 GATE ESE 2020 TARGET

ECE ENGINEERING

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GATE ESE PSU's 2019-20

GATEACADEMY

103 CONTROL SYSTEM_GATEACADEMY

TOTAL PAGE CONTROL SYSTEM-710 PGAE

LATEST GATEACADEMY CLASSROOM HANDWRITTING

CONTENT COVERED:

1. Theory Notes
2. Explanation
3. Derivation
4. Example
5. Shortcut & Formula Summary
6. Previous year Paper Q. Sol.

Noted-: Single Source Follow, Revise

Multiple Time Best key of Success

Syllabus

1. Basic Concept of Control System
2. Block Diagram and Signal flow graph
3. Time Response
4. Routh Hurwitz Stablity
5. Root Locus
6. Polar and Nyquist plot
7. Bode plot
8. Frequency Response of Second order System
9. Controller & Compensator
10. State space Analysis

Reference Book

1. Automatic Control System by kuo
2. Modern Control Engineering by Ogata.

15/0...

Chapter 01 Basic Concept of Control System

System :- System is a group of component which gives proper output for a given Input.

Control System :- Control System is a group of Component which give controlled output for a given Input

Classification of Control System depends on Controlling Mechanism in the System. There are two types of Control System.

1. Open Loop Control System
2. Closed Loop Control System.

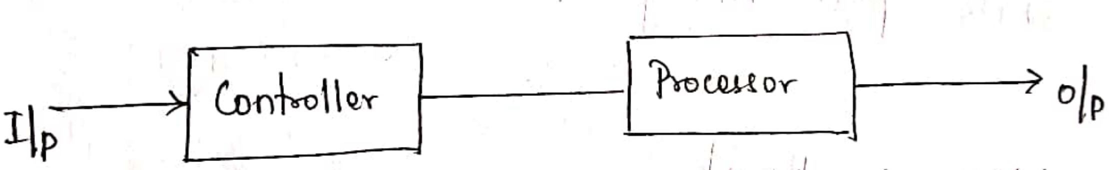

fig. open loop Control System / open system

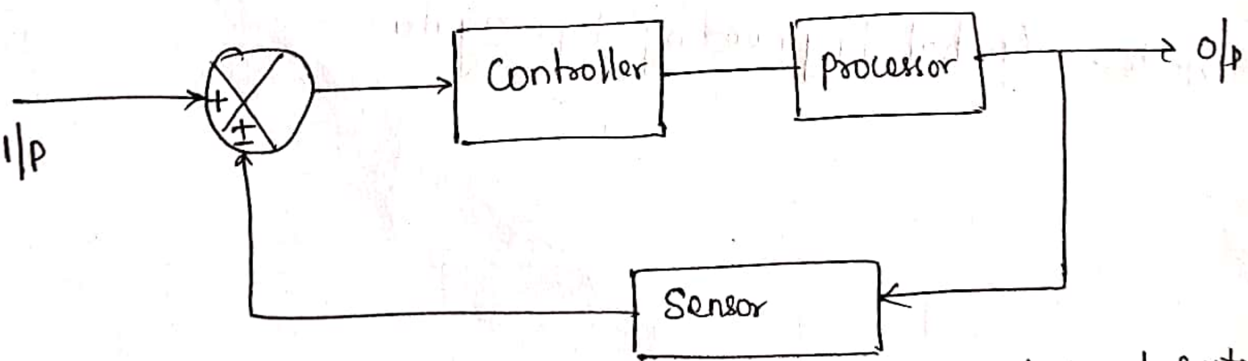

fig. Closed Loop Control System / closed system

Key points:

1. In Case of open Loop Control System or open System, Controlling action doesnot depends on output.
2. In Case of Closed Loop Control System Controlling action depends on output.

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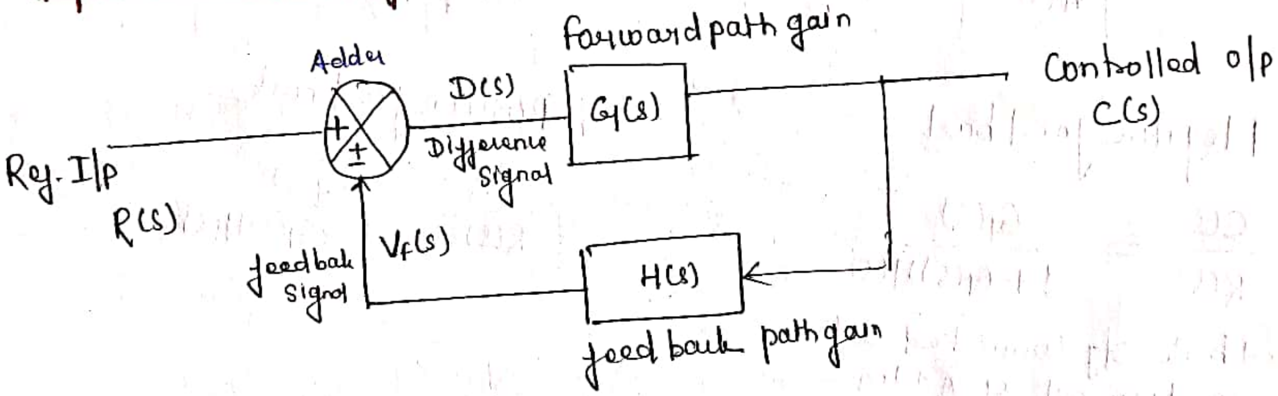
Examples

1. fan without regulator is a System
2. fan with regulator is an open loop control system
3. Air Conditioner (AC) is a closed loop control system.
4. Normal iron box is an openloop control system.
5. Automatic iron box is a closed loop control system.
6. Electric lamp is an openloop control system.
7. Traffic light signal is an open loop control system.

Key point

1. All devices having sensor are closed loop control systems.

* Representation of Closed loop Control System :-



$$G(s) = \frac{C(s)}{D(s)} \quad --- \ (1)$$

$$H(s) = \frac{V_f(s)}{C(s)} \quad --- \ (2)$$

$$D(s) = R(s) \pm V_f(s) \quad --- \ (3)$$

$$\frac{C(s)}{R(s)} = T(s) = \text{Closed loop transfer function (CLTF)}$$

from eq (1)

$$C(s) = G(s)\,D(s)$$

from eq (3) $C(s) = G(s)\left[R(s) \pm V_f(s)\right]$

from eq (2) $C(s) = G(s)\left[R(s) \pm H(s)C(s)\right]$

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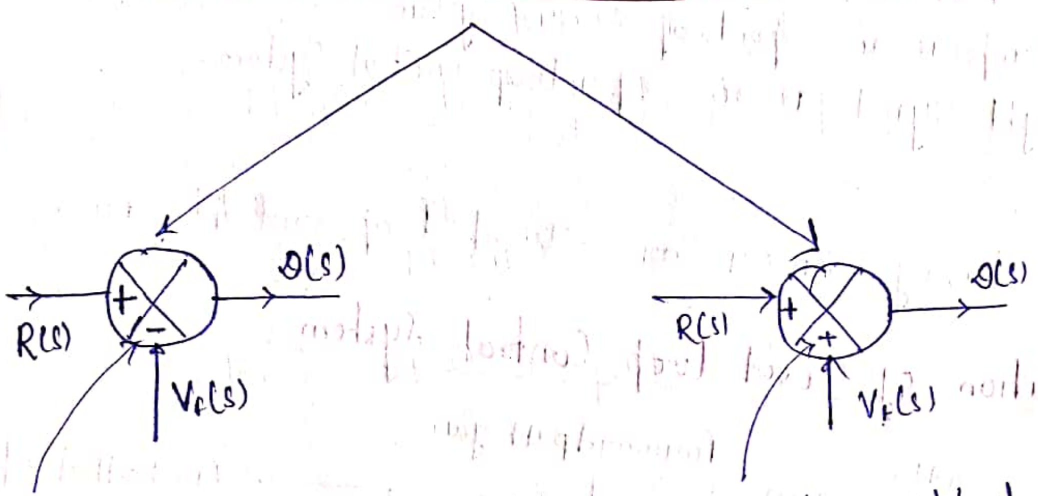
$\Rightarrow \quad C(s)\left[1 \mp G(s)H(s)\right] = R(s)G(s)$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}}$$ Transfer function



Negative feed back

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

feed back s/g connected with -ve terminal of Adder

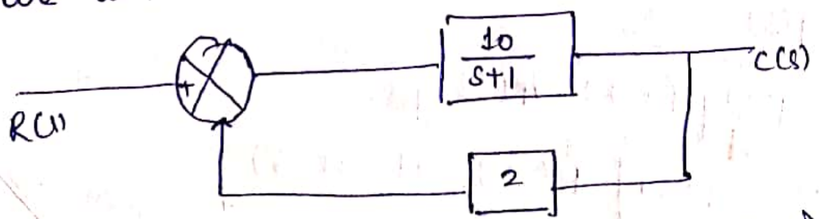
Positive feed back

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

Key points

1. Positive feedback is used for designing of oscillator and Multivibrator
1. Negative feedback is used for designing of Amplifier.
2. Negative feedback is used for designing of Amplifier.
3. positive feed back decreases the stablity of System.
4. Negative feedback increases the stablity of system
5. Positive feedback system is reffered as regenerative feed back system
6. Negative feedback system is reffered as degenerative feed back system
7. If Information of feedback (+ve / -ve) is not given then we will consider negative feed back system.

eg.



$$\frac{C(s)}{R(s)} = \frac{\frac{10}{s+1}}{1 + \frac{10 \times 2}{s+1}}$$

$$\frac{C(s)}{R(s)} \Rightarrow \frac{10}{s+21}$$

8. In case of unity feedback system we consider $H(s) = 1$

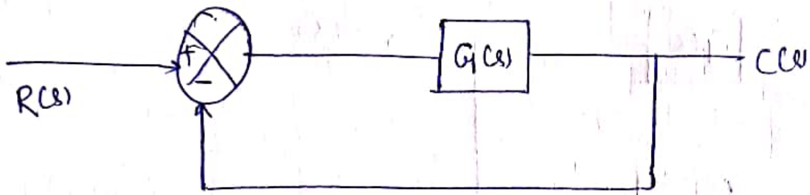


fig Negative unity feed back system

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

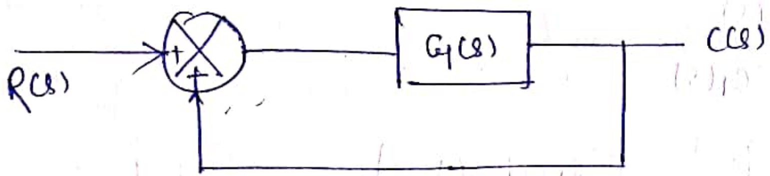


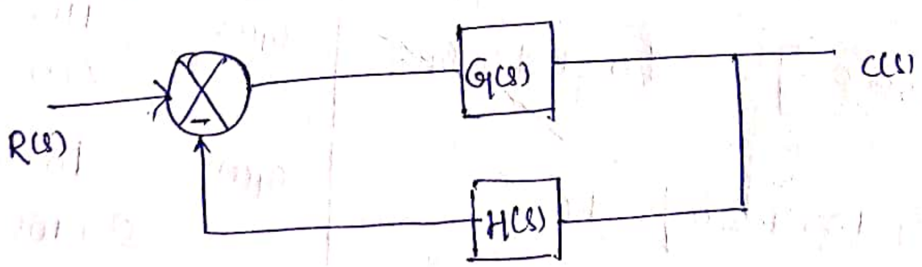
fig. positive unity feedback system

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)}$$

9. If information of feedback element H(s) is not given then we will consider unity feed back system that means $H(s) = 1$.

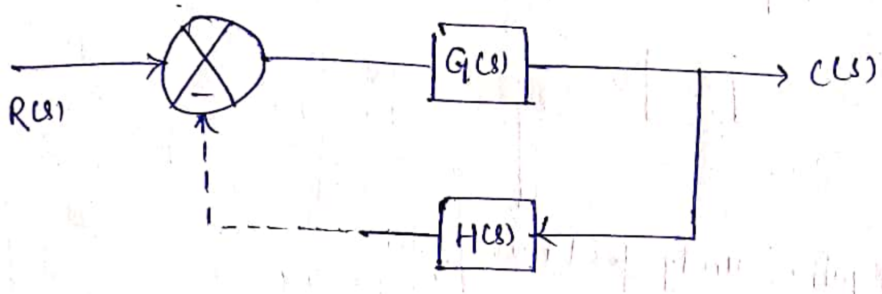
10. Mostly we use negative unity feedback system for analysis of control system.

+ <u>Open Loop Transfer function</u> :



$$CLTF = \frac{G(s)}{1 + G(s)H(s)}$$

$G(s)H(s)$ = open loop xfer function



$$OLTF = G(s)H(s)$$

In case of Negative Unity feed back System

$$CLTF = \frac{G(s)}{1+G(s)}$$

$$OLTF = G(s) \quad [H(s) = 1]$$

$$\boxed{CLTF = \frac{OLTF}{1+OLTF}}$$

only in case of Unity negative feedback system

eg. $$T(s) = \frac{10}{s^2+10s+30}$$

negative unity feed back

$$\frac{G(s)}{1+G(s)} = \frac{10}{s^2+10s+30}$$

$$\Rightarrow G(s)[s^2+10s+30] = 10 + 10\,G(s)$$

$$\Rightarrow G(s)[s^2+10s+20] = 10$$

$$G(s) = \frac{10}{s^2+10s+20}$$

$$\frac{G(s)}{1+G(s)} = \frac{10}{s^2+10s+30}$$

$$G(s) = \frac{N(s)}{D(s)-N(s)}$$

$$G(s) = \frac{10}{s^2+10s+20}$$

DC gain of CLTF :

DC = zero freq.

$f = 0$

$\omega = 0$

$s = 0$

$$T(s=0) = \frac{10}{0+0+30} = 1/3$$

Dc gain of open Loop Transfer function

$$G(s) = \frac{10}{s^2+10s+20}$$

$$G(s=0) = 0.5$$

Q. $G(s) = \dfrac{10}{s(s+2)}$ = DC gain = ∞ X (wrong)

Sensitivity :-

$$A = f(B)$$

$$\boxed{S_B^A = \frac{\partial A / A}{\partial B / B} = \frac{B}{A} \times \frac{\partial A}{\partial B}}$$

$$T = \frac{G}{1+GH}$$

$$T = f(G, H)$$

i) $S_G^T = \dfrac{\partial T / T}{\partial G / G} = \dfrac{G}{T} \times \dfrac{\partial T}{\partial G}$ --- (1)

ii) $S_H^T = \dfrac{\partial T / T}{\partial H / H} = \dfrac{H}{T} \times \dfrac{\partial T}{\partial H}$ --- (2)

$$\frac{\partial T}{\partial G} = \frac{(1+GH)\times 1 - G(0+H)}{(1+GH)^2}$$

$$= \frac{1+GH-GH}{(1+GH)^2}$$

$$\frac{\partial T}{\partial G} = \frac{1}{(1+GH)^2}$$

$$S^{T}_{G} = \frac{G}{\frac{G}{1+GH}} \times \frac{1}{(1+GH)2}$$

$$\boxed{S^{T}_{G} = \frac{1}{1+GH}}$$

now. $$S^{T}_{H} = \frac{H}{\frac{G}{1+GH}} \frac{\partial T}{\partial H}$$

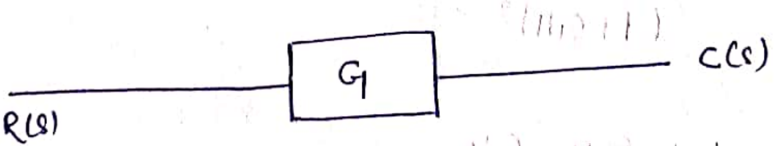
$$\frac{\partial T}{\partial H} = \frac{(1+GH)^{2}_{\times 0} - G^{2}}{(1+GH)2}$$

$$= \frac{-G^{2}}{(1+GH)^{2}}$$

$$S^{T}_{H} = \frac{H}{\frac{\cancel{G}}{1+GH}} \times \left(\frac{-\cancel{G}^{2}}{(1+GH)^{2}} \right)$$

$$\rightarrow \frac{-HG}{(1+GH)}$$

$$\boxed{S^{T}_{H} = \frac{-GH}{1+GH}}$$



$$T = \frac{C}{R} = G$$

$$S^{T}_{G} = \frac{\partial T | T}{\partial G | G} = \frac{G}{T} \times \frac{\partial T}{\partial G}$$

Chapter-2

Block Diagram

1. Block Diagram is used for determine the overall T.f of the system.
2. It is a pictorial representation of the system.
3. Analysis of T.f by using Block diag. is based on Block Diag. Reduction rule
4. In a Block diag. representn all system Variable are linked to eachother through functional block.

