1/8/24.

⇒ RADIATION

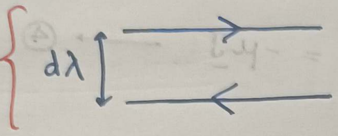
→ static charge produces electrostatic field.

→ When charged particles are moving with const. velocities or steady state current produce magnetostatic field.

→ When charge particles are moving with acceleration / decceleration then that means time varying current produces radiation.

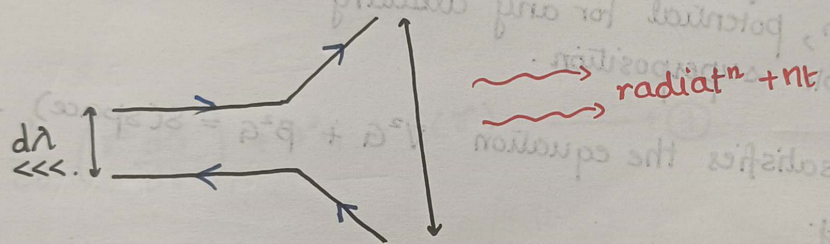
→ cause of radiation is time variation of current.

No radiat^n



2 current elements with opp. current flowing through them, in this case $d\lambda$ is small hence no radiat^n here. (or negligible)

→ Whenever there's spatial imbalance of ~~imbalance~~ of time varying current effectiv



⇒ Solution for potential f^ns

• wave equations :

$$\nabla^2 \underline{A} - \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu \underline{J} \quad \text{--- (1)}$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon} \quad \text{--- (2)}$$

A : magnetic vector potential (related to current only)
Wb/m ⟶ unit

V : Electric scalar potential (related to charge only)
unit ⟶ V (J/coulomb)

→ In the absence of ρ & J eq^n (1) & (2) becomes source free wave eq^n.

→ The 2 eq^{n}s are related to Laurent's conditions.

$$* \left[\nabla \cdot \underline{A} = -\mu\varepsilon \frac{\partial V}{\partial t} \right] *$$

→ time harmonic form of the eq^n:

$$\frac{\partial}{\partial t} = j\omega$$

$$\frac{\partial^2}{\partial t^2} = -\omega^2$$

∴ eq^n ① becomes (in time harmonic form):

$$\nabla^2 A - \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu J$$

subs $\frac{\partial^2}{\partial t^2} = -\omega^2 \Rightarrow \nabla^2 \underline{A} + \mu\varepsilon\omega^2 \underline{A} = -\mu \underline{J}$ —— ③

subs $\omega\sqrt{\mu\varepsilon} = \beta \Rightarrow \nabla^2 \underline{A} + \beta^2 \underline{A} = -\mu \underline{J}$ ⟶ ④

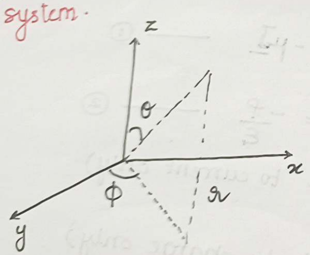
$$\nabla^2 G + \beta^2 G = \delta(\text{space})$$

→ The green f^n completely characterises the system at once. The green f^n is known, potential for any arbitrary excitation can be obtained by simple superposition.

→ The green's f^n ∴ satisfies the equation $\nabla^2 G + \beta^2 G = \delta(\text{space})$ —— ⑤

G = scalar quantity.

NOTE: for radiation we conventionally use spherical coordinate system.



→ In spherical coordinate system eq^n ⑤ becomes: (in sp. coordinate sys).

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial G}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial G}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 G}{\partial \phi^2} + \beta^2 G = \delta(\text{space})$$ —— ⑥

→ assume impulse is located @ origin and the problem is sym. in θ & φ.

→ consequently the derivative w.r.t θ & φ becomes zero.

$$\frac{\partial}{\partial \theta} = 0 \quad \& \quad \frac{\partial}{\partial \phi} = 0$$

subs. it in eq^n ⑥ we get,

$\frac{1}{r^2}\frac{\partial}{\partial r}$ → not req. to use partial derivative ∵ rest are = 0 ∴ we use full derivative

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dG}{dr}\right) + \beta^2 G = \delta(\text{space})$$ —— ⑦

5/8/24.

⇒

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dG}{dr}\right) + \beta^2 G = \delta(r)$$ ⟶ ⑦

$$\psi = G r$$

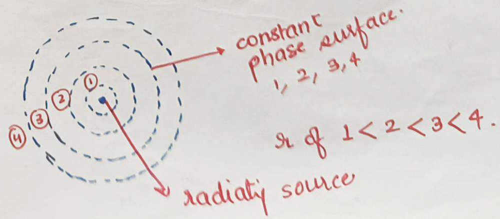
$$\frac{d^2\psi}{dr^2} + \beta\psi = \delta(r)$$ —— ⑧

sol^n → $\psi = \underbrace{Ce^{-j\beta r}}_{\text{wave in outward direc}^n} + \underbrace{De^{j\beta r}}_{\text{wave in inward direc}^n}$ —— ⑨

↳ part not feasible ∴ D = 0

$$\psi = Ce^{-j\beta r}$$ —— ⑩

$$G = C\frac{e^{-j\beta r}}{r}$$ —— ⑪



→ The arbitrary constant 'c' can be evaluated by substituting eqⁿ (11) in eqⁿ (7) and then integrating both side over a volume around r tends to 0. ∴ , we have

$$\lim_{r\to 0}\left[\text{subst. } G \text{ in } (7)\right]$$

$$\lim_{r\to 0}\left[\int \frac{1}{r^2}\frac{d}{dr}\left[r^2 \frac{d\left[c\frac{e^{-j\beta r}}{r}\right]}{dr}\right] + \beta^2\left[c\frac{e^{-j\beta r}}{r}\right]\right] = S(r)$$

vol. integral in sp. coo. sys.

$$\lim_{r\to 0}\left[\int_0^r\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{r^2}\frac{d}{dr}\left\{\left[r^2\frac{d}{dr}\, c\frac{e^{-j\beta r}}{r}\right]\right\} r^2 \sin\theta\, dr\, d\theta\, d\phi\right.$$

$$\left. + \beta^2\int_0^r\int_0^{\pi}\int_0^{2\pi} c\frac{e^{-j\beta r}}{r}\, r^2\sin\theta\, dr\, d\theta\, d\phi = \int_v S(r)\, dv\right]$$

Note : do this integration following are the hints :

1st part : I_1

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_0^r \frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{d}{dr}\left[c\frac{e^{-j\beta r}}{r}\right]\right] r^2\, dr\, \underbrace{\sin\theta\, d\theta\, d\phi}_{2\pi\times 2 = 4\pi}$$

$$r^2\frac{d}{dr}\left(c\frac{e^{-j\beta r}}{r}\right)$$

$$= -c\left[j\beta r + 1\right]e^{-j\beta r}$$

2nd part I_2.

$$\beta^2 4\pi\int_0^r c\, e^{-j\beta r}\, r\, dr$$

as $r\to 0$

$I_2 = 0$

$$\underset{r\to 0}{\mathrm{Lt}}\left[\int_0^r \frac{d}{dr}\, r^2\frac{d}{dr}\left[c\frac{e^{-j\beta r}}{r}\right] dr\, 4\pi + \beta^2\int_0^r\int_0^{\pi}\int_0^{2\pi} c\frac{e^{-j\beta r}}{r}\, r^2\sin\theta\, dr\, d\theta\, d\phi = \int_v S(r)\, dv\right]$$

$$= \underset{r\to 0}{\mathrm{Lt}}\left\{4\pi\left[-c(j\beta r+1)e^{-j\beta r}\right]\right\}$$

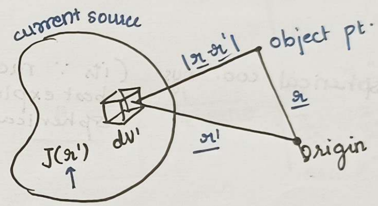
$$= \underset{r\to 0}{\mathrm{Lt}}\left\{4\pi\left[-c(j\beta r+1)e^{-j\beta r}\right] - 4\pi\beta^2\int_0^r e^{-j\beta r}\, r\, dr\right\} = 1 \quad (13)$$

→ In the first term continuous r goes to 0 ∴ term goes to zero.

→ ∴ the arbitrary constant is $-\frac{1}{4\pi} = c$.

→ ∴ the green's fⁿ is

$$\left[G = c\frac{e^{-j\beta r}}{r} = -\frac{e^{-j\beta r}}{4\pi r}\right] *$$



→ Instead of 'S', if the input (RHS of the eqⁿ) is any arbitrary i/p like (μJ) then A becomes : ~~volume~~

$$A = \int_v \frac{\mu J(r')\, e^{-j\beta|r-r'|}}{4\pi|r-r'|}\, dv$$

∴ the vector potential can be obtained as

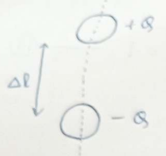
$$\boxed{A = \int_v \frac{\mu J(r')\, e^{-j\beta|r-r'|}}{4\pi|r-r'|}\, dv}$$

NOTE : to find A : first find 'G' or greens fⁿ → then 'c' → then subst. & find A.

6|8|24

⇒ THE HERTZIAN DIPOLE (H.D.)

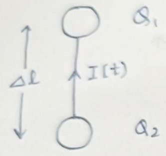
(a)



* consider 2 pt charge seperated by Δl, this arrangement is k.a an electrostatic dipole.

→ It gives a static electric field. (not useful for us ∴ we make some change to it as (b).

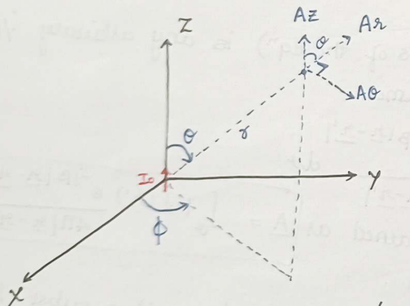
(b)



→ here charge is time variant
→ seperated by Δl & connected
∴ I(t) flows b/w them.

Note: this dipole placed @ center of spherical coo. sys. (its ∵ radiation is best explained in spherical coo. sys).

⇒ RADIATION FROM THE H.D.



$$I(t) = I_0 e^{j\omega t}\,\hat{a}_z \quad \text{——①}$$

ω = angular frequency of time varying current

$J = I\,dl$ = current density
$= I_0 e^{j\omega t}\,dl\,\hat{a}_z$ (current along az direction).

$$\underline{A} = A_z\hat{a}_z = \left(\frac{\mu}{4\pi} \frac{J e^{-j\beta r}}{r}\right)\hat{a}_z$$

from $A = \int \mu\, J(r') \frac{e^{-j\beta(r-r')}}{4\pi(r-r')}\,dv$

$$= \left(\frac{\mu}{4\pi} I_0\,dl\,\frac{e^{-j\beta r}}{r} e^{j\omega t}\right)\hat{a}_z \quad \text{——②}$$

$f = 100\,kHz$
$v = c\lambda$
$f = c/\lambda$
$\frac{100\times10^3}{3\times10^8} =$

$A_r = A_z\cos\theta$ ——③

$A_\theta = -A_z\sin\theta$ $[A_z\cos(90+\theta)]$ ——④

$A_\phi = A_z\cos 90° = 0$ (∵ ⊥) ——⑤

Now $\underline{H} = \frac{1}{\mu}\nabla\times\underline{A}$ ——⑥

We know $B = \nabla\times\underline{A}$
by 2nd maxwell eqⁿ $\nabla\cdot B = 0$
subs, $\nabla\cdot(\nabla\times\underline{A}) = 0$

We know $B = \mu\underline{H}$.
∴ $\mu\underline{H} = \nabla\times\underline{A}$
$\underline{H} = \frac{1}{\mu}(\nabla\times\underline{A})$

$$\therefore \underline{H} = \frac{1}{\mu}\nabla\times\underline{A} = \frac{1}{\mu}\cdot\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \cancel{\frac{\partial}{\partial\phi}}^{\,0} \\ A_r & rA_\theta & \cancel{r\sin\theta A_\phi}^{\,0} \end{vmatrix} \quad \text{——⑦}$$

$$\underline{H} = \frac{1}{\mu}\cdot\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & 0 \\ A_z\cos\theta & -rA_z\sin\theta & 0 \end{vmatrix} \quad \text{——⑧}$$

solving,

$H_r = 0$

$H_\theta = 0$

$$H_\phi = \frac{1}{\mu}\frac{1}{r}\left\{\frac{\partial}{\partial r}(-rA_z\sin\theta) - \frac{\partial}{\partial\theta}(A_z\cos\theta)\right\} \quad \text{——⑨}$$

2 steps ⑩, ⑪

$$* \left\{ H_\phi = \frac{I_0\,dl\,e^{j\omega t}\sin\theta}{4\pi r} e^{-j\beta r}\left\{j\beta + \frac{1}{r}\right\}\right\} * \quad \text{——⑫}$$

Note: H_ϕ = on opening bracket : 2 terms $\frac{(\;)}{4\pi r}$ & $\frac{(\;)}{4\pi r^2}$

$\frac{(\;)}{4\pi r}$ dominates when H measured at r↑ (or far pt) to dipole

$\frac{(\;)}{4\pi r^2}$ dominates when H measured at r↓ (near pt) to dipole.

