EC351 Wireless Communication (Shivnarayan Sir) ①

Books

① Theodore Rappaport, "Wireless Communication"

② Aditya K Jagannatham, "Principles of morderen wireless Comm"

GSM: Global System for Mobile Communication

2G family

① 2G	10 kbps	GSM / IS-95 CDMA (code division) - Essar 1995 (India)
② 2.5G	50 kbps	GPRS "General Packet Radio Services"
③ 2.7G	200 kbps	EDGE "Enhanced data for GSM Evolution"

Voice + data

3G family

a) 3G WCDMA (UMTS) (wideband), CDMA (2000) } 384 kbps

b) 3.5G HSPA, 1x EVDO, Rev - A, B etc } 5 to 30 Mbps — video Calling

UMTS: Universal mobile telecommunication Std

HSDPA: High speed dowlink packet Access

EVDO: Evolution Data optimized

$BS \underset{UL\ (Up\ link)}{\overset{DL\ (Down\ link)}{\rightleftarrows}} Mobile$

3UK: "Hutchision UK" - 2003

MTNL: MahaNagar telephone Nigam Ltd.

4G family

3rd Generation partnership project (3GPP)

ⓐ LTE. "Long Term Evolution"

ⓑ WiMax "world-wide interoperability for microwave access"

} 100 to 200 Mbps, "online-gamming, HDTV" etc

ⓒ LTE Advanced > 1 Gbps

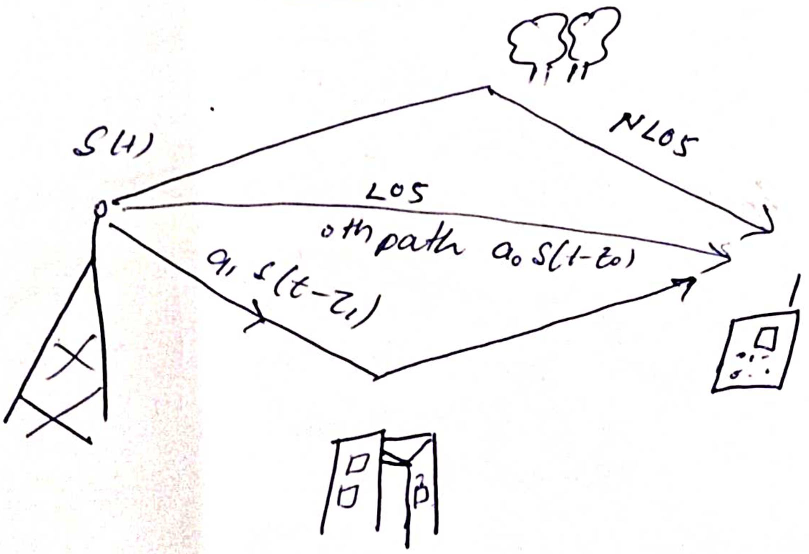
Summary

2G		
2.5G	10 to 100 kbps	Voice & Data
150x		
3G		
3.5G	300 kbps to 30 Mbps	Video Calling
6 to 7x		
4G	100 Mbps to 200 Mbps	Online gaming / HDTV
5G	100 Mbps to 300 Mbps	VR & AR

Cost/bit ↓

lower latency ↓ (delay)

Wireless Environment



BS

Model — channel / Signal

Line of Sight (LOS)

$$h(t)$$

$$s_b(t) \rightarrow \boxed{\text{Mod}} \rightarrow \underbrace{\boxed{\text{channel}}}_{y(t)} \rightarrow \boxed{\text{Demod}} \rightarrow y_b(t)$$

$s_b(t)$: baseband signal

$s(t)$: passband signal

LTI

$$y(t) = h(t) * s(t)$$

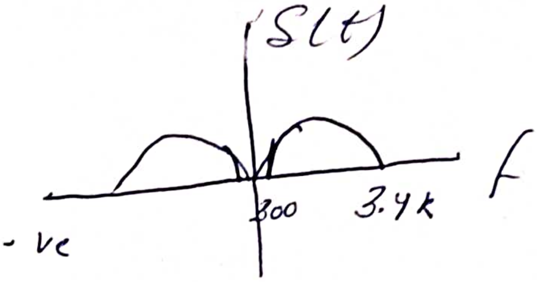
Voice Band

(a) NB 300 to 3.4 kHz

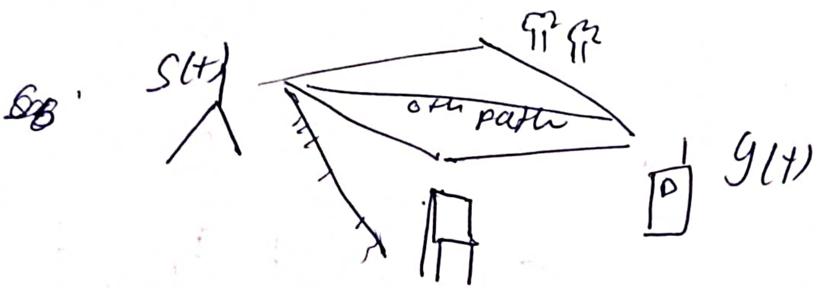
(b) WB 50 to 7 kHz

(c) Super WB 50 to 14 kHz

(d) full Band 20 to 20 kHz



Spectrum



L-path

$$\boxed{h(t) = \sum_{i=0}^{L-1} a_i\, \delta(t-\tau_i)}$$

(1) 2G: f_c = 900 MHz or 1800 MHz } GSM

(2) 3G/4G

$f_c = 2.4$ GHz

$S_b(t)$: message.

$$S(t) = Re\{S_b(t)\, e^{j2\pi f_c t}\}$$

LOS : $Re\{a_0\, S_b(t-\tau_0)\, e^{j2\pi f_c (t-\tau_0)}\}$

or (zeroth path)

NLOS : $Re\{a_1\, S_b(t-\tau_1)\, e^{j2\pi f_c (t-\tau_1)}\}$

⋮ L path

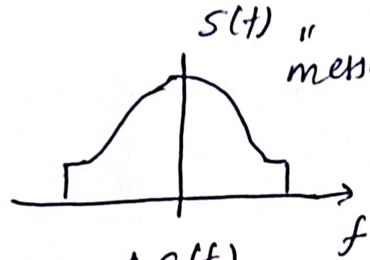
$$\boxed{y(t) = Re\left\{\sum_{i=0}^{L-1} a_i\, S_b(t-\tau_i)\, e^{j2\pi f_c (t-\tau_i)}\right\}}$$

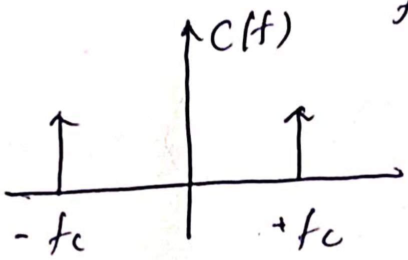
$$y(t) = Re\left\{ \left\{ \sum_{i=1}^{L-1} a_i\, s_b(t-\tau_i)\, e^{-j2\pi f_c \tau_i} \right\} e^{j2\pi f_c t} \right\}$$

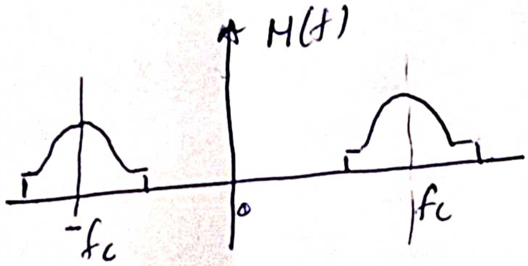
complex phase factor

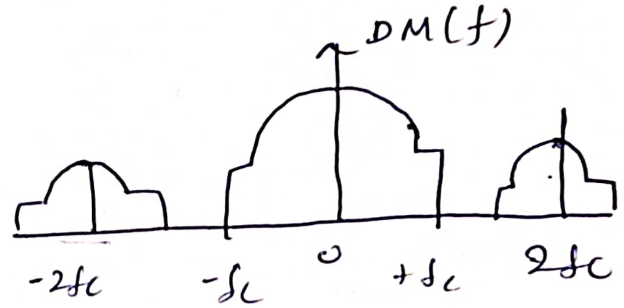
$$\boxed{y_b(t) = \sum_{i=0}^{L-1} a_i\, s_b(t-\tau_i)\, e^{-j2\pi f_c \tau_i}}$$

Modulation and Demodulation









low pass filtering

Recap:

$$S_b(t) \to \boxed{Mod} \xrightarrow{S(t)} \boxed{\text{channel } h(t)} \xrightarrow{y(t)} \boxed{Demod} \to y_b(t)$$

$$S(t) = Re\left\{ S_b(t)\, e^{j2\pi f_c t} \right\}$$

$$h(t) = \sum_{i=0}^{L-1} a_i \delta(t-\tau)$$

$$s(t) = Re\{ s_b(t)\, e^{j2\pi f_c t} \}$$

$$y(t) = h(t) * s(t) = \int h(t)\, h(t-\tau)\, d\tau$$

$$\downarrow$$

$$a_i\, s(t-\tau_i)$$

0th path

$$y_0(t) = \int a_0\, \delta(t-\tau_0)\, s(t-t)\, dt$$

$$= \int a_0 \underline{\delta(t-\tau_0)} \{ Re\{ s_b(t-t)\, e^{j2\pi f_c (t-t_0)} \} \} dt$$

$$= a_0\, Re\{ s_b(t-t_0)\, e^{j2\pi f_c (t-t_0)} \}$$

$$y_0(t) = Re\{ a_0\, s_b(t-\tau_0)\, e^{j2\pi f_c (t-\tau_0)} \}$$

$$\vdots$$

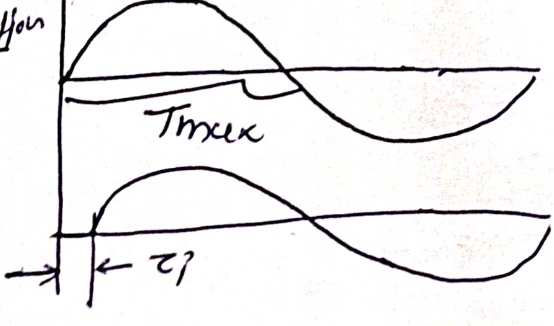
$$y_{L-1}(t) = Re\{ a_{L-1}\, s_b(t-\tau_{L-1})\, e^{j2\pi f_c (t-\tau_{L-1})} \}$$

$$y(t) = Re\{ \sum_{i=0}^{L-1} a_i\, s_b(t-\tau_i)\, e^{j2\pi f_c (t-\tau_i)} \}$$

$$y_b(t) \overset{?}{=} Re\{ \sum_{i=0}^{L-1} a_i\, s_b(t-\tau_i)\, e^{-j2\pi f_c \tau_i} \}$$