* Block Diagram Reduction Rules:-

1. Representation of closed System:-

i)

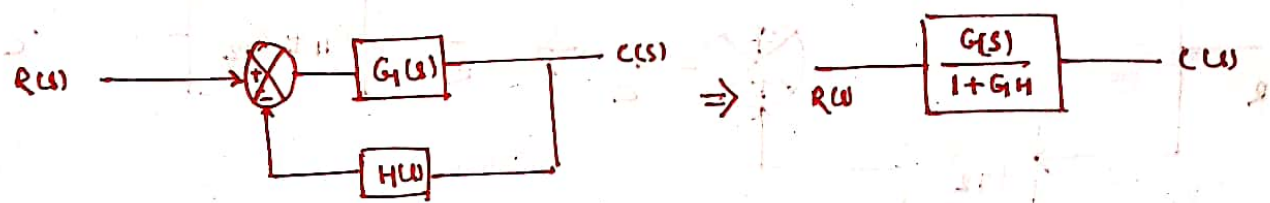


fig a

ii)

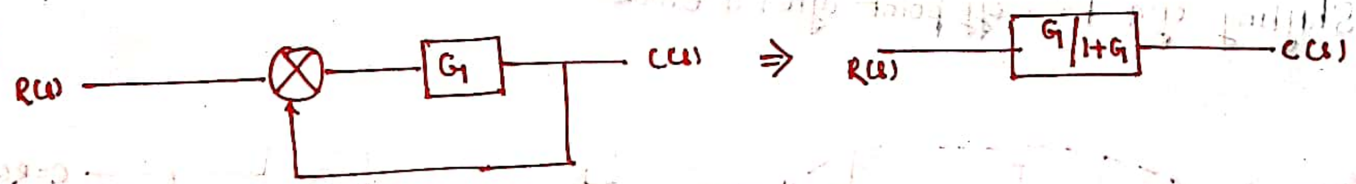


fig b

iii)

fig c

iv)

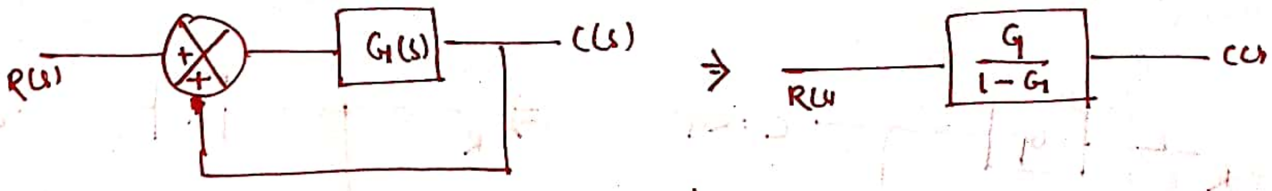
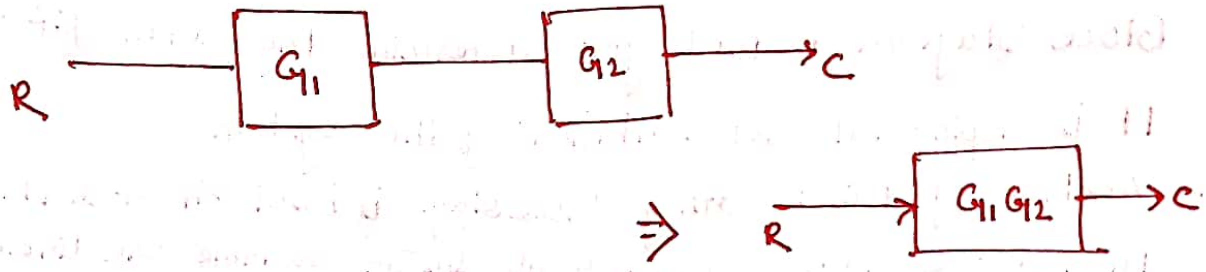
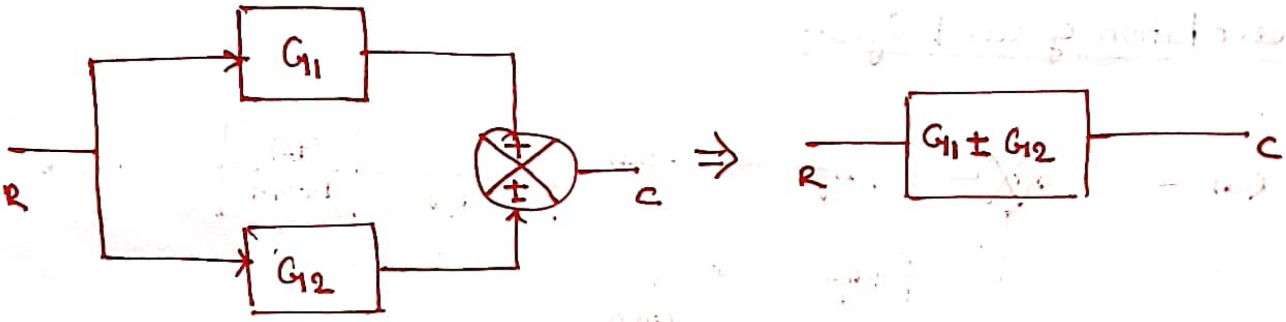


fig d

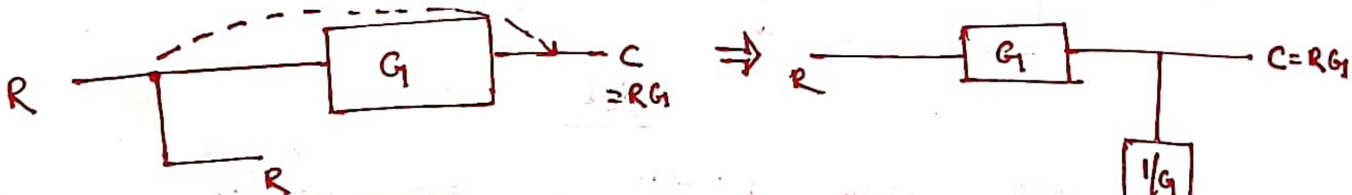
2. Blocks are Connected in Series | cascade :-



3. Blocks are Connected in parallel :-

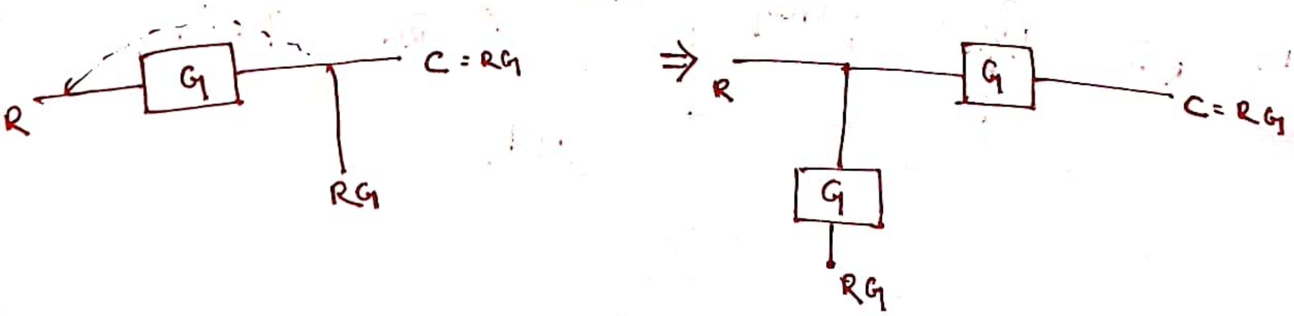


4. Shifting of a take off point after a Block :-



(Take off or pick off point as equipotential point)

5. shifting of a take off point before a block



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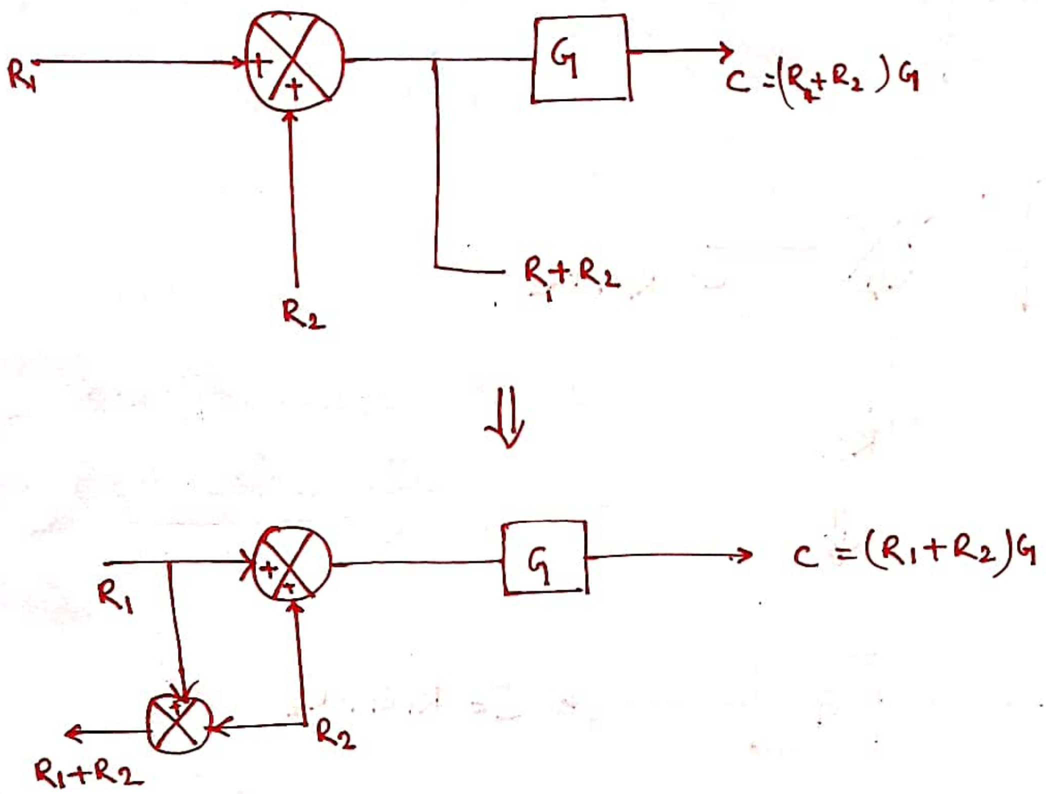
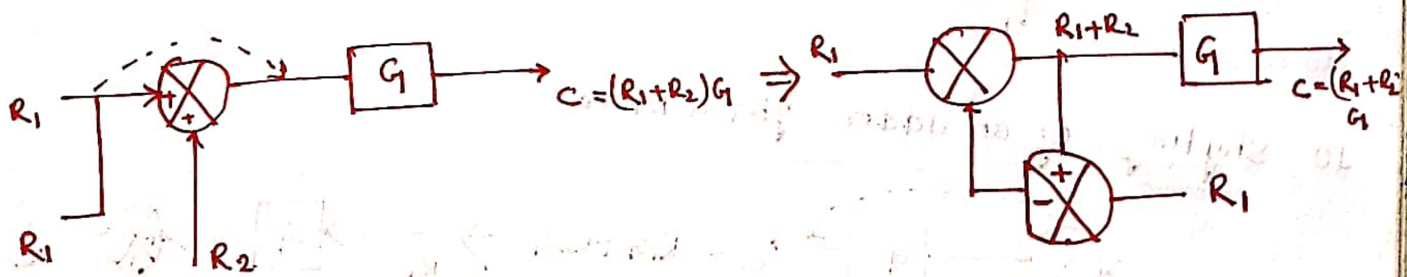
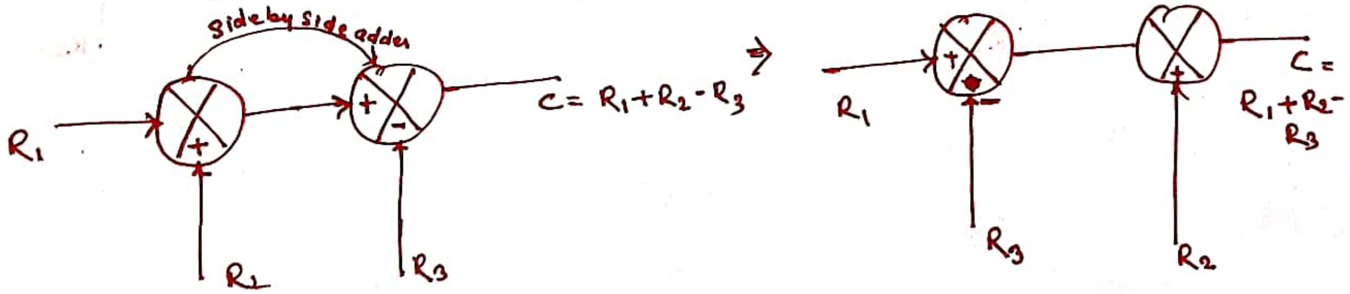
6. Shifting of a takeoff point before an Adder



