Bio medical 69

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QUESTIONS

1 Basics of Anatomy and Physiology

- 1.

1. For mammalian cells, typical values for the intracellular fluid (ICF) and extracellular fluid (ECF) concentrations of the major ion species in millimoles per liter are as follows:

Species	ICF	ECF
Na^+	5	145
K^+	140	5
Cl^-	4	110

Assuming room temperature of $37\,^{\circ}\mathrm{C}$ and typical values of the permeability coefficients P_{Na}=2x10^{-8} cm/s, P_K=2x10^{-6}cm/s, P_{Cl}=4x10^{-6}cm/s, calculate the equibrium resting potential for the membrane using Goldman equation. Assume gas constant $R = 8.31$ J/(mol. K) and Faraday's constant $F = 96,500$ C/equivalent.

Sol :

According to Goldman Hodgkin Katz (GHK) constant-field equation →

$$\boxed{E_M = \frac{RT}{F} \ln \frac{P_K[K^+]_{out} + P_{Na}[Na^+]_{out} + P_{Cl}[Cl^-]_{out}}{P_K[K^+]_{in} + P_{Na}[Na^+]_{in} + P_{Cl}[Cl^-]_{in}}}$$

Where,

E_M = Membrane potential = ?

R = Gas constant = 8.31 J/mol.K

T = Absolute temperature = $(37+273)$ K = 310 K

P_K = Relative permeability of K = 2×10^{-6} cm/s

P_{Na} = Relative permeability of Na = 2×10^{-8} cm/s

P_{Cl} = Relative permeability of Cl = 4×10^{-6} cm/s

$[K^+]_{out}$ = extracellular concentration = 5 mM/Lit

$[K^+]_{in}$ = Intracellular concentration = 140 mM/Lit

$[Na^+]_{out}$ = 145 mM/Lit

$[Na^+]_{in}$ = 5 mM/Lit

$[Cl^-]_{out}$ = 110 mM/Lit

$[Cl^-]_{in}$ = 4 mM/Lit

F = 96500 C/equivalent

⇒ E_M

$$= \frac{8.31\times310}{96500} \ln \frac{(2\times10^{-6}\times5) + (2\times10^{-8}\times145) + (4\times10^{-6}\times110)}{(2\times10^{-6}\times140) + (2\times10^{-8}\times5) + (4\times10^{-6}\times4)}$$

$$= \frac{2576.1}{96500} \ln \frac{10^{-5} + 2.9\times10^{-6} + 4.4\times10^{-4}}{2.8\times10^{-4} + 10^{-7} + 1.6\times10^{-5}}$$

$$= 0.0267 \ln \frac{4.529\times10^{-4}}{2.961\times10^{-4}}$$

$$= 0.0267 \times 0.4250$$

$$= 1.13475\times10^{-2} \text{ mV}$$

Answer → So the equilibrium resting potential will be $\underline{1.13475\times10^{-2}\text{ mV}}$.

2 Event Detection

▼ 2. **Describe the differences between the application of conventional first-order derivativebased filter and modified 3-point central difference equation-based filter on the**
removal of the baseline wandering in ECG signals? Use difference equations, transfer functions, and the amplitute and phase responses while answering the

question

▼ **3. Describe the Pan-Tompkins QRS detection algorithm with a block diagram. For**
all the involved filters, give the corresponding difference equation, filter cut-offs,
filter gain, and filter delay.

3 Adaptive Signal Processing

▼ **4. Describe Wiener filtering with a block diagram and derive the time as well as**
frequency domain equations for implementation of the same.

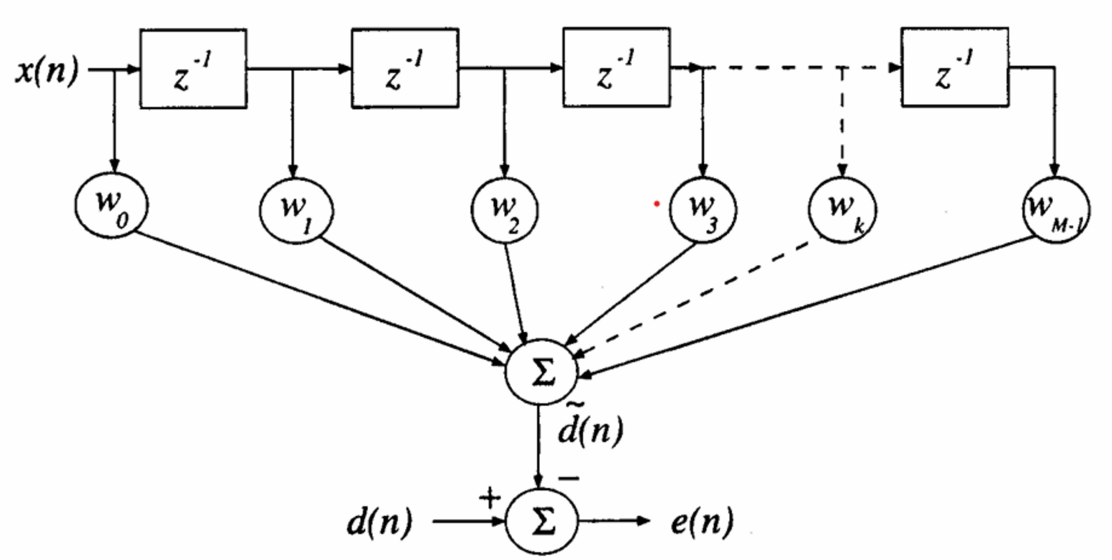
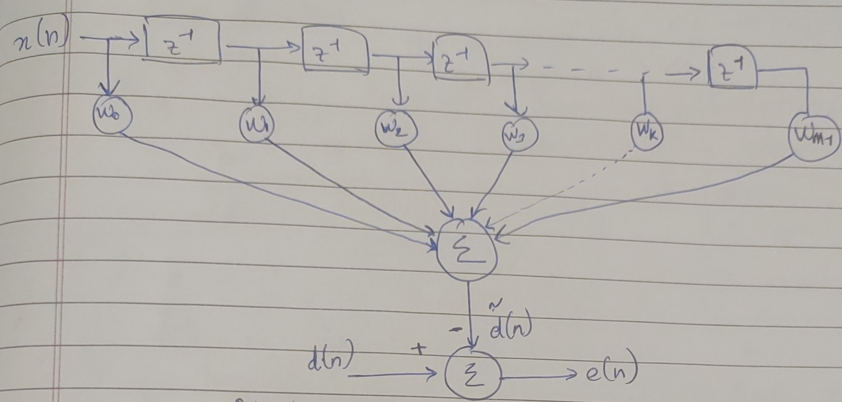


Figure 3.46 Block diagram of the Wiener filter.

Wiener Filter

Block diagram of Wiener Filter



~~Block diag.~~

Here, $x(n)$ = input to the filter
$w(n)$ = coefficient / Tap weight for each input
$\tilde{d}(n)$ = Output of wiener filter
$d(n)$ = The desired output from the filter.

$$e(n) = d(n) - \tilde{d}(n)$$

$e(n)$ = estimation error b/w output and desired signal.

since $\tilde{d}(n)$ is output of Linear FIR filter it can be expressed as convolution of inputs ~~as~~ and Tap weight as

$$\cancel{\tilde{d}(n) = \sum_{k=0}^{M-1} w_k x(n-k)} \quad \tilde{d}(n) = \sum_{k=0}^{M-1} w_k \, x(n-k)$$

In vector form we represent $\vec{w}$ as $M \times 1$ tap weight vector

$$w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{M-1} \end{bmatrix}$$

similarly $x(n) = \begin{bmatrix} x(n) \\ x(n-1) \\ x(n-2) \\ \vdots \\ x(n-M+1) \end{bmatrix}$