Narrow Band Assumption

$$- s_b(t)\, Re\{ \quad " \quad \}$$



$\tau \ll T_{max}$

$\frac{1}{\tau} \gg f_{max}$

typically $\tau \simeq 1\mu s$

1MHz > 100kHz (GSM)

Complex fadding coefficient

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

$S_b(t)$ — [Channel] — $y_b(t)$

$$\boxed{h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}}$$

$y_b(t) = \underbrace{S_b(t)\, h}$

multiplication (not convolution)

"Narrow Band Signal Assumption"

(GSM 2G)

$y_b(t) = S(t) \cancel{*} h$

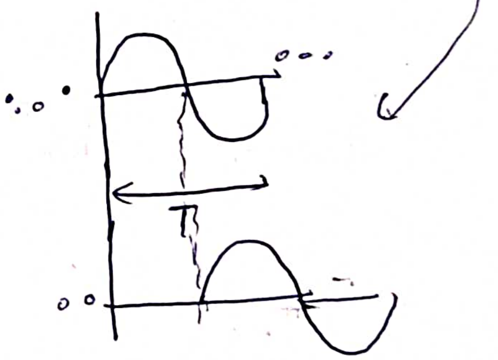
Example: $L = 2$

case ① $\begin{cases} a_0 = 1, & a_1 = 1 \\ \tau_0 = 0, & \tau_1 = \frac{1}{2f_c} = \frac{T}{2} \end{cases}$

case ② $\begin{cases} a_0 = 1, & a_1 = 1 \\ \tau_0 = 0, & \tau_1 = \frac{1}{f_c} = T \end{cases}$

a_i = Large Scale attenuation

$\left(\propto \frac{1}{r^2}\right)$



h for case 1

$$h = \sum_{i=0}^{1} a_i e^{-2\pi f_c \tau_i}$$

$$= a_0 e^{-2\pi f_c \tau_0} + a_1 e^{-2\pi f_c \tau_1}$$

$$= 1 + 1 \cdot e^{-2\pi f_c \frac{1}{2f_c}}$$

$$= 1 + (-1) = 0$$

Power is varying

h for case 2

$$h = a_0 e^{-2\pi f_c \tau_0} + a_1 e^{-2\pi f_c \tau_1}$$

$$= 1 + 1 \cdot e^{-2\pi \frac{1}{f_c} f_c} = 2$$

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

Power of recived Signal.

$$h = a e^{j\phi} = X + jY$$

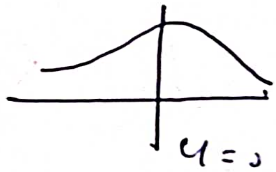
$a(\cos\phi + j\sin\phi)$ $\qquad$ a^2: gain of channel

$X = a\cos\phi \qquad Y = a\sin\phi$

Expanding $h = \sum \left(a_i \cos(2\pi f_c \tau_i) - j\, a_i \sin(2\pi f_c \tau_i)\right)$

$$X = \sum_{i=0}^{L-1} a_i \cos(2\pi f_c \tau_i)$$

$$Y = \sum_{i=0}^{L-1} a_i \sin(2\pi f_c \tau_i)$$



$X \sim N(\mu, \sigma^2)$

$Y \sim N(\mu, \sigma^2)$

$\text{Var } h = \text{Var } x + \text{Var } y = \frac{1}{2} + \frac{1}{2} = 1$

$N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

$$N\left(0, \tfrac{1}{2}\right) \quad f_X(x) = \frac{1}{\sqrt{2\pi}\frac{1}{\sqrt{2}}} e^{-x^2/2\cdot\frac{1}{2}} = \frac{1}{\sqrt{\pi}} e^{-x^2}$$

$$f_Y(y) = \frac{1}{\sqrt{\pi}} e^{-y^2}$$

Joint PDF of X \$ Y (Independent)

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{-(x^2+y^2)}$$

$$f_{A,\phi}(a,\phi) = f_{X,Y}(x,y) \det(J_{X,Y})$$

$$J_{X,Y} = \begin{bmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} \end{bmatrix}$$

$h = ae^{j\phi}$

$= X + jY$

$x = a\cos\phi$

$y = a\sin\phi$

$x^2 + y^2 = a^2$

$$= \det \begin{bmatrix} \cos\phi & \sin\phi \\ -a\sin\phi & a\cos\phi \end{bmatrix} = a$$

$$\boxed{f_{A,\phi}(a,\phi) = \frac{a}{\pi} e^{-a^2}}$$

$f_A(a)\, f_\phi(a)$

<u>Recap</u>:

$$h = \sum_{i=0}^{L-1} a_i e^{-2\pi f_c \tau_i} = X + iY = a e^{i\phi}$$

$$\downarrow_{r.v}$$

$$\sum_{i=0} a_i \cos(2\pi f_c \tau_i)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\left.\begin{array}{l} X \sim N(0, \tfrac{1}{2}) \\ Y \sim N(0, \tfrac{1}{2}) \end{array}\right\}$$ Gaussian Random variable

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$ | Standard normal

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{-(x^2+y^2)} = \frac{1}{\pi} e^{-a^2}$$

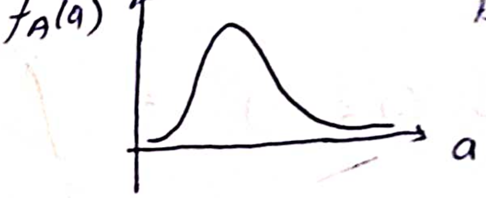
$$f_{A,\phi}(a,\phi) = f_{XY}(x,y) \det(J_{x,y})$$

$$= \frac{a}{\pi} e^{-a^2}$$

pdf

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\phi}(a,\phi)\, d\phi = \int_{-\pi}^{\pi} \frac{a}{\pi} e^{-a^2} d\phi$$

$$= 2ae^{-a^2}$$ "Rayleigh Distribution"



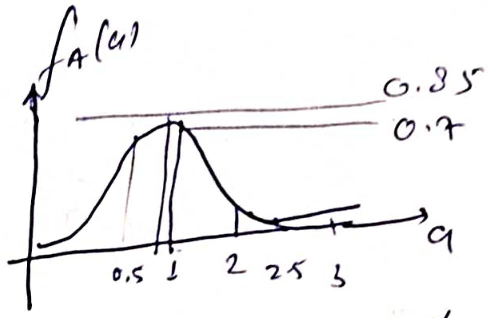
$$f_\phi(a) = \int_0^a f_{A,\phi}(a,\phi)\, da$$

$$= \int_0^a \frac{a}{\pi} e^{-a^2} da \quad \text{let } a^2 = t,\ d_t = 2a\, dt$$

$$\int_0^{\infty} \frac{g}{\pi} e^{-a^2} \, da \qquad a^2 = t \qquad 2a\,da = dt$$
$$a\,da = \frac{1}{2} dt$$

$$= \int_0^{\infty} \frac{1}{2\pi} e^{-t} \, dt$$

$$\Rightarrow \left(\frac{1}{2\pi}\right) - \int_0^{\infty} \left[e^{-t} \right]$$



Deep fade.

$$= \frac{1}{2\pi} \quad \text{"Uniform distribution"}$$

Ex - What is the probability that the transmitted Signal power attenuates more than 20dB?

$$g = \text{gain} = a^2 \qquad \boxed{g < 20dB} \; ? \qquad 20 \log() = 20 \,; \Rightarrow A = 10$$

$$g_{dB} = 10 \log_{10} g < -20dB$$

$$\log_{10} g < -2 \quad \text{attenuation.} \qquad \boxed{\sqrt{g} = a}$$

$$< -2 \log_{10} 10 \qquad \sqrt{0.01} = a$$

$$g < \cancel{100} \; 0.01 \qquad g = \frac{P_r}{P_t}$$

$$a < 0.1$$

$$P_r(a < 0.1) = \int_0^{0.1} 2a\, e^{-a^2} \, da \qquad \begin{matrix} 0.1 \\ 1 - 0.9 \end{matrix}$$

$$= \frac{-1}{2\pi} \Big[e^{-t} \Big]_0^{0.1} = \frac{1}{2\pi} \left[1 - e^{-0.1}\right]$$

$$= 0.01$$

Q: Consider a wireless signal with a carrier freq (8)
of 850 MHz which is transmitted over a wireless channel
that results in $L=4$ multipath components at
delay of 201, 513, 819, 1223 ns and corresponding
to recieved signal of amplitude 1, 0.6, 0.3, 0.3
Now you have to derive the expression for received

τ_i = 201, 513, 819, 1223 baseband given
a_i = 1, 0.6, 0.3, 0.3 if transmitted band signal is $S_p(t)$

↳ repeat same question under narrow band assumption.

↳ Derive the pdf of channel power gain
$g = a^2$ where a is ~~delay~~ amplitude of delay fadding channel.

Performance measures

1. Bit Error Rate ✓
2. Channel capacity
3. Outage probability

Wireline Commonication System

Line coding '1' Symbol → 1 Volt $\sqrt{P}$
(NRZ) '0' → -1 Volt $-\sqrt{P}$

Non-returning to zero

BPSK
Binary phase shift keying

Each Symbol: Power P

$y = x + n$ "AWGN"
↳ Additive ~~Wite~~ White Gaussian Noise.