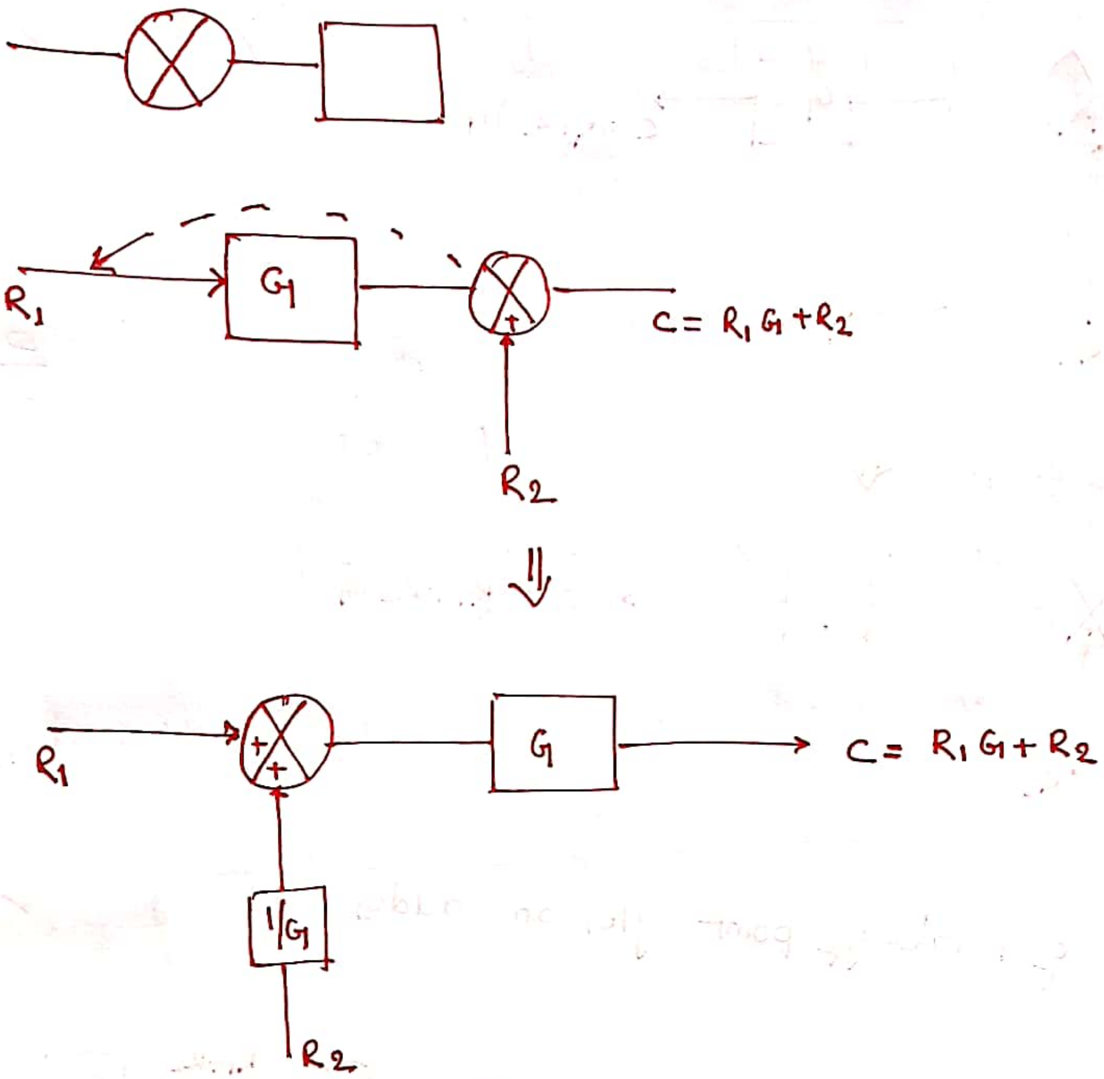
7. Shifting of a takeoff point after an adder.



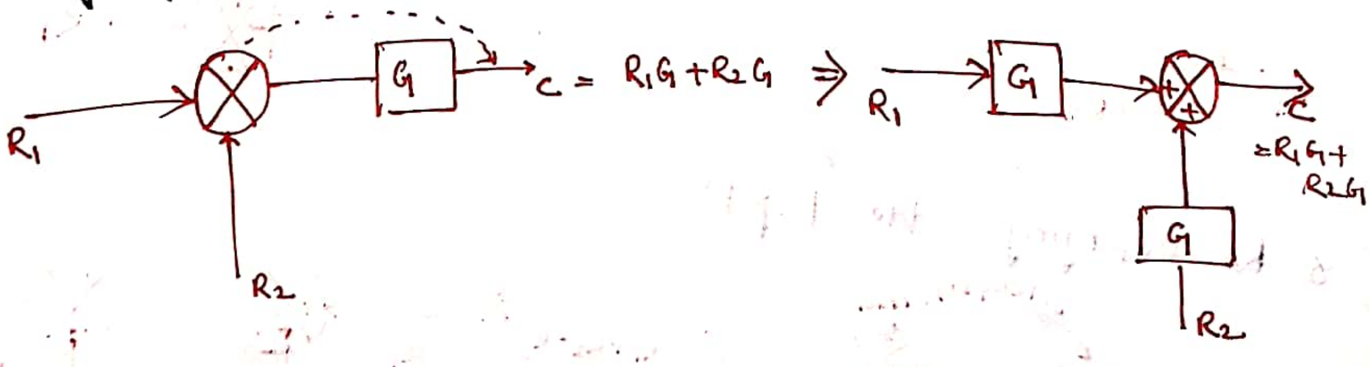
8. Re-arranging adder Input:



9. Shifting of an adder before a block:



10. shifting of an adder after a block:



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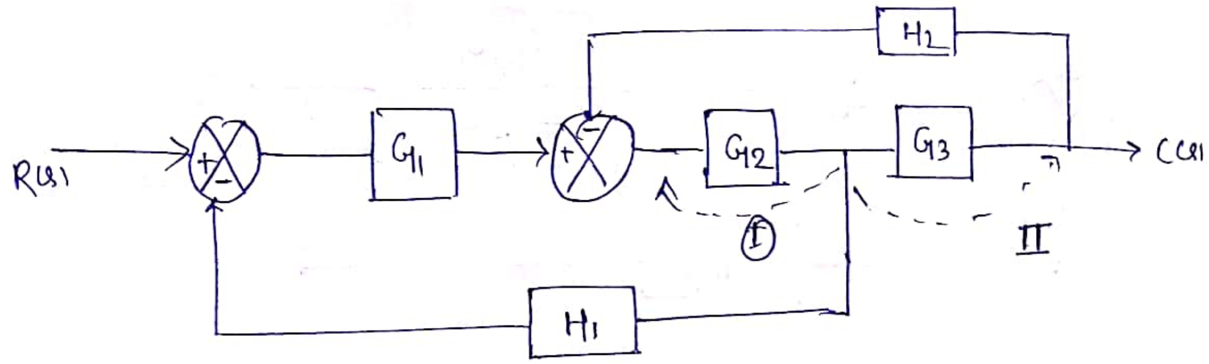
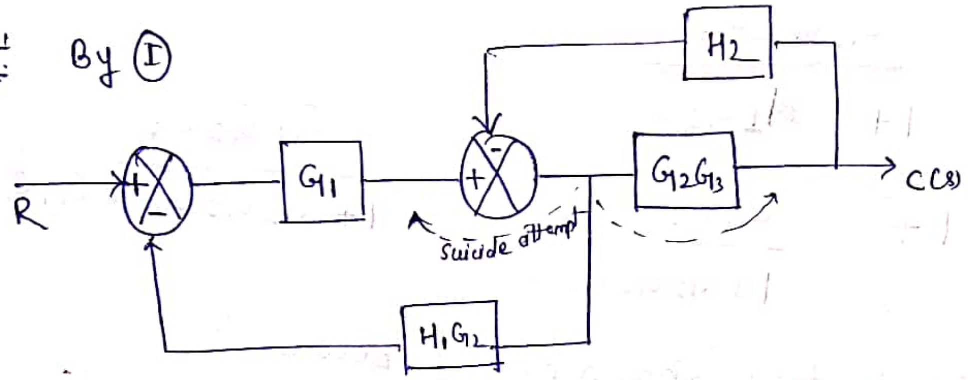
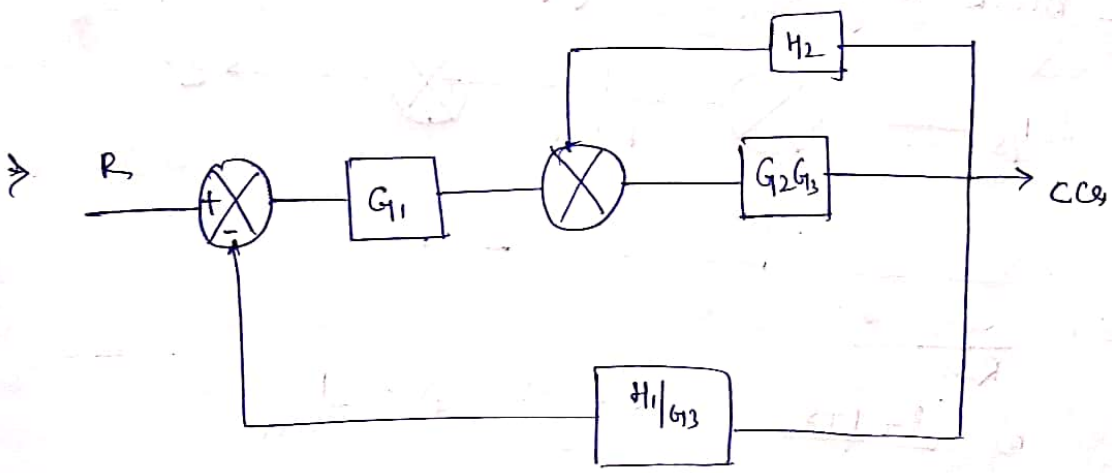
Q1. Find transfer function C(s)/R(s) for a diagram given below:-



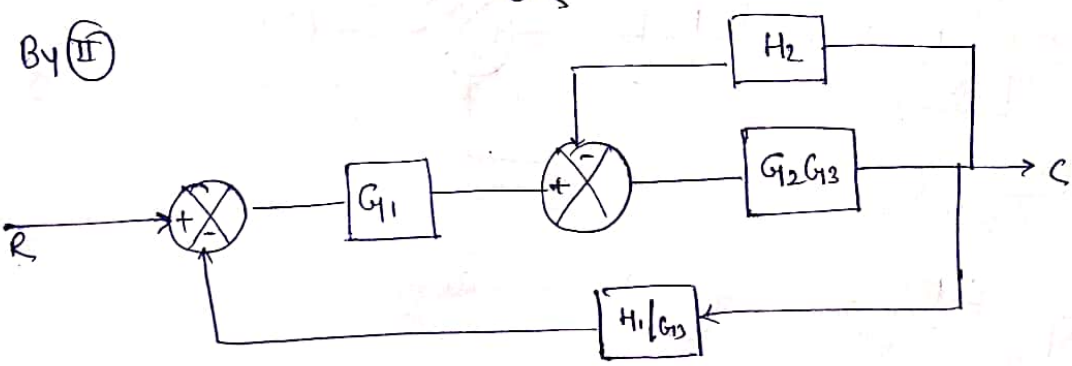
Sol:- By (I)





OR

By (II)



By (I) 2 step
By (II) 1 step

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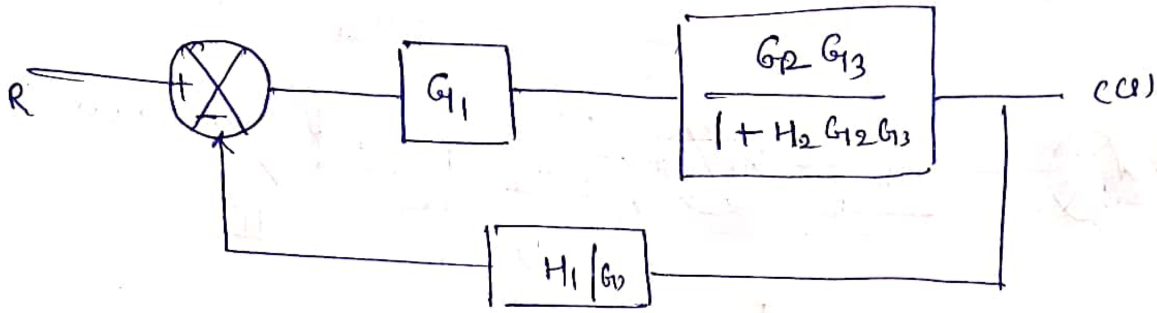
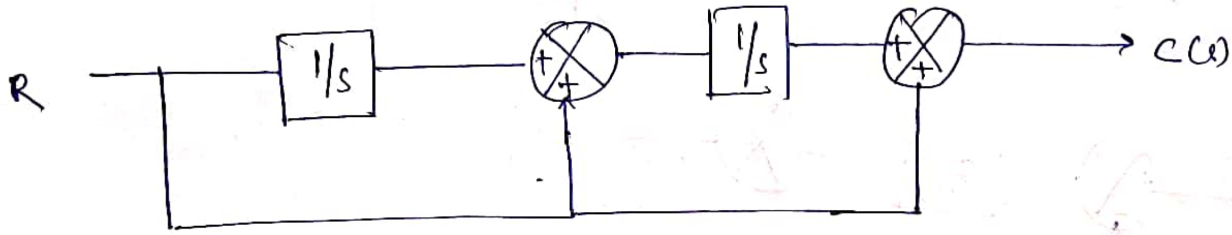


$$\frac{C}{R} = \frac{\dfrac{G_1G_2G_3}{1+H_2G_2G_3}}{1+\dfrac{G_1G_2G_3}{1+H_2G_2G_3}\times\dfrac{H_1}{G_3}} = \frac{G_1G_2G_3}{1+H_2G_2G_3+G_1G_2H_1}$$

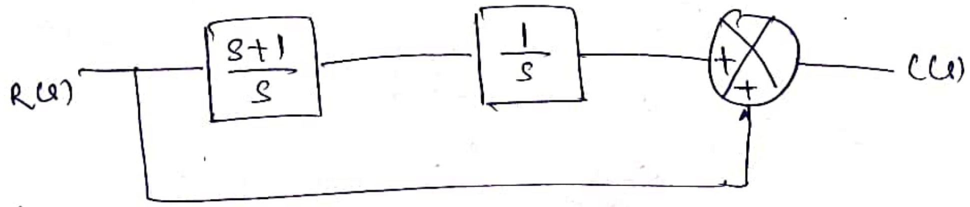
Q2. For the block diag. shown in fig. below



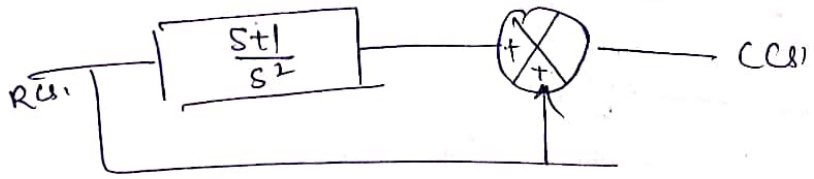
The xfer function $\frac{C(s)}{R(s)} = ?$

a) $\frac{s^2+1}{s^2}$ b) $\frac{s^2+1+s}{s^2}$ c) $\frac{s^2+s+1}{s}$ d) $\frac{1}{s^2+s+1}$