$x(n)$ varies with ~~bcaa~~ time

so,

$$\tilde{d}(n) = w^T x(n) = x^T(n) w = \langle x, w \rangle$$

estimation error $\Rightarrow e(n) = d(n) - w^T x(n)$

To ~~calculate~~ ~~the~~ minimise the estimation error in Wiener Filter we calculate the Minimum mean squared error (MMSE) To optimise the filter

$$J(w) = E[e^2(n)]$$

$$= E[\{d(n) - w^T x(n)\}\{d(n) - w^T x(n)\}]$$

$$= E[d^2(n) - w^T E[x(n) d(n)] - E[d(n) x^T(n)] w + w^T E[x(n) x^T(n)] w]$$

Assuming $x(n)$ and $d(n)$ are jointly stationary and $E[d(n)] = 0$ we get

1) $E[d^2(n)]$ = varience of $d(n)$ = σ_d^2

2) $E[x(n) d(n)]$ = cross correlation b/w $x(n)$ and $d(n)$ which is $M \times 1$ vector

$$\Theta = E[x(n) d(n)]$$

$$\Theta = [\theta(0), \theta(-1), \ldots \theta(1-M)]^T$$

where $\theta(-k) = E[x(n-k) d(n)]$ $k = 0, 1, 2$

3) $E[x(n) x^T(n)] = \Phi$ = auto correlation of input vector ~~(x)~~ $x(n)$ computed as outer product with itself

$$\Phi = \begin{bmatrix} \phi(0) & \phi(1) & \cdots & \phi(M-1) \\ \phi(-1) & \phi(0) & \cdots & \phi(M-2) \\ \vdots & & & \vdots \\ \phi(-M+1) & \phi(-M+2) & \cdots & \phi(0) \end{bmatrix}_{M \times M}$$

~~where~~ where $\phi(i-k) = E[x(n-k) x(n-i)]$

so,

$$J = \sigma_d^2 - w^T \Theta - \Theta^T w + w^T \Phi w$$

This indicates MMSE is second order function of Tap weight vector w. To denote optimal Tap-weight w_o we differentiate $J(w)$ wrt w, set it to 0 and solve the resulting equation.

$$\frac{d}{dw}\left(\Theta^T w\right) = \Theta = \frac{d}{dw}\left(w^T \Theta\right)$$

$$\frac{d}{dw}\left(w^T \Phi w\right) = 2\Phi w \qquad \text{so,}$$

$$\frac{d}{dw} J(w) = -2\Theta + 2\Phi w \qquad \left(\text{setting } \frac{d}{dw}J(w) = 0\right)$$

$$\Rightarrow \boxed{\Phi w_o = \Theta}$$

This equation is called Wiener-Hopf equation so,

$$\boxed{w_o = \Phi^{-1} \Theta}$$

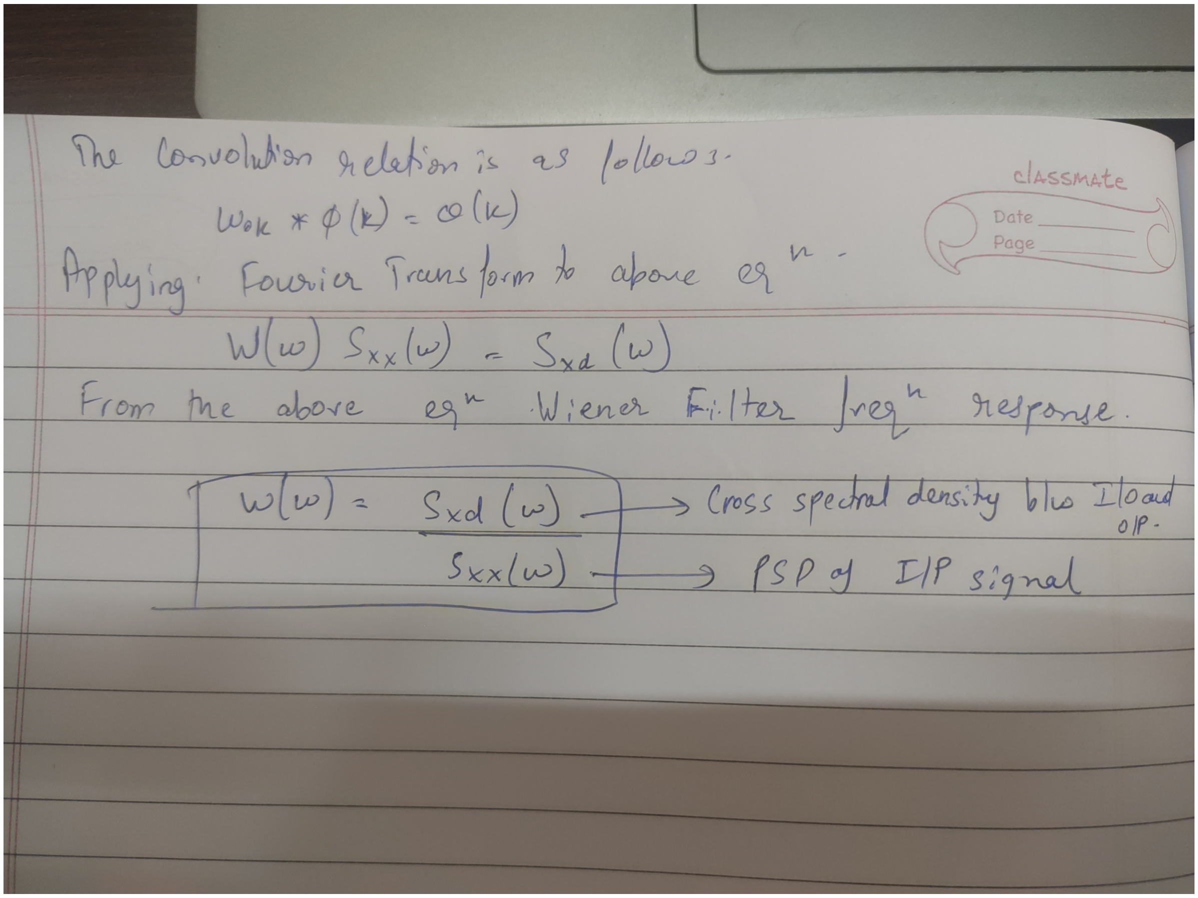
$$\Rightarrow \begin{bmatrix} \phi(0) & \phi(1) & \cdots & \phi(M-1) \\ \phi(-1) & \phi(0) & \cdots & \phi(M-2) \\ \vdots & & & \vdots \\ \phi(-M+1) & \phi(-M+2) & & \phi(0) \end{bmatrix} \begin{bmatrix} w_o 0 \\ w_o 1 \\ w_o 2 \\ \vdots \\ w_o(M-1) \end{bmatrix} = \begin{bmatrix} \Theta(0) \\ \Theta(-1) \\ \vdots \\ \Theta(1-M) \end{bmatrix}$$

$$\Rightarrow \sum_{i=0}^{M-1} w_{oi}\, \phi(i-k) = \Theta(-k) \qquad k = 0, 1, 2 \ldots M-1$$

$$J_{min} = \sigma_d^2 - \Theta^T \Phi^{-1} \Theta$$

Since signals are stationary $\phi(i-k) = \phi(k-i)$ and $\Theta(-k) = \Theta(k)$

$$\sum_{i=0}^{M-1} w_{oi}\, \phi(k-i) = \Theta(k), \quad k = 0, 1, 2, \ldots$$



The Convolution relation is as follows.

$$w_{ok} * \phi(k) = \theta(k)$$

Applying Fourier Transform to above eq^n.

$$W(\omega)\, S_{xx}(\omega) = S_{xd}(\omega)$$

From the above eq^n Wiener Filter freq^n response.

$$\boxed{W(\omega) = \frac{S_{xd}(\omega)}{S_{xx}(\omega)}}$$

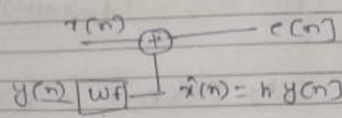
$S_{xd}(\omega)$ → Cross spectral density b/w I/P and O/P.

$S_{xx}(\omega)$ → PSD of I/P signal

▼ 5. For single weight-case, describe the steepest-descent algorithm with analytical
steps for solving Wiener filtering. Why iterative steepest-descent algorithm is preferred than solving basic equation wopt=R−1r.