8/8/24

Whenever $\frac{\partial}{\partial t} \to j\omega$

maxwells 4th eqn :
$$\nabla \times H = J + J_d$$
$$= \sigma E + \frac{\partial D}{\partial t}$$
$$= \sigma E + j\omega D$$
$$= j\omega D$$
$$\nabla \times H = j\omega \varepsilon E$$
$$E = \frac{1}{j\omega\varepsilon}(\nabla \times H) \quad \text{--- (13)}$$
$$= \frac{1}{j\omega\varepsilon} \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & H_\phi r\sin\theta \end{vmatrix} \quad \text{--- (14)}$$

(do this derivation step by step also solve $\nabla \times H$ to get)

solving (14)

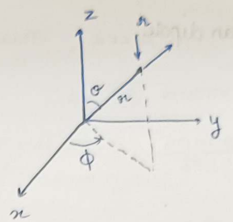
$$E_r = \frac{2 I_0 dl\, e^{j\omega t} \cos\theta\, e^{-j\beta r}}{4\pi\omega\varepsilon} \left\{ \frac{\beta}{r^2} - \frac{j}{r^3} \right\} \quad \text{--- (15)}$$

$$E_\theta = \frac{I_0 dl \sin\theta\, e^{j\omega t} e^{-j\beta r}}{4\pi\varepsilon} \left\{ \frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3} \right\} \quad \text{--- (16)}$$

$$E_\phi = 0$$

NOTE : Electric field E
Inference $E_\phi = 0$ has E_r, E_θ
but magnetic field has H_ϕ, but $H_r, H_\theta = 0$
∴ whats relatn b/w E & H for hertzian dipole ?

→ The 3 fields have spatial variation at $\frac{1}{r}$ (radiation field), $\frac{1}{r^2}$ (induction field) and $\frac{1}{r^3}$ (electrostatic field) (∵ of charge at tip of dipole).



if we check at pt r & ↑ frequency what happens to
i] $1/r$ (radiation field)
ii] $1/r^2$ (inductn ")
iii] $1/r^3$ (electrostatic ")

NOTE: (iii) $\beta = \omega\sqrt{\mu\varepsilon}$ ∴ $\beta \propto \omega$, we observe that the magnitude of the field that has $\frac{1}{r^3}$ is inversely prop to freq.

∴ electrostatic field (mag) $\propto \frac{1}{f}$

(ii) for $\frac{1}{r^2}$ or induction field in eqn (16) $\frac{\beta}{\omega r^2} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega r^2}$
∴ in independent of ω
" f
∴ if f ↑ $\frac{1}{r^2}$ (inductn field) has no effect.

(i) for radiation field. from eqn (16) $\frac{j\beta^2}{\omega r} = \frac{\omega^2(\mu\varepsilon)}{\omega r}$ ⇒ directly prop to ω

→ $\frac{1}{r^2}$ & $\frac{1}{r^3}$ fields are dominant only for small values of r whereas $\frac{1}{r}$ fields are dominant for large values of r.

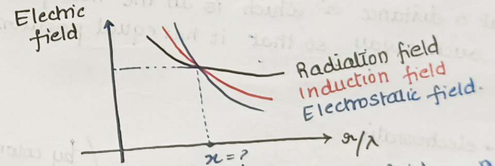
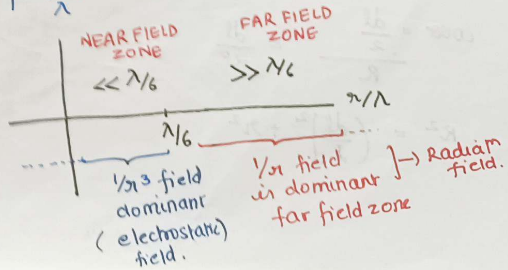


Fig: 1
→ Variatn of radiatn, inductn & electrostatic field as a fn of distance from antenna.

→ r = ?
$$\frac{\beta^2}{r} = \frac{\beta}{r^2} = \frac{1}{r^3} \qquad \text{since } \beta = \frac{2\pi}{\lambda}, \text{ so approx} \sim \frac{\lambda}{6}$$
$$\therefore \boxed{r = \frac{\lambda}{6}}^*$$
far



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→ The model to illustrate $\frac{1}{r^3}$ variation hertzian dipole

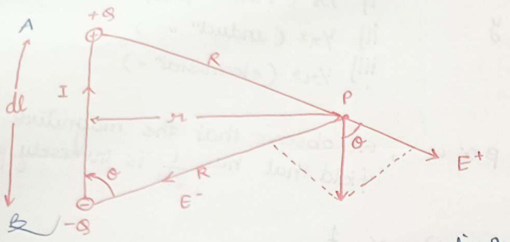
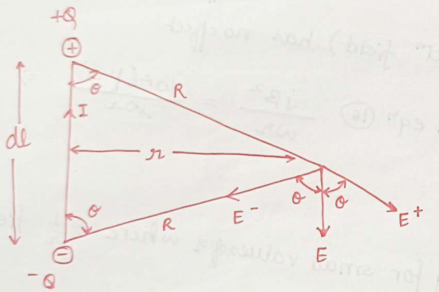


Fig: Electrostatic field due to a Hertz dipole

since $r >>> dl$, dl is very small $<< \frac{\lambda}{6}$



→ w.r.t ~~dl~~ λ, dl is v. small.
→ e^- flows top to bottom
→ I " bottom to top.
→ e's deficient at the tip of dipole.

$r >> dl$.

• Assumption:

→ The pt 'P' is placed at a distance 'r' which is in the near field zone. the r is chosen in such a way so that it has equal position from pt 'A' & 'B'.

At pt P, field → electrostatic

$\underline{E} = \underline{E}^+ + \underline{E}^-$ (vector addition)

magnitude of E $= \frac{2Q\cos\theta}{4\pi\varepsilon R^2}$ —— (18)

by colombs laws
$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$
$E = \frac{F}{q}$
$E = \frac{q}{4\pi\varepsilon_0 r^2}$

$\cos\theta = \frac{dl/2}{R} = \frac{dl}{2R}$

$R^2 = \left(\frac{dl}{2}\right)^2 + r^2$

Assume, $r >> dl$ ∴ $R^2 \approx r^2$

∴ eq (18) becomes :

$E = \frac{2Q\cos\theta}{4\pi\varepsilon R^2} = \frac{2Q\,dl}{4\pi\varepsilon \times 2R \times r^2}$

$= \frac{Q\,dl}{4\pi\varepsilon r^3}$

$* \left[E = \frac{Q\,dl}{4\pi\varepsilon} \cdot \frac{1}{r^3}\right] *$ —— (19)

12/8/24

$r << \frac{\lambda}{6}$ ⟹ the dominant field is $\frac{1}{r^3}$ related field (electrostatic)

∴ $E_r \approx \frac{-2jI_0 dl\cos\theta e^{j\omega t}}{4\pi\varepsilon r^3} e^{-j\beta r}$ —— (20)

$\approx \frac{-j2I_0 dl\cos\theta}{4\pi\varepsilon\omega r^3} e^{j\omega t}$ —— (21)

and, $E_\theta \approx \frac{-jI_0 dl\sin\theta\, e^{j\omega t}}{4\pi\varepsilon\omega r^3} e^{-j\beta r}$ —— (22)

$\approx \frac{-jI_0 dl\sin\theta}{4\pi\varepsilon\omega r^3} e^{j\omega t}$ —— (23)

since $r << \frac{\lambda}{6}$, $e^{-j\beta} \approx 1$

∴ Now $E_r = E_\theta$

∴ $|E| = \sqrt{|E_r^2| + |E_\theta^2|} = \frac{I_0 dl}{4\pi\varepsilon\omega r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$ —— (24)

$\boxed{|E| = \frac{I_0 dl}{4\pi\omega\varepsilon r^3}\sqrt{1+3\cos^2\theta}}$ —— (25)

θ vs |E| plot on matlab / python :

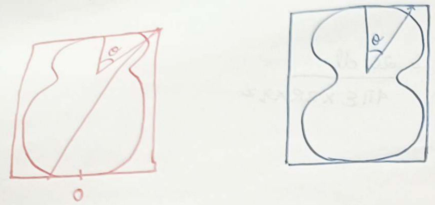


Fig : angular variation of the electric field (near zone)

$r \gg \frac{\lambda}{6}$

$$E_\theta = \frac{j I_o dl \beta^2 \sin\theta \, e^{-j\beta r} e^{j\omega t}}{4\pi \epsilon \omega r} \quad \text{——— (26)}$$

$$H_\phi = \frac{j I_o dl \sin\theta \, \beta \, e^{-j\beta r} e^{j\omega t}}{4\pi r} \quad \text{——— (27)}$$

NOTE : θ, & φ direcⁿ are orthogonal ⟶ causing transverse electromagnetic wave.

$e^{-j\beta r}$ → phase part of field.

i.e equal ∴ (26), (27) are inphase.

→ The electric & magnetic fields are inphase with each other and they are 90° out of phase with each other.

(Reason? :)

→ The constant phase surface are given by ~~$\theta_r = 0$~~ βr = constant, and therefore are spherical in shape.

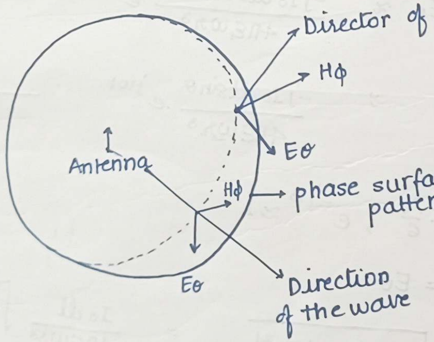
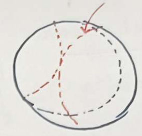


Fig: spherical wave direcⁿ w.r.t the electric & magnetic fields due to an antenna.

→ The travel in the +r direcⁿ, away from the current element, this wave is called spherical wave.



$$\frac{E_\theta}{H_\phi} = \frac{\beta}{\omega\epsilon} = \frac{\omega\sqrt{\mu\epsilon}}{\omega\epsilon} = \sqrt{\frac{\mu}{\epsilon}}$$

(units
μ = H/m
ϵ = F/m)

$$= \sqrt{\frac{\mu_o}{\epsilon_o}} = \sqrt{\frac{4\pi\times10^{-7}}{8.85\times10^{-12}}} = \sqrt{1.419\times10^{5}} = 10^3 x$$

$$= \eta$$

* Imp
* direcⁿal dependence of hertzian dipole
(ans: radiation pattern)

19/8/24.

$$P_{average} = \frac{1}{2}\mathrm{Re}\,(\underline{E}\times\underline{H}^*) \quad \text{——— (29)}$$

$$= \frac{1}{2}\mathrm{Re}\,(E_\theta H_\phi^* \hat{a}_r - E_r H_\phi^* \hat{a}_\theta)$$

(E_r & H_ϕ^* are 90° out of phase)

$$P_{ave} = \frac{1}{2}\mathrm{Re}\left\{E_\theta H_\phi^*\right\}\hat{a}_r$$

after substituting :

$$= \frac{1}{2}\mathrm{Re}\left\{\left(\frac{I_o dl \sin\theta}{4\pi r}\right)^2 \frac{1}{\omega\epsilon}\left(j\beta^2 + \frac{\beta}{r} - \frac{j}{r^2}\right)\left(-j\beta + \frac{1}{r}\right)\right\}\hat{a}_r$$

$$= \frac{1}{2}\left(\frac{I_o dl \sin\theta}{4\pi r}\right)^2 \frac{\beta^3}{\omega\epsilon}\hat{a}_r \quad \text{——— (30)}$$

$$= \frac{1}{2}\eta\,|H_\phi|^2\hat{a}_r \quad \text{——— (31)}$$

→ in eqⁿ (30) we know the net real product is only due to radialⁿ term.
→ The other fields give net imaginary product and therefore donot contribute to average power flow.

→ Therefore if there's a power flow, it should be through radialⁿ. since the electrostatic & induction fields contribute to only the reactive fields. there are also called reactive power,

* imp. → The total radiated power can be obtained by $\int$ the pointing vector over a sphere enclosing the antenna and the directional dependance of the fields is given by the radiation pattern of the antenna. (generally electrical field.)

→ The total power radiated by the hertz dipole :

$$W = \iint P_{avg}\, r^2 \sin\theta \, d\theta \, d\phi$$

(gives total power at a pt at a distance experienced)

* study elemental surface area concept.

