353 Digital Communication. (Trilochan Sir) | 4 credit | Repeat MidSem (20)

1

↳ 4 Modules → 1] Converting Analog to Digital (Source Coding)
→ 2] Baseband Communication
→ 3] Modulation technique & Coding

Internal Evaluation
- Surprise Quiz
- minor
- Assignment
- Probject* (10%) Harware or Simulation for both lab & theory

4-5 slides Project records

Text Book - 1. Communication System Symon Haykin
2. Digital Communication by J. Proakish (Random process)
3. Morden Digital & analog comm. syst, BP Lathi
4. Digital Communication: Fundamentals and application by Vish B. Sklar (Exercise.)

Objectives:

1. Knowledge about random signal and their proporties
2. Genralise the concept of Digital Source generation.
3. Analytic knowledge on baseband and passband communication.
4. Understand the different digital modulation scheme.
5. Able to realise an efficient Digital commn system in terms of BER and SNR
6. Basic knowledge of Coding theorm

Purpose: to transmit a signal with information to a destination

S:

Source → Transmitter → Channel → Reciver → Destination

Transmitter: Tx Reciver: Rx

Eg: fm radio

Source : microphone
Channel : free space
Receiver : speaker

TV

Sou: Camera ← Image, Video
microphone
free Space : Channel

Channel : wired, wireless

Randomness in channel deterministi

↓
noise ← Thermal Noise, flicker Noise

↳ Distribution : probability.

AWGN - Additive white Gaussian Noise

$$\text{Source} \xrightarrow{s(t)} \Sigma \rightarrow y(t) = s(t) + n(t)$$

$n(t)$ → Σ

Early day's digital Communication

Telegraph

A - Z
0 - 9
'.' , ',' , ')' , '?'

using Morse code encoded using
(.) & (—)
short pulse, long pulse.

most frequent - short code
rare occurance - long code

Aim - average code length is to minimum

Redudance - Remove unused information

3

E →
Z → ---...

→ Remove the redundancy
→ compression

Source → Source Encoder → Channel Encoder → Digital Modulation → Channel → Digital Demodulation → channel demo → Source decoder → Destination

- We are adding redundancy to improve error correction/ Error detection because of channel noise

$0 \longrightarrow 000$ $\qquad$ $000 \rightarrow 0$

$1 \longrightarrow 111$ $\qquad$ $001 \rightarrow 0$

$010 \rightarrow 0$

$011 \rightarrow 1$

$100 \rightarrow 0$

$101 \rightarrow 1$

$110 \rightarrow 1$

$111 \rightarrow 1$

Probability And Random Process

Experiment
↓
outcome

* All the outcome in a set is called Sample Space

eg Throwing a Dice

$S = \{1,2,3,4,5,6\}$

Event - Subset of Sample Space

$S = \{1,2,3,4,5,6\}$

$A_1 = \{2,4,6\}$ $\quad A_2 = \{1,2,3\}$

$P(A_1) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

$P(A_2) = \frac{1}{2}$

$$A = \bigcup_{i=1}^{N} A_i \qquad P(A) \leq \sum_{i=1}^{N} P(A)$$

Exp 1

$S = \{x_1, x_2, \cdots x_N\}$

Exp 2

$S = \{y_1, y_2, \cdots y_N\}$

$P(x_i, y_i) \Rightarrow$ joint probability

$\Rightarrow$ probability of x_i from Exp 1 & probability y_i from Exp 2

$$\sum_{i=1}^{N} \sum_{j=1}^{M} P(x_i, y_i) = 1 \qquad \int\int P(x, y)\,dx\,dy = 1$$

$$\sum_{j=1}^{M} P(x_i, y_j) = P(x_i) \qquad \int P(x, y)\,dy = P(x)$$

$\hookrightarrow$ marginal probability

$$\sum_{i=1}^{N} (x_i, y_j) = P(y_j)$$

Conditional Probability

$P(A/B) \Rightarrow$ probability of A when B occured.

$$= \frac{P(A, B)}{P(B)}$$

$$P(A, B) = P(A)\,P(B/A)$$

$$= P(B)\,P(A/B)$$

$$P(A,B,C) = P(A)\,P(B/A)\,P(C/_{A,B})$$

$$P(A_1, A_2, \cdots A_n) = P(A_1) + P(A_2 | A_1) + P(A_3/_{A_1 A_2}) + \cdots P(A_n/_{A_1, A_2 \cdots A_1})$$

→ If A & B are Independent

$$\boxed{P(A|B) = P(A)} \quad \& \quad P(A,B) = P(A)P(B)$$

Random Variable: It is mapping from sample space to real no.

$$X : S \to \mathbb{R}$$

X = Random process
x - random variable

$\{H, T\}$ $H \to -L$, $T \to L$ or $H \to 0$, $T \to L$

Continous Random Variable

when the image of sample space is continous then the rand. varible is called conti. rand varible

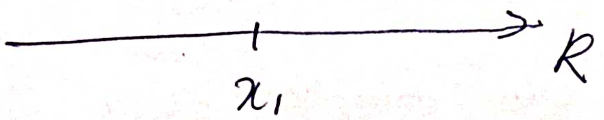
discrete random variable

when image is disconti. then discrete rand var.

Probability Distribution function/ Cummulative Distribution.

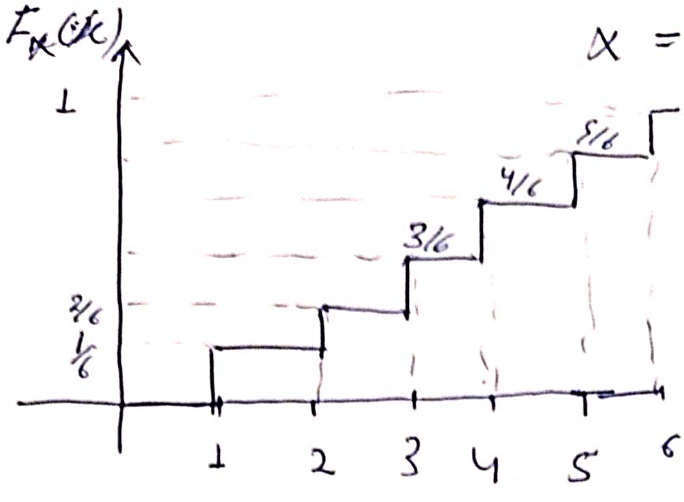
$$F_X(x) = P(X \le x_1)$$

probability of random process from $-\infty$ to x_1



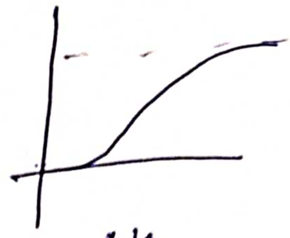
CDF → non decreasing function

$x = \{1, 2, 3, 4, 5, 6\}$



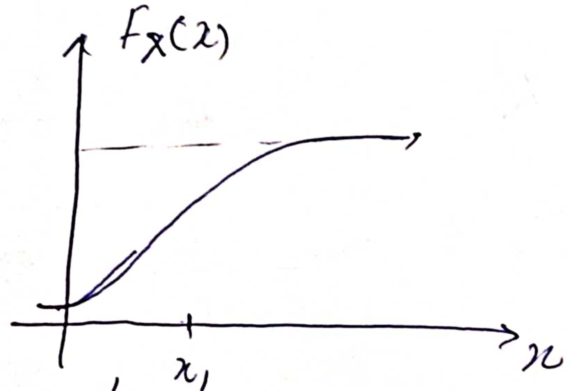
x DRV

$F_X(x)$



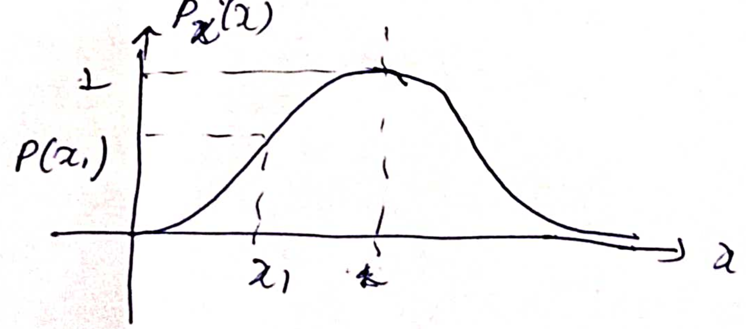
Contⁿ R.V

Pdf : probability density function



$$p_X(x) = \frac{d}{dx}[F_X(x)]$$

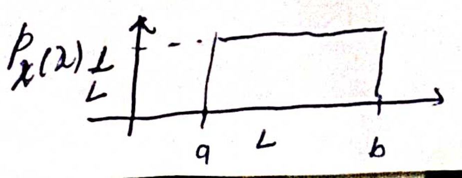
$$F_X = \int_{-\infty}^{x} p(x)\,dx$$

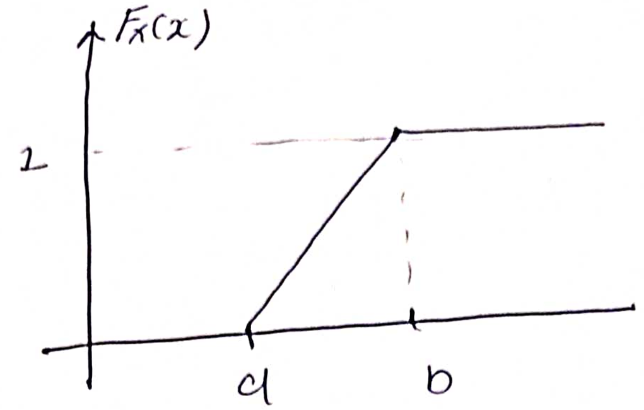


$$P(x_1 \leq X \leq x_2) = F_X(x_2) - F_X(x_1)$$

$$= \int_{x_1}^{x_2} p(x)\,dx$$

Ex Uniform Distribution $\int_a^b p(x)\,dx = 1$



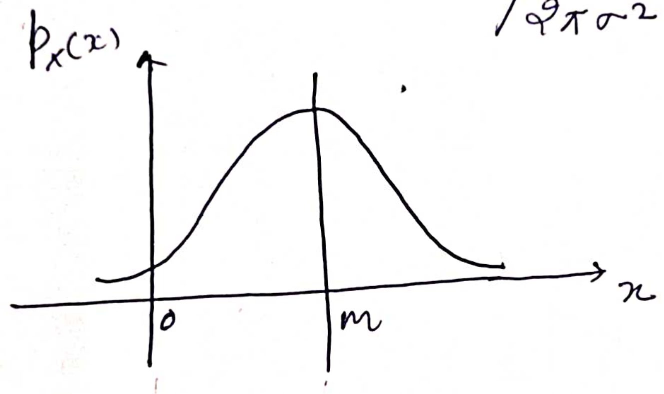


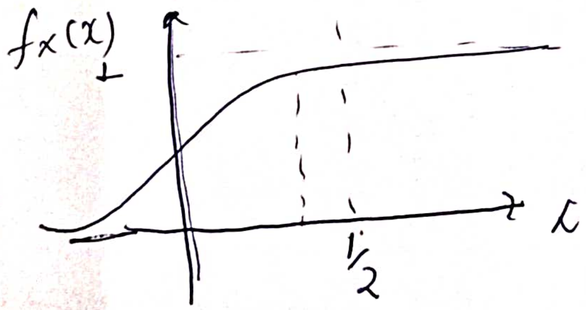
Gaussian distribution

$$P_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

m = mean

σ^2 = varian





multiple r.v.

x_1, x_2

$$F_{X_1 X_2}(x_1, x_2) = P(X_1 \le x_1, X_2 \le x_2)$$

$$p_{X_1 X_2} = \frac{\partial}{\partial x_1 \partial x_2} F_{X_1 X_2}(x_1, x_2)$$

$$F_{X_1 X_2} = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} p_{X_1 X_2}(y_1, y_2)\, dy_1\, dy_2$$

$$F_{X_1/X_2}(x_1 \mid X_2 = x_2) = \frac{\int_{-\infty}^{x_1} P_{X_1 X_2}(y, x_2)\, dy}{P_{X_2}(x_2)}$$

$$P_{X_1|X_2}(x_1 \mid X_2 = x_2) = \left. \frac{\partial F(x/x_2)}{\partial x} \right|_{x = x_1}$$

$$= \frac{P_{X_1 X_2}(x_1, x_2)}{P_{X_2}(x_2)}$$

<u>Independent r.v</u>

$$P_{X_1 X_2}(x_1, x_2) = P_{X_1}(x_1)\, P_{X_2}(x_2)$$

$$F_{X_1 X_2}(x_1, x_2) = F_{X_1}(x_1)\, F_{X_2}(x_2)$$

Mean / Expectation

$\frac{2}{40}$ $\frac{5}{40}$

$$\frac{0 \times 2 + 1 \times 5 + 2 \times 6 + \cdots 10 \times 15}{40}$$

$x,\ P_X(x)$

$$\mu_x = E(X) = \bar{x} = \sum_{i=1}^{n} x_i P_i \qquad \mu_x = \int_{-\infty}^{\infty} x p(x)\, dx$$