(5) single-weight describle steepest decent algorithm with wiener coifilter: $w_{opt} = \bar{R}^1 \gamma$

$\Rightarrow$ $h_{opt} = \bar{R}^1 \gamma$



for stepeest algo:

$$\mathcal{E}(h_{opt}) \leq \mathcal{E}(h)$$

$$\hat{x}(n) = h(n)\, y(n)$$

$$\hat{x}(n+1) = h(n+1)\, y(n+1)$$

$$\Rightarrow \mathcal{E}(h+\Delta h) \leq \mathcal{E}(h)$$

using taylor expansion

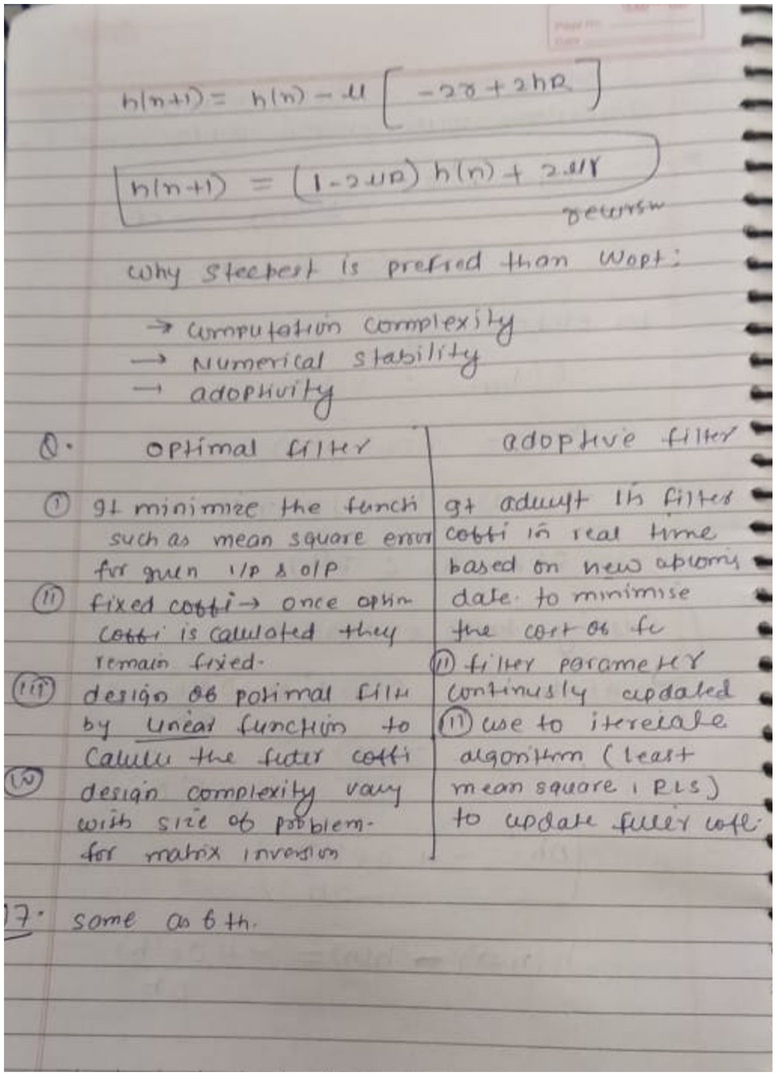
$$\mathcal{E}(\Delta h + h) = \mathcal{E}(h) + \Delta h \frac{\partial \mathcal{E}(h)}{\partial h} \leq \mathcal{E}(h)$$

$$\Delta h \propto -\frac{\partial \mathcal{E}(h)}{\partial h}$$

$\bar{R}^1 \gamma$

$$\boxed{\Delta h = -\mu \frac{\partial \mathcal{E}(h)}{\partial h}}$$

$$h(n+1) - h(n) = -\mu \frac{\partial \mathcal{E}(h)}{\partial h}$$



$h(n+1) = h(n) - \mu\left[-2r + 2hR\right]$

$\boxed{h(n+1) = (1-2\mu R)\, h(n) + 2\mu r}$

recursive

why steepest is prefred than Wopt:

→ computation complexity
→ Numerical stability
→ adaptivity

Q. optimal filter	adaptive filter
(i) It minimize the functi such as mean square error for given i/p & o/p	It adjust its filter coffi in real time based on new incoming date to minimise the cost of fc
(ii) fixed coffi → once optim coffi is calculated they remain fixed.	(ii) filter parameter continusly updated
(iii) design of potimal filt by linear function to calculi the filter coffi	(iii) use to itereiate algorithm (least mean square, RLS) to update filter cofe.
(iv) design complexity vary with size of problem. for matrix inversion	

17. some as 6th.

▼ 6.List out the differences between the optimal filter and the adaptive filter.

Optimal Filters:
1.Optimal filters are designed to provide the best performance based on a specific performance criterion.
2.They are typically designed for stationary signals where the statistical

properties do not change over time.
3.The Wiener-Hopf equations are used to derive the optimal filter.
4.The optimal filter design involves structures like prediction and interpolation.

Adaptive Filters:
1.Adaptive filters are digital filters whose coefficients change with an objective to make the filter converge to an optimal state.
2.They are particularly useful when the statistical moments relevant to solving the optimal filtering problem are unknown and should be estimated from the incoming data and a training sequence.
3.Adaptive filters are often used in applications where the filter's characteristics must be able to adapt to changes in the signal.
4.Algorithms like Recursive Least Squares (RLS) and Least Mean Square (LMS) are used in adaptive filters.
5.Adaptive filters can be regarded as approximations to the Wiener filter.

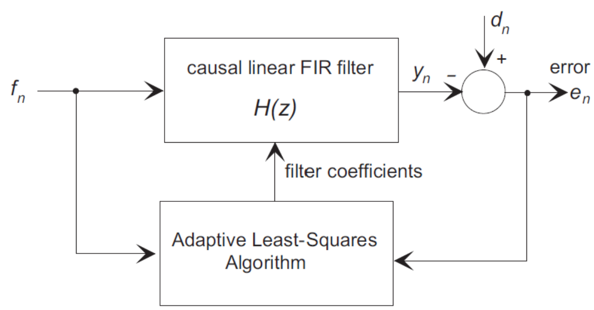
▼ 7. What is an adaptive filter? How it differs from the ordinary filter.

1 Adaptive Filtering

In Lecture 24 we looked at the least-squares approach to FIR filter design. The filter coefficients b_m were generated from a one-time set of experimental data, and then used subsequently, with the assumption of stationarity. In other words, the design and utilization of the filter were decoupled.

We now extend the design method to *adaptive FIR filters*, where the coefficients are continually adjusted on a step-by-step basis during the filtering operation. Unlike the static least-squares filters, which assume stationarity of the input, adaptive filters can track slowly changing statistics in the input waveform.

The adaptive structure is shown in below. The adaptive filter is FIR of length M with coefficients b_k, $k = 0, 1, 2, \ldots, M-1$. The input stream $\{f(n)\}$ is passed through the filter to produce the sequence $\{y(n)\}$. At each time-step the filter coefficients are updated using an error $e(n) = d(n) - y(n)$ where $d(n)$ is the desired response (usually based of $\{f(n)\}$).



The filter is not designed to handle a particular input. Because it is adaptive, it can adjust to a broadly defined task.

▼ 8. When are adaptive filters preferred?

▼ 9. Describe LMS algorithm with steps and a signal flow graph?