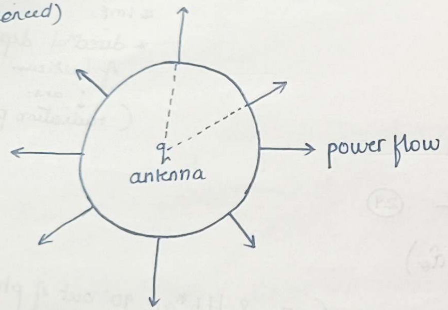


Fig : Power flow from an antenna.

$$W = \int_{\phi=0}^{2\pi}\int_{0}^{\pi} P_{avg}\, r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_{\phi=0}^{2\pi}\int_{0}^{\pi} \frac{1}{2}\left(\frac{I_0 dl \sin\theta}{4\pi r}\right)^2 \frac{\beta^3}{\omega\varepsilon}\, r^2 \sin\theta \, d\theta \, d\phi \quad \text{——— (32)}$$

$$= \frac{1}{2}\left[\frac{I_0 dl}{4\pi}\right]^2 \frac{\beta^3}{\omega\varepsilon}\int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta \, d\theta \quad \text{——— (33)}$$

$$= \frac{1}{2}\left[\frac{I_0 dl}{4\pi}\right]^2 \frac{\beta^3}{\omega\varepsilon}\, 2\pi \int_0^{\pi} (1-\cos^2\theta)\sin\theta \, d\theta \quad \text{——— (34)}$$

(do this integral v.imp)
do not write by rowing.

$$= \frac{1}{2}\left[\frac{I_0 dl}{4\pi}\right]^2 \frac{\beta^3}{\omega\varepsilon}\, 2\pi \int_0^{\pi} \sin^3\theta \, d\theta$$

=

OR

$$= \frac{1}{2}\left[\frac{I_0 dl}{4\pi}\right]^2 \frac{\beta^3}{\omega\varepsilon}\, 2\pi \int_0^{\pi} (\sin\theta - \sin\theta\cos^2\theta)\, d\theta$$

$\cos\theta = x$

$\therefore \cos^2\theta = x^2$

$-\sin\theta = \frac{dx}{d\theta}$

$-\sin\theta \, d\theta = dx$

| @ $\theta = 0$, $\cos\theta = x$, $1 = x$ | $\theta = \pi$, $\cos\pi = x$, $-1 = x$.

$$\therefore -\int_{-1}^{+1} (1-x^2)\, dx$$

$$= -\left[x - \frac{x^3}{3}\right]_{-1}^{1}$$

$$= -\left[\left(-1+\frac{1}{3}\right) - \left(1-\frac{1}{3}\right)\right]$$

$$= -\left[-\frac{2}{3} - \frac{2}{3}\right]$$

$$= -\left[\frac{-4}{3}\right] = \frac{4}{3}.$$

$$\therefore W = \frac{1}{2}\left[\frac{I_0 dl}{4\pi}\right]^2 \frac{\beta^3}{\omega\varepsilon}\, 2\pi \left[\frac{4}{3}\right] \quad \text{——— (35)}$$

using $\beta = w = \sqrt{\mu\varepsilon}$

we get $\frac{\beta^3}{\omega\varepsilon} = \frac{4\pi^2}{\lambda^2}\eta_0$

* In free space $\eta = 120\pi$

$$* \left[\therefore W = 40\pi^2 \left[\frac{dl}{\lambda}\right]^2 I_0^2\right] * \quad \text{——— (36)} \text{ v.imp}$$

→ RADIATION RESISTANCE

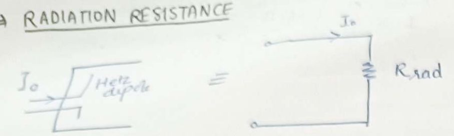


Fig: Hertz dipole equivalent to a resistance when seen from i/p side

$I = I_o \cos\omega t$

I_{rms} Rrad

$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$

$W = 40\pi^2 I_o^2 \left[\frac{dl}{\lambda}\right]^2 = \frac{1}{2} I_o^2 Rad$ ——— (37)

$R_{rad} = 80\pi^2 \left[\frac{dl}{\lambda}\right]^2$ ——— (38)

since $dl \ll \lambda$ → for hertz dipole we can say that Rrad and consequently the radiated power is generally small.

→ For a 0.1λ long dipole the radiation resistance is nearly = 8Ω.

→ Radiation pattern of the hertz dipole is :

$F(\theta, \phi) = \sin\theta$ ——— (39)

→ In most cases we perform normalisation of radiatn pattern.

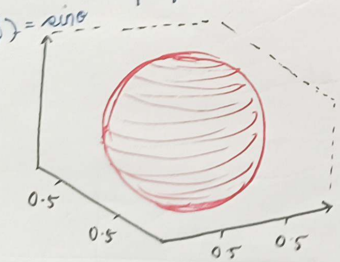
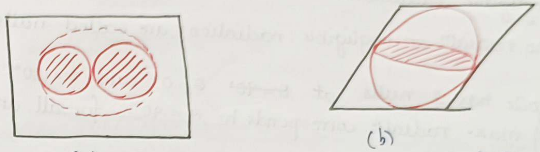


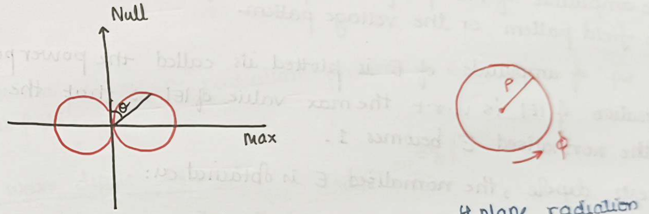
Fig Radiatn pattern after hertz dipole.

eq. (39) when plotted in the spherical coordinate system (r, θ, ϕ) with $r = |F(\theta,\phi)|$ generates a shape shown in fig. 9. This shape is similar to an apple. The radiation pattern of a hertz dipole ∴ appears like an apple.



(a) Vertical plane

(b) Horizontal plane.



E plane radiation pattern

H plane radiation pattern

Fig : H & E plane radiation pattern.

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Q. Why H field ? (ϕ component only) but still k.a H field. (do)

→ Radiation pattern is a very useful description of an antenna as it indicates the directions of maximum radiation as well as the directions of no radiation or negligible radiation.

→ The direcⁿ of no radiatⁿ or negligible radiation are called nulls of radiation pattern.

eg: The hertz dipole has 2 nulls at ~~$\theta = 90°$~~ $\theta = 0°$ and $\theta = 180°$ and the direcⁿ of max. radiatⁿ corresponds to $\theta = 90°$. for all angle of ϕ.

⇒ ANTENNA PATTERNS

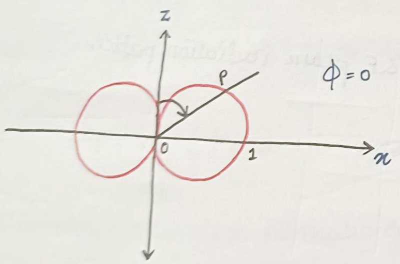
→ When the amplitude of the specific component of the E field is plotted its called the field pattern or the voltage pattern.

→ When the sq. of amplitude of E is plotted its called the power pattern.

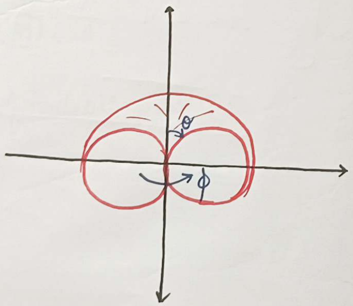
→ The normalisation of $|E|$ is w.r.t the max value of $|E|$ so that the max value of the normalised E becomes 1.

For the hertz dipole, the normalised E is obtained as:

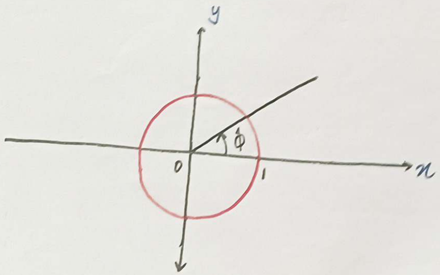
$$f(\theta) = |\sin\theta|$$



(a)



(c)



(b)

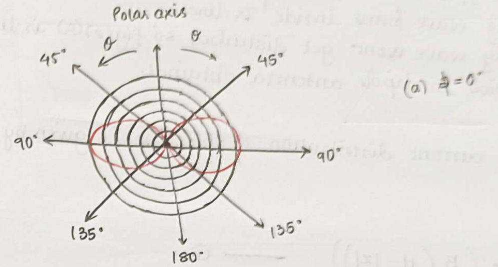
Fig: field pattern of hertz dipole

a) normalised E plane (ϕ = const = 0°)

b) normalised H plane ($\theta = \pi/2$)

c) 3D plane

→ For a hertz dipole the normalised power pattern is $f^2(\theta) = \sin^2\theta$



→ An antenna pattern or radiation pattern is a 3D plot of its radiation at far field.

⇒ Half wave dipole ANTENNA

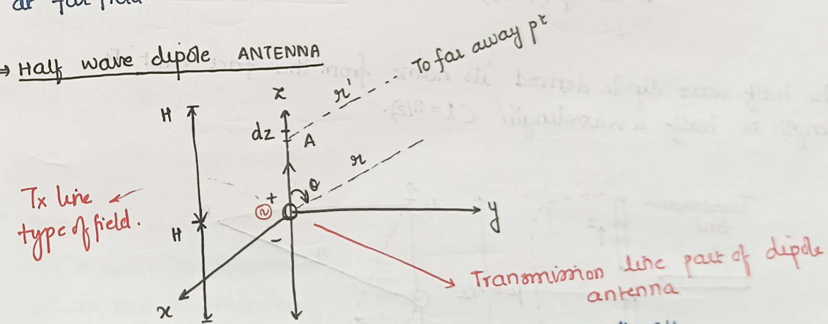
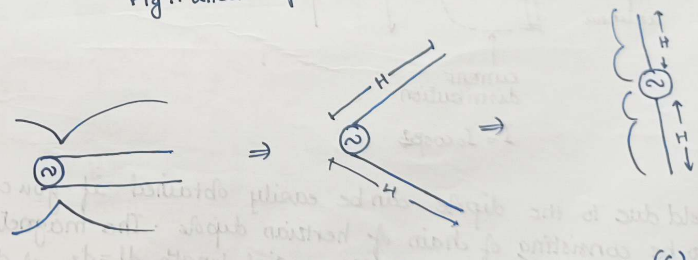


Fig 1: linear dipole antenna of length 2H.



(a) (b) (c)

fig. 2 A dipole antenna as a flared version of a transmission line

* no load conn. for this, open ckt ended so current = 0 large imp. $Z_L = \infty$

$$\left[Z_L = \frac{Z_L - Z_0}{Z_L + Z_0} \right]$$ Reflection coeff.

→ In Tx line current & V are in form of waves when these waves interact standing wave forms inside Tx line. once flared standing wave wont get disturbed so fig(2)(c) is like fig(1) so current distribution in dipole antenna obtained.

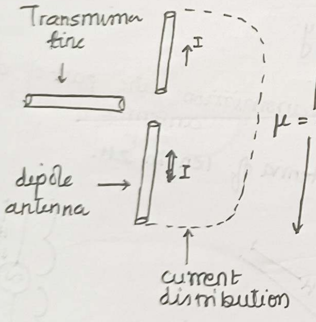
→ The standing wave current distribution of the line is given by:

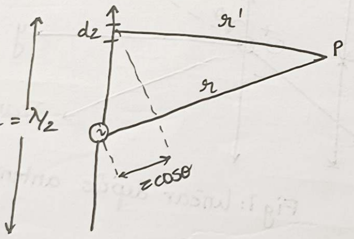
I(z):

$$I(z) = I_0 \sin(\beta(H - |z|)) \quad \text{--- ①}$$

$$\left.\begin{aligned} &= I_0 \sin(\beta(H - z)) && z > 0 \\ &= I_0 \sin(\beta(H + z)) && z < 0 \end{aligned}\right\} ②$$

→ The half wave dipole derived its name from the fact that its length is half a wavelength ($l = \lambda/2$).