D. r.v $\qquad$ C. r.v

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 p(x)\, dx = E\left[(x - \mu_x)^2\right] = \sigma_x^2$$

$E(x^2) = \int x^2 . p(x) dx$ - second moment of r.v x

$\int x^n p(x) dx$ - n^{th} moment of r.v

$$E[(x-\mu_x)^2] = E(x^2+\mu_x^2-2x\mu_x)$$

$$= E(x^2) + E(\mu_x^2) - 2\mu_x E(x)$$

$$= E(x^2) + \mu_x^2 - 2\mu_x^2$$

$$\boxed{\sigma_x^2 = \underbrace{E(x^2)}_{\text{2nd mo}} - \mu_x^2}$$

<u>Covariance</u> – varies w.r.t mean

x y

$E[xy] = r_{xy}$ Correlⁿ

$$C_{xy} = E[(x-\mu_x)(y-\mu_y)^*]$$

$$= E(xy - x\mu_y - y\mu_x + \mu_x\mu_y]$$

$$= E(xy) - \mu_y \underset{\mu_x}{\underbrace{E(x)}} - \mu_x \underset{\mu_y}{\underbrace{E(y)}} + \mu_x\, xy$$

$$= E(xy) - 2\mu_x\mu_y + \mu_x\mu_y$$

$$\boxed{C_{xy} = E[xy] - \mu_x\mu_y}$$

if C_{xy} then x and y are uncorrelated

$E(xy) = r_{xy} = 0$ then orthogonal / Independent vector → underline

Covariance Matrix

$$x_1 \quad x_2 \quad \cdots \quad x_n$$
$$\downarrow \quad \downarrow \quad\quad \downarrow$$
$$\mu_1 \quad \mu_2 \quad\quad \mu_n$$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \underline{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}$$

covariance b/w x_i, x_j

$$E[(x_i - \mu_i)(x_j - \mu_j)]$$

$$E[(\underline{x} - \underline{\mu})(\underline{x} - \underline{\mu})^T]$$

$$\begin{bmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{bmatrix}_{n \times 1} \begin{bmatrix} (x_1 - \mu_1 \cdots x_n - \mu_n) \end{bmatrix}_{1 \times n}$$

$$= \begin{bmatrix} E[(x_1-\mu_1)(x_1-\mu_1)] & E[(x_1-\mu_1)(x_2-\mu_2)] & \cdots & E[(x_1-\mu_1)(x_n-\mu_n)] \\ E[(x_2-\mu_2)(x_1-\mu_1)] & \cdots & & E[(x_2-\mu_2)(x_n-\mu_n)] \\ \vdots & & & \\ E[(x_n-\mu_n)(x_1-\mu_1)] & \cdots & & E[(x_n-\mu_n)(x_n-\mu_n)] \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{x_1}^2 & C_{x_1 x_2} & \cdots & C_{x_1 x_n} \\ C_{x_2 x_1} & \sigma_{x_2}^2 & \cdots & C_{x_2 x_n} \end{bmatrix}$$

$p \to$ probability distribution
$P \to$ probability

$x_1, x_2 \cdots x_n$

$\mu_1, \mu_2 \cdots \mu_n$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \underline{\mu} = \begin{bmatrix} \mu_1 \\ \circ \\ \circ \\ \cdot \\ \mu_n \end{bmatrix}$$

$$C_{xx} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & & \vdots \\ \mu_{n1} & \vdots & & \mu_{nn} \end{bmatrix}$$

$$\mu_{ij} = E_x[(x_i - \mu_i)(x_j - \mu_j)]$$

$\mu_{ij} = \mu_{ji}$

$\mu_{ii} = \sigma_i^2$ Symmetric Matrix

$$F_y(y) = P(Y \le y)$$

$$y = ax + b$$

$$< P(ax + b \le y)$$

$$P\left(x \le \frac{y-b}{a}\right) = F_x\left(\frac{y-b}{a}\right)$$

$$F_y(y) = F_x\left(\frac{y-b}{a}\right)$$

$$p_y(y) = \frac{d}{dy} F_y(y)$$

$$= \frac{d}{dy}\left[F_x\left(\frac{y-b}{a}\right)\right]$$

$$= \frac{1}{a} P_x\left(\frac{y-b}{a}\right)$$

multiple r.v

$$x = [x_1 \cdots x_n]$$

$$y_i = g_i(\underline{x})$$

$$g_i^{-1}(y_i) = x_i$$

$$x = f_i(y)$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \frac{\partial f_2}{\partial y_2} & \cdots & \frac{\partial f_n}{\partial y_1} \\ \frac{\partial f_1}{\partial y_2} & & \cdots & \frac{\partial f_n}{\partial y_2} \\ \vdots & & & \\ \frac{\partial f_1}{\partial y_n} & & \cdots & \frac{\partial f_n}{\partial y_n} \end{bmatrix}_{n \times n}$$

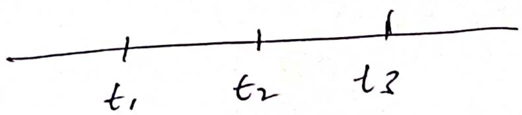
$$\cancel{P_Y(y) = P_X(y) \times |J|}$$

$$P_Y(y) = P_X[f(y)]\,|J|$$

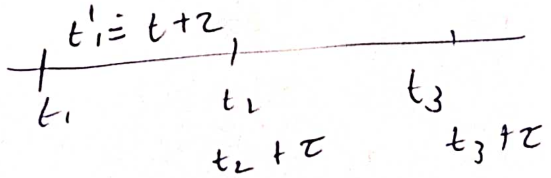
Jacobian Matrix

Discrete Random process

$X(t)$ $\qquad t_1 < t_2 < t_3$

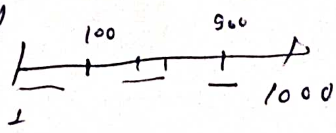


$P(X_{t_1}, X_{t_2}, X_{t_3})$



$P(X_{t'_1}, X_{t'_2}, X_{t'_3})$

if PDf are equal then we call random process strictly stationary



$E[x(n)] = \mu \rightarrow$ first order stationary

Correlation

$$r_{xy} = E[XY]$$

$$r_{xy}(k) = E[x(n)y(n-k)$$

1st & 2nd by kunil

discrete $\phi(t_1 t_2) = E(x_{t_1} y_{t_2})$

Auto correlation

$$R_x(k) = E[x(n)x(n-k)]$$

$$\phi_{xx}(t_1 t_2) = E(x(t_1)\, x(t_2))]$$

$$\phi_{xx}(t_1, t_1+\tau) = E[x(t)x(t-\tau)]$$

$$= \phi(\tau),\ R_{xx}(k)$$

~~system~~.
1st order - μ const
2n - correlⁿ

$R_{xx}(k)$ corrlⁿ depend only on time gap or time delay τ in cont signal

R in discrete

$$\phi_{xx}(\tau) = \phi(\tau) \quad - 2^{nd} \text{ order stationary}$$

If any random process ~~delay~~ ~~obeys~~ obeys.

- 1st order stationary

$$E[x(k)] = \mu_x$$

$$E[x(t)] = \mu_x$$

LPC

and 2nd order

$$\phi(\tau) = E[x(t)x(t-\tau)]$$

$$r_{xx}(k) = E[x(n)x(n-k)]$$

if both satisfy then we call process is called WSS - wide sense stationary (by default)

IID - Independent & Identical Distribution.

$x_1, x_2 \dots x_n \rightarrow x_1 <$ mean $= \mu$, variance $= \sigma^2$

$x_2 <$ mean μ, var σ^2

~~covariance~~

$$E[x(n)x(n-k)]$$

$$= N\delta(k)$$

$$k=0, \; E[x(n)^2] = \sigma^2$$

$$r_{xx}(k) = N\delta(k)$$

$$P(x_1, x_2 \dots x_n) = p(x_1)p(x_2) \dots p(x_n)$$

~~$x_1, x_2 \dots$~~

Uncorrelated $\Rightarrow$ covariance $= 0$

Independence $=$ corr^n $= 0$

#

$x_1 \quad x_2 \quad \dots \quad x_n$

$x_i \sim N(0, \sigma^2)$

$N(\mu, \sigma^2)$

$$M = \begin{bmatrix} \sigma^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma^2 & 0 & \cdots & 0 \\ 0 & & \cdots & & \\ & & & \sigma^2 & 0 \\ 0 & 0 & \cdots & & \sigma^2 \end{bmatrix} = \sigma^2 I_n$$

$$r_{11} = E[x_1 x_1]$$

$$R_{xx}(k) = \frac{N}{2}\delta(x)$$

$$, \sigma^2$$

$$= E[|x_1|^2]$$

$$= \sigma_1^2 = r$$

$$P_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$P_{x_1 x_2 \cdots x_n}(x_1, x_2, \cdots x_n)$$

General

M = covariance matrix

$$P_{x_1 x_2 \cdots x_n}(x_1, x_2 \cdots x_n) = \frac{1}{[2\pi \det[M]]^{\frac{N}{2}}} e^{-\underset{1\times n}{(x-u)^t} \underset{n\times n}{M^{-1}} \underset{n\times 1}{(x-u)}}$$

IID

$$P_{x_1 x_2 \cdots x_n}(x_1, x_2 \cdots x_n) = P_{x_1}(x_1) P_{x_2}(x_2) \cdots P_{x_n}(x_n)$$

$$= \left[\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-u)^2}{2\sigma^2}} \cdots \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-u)^2}{2\sigma^2}}\right.$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{N}{2\sigma^2}(x-u)^2}$$

Power Spectral Density

$$\phi(\tau) = E[x(t)\, x(t-\tau)]$$

FT of $\phi(t)$

$$\phi(f) = \int_{-\infty}^{\infty} \phi(\tau)\, e^{-2\pi f \tau}\, d\tau$$

$$\phi(\tau) = \int_{-\infty}^{\infty} \phi(f)\, e^{j2\pi ft}\, df$$

$$\phi(0) = \int_{-\infty}^{\infty} \phi(f)\, e^{0}\, df$$

Small

$$= \int_{-\infty}^{\infty} \phi(f)\, df$$

$$\phi(0) = E[|x(t)|^2]$$

= power ~~spectral density~~ of signal

$\phi(f) \rightarrow$ is called PSD / FT of $\phi(\tau)$ / FT of Autocorrln

$$\phi(\tau) = E[x(t)\, x(t-\tau)]$$

$$\phi(-\tau) = E(x(t)\, x(t+\tau)]$$

$$= E[x(t-\tau)\, x(t)]$$

$$= \phi(\tau)$$

for Complex Signal

$$\phi(t_1, t_2) = \frac{1}{2} E[x(t_1)\, x^*(t_2)]$$

$$= \frac{1}{2} E[x(t)\, x^*(t-\tau)]$$

Q: What is PSD of IID distributed random process.