1.1 The Adaptive LMS Filter Algorithm

1.1.1 Simplified Derivation

In the length M FIR adaptive filter the coefficients $b_k(n)$, $k = 1, 2, \ldots, M-1$, at time step n are adjusted continuously to minimize a step-by-step squared-error performance index $J(n)$:

$$J(n) = e^2(n) = (d(n) - y(n))^2 = \left(d(n) - \sum_{k=0}^{M-1} b(k) f(n-k) \right)^2$$

$J(n)$ is described by a quadratic surface in the $b_k(n)$, and therefore has a single minimum. At each iteration we seek to reduce $J(n)$ using the "steepest descent" optimization method, that is we move each $b_k(n)$ an amount proportional to $\partial J(n)/\partial b(k)$. In other words at step $n+1$ we modify the filter coefficients from the previous step:

$$b_k(n+1) = b_k(n) - \Lambda(n) \frac{\partial J(n)}{\partial b_k(n)}, \qquad k = 0, 1, 2, \ldots M-1$$

where Λ(n) is an empirically chosen parameter that defines the step size, and hence the rate of convergence. (In many applications $\Lambda(n) = \Lambda$, a constant.) Then

$$\frac{\partial J(n)}{\partial b_k} = \frac{\partial e^2(n)}{\partial b_k} = 2e(n) \frac{\partial e(n)}{\partial b_k} = -2e(n) f(n-k)$$

and the fixed-gain FIR *adaptive Least-Mean-Square (LMS) filter algorithm* is

$$\boxed{b_k(n+1) = b_k(n) + \Lambda e(n) f(n-k), \qquad k = 0, 1, 2, \ldots M-1}$$

or in matrix form

$$\boxed{\mathbf{b}(n+1) = \mathbf{b}(n) + \Lambda e(n) \mathbf{f}(n),}$$

where

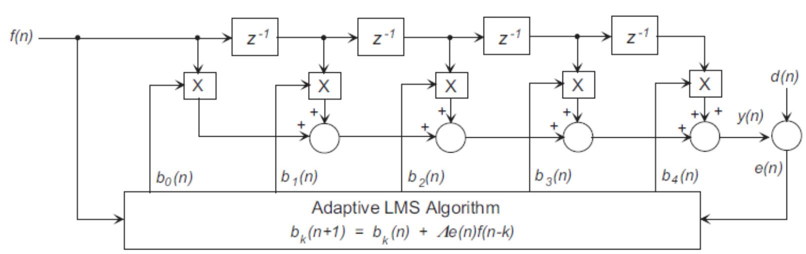
$$\mathbf{b}(n) = [b_0(n) \quad b_1(n) \quad b_2(n) \quad \cdots \quad b_{M-1}]^T$$

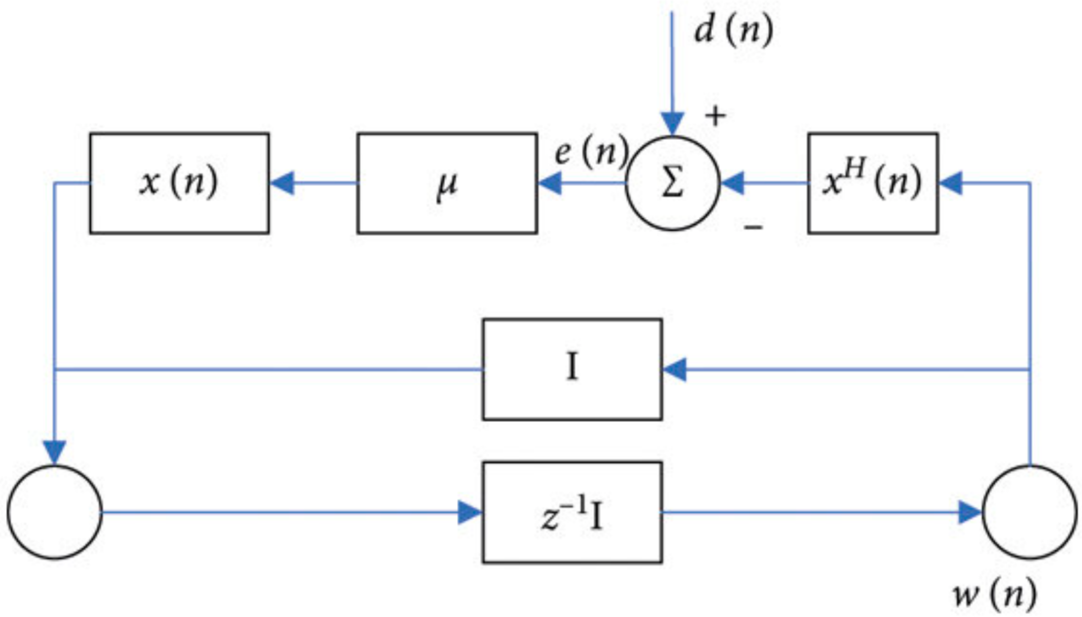
is a column vector of the filter coefficients, and

$$\mathbf{f}(n) = [f(n) \quad f(n-1) \quad f(n-2) \quad \cdots \quad f(n-(M-1))]^T$$

is a vector of the recent history of the input $\{f(n)\}$.

A Direct-Form implementation for a filter length $M = 5$ is





The signal flow diagram of the LMS algorithm.

▼ 10. What do you mean by least square estimation?

▼ 11. List out the variants of adaptive algorithms.

1. Least Mean Squares (LMS) Algorithm Variants

- **Standard LMS:** The simplest form, using a straightforward implementation but sensitive to step-size selection.
- **Normalized Least Mean Squares (NLMS):** Adjusts the step size according to the input signal's energy, improving stability and convergence.
- **Sign LMS:** Uses only the sign of the error and the sign of the input, reducing computational complexity.
- **Leaky LMS:** Incorporates a leakage factor to ensure stability by penalizing large weights.
- **Affine Projection Algorithm (APA):** Generalizes the NLMS algorithm by using multiple data vectors and desired responses to improve convergence rate.

2. Recursive Least Squares (RLS) Algorithm Variants

- **Standard RLS:** Provides fast convergence by recursively updating weight vectors based on all past data.
- **Exponentially Weighted RLS:** Introduces a forgetting factor to give more importance to recent data, useful in non-stationary environments.
- **Sliding Window RLS (also known as Windowed or Finite Window RLS):** Limits the window of data points considered for computation, balancing between performance and computational load.

▼ 12. How the step size impacts the LMS algorithm?

▼ 13. What is the RLS algorithm, and how it differs from LMS?

2 The Recursive-Least-Squares Filter Algorithm

For a filter as shown in Fig. 1, the total-squared-error $\mathcal{E}(n)$ at the nth iteration is defined as

$$\mathcal{E}(n) = \sum_{i=0}^{n} e^2(i) = \sum_{i=0}^{n} (d(i) - y(i))^2 \tag{1}$$

We modify the standard least-squares approach by including an exponential "forgetting factor" λ^{n-i}, $(0 < \lambda \leq 1)$ to each error term, and modify Eq. 1 as follows

$$\mathcal{E}'(n) = \sum_{i=0}^{n} \lambda^{n-i} e^2(i) = \sum_{i=0}^{n} \lambda^{n-i} (d(i) - y(i))^2$$

[1]D. Rowell December 9, 2008

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$$= \sum_{i=0}^{n} (d'(i) - y'(i))^2 \tag{2}$$

where $d'(i) = \sqrt{\lambda^{n-i}} d(i)$, and $y'(i) = \sqrt{\lambda^{n-i}} y(i)$. The purpose of the factor λ is to weight recent data points more heavily, and thus allow the filter to track changing statistics in the input data.

The FIR filter output is given by the convolution sum

$$y(n) = \sum_{k=0}^{M-1} b_k f(n-k) \tag{3}$$

and for stationary waveforms, Eq. (1) at time step n reduces to

$$\begin{aligned}
\mathcal{E}'(n) &= \sum_{i=0}^{n} d'^2(i) + \sum_{i=0}^{n} y'^2(i) - 2\sum_{i=0}^{n} d'(i) y'(i) \\
&= \sum_{i=0}^{n} \lambda^{n-i} d^2(i) + \sum_{i=0}^{n} \lambda^{n-i} \sum_{k=0}^{M-1} \sum_{m=0}^{M-1} b_k b_m f(n-k) f(n-m) \\
&\quad -2 \sum_{i=0}^{n} \lambda^{n-i} \sum_{k=0}^{M-1} b_k f(n-k) d(i)
\end{aligned} \tag{4}$$

▼ 14. Describe the main differences among Wiener, LMS and RLS algorithms

▼ 15.Describe at least three biomedical applications of the Wiener filtering or adaptive
signal processing with block diagram.

▼ 16. Explain how cancellation of maternal ECG in fetal electrocardiograpy is achieved by using adaptive signal processing with block diagram?