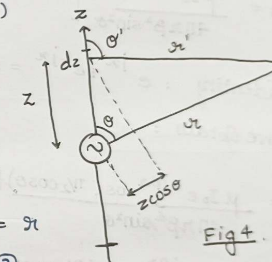


$I = I_0 \cos\beta z$

→ The field due to the dipole can be easily obtained if you consider it to be consisting of chain of hertzian dipole. The magnetic vector potential at P. due to a differential length $dl = dz$ of dipole carrying a phasor current $I_s = I_0 \cos\beta z$ is

<u>27|8|24</u>

$$dA_{zs} = \frac{\mu I_0 \cos\beta z \, dz}{4\pi r'} e^{-j\beta r'} \quad \text{--- (3)}$$



<u>Fig 4</u>

if $r \gg l$ then,

$r - r' = z\cos\theta$

$r' = r - z\cos\theta$

→ thus, we may substitute r' near $= r$ ($r' \approx r$) in denominator of eqⁿ ③ where the magnitude of the distance is needed, for the phase term in numerator the diff b/w βr & $\beta r'$ is significant so we replace r' by $r - z\cos\theta$ & not by r.

$$\therefore A_{zs} = \frac{\mu_0 I_0}{4\pi r} \int_{-\lambda/4}^{\lambda/4} e^{-j\beta(r - z\cos\theta)} \cos\beta z \, dz$$

$$= \frac{\mu I_0}{4\pi r} e^{-j\beta r} \int_{-\lambda/4}^{\lambda/4} e^{j\beta z\cos\theta} \cos\beta z \, dz \quad \text{--- ④}$$

assume $j\beta\cos\theta = a$ $\quad \beta = b$

so eqⁿ ④ : $\int e^{az} \cos bz \, dz$ form

$$= \frac{e^{az}(a\cos bz + b\sin bz)}{a^2 + b^2} + C$$

apply this in eqⁿ ④

$$A_{zs} = \frac{\mu_0 I_0}{4\pi r} e^{-j\beta r} \int_{-\lambda/4}^{\lambda/4} e^{j\beta z\cos\theta} \cos\beta z \, dz \quad — ④$$

$$= \frac{\mu I_0}{4\pi r} e^{-j\beta r}\left[e^{j\beta\cos\theta z}\frac{(j\beta\cos\theta\cos\beta z + \beta\sin\beta z)}{j^2\beta^2\cos^2\theta + \beta^2}\right]_{-\lambda/4}^{\lambda/4}$$

$$= \frac{\mu I_0}{4\pi r} e^{-j\beta r}\left[\frac{e^{j\beta\cos\theta z}(j\beta\cos\theta\cos\beta z + \beta\sin\beta z)}{\beta^2\sin^2\theta}\right]_{-\lambda/4}^{\lambda/4} \quad — ⑤$$

→ since $\beta = \frac{2\pi}{\lambda}$ or $\frac{\beta\lambda}{4} = \frac{\pi}{2}$ and $-\cos^2\theta + 1 = \sin^2\theta$

eqⁿ ⑤ becomes :

$$A_{zs} = \frac{\mu I_0 e^{-j\beta r}}{4\pi r\beta^2\sin^2\theta}\left[e^{j(\pi/2)\cos\theta}(0+\beta) - e^{-j(\pi/2)\cos\theta}(0-\beta)\right] \quad — ⑥$$

using the identity : $e^{jx} + e^{-jx} = 2\cos x$

we obtain :

$$A_{zs} = \frac{\mu I_0 e^{-j\beta r}\cos(\pi/2\cos\theta)\beta}{4\pi r\beta^2\sin^2\theta}$$

$$A_{zs} = \frac{\mu I_0 e^{-j\beta r}\cos(\pi/2\cos\theta)}{2\pi r\beta\sin^2\theta} \quad — ⑦$$

• NOTE :

→ We cal. A ∵ ~~H~~ E & H is directly related to A & for antenna we want E & H of farfield.

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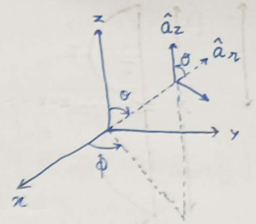
$A_s = (A_{rs}, A_{\theta s}, A_{\phi s})$

$A_{rs} = (A_{zs}\cos\theta)$, $A_{\theta s} = (-A_{zs}\sin\theta)$, $A_{\phi s} = (0)$



→ Then following the previous procedure (hertzian dipole)

$B_s = \mu H_s = \nabla \times A_s$

$\nabla \times H_s = j\omega\varepsilon E_s$

$$H_{\phi s} = \frac{jI_0 e^{-j\beta r}\cos[\pi/2\cos\theta]}{2\pi r\sin\theta}, \quad E_{\theta s} = \eta H_{\phi s} \quad — ⑧ \quad \text{(farfield)}$$

$$E_{\theta s} = \frac{j60 I_0 e^{-j\beta r}\cos[\pi/2\cos\theta]}{r\sin\theta} \rightarrow F(\theta)$$

↳ shows how field flows

$$E_{\theta s} = \frac{j60 I_0 e^{-j\beta r}}{r} F(\theta) \quad — (9)$$

$$\text{Where, } F(\theta) = \frac{\cos(\pi/2\cos\theta)}{\sin\theta} \quad — (10)$$

→ If we perform ∫ of eqⁿ ④ by considering the limit −H to +H F(θ) becomes ~~cos (βH~~ :

$$F(\theta) = \frac{\cos(\beta H\cos\theta) - \cos(\beta H)}{\sin\theta} \quad — ⑪ \quad \text{(v. imp)}$$

↓

E plane radiation pattern.

→ In the H plane E_θ is const (as its not a fⁿ of φ), and hence the H plane radiation pattern of the dipole is a circle, which is same as that of hertz dipole.

→ The E plane radiation pattern is different than that of the hertz dipole & it varies with length of dipole.

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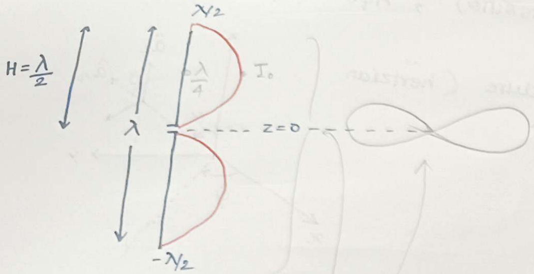
⟹ CURRENT DISTRIBUTION



Note:

$I = I_0 \sin(\beta H - \beta z)$

We know $\beta = \frac{2\pi}{\lambda}$

& $H = \frac{\lambda}{2}$

$I = I_0 \sin\left[\frac{2\pi}{\lambda}\times\frac{\lambda}{2} - \frac{2\pi z}{\lambda}\right]$

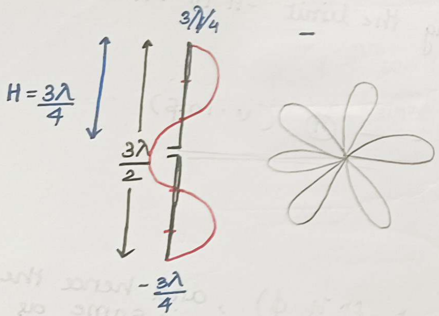
$= I_0 \sin\left[\pi - \frac{2\pi z}{\lambda}\right]$

$I = I_0 \sin\beta z$

z	I(value)
0	0
$\lambda/4$	$I = I_0 \sin\left[\frac{2\pi}{\lambda}\times\frac{\lambda}{4}\right]$ $= I_0$
$\frac{\lambda}{2}$	$I = I_0 \sin\left[\frac{2\pi}{\lambda}\times\frac{\lambda}{2}\right]$ $= 0$

$F(\theta) = \frac{\cos(\pi\cos\theta) - \cos(\pi)}{\sin\theta}$

⟶ putting value of θ we get radiation pattern.



$I = I_0 \sin\left[\beta\left[\frac{3\lambda}{4}\right] - \beta z\right]$

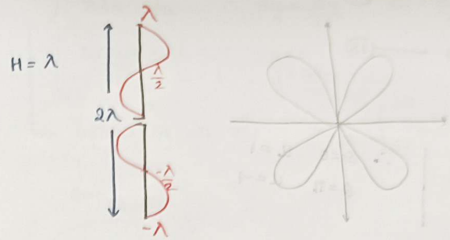
$= I_0 \sin\left[\frac{2\pi}{\lambda}\times\frac{3\lambda}{4} - \beta z\right]$

$= I_0 \sin\left[\frac{3\pi}{2} - \beta z\right]$

z	I
0	$I_0 \sin\left[\frac{3\pi}{2}\right] = -I_0$
$\frac{3\lambda}{8}$	$I_0 \sin\left[\frac{3\pi}{2} - \frac{3\lambda}{8}\times\frac{2\pi}{\lambda}\right]$ $= I_0 \sin\left[\frac{3\pi}{4}\right]$ $= \frac{I_0}{25}$
$\frac{3\lambda}{12}$	$I_0 \sin\left[\frac{3\pi}{2} - \frac{2\pi}{\lambda}\times\frac{3\lambda}{12}\right]$ $I_0 \sin\left[\frac{2\pi}{2}\right] = 0$
$\frac{3\lambda}{4}$	$I_0 \sin\left[\frac{3\pi}{2} - \frac{2\pi}{\lambda}\times\frac{3\lambda}{4}\right] = 0$

instead of this find the zero crossings

dipole of length 2λ.



$I = I_0 \sin[\beta H - \beta z]$

$= I_0 \sin\left[\frac{2\pi}{\lambda}\times\lambda - \beta z\right]$

$= I_0 \sin[2\pi - \beta z]$

z	I
0	$I_0 \sin[2\pi]$ $= 0$
$z = \frac{\lambda}{2}$	$I_0 \sin\left[2\pi - \frac{2\pi}{\lambda}\times\frac{\lambda}{2}\right]$ $= I_0 \sin[2\pi]$ $= 0$
$\frac{\lambda}{4}$	$I_0 \sin\left[2\pi - \frac{2\pi}{\lambda}\times\frac{\lambda}{4}\right]$ $= I_0 \sin\left[\frac{3\pi}{2}\right]$

$F(\theta) = \frac{\cos(\beta H\cos\theta) - \cos(\beta H)}{\sin\theta}$

$= \frac{\cos\left[\frac{2\pi}{\lambda}\times\lambda\cos\theta\right] - \cos\left[\frac{2\pi}{\lambda}\times\lambda\right]}{\sin\theta}$

$= \frac{\cos[2\pi\cos\theta] - \cos[2\pi]}{\sin\theta}$

→ POWER CALCULATION

$P_{avg} = \frac{1}{2}\eta\,|H_{\phi s}|^2\,\hat{a}_r$

$= \frac{\eta I_0^2\cos^2\left[\frac{\pi}{2}\cos\theta\right]}{8\pi^2 r^2\sin^2\theta}\,\hat{a}_r \longrightarrow (12)$

→ The time avg. radiated power can be determined as

$P_{rad} = \int P_{avg}\cdot ds = \int_{\phi=0}^{2\pi}\left[\int_{\theta=0}^{\pi}\frac{\eta I_0^2\cos^2\left[\frac{\pi}{2}\cos\theta\right]}{8\pi^2 r^2\sin^2\theta}\,r^2\sin\theta\,d\theta\right]\underbrace{d\phi}_{2\pi}$ — (13)

(total power ∴ its scalar)

$= \int_{\theta=0}^{\pi}\frac{2\pi\eta I_0^2\cos^2\left[\frac{\pi}{2}\cos\theta\right]}{8\pi^2 \cancel{r^2\sin\theta}}\,\cancel{r^2\sin\theta}\,d\theta$

$= \frac{\eta I_0^2}{4\pi}\int_0^{\pi}\frac{\cos^2\left[\frac{\pi}{2}\cos\theta\right]}{\sin\theta}\,d\theta$ $\quad(\eta = 120\pi)$

$= 30 I_0^2\int_0^{\pi}\frac{\cos^2\left[\frac{\pi}{2}\cos\theta\right]}{\sin\theta}\,d\theta = 1.219$ (on solving)

→ solve this ∫.

∴ sub. $u = \cos\theta$.

let $I = \int_0^{\pi} \frac{\cos^2[\pi/2 \cos\theta]}{\sin\theta} d\theta$ —— (14)

$= \frac{1}{2}\int_0^{\pi} \frac{1+\cos(\pi\cos\theta)}{\sin\theta} d\theta$ —— (15)

subs. $\cos\theta = u$

$\frac{du}{d\theta} = -\sin\theta$

$\therefore \frac{-du}{\sin\theta} = d\theta$

$\therefore \theta = 0 \quad u = 1$

$\theta = \pi \quad u = -1$

$= \frac{1}{2}\int_0^{\pi} \frac{1+\cos(\pi\cos\theta)}{\sin\theta} \left[\frac{-du}{\sin\theta}\right]$

$= \frac{-1}{2}\int_0^{\pi} \frac{1+\cos(\pi u)}{1-u^2} du$

$= \frac{+1}{2}\int_{-1}^{1} \frac{1+\cos(\pi u)}{1-u^2} du$ —— (16)

(We know)

$\frac{1}{1-u^2} = \frac{1}{2}\left[\frac{1}{1+u} + \frac{1}{1-u}\right]$

$= \frac{+1}{4}\left[\int_{-1}^{1} \frac{1+\cos(\pi u)}{1+u} du + \int_{-1}^{1} \frac{1+\cos(\pi u)}{1-u} du\right]$ —— (17)

subs $u = -t$ in & interchanging the limit

$\left[\frac{+1}{4}\int_{-1}^{1} \frac{1+\cos(\pi u)}{1-u} du\right] \quad u = -t$

$= \frac{+1}{4}\int_{-1}^{1} \frac{1+\cos(\pi t)}{1+t} dt$

we see that (13) is double of 17

$\therefore \frac{2}{4}\int_{-1}^{1} \frac{1+\cos(\pi u)}{1+u} du$

$= \frac{1}{2}\int_{-1}^{1} \frac{1+\cos(\pi u)}{1+u} du$ —— (18)

$\pi u = y - \pi$

$\pi du = dy \quad \therefore dy = \frac{dy}{\pi} \quad du = \frac{dy}{\pi}$

limits $u = -1 \; ; y = 0$

$u = 1 \; ; y = 2\pi$.

applying in (18) $= \frac{1}{2}\int_0^{2\pi} \frac{1+\cos(y-\pi)}{y} dy$

* Harsh look for ques

cosy

taylor expansion of $\cos y = 1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \frac{y^6}{6!} + \cdots$ → can be used to rewrite the integral

$I = \frac{1}{2}\int_0^{2\pi}\left[\frac{y}{2!} - \frac{y^3}{4!} + \frac{y^5}{6!} - \frac{y^7}{8!} \cdots\right]_0^{2\pi} dy$ (20)

$= \frac{1}{2}\int_0^{2\pi} \frac{1+\cos(y-\pi)}{y} dy$

$= \frac{1}{2}\int_0^{2\pi} \frac{1-\cos y}{y} dy$ —— (19)

$= \frac{1}{2}[9.8696 - 16.235 + 14.2428 - 7.5306 + 2.6426 - 0.6586 + 0.1225 - 0.01763 \cdots)$

$= 1.2179$.