$R_{xx}(k) = \frac{N_0}{2}\, \delta(k)$

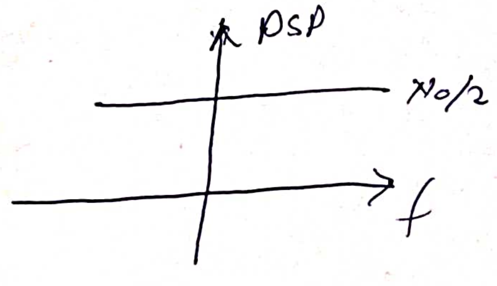
$\phi(\tau) = \frac{N_0}{2}\, \delta(\tau)$

PSD $\Phi(f) = \int_{-\infty}^{\infty} \phi(\tau)\, e^{-j2\pi f \tau}\, d\tau$

$= \int_{-\infty}^{\infty} \frac{N_0}{2}\, \delta(\tau)\, e^{-j2\pi f \tau}\, d\tau$

$= \frac{N_0}{2}$ costant

$= 10^{-5}$ watt/Hz



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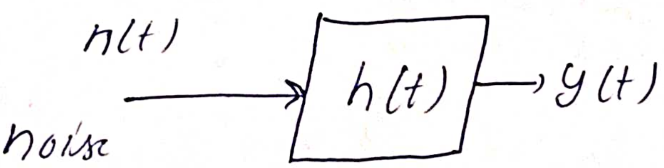


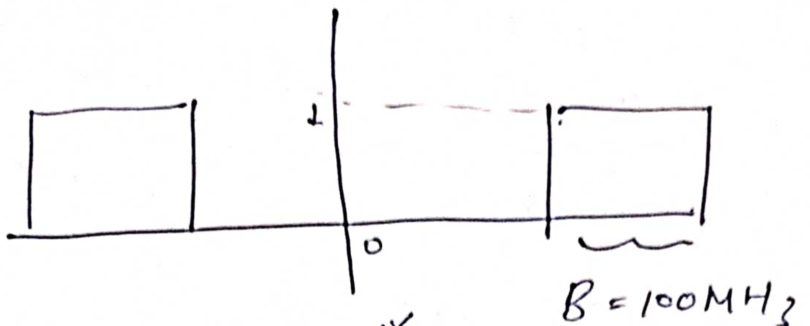
$\phi_y(f) = |H(f)|^2\, \phi_n(f)$

Q: A Noise signal having PSD 10^{-12} watt/Hz given to a ideal BPF with 100 MHz BW find output power

10^{-11}

100×10^{6}

10^{8}



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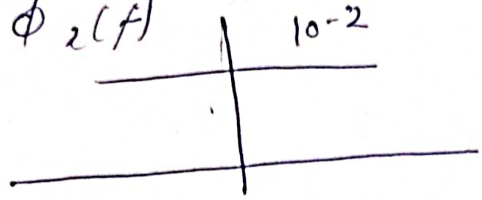
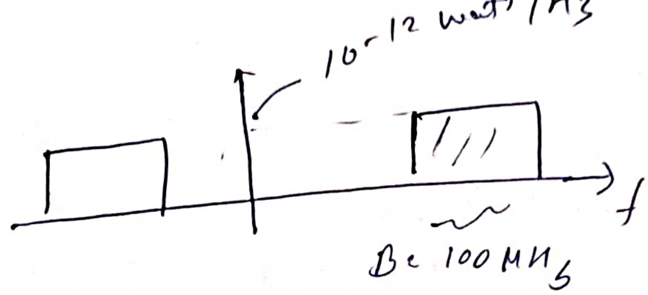




$$P_o = \int_{-\infty}^{\infty} \phi(f)\,df$$

$$= 2 \times 10^{-12} \times 10^{8}$$

$$P = 2 \times 10^{-4} \text{ watt}$$

$$P = 0.20 \text{ mW}$$

Noise is uncorrelate PSD

Question: Prove that PSD $\phi(e^{j\omega})$ of a signal $x(n)$ is real for (real valued signal)

$$\phi(e^{j\omega}) = \text{DTFT of } \cancel{x(n)}\ R_{xx}(k)$$

$$= \sum_{k=-\infty}^{\infty} R_{xx}(k)\, e^{-j\omega k}$$

$$= R_{xx}(0) + \sum_{k=1}^{\infty} R_{xx}(k)\, e^{-j\omega k} + \sum_{k=-1}^{-\infty} R_{xx}(k)\, e^{-j\omega k}$$

$$= \text{''} + \text{''} + \sum_{k=1}^{\infty} R_{xx}(-k)\, e^{j\omega k}$$

$$= \text{''} + \text{''} + \left(\sum_{k=1}^{\infty} R_{xx}(k)\, e^{-j\omega k}\right)^{*}$$

Hemissian Symmetry

$$R_{xx}(-k) = E[x(n)\, x(n+k)]$$

$$= E[x(n-k)\, x(k)] = R_{xx}(k)$$

$$R_{xx}(k) = R_{xx}^{*}(-k)$$

$$\phi(e^{j\omega}) = R_{xx}(0) + \underline{\sum_{k=1}^{\infty} R_{xx}(k) e^{-j\omega k}} + \left(\sum_{k=1}^{\infty} \{R_{xx}(k)\, e^{-j\omega k}\right)^{*}$$

$$= R_{xx}(0) + 2\sum_{k=1}^{\infty} Re[R_{xx}(k) e^{-j\omega k}]$$

Q: 2: A_i $i = 1,2,3,4$ $B_j = 1,2,3$ → outcome

Event 1 E 2

$P(A_1, B_1) = 0.1$ $P(A_1, B_2) = 0.08$ $P(A_1, B_3) = 0.13$

$P(A_2, B_1) = 0.05$ $P(A_2, B_2) = 0.03$ $P(A_2, B_3) = 0.09$

$P(A_3, B_1) = 0.05$ $P(A_3, B_2) = 0.12$ $P(A_3, B_3) = 0.14$

$P(A_4, B_1) = 0.11$ $P(A_4, B_2) = 0.04$ $P(A_4, B_3) = 0.06$

Determine marginal probability $P(A_i)$ $i = 1,2,3,4$ & $P(B_j)$ $j = 1,2,3$

Soln

$$P(A_i) = \sum_{j=1}^{3} P(A_i, B_j) \quad i = 1,2,3,4$$

$$P(A_1) = \sum_{j=1}^{3} P(A_1, B_j) = P(A_1, B_1) + P(A_2, B_2) + P(A_3, B_3)$$

$$= 0.31$$

$P(A_2) = 0.17$ $P(A_3) = 0.31$ $P(A_4) = 0.21$

$$P(B_j) = \sum_{i=1}^{4} P(A_i, B_j)$$

$$P(B_1) = \sum_{i=1}^{4} P(A_i, B_1) = 0.1 + 0.05 + 0.05 + 0.11 = 0.31$$

$P(B_2) = 0.27$ $\quad$ $P(B_3) = 0.42$

$H/W \rightarrow P(A_i | B_j)$

Q: Consider a r.v Y defined as $Y = ax^3 + b$ $a>0$ where x is a Gaussian r.v with pdf $P_x(x)$. Determine PDF of Y in terms of pdf of X

Sol^n.
$$F_Y(y) = P(Y \le y)$$
$$= P(ax^3 + b \le y)$$
$$= P\left(X \le \left(\frac{y-b}{a}\right)^{1/3}\right)$$
$$= F_X\left[\left(\frac{y-b}{a}\right)^{1/3}\right]$$

Now
$$p_Y(y) = \frac{d}{dy} F_X(y) = \frac{d}{dy} F_X\left[\left(\frac{y-b}{a}\right)^{1/3}\right]$$
$$= P_X\left[\left(\frac{y-b}{a}\right)^{1/3}\right] \frac{d}{dy}\left(\frac{y-b}{a}\right)^{1/3}$$
$$= \frac{1}{3a\left[\frac{y-b}{a}\right]^{2/3}} \times P_X\left[\left(\frac{y-b}{a}\right)^{1/3}\right]$$

Q: X_R, X_i (Real, Imag) - statistical independent gaussian r.v

Rotational transformation of form
$$Y_R + jY_i = (X_R + jX_i)e^{j\phi}$$

results another pair (Y_R, Y_i) of gaussian r.v

Show that (Y_R, Y_i) pair has same joint pdf as (X_R, X_i)

Sol^n. $p(X_R, X_i) = P_X(X_R)\,p(X_i)$ $\because$ Independent

$$p_X(x_R) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x_R^2}{2\sigma^2}} \quad \because \text{assume } \mu = 0$$

$$p_X(x_R, x_i) = \frac{1}{2\pi\sigma^2} \cdot e^{-\left(\frac{x_R^2 + x_i^2}{2\sigma^2}\right)}$$

$$y_R + j y_i = (x_R + j x_i) e^{j\phi}$$

$$x_R + j x_i = (y_R + j y_i) e^{-j\phi}$$

$$= (y_R + j y_i)(\cos\phi - j\sin\phi)$$

$$x_R = y_R \cos\phi + y_i \sin\phi$$

$$x_i = -y_R \sin\phi + y_i \cos\phi$$

$$J = \begin{vmatrix} \frac{\partial x_R}{\partial y_R} & \frac{\partial x_i}{\partial y_R} \\ \frac{\partial x_R}{\partial y_i} & \frac{\partial x_i}{\partial y_i} \end{vmatrix} = \begin{vmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{vmatrix} = 1$$

$$p_Y(y_R, y_i) = \frac{1}{\cancel{|J|}1} p_X(y_R\cos\phi + y_i \sin\phi, -y_R \sin\phi + y_i \sin\phi)$$

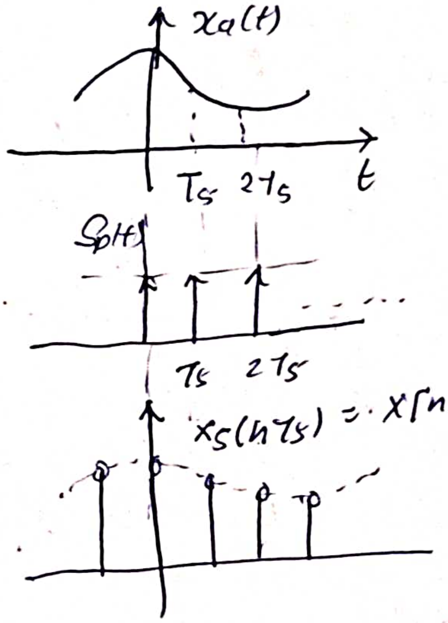
$$\Rightarrow \frac{1}{2\pi\sigma^2} e^{-\left[(y_R\cos\phi + y_i\sin\phi)^2 + (-y_R\sin\phi + y_i\sin\phi)^2\right]}$$

$$\Rightarrow \frac{1}{2\pi\sigma^2} e^{-\frac{(y_R^2 + y_i^2)}{2\sigma^2}}$$

2nd part

I. G. Prakish → 2nd Chapter
Simon Hyking + Sampling

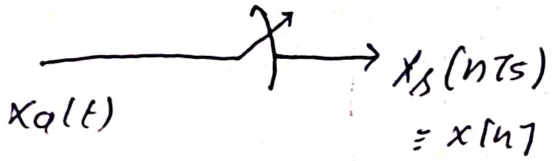
Sampling : Continous time signal $\xrightarrow{\text{Sampling}}$ Discrete Time Signal
↳ Discretizating time



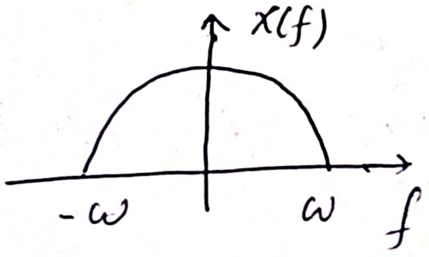
uniform sampling

T_s : Sampling period

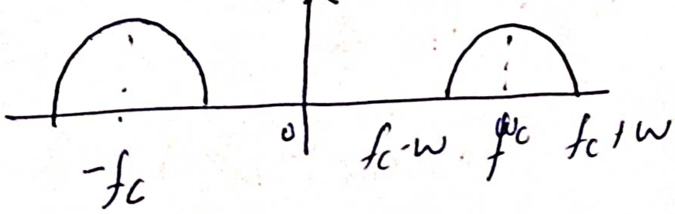
$\frac{1}{T_s} = f_s$: Sampling freq.



Question 1) Can we ~~rec~~ recover the original signal
2) Is their any loss of information.
loss can be minimize if signal is Bandlimited.