3.9 APPLICATION: ADAPTIVE CANCELLATION OF THE MATERNAL ECG TO OBTAIN THE FETAL ECG

Problem: *Propose an adaptive noise cancellation filter to remove the maternal ECG signal from the abdominal-lead ECG shown in Figure 3.9 to obtain the fetal ECG. Chest-lead ECG signals of the mother may be used for reference.*

Solution: Widrow et al. [62] describe a multiple-reference ANC for removal of the maternal ECG in order to obtain the fetal ECG. The combined ECG was obtained from a single abdominal lead, whereas the maternal ECG was obtained via four chest leads. The model was designed to permit the treatment of not only multiple sources of interference, but also of components of the desired signal present in the reference inputs, and further to consider the presence of uncorrelated noise components in the reference inputs. It should be noted that the maternal cardiac vector is projected onto the axes of different ECG leads in different ways, and hence the characteristics of the maternal ECG in the abdominal lead would be different from those of the chest-lead ECG signals used as reference inputs.

Each filter channel used by Widrow et al. [62] had 32 taps and a delay of 129 ms. The signals were pre-filtered to the bandwidth $3 - 35$ Hz and a sampling rate of 256 Hz was used. The optimal Wiener filter (see Section 3.5) included transfer functions and cross-spectral vectors between the input source and each reference input. Further extension of the method to more general multiple-source, multiple-reference noise cancelling problems was also discussed by Widrow et al.

The result of cancellation of the maternal ECG from the abdominal lead ECG signal in Figure 3.9 is shown in Figure 3.58. Comparing the two figures, it is seen that the filter output has successfully extracted the fetal ECG and suppressed the maternal ECG. See Widrow et al. [62] for details; see also Ferrara and Widrow [91].

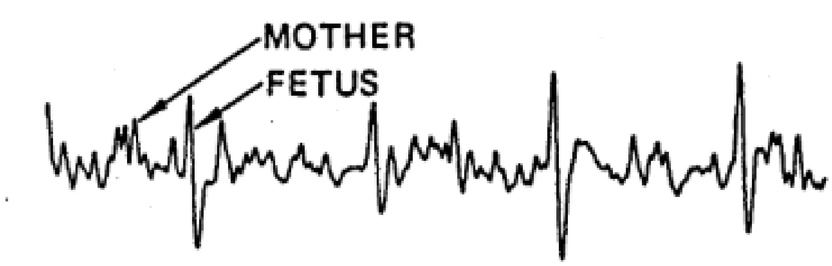


Figure 3.58 Result of adaptive cancellation of the maternal chest ECG from the abdominal ECG in Figure 3.9. The QRS complexes extracted correspond to the fetal ECG. Reproduced with permission from B. Widrow, J.R. Glover, Jr., J.M. McCool, J. Kaunitz, C.S. Williams, R.H. Hearn, J.R. Zeidler, E. Dong, Jr., R.C. Goodlin, Adaptive noise cancelling: Principles and applications, *Proceedings of the IEEE*, 63(12):1692–1716, 1975. ©IEEE.

▼ 17.

17. Consider a sinusoidal signal $s[n]=A\cos(\omega_0 n + \phi)$ with $A = 5$, $\omega_0 = \pi/4$, and ϕ is uniformly distributed random variable in the range 0 to 2π. Assume $v[n]$ as zero mean and unit variance white noise which corrupts the uncorrelated signal $s[n]$ to form $x[n]$. Design a second-order Wiener filter to filter $x[n]$ with desired signal $d[n]=s[n]$.

EXAMPLE 16.3 Consider the block diagram as shown in Figure 16.7. In the figure, $s[n]$ is a sinusoidal signal which is expressed as $s[n] = A\cos(\omega_0 n + \phi)$ with the amplitude $A = 5$ V and $\omega_0 = \frac{\pi}{4}$, the phase angle 'ϕ' is a random variable which follows uniform distribution in the range 0 to 2π. '$v[n]$' represents a white noise zero mean and unit variance. The objective is to design a second-order Wiener filter such that the desired signal $d[n]$ resembles the signal $s[n]$. Assume that the signal and the noise are uncorrelated.

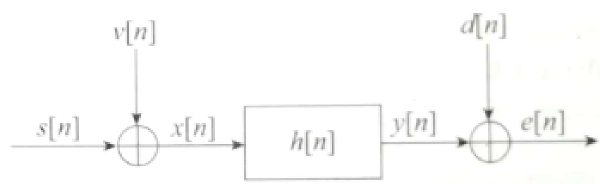


FIGURE 16.7 Block diagram

Solution

Given data

(i) Expression for the signal $s[n]$ is $s[n] = A\cos(\omega_0 n + \phi)$
(ii) The additive noise $v[n]$ has zero mean and unit variance.
(iii) The desired signal is same as the uncorrupted signal $s[n]$. That is $d[n] = s[n]$.

To find: (i) Second-order optimal Wiener filter that will minimise the mean square error between the desired signal and the observed signal.

Step 1: According to the Wiener–Hopf relation, the expression for optimal filter is given by

$$h_{\text{opt}} = R^{-1} p$$

In the above expression, 'R' represents the autocorrelation matrix of the observed sequence $x[n]$ and 'p' denotes the cross-correlation vector between the desired signal and the observed signal. The expression for optimal filter can be given as

$$h_{\text{opt}} = R_{xx}^{-1} \times R_{dx}$$

It is desired to design second-order linear phase filter. We know that second-order filter will have three coefficients, hence the above expression can be written in matrix form as

$$h_{opt} = \begin{bmatrix} R_{xx}(0) & R_{xx}(1) & R_{xx}(2) \\ R_{xx}(1) & R_{xx}(0) & R_{xx}(1) \\ R_{xx}(2) & R_{xx}(1) & R_{xx}(0) \end{bmatrix}^{-1} \begin{bmatrix} R_{dx}(0) \\ R_{dx}(1) \\ R_{dx}(2) \end{bmatrix}$$

Step 2: To find the autocorrelation matrix R_{xx}

As per Figure 16.7, the expression for the observed sequence is

$$x[n] = s[n] + v[n]$$

From the above equation, the expression for the autocorrelation of the signal $x[n]$ is given by

$$R_{xx}(k) = R_{ss}(k) + R_{vv}(k) \tag{16.22}$$

Step 2a: To find the expression for $R_{ss}(k)$

The expression for the signal $s[n]$ is $s[n] = A\cos(\omega_0 n + \phi)$. The autocorrelation of this signal is given by

$$R_{ss}(k) = E\{s[n]s^*[n-k]\}$$

If the signal is real, the expression for the autocorrelation function is given by

$$R_{ss}(k) = E\{s[n]s[n-k]\}$$

Substituting the expression for $s[n]$ and $s[n-k]$ in the above expression we get

$$R_{ss}(k) = E\{A\cos(\omega_0 n + \phi)A\cos(\omega_0(n-k) + \phi)\}$$

Simplifying the above expression, the expression for the autocorrelation is given by

$$R_{ss}(k) = \frac{A^2}{2}\cos(\omega_0 k) \tag{16.23}$$

Step 2b: To find the expression for $R_{vv}(k)$

The signal $v[n]$ is an additive noise with zero mean and unit variance. The expression for the autocorrelation of white noise is given by

$$R_{vv}(k) = \sigma_v^2\delta(k) \tag{16.24}$$

It is mentioned in the problem statement that the variance value is unity; hence the above expression is given by

$$R_{vv}(k) = \delta(k)$$

Combining the results of step 2a and step 2b, the expression for autocorrelation of observed sequence is given by

$$R_{xx}(k) = \frac{A^2}{2}\cos(\omega_0 k) + \delta(k) \tag{16.25}$$

Substituting $k = 0$ in the above expression we get

$$R_{xx}(0) = \frac{A^2}{2}\cos(\omega_0 \times 0) + \delta(0)$$

Substituting $\cos(0) = 1$ and $\delta(0) = 1$ in the above expression we get

$$R_{xx}(0) = \frac{A^2}{2} + 1 \tag{16.26}$$

Substituting $k = 1$ in the above expression (16.25) we get

$$R_{xx}(1) = \frac{A^2}{2}\cos(\omega_0) + \delta(1)$$

Substituting $\delta(1) = 0$ in the above expression we get

$$R_{xx}(1) = \frac{A^2}{2}\cos(\omega_0) \tag{16.27}$$

Substituting $k = 2$ in the above expression (16.25) we get

$$R_{xx}(2) = \frac{A^2}{2}\cos(2\omega_0) + \delta(2)$$

Substituting $\delta(2) = 0$ in the above expression we get

$$R_{xx}(2) = \frac{A^2}{2}\cos(2\omega_0) \tag{16.28}$$

Substituting $A = 5\text{V}$ and $\omega_0 = \frac{\pi}{4}$ in Eqs (16.26)–(16.28), the values of $R_{xx}(0)$, $R_{xx}(1)$ and $R_{xx}(2)$ are obtained as $R_{xx}(0) = 13.5$, $R_{xx}(1) = \frac{12.5}{\sqrt{2}}$ and $R_{xx}(2) = 0$.

