= \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, \underbrace{d\Omega}_{\text{differential solid angle}}$

$U(\theta,\phi)$ $U_{avg} = \frac{P_{rad}}{4\pi}$



Directive Gain

$G_d(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}}$

$G_d = \frac{4\pi U(\theta,\phi)}{P_{rad}} = 4\pi \frac{r^2 P_{avg}}{P_{rad}}$

$$\boxed{P_{avg} = \frac{G_d \cdot P_{rad}}{4\pi r^2}}$$

For isotropic antenna

$\boxed{G_d = 1}$

Directivity

$D = \frac{U_{max}}{U_{avg}} = G_{d\,max}$

$= \frac{4\pi U_{max}}{P_{rad}}$

$= \frac{4\pi U_{max}}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) d\Omega}$

$D = \frac{4\pi E_{max}^2}{\int_0^{2\pi}\int_0^{\pi} |E(\theta,\phi)|^2 \sin\theta\, d\theta\, d\phi}$

For Hertzian dipole

$G_d = 1.5 \sin^2\theta$

$G_{d\,max} = 1.5$

→ For $\frac{\lambda}{2}$ diapole

$G_d(\theta,\phi) = \frac{\eta}{\pi R_{rad}} f^2(\theta)$

$D = 1.64$

Power Gain

$\rightarrow$ P_{in} P_l $\rightarrow P_{rad}$

$P_{in} = P_l + P_{rad}$

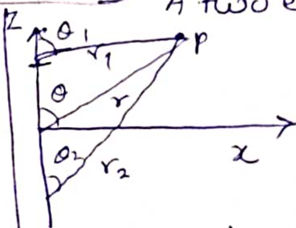
$P_{in} = \frac{1}{2} |I_{in}|^2 (R_l + R_{rad})$

$G_p(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{in}}$

$$\boxed{G_p = \eta_r G_d}$$

Antenna Array

A two element array



$I_1 = I_0 \angle \alpha$

$I_0 = I_0 \angle 0$.

$E_s = E_{1s} + E_{2s}$

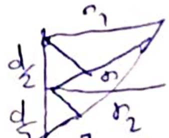
$E_{1s} = \frac{j\eta\beta I_0 dl \cos\theta}{4\pi r_1} e^{-j\beta r_1} e^{j\frac{\alpha}{2}} a_\theta$

$+ \frac{j\eta\beta I_0 dl \cos\theta_2}{4\pi r_2} e^{-j\beta r_2}$

For amplitude $r_1 \approx r_2 \approx r$ & $\theta_1 \approx \theta_2 \approx \theta$

$E_s = \frac{j\eta\beta I_0 dl \cos\theta}{4\pi r_1}\left[e^{-j\beta r_1} e^{j\alpha} + e^{-j\beta r_2}\right]$

$r_1 = r - \frac{d}{2}\cos\theta$ $r_2 = \frac{d}{2} + \cos\theta$



$= \frac{j\eta\beta I_0 dl \cos\theta}{4\pi r}\left[e^{-j\beta(r - \frac{d}{2}\cos\theta)} e^{j\alpha} + e^{-j\beta(r + \frac{d}{2}\cos\theta)}\right]$

$\Rightarrow \frac{j\eta\beta I_0 dl \cos\theta}{4\pi r} e^{-j\beta r} e^{j\alpha/2}\left[e^{-j(\frac{\alpha + d\cos\theta}{2})} + e^{-j(\frac{\alpha + d\cos\theta}{2})}\right]$

$\Rightarrow \frac{j\eta\beta I_0 dl \cos\theta}{4\pi r} e^{-j\beta r} e^{j\frac{\alpha}{2}} \times \cos\left(\frac{\alpha + d\cos\theta}{2}\right)$

$E_s = \frac{j\eta\beta dl \cos\theta}{4\pi r} e^{-j\beta r} e^{j\frac{\alpha}{2}} \cos(\alpha + d \ldots$