10/9/24.

Prad $= 30 I_0^2 \times 1.2179$

$\approx 36.54\, I_0^2$ —— (21)

$= \frac{1}{2} I_0^2 R_{rad}$

$R_{rad} = \frac{2 P_{rad}}{I_0^2} \approx 73\,\Omega$ (radiatn resist. of $\lambda/2$ wave dipole antenna)

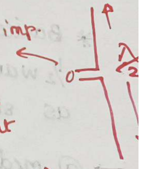
→ The total i/p impedance Zin of antenna, sink @ terminal of antenna,

$Z_{in} = R_{in} + jX_{in}$ —— (23)

→ Its found that for typical $\frac{\lambda}{2}$ antenna $X_{in} = 42.5\,\Omega$, and $R_{in} = R_{rad}$ if the antenna is lossless.

$\therefore$ for a typical $\frac{\lambda}{2}$ wave dipole antenna

$Z_{in} = (73 + j\,42.5)\,\Omega$] → problem? we see R here & its nothing but Tx line (Mismatch happens) antenna doesnt accept the power of i/p.

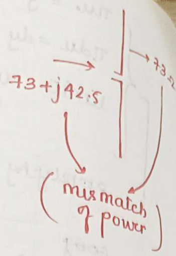


→ solution to this :

$Z_{in} = (73 + j\cancel{42.5}^{\,0})\ \Omega$

└→ nullify this component

done by slightly changing length of antenna.



$[\ l = 0.485\lambda$

@ this, $jX_{in} = 0$

12/9/24 ⇒ Quarter wave Monopole Antenna

→ length of antenna = $\frac{\lambda}{4}$

→ in monopole radiation pattern is ½ ~~hemisphere~~

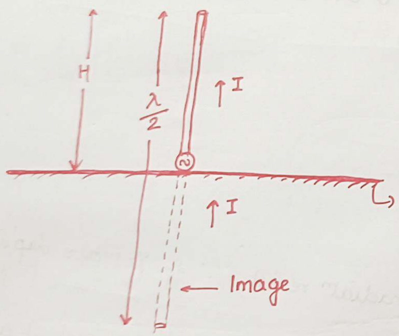


Fig : The monopole antenna.

Uses.

* this was used in phones (20 years ago)
* used in walkie - talkie
* " " radio station

* Basically the quarter wave monopole antenna consist of a ½ of a ½ wave dipole antenna, located on a conducting ground plane as shown in the fig.

→ @ medium wave (medium) when the wavelength is of the order of ~~200~~ few 100 m the realization of a dipole antenna becomes practically difficult.

→ The radiation characteristics of a monopole antenna are identical to that of a dipole antenna of double length except the power radiated by the monopole is ½ of that of ~~that~~ the corresponding dipole.

→ Most of the radiobroadcasting stations at medium wave use monopole antenna, also its commonly used in walkie-talkie handsets.

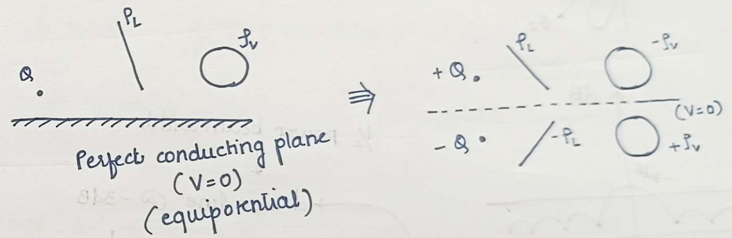
* if in exam derivation comes, derive the formula using same steps as that of hertzian dipole.

• Method of image:

↳ (basically we use dipole antenna concept to estimate for monopole antenna).

→ The image theory states that a given charge config. above an infinite perfect conducting plate may be replaced by the charge config. itself, its image and an equipotential surface, in the place of conducting plane.

(a) (b)

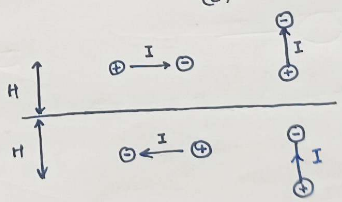


→ To apply this theory 2 conditions must be satisfied:

1) The image charges must be int in the conducting region
2) " " " " " such that on the " surfaces the potential is 0 or constant.

Note : (pattern is opp but current direc^n is same)

(c)



→ for a $\frac{\lambda}{4}$ monopole the integration in eq^n (14) is only over hemispherical surface above the ground plane i.e $0 \le \theta \le \pi/2$, because the monopole radiates only through that surface. Thus for a $\lambda/4$ monopole:

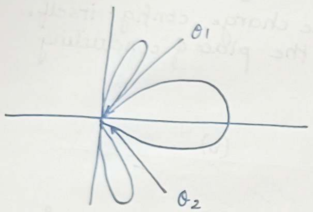
$$P_{rad} \simeq 18.28\, I_0^2 \quad \text{—— (24)}$$

$$R_{rad} = \frac{2 P_{rad}}{I_0^2} \simeq 36.5\,\Omega \quad \text{—— (25)}$$

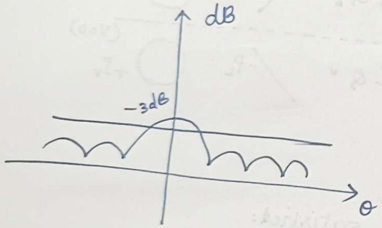
$\left(\frac{P_{rad}(\text{dipole})}{2} = \frac{36.5}{2} \right.$

$\left(\frac{R_{rad}(\text{dipole})}{2} = \frac{73}{2} = 36.5 \right.$

NOTE:



$(\theta_2 - \theta_1)$ = main beam width



½ power beam width

→ draw a line @ -3dB

normalize : $\left| \frac{\text{rad}}{\text{max rad}} \right| \log$

then

Questions

1) A 0.1m long thin wire is carrying 10A peak current at 30 MHz and is oriented along the z-direction. Find the magnetic vector potential at a distance of i) 1m
ii) 10m
iii) 100m from antenna.

Wavelength = $\lambda \approx 10m$

$l = 0.1m$

Hertzian dipole

$$A = A_z \hat{a}_z$$
$$= \frac{\mu}{4\pi} \frac{I_0 dl}{r} e^{-j\beta r} e^{j\omega t} \hat{a}_z$$

$\omega = 2\pi \times 30 \times 10^6$

$= 6\pi \times 10^7$ rad/s

$\mu = 4\pi \times 10^{-7}$ H/m

$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{c} = 0.2\pi$ rad/m

$\frac{2\pi}{\lambda} = 0.6283$ rad/m

i) @ $r = 1m$

$A_z = 10^{-7} e^{-j0.2\pi} e^{j\omega t}$ Wb/m

ii) @ $r = 10m$

$A_z = 10^{-7} \times 10 \times \cancel{10} \times \frac{e^{-j0.2\pi \times 10}}{\cancel{10}} \cdot e^{j6\pi \times 10^7 t}$

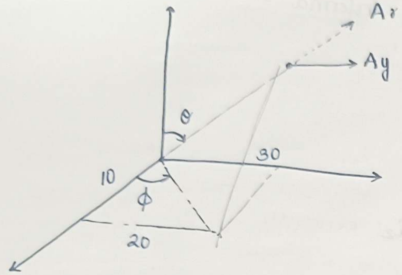
$= 10^{-6} \times e^{-j2\pi} e^{j6\pi \times 10^7 t}$

iii) $100m = r$

$A_z = 10^{-7} \times 10 \times \cancel{100} \times \frac{e^{-j0.2\pi \times 100}}{\cancel{100}} \cdot e^{j6\pi \times 10^7 t}$

$= 10^{-6} \times e^{-j20\pi} e^{j6\pi \times 10^7 t}$

Q) A 0.1m long thin wire is carrying a 10A peak current @ 30 MHz and is oriented along y direction. Its located at the origin. Find the components of magnetic vector potential at point (10, 20, 30m) also find the component in the spherical coordinate system.



The radial distance : $r = \sqrt{10^2+20^2+30^2}$

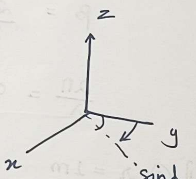
$= 37.41$

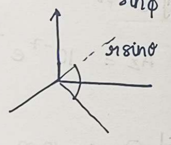
$= 10\sqrt{14}$

$A = A_y \hat{a}_y$

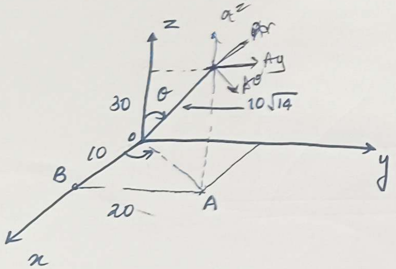
$A = \frac{\mu}{4\pi} I_0 dl \frac{e^{-j\beta r}}{r} e^{j\omega t} \hat{a}_y$

$A_y = \frac{10^{-8}}{\sqrt{14}} e^{-j2\pi\sqrt{14}} e^{j\omega t}$ wb/m





component in spherical coordinate system



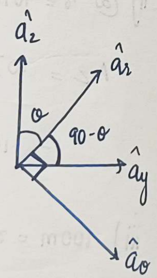
$OA = \sqrt{10^2+20^2} = 10\sqrt{5}$

$\cos\phi = \frac{OB}{OA} = \frac{1}{\sqrt{5}} = 63°$

$\sin\phi = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{30}{10\sqrt{14}} = \frac{3}{\sqrt{14}}$

$\sin\theta = \frac{\sqrt{5}}{\sqrt{14}}$



$A_r = A_y \sin\theta \sin\phi = A_y \frac{2}{\cancel{\sqrt{5}}} \times \frac{\cancel{\sqrt{5}}}{\sqrt{14}} = \frac{2}{\sqrt{14}} A_y$

$A_\theta = A_y \cos\theta \sin\phi = A_y \frac{3}{\sqrt{14}} \times \frac{2}{\sqrt{5}} = \frac{3}{7\sqrt{5}} A_y$.

$A_\phi = A_y \cos\phi$

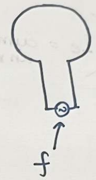
1/10/24

⇒ SMALL LOOP ANTENNA

→ antenna forming some kind of loop: square loop / circle / elliptical

→ 2 types : → electrically larger loop
  circumference $\gtrsim \frac{\lambda}{10}$
  $c > \lambda/10$
  [λ of corresponding to operating frequency]
  → electrically smaller loop
  $c < \frac{\lambda}{10}$

→ they are not good radiator
→ Its used for receiving purpose
  eg: Pagers (where antenna efficiency is not a problem)
→ good ~~radi~~ recievers



→ Most of the applications of loop antennas are in the HF (3-30MHz), VHF (30-300MHz) and UHF (300-300 GHz) Bands

→ In electrically small circumference loop antenna, they have small radiation resistance that are usually smaller than their loss resistance.

→ Application of these small loops;
→ portable radio
→ pagers
] → where antenna efficiency is not as important as SNR.

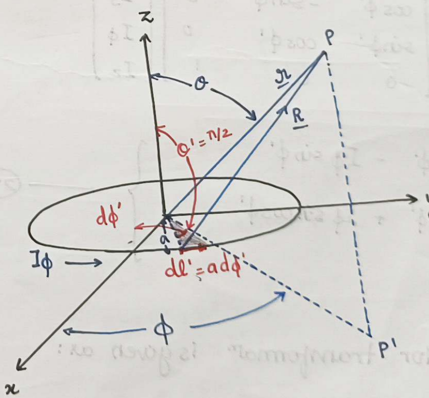


Fig-1

• cylindrical system used ∵ (primed)
→ for loop related things (∵ of cylindrical symmetricity)
• spherical : ∵ radiation pattern best explained (unprimed)

→ Assume the antenna is located symmetrically on the xy plane at z = 0 (fig 1)

The wire is assumed to be very thin and the current spatial distribution is given by:

$$I_\phi = I_0 \quad \text{—— (26)}$$

→ The magnetic vector potential is given by

$$A(x,y,z) = \frac{\mu}{4\pi} \int_C \frac{I_e(x',y',z')\, dl'\, e^{-jBR}}{R} \quad \text{—— (27)}$$

(I_e = current in rect. syst)

where, R = distance from any point on the loop to the observation point
(always const ∵ loop = small & obs. pt is very far)

dl' = infinity small section of the loop.