Step 3: To obtain the expression for cross-correlation vector $R_{dx}(k)$

The expression for the cross-correlation vector $R_{dx}(k)$ is given by

$$R_{dx}(k) = E\{d[k]x^*[n-k]\}$$

For a real signal, the above expression can be written as

$$R_{dx}(k) = E\{d[k]x[n-k]\} \tag{16.29}$$

From Eq. (16.21), $x[n] = s[n] + v[n]$, this implies

$$x[n-k] = s[n-k] + v[n-k] \tag{16.30}$$

Substituting Eq. (16.30) in Eq. (16.29) we get

$$R_{dx}[k] = E\left\{d[k]\left[s[n-k] + v[n-k]\right]\right\} \tag{16.31}$$

The above expression can be written as

$$R_{dx}(k) = E\left\{d[k]s[n-k]\right\} + E\{d[k]v[n-k]\} \tag{16.32}$$

It is mentioned in the problem statement that $d[n] = s[n]$, hence the above expression can be written as

$$R_{dx}(k) = E\{s[k]s[n-k]\} + E\{s[k]v[n-k]\} \qquad (16.33)$$

Since the signal is independent of the noise, we have

$$E\{s[k]v[n-k]\} = 0 \qquad (16.34)$$

Substituting Eq. (16.34) in Eq. (16.33) we get

$$R_{dx}(k) = E\{s[k]s[n-k]\}$$

From the definition of autocorrelation, the above expression can be written as

$$R_{dx}(k) = R_{ss}(k)$$

where $R_{ss}(k) = \frac{A^2}{2}\cos(\omega_0 k)$, hence

$$R_{dx}(k) = \frac{A^2}{2}\cos(\omega_0 k) \qquad (16.35)$$

Substituting $A = 5$ and $\omega_0 = \frac{\pi}{4}$ in Eq. (16.35) we get

$$R_{dx}(k) = 12.5\cos\left(\frac{\pi}{4}k\right) \qquad (16.36)$$

Substituting $k = 0$ in the above equation we get $R_{dx}(0) = 12.5$. Substituting $k = 1$, the value of $R_{dx}(1) = \frac{12.5}{\sqrt{2}}$ and substituting $k = 2$ we get $R_{dx}(2) = 0$.

Step 4: To find the optimal filter coefficient

Substituting the expression of autocorrelation matrix from step 2 and the cross-correlation vector from step 3, the value of optimal filter coefficient is obtained as

$$h_{opt} = \begin{bmatrix} 13.5 & \frac{13.5}{\sqrt{2}} & 0 \\ \frac{13.5}{\sqrt{2}} & 13.5 & \frac{13.5}{\sqrt{2}} \\ 0 & \frac{13.5}{\sqrt{2}} & 13.5 \end{bmatrix}^{-1} \begin{bmatrix} 12.5 \\ \frac{12.5}{\sqrt{2}} \\ 0 \end{bmatrix}$$

Solving the above expression, the optimal filter coefficient is computed as

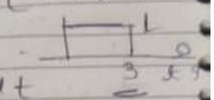
$$h_{opt} = \begin{bmatrix} 0.707 \\ 0.34 \\ -0.226 \end{bmatrix}$$

▼ 18.

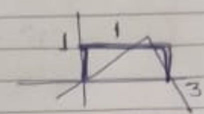
18. State the significance of autocorrelation function with respect to power content in the signal. Determine and plot the autocorrelation functions for the following signals: (a) $x(t) = u(t)- u(t-3), 0 \leq t \leq 9$; (b) $z(t)=e^{-2t}\ u(t)$.

Q.(18) signal → autocorrelation

(a) $x(t) = u(t) - u(t-3)$, $[0, 9]$



$R_{xx}(t) = \int_{-\infty}^{\infty} x(t) \cdot x(t+\tau)\, dt$



$\cancel{R_x(\tau) = \int_0^{9-\tau} 1\, dt}$

$= \cancel{3}-\tau$

$(t)_0^{9-\tau}$

$R_x(\tau) = \int_0^{3-\tau} 1\, dt + \int_3^{9} 0\, dt$

$\boxed{R_x(\tau) = 3-\tau}$

(b) $x(t) = e^{-2t} u(t)$

$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t+\tau)\, dt$

$e^{-2t} \cdot e^{-2(t+\tau)}\, dt$

$e^{-2t-2t-2\tau}\, dt$

$e^{-4t-2\tau}$

$e^{-2(2t+\tau)}$

$R_{xx}(t) = \int_{-\infty}^{\infty} e^{-2(2t+\tau)}\, dt \Rightarrow \left[-\frac{1}{4} e^{-2(2t+\tau)}\right]_{-\infty}^{\infty}$

$$-\frac{1}{4}\left[e^{-2(2t+\tau)}\right]_0^{\infty}$$

$$-\frac{1}{4}\left[-e^{-2\tau}\right]$$

$$\boxed{R_x(\tau) = \frac{1}{4}e^{-2\tau}}$$

4 PSD Estimation

▼ 19.

EXAMPLE 15.12 Find the power spectrum of the wide-sense stationary random process whose autocorrelation function is given by

$$R_{xx}(k) = \begin{cases} 1, & k = 0 \\ 0.5, & |k| = 1 \\ 0.25, & |k| = 2 \\ 0, & |k| > 2 \end{cases}$$

Solution

The power spectrum is obtained by taking the Fourier transform of the autocorrelation function.

Step 1: To compute $S_{xx}(z)$:
Upon taking the Z-transform of the autocorrelation function we get $S_{xx}(z)$ which is given by

$$S_{xx}(z) = Z\{R_{xx}(k)\}$$

$$S_{xx}(z) = \sum_{k=-\infty}^{\infty} R_{xx}(k) z^{-k}$$

From the problem specification, it is obvious that the autocorrelation exists between −2 and +2, hence the above expression can be written as

$$S_{xx}(z) = \sum_{k=-2}^{2} R_{xx}(k) z^{-k}$$

The above expression can be expanded as

$$S_{xx}(z) = R_{xx}(-2)z^{2} + R_{xx}(-1)z^{1} + R_{xx}(0)z^{0} + R_{xx}(1)z^{-1} + R_{xx}(2)z^{-2}$$

Substituting the expression for the autocorrelation values we get

$$S_{xx}(z) = \frac{1}{4}z^{2} + \frac{1}{2}z^{1} + 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}$$

The above expression can be written as

$$S_{xx}(z) = \frac{1}{4}(z^{2} + z^{-2}) + \frac{1}{2}\left(z^{1} + z^{-1}\right) + 1$$

Substituting $z = \mathrm{e}^{j\omega}$ in the above expression we get

$$S_{xx}\left(\mathrm{e}^{j\omega}\right) = \frac{1}{4}\left(\mathrm{e}^{j2\omega} + \mathrm{e}^{-j2\omega}\right) + \frac{1}{2}\left(\mathrm{e}^{j\omega} + \mathrm{e}^{-j\omega}\right) + 1$$

Simplifying the above expression we get

$$S_{xx}(\mathrm{e}^{j\omega}) = \frac{1}{4} \times 2\cos(2\omega) + \frac{1}{2} \times 2\cos(\omega) + 1$$

The above expression can be simplified as

$$S_{xx}(\mathrm{e}^{j\omega}) = \frac{1}{2}\cos(2\omega) + \cos(\omega) + 1$$

▼ 20.