$|X(f)| = 0 \quad |f| \geq W$



Nyquiest Theorem / Rate

$f_s \geq 2W$ if sampling freq satisfy this condn then we can recover the original signal from sampled sign

$f_s = 2W$ [critical rate]

· DC Trilochan Sir

(15)

Q: The Covariance matrix of r.v is X_1, X_2, X_3 is

$$\begin{bmatrix} \mu_{11} & 0 & \mu_{13} \\ 0 & \mu_{22} & 0 \\ \mu_{31} & 0 & \mu_{33} \end{bmatrix}$$ The linear transformation $\underline{Y} = A\underline{X}$ $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

find covariance matrix of y

Sol^n $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\mu_x = \begin{bmatrix} \mu_{x_1} \\ \mu_{x_2} \\ \vdots \\ \mu_{x_3} \end{bmatrix}$

Covarian matrix $M_X = E[(\underline{x} - \mu_x)(x - \mu_x)^T]$

$$M_y = E[(Y - \mu_y)(Y - \mu_y)^T]$$

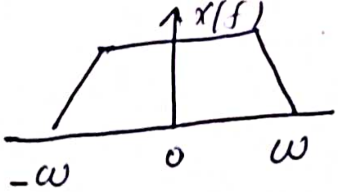
$$= E[(AX - A\mu_x)(AX - A\mu_x)^T]$$

$$\Rightarrow M_y = E[A(X - \mu_x)(X - \mu_x)A^T]$$

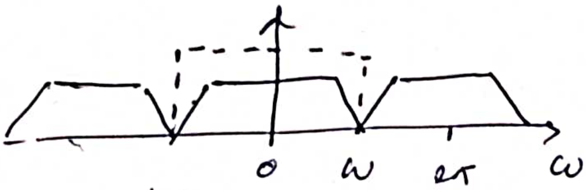
$$= A\, E[(X - \mu_x)(X - \mu_x)^T] A^T$$

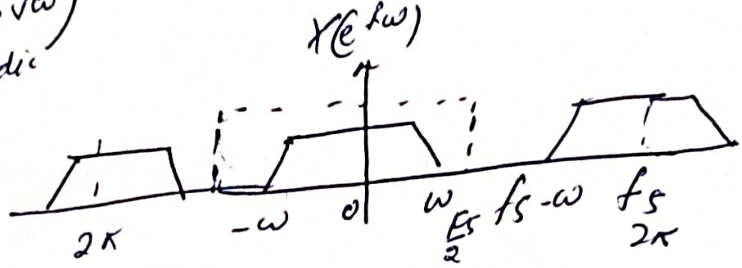
$\boxed{\text{S-6 Eg Sum}}$

$x_a(t) \xleftarrow{F^{-1}} X(f)$



$x[n] \xleftrightarrow{DTFT} X(e^{j\omega})$ periodic





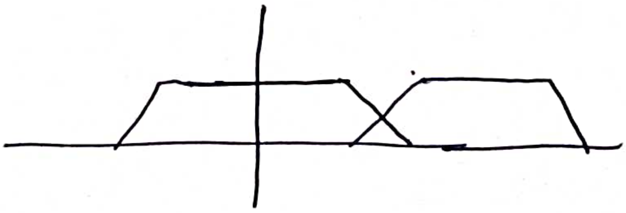
Critical

$f_s = 2W$

$\frac{f_s}{2} = W$

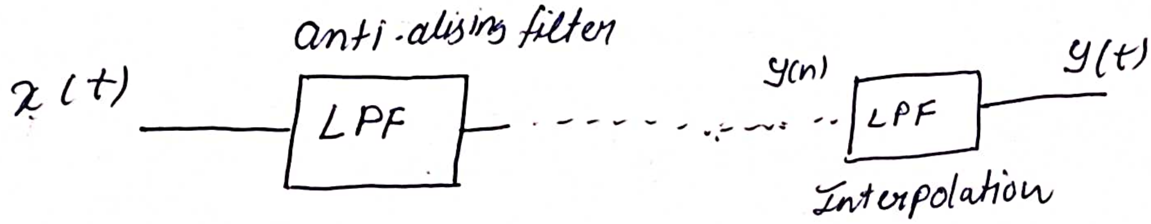
$f_s - W \geq W$

$\Rightarrow \boxed{f_s \geq 2W}$



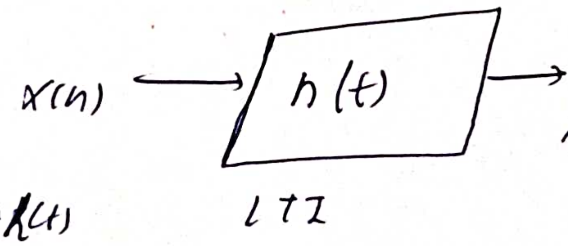
$\Leftarrow f_s < 2W$

↳ aliasing



$x_s[n] \xleftrightarrow{LPF} \hat{x}(t)$ $\quad x_s(t) = \sum_{-\infty}^{\infty} \delta(t - nT_s)$

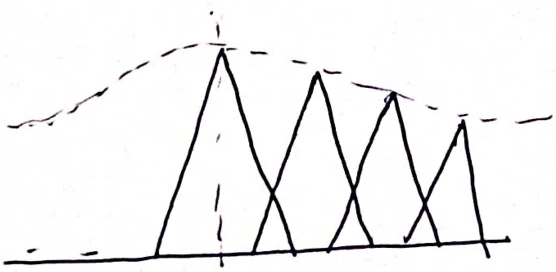
$$x[n] = \sum_{n=-\infty}^{\infty} x[nT_s]\,\delta(t - nT_s)$$ Ideal sampling



$\hat{x}(t) = \sum x(nT_s)\,h(t - nT_s)$

Let

$\longrightarrow$



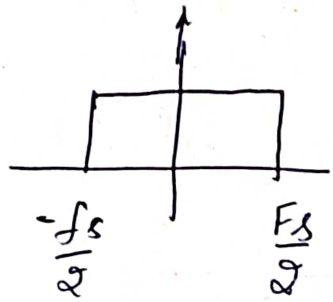
$\hat{x}(t) = \sum x(nT_s) h(t - nT_s)$

$h(t) = ?$ find ~~Satisfy~~ such that $\hat{x}(t) = x(t)$

$\hat{x}(f) = X(f)$

$h(t) = \text{sinc}(f_s t) = \dfrac{\sin \pi f_s t}{\pi f_s t}$

$H(f) =$



$\hat{x}(f) = \mathcal{F}\left(\sum \underbrace{x(nT_s)}_{\text{Constant}} h(t - nT_s)\right)$

$= \underbrace{\sum_{-\infty}^{\infty} x[n] e^{-j2\pi \left(\frac{f}{f_s}\right) n}}_{X(e^{j\omega})}$

$= X(f)$

Ex Undersampling

$f_m = 4kHz$

$f_o = ?$ $2kHz$

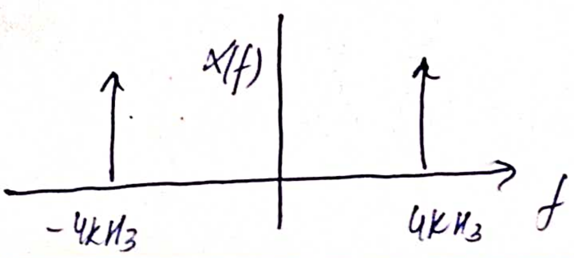
Sinusoidal

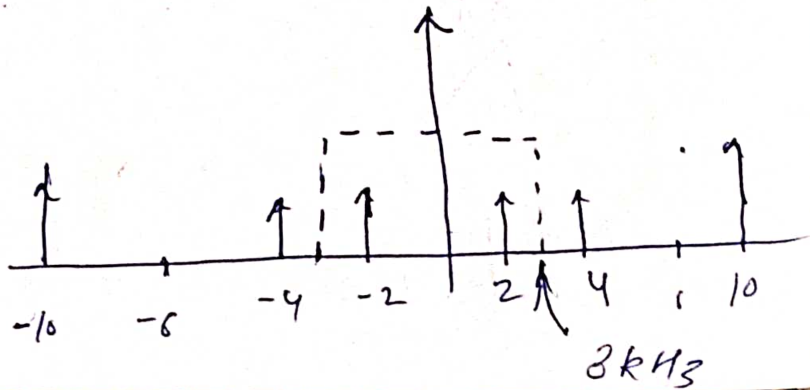
$f_s = 6kHz$

$f_c = \frac{f_s}{2} = \frac{6kHz}{2}$

Supposed to 8kHz

$x(t) = A \cos(2\pi f_m t)$





↳ Anti alising filter used to check sampling freq.

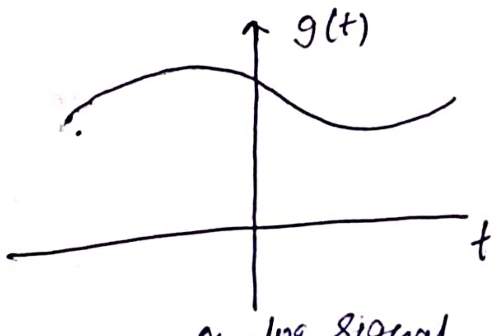
Practical Aspect of Sampling

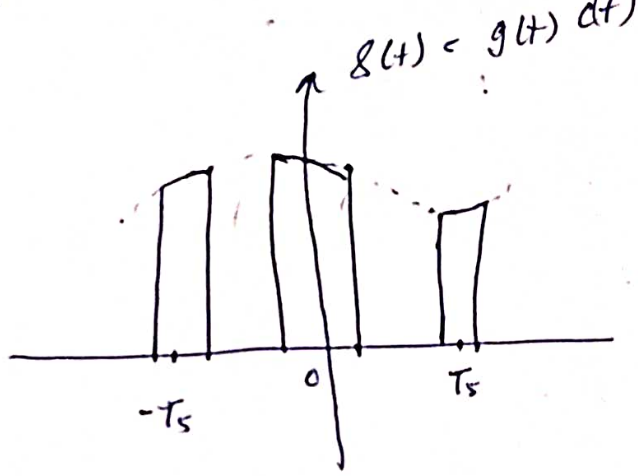
↳ sampling is accomplished by means of high speed switching transition circuit

↳ practically the switching time can not be zero !!!.

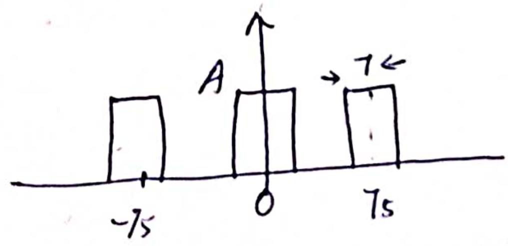
type of sampling

(1) Natural Sampling





(a) Analog Signal



Sampling function

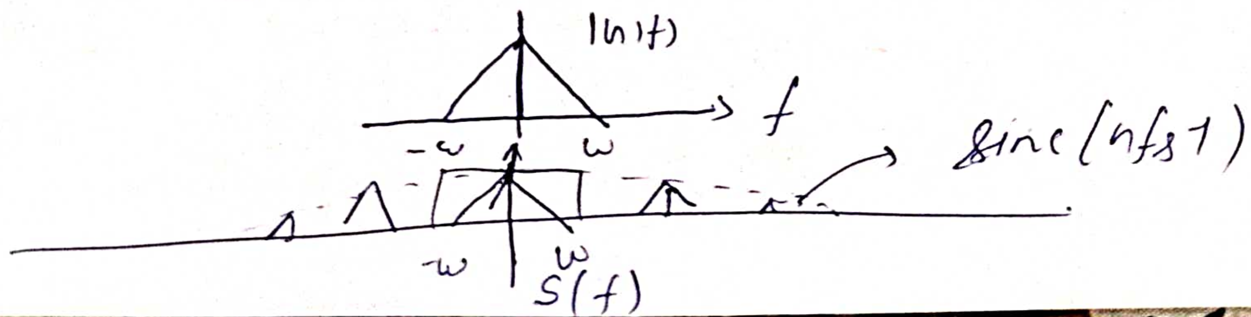
$c(t)$ = fourier Series

$\rightarrow f_s \tau A \cdot \sum C_n e^{j 2\pi n f_s t}$

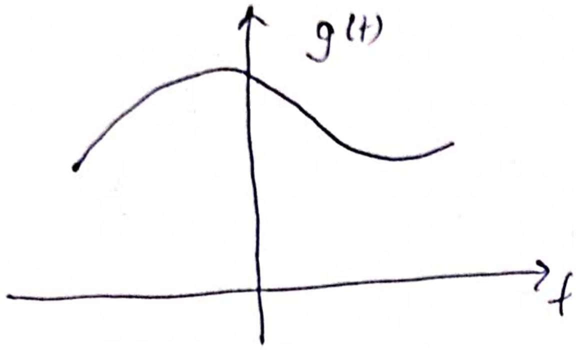
$C_n = \text{sinc}(n f_s \tau)$

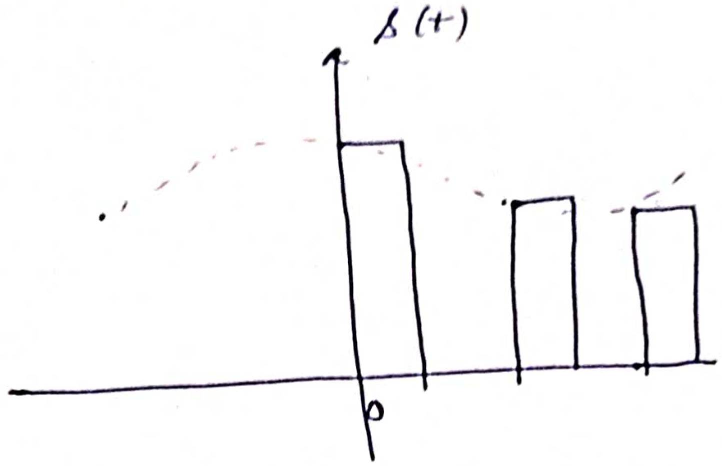
$\rightarrow s(t) = f_s \tau A \sum_{-\infty}^{\infty} \text{sin}(n f_s \tau) e^{j 2\pi n f_s t} g(t)$

