Sub- Information Theory and Coding (EC402)

Assignment-01

Date of Submission: 16-08-2024

Q.1 When is $H(X|Y) = H(X)$?

Q2. Does Conditioning reduce entropy? Is $H(X|Y) \leq H(X)$?

Q3. What is the minimum vale of entropy?

Q4. Let (X, Y) have the following joint distribution

Y \ X	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

The marginal distribution of X is (½,1/4, 1/8, 1/8) and the marginal distribution of Y is (¼,1/4, ¼,1/4). Find out H(X), H(Y), H(X,Y), H(X|Y) and H(Y|X).

Q5. Entropy of a functions. Let X be a random variable taking on a finite number of values. What is the (general) inequality relationship of H(X) and H(Y) if

(a) $Y = 2^x$?

(b) Y = cos X?

Q6. Give an example of joint random variable X and Y such that

(i) H(Y|X=x) < H(Y)

(ii) H(Y|X=x) > H(Y)

Q7. Give an example of joint random variables X, Y and Z such that

(i) I(X; Y|Z) < I(X;Y)

(ii) I(X; Y|Z) > I(X;Y)

Q8. Consider a discrete memoryless channel with inputs X and outputs Y. The input X takes value from a ternary set with equal probability and its known that the probability of error is for the system is p. Using Feno's lemma, find a lower bound to that mutual information I(X;Y) as a function of p.

Q9. Let $p(x,y)$ be given as

X \ Y	0	1
0	$\frac{1}{3}$	$\frac{1}{3}$
1	0	$\frac{1}{3}$

Find:

(a) H(*X*) and H(*Y*)
(b) H(*X*|*Y*), H(*Y*|*X*)
(c) H(*X*, *Y*)
(d) H(*Y*)-H(*Y* |*X*)
(e) I(*X*;*Y*)
(f) Draw a Venn diagram for the quantities in parts (a) through (e).

Q10. Let the random variable X have three possible outcomes {a,b,c}. Consider two distributions on this random variable:

Symbol	*p*(*x*)	*q*(*x*)
a	½	1/3
b	¼	1/3
c	¼	1/3

Calculate H(*p*), H(*q*), D(*p*||*q*), and D(*q*||*p*). Verify that in this case, relative entropy is not symmetric.

Q11. Prove that the entropy for a discrete source is a maximum when the output symbols are equally probable.

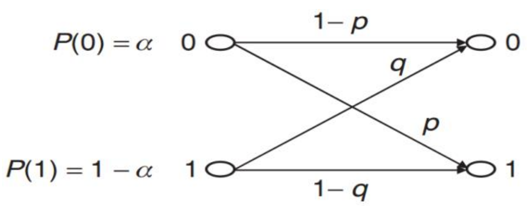
Q12. Consider tossing of a coin. Let X be a random variable which denotes number of tosses required until the first 'head' appears. Find
i) Entropy H(X) with a fair coin.
ii) Entropy H(X) with an unfair coin with p as probability of occurring a head.

Q13. Let a random variable X with probability mass function as

$$P(x) = \begin{cases} 1/4, & x = 0,1 \\ 1/2, & x = 2 \end{cases}$$

Y and Z are two random variables generated as follows. When X = 0, we have Y = Z = 0; For X = 1, then Y = 1, Z = 0; when X = 2, we have Z = 1 while Y is randomly chosen from 0 and 1 with equal probability. Find the values of the following quantities: H(X), H(Y), H(Z), H(Y|X), H(X, Y), H(X|Y), H(X, Z), H(X|Z), H(Y, Z), H(Z|Y).

Q14. The below figure shows a non-symmetric binary channel. Prove that in this case I(X,Y)=Ω[q+(1−p−q)α]−αΩ(p)−(1−α)Ω(q) where the function $\Omega\,(\mathrm{p}) = plog_2\frac{1}{p} + (1-p)log_2\frac{1}{1-p}$.



Q15. Consider a 1st order Markov source having three symbols. The transition probabilities are given as

$$p(s_j|s_i) = \frac{p}{2} \quad for \quad i \neq j$$

(a) Sketch the state diagram

(b) determine the probabilities of the symbols
(c) Determine amount of information with respect to an arbitrary transition.
(d) Determine the value of *p* for which this amount of information i.e. $H(s_2|s_1)$ achieves a maximum.
(e) Find maximum value of $H(s_2|s_1)$.

Q16.

Entropy of a sum. **Let X and Y be random variables that take on values $x_1, x_2, \ldots, x_r$ and $y_1, y_2, \ldots, y_s$, respectively. Let $Z = X + Y$.**

(a) Show that $H(Z|X) = H(Y|X)$. Argue that if X, Y are independent, then $H(Y) \leq H(Z)$ and $H(X) \leq H(Z)$. Thus the addition of *independent* random variables adds uncertainty.

(b) Give an example (of necessarily dependent random variables) in which $H(X) > H(Z)$ and $H(Y) > H(Z)$.

(c) Under what conditions does $H(Z) = H(X) + H(Y)$?

Q17. Prove that

$$I(X;Z|Y) = I(Z;Y|X) - I(Z;Y) + I(X;Z)$$

Where X, Y and Z are joint random variables.

Q18. Determine the redundancy *R* of source, $S = \{a, b, c, d\}$ with probabilities *p*(*a*) = 0.5, *p*(*b*) = 0.25, *p*(*c*) = *p*(*d*) = 0.125.