→ $I_e(x',y',z') = I_x(x',y',z')\hat{a}_x + I_y(x',y',z')\hat{a}_y + I_z(x',y',z')\hat{a}_z$ —— (28)

→ ∴ writing the cylindrical components using the transformation matrix :

$$\begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} \cos\phi' & -\sin\phi' & 0 \\ \sin\phi' & \cos\phi' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_\rho \\ I_\phi \\ I_z \end{bmatrix}$$

$$\left.\begin{aligned} I_x &= I_\rho \cos\phi' - I_\phi \sin\phi' \\ I_y &= I_\rho \sin\phi' + I_\phi \cos\phi' \\ I_z &= I_z \end{aligned}\right\} \quad \text{—— (29)}$$

→ ~~unit~~ ⁿˡʸ unit vector transformation is given as:

→

$$\left.\begin{aligned} \hat{a}_x &= \hat{a}_r \sin\theta\cos\phi + \hat{a}_\theta \cos\theta\cos\phi - \hat{a}_\phi \sin\phi \\ \hat{a}_y &= \hat{a}_r \sin\theta\sin\phi + \hat{a}_\theta \cos\theta\sin\phi + \hat{a}_\phi \cos\phi \\ \hat{a}_z &= \hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta \end{aligned}\right\} \quad \text{—— (30)}$$

→ substituting (29) & (30) in (28)

~~$I_e = \hat{a}_\theta [I_\rho \sin\theta \cos(\phi-\phi') + I_\phi \cos\theta \sin(\phi-\phi') - I_z \sin\theta] +$~~
~~$\hat{a}_\phi [-I_\rho \sin(\phi-\phi') + I_\phi \cos(\phi-\phi')] +$~~
~~$\hat{a}_r [I_\rho \sin\theta \cos(\phi-\phi') + I_\phi (\sin\theta \sin(\phi-\phi')) + I_z \cos\theta]$~~

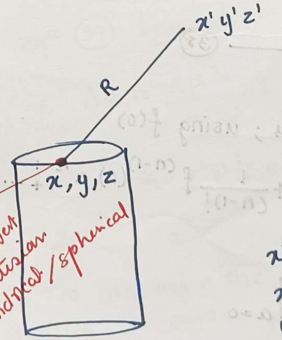
$$\left.\begin{aligned} I_e = {} & \hat{a}_r [I_\rho \sin\theta \cos[\phi-\phi'] + I_\phi \sin\theta \sin(\phi-\phi') + I_z\cos\theta] + \\ & \hat{a}_\theta [I_\rho \cos\theta\cos[\phi-\phi'] + I_\phi \cos\theta \sin(\phi-\phi') - I_z \sin\theta] + \\ & \hat{a}_\phi [-I_\rho \sin(\phi-\phi') + I_\phi \cos(\phi-\phi')] . \end{aligned}\right\} \quad \text{—— (31)}$$

3/10/24
→ It should be emphasized that the source coordinates are designated as (ρ', ϕ', z') and the observation coordinates as ~~(ρ, ϕ, z)~~ (unprimed) (r, θ, ϕ). For the circular loop the current is flowing in the ϕ direction ∴ eq (31) is reduced to:

$$I_e = \hat{a}_r I_\phi \sin\theta \sin(\phi-\phi') + \hat{a}_\theta I_\phi \cos\theta \sin(\phi-\phi') + \hat{a}_\phi I_\phi \cos(\phi-\phi') \quad \text{—— (32)}$$

→ The distance R can be written as :

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \quad \text{—— (33)}$$



also in cylindrical system ∴ we convert from cartesian to cylindrical/spherical

now,
$$\left.\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}\right\} \text{cartesian to spherical.}$$

$$\left.\begin{aligned} & x^2+y^2+z^2 = r^2 \\ & x' = a\cos\phi' \\ & y' = a\sin\phi' \\ & z' = 0 \} \text{ as we consider } z = 0 \text{ dB plane} \end{aligned}\right\} \text{cartesian to cylindrical}$$

$$\therefore x'^2 + y'^2 + z'^2 = a^2$$

So R becomes:

$$R = \sqrt{r^2 + a^2 - 2ar\sin\theta\cos(\phi-\phi')} \quad \text{--- (34)}$$

→ In the local coordinate system : $dl' = a\,d\phi'$ ——— (35)

(dl in cylindrical system: $d\cancel{\rho}\,\hat{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + \cancel{dz}\,\hat{a}_z = dl$

(ρ = const) (z = 0 plane)

$\rho = a \therefore dl' = a\,d\phi'\,\hat{a}_\phi$

→ Now using above eqⁿ we get from eq (32) as:

$$* \left[A_\phi = \frac{a\mu}{4\pi}\int_0^{2\pi} I_\phi \cos(\phi-\phi')\, \frac{e^{-j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}}\,d\phi' \right] *$$

→ since the spatial current I_ϕ given in eqⁿ (26) is const. the field radiated by the loop will not be a fⁿ of the obs. angle ϕ. Thus any obs. angle ϕ can be chosen. For simplicity take $\phi = 0$. ∴ eqⁿ (36) becomes : ~~A, φ, 3~~

$$A_\phi = \frac{a\mu I_0}{4\pi}\int_0^{2\pi} \cos\phi'\, \frac{e^{-j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}\,d\phi' \quad \text{——— (37)}$$

→ for small loops the fⁿs :

$$f(a) = \frac{e^{-j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}} \quad \text{——— (38)}$$

→ it can be expanded in a Mc Laurin series ; using f(0)

$$f(a) = f(0) + f'(0)a + \frac{1}{2!}f''(0)a^2 + \cdots + \frac{1}{(n-1)!}f^{(n-1)}(0)\,a^{n-1} + \cdots$$

where $f'(0) = \partial f/\partial a\big|_{a=0}$ $\quad f''(0) = \partial^2 f/\partial a^2\big|_{a=0}$

$$\therefore f(0) = \frac{e^{-j\beta r}}{r}$$

~~$f'(0) = \frac{1}{r} f e^{-j\beta} \frac{1}{r}$~~

for $f'(0)$, $f = \dfrac{e^{-j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}$

$$\log f = j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}\,\log e - \frac{1}{2}\log\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}$$

$$= j\beta\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'} - \frac{1}{2}\log\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}$$

$$\frac{1}{f}\frac{\partial f}{\partial a} = j\beta\,\frac{2a-2r\sin\theta\cos\phi'}{2\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}} - \frac{1}{2}\,\frac{2a-2r\sin\theta\cos\phi'}{r^2+a^2-2ar\sin\theta\cos\phi'}$$

$\dfrac{df}{da}$ at ($f=0$) : $f[0]\left[\dfrac{j\beta r\sin\theta\cos\phi'}{r} + \dfrac{r\sin\theta\cos\phi'}{r^2}\right]$

$$= \left[\frac{j\beta}{r} + \frac{1}{r^2}\right]e^{-j\beta r}\sin\theta\cos\phi'$$

$$\therefore f'(0) = \left[\frac{j\beta}{r} + \frac{1}{r^2}\right]e^{-j\beta r}\sin\theta\cos\phi' \quad \text{——— (41)}$$

8/10/24.

∴ taking into account only the 1st 2 terms :

f becomes :

$$f \simeq \left[\frac{1}{r} + a\left(\frac{j\beta}{r} + \frac{1}{r^2}\right)\sin\theta\cos\phi'\right]e^{-j\beta r}$$

∴ eqⁿ (37) becomes :

$$* \quad A_\phi \approx \frac{a\mu I_0}{4\pi}\int_0^{2\pi}\cos\phi'\left[\frac{1}{r} + a\left(\frac{j\beta}{r} + \frac{1}{r^2}\right)\sin\theta\cos\phi'\right]e^{-j\beta r}\,d\phi'$$

$$\simeq \frac{a^2\mu I_0}{4}\,e^{-j\beta r}\left[\frac{j\beta}{r} + \frac{1}{r^2}\right]\sin\theta \quad \text{——— (42)}$$

→ In ~ manner the r and θ component of A, when integrated reduce to zero.

→ Thus $A = \hat{a}_\phi A_\phi = \hat{a}_\phi \frac{a^2 \mu I_0}{4} e^{-j\beta r}\left(\frac{j\beta}{r} + \frac{1}{r^2}\right) \sin\theta$ — (43)

→ And the magnetic field component can be obtained as :

~~$H_r - j\beta$~~ $H = \frac{1}{\mu}(\nabla \times A)$

$$= \frac{1}{\mu r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r\sin\theta A_\phi \end{vmatrix}$$

* $H_r = j\frac{\beta a^2 I_0 \cos\theta}{2r^2}\left[1 + \frac{1}{j\beta r}\right] e^{-j\beta r}$ — (44) a

* $H_\theta = -\frac{(\beta a)^2 I_0 \sin\theta}{4r}\left[1 + \frac{1}{j\beta r} - \frac{1}{(\beta r)^2}\right] e^{-j\beta r}$ — (44) b

* $H_\phi = 0$ — (44) c

$E = \frac{1}{j\omega\varepsilon}(\nabla \times H)$

$$= \frac{1}{j\omega\varepsilon r^2 \sin\theta}\begin{vmatrix} a_r & r\,a_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & r H_\theta & r\sin\theta H_\phi \end{vmatrix}$$

* $E_r = E_\theta = 0$ — (45) a.

* $E_\phi = \eta \frac{(\beta a)^2 I_0 \sin\theta}{4r}\left[1 + \frac{1}{j\beta r}\right] \cdot e^{-j\beta r}$ — (45) b.

10/10/24.

Q. → A magnetic of $5\,\mu A/m$ is required @ a pt on $\theta = \pi/2$, which is 2km from an antenna in air, neglecting ohmic loss, how much power the antenna transmit if it's:

a) Hertzian dipole of length ~~$\lambda/2$~~ $\lambda/25$

b) a ½ wave "

c) a quarter wave monopole

d) a 10 turn loop antenna of radius $\lambda/2$.

* do exercise from sadiku & S. Gaonkar

a) $|H_{\phi s}| = \frac{I_0 \beta dl \sin\theta}{4\pi r}$

given $dl = \lambda/25$

$\beta dl = \frac{2\pi}{25}$

$I_0 = 0.5A$.

$P_{rad} = 40\pi^2\left[\frac{\lambda^2}{dl^2}\right] I_0^2 = 40\pi^2\left[\frac{dl}{\lambda}\right]^2 I_0^2$ (subs. I_0)

$= \underline{\underline{158\,mW}}$.

b) $|H_{\phi s}| = \frac{I_0 \cos(\pi/2 \cos\theta)}{2\pi r \sin\theta}$

$I_0 = 20\pi\,mA$. (by subs. $5\mu A = \frac{I_0 \cos(\pi/2 \cos\pi/2)}{2\pi r \sin(\pi/2)}$)

$P_{rad} = \frac{1}{2} I_0^2 R_{rad}$ ↳ $73\,\Omega$

$= \frac{1}{2}(20\pi m)^2 \times 10^{-6}(73)$

$= 144\,mW$

c) I_0 = same as monopole. $= 20\pi\,mA$

$P_{rad} = \frac{1}{2} I_0^2 R_{rad}$ ↳ 36.56

$= \frac{1}{2}(20\pi)^2 \times 36.56 \times 10^{-6} = 72\,mW$

d) $|H_\theta| = \frac{\beta^2 a^2 I_0 \sin\theta}{4r} = \frac{4\pi^2 a^2 I_0 \sin\theta}{\lambda^2 4r}$

$S = \pi a^2$

$|H_\theta| = \frac{\pi I_0}{r} \cdot \frac{S}{\lambda^2} \sin\theta$

for N turns;

$S = N\pi \rho_0^2$ (say ρ_0 = radius of ring)

$\therefore 5\times10^{-6} = \frac{\pi I_0\, 10\pi}{2\times10^3}\left[\frac{\rho_0}{\lambda}\right]^2$

$I_0 = 40.53\,mA$.

$R_{rad} = \frac{320\pi^4 S^2}{\lambda^4} = 192.3\,\Omega$

$P_{rad} = \frac{1}{2} I_0^2 R_{rad} = 158\,mW$

14/10

⇒ Radiation Intensity

↳ its defined as :

$$U(\theta,\phi) = r^2 P_{avg} \quad \text{—— (46)}$$

where P_{avg} is the time average power density.

→ The total avg. power radiated can be expressed as:-

$$P_{rad} = \oint_s P_{avg}\, ds$$
$$= \int_s P_{av}\, r^2 \sin\theta\, d\theta\, d\phi$$
$$= \int_s U(\theta,\phi) \sin\theta\, d\theta\, d\phi$$
$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta,\phi)\, d\Omega$$

where $d\Omega = \sin\theta\, d\theta\, d\phi$ is the differential solid angle in steriadian (sr)

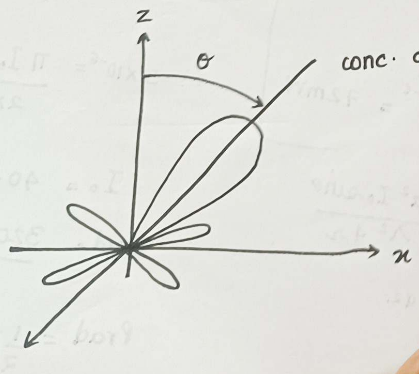
→ The radiation intensity $U(\theta,\phi)$ is measured in watts/steradian (W/sr).