Sol

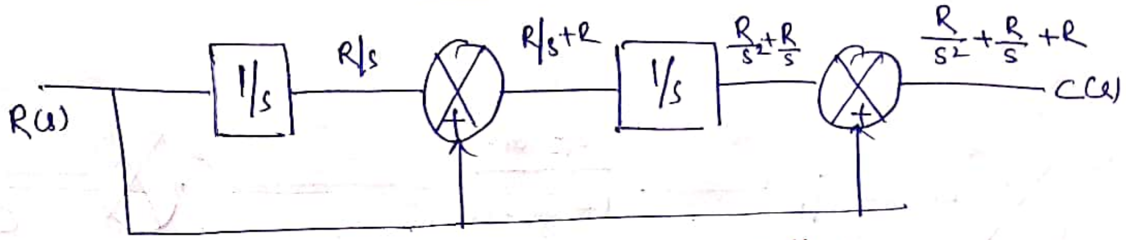


⇒



$$\Rightarrow \frac{C}{R} = \frac{s+1}{s^2}+1 \Rightarrow \frac{s+1+s^2}{s^2}$$

OR



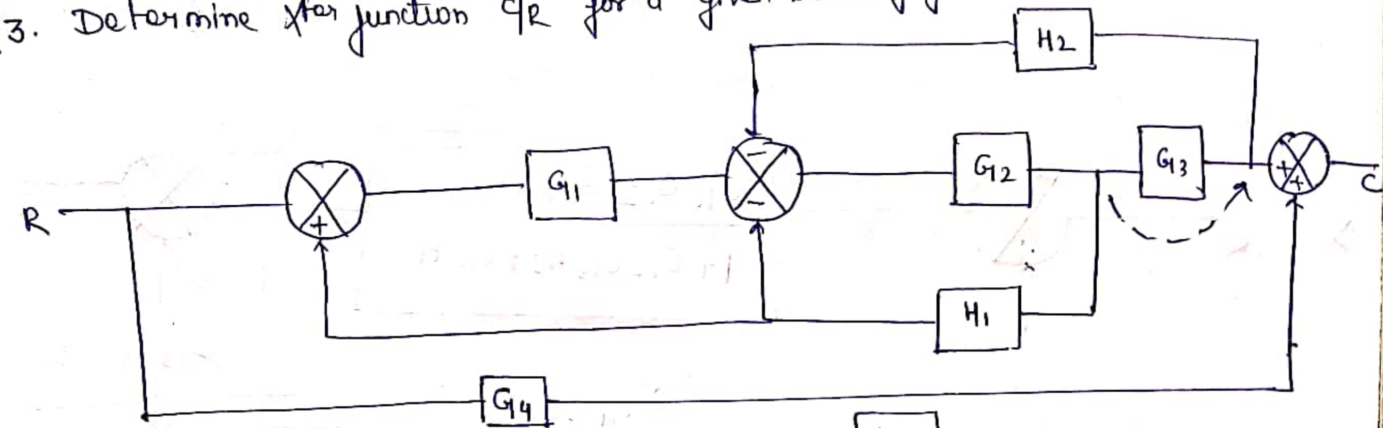
(Other good approach)

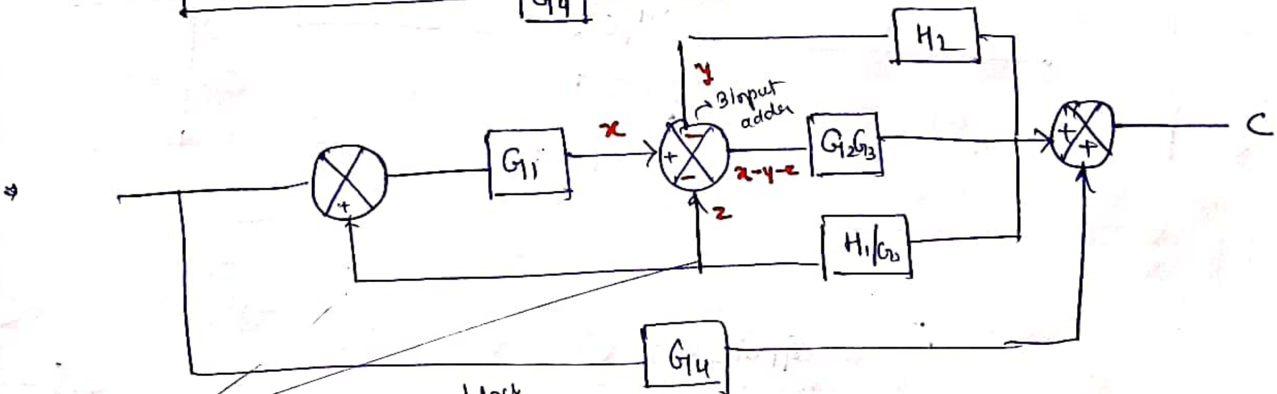
$$C = \frac{R}{s^2} + \frac{R}{s} + R$$

$$\frac{C}{R} = \frac{1}{s^2} + \frac{1}{s} + 1$$

$$\frac{C}{R} = \frac{s^2+s+1}{s^2}$$

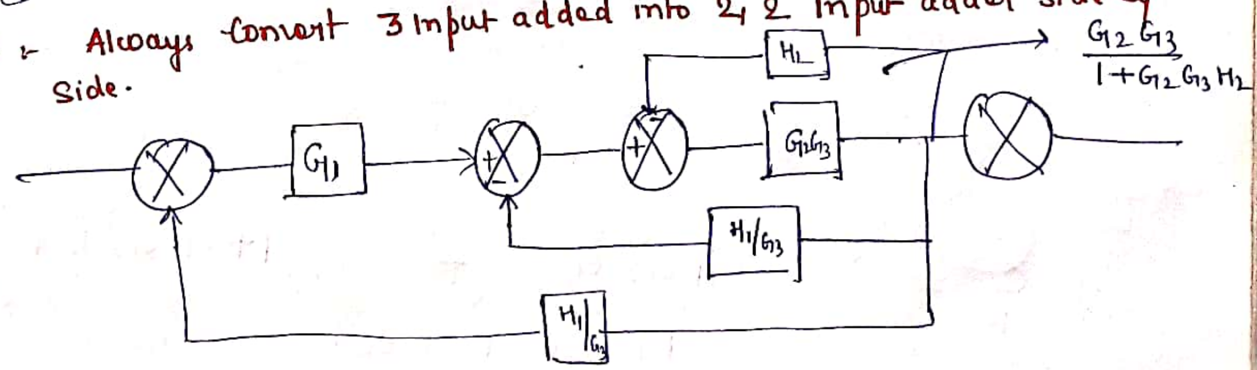
Q3. Determine xfer junction C/R for a given below figure.

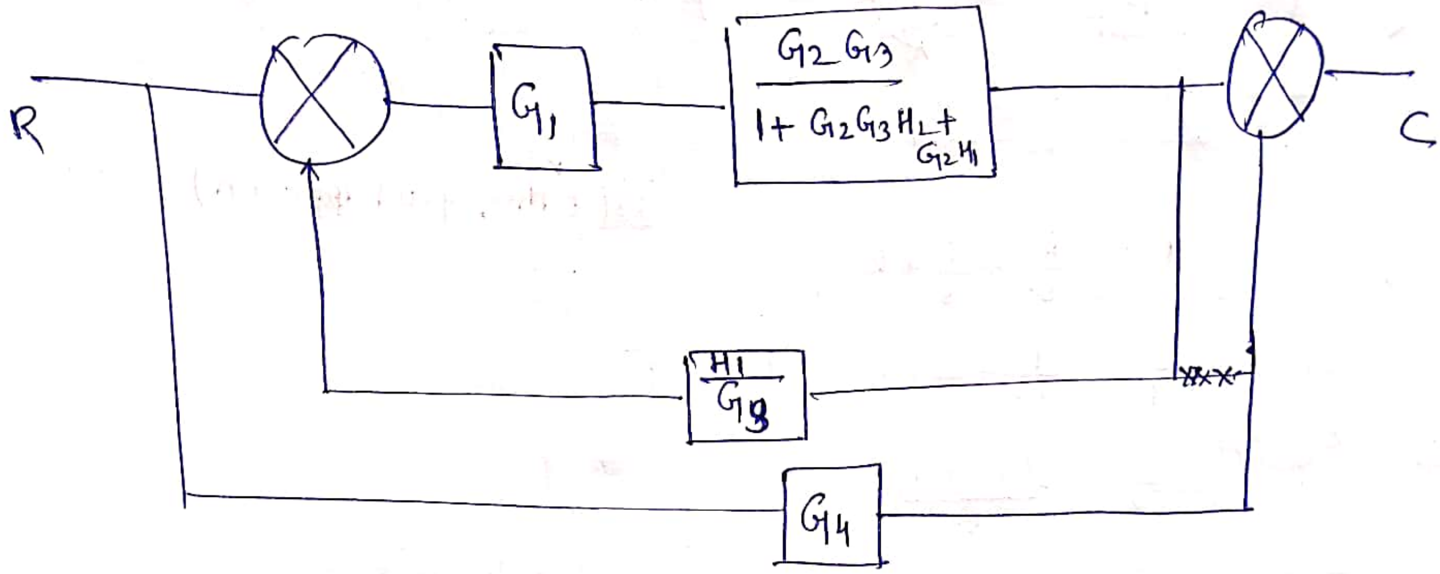


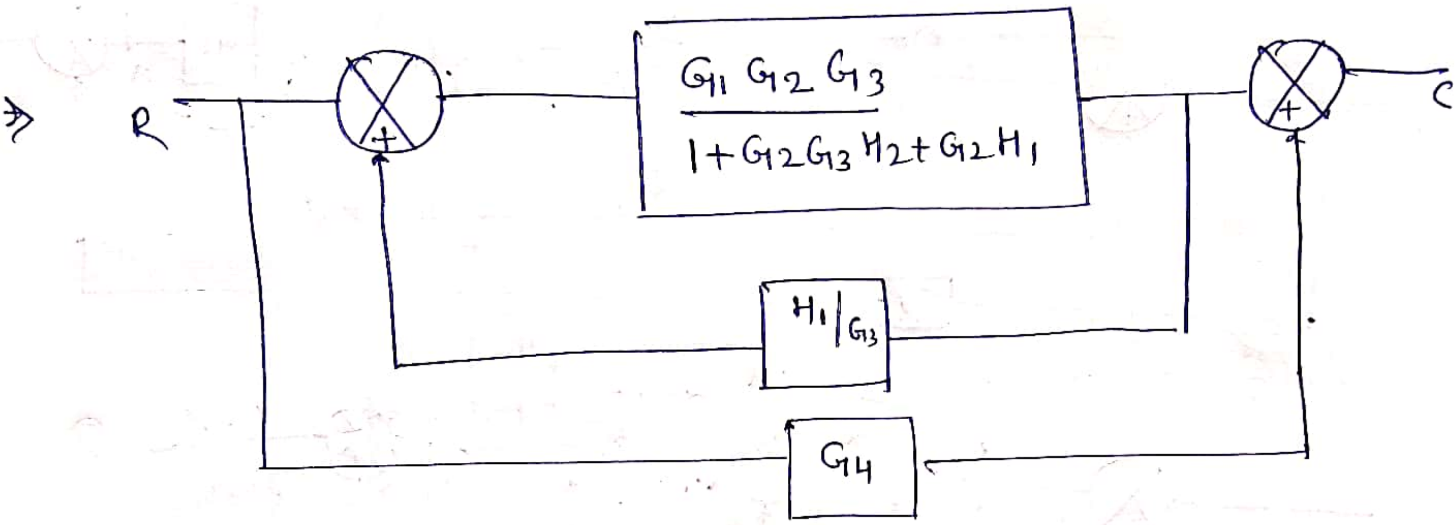


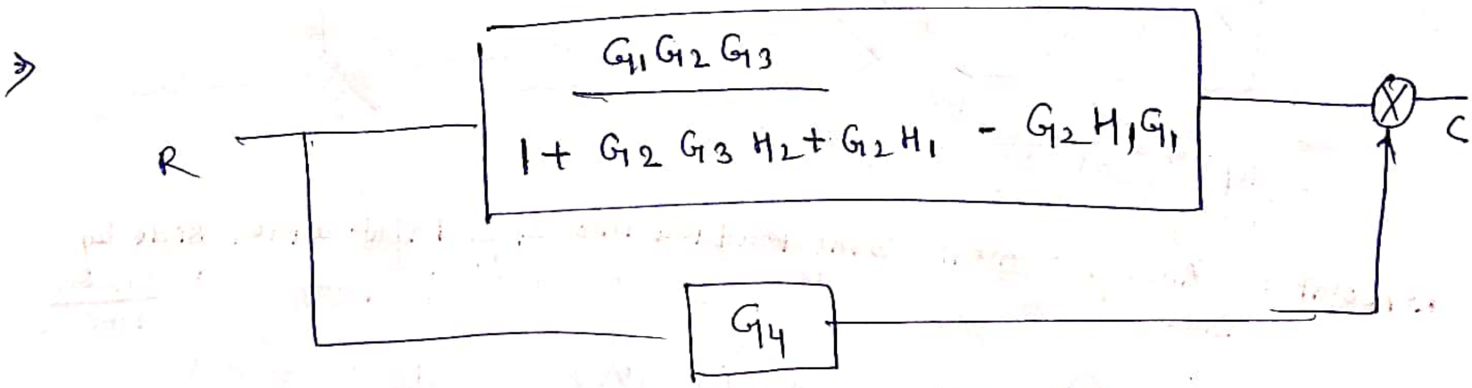
equipotential can be replaced by two block H1/G3

Keypoint :- Always Convert 3 input added into 2, 2 input adder Side by Side.







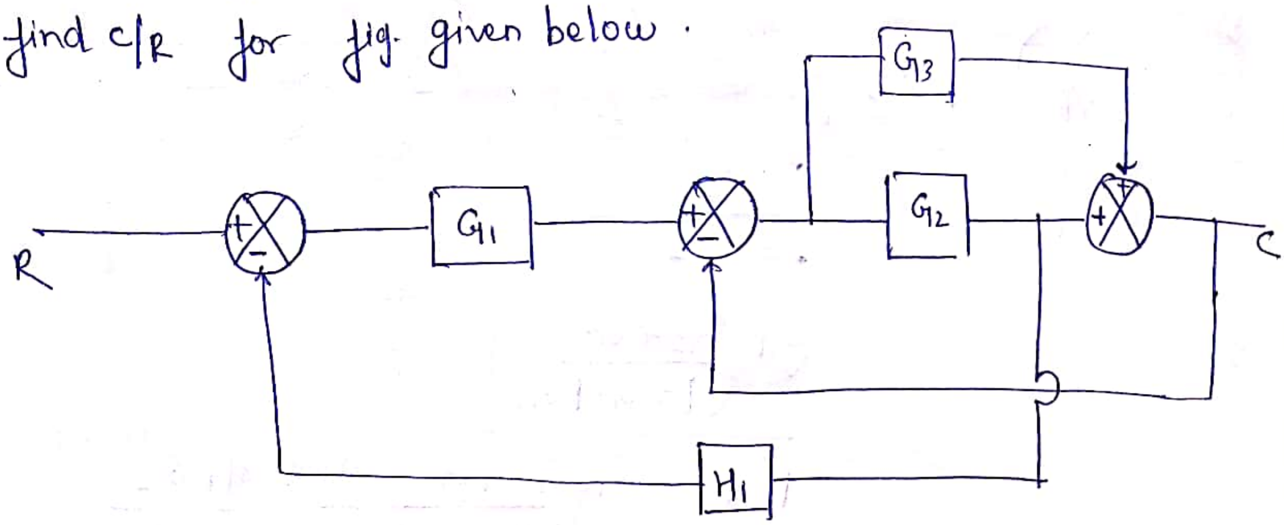


$$\frac{C}{R} = \frac{G_1 G_2 G_3 + G_4 \left(1 + G_2 G_3 H_2 + G_2 H_1 - G_2 H_1 G_1\right)}{1 + G_2 G_3 H_2 + G_2 H_1 - G_2 H_1 G_1}$$

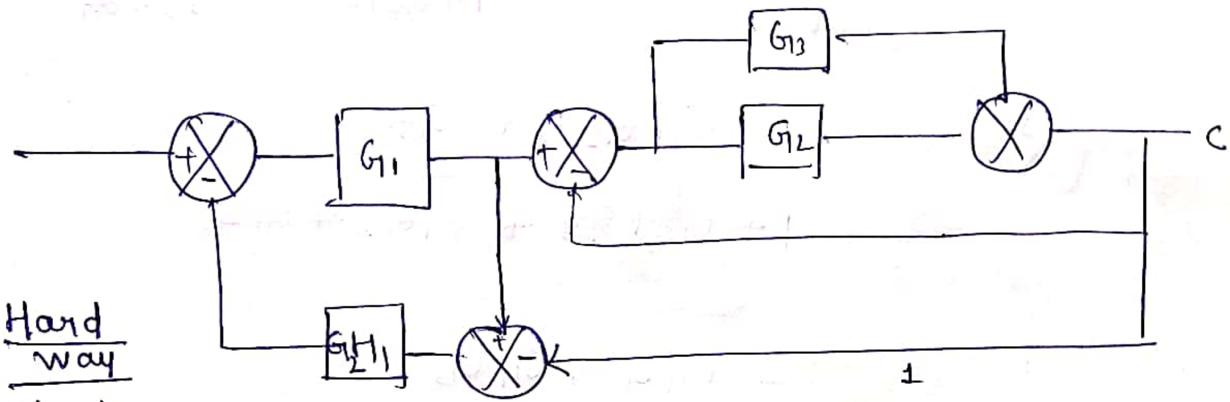
//

H.W
without converting 3 input adder in 2,2 input adder and check Ans.