Additive White Gaussian Noise

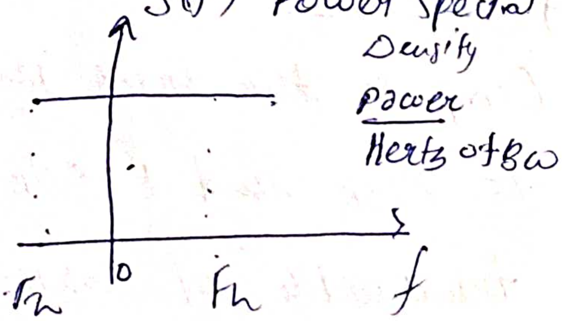
↳ If n is a Sample n(t)

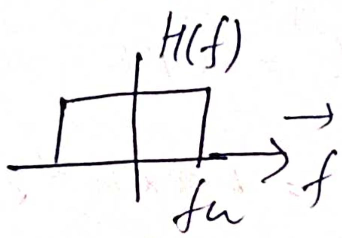
$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}}$$ } Time Domain

Probability Density function

$$E[n(t)\, n(t-\tau)] = \delta(\tau)$$

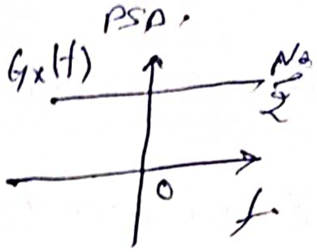
$\tau = 0$ $\quad E[n(t)^2] \geq 0$





noise can effect all freq.

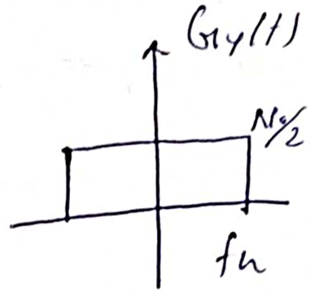
AWGN



$$E[x(t)x(t-\tau)] = \delta(t)$$

normalized noise (-1 to 1)

$$G_{noise}(f) \xleftrightarrow{f} R_n(\tau) = \frac{N_0}{2}\delta(\tau)$$



$G_{noise}(f) = \frac{N_0}{2}$

"PSD"

Square of FFT of $x(t)$

$$\boxed{G_y(f) = G_x(f)\,|H_{LPF}|^2}$$

$$R_y(\tau) = N_0 f_u \operatorname{Sinc}(2 f_u \tau)$$

TD FD

Power $\sigma_n^2 = \frac{N_0}{2} \times BW$

"TD" FD

Q: Thermal noise at the reciver, $n_0 = KTF$ (PSD, kelvin)

Compute the noise power at $T = 293k$ and $F = 5dB$ for bandwidth of 30kHz

Defination: $K = 1.38 \times 10^{-23}$, (Boltzman constant)

$F = \frac{SNR_i}{SNR_0}$ "noise factor"

$NF = 10\log_{10} F$ "noise figure"

Valid at 290k

Sol^n $n_0 = \frac{N_0}{2} = KTF$

$= 1.38 \times 10^{-23} \times 293 \times 5 = 6.06 \times 10^{-16}$

$\sigma_n^2 = n_0 \times BW = $ ~~10 log (1.5 21)~~

$10 \log(6.06 \times 10^{-16}) = -152.17$ dB.

$NF = 10 \log_{10} F$

$5 = 10 \log_{10} F$

$F = 10^{0.5}$

$\eta_0 = 1.38 \times 10^{-23} \times 10^{0.5} \times 293 = 1.27 \times 10^{-20}$

$\eta_0 BW = 3.83 \times 10^{-16}$

$10 \log_{10} \eta_0 B = 10 \log 3.83 \times 10^{-16} = -154.16$ dB

Performance of wireless Communication System

NRZ line coding

BPSK modulance $\{ \underset{1}{+\sqrt{P}}, \underset{0}{-\sqrt{P}} \}$

$y = \sqrt{P} + n$ "AWGN"

$\underset{\sqrt{P}}{y} > 0$ $\quad 0 \to 1$

Error

0 to 1	1, to 0
$-\sqrt{P} + n > 0$	$\sqrt{P} + n < 0$
$n > \sqrt{P}$	$n < -\sqrt{P}$
P_n (Error)	$Pr(n < -\sqrt{P}$
$Pr(n > \sqrt{P}) = \int_{\sqrt{P}}^{\infty} \underbrace{\frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{x^2}{2\sigma^2}}}_{\text{pdf of Noise}} dx$	$Pr \int_{-\infty}^{-\sqrt{P}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx$

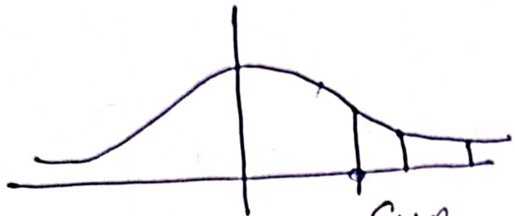
$x = \sigma_n t$

$dx = 2\sigma n \, dx$

$\Rightarrow \int_{\sqrt{P}/\sigma^2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$

$BER = P(n > \sqrt{P})$ or $P(n < -\sqrt{P}) = Q(\sqrt{SNR})$

$SNR \uparrow \rightarrow$ area under curve $\downarrow$ $\rightarrow$ Bit rate $\downarrow$



Q: At SNR = 10dB what is BER of wireless Comm^n system

$$10 \log SNR = 10$$

$$SNR = 10$$

$$BER = Q(\sqrt{10}) = Q(3.16)$$

$$Q(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \left[\frac{1}{(1-a)t + a\sqrt{t^2+b}} \right]$$

$$\cancel{Q(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}} \qquad a = \frac{1}{\pi} \qquad b = 2\pi$$

BER performace of wireless System "single antenna"

Model $\qquad h = a\, e^{j\phi}$ gain $= a^2$

$\underline{a} \hookrightarrow$ voltage

$y = hx + n \qquad SNR_{Received} = \frac{a^2 P}{\sigma_n^2} \qquad$ Received power $= a^2 P$

$$BER = Q(\sqrt{SNR}) = Q\sqrt{\frac{a^2 P}{\sigma_n^2}}$$

$$BER_{Average} = \int_0^\infty Q\left(\sqrt{\frac{a^2 P}{\sigma_n^2}}\right) 2a e^{-a^2} da$$

$$\boxed{\text{Average of } g(a) = \int_0^\infty g(a) f_A(a)\, dA}$$

(11)

$$BER_{avg} = \int_0^\infty \int_{\sqrt{a^2 u}}^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \cdot 2a e^{-a^2} \, da$$

$$Q(r) = \int_r^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

$$u = \frac{P}{\sigma_n^2}$$

$$\frac{x}{a\sqrt{u}} = u$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty \int_1^\infty 2a e^{-a^2} \cdot a\sqrt{u}\, e^{-\mu a^2 \frac{u^2}{2}} \, du \, da$$

$$= \sqrt{\mu} \int_1^\infty \int_0^\infty \frac{2a^2}{\sqrt{2\pi}} e^{-\frac{a^2}{2}(2+\mu u^2)} \cdot da \, du$$

$$= \sqrt{\mu} \int_1^\infty \left(\frac{1}{2+\mu u^2}\right)^{\frac{3}{2}} du$$

$$\int_0^\infty \frac{2y^2}{\sqrt{2\pi\sigma^2}} e^{\frac{-y^2}{2\sigma^2}} dy = \sigma^2$$

$$\sigma = \left(\frac{1}{2+\mu u^2}\right)^{3/2}$$

$$u = \sqrt{\frac{2}{\mu}} \tan\theta = \infty$$

$$du = \sqrt{\frac{2}{\mu}} \sec^2\theta \, d\theta$$

$$= \sqrt{\mu} \int_{\tan^{-1}\sqrt{\frac{\mu}{2}}}^{\pi/2} \left(\frac{1}{2\sec^2\theta}\right)^{3/2} \sqrt{\frac{2}{\mu}} \sec^2\theta \, d\theta$$

$$\int_{\tan^{-1}\sqrt{\frac{\mu}{2}}}^{\pi/2} \left(\frac{1}{2}\sec^2\theta\right)^{3/2} \sqrt{\frac{2}{\cancel{\mu}}}\ \sec^2\theta\, d\theta$$

$$\Rightarrow \int \frac{1}{2\,\cancel{\sec^2\theta}\ \cancel{\sqrt{2}}\ \sec\theta}\ \cancel{\sqrt{2}\ \sec^2\theta}\, d\theta$$

$$\Rightarrow \frac{1}{2}\int_{\tan^{-1}\sqrt{\frac{\mu}{2}}}^{\pi/2} \cos\theta\, d\theta \Rightarrow \frac{1}{2}\left\{1 - \sin\left(\tan^{-1}\sqrt{\frac{\mu}{2}}\right)\right\}$$

$$\Rightarrow \frac{1}{2}\left\{1 - \sqrt{\frac{\mu}{\mu+2}}\right\} \quad \because \sin^2\theta = \frac{\tan^2\theta}{1+\tan^2\theta}$$

$$= \frac{\mu/2}{1+\mu/2} = \frac{\mu}{\mu+2}$$

$$\Rightarrow \frac{1}{2}\left\{1 - \left(\frac{1}{1+2/\mu}\right)^{1/2}\right\} \Rightarrow \frac{1}{2}\left\{1 - \left(1+\frac{2}{\mu}\right)^{-1/2}\right\}$$

$$\Rightarrow \frac{1}{2}\left\{\cancel{1} - \left(\cancel{1} - \frac{1}{2}\cdot\frac{2}{\mu}\right)\right\} = \frac{1}{2\mu}$$

$$\boxed{BER_{average} = \frac{1}{2\mu} = \frac{1}{2SNR}}$$ $\mu \uparrow$

Q: Compare BER of wireless sys at SNR 20dB to the wireline with ~~at~~ SNR 10 dB

$$BER_{wireless} = \frac{1}{\underset{\substack{\downarrow \\ 20dB}}{2.100}} = 0.5\times10^{-2} \cdot sm$$

wireline = 7.8×10^{-4} [table?