→ The avg value of $U(\theta,\phi)$ is the total radiated power divided by 4π steradian.

$$U_{av} = \frac{P_{rad}}{4\pi} \quad \text{—— (48)}$$

⇒ Directive Gain

The directive gain $G_d(\theta,\phi)$ of an antenna is a measure of the concentration of the radiated power in a particular direction.



conc. as a f^n of θ & ϕ.

→ Its usually obtained as the ratio of radiation intensity in a given direction (θ,ϕ) to the avg rad. intensity i.e

$$G_d(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}}$$
$$= \frac{r^2 P_{avg}\, 4\pi}{P_{rad}} = \frac{4\pi\, U(\theta,\phi)}{P_{rad}} \quad \text{—— (49)}$$

subs. (46) in P_{avg} can be expressed in G_d as

$$P_{avg} = \frac{G_d(\theta,\phi)\, P_{rad}}{4\pi r^2} \quad \text{—— (50)}$$

→ The directive gain $G_d(\theta,\phi)$ depends on antenna pattern.

→ For Hertzian dipole we notice P_{avg} is max at $\theta = \pi/2$ and minimum at $\theta = 0$ or π.

→ Thus Hertzian dipole radiates power in a direction broad side to its length.

→ For an isotropic (Its point sized radiates equally in all direction) antenna.

∴ $G_d = 1$

→ However, such an antenna is not a physical one but an ideal one.

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*IMP. → The directivity D of the antenna is the ratio of the maximum ~~integrity~~ intensity to the avg radiation intensity.

→ Another way D: is the max. directive gain $G_d max$

$$\therefore * \left\{ D = \frac{U_{max}}{U_{avg}} = G_{d,max} \right\} * \quad —— (51)$$

from eqn (48) we can derive :

$$D = \frac{4\pi U_{max}}{P_{rad}} \quad —— (52)$$

$$= \frac{4\pi U_{max}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\, d\Omega}$$

$$\therefore D = P_{avg} = \frac{1}{2} Re\left(\underline{E_s} \times \underline{H_s}^* \right)$$

$$= \frac{1}{2} Re\left(E_{\theta s} \times \underline{H_{\phi s}^*} \right) \hat{a}_r$$

$$= \frac{1}{2} \eta |H_{\phi s}|^2 \hat{a}_r$$

$$P_{avg} = \frac{1}{2} \eta |H_{\phi s}|^2 \hat{a}_r$$

$$E_{\theta s} = \eta H_{\phi s}$$

$$P_{avg} = \frac{1}{2} \eta \frac{|E_{\theta s}|^2}{\eta^2} \hat{a}_r$$

$$= \frac{1}{2} \frac{|E_{\theta s}|^2 \hat{a}_r}{\eta}$$

$$\left\{ \therefore D = \frac{4\pi E_{max}^2}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} |E(\theta,\phi)|^2 \sin\theta\, d\theta\, d\phi} \right\} * \quad —— (53)$$

v. imp

→ For hertzian dipole, $G_d(\theta,\phi) = 1.5\sin^2\theta$

$$\therefore \boxed{D = G_{d\,max} = 1.5}$$

similarly for $\lambda/2$ dipole $G_d(\theta,\phi) = \frac{\eta}{\pi R_{rad}} f^2(\theta)$

where $f(\theta) = \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$ & $R_{rad} = 73\Omega$

substituting ↓

$$\boxed{D = 1.64}$$

⇒ **Power Gain :**

$P_{in} \longrightarrow$ P_l $\longrightarrow P_{rad}$

→ $P_{in} = P_l + P_{rad}$

where : P_l = ohmic loss
P_{in} = i/p power
P_{rad} = radiated power

NOTE
* prev. we assume antenna made up of metal, & we only saw & taken into account the radiated power
& ∵ its metal made there is loss of surface i.e. ohmic loss
(expelled by heat)

$$P_{in} = P_l + P_{rad}$$
$$= \frac{1}{2} [I_{in}]^2 (R_l + R_{rad})$$

where, I_{in} = Current of i/p terminal
R_l = loss / ohmic resistance of the antenna (loss resistance of antenna)

→ The power gain $G_p(\theta,\phi)$ is defined of antenna as :

$$* \left\{ G_p(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{in}} \right\} * \quad —— (56)$$

NOTE :
* heat doesn't affect the radiation but still its a problem ∵ heat affects the IC's connected nearby these antennas.

→ The ratio of the power gain in any specific direction (θ,ϕ) to the directive gain, in that direction, is refered to the radiation efficiency. * $[\eta_r]$

$$* \left[\eta_r = \frac{G_p}{G_d} = \frac{P_{rad}}{P_{in}} \right] * \quad —— (57)$$

↳ ~ nearer to 1%

→ introducing eqⁿ (55) leads :

$$* \left\{ \eta_r = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_L} \right\} * \text{ imp} \quad \text{—— (58)}$$

→ for many antennas η_r is close to 100% so that G_p near equal to G_d; its customary to express 'D' directivity & gain decible (dB) thus

$$\rightarrow D(dB) = 10\log_{10} D \quad \text{—— (59)}$$

$$\rightarrow G(dB) = 10\log_{10} G \quad \text{—— (60)}$$

⇒ ANTENNA ARRAY

↳ why is it req? (learn)

→ Some problems: (of single antenna)

1. bulky, huge gain required, cannot be met
2. direction (rotated version)
3. diff radiaⁿ pattern req: eg: $\csc^2\theta$ pattern (flat top) → hot possible ∴ (flexibility not there)

↳ to overcome this we have antenna array

→ In an array of identical elements there are atleast 5 controls that can be used to save the overall pattern of the antenna.

→ These are :

1) Geometrical configuration of overall array (linear, circular, rect, sq)
2) Relative displacement between the elements (space b/w elements).
3) The excitation amplitude of individual elements.
4) " " phase " "
5) The relative pattern of the individual element.

→ An antenna array is a group of radiating elements arranged to produce particular radiaⁿ characteristic.

→ A two element array

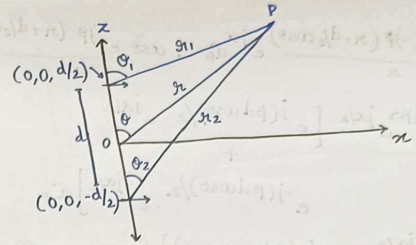


Fig 1: A two element array.

→ Consider an antenna consisting of 2 hertzian dipole placed in free space along the z axis but oriented parallel to the x axis, assume

$I_{1s} = I_0 \angle \alpha$

$I_{2s} = I_0 \angle 0$

→ If P is the far field zone, we obtain the total electric field at P as:

$$E_s = E_{1s} + E_{2s}$$

(due to 1st element) + (due to 2nd element)

We know: $$H_{\phi s} = \frac{jI_0\beta dl \sin\theta\, e^{-j\beta r}}{4\pi r} \quad \text{—— (I)}$$

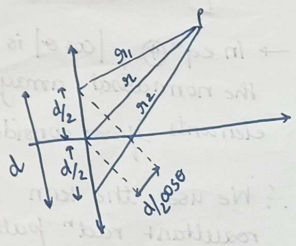
$$E_s = \frac{j\eta\beta I_0 dl}{4\pi}\left[\cos\theta_1 \frac{e^{-j\beta r_1}}{r_1} e^{j\alpha}\hat{a}_{\theta_1} + \cos\theta_2 \frac{e^{-j\beta r_2}}{r_2}\hat{a}_{\theta_2}\right] \quad \text{—— (1)}$$

Note that sine in equation (1) has been replaced by cos θ ∵ here the elements are along x direction.

∴ P is far away $\theta_1 = \theta \approx \theta_2$;

$\hat{a}_{\theta_1} \simeq \hat{a}_\theta \simeq \hat{a}_{\theta_2}$

$r_1 \simeq r \simeq r_2$



But in phase ; $r_1 \simeq r - \frac{d}{2}\cos\theta$ —— (2)

$r_2 \simeq r + d/2 \cos\theta$ —— (3)

→ Now eqⁿ ① becomes

$$E_s = j\eta \frac{\beta I_0 dl}{4\pi} \left[\cos\theta \frac{e^{-j\beta(r - d/2 \cos\theta)}}{r} e^{j\alpha} \hat{a}_\theta + \cos\theta \frac{e^{-j\beta(r + d/2\cos\theta)}}{r} \hat{a}_\theta \right]$$

$$= j\eta \frac{\beta I_0 dl}{4\pi r} \cos\theta\, e^{-j\beta r} e^{j\alpha/2} \left[e^{j(\beta d\cos\theta)/2} \cdot e^{j\alpha/2} + e^{-j(\beta d\cos\theta)/2} \cdot e^{-j\alpha/2} \right] \hat{a}_\theta$$

$$E_s = \frac{j\eta\beta I_0 dl}{4\pi r} \cos\theta\, e^{-j\beta r} e^{j\alpha/2}\, 2\cos\left[\frac{1}{2}(\beta d\cos\theta + \alpha)\right] \hat{a}_\theta \quad \text{—— ④}$$

comes ∵ mag. of current (I_0) is same.

$$\left[\begin{array}{l} I_{1s} = I_0 \angle 0^\circ \\ I_{2s} = I_0 \angle \alpha^\circ \end{array} \right]$$

→ comparing ④ with eqn ① shows that the total field of an array is equal to the field of single element located @ the origin, multiplied by an array factor ~~K~~ (AF):

$$* \left[\left[AF = 2\cos\left[\frac{1}{2}(\beta d\cos\theta + \alpha)\right] e^{j\alpha/2} \right]\right] * \quad \text{—— ⑤}$$

→ ∴ $(AF)_n = \left| \cos\left[\frac{1}{2}(\beta d\cos\theta + \alpha)\right] \right|$ —— ⑥

(normalised array factor)

→ Thus in general, the far field due to an 2 element array is given be :

$$* \left[\underline{E}(\text{total}) = (\underline{E} \text{ due to a single element}) \times (\text{Array factor}) \right] * \quad \text{—— ⑦}$$

In eqⁿ ④ $|\cos\theta|$ is the radiation pattern due to single element. whereas the normalised array factor is the radⁿ pattern of the group of elements if we consider elements were isotropic.

We use the term ~~unique~~ unit pattern /or/ group pattern to denote the resultant radⁿ pattern.

$$* \left[\text{resultant pattern} = \text{unit pattern} \times \text{group pattern} \right] \quad \text{—— ⑧}$$

→ k.a pattern multiplication.

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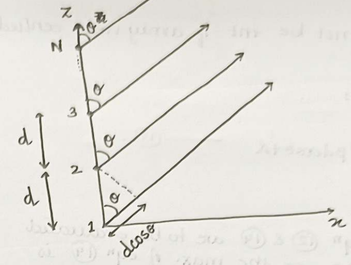
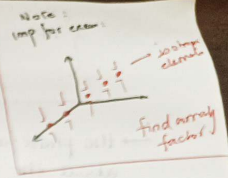
→ An nth N element uniform linear array :



Note : imp for exam



→ Assume that the array is uniform and the current distribution in the isotropic elements are as follows :

$I_{1s} = I_0 \angle 0^\circ$

$I_{2s} = I_0 \angle \alpha^\circ$

$I_{3s} = I_0 \angle 2\alpha^\circ$

⋮ ⋮

so on.

→ For the uniform linear array, the array factor is the sum of the contributions of the all elements, thus array factor =

$$AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} \dots + e^{j(N-1)\psi} \quad \text{—— ⑨}$$

where, ~~sum~~ $\psi = \beta d\cos\theta + \alpha$ —— ⑩

→ In equation ⑩ $\beta = 2\pi/\lambda$ and d & α are interelement spacing and progressive phase (or interelement phase).

→ Notice that RH eqⁿ of ⑨ is a geometric series of the form :

$$1 + x + x^2 + x^3 + \dots x^{N-1} = \frac{1 - x^N}{1 - x} \quad \text{—— ⑪}$$

∴ eqⁿ ⑨ becomes :

$$AF = \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \quad \text{—— ⑫}$$

$$AF = \frac{e^{jN\psi} - 1}{e^{jN\psi} - 1} = \frac{e^{jN\psi/2}\left(e^{jN\psi/2} - e^{-jN\psi/2}\right)}{e^{j\psi/2}\left(e^{j\psi/2} - e^{-j\psi/2}\right)} = e^{j(N-1)\psi/2} \left[\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin(\psi/2)} \right]$$

↓ ⑬

$$\left\{ AF = e^{j(N-1)\psi/2} \left[\frac{\sin(\frac{\psi N}{2})}{\sin(\psi/2)} \right] \right\} \quad \text{—— (13)}$$

→ the phase factor : $e^{jN-1\psi/2}$ would not be tnt if array were centred about the origin.

∴ neglecting this unimp. term:

$$* \left\{ |AF| = \left| \frac{\sin N\psi/2}{\sin \psi/2} \right| \right\}, \quad \psi = \beta d\cos\theta + \alpha \quad \text{—— (14)}$$

└ for $\psi = 0$ the exp in eqn (12) & (14) are to be evaluated in limits $\psi \to 0$ so the max. of eqn (14) is 'N'.