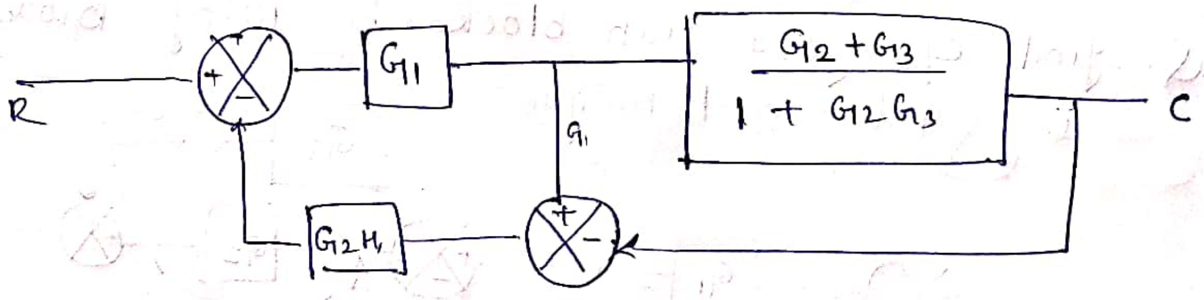
Q. find C/R for fig. given below.



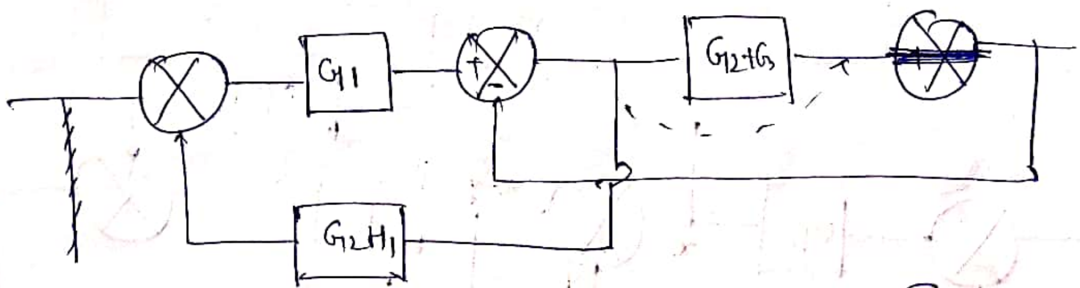
Sol

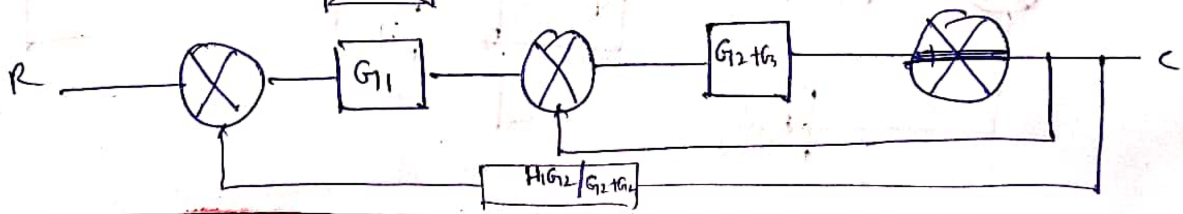


Hard way
Adder before, ayh



(Simple)
Other Method





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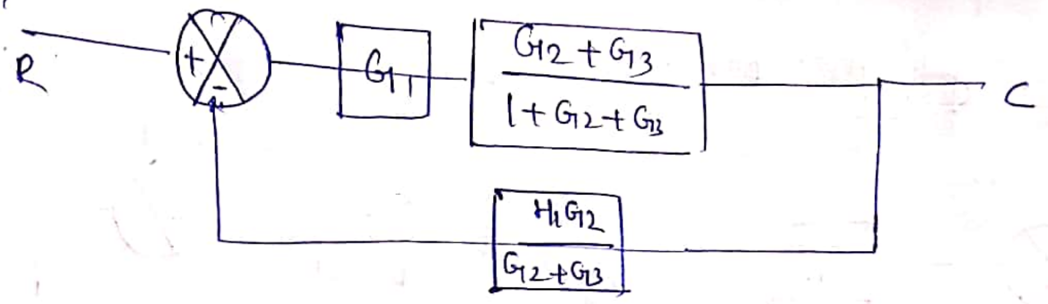
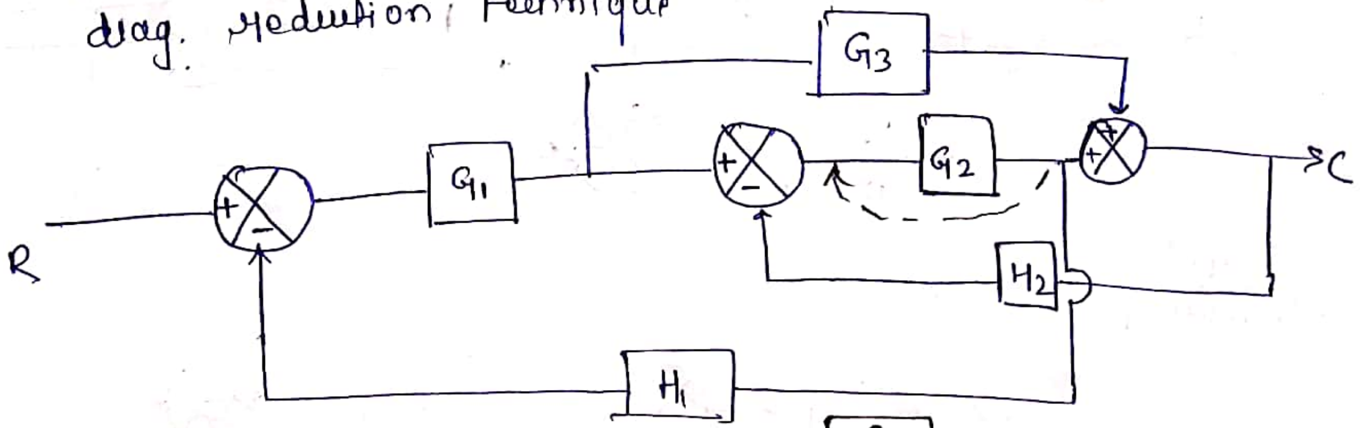


⇒
$$\frac{G_1\left(\frac{G_2+G_3}{1+G_2+G_3}\right)}{1+G_1\left(\frac{G_2+G_3}{1+G_2+G_3}\right)\times\frac{H_1G_2}{G_2+G_3}}$$

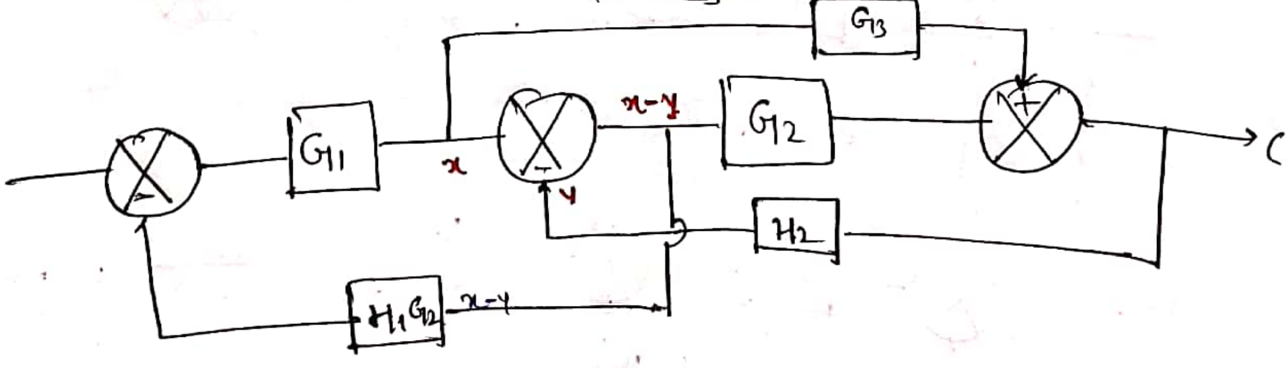
⇒
$$\frac{\cancel{G_1G_2+G_1G_3}}{1\cancel{+G_2+G_3}+\cancel{G_1G_2+G_1G_3}}$$

⇒
$$\frac{G_1G_2+G_1G_3}{1+G_2+G_3+G_1G_2H_1}$$ //

Q- find C/R for a given block by using Block diag. reduction technique



Sol

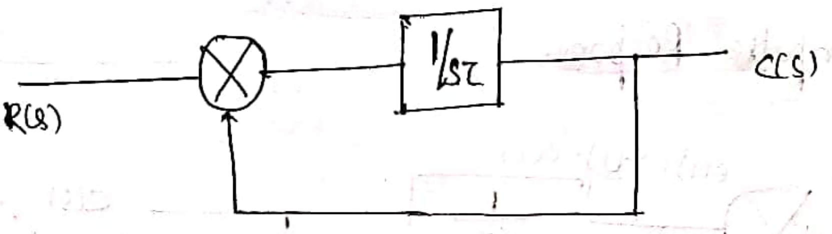


Chapter - 3

Time Response

1. Time Response discribe the performance of the System wrt time.
2. Time Response is a Combination of $x$sient and steady state Response.
3. Time Response gives the speed and error of the System.

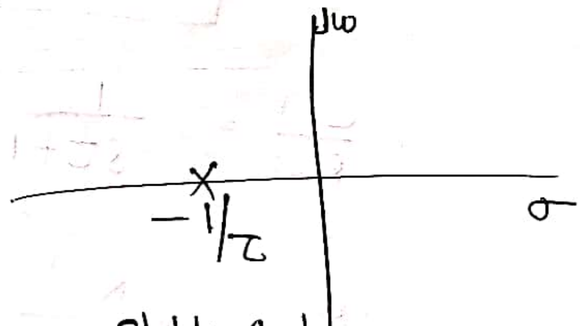
* Analysis of first Order System -



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$T(s) = \frac{1/s\tau}{1+1/s\tau}$$

$$T(s) = \frac{1}{1+s\tau}$$



Bandwidth $= \frac{1}{T_c} = \frac{1}{\tau}$ rad/sec

Stable System

Time Constant $= \tau$

first order System

If DC gain = 1

ii) τ = 2sec

$$T(s) = \frac{1}{1+2s}$$

first order System

i) DC gain 10

ii) τ = 2sec

$$T(s) = \frac{1}{1+2s} \times$$

$$T(s) = \frac{10}{1+2s} \checkmark$$

$$T(s) = \frac{k e^{-T_d s}}{1+s\tau} \qquad \text{DC gain} = k$$

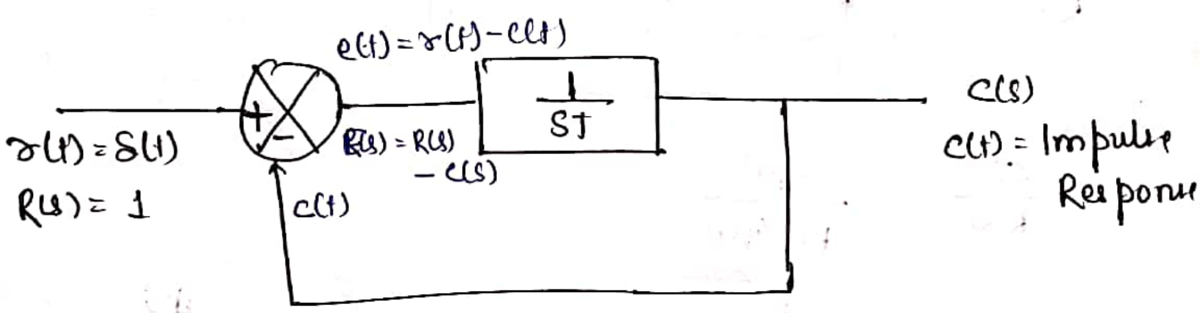
iii) DC gain = 5

$\tau = 3$ sec

Delay time = 2 sec

$$T(s) = \frac{5e^{-2s}}{1+3s} \quad //$$

Case I   Unit Impulse Response



$$\frac{C(s)}{R(s)} = \frac{1}{s\tau+1}$$

$$C(s) = \frac{R(s)}{1+s\tau}$$

$$C(s) = \frac{1}{1+s\tau}$$

$$c(t) = \frac{1/\tau}{s+1/\tau} = \frac{1}{\tau}e^{-t/\tau}u(t) \quad //$$

$$e(t) = r(t) - c(t)$$

$$E(s) = R(s) - C(s)$$

steady state error

$$e_{ss} = \lim_{t \to \infty} e(t)$$

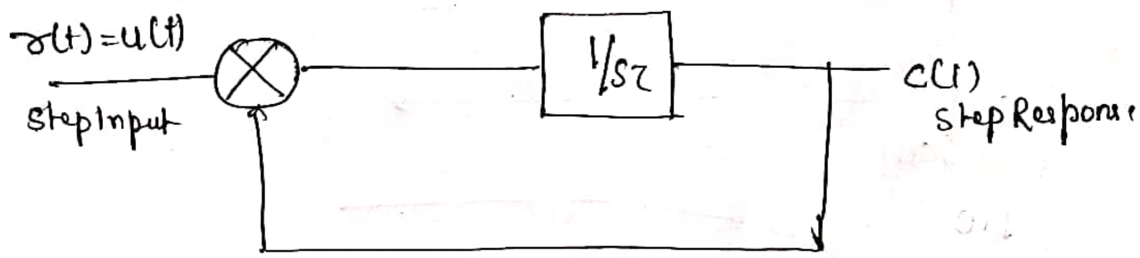
$$e_{ss} = \lim_{s \to 0} s\,E(s)$$

$$e(t) = \delta(t) - \frac{1}{\tau} e^{-t/\tau} u(t)$$

Key point

1. Steady state error doesnot exist for Impulse response because we cannot compare Input & output at $t = \infty$.

Case II Unit Step Response :