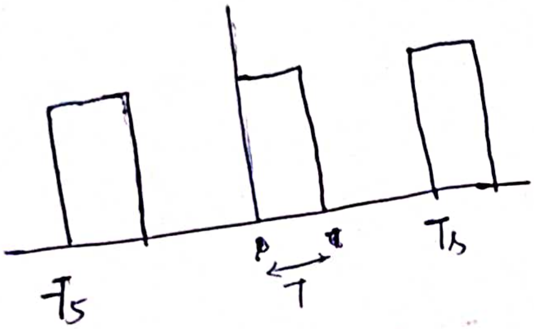
$\Rightarrow S(f) = f_s \tau A \sum_{-\infty}^{\infty} \text{sinc}(n f_s \tau) \cdot G(f - n f_s)$ $\quad \tau A = 1$

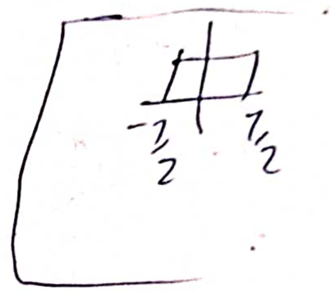


Q flat-Top Sampling / PLOSE Amplitude modulation / Sample & Hold (S/H)









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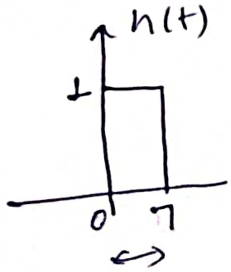


$$h(t) = \text{rect}\left(\frac{t - T/2}{T}\right)$$

$$= \text{rect}\left(\frac{t}{T} - \frac{1}{2}\right)$$

$$h(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{other} \end{cases}$$

* Ideal sampling / Impulse sampling

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)$$

$$G_\delta(f) = f_s \sum_{-\infty}^{\infty} G(f - nf_s)$$

$$g_s(t) = \sum_{-\infty}^{\infty} g_\delta(t) \circledast h(t)$$ ← flat top sampling

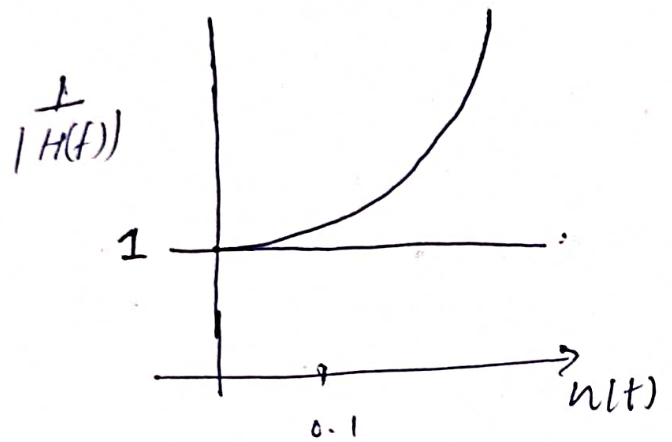
$$= \sum g(nT_s)\,\delta(t - nT_s) \circledast h(t)$$

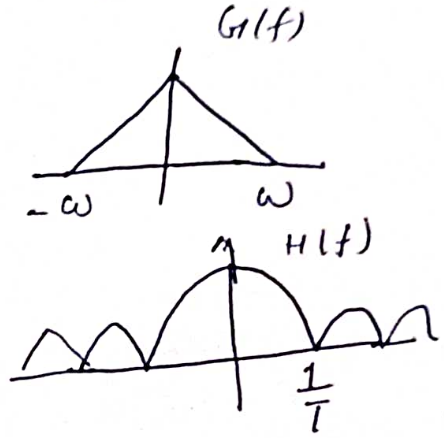
$$= \sum_{-\infty}^{\infty} g(nT_s)\,h(t - nT_s)$$

$$S(f) = G_s(f)\,H(f)$$

$$\Rightarrow f_s \sum H(f)\, G(f - nf_s)$$

$$H(f) = T \operatorname{sinc}(fT)\, e^{-2\pi \frac{fT}{2}}$$





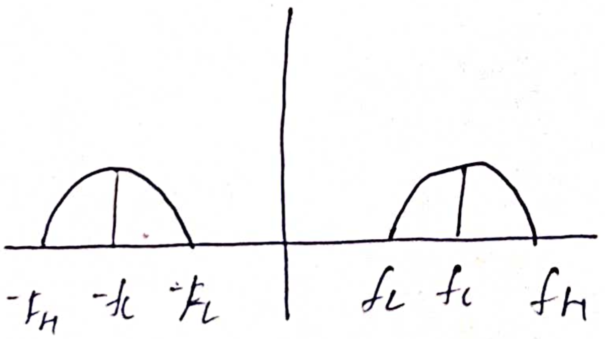
↳ allowed duty cycle 10% $= \frac{T}{T_s} = 0.1$

Sampling of Band pass Signal

Sampling Range: f_H = highest freq

f_L = Lowest freq

$B = f_H - f_L$



$f_s = 2B$

If f_H or f_L is one of the Harmony of f_s

$f_H = 100\ MHz$

$f_L = 90\ MHz$

$B = 10\ MHz$

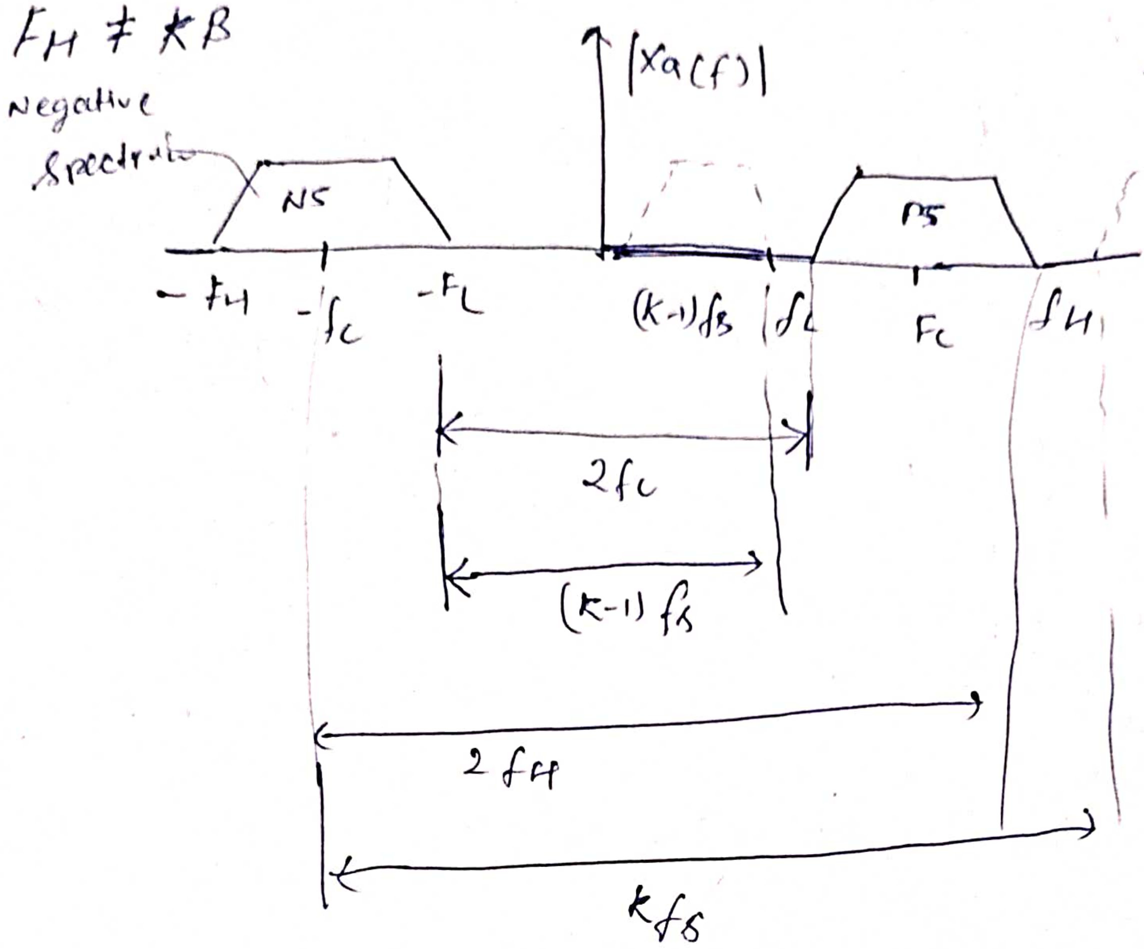
Minimum Sampling frequency allowed is

$$F_s = 2(f_H - f_L)$$

provided that either f_L or f_H is harmonic of f_s

$F_H \neq kB$

$f_s << f_c$



$$2f_L \geq (k-1) f_s$$

$$2F_H \geq k f_s$$

$$\Rightarrow \boxed{\frac{2f_H}{k} \leq f_s} \leq \frac{2f_L}{k-1} \qquad \therefore f_L = f_H - B$$

$$\frac{1}{f_s} \leq \frac{k}{2f_H} \qquad \Rightarrow \quad (k-1) \leq \frac{k}{2f_H} \cancel{2}(f_H - B)$$

$$(k-1) f_s \leq 2 f_L \Rightarrow (k-1) f_s \leq 2(f_H - B)$$

$$\boxed{k \leq \frac{f_H}{B}}, \quad \boxed{k_{max} = \frac{f_H}{B}}$$

$$\Rightarrow \frac{k-1}{k} \leq \frac{f_H - B}{f_H} \Rightarrow \cancel{1} - \frac{1}{k} \leq \cancel{1} - \frac{B}{f_H} \Rightarrow \frac{1}{k} \geq \frac{B}{f_H} \Rightarrow \boxed{k \leq \frac{f_H}{B}}$$

$$\therefore \quad \boxed{k_{max} = \frac{f_H}{B}}, \quad \boxed{f_s \geq \frac{2f_H}{k}}$$

Band pass Sampling Theorem

A Band pass signal with Higest freq f_H and Bandwidth B can be recovered from its samples through a BPF filtering by sampling it with frequency

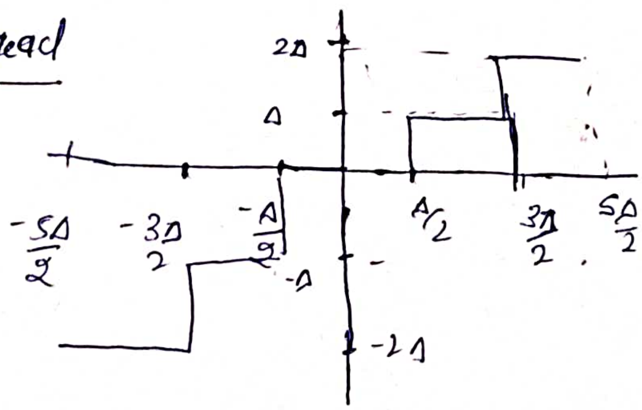
$$f_s = \frac{2f_H}{k}$$ where k is the largest integer Not exceeding f_H/B

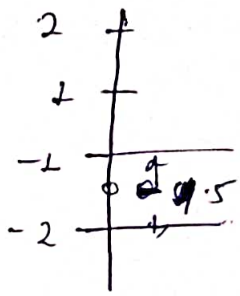
$\boxed{\text{Summery - 4 sampling Ideal, Normal, flat top, BP}}$