Q: Compute SNR require for wireless system to BER of 5×10^{-6}.

$$\frac{1}{2 SNR} = 5 \times 10^{-6} \Rightarrow SNR = \frac{10^6}{2} \Rightarrow 10[\log 10^6 - \log 2]$$

$$= 6 - 0.3$$

$$= 57 \text{ dB}$$

$\sqrt{SNR}_{wireline} = 4.75$

Q: Compute SNR_{dB} for wireless Communication System for a probability of bit error 10^{-6} is BER. Compare with wireline system

Sol^n^ $BER = 10^{-6} = \frac{1}{2 SNR} = SNR_{dB} = 57 dB$ "High SNR"

$$BER_{wired} = Q(\sqrt{SNR}) = 10^{-6}$$

N for wired : $BER = Q(\sqrt{SNR}) = 10^{-6} \rightarrow$ Graph

$$\sqrt{SNR} = 4.75$$

$$SNR = 22.595$$

$$SNR_{dB} = 10 \log_{10}(22.595) = 13.6 \text{ dB}$$

$$\frac{\text{SNR wireless}}{\text{SNR wired}} = \cancel{\frac{57}{13.6}} = \cancel{4.19} \quad \text{linear} = \frac{10^{5.7}}{10\,4636} = 10$$

$10 \log BER = 57$
$10^{5.7}$

Summary

wireline	wireless
① $y = x + n$	① $y = hx + n$
② $BER = Q(\sqrt{SNR})$	② $BER_{avg} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$
③ $BER \leq \frac{1}{2} e^{-\frac{SNR}{2}}$	$BER_{High(SNR)} = \frac{1}{2\,SNR}$
Chernoff Bound	Note: SNR (raw)
$Q(x) < \frac{1}{2} e^{-\frac{1}{2}x^2}$	$\neq \frac{\text{Signal pow}}{\text{Noise pow}} \neq \frac{a^2 P}{\sigma^2}$
	$= \frac{P}{\sigma^2}$
	③ $BER_{Low SNR} = \frac{1}{SNR}$

DEEP fading

$$BER = P_r(a^2 P \ll \sigma^2)$$

$$= P_r\left(a^2 \ll \frac{\sigma^2}{P}\right) = P_r\left(a \ll \frac{1}{\sqrt{SNR}}\right)$$

$$= \int_0^{\frac{1}{\sqrt{SNR}}} f_A(a)\,da = \int_0^{\frac{1}{\sqrt{SNR}}} 2a e^{-a^2}\,da$$

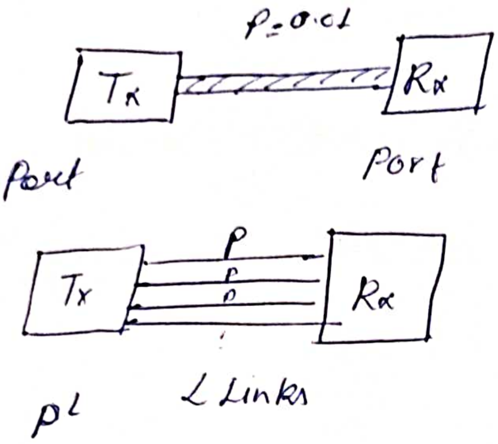
$f_A(a) = 2ae^{-a^2}$

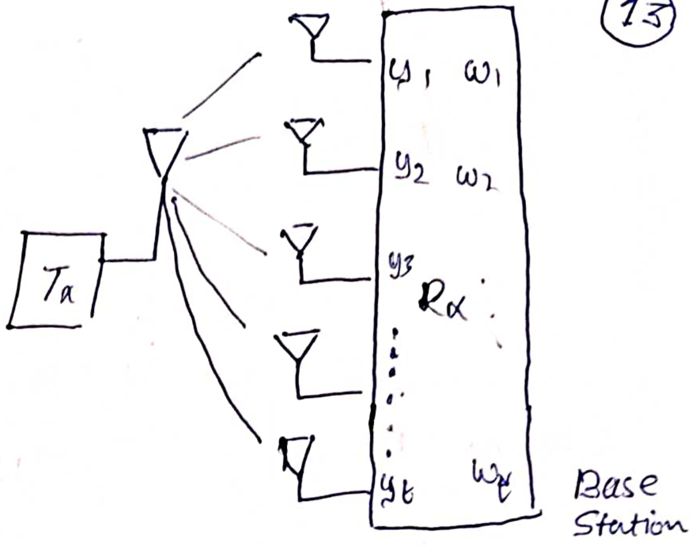
Rayleigh r.v

$$\approx \int^{\frac{1}{\sqrt{SNR}}} 2a\,da = \frac{1}{SNR}$$

Diversity

$y = hx + n$





Multiple Antena System

r : recive antena

Constrains Not allow on Cellphone
1. Space constraint
2. Battery
3. technology in use

$$\left.\begin{aligned} y_1 &= h_1 x + n_1 \quad \text{"AWGN"} \\ y_2 &= h_2 x + n_2 \\ &\vdots \\ y_r &= h_r x + n_r \end{aligned}\right\} \text{SNR received}$$

$$\boxed{\bar{y} = \bar{h}x + \bar{n}} \quad \text{vector form}$$

$$\bar{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad \bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix} \quad \bar{h} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_r \end{bmatrix}$$

Diversity Combining Technique

- ↳ selection Combining
- ↳ maximum Ratio Combining ✓ → maximizing ratio of SNR
- ↳ Equal gain Combining

MRC

$$[w_1^*, w_2^*, w_3^*, \ldots\ldots w_r^*] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \bar{w}^H \bar{y} \qquad \bar{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_r \end{bmatrix}$$

Beam - forming vector → from different attenna we are getting inform

Hermition

$$w^H \bar{y} = \bar{\omega}^H (\bar{h}x + \bar{n})$$

$$= \underbrace{\bar{\omega}^H \bar{h}x}_{\text{Signal}} + \bar{\omega}^H \bar{n} \rightarrow \text{noise}$$

Signal Power $= |\bar{\omega}^H h|^2 p$

Noise power $= E\{|\bar{\omega}^H n|^2\}$ $\quad Var(X) = E(x^2) - (E(x))^2$

$$= E\{(\bar{\omega}^H \bar{n})(\bar{\omega}^H \bar{n})^*\}$$

$$[\quad][\;] \rightarrow w_1 n_1^* + w_2 n_2^* + \dots w_r n_r^*$$

$$\Rightarrow w_1^* n_1 + w_2^* n_2 + \dots w_r^* n_r$$

$$E\{(\omega^H \bar{n})(\bar{\omega}^H \bar{n})^*\} \Rightarrow \sum_{i=1}^{r} |w_i|^2 |n_i|^2 + \sum_i \sum_j w_i w_j^* n_i^* n_j \quad i \neq j$$

$$\Rightarrow \sum_{i=1}^{r} E(|w_i|^2 |n_i|^2) + \sum\sum E(w_i w_j^* n_i^* n_j)$$

$$\Rightarrow \sum_{i=1}^{r} |w_i|^2 E(|n_i|^2) + \sum_i \sum_j w_i w_j^* \underbrace{E(n_i^* n_j)}_{0}$$

$\cancel{E(n_i^*) E}(n_j)$

samples uncorrelated

$$\Rightarrow \sigma_n^2 \sum_{i=1}^{r} |w_i|^2$$

$$\Rightarrow \sigma_n^2 \|\bar{\omega}\|^2 \qquad \therefore \text{Norm } \|x\|^2 = x_1^2 + x_2^2 \dots x_n^2$$

$$SNR_{opt} = \left.\frac{|\bar{\omega}^H h|^2 p}{\sigma_n^2 \|w\|^2}\right|_{w = w_{opt}}$$

"optimisation"

Recap: $\bar{y} = \bar{h}x + \bar{n}$ "Multiple Receive Antenna System"

MRC

$\bar{w}^H \bar{y}$ "Output of MRC"

$$SNR_{recived} = \frac{\text{Signal Power}}{\text{Noise power}} = \frac{|\bar{w}^H \bar{h}|^2 P}{?}$$

analogous to $|h|^2 = a^2$ Simple Antenna system.

$$\bar{w}^H(\bar{h}x + \bar{n}) = \underbrace{\bar{w}^H \bar{h} x}_{\text{signal}} + \underbrace{\bar{w}^H \bar{n}}_{\text{noise}}$$

$$E\{(\bar{w}^H \bar{n})(\bar{w}^H \bar{n})^*\}$$

$$\Rightarrow E\{\bar{w}^H \bar{n}\bar{n}^H \bar{w}\} \Rightarrow \bar{w}^H E(\underset{L\times 1 \text{ or } r\times 1}{\bar{n}}\ \underset{1\times L \text{ or } 1\times r}{\bar{n}^H})\bar{w}$$

$$E\{\bar{n}\bar{n}^H\} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix} [\bar{n}_1^*, \bar{n}_2^* \cdots \bar{n}_r^*]$$

$$= \bar{w}^H \sigma_n^2 I \bar{w}$$

$$= \sigma_n^2 \bar{w}^H I \bar{w} = \sigma_n^2 \|w\|^2$$

$$\Rightarrow E \begin{bmatrix} |n_1|^2 & n_1 n_2^* & n_1 n_3^* & \cdots n_1 n_r^* \\ n_2 n_1^* & |n_2|^2 & \cdots & \\ & & & \\ \cdots & \cdots & \cdots & |n_r|^2 \end{bmatrix} = \begin{bmatrix} \sigma_n^2 & 0 & 0 \cdots & 0 \\ 0 & \sigma_n^2, & 0 \cdots & 0 \\ 0, & 0, & \cdots & \sigma_n^2 \end{bmatrix}$$

$$= \sigma_n^2 I_{r\times r}$$

$$E\{n_1 n_2^*\} = \overset{0}{E(n_1)}\ \overset{0}{E(n_2)} = 0$$

$$SNR_{recived} = \frac{|\bar{w}^H \bar{h}|^2 P}{\|\bar{w}\|^2 \sigma_n^2}$$

$\bar{w}_n = k\bar{w}$. ratio same

maximizing SNR w?