$$\frac{c(s)}{R(s)} = \frac{1}{s\tau + 1}$$

$$C(s) = \frac{1/s}{1 + s\tau}$$

$$= \frac{1/s}{\frac{\tau}{1}\left(s + 1/\tau\right)}$$

$$= \frac{1/\tau}{s\left(s + 1/\tau\right)}$$

$$C(s) = \frac{1}{s} - \frac{1}{s + 1/\tau}$$

$$c(t) = 1 - e^{-t/\tau}$$

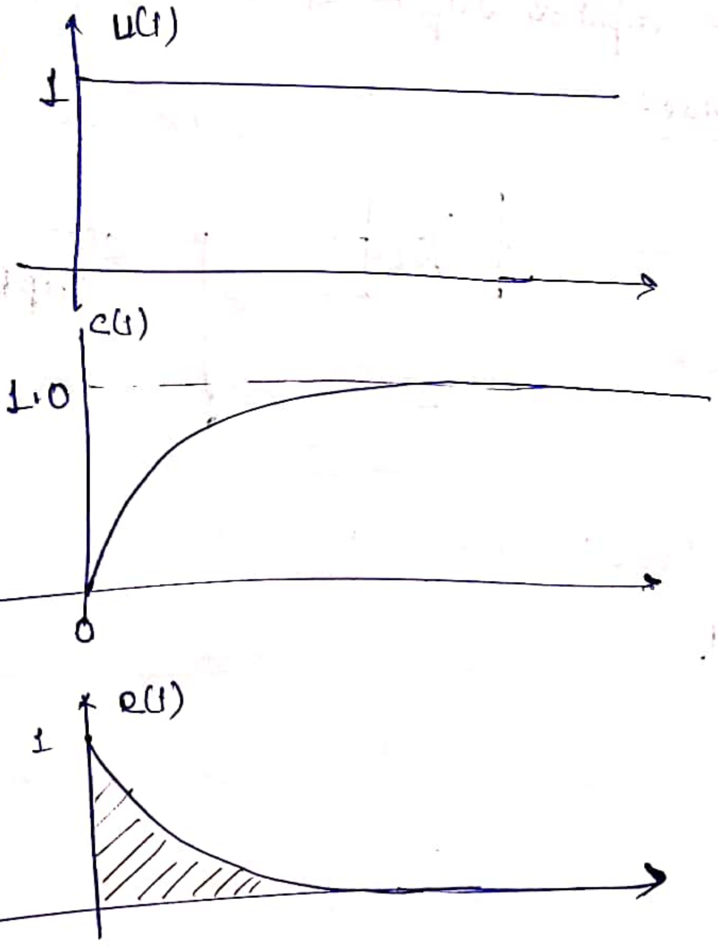
$$c(t) = u(t) - e^{-t/\tau} u(t)$$

$$c(t) = \left(1 - e^{-t/\tau}\right) u(t)$$

error $e(t) = u(t) - u(t) + e^{-t/\tau} u(t)$

$$e(t) = e^{-t/\tau} u(t)$$

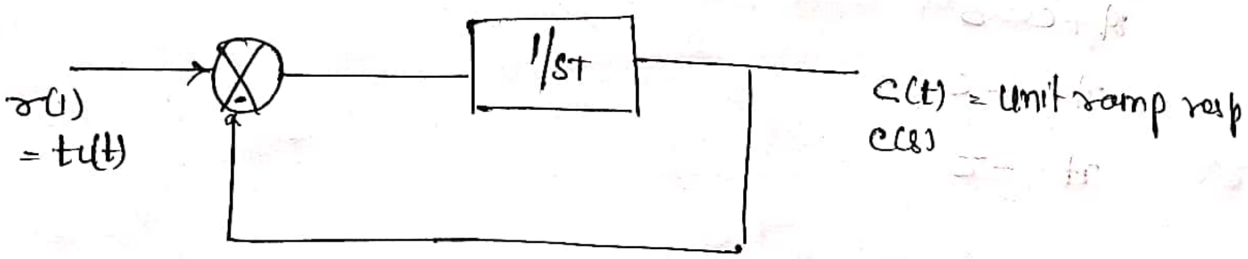
$r(t) = u(t)$



$$e_{ss} = \lim_{t \to \infty} e^{-t/\tau} u(t)$$

$$e_{ss} = 0$$

Case III Unit Ramp Response



$$\frac{C(s)}{R(s)} = \frac{1}{1+sT}$$

$$C(s) = \frac{1}{s^2(1+s\tau)}$$

$$C(s) = \frac{1/\tau}{s^2(s+1/\tau)}$$

$$\frac{1}{\tau} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1/\tau}$$

$s = 0$

$$\frac{1}{\tau} = A\left(s+\frac{1}{\tau}\right)s + B\left(s+1/\tau\right) + Cs^2$$

$$\frac{1}{\tau} = B\left(\frac{1}{\tau}\right)$$

$B = 1$

$s = -1/\tau$

$$\frac{1}{\tau} = C\frac{1}{\tau^2}$$

$C = \tau$

s^2 Compare

$A + C = 0$

$A = -C$

$A = -\tau$

$$C(t) = \frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau}{s + 1/\tau}$$

$$c(t) = -\tau u(t) + t u(t) + \tau e^{-t/\tau} u(t)$$

$$e(t) = r(t) - c(t)$$

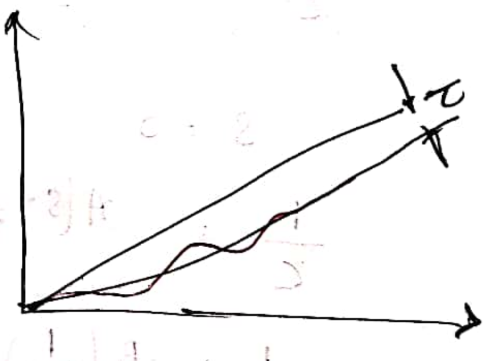
$$= t u(t) + \tau(u(t)) - t u(t) - \tau e^{-t/\tau} u(t)$$

$$e(t) = \tau u(t) - \tau e^{-t/\tau} u(t)$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{t \to \infty} \tau u(t) - \tau e^{-t/\tau} u(t)$$

$$= \tau u(\infty)$$

$$e(\infty) = e_{ss} = \tau$$



Case IV parabolic Input

$$R(s) = 1/s^3 \qquad r(t) = \frac{t^2}{2}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s\tau + 1}$$

$$C(s) = \frac{1/\tau}{s^3(s + 1/\tau)}$$

Chapter-4 Routh Hurwitz Stablity

- Routh Hurwitz describe the stablity of Control System.
- Stablity is defined in two parameters—
  1) Absolute stablity
  11) Relative stablity.
- In Case of Absolute stablity, It is defined in terms of location of poles & for CLS to be stable all pole should lie on left hand side of S-plane.

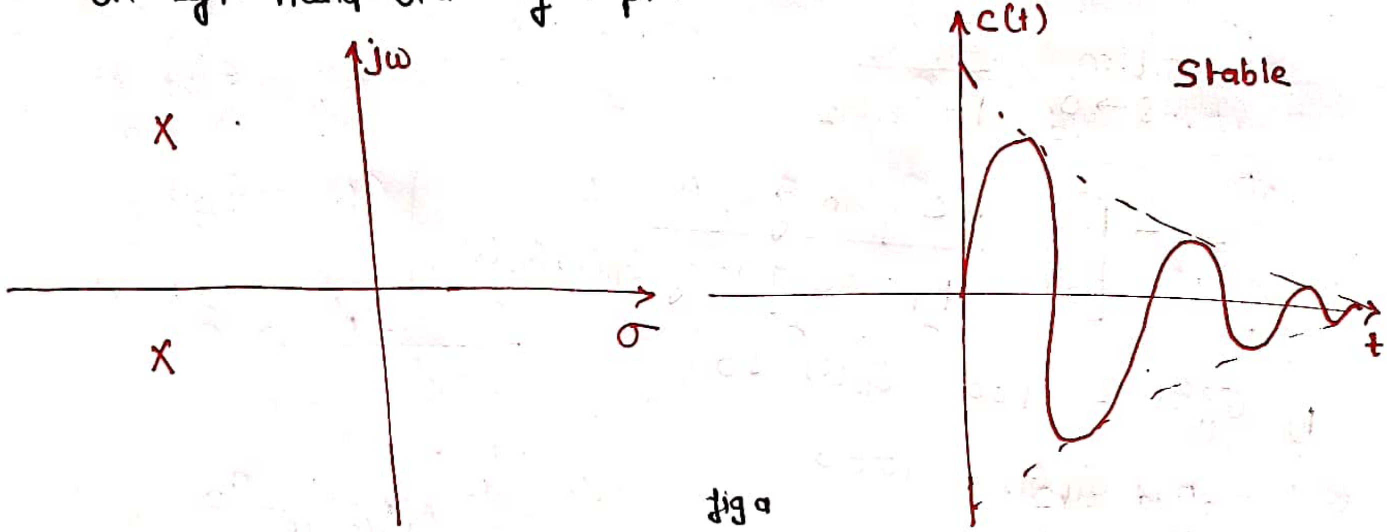
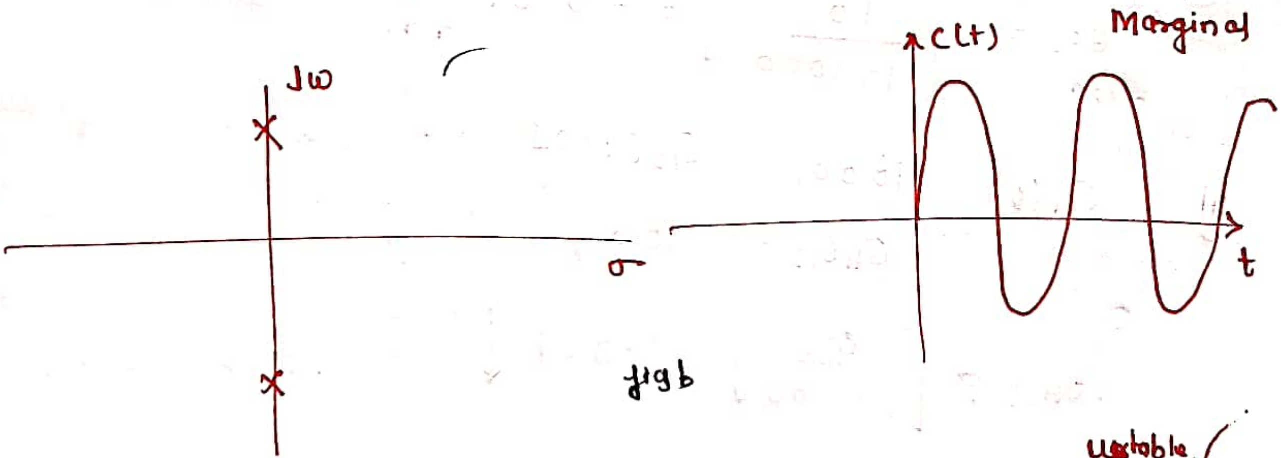


fig a



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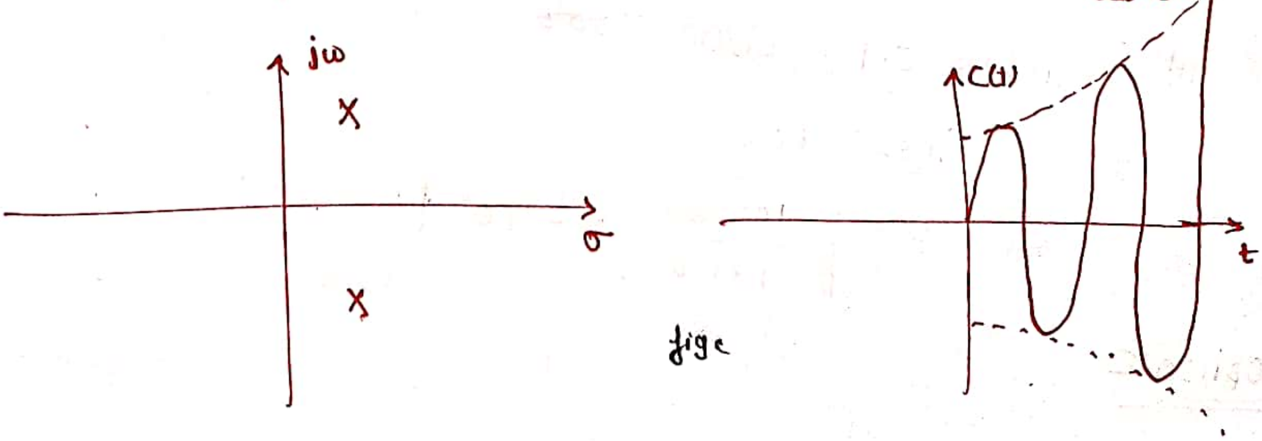

fig c

• In case of Relative stablity it is defined in terms of damping ration, Gain Margin, phase Margin.

The Properties of R-H stablity Criterion:

1. The R-H Stablity Criterion is applicable for closed loop System because formation of Routh table we use characteristic eqn which is defined for closed loop system.

Key point :- It is also applicable for determining stablity of OLTF or stablity of any polynomial

2. For closed loop system to be stable, all elements of first column in Routh table must be same sign (+ve / -ve).

*3. Number of sign changes in first Column represent the number of poles in RHS of s plane. & System will become Unstable.

4. If any power of 's' is missing in characteristic eqn that represents the presence of atleast one pole in the RHS of s plane & System will become Unstable.

5. For R-H Stablity, Coefficient of characteristic equation must be real that means neither Imaginary nor Complex nor Sinusoidal

6. If characteristic eqn Contain only even power of s that represent presence of pole on Imaginary axis & System become Marginal stable.

7. If Imaginary axis Contain repeated poles then System will become Unstable.

*8. Routh table give Information of number of Right-hand pole in the s plane.

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Formation of Routh table:-

Consider characteristic eqn -

$$1 + G(s)H(s) = a_0 s^6 + a_1 s^5 + a_2 s^4 + a_3 s^3 + a_4 s^2 + a_5 s^1 + a_6 s^0 = 0$$

Given Rows

s^6	a_0	a_2	a_4	a_6	
s^5	a_1	a_3	a_5	0	
s^4	b_1	b_2	b_3	0	
s^3	c_1	c_2	c_3	0	
s^2	d_1	d_2	0	0	
s^1	$e_1 = \frac{d_1 c_2 - c_1 d_2}{d_1}$	$c_3 = e_2$	0	0	
s^0	$\frac{e_1 d_2 - d_1 e_2}{e_1}$				

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 \times 0}{a_1} = a_6$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

$$c_3 = \frac{b_1 \times 0 - 0}{b_1} = 0$$

$$d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1}$$

$$d_2 = \frac{c_1 b_3 - b_1 c_3}{c_1}$$

eg 1 $P(s) = s^3 + 2s^2 + 8s + 10$

eg 2 $P(s) = s^3 + 2s^2 + 8s + 20$

eg 3 $P(s) = s^3 + 2s^2 + 8s + 16$

Sol 1) $P(s) = s^3 + 2s^2 + 8s + 10$

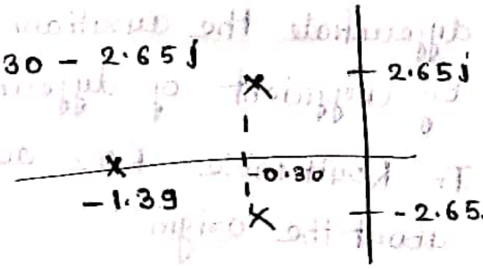
s^3	+1	8
s^2	+2	10
s^1	+3	
s^0	+10	

s^3	1	no sign change
s^2	2	no sign change
s^1	3	no sign change
s^0	10	

i) Number of Sign changes in first Column = 0
ii) Number of Right hand poles = 0
iii) Number of left hand poles = 3
iv) Number of Imaginary axis poles = 0
v) P(s) = stable

$s^3 + 2s^2 + 8s + 10 = 0$ {using Calculator}

$s = -1.39$, $-0.301 + 2.65j$, $-0.30 - 2.65j$



eg 2. $P(s) = s^3 + 2s^2 + 8s + 20$

Sol

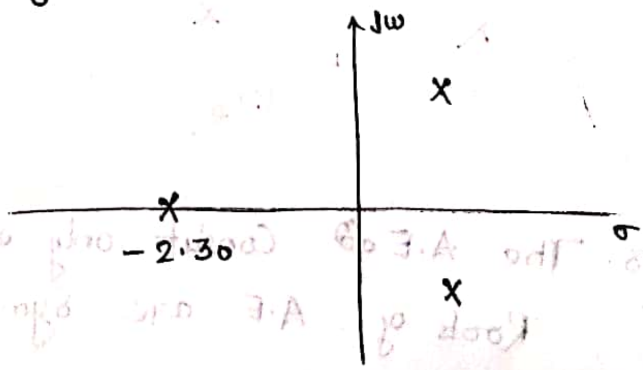
s^3	1	8
s^2	2	20
s	-2	
s^0	20	

s^3	1	No
s^2	2	1 Sign change
s	-2	1 Sign change
s^0	20	

i) Number of Sign changes in first Column = 2
ii) Number of Right hand pole = 2
iii) Number of left hand pole = 1
iv) Number of Imaginary pole = 0
v) P(s) = unstable.