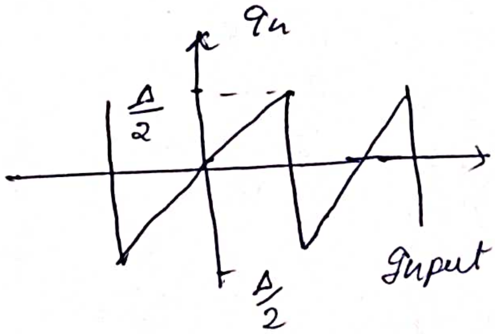
Quantization

↳ Descritizing Sample in Amplitude

Midtread

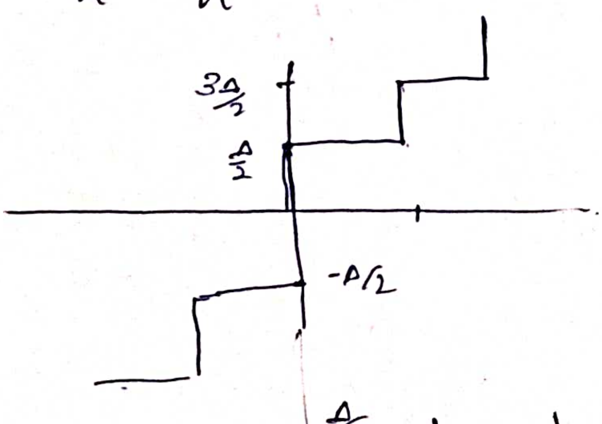




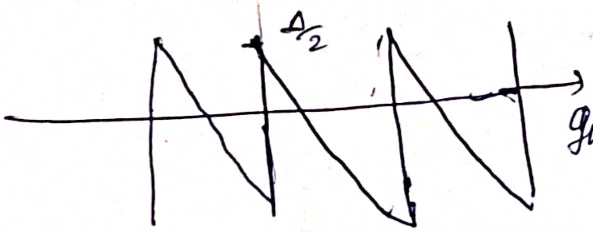


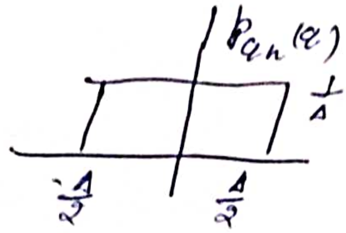
$$\tilde{x} = x_n + q_n$$

Quantization Error. $q_n = \tilde{x}_n - x_n$



Mid Rize
↳ origin mid b/w rize path





Max^m quantization Error = $\Delta/2$

Range $-\frac{\Delta}{2}$ to $\frac{\Delta}{2}$

$$p_{q_n}(q) = \text{pdf of } q_n = \begin{cases} \frac{1}{\Delta} & -\frac{\Delta}{2} \le q_n \le \frac{\Delta}{2} \\ 0 & \text{otherwise} \end{cases}$$

Mean of $q_n = 0$

i.e. mean sqr Error

$$MSE = \sigma_{q_n}^2 = E(q_n^2) = \int_{-\Delta/2}^{\Delta/2} q^2 \, p(q)\, dq$$

$$= \int_{-\Delta/2}^{\Delta/2} q^2 \times \frac{1}{\Delta}\, dq = \frac{\Delta^2}{12}$$

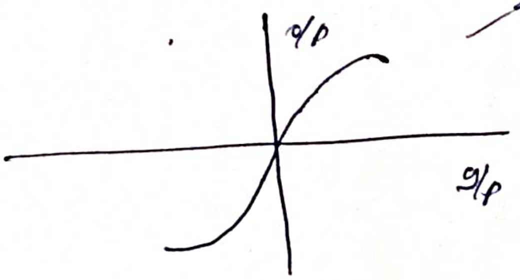
less Δ → Bits ↑↑

$$\boxed{\sigma_{q_n}^2 = \frac{\Delta^2}{12}}$$

Non-linear Quantization

Compander

$x(t)$ → Compression — Uniform quantization — Expander

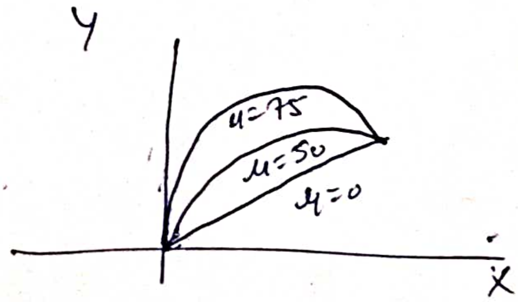


A-low- European

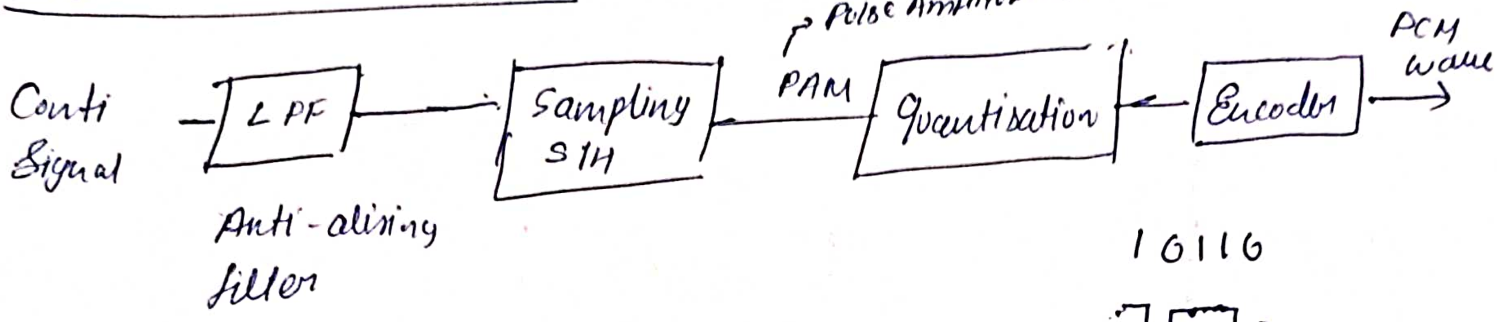
μ-low → American

$$|y| = \frac{\log(1+\mu|x|)}{\log(1+x)} \operatorname{Sin}(x)$$

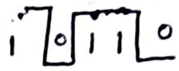
$0 \le \mu \le 255$

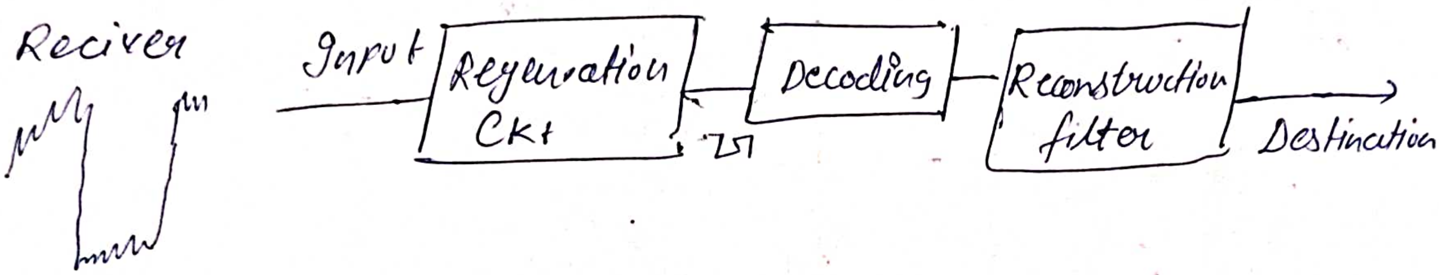


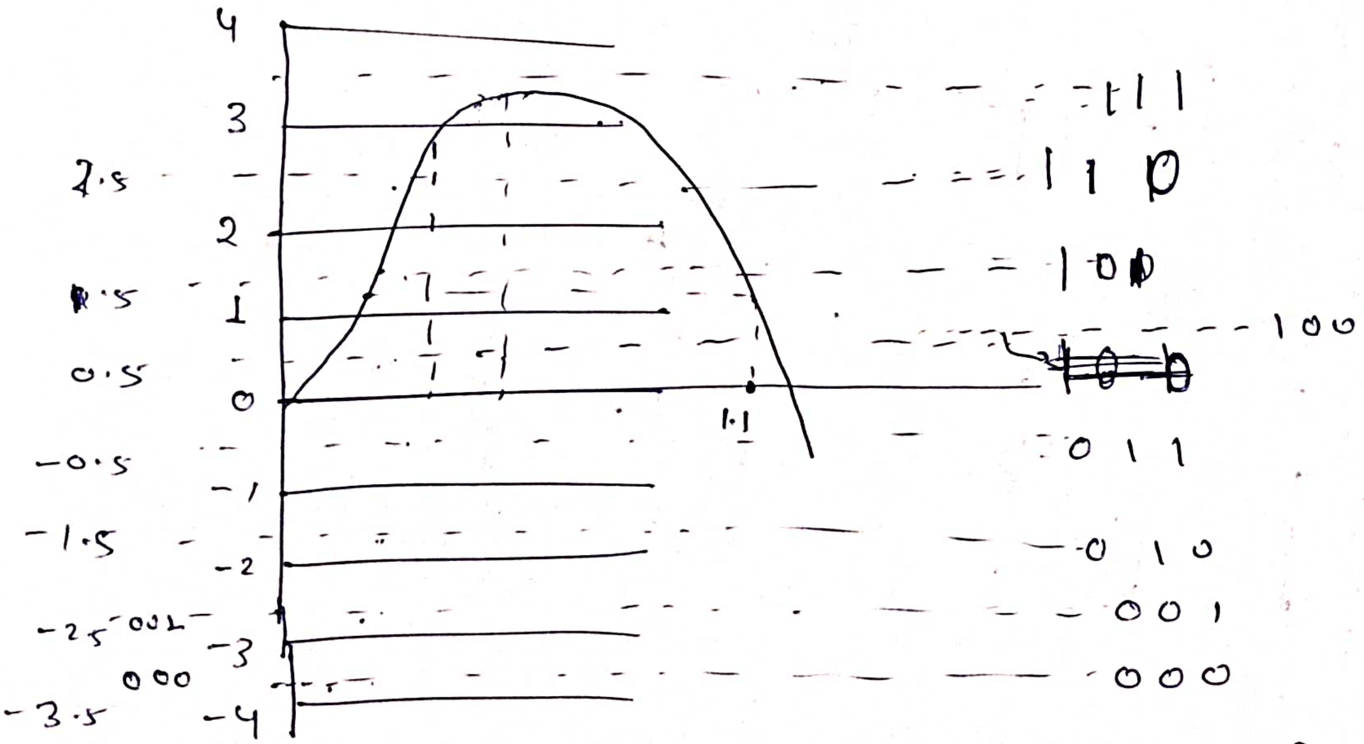
Pulse Code Modulation PCM



10110







$x_n = 0$ $\hat{x}_n = 0.5$ code = 100

$x_n = 2.8V$ $\hat{x}_n = 2.5$ code = 110

$x_n = 3.2V$ $\hat{x}_n = 3.5$ code = 111

$x_n = 1.1V$ $\hat{x} = 1.5$ code = 101

Line coding

Eg Audio Signal

Speech Signal

0.3 – 3.3 kHz

$f_s = 8\,kHz$ 8-bit PCM 256-levels

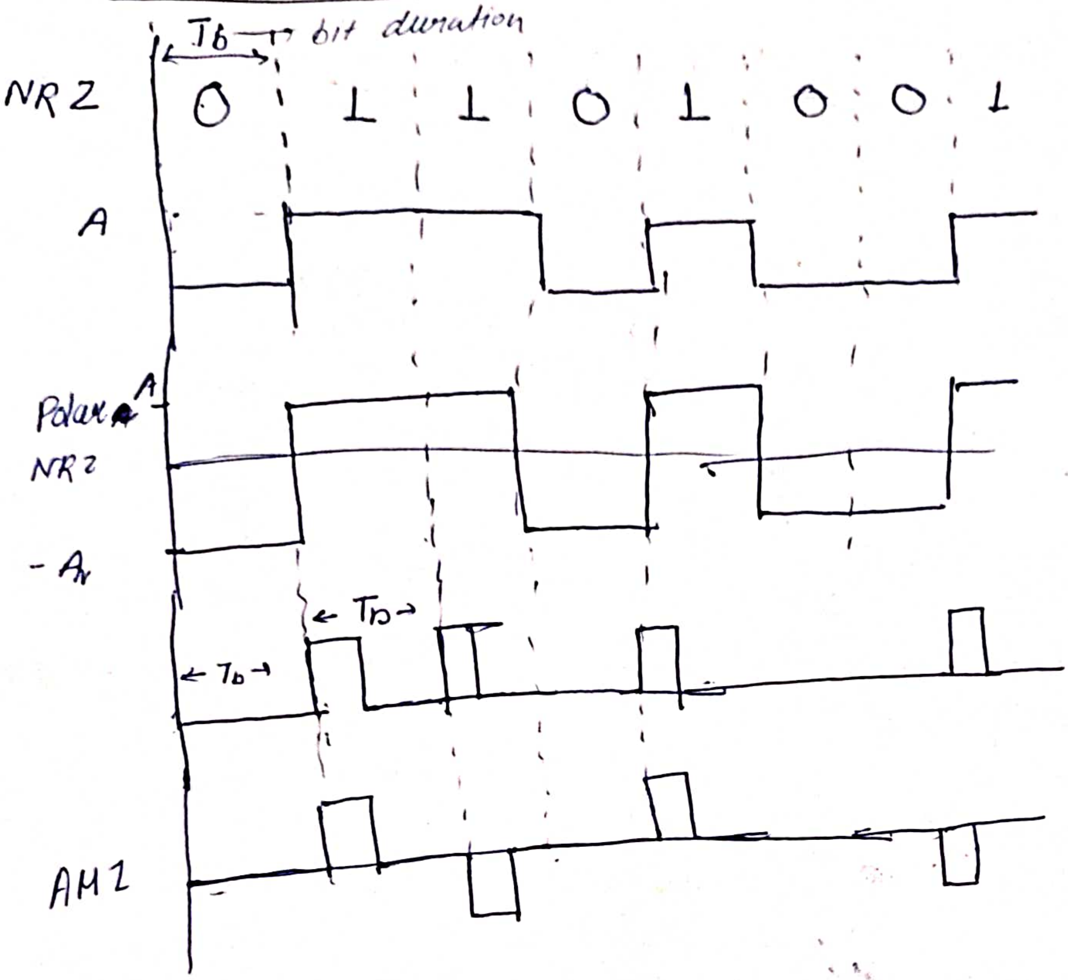
Output bit rates = $R = 8\,kHz \times 8\,bit$