EXAMPLE 15.7 A linear time-invariant system is excited by a wide-sense stationary random process whose mean value is two. The relationship between the input and output of the system is given by the difference equation $y[n] = \frac{1}{2} y[n-1] + x[n]$. Determine the mean value of the output signal

Solution

When an LTI system is excited by a WSS input, the relationship between mean value of the input signal and the mean value of the output signal is given by $\mu_y = \mu_x \times H(0)$

In the above equation, μ_x, μ_y represent the mean value of the input and output signal, respectively, $H(0)$ represents the DC response of the system.

Step 1: To determine the DC response of the system

The DC response of the system is obtained by obtaining the frequency response of the system and then substituting $\omega = 0$ in the frequency response, one obtains the DC response of the system. The given difference equation of the system is $y[n] = \frac{1}{2} y[n-1] + x[n]$

Taking Z-transform on both sides of the above equation we get

$$Y(z) = \frac{1}{2} z^{-1} Y(z) + X(z)$$

From the above expression, the transfer function of the system is given by

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2} z^{-1}}$$

Replacing z by $e^{j\omega}$ we get the frequency response of the system as

$$H\left(e^{j\omega}\right) = \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

Substituting $\omega = 0$ in the above expression, the DC response of the system is obtained as

$$H(0) = \frac{1}{1 - \frac{1}{2}} = 2$$

Step 2: To obtain the mean value of the output signal

The mean value of the output signal is given by

$$\mu_y = \mu_x \times H(0)$$

The mean value of the input signal is given in the problem as two, the DC response of the system is calculated in Step 1 as two; hence the mean value of the output signal is given by

$$\mu_y = 2 \times 2 = 4$$

EXAMPLE 15.13 Suppose $x[n]$ is an AR(1) process with $a_1 = 0.2$. Derive the expression for the PSD of $x[n]$. Assume the input is a white noise which has zero mean and unit variance

Solution

Given data

(i) The mean of the input white noise is zero. This implies $E\{e[n]\} = 0$.
(ii) The variance of the input white noise is unity. This implies $\sigma_e^2 = 1$.
(iii) The parameter of AR(1) process $a_1 = 0.2$.

To find: PSD of $x[n]$. This is represented as $S_{xx}(e^{j\omega})$
Formula: The expression for PSD is given by

$$S_{xx}\left(e^{j\omega}\right) = \left|H\left(e^{j\omega}\right)\right|^2 S_{ee}\left(e^{j\omega}\right) \tag{15.169}$$

Step 1: To find the expression for $H(e^{j\omega})$
The transfer function of AR(1) process is given by

$$H(z) = \frac{1}{1 + a_1 z^{-1}} \tag{15.170}$$

Substituting the value of $a_1 = 0.2$ in the above expression we get

$$H(z) = \frac{1}{1 + 0.2z^{-1}}$$

Substituting $z = e^{j\omega}$ in the above expression, the frequency response is obtained as

$$H\left(e^{j\omega}\right) = \frac{1}{1 + 0.2e^{-j\omega}}$$

Step 2: PSD of the white noise
The PSD of the white noise is given by

$$S_{ee}\left(e^{j\omega}\right) = \sigma_e^2$$

In the problem statement, the variance of the white noise is given as unity, hence the expression for the PSD is

$$S_{ee}\left(e^{j\omega}\right) = 1$$

Step 3: Expression for output PSD
The expression for the output PSD is given by

$$S_{xx}\left(e^{j\omega}\right) = \left|H\left(e^{j\omega}\right)\right|^2 S_{ee}\left(e^{j\omega}\right)$$

Substituting $H\left(e^{j\omega}\right) = \dfrac{1}{1 + 0.2e^{-j\omega}}$ and $S_{ee}\left(e^{j\omega}\right) = 1$ in the above expression we get

The above expression can be written as

$$S_{xx}\left(e^{j\omega}\right) = \frac{1}{\left|1 + 0.2e^{-j\omega}\right|^2}$$

▼ 21.

EXAMPLE 15.24 If $x[n]$ is a AR(1) process with $a_1 = 0.25$. Obtain the expression for the power spectral density of $x[n]$, assuming the input to be a zero mean and unit variance white noise

Solution

Given data

(1) Process is AR(1) with the process parameter $a_1 = 0.25$.

(2) Input is zero mean, unit variance white noise $E\left\{e^2[n]\right\} = 1$.

To find: Expression for the PSD of $x[n]$.
Formula The equation of AR(1) process is given by

$$x[n] + a_1 x[n-1] = e[n] \qquad (15.361)$$

Substituting $a_1 = 0.25$ in the above equation we get

$$x[n] + 0.25x[n-1] = e[n] \qquad (15.362)$$

Taking Z-transform on both sides of the above equation we get

$$X(z) + 0.25z^{-1}X(z) = E(z)$$

From the above expression, the transfer function of the system is given by

$$H(z) = \frac{X(z)}{E(z)} = \frac{1}{1+0.25z^{-1}}$$

Substituting $z = e^{j\omega}$ in the above expression we get
The expression for the PSD is given by

$$S_{xx}(e^{j\omega}) = \left|H(e^{j\omega})\right|^2 S_{ee}(e^{j\omega}) \qquad (15.363)$$

Substituting $H\left(e^{j\omega}\right) = \dfrac{1}{1+0.25e^{-j\omega}}$ and $S_{ee}\left(e^{j\omega}\right) = \sigma^2(e)$ we get

$$S_{xx}\left(e^{j\omega}\right) = \left(\frac{1}{1+0.25e^{-j\omega}}\right)^2 \sigma^2(e) \qquad (15.364)$$

In the problem statement, the variance of the white noise is mentioned as unity; hence Eq. (15.364) can be expressed as

$$S_{xx}(e^{j\omega}) = \left(\frac{1}{1+0.25e^{-j\omega}}\right)^2$$

22.

EXAMPLE 15.25 Let $x[n]$ represent a wide-sense stationary random process. The process is modelled as AR(1) model which is excited by white noise. Four measurements are obtained from the process which are $x[0] = 0.2$, $x[1] = -0.1$, $x[2] = 0.2$ and $x[3] = -0.1$. Estimate the process parameter and the variance of the white noise

Solution

Given data

(i) Four measurements from the process are $x[n] = \{0.2, -0.1, 0.2, -0.1\}$.
(ii) Process is modelled as AR(1) process which is excited by white noise

To find: Process parameter a_1 and the noise variance.
Step 1: Yule–Walker equation for AR(1) process
The AR(1) process is given by

$$x[n] + a_1 x[n-1] = e[n]$$

Multiplying both sides by $x[n-k]$ we get

$$x[n]x[n-k] + a_1 x[n-1]x[n-k] = e[n]x[n-k]$$

Taking expectation operator on both sides of the above expression we get

$$E\left[x[n]x[n-k]\right] + a_1 E\left[x[n-1]x[n-k]\right] = E\left[e[n]x[n-k]\right]$$

The above equation can be expressed as

$$R_{xx}(k) + a_1 R_{xx}(k-1) = \sigma_e^2 \delta(k) \qquad (15.365)$$

Substituting $k = 0$ in Eq. (15.365) we get

$$R_{xx}(0) + a_1 R_{xx}(-1) = \sigma_e^2 \delta(0)$$

Since autocorrelation is even symmetric $R_{xx}(-1) = R_{xx}(1)$. Substituting this in the above expression we get

$$R_{xx}(0) + a_1 R_{xx}(1) = \sigma_e^2 \delta(0)$$

Substituting $\delta(0) = 1$ in the above expression we get

$$R_{xx}(0) + a_1 R_{xx}(1) = \sigma_e^2 \tag{15.366}$$

Substituting $k = 1$ in Eq. (15.365) we get

$$R_{xx}(1) + a_1 R_{xx}(0) = \sigma_e^2 \delta(1)$$

Substituting $\delta(1) = 0$ in the above expression we get

$$R_{xx}(1) + a_1 R_{xx}(0) = 0 \tag{15.367}$$

Combining Eqs (15.366) and (15.367) in matrix form we get

$$\begin{bmatrix} R_{xx}(0) & R_{xx}(1) \\ R_{xx}(1) & R_{xx}(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sigma_e^2 \\ 0 \end{bmatrix} \tag{15.368}$$

Eq. (15.368) represents the Yule–Walker equation of AR(1) model.