$s^3 + 2s^2 + 8s + 20 = 0$

$s = -2.30$, $0.18 + j2.94$
$0.18 - j2.94$



eg 3 $P(s) = s^3 + 2s^2 + 8s + 16$

Sol

s^3	1	8
s^2	2	16
s^1	0	
s^0		

$\boxed{s^1 \quad 0}$ (Row of zeros) ROZ

Row of Zero (ROZ)

1. Whenever in Routh table the entire row become zero then we require to form Auxiliary eqn and differentiate the auxiliary equation and replace the '0' by coefficient of differential Auxiliary Eqn.

2. In Routh table ROZ occur when poles are located symmetrical about the origin.

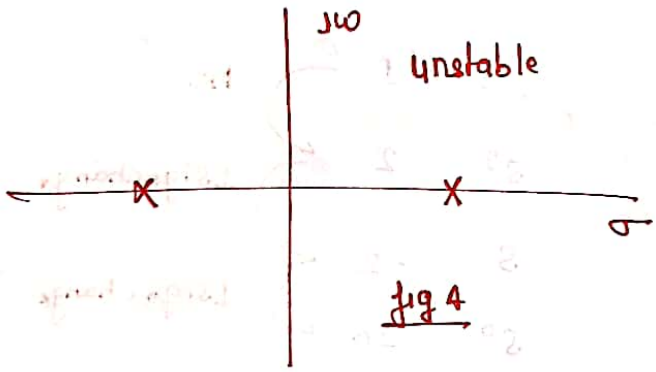
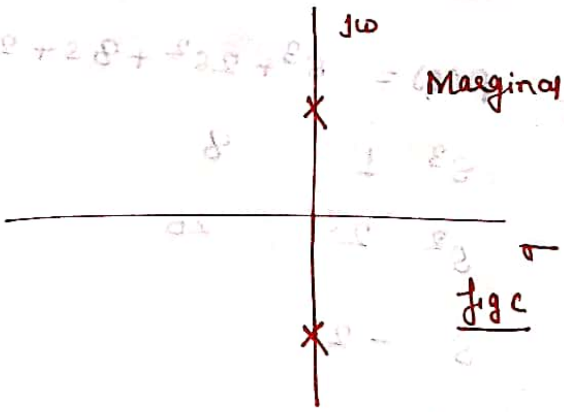


fig a



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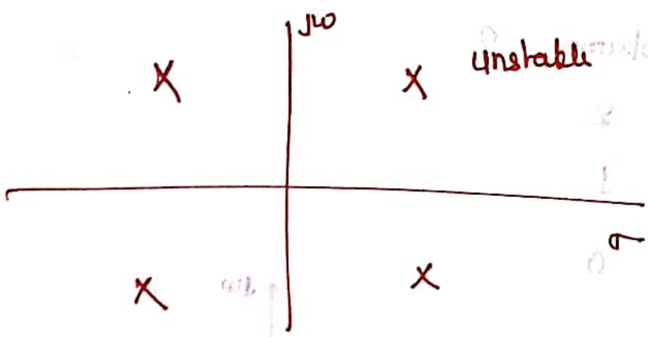

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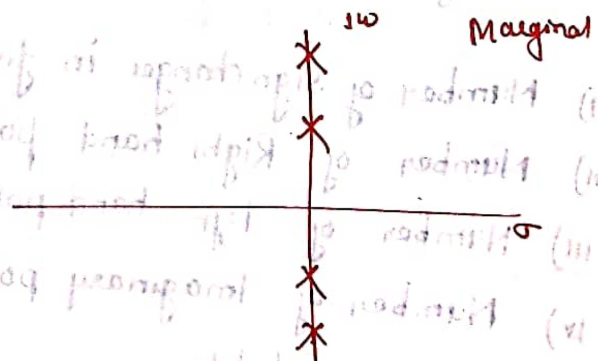


fig d

3. The A.E eqn consist only even power of 's' because roots of A.E are symmetrical about origin.

4. The Roots of A.Eqⁿ are poles of Closed loop System (Normally we consider closed loop System in Control System).

** 5. The ROZ occurs only odd power of s.

6. Whenever in Routh table only 1 ROZ occur & all the Coefficient in first Column are positive or same sign then System is marginal stable. because the poles are on Imaginary axis which are not repeated. (Conditional)

7. If Imaginary axis Contain repeated poles then System will be Unstable.

* 8. If more than one ROZ occur that represents presence of repeated poles & No. of Repeated poles = No. of ROZ.

9. Imaginary axis poles is related with Undamped Natural frequency 'ω_n'.

10. Routh table gives the exact location of Imaginary axis poles.

eg.3 $\quad p(s) = s^3 + 2s^2 + 8s + 16$

s^3	1	8
s^2	2	16
s^1	0	
s^0		

$\longleftarrow$ A.Eqⁿ (s^2 row)

$\longleftarrow$ ROZ (s^1 row)

$A(s) = 2s^2 + 16$

$\frac{dA(s)}{ds} = 4s + 0 = 4s$

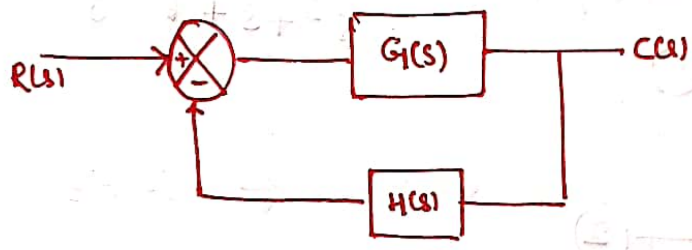
Replace $\frac{dA(s)}{ds}$ Coeff with ROZ

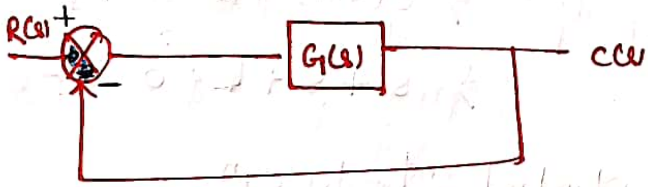
s^3	1	8
s^2	2	16
s^1	4	0.
s^0	16	0

Chapter - 5

Root Locus

- In Root Locus, Root represents poles of closed Loop X^{fer} function & locus represents path of closed Loop poles.
- Root Locus describes the path of poles of closed Loop X^{fer} function for different values of Gain of open loop X^{fer} function (k).
- Root locus is valid for negative feedback System.





$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

char. eq^n $1+GH=0$

char. eq^n $1+G=0$

- for negative feed back System Range of k is 0 to ∞ i.e $0<k<\infty$
- Inverse Root locus or Complementary Root locus is valid for positive feed back System.

Positive feedback System :-

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)\,H(s)}$$

Non unity +ve feed back

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)}$$

unity positive feed back

char. eq^n $1-G(s)H(s)=0$

Char. eq^n $1-G(s)=0$

- for positive feedback System range of k :- $-\infty<k<0$
- MATLAB Software is used for Sketching the Root Locus. Before MATLAB, SPIRULE Instrument is used for sketching Root locus.

eg. $G(s)H(s) = \frac{k}{s(s+1)}$ or $G(s) = \frac{k}{s(s+1)}$

$T(s) = \frac{G(s)}{1+G(s)H(s)} = ?$ $\quad$ $T(s) = \frac{G(s)}{1+G(s)} = ?$

$C.E = 1 + GH = 0$ $\quad$ $C.E = 1 + G = 0$

$= 1 + \frac{k}{s(s+1)} = 0$ $\quad$ $= 1 + \frac{k}{s(s+1)} = 0$

$\Rightarrow s(s+1) + k = 0$ $\quad$ $= s(s+1) + k = 0$

$\Rightarrow s^2 + s + k = 0$ —— (1) $\quad$ $= s^2 + s + k = 0$

Standard 2nd order eqn

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \text{ —— (2)}$$

On Comparison (1) & (2)