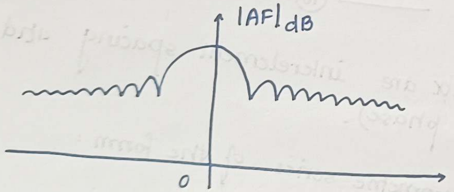
→ normalised AF =

$$\left[\frac{|AF|}{N} = \frac{1}{N} \times \left| \frac{\sin(N\psi/2)}{\sin(\psi/2)} \right| \right] * \quad \text{—— 15}$$

→ since mod AF has the max value of N. the normalised array factor is obtained by dividing mod AF by N.

The max value of eqn 15 occurs when:

$$\frac{\psi}{2} = \frac{1}{2}(\beta d\cos\theta + \alpha)\Big|_{\theta = \theta m} = \pm m\pi$$



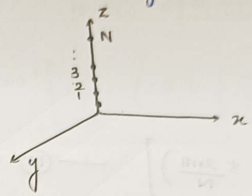
$$\theta m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\alpha \pm 2m\pi)\right] \quad \text{—— (16)}$$

$m = 0, 1, 2 \ldots$

— since the array factor has only 1 global maxima and occurs when $m=0$ in eqn 16

$$\therefore \theta m = \cos^{-1}\left[\frac{-\lambda\alpha}{2\pi d}\right] \quad \text{—— (17)}$$

→ which is the observation angle that makes $\psi = 0$



→ The direction of max radiation is independent of the no. of elements in the uniform array.

→ If the max. radiation appears along the axis of array the array is called end-fire array.

→ If max radiation appear along the direction perpendicular to the axis of array its called broadside array.

condition for end-fire array : $\psi = 0$, $\theta = 0$

$$0 = \cos^{-1}\left[\frac{-\lambda\alpha}{2\pi d}\right] = \cos^{-1}\left[\frac{-\alpha}{\beta d}\right]$$

$$\frac{-\alpha}{\beta d} = 1$$

$$\boxed{\alpha = -\beta d}$$

→ similarly for $\theta = \pi$, $\alpha = \beta d$

$\psi = 0$; $\theta = 0$ or π so that $\alpha = -\beta d$ or $\alpha = \beta d$

broadside condition : $\psi = 0$ (maxima), $\theta = \pi/2$

$$\boxed{\alpha = 0} *$$

• direction of nulls: The dirn in which no radiation goes i.e in which the EF = 0 is called a null of radiation pattern (Ideally 0, practically not possible).

→ we find that the EF = 0 whenever the numerator of eqn 15 goes to 0.

→ The nulls of radiation pattern therefore corresponds to

$$\sin\left(\frac{N}{2}\psi\right) = 0$$

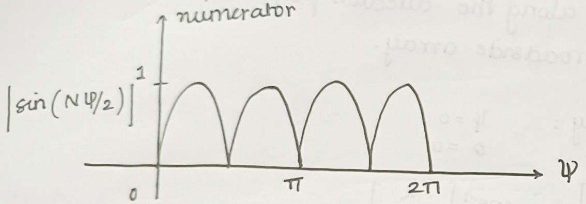
$$\left.\frac{N\psi}{2}\right|_{\theta=\theta_0} = \pm n\pi$$

$$\therefore \theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\alpha \pm \frac{2n\pi}{N}\right)\right] \quad \text{— (18)}$$

where $n = 1, 2, 3 \ldots$

$n \neq N, 2N, 3N$

• The direction of sidelobes:



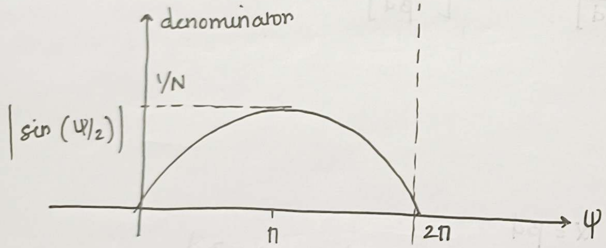


Fig 3: shows the numerator & deno of eqⁿ (15) as a fⁿ of ψ for large N the num is a rapidly varying fⁿ of ψ compared to the deno. we can then say approx whenever the num. is max. the radⁿ pattern has a local max. Approx then then direcⁿ of sidelobe lies midway b/w 2 adj nulls.

Num: $\sin\left(\frac{N}{2}\psi\right) = \sin\left(\frac{N}{2}(\beta d\cos\theta + \alpha)\right)\Big|_{\theta=\theta_s}$

$\simeq \pm 1$

$$\frac{N}{2}(\beta d\cos\theta + \alpha)\Big|_{\theta=\theta_s} = \left(\frac{\alpha s+1}{\alpha}\right)\pi$$

$$\theta_s \simeq \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\alpha \pm \left(\frac{\alpha s+1}{N}\right)\pi\right)\right] \quad \text{— (20)}$$

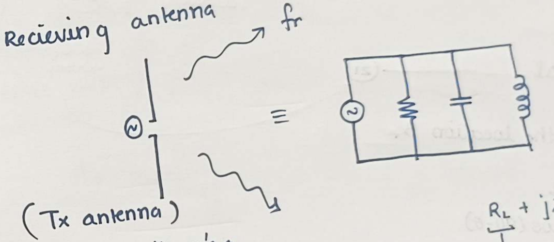
$s = 1, 2, 3$

g:

BWFN → Beam width 1st null
FNBW (degree)

HPBW – Half power beam width (side, main lobe)

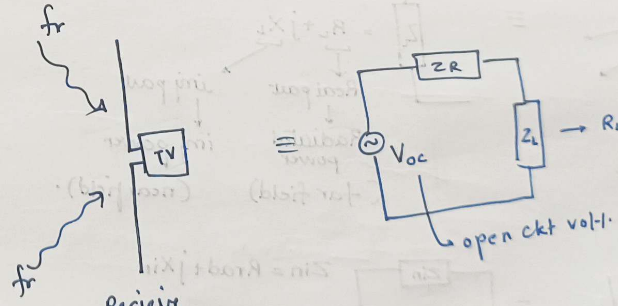
→ Recieving antenna



it Tx when there's a spatial imbalance of current

$R_L + jX_L = Z_R$

Real part ↓ corresponds to Radiated power



$\rightarrow R_L - jX_L$ (complex conj of Z_R for max power)

(it's a reciprocal device)

When the Tx signal reaches the Rx antenna, the surface current @ Rx antenna tends to flow that causes time varying voltage.

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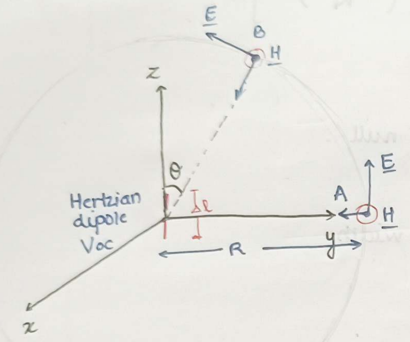
⇒ RECEIVING ANTENNA



identify eq^n (20)

Receiving can be written as

→ At A : $V_{oc} = V_A = E \cdot \Delta l$ ——— (21)

→ say the source is at the location B.

$$V_{oc} = V_B = E \cdot \Delta l$$
$$= E \Delta l \cos(90-\theta)$$
$$= E \Delta l \sin\theta$$

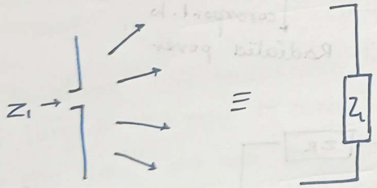
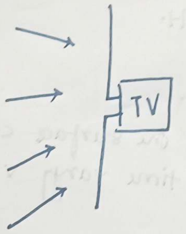


Fig.2

$= R_L + jX_L$

R_L → Real part → Radiated power (far field)

X_L → img part → img power (near field).



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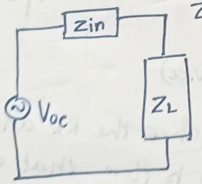

$Z_{in} = R_{rad} + jX_{in}$

$Z_L = R_{rad} - jX_{in}$

∴ power delivered →

power delivered to the matched load $= P_r = \frac{1}{2}\left[\frac{V_{oc}}{2R_{rad}}\right]^2 R_{rad}$

$$\boxed{P_r = \frac{|V_{oc}|^2}{8R_{rad}}} \quad \text{——— (23) imp.}$$

→ When the incoming em wave is normal to the entire surface of an antenna, the power recieved is :

$$P_r = \int P_{avg} \cdot ds = P_{avg} S \quad \text{——— (24)}$$

Note : P_{rad} → total avg. power radiated
P_{avg} → time avg " density

→ The effective area A_e of a recieving antenna is the ratio of the time average power recieved P_r (or delivered to the load) to the time average power density P_{avg} of the incident wave of the antenna.

$$A_e = \frac{P_r}{P_{avg}} \quad \text{——— (25)}$$

(effective area) ↓ is the ability of an antenna to recieve/extract power

derivation

$$P_r = \frac{P_{rad}}{4\pi r^2} A_e$$

if the source antenna is isotropic ($G_d = 1$)

again, $P_{avg}\,(\text{max}) = \left(\frac{G_d}{4\pi r^2}\right) P_{rad}$

∴ ~~P_{rad}~~ $P_r = P_{avg} \cdot A_e$ recieved power.

$$A_e = \frac{P_r}{P_{avg}}$$

4/11/24.

→ from eqn (25) we notice that the effective area is a measure of the ability of the antenna to extract energy from the passing EM wave.

→ Now for Hertzian dipple : (exam : for $\lambda/2$ antenna can be asked).

$P_r = ?$

$$\left.\begin{aligned} R_r &= 80\pi^2\left[\frac{dl}{\lambda}\right]^2 \\ V_{oc} &= E\,dl \end{aligned}\right] \rightarrow \text{subst. in (23)} \quad P_r = \frac{|V_{oc}|^2}{8 R_{rad}}$$

∴ eqn (23) becomes :

$$\frac{|E\,dl|^2}{8\times 80\pi^2\left[\frac{dl}{\lambda}\right]^2} = \frac{E^2 dl^2 \lambda^2}{8\times 80\times \pi^2 \times dl^2} = \frac{E^2\lambda^2}{8\times 80\times\pi^2}$$

$$* \left\{ P_r = \frac{E^2\lambda^2}{640\pi^2} \right\} * \quad \text{— (26)}$$

→ Again, we know the time average power @ antenna is :

$$* \left\{ P_{avg} = \frac{E^2}{2\eta} = \frac{E^2}{240\pi} \right\} * \quad \text{— (27)}$$

subs. eqn (26) & (27) in (25)

$$A_e = \frac{P_r}{P_{avg}} = \frac{E^2\lambda^2}{640\pi^2} \times \frac{240\pi}{E^2} = \frac{3\lambda^2}{8\pi}$$

$$A_e = \frac{3\lambda^2}{8\pi} = \frac{1.5\,\lambda^2}{4\pi}$$

└→ directivity of hertzian dipole.

$$* \left[A_e = \frac{D\lambda^2}{4\pi} \right] * \quad \text{— (28)}$$

(if D value given in dB convert & then subst. here).

where
→ D = 1.5 is the directivity of the hertzian dipole.

→ Although eqn (28) was derived for the hertzian dipole it holds for any antenna if D is replaced by $G_d(\theta,\phi)$ (directive gain).

→ ∴ in general,

$$* \left[A_e = \frac{\lambda^2}{4\pi} G_d(\theta,\phi) \right] * \quad \text{— (29)}$$

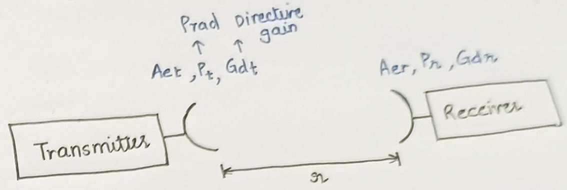

Fig : Tx and Rx antennas in free space.

(u = radial intensity) = $r^2 P_{avg}$

→ At the transmitter, $G_{dt} = \frac{4\pi U}{P_t} = \frac{4\pi r^2 P_{avg}}{P_t}$

$$\therefore \left[P_{avg} = \frac{P_t}{4\pi r^2} G_{dt} \right] \quad \text{— (30)}$$

$P_r = P_{avg} A_{er}$ (from eqn (29))

from eqn 25

$= P_{avg} \frac{\lambda^2}{4\pi} G_{dr}(\theta,\phi)$

$= \frac{\lambda^2}{4\pi} G_{dr} \underbrace{P_{avg}}_{\frac{P_t}{4\pi r^2} G_{dt}}$ — (31)

subst. eq (30) in (31) :

$$* \left\{ P_r = G_{dr} G_{dt} \left[\frac{\lambda}{4\pi r}\right]^2 P_t \right\} * \quad \text{— (32).}$$

* friis eqn of transmission * v.imp.

it relates the power recieved by one antenna to the power transmitted by the other antenna provided the two antennas are seperated by "r" where

~~$r \geq \frac{2d^2}{\lambda^2}$~~ $r \geq \frac{2d^2}{\lambda}$.

d = largest dimension of either antenna

→ ∴ to apply friis eqn we must make sure that each antenna is
is in the far field of other