$a^H b = |a||b| \cos\theta$

$\theta \to 0$

$a \to b$ "alinged" $\Rightarrow \bar{a} = c\vec{b}$

$w^H = c\bar{h}$

Optimization Problem

Choose $\bar{w}$ such that $\|w\|^2 = 1$ and SNR maximizes!

max $SNR_{received}$ = $\frac{\text{Signal Power}}{\text{Noise Power}}$ $\Rightarrow$ $|\bar{w}^H \bar{h}|^2 P \Big|$ such that $\|w\|^2 = 1$

$w^H = c\bar{h}$

$\|w\|^2 = 1$

$c^2 \|h\|^2 = 1 \Rightarrow c = \frac{1}{\|\bar{h}\|}$

$w^H = \frac{\bar{h}}{\|\bar{h}\|}$

$$\rightarrow \frac{\left|\frac{\bar{h}^*}{\|\bar{h}\|} \bar{h}\right|^2 P}{\sigma_n^2}$$

$$= \frac{\left|\frac{\|\bar{h}\|^2}{\|\bar{h}\|}\right|^2 P}{\sigma_n^2}$$

$$= \frac{\|\bar{h}\|^2 P}{\sigma_n^2} \Bigg| \bar{w}_{op} = \frac{h}{\|h\|}$$

$h_1^2 + h_2^2$

Recap

$$SNR_{max} = \frac{\|h\|^2 P}{\sigma_n^2} \Bigg|_{\|w\|^2 = 1}$$

for $\bar{w}_{opt} = \frac{\bar{h}}{\|h\|}$

Example : $L = 2$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Q: (a) obtain the received signal at the output of MRC "$w^H y$" (b) obtain SNR

given: $h_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j$

$h_2 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j$

$\bar{y} = \bar{h}x + \bar{n}$

Sol^n $\|h\| = \sqrt{|h_1|^2 + |h_2|^2}$

$= \sqrt{1 + 1}$

$= \sqrt{2}$

$\bar{w}_{opt} = \frac{\bar{h}}{\|h\|}$

$$\bar{h} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix}$$

(15)

$$\bar{w}_{op} = \frac{\bar{h}}{\|h\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} j \\ \frac{1}{2} - \frac{1}{2} j \end{bmatrix} \xrightarrow{\text{Hermi}} \begin{bmatrix} \frac{1}{2} - \frac{1}{2} j \\ \frac{1}{2} + \frac{1}{2} j \end{bmatrix}^T$$

$$\Rightarrow \frac{1}{2}[1-j, 1+j]$$

$$\bar{w}_{opt}^H \bar{y} = \frac{1}{2}[1-j, 1+j]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$ "Detected Signal after MRC".

Hermition :- Conjugate + Transpose.

$$SNR = \frac{\|h\|^2 P}{\sigma^2} = \frac{(\sqrt{2})^2 \cdot P}{\sigma^2} = \frac{2P}{\sigma^2}$$

#

$y =$ $$F_G(g) = \frac{1}{(L-1)!} g^{L-1} e^{-g}$$ $g = a^2 = \|h\|^2$

$$BER = Q(\sqrt{SNR})$$
instantaneous

$$BER_{avg} = \int_0^P Q(\sqrt{SNR})\, f_G(g)\, dg$$

$$= \left(\frac{1-\lambda}{2}\right)^L \sum_{l=0}^{L-1} {}^{L+l-1}C_l \left(\frac{1+\lambda}{2}\right)^l$$

where ${}^nC_k = \frac{\lfloor n}{\lfloor k \lfloor n-k}$

$$\lambda = \sqrt{\frac{SNR}{SNR+2}}$$

Case 1: $L = L$ $\quad BER = \left(\frac{1-\lambda}{2}\right)$

for Single Antenna System

$$BER_{avg} = \frac{1}{2}(1-\lambda)$$

case 2: High SNR term by term

$$\frac{1}{2}(1-\lambda) = \frac{1}{2SNR}$$

$$\frac{1}{2}(1+\lambda) \to 1$$

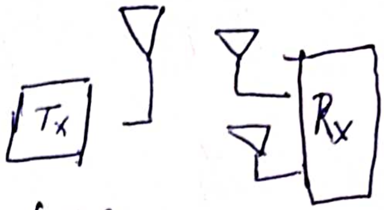
$$BER_{avg} = \frac{1}{2^L} \times \frac{1}{SNR^L} \times \left(\sum_{l=0}^{L-1} {}^{L+l-1}C_l\right) =$$

$$= \frac{1}{2^L} \times \frac{1}{(SNR)^L}\, {}^{2L-1}C_L$$

for BER of 1ppm i.e 10^{-6}

① Single Antenna, SNR = 57 dB

② Two Antenna



$L = 2$

Example

$$BER_{avg} = 10^{-6} = \frac{1}{2^2} \frac{1}{(SNR)^2} {}^{3}C_{2}$$

$$\Rightarrow 10^{-6} = \frac{1}{4} \times \frac{1}{SNR^2} 3$$

$$\Rightarrow SNR = \frac{3 \times 10^6}{4} \Rightarrow SNR_{dB} = 10 \log\left(\sqrt{\tfrac{3}{4} \times 10^6}\right)$$

$$= 29.37 \text{ dB}$$

Independent links

well seprated among antenna's can ensure

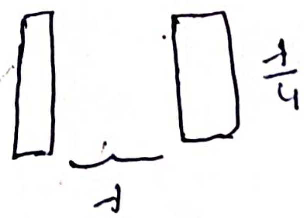
$$d = \frac{c}{f_c}$$ Antenna Size

• GSM "2G" $f_c = 900$ MHz

$$d = \frac{3 \times 10^8}{900 \times 10^6}$$

$d = 33.33$ cm

Size $= \frac{d}{4} = 8.3$ cm



• 3G/4G - $f_c = 2.4$ GHz

$$d = \frac{3 \times 10^8}{2.4 \times 10^9}$$

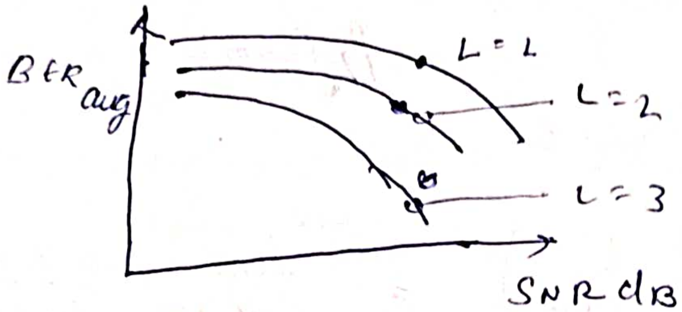
$= 12.5$ cm

• 5G $f_c = 5$ GHz

$$d = \frac{3 \times 10^8}{5 \times 10^9} = 6 \text{ cm}$$

How the BER_{avg} varies with antenna

$$BER_{avg} = \frac{1}{2^L} \times \frac{1}{SNR^2} \times {}^{2L-1}C_L$$



Deep fade ~~for multiple~~ (~~multiple~~ multiple Receive Antenna System)

$$\bar{y} = \bar{h}\bar{x} + \bar{n}$$

Chi Sqr PdF

$$\|h\|^2 P < \sigma_n^2$$

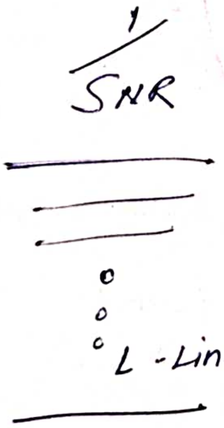
$$gP < \sigma_n^2$$

g is very less

$$\therefore e^{-g} \approx 1$$

$$BER_{avg} = P_r\left(g < \frac{1}{SNR}\right) = \int_0^{1/SNR} \underbrace{\frac{g^{L-1} e^{-g}}{\lfloor L-1}}_{f_G(g)} dg$$

E : Deep fade



$$= \int_0^{\frac{1}{SNR}} \frac{g^{L-1}}{\lfloor L-1} dg = \frac{1}{\lfloor L-1} \left(\frac{g^{L-1+1}}{L-1+1} \right)_0^{\frac{1}{SNR}}$$

$$\Rightarrow \frac{1}{\lfloor L} \left(\frac{1}{SNR} \right)^L$$

$$P(E) = P(E_1 \cap E_2 \cap E_3 \cap \ldots E_L)$$

$$= P(E_1)\, P(E_2)\, P(E_3) \cdots P(E_L)$$

$$= \frac{1}{SNR} \times \frac{1}{SNR} \cdots\cdots L \text{ terms}$$

24 Feb

Diversity :

$\downarrow$

no of links independent

$$BER = P_e(SNR)$$

$$d = -\lim_{SNR \to \infty} \frac{\ln\left(\overset{\bar{P}_e}{P_e}(SNR)\right)}{\ln(SNR)}$$