Step 2: To find the autocorrelation values from the data
The expression for the autocorrelation function is given by

$$R_{xx}(k) = \frac{1}{N} \sum_{n=0}^{N-1-k} x[n] x^*[n+k] \tag{15.369}$$

The number of samples of $x[n]$ given in the problem is four. Substituting $N = 4$ in the above expression we get

$$R_{xx}(k) = \frac{1}{4} \sum_{n=0}^{4-1-k} x[n] x^*[n+k]$$

For a real sequence $x[n] = x^*[n]$, hence the above expression can be written as

$$R_{xx}(k) = \frac{1}{4} \sum_{n=0}^{4-1-k} x[n] x[n+k] \tag{15.370}$$

Substituting $k = 0$ in the above expression we get

$$R_{xx}(0) = \frac{1}{4} \sum_{n=0}^{3} x[n] x[n]$$

The above equation can be expanded as

$$R_{xx}(0) = \frac{1}{4}\{x[0]x[0] + x[1]x[1] + x[2]x[2] + x[3]x[3]\}$$

Substituting the values of $x[0]$, $x[1]$, $x[2]$ and $x[3]$ in the above expression we get

$$R_{xx}(0) = \frac{1}{4}\{0.2\times 0.2 + -0.1\times -0.1 + 0.2\times 0.2 + -0.1\times -0.1\}$$

Simplifying the above expression we get

$$R_{xx}(0) = 0.0925$$

Substituting $k = 1$ in Eq. (15.370) we get

$$R_{xx}(1) = \frac{1}{4}\sum_{n=0}^{2} x[n]x[n+1]$$

The above expression can be written as

$$R_{xx}(1) = \frac{1}{4}\{x[0]x[1] + x[1]x[2] + x[2]x[3]\}$$

Substituting the values of $x[0]$, $x[1]$, $x[2]$ and $x[3]$ in the above expression we get

$$R_{xx}(1) = \frac{1}{4}\{0.2\times -0.1 + -0.1\times 0.2 + 0.2\times -0.1\}$$

Simplifying the above expression we get

$$R_{xx}(1) = -0.045$$

Step 3: To determine the process parameter
From Eq. (15.367) the expression for the process parameter is given by

$$a_1 = -\frac{R_{xx}(1)}{R_{xx}(0)}$$

Substituting the values of $R_{xx}(0)$ and $R_{xx}(1)$ in the above expression we get

$$a_1 = -\frac{(-0.045)}{0.0925} = 0.4865$$

Step 3a: To determine the variance of the noise
The expression for the variance of the noise is given by

$$\sigma_e^2 = R_{xx}(0) + a_1 R_{xx}(1)$$

Substituting the values of $R_{xx}(0)$, $R_{xx}(1)$ and the process parameter a_1 in the above expression we get
Simplifying the above expression the value of noise variance is obtained as

$$\sigma_e^2 = 0.0706$$

▼ 23.

23. The autocorrelation function of a real val- ued zero mean random process is given by $R_{xx}(k)$=2$\delta[k+1]$+10$\delta[k]$+2$\delta[k-1]$. Determine the (i) power spectral density and (ii) variance of $x[n]$.

Solution:

According to Wiener-Khintchine theorem, Fourier transform of autocorrelation function gives power spectral density.

Step 1: To compute the Z transform of the autocorrelation function

The expression for the autocorrelation function (given in the problem) is

$$R_{xx}(k) = 2\delta[k+1] + 10\delta[k] + 2\delta[k-1]$$

Taking Z transform on both sides of the above expression, we get

$$S_{xx}(z) = Z\{2\delta[k+1] + 10\delta[k] + 2\delta[k-1]\}$$

$$S_{xx}(z) = 2Z(\delta[k+1]) + 10Z(\delta[k]) + 2Z(\delta[k-1])$$

Upon simplifying the above expression we get

$$S_{xx}(z) = 2z + 10 + 2z^{-1}$$

Substituting $z = e^{j\omega}$ in the above expression we get

$$S_{xx}(e^{j\omega}) = 2e^{j\omega} + 10 + 2e^{-j\omega}$$

The above expression can be written as

$$S_{xx}(e^{j\omega}) = 10 + 2[e^{j\omega} + e^{-j\omega}]$$

The above expression is simplified as

$$S_{xx}(e^{j\omega}) = 10 + 4\cos(\omega)$$

Step 2: To compute the variance of $x[n]$

The expression for variance of $x[n]$ is given by

$$\sigma_x^2 = E\{x^2\} - [E\{x\}]^2$$

The given process is zero mean process; hence the expression for variance is given by

$$\sigma_x^2 = E\{x^2\}$$

One can compute $E\{x^2\}$ from the autocorrelation function. This is given by

$$R_{xx}(k) = E\{x(n)x^*(n-k)\}$$

The given process is a real process hence the above expression is written as

$$R_{xx}(k) = E\{x[n]x[n-k]\}$$

At zero lag, the above expression is written as

$$R_{xx}(0) = E\{x[n]x[n]\} = E\{x^2\}$$

Thus Autocorrelation function at zero lag gives the value of $E\{x^2\}$ which in-turn gives the variance of the process. The variance of the process is computed as $R_{xx}(0)$. This is given by

$$R_{xx}(k) = 2\delta[k+1] + 10\delta[k] + 2\delta[k-1]$$

Substituting $k = 0$ in the above expression we get

$$\sigma_x^2 = E(x^2) = R_{xx}(0) = 2\delta[1] + 10\delta[0] + 2\delta[-1]$$

23

▼ 24.

24. Consider an ARMA(1,1) process $x[n] - 0.5x[n-1] = e[n] - 0.7\,e[n-1]$, where $e[n]$ is a wide-sense stationary white noise process with variance $\sigma_e^2 = 1$. Compute the power spectral density of the process.

Solution:

Step 1: To obtain the transfer function of ARMA(1,1) process:

The given ARMA(1,1) difference equation is

$$x[n]-0.5x[n-1]=e[n]-0.7e[n-1]$$

Taking Z-transform both sides

$$H(z)=\frac{X(z)}{E(z)}=\frac{1-0.7z^{-1}}{1-0.5z^{-1}}$$

The frequency response is obtained by substituting $z=e^{j\omega}$ in the above expression to yield

$$H(e^{j\omega})=\frac{1-0.7e^{-j\omega}}{1-0.5e^{-j\omega}} \tag{15.17}$$

Step 2: Computing the power spectral density

The expression for the power spectral density of the process is given by

$$S_{xx}(e^{j\omega})=\left|H(e^{j\omega})\right|^2 \times S_{ee}(e^{j\omega}) \tag{15.18}$$

Substituting the value of $H(e^{j\omega})$ from expression (15.17) and the value of $S_{ee}(e^{j\omega})$ as σ_e^2 in expression (15.18) we get

$$S_{xx}(e^{j\omega})=\frac{\left|1-0.7e^{-j\omega}\right|^2}{\left|1-0.5e^{-j\omega}\right|^2}\sigma_e^2$$

Substituting the value of $\sigma_e^2=1$ in the above expression we get

$$S_{xx}(e^{j\omega})=\frac{\left|1-0.7e^{-j\omega}\right|^2}{\left|1-0.5e^{-j\omega}\right|^2}.$$

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Before midsem:

Bio | Notion

1.Write short note on (a) Electroneurogram (ENG) signals, (b) Electrooculogram (EOG) signals and (c) Electroencephalogram (EEG) signals mentioning their time-frequency characteristics

https://biomedical6969.notion.site/Bio-b5898f6678a245aba8651386d5c45367?pvs=4

Notion