$= 64\,k\,bit/sec$

$= 64\,kbps$

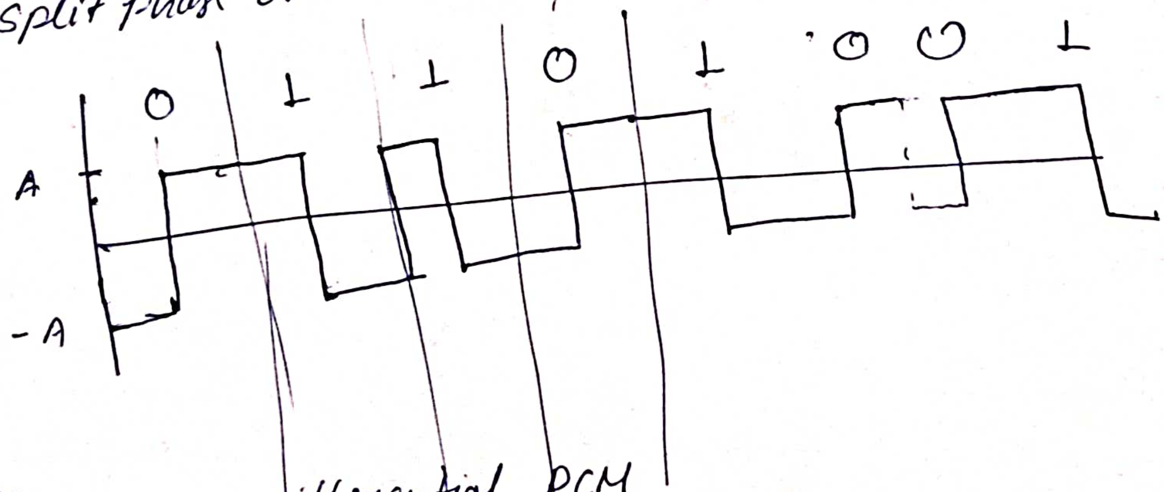
Line Coding

NRZ
Non Return to zero



Alternate mark Inversion
or
Bipolar return to Zero

Split phase or manchester Code



DPCM - differential PCM

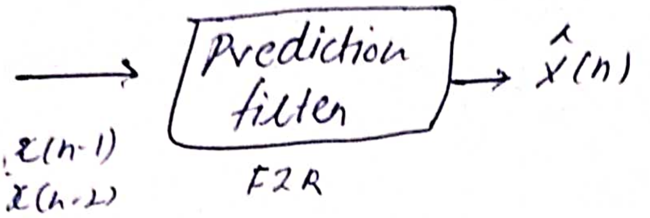
PCM — $R = nf_s$

n = no of bits used to Encode each Sample

f_s = Sampling freq.

$$e(n) = m(nT_s) - m[(n-1)T_s]$$

$$m(n-1) + e(n) = m(n)$$



prediction ~~present symbol~~ signal from past

$$e(n) = m(n) - \hat{m}(n)$$ predicted.

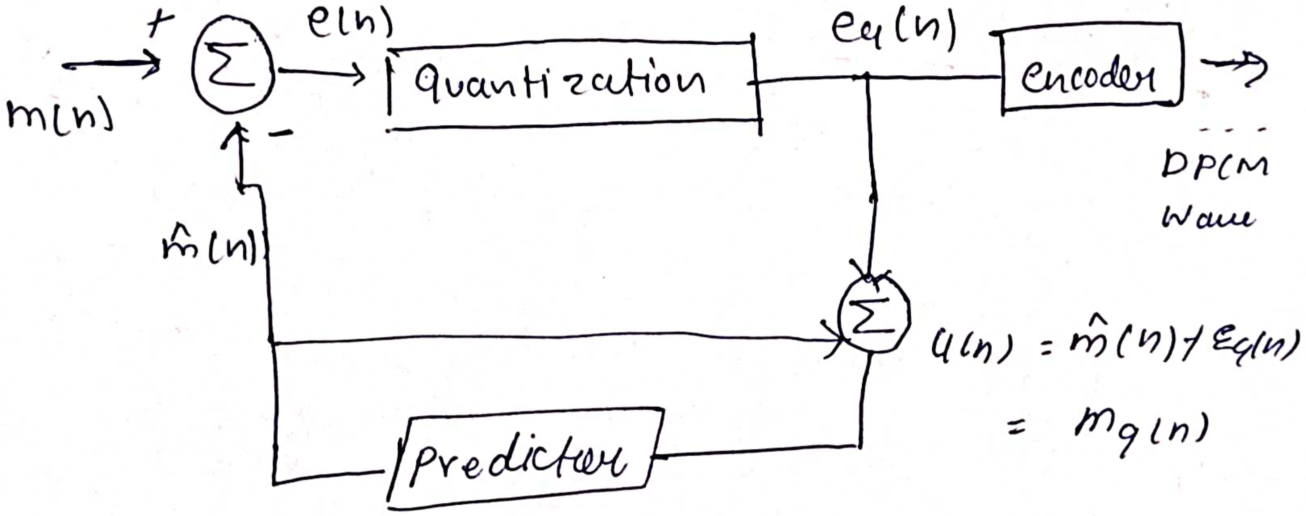
$$e_q(n) = e(n) + q(n)$$

Quantization Error

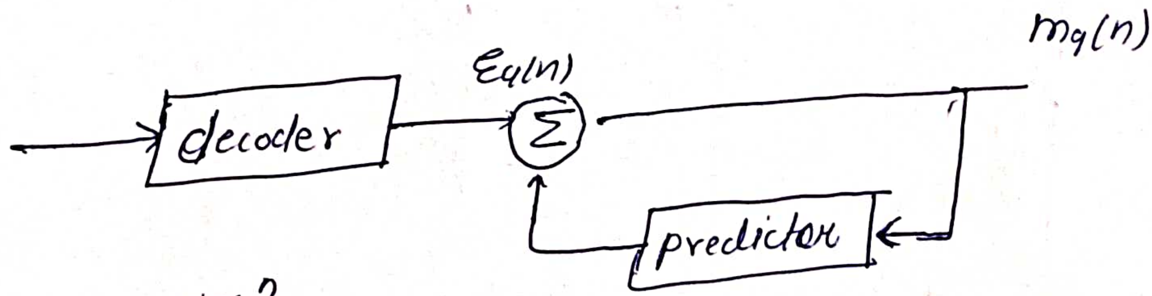
Quantized signal $$m_q(n) = \hat{m}(n) + e_q(n)$$

$$= \hat{m}(n) + e(n) + q(n)$$

$$m_q(n) = m(n) + q(n)$$



$$u(n) = \hat{m}(n) + E_q(n)$$
$$= m_q(n)$$



$$(SNR)_o = \frac{\sigma_m^2}{\sigma_Q^2}$$

Error of Prediction

$$= \left(\frac{\sigma_m^2}{\sigma_E^2}\right)\left(\frac{\sigma_E^2}{\sigma_q^2}\right)$$

$$= G_p \, G_q$$

Processing gain

Delta Modulator (DM) → why we call delta- becuz error is Bounded by Δ

$x_n > x_{n+1}$ ← quantized. ⇒ ~~binary~~ transmit binary '1'

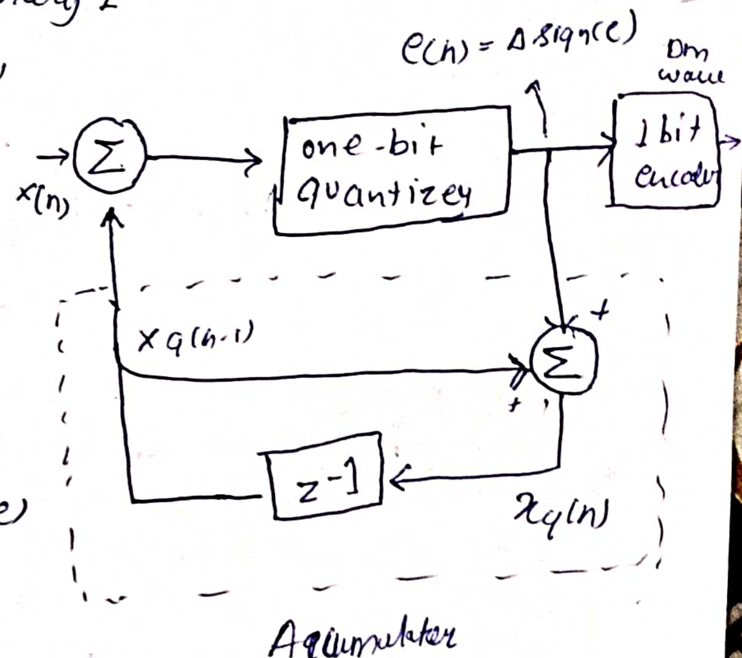
$x_n < x_{n+1}$ → trasmit binary '0'

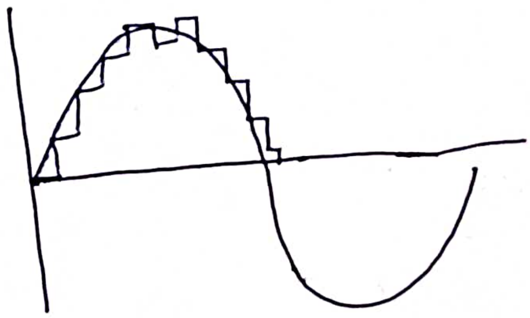
$$e(n) = x(n) - x_q(n-1)$$

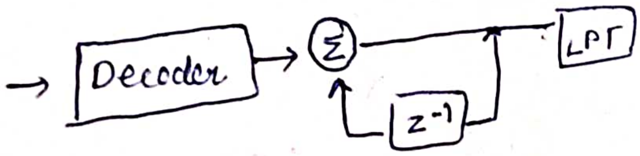
$$= \Delta \operatorname{sign}(e)$$

$$x_q(n) = x(n) + q(n)$$

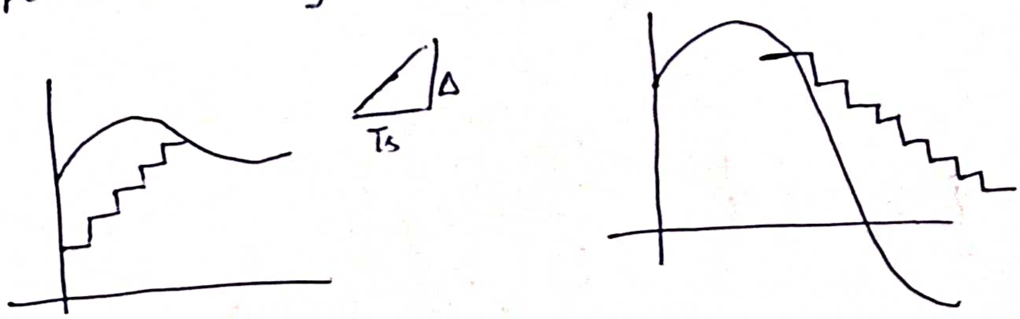
$$= x_q(n-1) + \Delta \operatorname{sign}(e)$$







↳ if slope of signal is very less or very more then slope overloading



① Slope Overloading

A.

$$\frac{\Delta}{T_s} \geq \left| \frac{dx(t)}{dt} \right|_{max}$$

	Slope overloading	Granular Noise
$\Delta\uparrow$	$\downarrow$	$\uparrow$
$\Delta\downarrow$	$\uparrow$	$\downarrow$

→ So we have to change Δ value with slope → adaptive

Adaptive Delta modulation, ADM

$$\Delta_n = \Delta_{n-1}\, K^{e_n e_{n-1}}$$

$\Delta_n < \Delta_{max}$

e_n = error

K = Constant > 1

1.5 common choice.