$BER = Q(\sqrt{SNR})$

$BER = P_e(SNR)$

Case 1 : Single Antenna System

$$d = -\lim_{SNR \to \infty} \frac{\ln\left(\frac{1}{2SNR}\right)}{\ln SNR} = \lim_{SNR \to \infty} \frac{\ln 2SNR}{\ln SNR}$$

$$= \lim_{SNR \to \infty} \frac{\ln 2 + \ln SNR}{\ln SNR}$$

$$= \lim_{SNR \to \infty} \frac{\ln 2}{\ln SNR} + 1$$

$$\boxed{d = 1}$$

case 2: L- antenna

$$d = -\lim_{SNR \to \infty} \frac{\ln\left(\frac{1}{2^L} \times \frac{1}{SNR^L} \times {}^{2L-1}C_L\right)}{\ln(SNR)}$$

$$= -\lim_{SNR \to \infty}\left[\frac{\cancel{\ln\left(\frac{1}{2}\right)^L}}{\ln SNR} - \frac{L \cdot \ln SNR}{\ln SNR} + \frac{\cancel{\ln {}^{2L-1}C_L}}{\ln SNR}\right]$$

$$\boxed{d = L}$$

Case 3: Wireline

$$d = -\lim_{SNR \to \infty} \frac{\ln \frac{1}{2} e^{-SNR/2}}{\ln SNR}$$

$$= -\lim_{SNR \to \infty} \frac{\ln \frac{1}{2} - SNR/2}{\ln SNR}$$

$$= \frac{SNR}{2 \ln SNR} = \frac{1}{2 \frac{1}{SNR}}$$

$$\boxed{d = \infty}$$

Chernoff bound

$$Q(x) \le \frac{1}{2} e^{-x^2/2}$$

$$BER = Q(\sqrt{SNR}) \le \frac{1}{2} e^{-SNR/2}$$

Power Profile

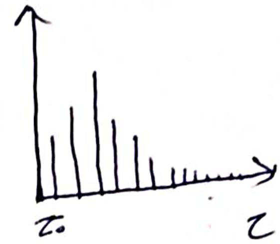
$$\boxed{h = \sum_{i=0}^{L-1} a_i \delta(t-\tau_i)}$$ $f(\tau)$

- Delay Spread

"Retriving info time"

↳ Max or $\sigma_{\tau max} = \tau_{L-1} - \tau_0$

$\phi(\tau)$



Power = $|h|^2$ or g

$\phi(\tau) = |h(\tau)|^2$

L - Components

↳ Root Mean Square (RMS) delay spread (σ_τ)

$\sigma_\tau = \sqrt{\text{Variance}}$

Gain	Delay
$\lvert h_0\rvert^2$ or $\lvert a_0\rvert^2$	τ_0
	τ_1
$\lvert a_1\rvert^2$	
⋮	
$\lvert a_{L-1}\rvert^2$	τ_{L-1}

$$\sigma_\tau = \sqrt{b_0(\tau_0-\bar{\tau})^2 + b_1(\tau_1-\bar{\tau})^2 + \dots b_{L-1}(\tau_{L-1}-\bar{\tau})^2}$$

$$b_0 = \frac{|a_0|^2}{\sum_{i=0}^{L-1} |a_i|^2}$$

$$b_i = \frac{|a_i|^2}{\sum |a_i|^2} = \frac{g_i}{\sum g_i}$$

$$\bar{\tau} = b_0\tau_0 + b_1\tau_1 + \dots b_{L-1}\tau_{L-1}$$

$$\bar{\tau} = \sum_{i=0}^{L-1} b_i \tau_i$$

Power profile (recap)

$$\phi(\tau) = |h(\tau)|^2$$

for, $h(\tau) = \sum_{i=0}^{L-1} a_i \delta(t-\tau_i)$

$$\phi(\tau) = \sum_{i=0}^{L-1} |a_i|^2 \delta(t-\tau_i)$$

$$\phi(\tau) = \sum_{i=0}^{L-1} g_i \delta(t-\tau_i)$$

① $\sigma_{\tau max}$

② $\sigma_\tau = \sqrt{\dfrac{\sum_{i=0}^{L-1} |a_i|^2 (\tau-\tau_i)^2}{\sum_{i=0}^{L-1} |a_i|^2}}$

where $\bar{\tau} = \dfrac{\sum g_i \tau_i}{\sum g_i}$

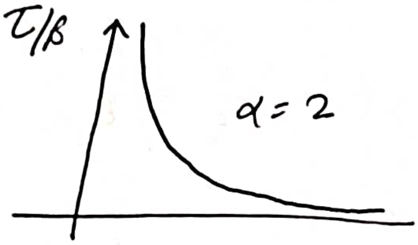
$\overline{\phi}(\tau) = E\{|h(\tau)|^2\}$ for all antennas/users

fractional power $= f(\tau) = \dfrac{\overline{\phi}(\tau)}{\int \overline{\phi}(\tau)\,d\tau}$ "power contained at τ" / "Total power"

$$\sigma_\tau = \sqrt{\int_0^\infty f(\tau)(\tau-\bar{\tau})^2}$$

Q: Assume $\overline{\phi}(\tau) = 2e^{-\tau/\beta}$ where $\beta = 1\,us$

$\propto e^{-\tau/\beta}$



find Rms delay spread of σ_τ

Sol^n

Rms delay spread $\sigma_\tau = \sqrt{\int_0^\infty f(\tau)(\tau-\bar{\tau})^2}$

↳ $f(\tau) = \dfrac{\overline{\phi}(\tau)}{\int_0^\infty \phi(\tau)\,d\tau}$ ↳ $\int_0^\infty \phi(\tau)\,d\tau = \int_0^\infty 2e^{-\tau/\beta}\,d\tau$

$f(\tau) = \dfrac{2e^{-\tau/\beta}}{2\beta} = \dfrac{e^{-\tau/\beta}}{\beta}$

$= \left[\dfrac{2e^{-\tau/\beta}}{-1/\beta}\right]_0^\infty$

$= 0 + \dfrac{2\beta}{e^{\tau/\beta}}\Big|_{\tau=0}$

$= 2\beta$

↳ $\bar{\tau} = \int_0^\infty \dfrac{\tau e^{-\tau/\beta}}{\beta}\,d\tau$

u.v

$= \cancel{\dfrac{\tau\, e^{\tau}}{\beta}} = x$

$= -\dfrac{1}{\beta}$

$= \tau e^{-\tau/\beta}\Big|_0^\infty + \int_0^\infty e^{-\tau/\beta}\,d\tau$

$= \beta e^{-\tau/\beta}\Big|_0^\infty \Rightarrow \beta = 1\,\mu s$

① Write a Short Note on time-domain & frequency-domain properties of AWGN

② Explain source & line coding. Give the classification of the same.

③ Show that

$$\sigma_x^2 = E(x^2) - [E(x)]^2$$

④ Determine CDF of Rayleigh r.v. Find the mean & variance of the same.

assume

$$f_X(x) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} & x > 0 \\ 0 & x < 0 \end{cases}$$

⑤ find the mean & variance of uniform distribution in discrete & continuous sense.

$$\sigma_\tau = \sqrt{\int_0^\infty \frac{e^{-\tau/\beta}}{\beta}(\tau-\beta)^2}$$

$$= \sqrt{\int_0^\infty \frac{e^{-\tau/\beta}}{\beta}(\tau^2+\beta^2-2\beta\tau)\,d\tau}$$

$$= \int_0^\infty -\frac{2\tau e^{-\tau/\beta}}{\beta} + \frac{\tau^2 e^{-\tau/\beta}}{\beta} + \beta^2 \frac{e^{-\tau/\beta}}{\beta}$$

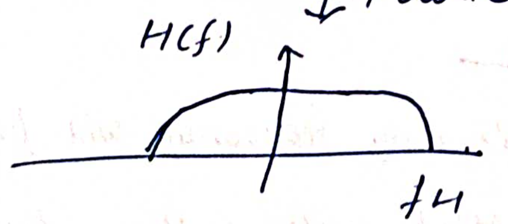
$$= \quad + 2\beta^2 -$$

$$= \quad \sigma_\tau^{RMS} = \beta = 1\mu s$$

frequency domain

$$\text{channel} \quad \underbrace{h(\tau) = \sum_{i=0}^{L-1} a_i\, \delta(t-\tau_i)}_{\text{LTI System}}$$ " Time domain)

↓ Fourier Transform



channel/coherance Bandwidth = BW_c

↳ Signal Bandwidth should be less than or Equal to Channel/coherance bandwidth.

$$BW_{signal} \le BW_c$$

Relationship b/w RMS delay spread & channel Bandwidth

$$\boxed{BW_c = \frac{1}{2\sigma_\tau}}$$

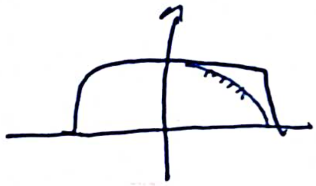
$\downarrow$ FD $\qquad\qquad$ $\downarrow$ TD

7 Feb

Implications of Delay Speed

Case ① frequency domain

$$H(f) = \int_0^\infty h(\tau)\, e^{-2j\pi f \tau}\, d\tau$$



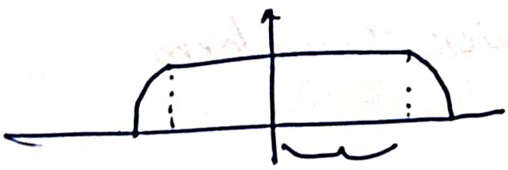
$$X(f) \rightarrow \boxed{H(f)} \rightarrow Y(f) = X(f)\,H(f)$$

: B → Bandwidth

$B_S < B_C$: flat fading

Signal ↙ $\qquad$ ↳ Channel

otherwise : frequency selective fading $\qquad$ $\therefore h(\tau) = \sum_{i=0}^{L-1} a_i \delta(t-\tau_i)$



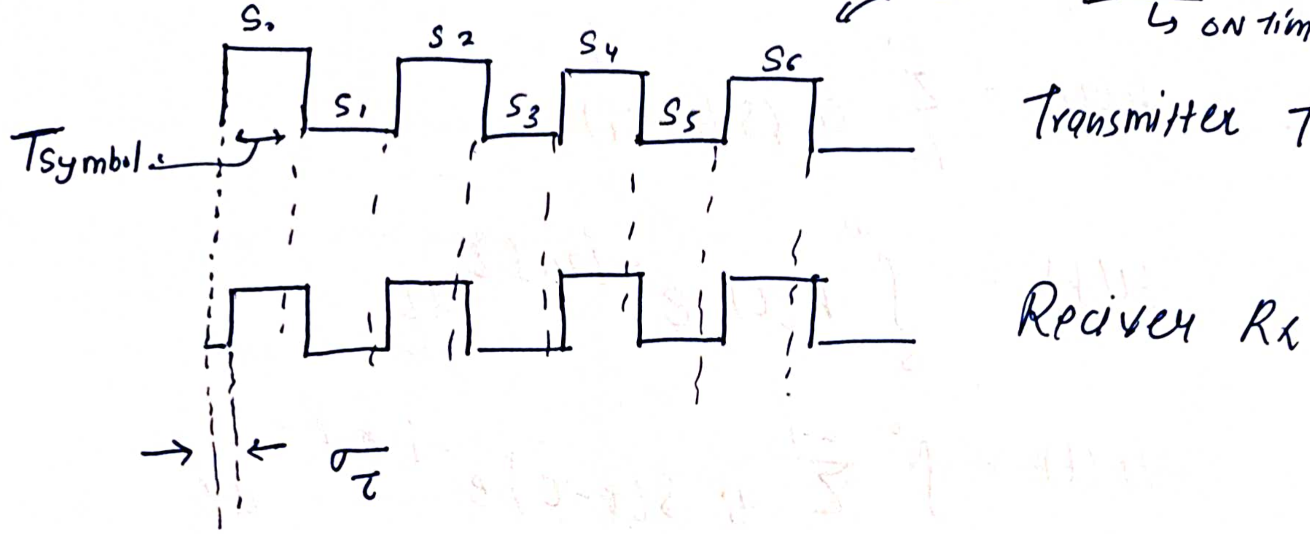
till this only frequency will properly pass rest will suffer loss.

↳ Signal should be Bandlimited

$B_{signal} < B_{channel\ or\ coherence}$

Case 2: Time domain.

50% duty cycle
↳ ON time / OFF time



If delay σ_τ > Tsymbol → Intersymbol Interference (ISI)

Take reciprocal

$$\frac{1}{\sigma_\tau} < \frac{1}{T_{symbol}}$$

$$\therefore B_c \propto \frac{1}{\sigma_\tau}$$

$$\boxed{B_c < f_s}$$

Summary

$\boxed{B_s < B_c}$ → flat fading → No ISI

$\boxed{B_s > B_c}$ → frequency selective fading → ISI

$\sigma_\tau \xrightarrow{\text{limits}}$ Channel BW $\xrightarrow{\text{limits}}$ Signal frequencies

(Time domain) → Narrow

3G/4G

Suppose $\sigma_\tau = 1ms$

$$B_c = \frac{1}{2\sigma_\tau} = 0.5\ MHz = 500\ kHz$$

① GSM

$B_s = 200\ kHz$

$B_s < B_c$ → flat top fading

② 3G wide CDMA

$B_s = 5\ MHz = 5000\ kHz$

$B_s > B_c$ → freq selective fading and ISI we will try to find Bandwidth (B)

$$h(\tau) = \sum_{i=0}^{L-1} a_i * \delta(t-\tau_i)$$

$$H(f) = \int_0^\infty h(\tau)\, e^{-j2\pi f \tau}\, d\tau$$

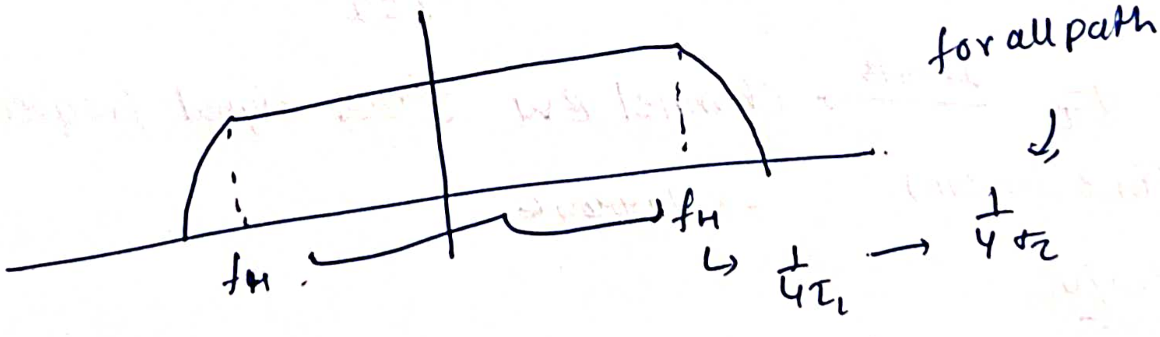
$$H(f) = \int_0^\infty \sum_{i=0}^{L-1} a_i\, \delta(t-\tau)\, e^{-j2\pi f \tau}\, d\tau$$

$$H(f) = \sum_{i=0}^{L-1} \int_0^\infty a_i\, \delta(t-\tau)\, e^{-j2\pi f \tau}\, d\tau$$

$$\boxed{H(f) = \sum_{i=0}^{L-1} a_i \left[e^{-j2\pi f \tau_i}\right]}$$

$$f = 0 \longrightarrow H(0) = \sum_{i=0}^{L-1} a_i$$

$$f = \frac{1}{4\tau_1} \rightarrow H\left(\frac{1}{4\tau_1}\right) = \sum_{i=0}^{L-1} -j a_i$$

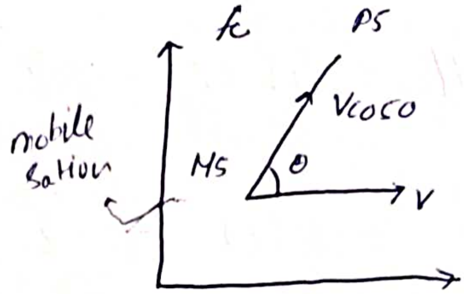


Band width

$$B_c = 2 \times \frac{1}{4\sigma_\tau} \Rightarrow \boxed{B_c = \frac{1}{2\sigma_\tau}}$$

8 feb

Doppler effect ' Mobility



$h(t) \longrightarrow H(t,f)$ "Time Varying"

Doppler Effect

$$f_d = \left(\frac{V\cos\theta}{c}\right) f_c$$

Doppler shift

$$\boxed{f_r = f_c + f_d}$$ Recived carrier freq

Case ① MS $\xrightarrow{\text{towards}}$ BS $\quad 0 \le \theta \le \pi/2$

$\theta = 0 \longrightarrow f_d = \frac{V}{c} f_c \longrightarrow f_r = \left(1+\frac{V}{c}\right) f_c > 0 \uparrow$

$\theta = \pi/2 \longrightarrow f_d = 0 \quad f_r = f_c$

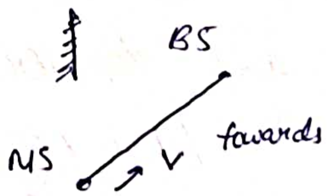
Example ①: If $f_c = 1850\,MHz$ and user is in car moving at 60 mph directly towards base-station. Compute Doppler shift and recived freq?

$$f_d = \frac{V\cos(0)}{c} f_c = \frac{60 \times 0.447 \times 1850M}{3\times 10^8}$$

$$= 165.39\,Hz$$

$$f_r = f_c + f_d$$

$$= 1850 \times 10^6 + 165.39$$



⇒ After time t, distance ↓ by Vt

① Straight path $\quad$ delay ↓ $\quad - \tau_i - Vt/c$

② $\theta \ne 0$ $\qquad \tau_i(t) = \tau_i - \frac{V\cos\theta}{c} t$

Recall $h(\tau) = \sum_{i=0}^{L-1} a_i \, \delta(t - \tau_i)$ "L paths"

$$H(f) = \sum_{i=0}^{L-1} a_i \, e^{-j2\pi f_c \tau_i}$$

$\downarrow$

$$H(f,t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \left(\tau_i - \frac{v\cos\theta}{c} t\right)}$$

$$= \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} \, \underline{e^{j2\pi \overbrace{f_c \frac{v\cos\theta}{c}}^{f_d} t}}$$

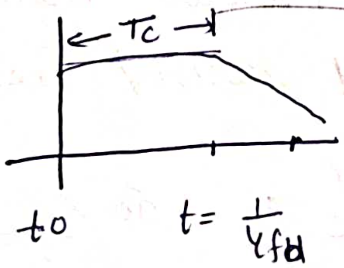
time varying phase factor

Let i^{th} component,

$$a_i(t) = a_i e^{-j2\pi f_c \tau_i} \cdot e^{j2\pi f_d t}$$

$t = 0 \longrightarrow a_i e^{-j2\pi f_c \tau_i}$

$t = \frac{1}{4f_d} \longrightarrow a_i(t) = j a_i e^{-j2\pi f_c \tau_i}$



T_c = Coherence Time — time for which channel is constant

$$T_c = \frac{1}{4f_d}$$

Example: consider a vehicle moving towards base station at 60 mph. Assume $f_c = 1850\ MHz$. What is coherence time and doppler bandwidth

$f_d = \frac{v\cos(0)}{c} f_c$

$f_d = 165\ Hz$

$f_r = f_c + f_d$

$T_c = \frac{1}{4f_d} = \frac{1}{4 \times 165} = 0.0015 = 1.5\ ms$

$B_d = \frac{1}{2T_c}$

$= \frac{4f_d}{2} = 2f_d$

$= 330\ Hz$

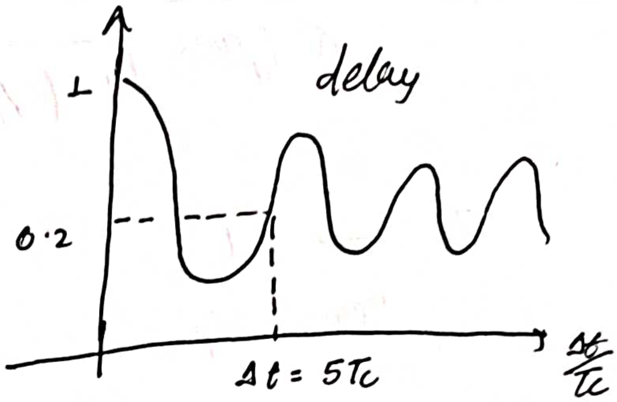
9 feb

Random Variable

Correlation of RVs X & Y

$E\{XY\}$

$a_i(t) = a_i e^{-j2\pi f_i \tau_i} \cdot e^{j2\pi f_d t}$



Time - Correlation of channel

$E\{a_i(t)\, a_i^*(t+\Delta t)\}$

$\psi(\Delta t) = E\{\ \} = \{ |a_i|^2 \times 1 \times e^{-2\pi f_d \Delta t} \} = E\{ e^{-j2\pi f_d \Delta t} \}$