$e_n = 1 \quad x_n > x_q(n-1)$

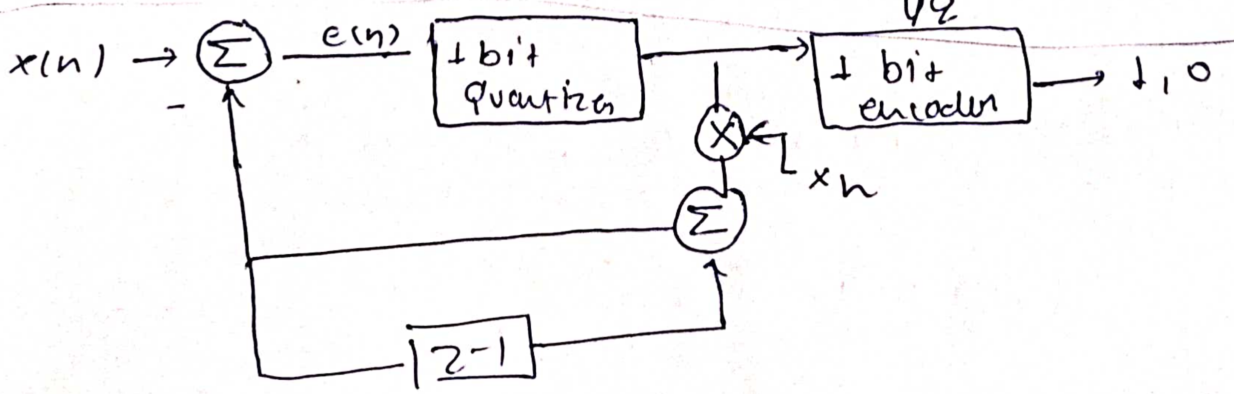
$e_n = -1 \quad x_n < x_q(n-1)$

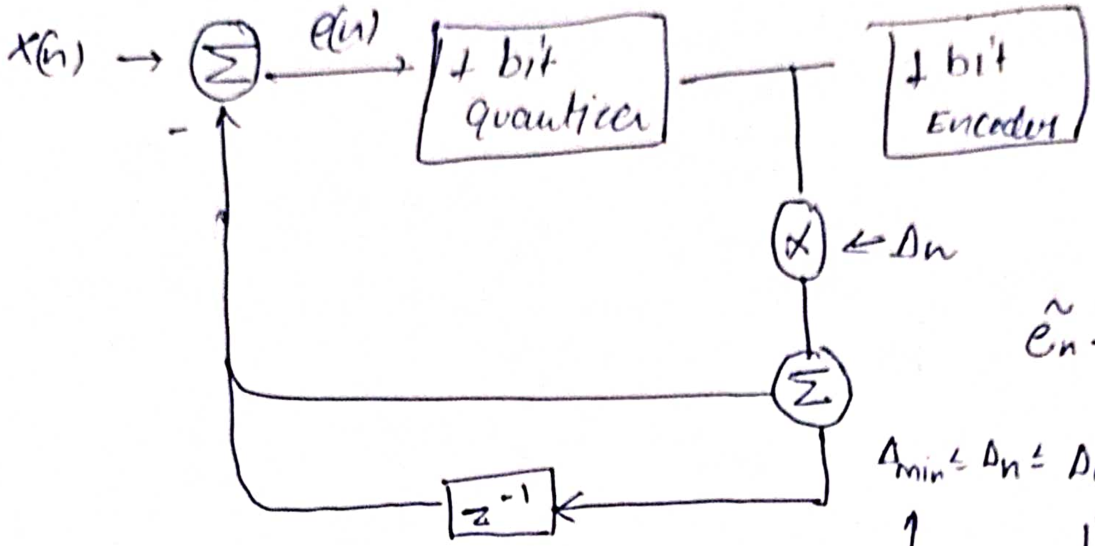
PCM, DM, DPCM, ADM

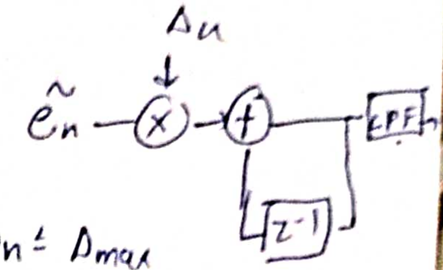
SQNR → Signal to Quantisation noise ratio

$$SQNR = \frac{\sigma_x^2}{\sigma_q^2}$$

$$SQNR_{dB} = 10 \log\left(\frac{\sigma_x^2}{\sigma_q^2}\right)$$







$\Delta_{min} \leq \Delta_n \leq \Delta_{max}$

↑ Granular Noise

↓ Slope overloading

ΔDM Rx

ADM Tx

$|x_{max}| = |x_{min}|$

$n \rightarrow$ no of bits

$$\Delta = \frac{x_{max} - x_{min}}{2^n}$$

$$= \frac{2x_{max}}{2^n}$$

SQNR for Sinusoidal Signal

Signal Power $= \frac{A^2}{2}$

Quantisation noise $= \frac{\Delta^2}{12} = \left(\frac{2A}{2^n}\right)^2 / 12$

$$\Rightarrow \quad SQNR = 10 \log \left(\frac{A^2 \times \cancel{12}}{2 \cdot \frac{4A^2}{2^{2n}}} \;\; 12^2 \quad \frac{2^{2n}}{8} \; 12^2 \right.$$

0.45
0.5
15

$$= 10 \log\left(\frac{A^2}{2} \times \frac{12}{\frac{4A^2}{2^{2n}}}\right)$$

$$= \log\left(2^{2n} * \frac{3}{2}\right)$$

$$= 10 \log\left(\frac{3}{2}\right) + 2n(\log 2)10$$

$$= 6n + 1.8 \text{ dB}$$

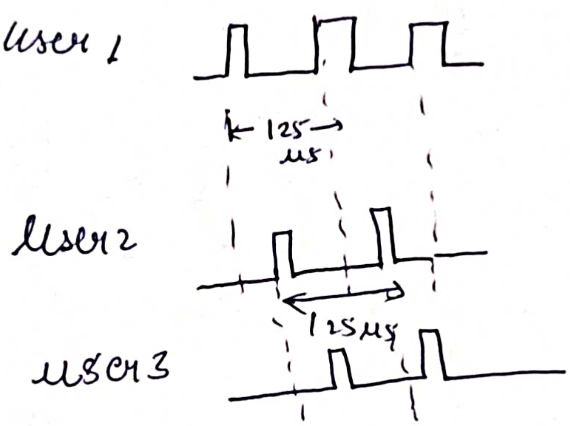
$\tilde{e}$ → Error difference

Time-Division Multiplexing

10 user → concealm → 1 signal   found a

$f_s = 8$ kHz

$T_s = 125\ \mu s$



Sq wave contain all freq

transition smooth

GSM = 8

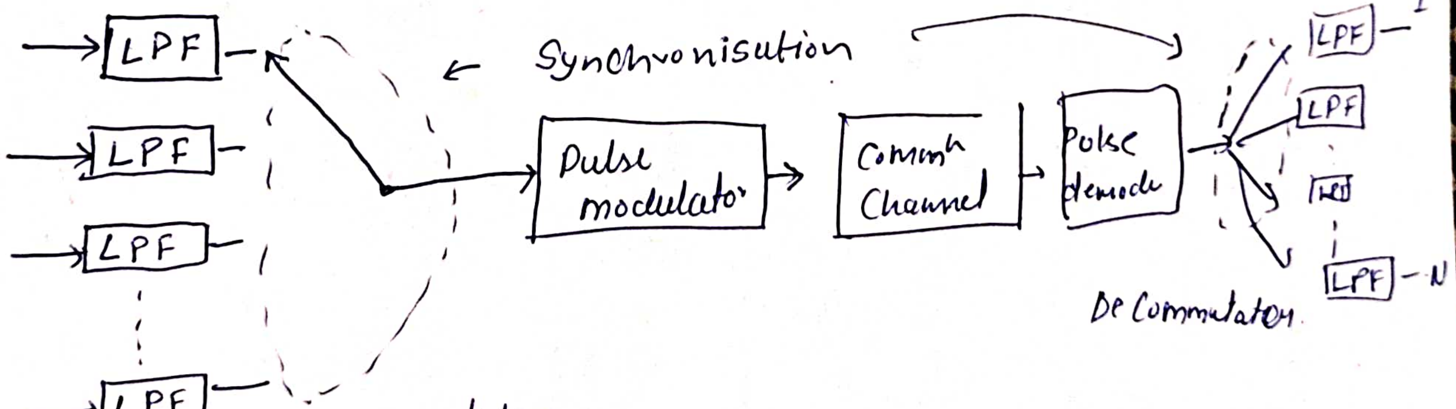
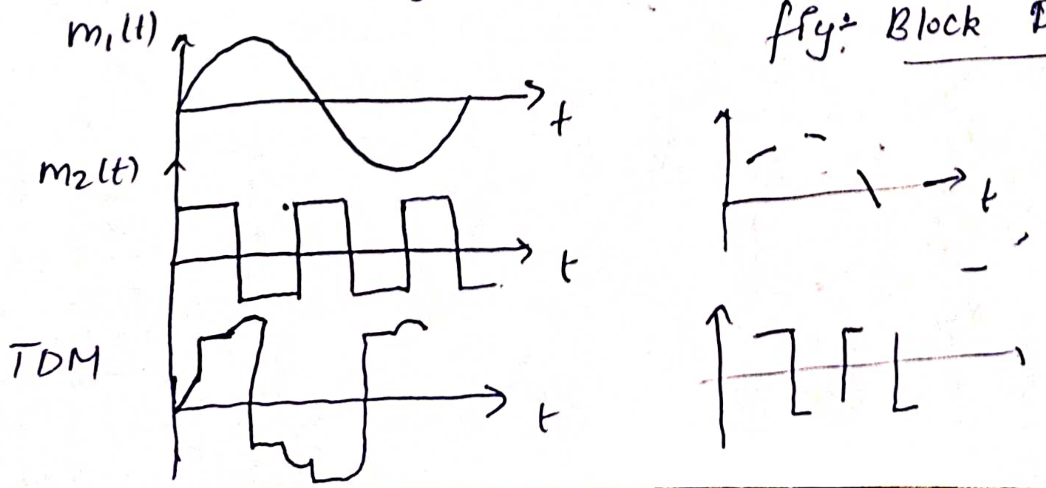


fig: Block Diagram of TDM System



$$\Delta = \frac{\overset{\text{Range}}{2V}}{M} \qquad M = \text{No of levels} = 2^n$$

$$\sigma_q^2 = \frac{\Delta^2}{12}$$

$$= \frac{4V^2}{M^2} \times \frac{1}{12}$$

$$= \frac{V^2}{3M^2} = \frac{V^2}{3(2^n)^2}$$

Quantisation Noise $\boxed{\sigma_q^2 = \frac{V^2}{3 \cdot 4^n}}$

Signal is Random (Assume zero mean)

$$\text{Signal Power} = \sigma_x^2$$

$$SQNR = 10 \log_{10} \frac{\sigma_x^2}{\sigma_q^2}$$

$$= 10 \log \frac{\sigma_x^2}{V^2 / 3 \cdot 4^n}$$

$$= \log \frac{\sigma_x^2}{V^2} + 10 \log_{10} 3 + 10 \log 2^{2n}$$

$$= 10 \log \frac{\sigma_x^2}{V^2} + 6n + 4.8$$

$$V_{max} = 4 \sigma_x \rightarrow 99.99\%$$

$$= 10 \log \frac{\sigma_x^2}{4^2 \sigma_x^2} + 6n + 4.8$$

$$\boxed{SQNR = 6n - 7.24}$$