Assume $|a_i|^2 = $ unity

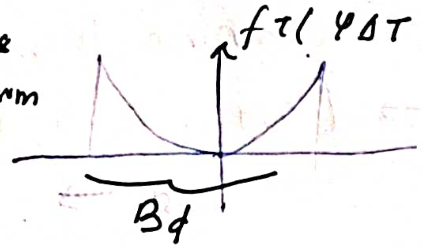
$\Rightarrow E\{ e^{-j2\pi \left(\frac{v}{c}\cos\theta\right) f_c \Delta t} \}$

↳ Replace $f_d^{max} = \frac{v}{c} f_c$ " when $\theta = 0$ "

$\psi(\Delta t) = E\{ e^{-j2\pi f_d^{max} \cos\theta\, \Delta t} \} = \int_0^{\pi} \frac{1}{\pi} e^{-j2\pi f_d^{max} \cos\theta\, \Delta t}\, d\theta$

$\Rightarrow J_0(2\pi f_d^{max} \Delta t) = J_0\left(2\pi \frac{1}{4T_c} \Delta t\right) = J_0\left(\frac{\pi}{2} \frac{\Delta t}{T_c}\right)$

Jakes Spectrum



Date 12/02/2024

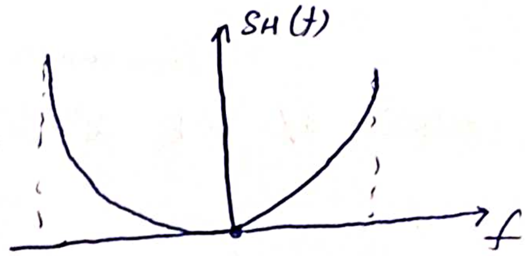
Doppler Spectrum " Jake's Spectrum"

$\phi(\Delta t) = E\{ a_i(t) \cdot a_i(t+\Delta t) \}$ "Time - correlation"

↳ FT → $S_H(f)$ "Jakes Spectrum"

$S_H(f) = \int_{-\infty}^{\infty} \phi(\Delta t)\, e^{-2\pi f(\Delta t)}\, d\Delta t = \int_{-\infty}^{\infty} J_0(2\pi f_d^{max} \Delta t)\, e^{-2\pi f \Delta t}\, d\Delta t$

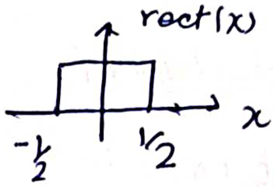
$$S_H(f) = \frac{1}{\pi f_d^{max} \sqrt{1-(f/f_d)^2}} \, rect(f/2f_d)$$



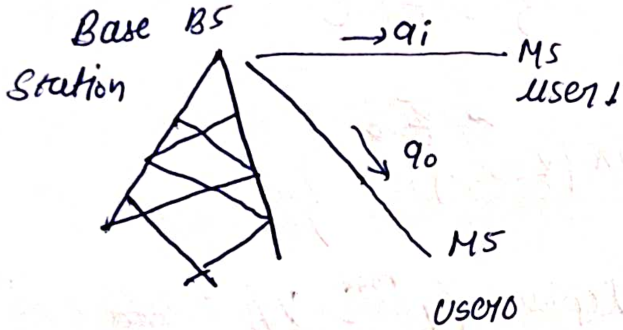
$B_d = 2f_d$

$T_c \longrightarrow B_d$

Relationship



CDMA Code Division for Multiple Acess



$C_0 = [1, 1, 1, 1]$

$C_1 = [1, -1, -1, 1]$

↳ Codes should be unique and their dot product should be 0

↳ a_i & a_0 is information, multiply them with the codes

$C_0 = [1, 1, 1, 1] \xrightarrow{\times a_0} [a_0, a_0, a_0, a_0]$

$C_1 = [1, -1, -1, 1] \xrightarrow{\times a_1} [a_1, -a_1, -a_1, a_1]$

Combine them

this signal has been recived from BS $\rightarrow$ $[a_0 + a_1, a_0 - a_1, a_0 - a_1, a_0 + a_1]$

user 1

$[a_0 + a_1, a_0 - a_1, a_0 - a_1, a_0 + a_1]$

↓ multiply C_1 $[1, -1, -1, 1]$

$[a_0 + a_1, -a_0 + a_1, -a_0 + a_1, a_0 + a_1]$

User 2

$[a_0 + a_1, a_0 - a_1, a_0 - a_1, a_0 + a_1]$

↓ multiply $C_0 [1, 1, 1, 1]$

$[a_0 + a_1, a_0 - a_1, a_0 - a_1, a_0 + a_1]$

$[a_0+a_1, -a_0+a_1, -a_0+a_1, a_0+a_1]$

add them

$4a_1$

this information going to user1

$[a_0+a_1, a_0-a_1, a_0-a_1, a_0+a_1]$

add them

$4a_0$

to user 2

Steps to transmit information from BS to MS

1. Genrate Code
~~② Multiply information with Codes~~
2. Multiply codes with individual information
3. Combine these and air it

Tx

Rx

4. Multiply code with the Rx signal
5. Added each ~~them~~ term of tupple

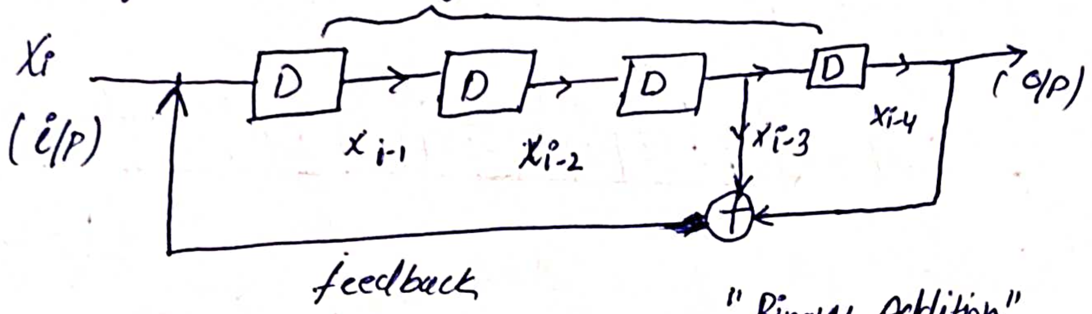
— x — x —

13 feb

(22)

Linear feedback Shift Registers (LFSR)

- Register as delay element



"Binary Addition" in field 2 (or XOR operⁿ)

XoR

0	0	0
0	1	1
1	0	1
1	1	0

"odd priority detector"

x_i	x_{i-1}	x_{i-2}	x_{i-3}	x_{i-4}
1	1	1	1	1
2	0	1	1	1
3	0	0	1	1
4	0	0	0	1
5	1	0	0	0
⋮				
15	1	1	1	0

States : $2^D - 1$ "code Length"

Code : { 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0 }

Symbols :

1 → 1 } Convert
0 → -1

Property ① Randomness or "Balancing" ⇒ no of zero ≃ ones & long lengths → hacking

$P(0) = \frac{7}{15}$, $P(1) = \frac{8}{15}$

② Run Length

{ $\boxed{1, 1, 1, 1}$, $\boxed{0, 0, 0}$, $\boxed{1}$, $\boxed{0, 0}$, $\boxed{1, 1}$, $\boxed{0}$, $\boxed{1}$, $\boxed{0}$ }

patterns (or Runs)

- 4bit pattern : 1
- 3bit pattern : 1
- 2bit : 2
- 1 bit : 4

Probability

4 bit $\rightarrow$ $1/8$

3 bit $\rightarrow$ $1/8$

2 bit $\rightarrow$ $2/8$

1 bit $\rightarrow$ $4/8 = \frac{1}{2}$

③ cyclic shift property

Correlation for lag 'l' $= \frac{1}{N} \sum_{\forall \text{bits}} c[n]\, c[n-l]$

$c[n] = \{$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	-1	-1	-1	1	-1	-1	1	1	-1	1	-1

$\}$

$c[n-2] =$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	-1	1	1	1	1	-1	-1	-1	1	-1	-1	1	1	-1

$c[n]\,c[n-2] = [1, -1, 1, 1, -1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1]$

(1 2 3 4 5 6 7 8 9 10 11 12 13 14)

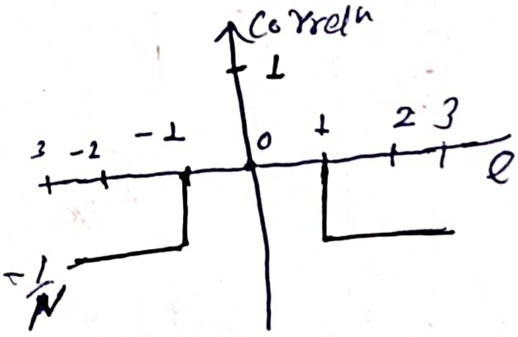
$\sum_{\forall \text{bit}} c[n]\,c[n-2] = +1 + (-1) + \cdots (1)$

$= -1$

$8 \rightarrow -1$

$7 \rightarrow +1$

Correlation $= \frac{1}{N} \sum c[n]\,c[n-2] = \frac{-1}{15}$



↳ for only zero lag corr^n = 1.

↳ for others corr^n $= \frac{-1}{N}$