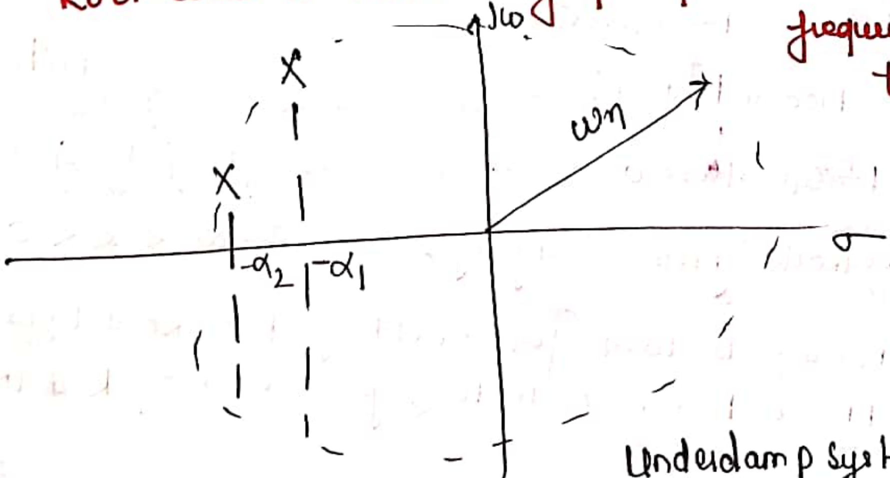
$2\xi\omega_n = 1$ & $\omega_n^2 = k$

$\xi = \frac{1}{2\sqrt{k}}$ = Variable $\quad$ $\omega_n = \sqrt{k}$ = Variable

$\Rightarrow \xi\omega_n = \frac{1}{2\sqrt{k}}\sqrt{k} = \frac{1}{2}$ = Constant

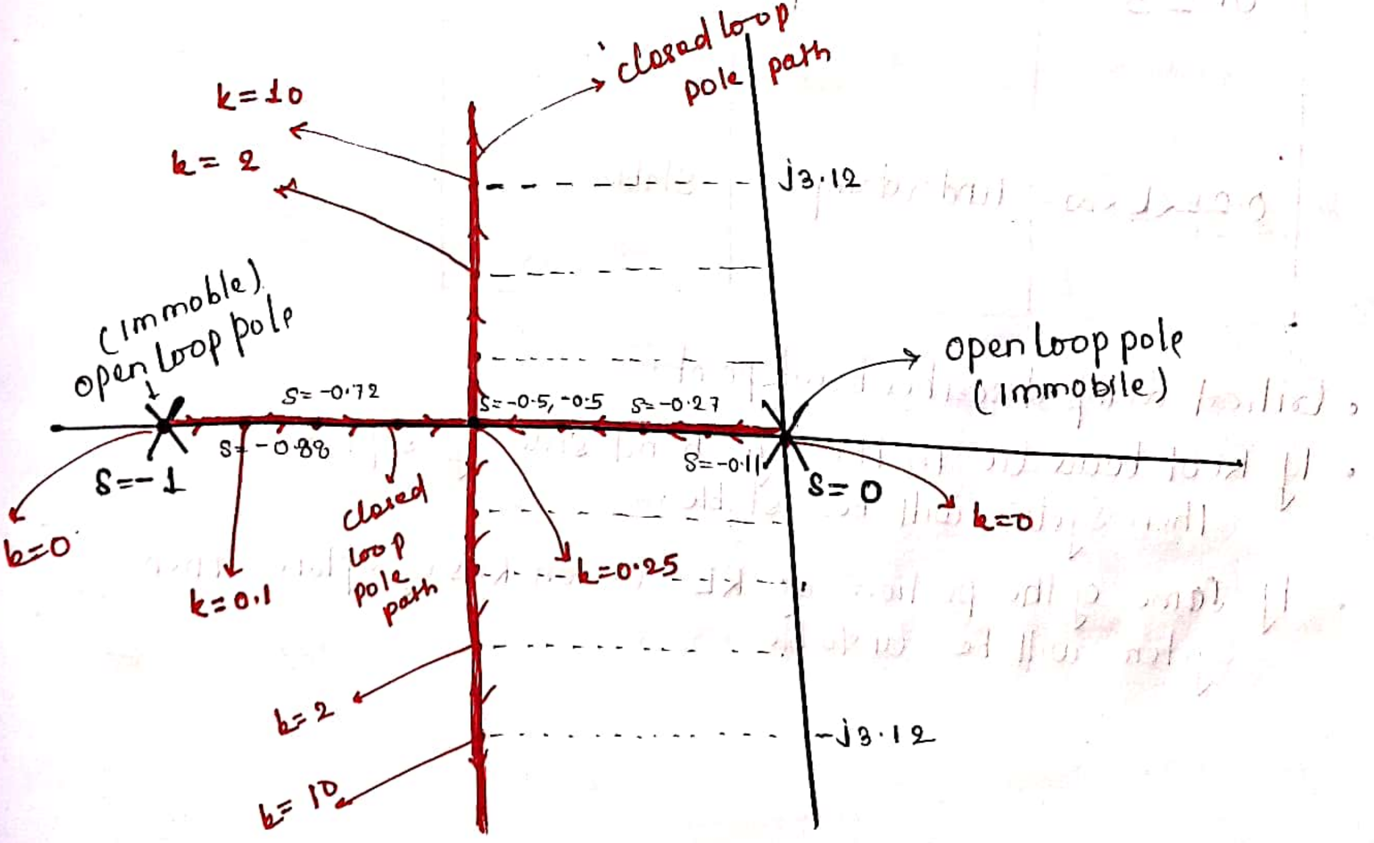
Hence. $T = 2$ Sec

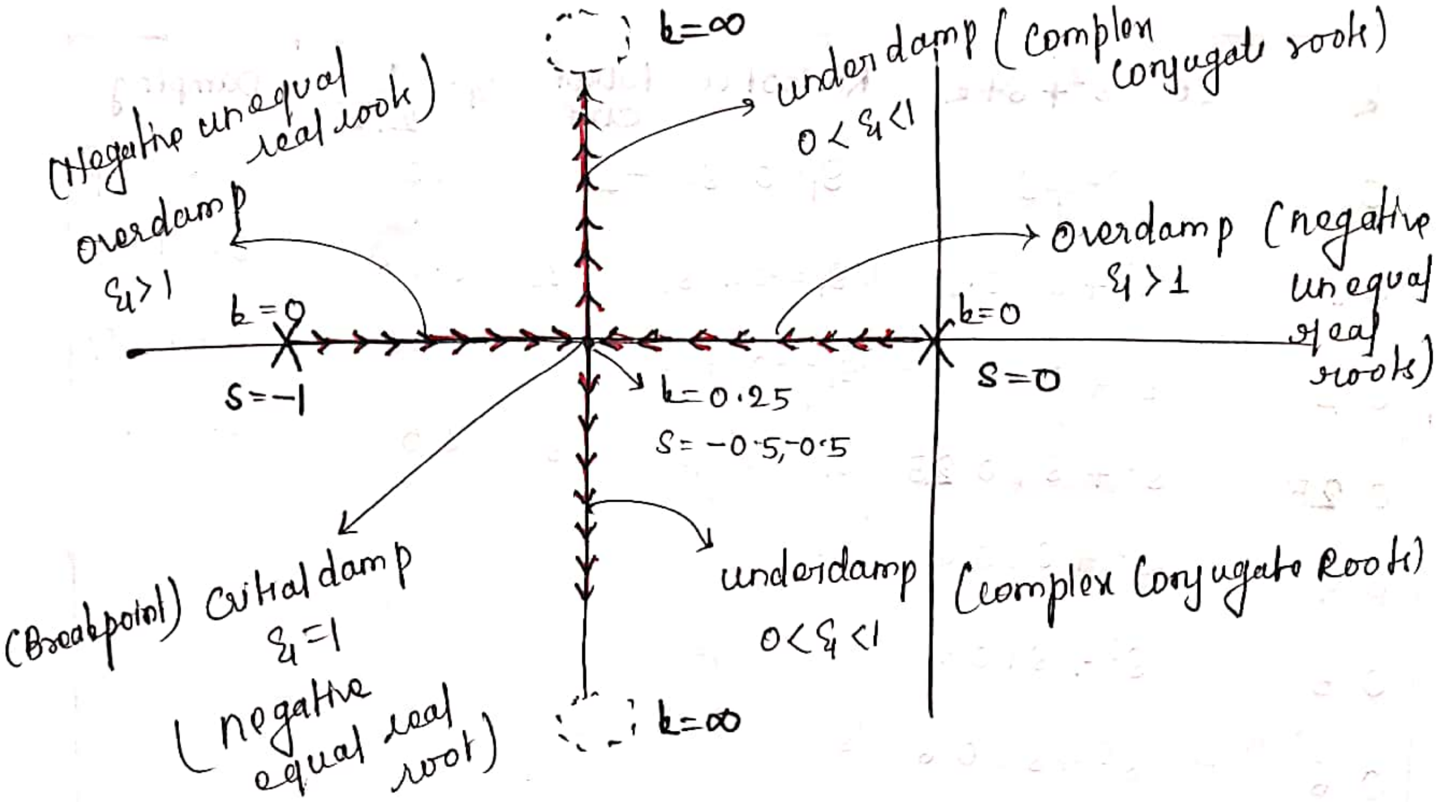
Key point: Root Locus is based on frequency domain Analysis because frequency is Variable & time Constant is fixed.



Underdamp system

k	$CE = s^2+s+k$	Roots of CE = Pole of CLTF	$\xi = \frac{1}{2\sqrt{k}}$	Damping
0	s^2+s	$s_1=0,\ s_2=-1$	∞	Overdamp
0.1	$s^2+s+0.1$	$s_1=-0.11\ \ s_2=-0.88$	1.58	Overdamp
0.2	$s^2+s+0.2$	$s_1=-0.27,\ s_2=-0.723$	1.11	Overdamp
0.25	$s^2+s+0.25$	$s_1=-0.5,\ s_2=-0.5$	1.0	Critical
0.4	$s^2+s+0.4$	$s_1=\quad ,s_2=$		
0.5	$s^2+s+0.5$	$s_1=\quad ,s_2=$		
0.6	$s^2+s+0.6$	$s_1=\quad ,s_2=$		
1.0	s^2+s+1	$s_1=\quad ,s_2=$		
2.0	s^2+s+2	$s_1=\quad ,s_2=$		
10.0	s^2+s+10	$s_1=-0.5+j3.12,\ s_2=-0.5-j3.12$		





k	Damping	Stablity
0	Overdamp	stable
0.25	Critical damp	stable
$0.25 < k < \infty$	under damp	stable

- Critical damp describes break point.
- If Root locus lie in the left hand side of s plane then system will be stable.
- If some of the portion of RL lie on RHS of s plane then System will be unstable.

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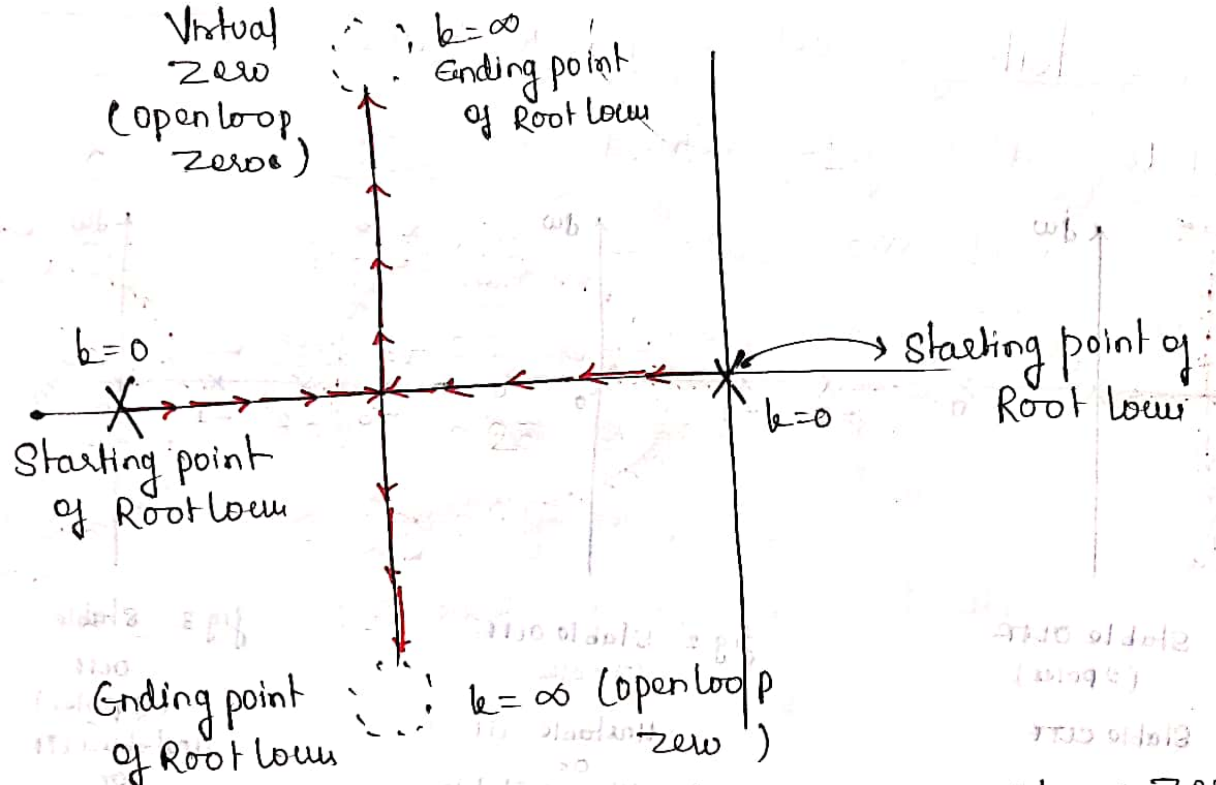


- In Case of proper Xfer function (No. of poles 'P' > No. of Zeros 'Z'), No. of Virtual Zeros is given by:-

$$Z^* = P - Z$$

eg. $GH = \frac{k}{s^2+s}$

$P = 2, \; Z = 0$

$Z^* = 2 - 0 = 2$ Hence No. of Virtual Zeros are 2

Rules for Sketching the Root locus

1. Root locus is always Symmetrical about real axis.

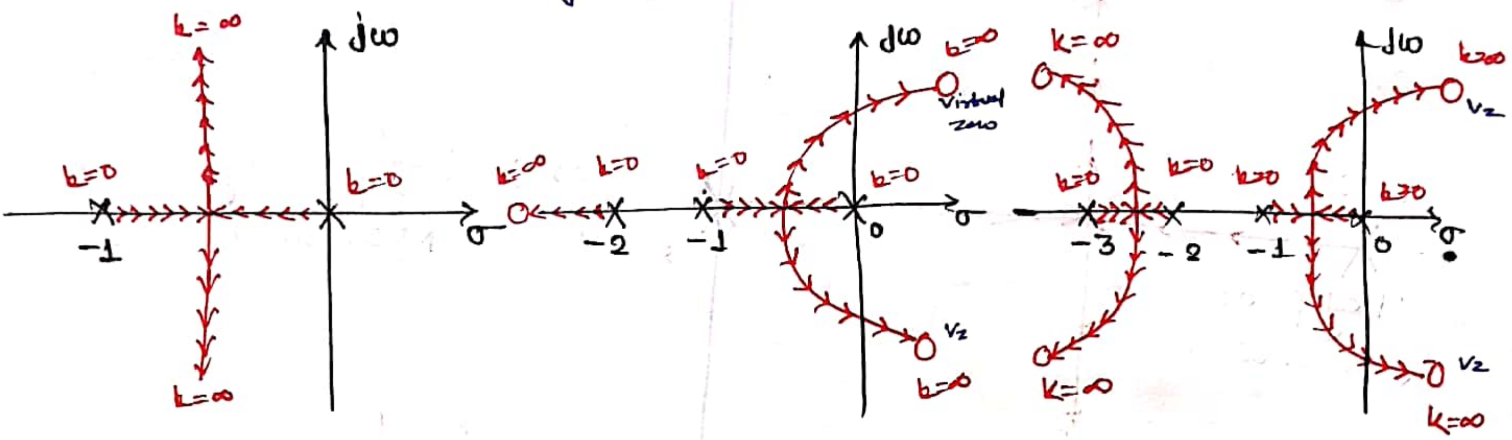


fig 1 Stable OLTF (2 poles)
Stable CLTF

fig 2 Stable OLTF (3 poles)
Unstable CLTF or Conditional Stable

fig 3 Stable OLTF (4 poles)
Unstable CLTF or Conditional stable

2. Root locus always start from open loop pole ($k=0$) and terminate either finite open loop zero or Infinity i.e Virtual zero ($k=\infty$)

Key point :- If $P>Z$ the Root locus will terminate at ∞ (Virtual Zero)

Consider

$$G(s)\,H(s) = \frac{k\,N(s)}{D(s)} \quad \text{——— (1)}$$

Zeros of OLTF $N(s) = 0$

poles of OLTF $D(s) = 0$

$$CLTF = \frac{G(s)}{1+G(s)H(s)}$$

Poles of CLTF = $1+G(s)\,H(s) = 0$

$$\Rightarrow 1 + k\frac{N(s)}{D(s)} = 0$$

$$\Rightarrow \frac{D(s) + kN(s)}{D(s)} = 0 \quad \text{——— (2)}$$

Starting point of Root locus, $k = 0$

$$D(s) = 0$$

Hence Root locus starts with poles of OLTF.

Chapter - 6

Polar plot

Plot	Variable	Range
Root locus	k	$0 < k < \infty$
Inverse Root locus	k	$-\infty < k < 0$
Root Contour	k	$-\infty < k < \infty$
Polar plot	ω	$0 < \omega < \infty$
Nyquist plot	ω	$-\infty < \omega < \infty$

Ques. i) $GH(s) = \frac{1}{s}$

ii) $GH(s) = \frac{1}{s^2}$

iii) $GH(s) = \frac{1}{s^3}$

iv) $GH(s) = \frac{1}{s^4}$

i) $GH(s) = 1/s$

Put $s = j\omega$

$GH(j\omega) = 1/j\omega$

$|GH(j\omega)| = \frac{1}{\omega}$ (Magnitude)

$\angle GH(j\omega) = -90^\circ$ (phase)

$\angle j = 90^\circ$ $\quad \angle \frac{1}{j} = -90^\circ$

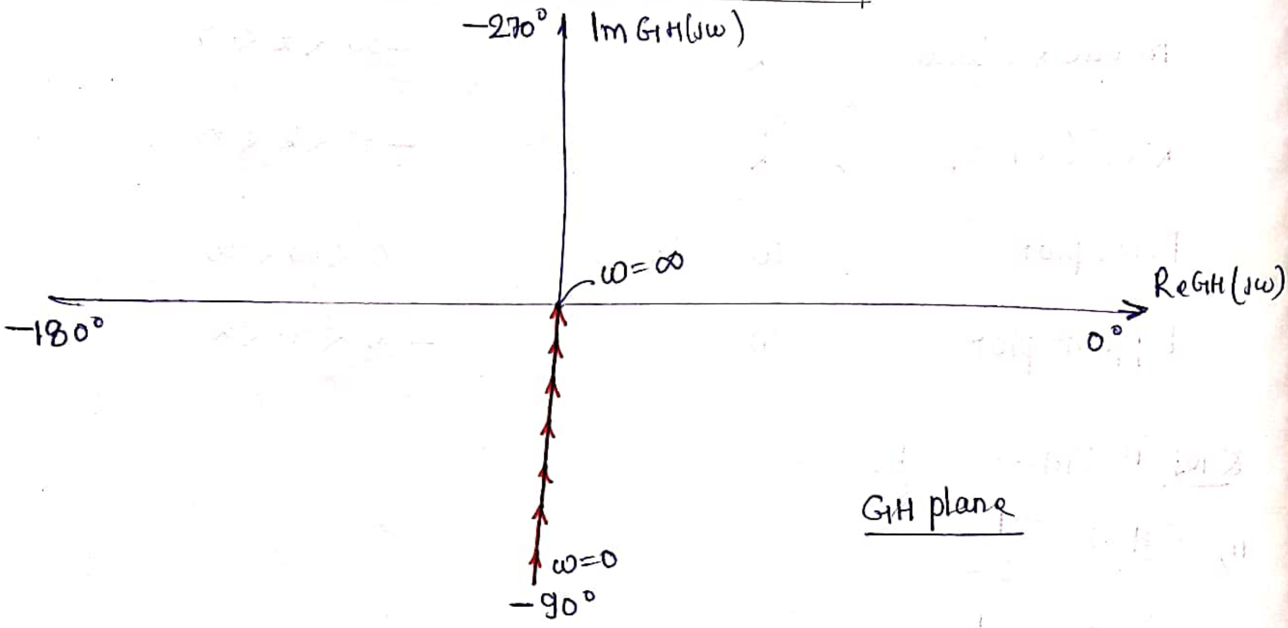
$\angle j^2 = 180^\circ$ $\quad \angle 1/j^2 = -180^\circ$

$\angle j^3 = 270^\circ$ $\quad \angle 1/j^3 = -270^\circ$

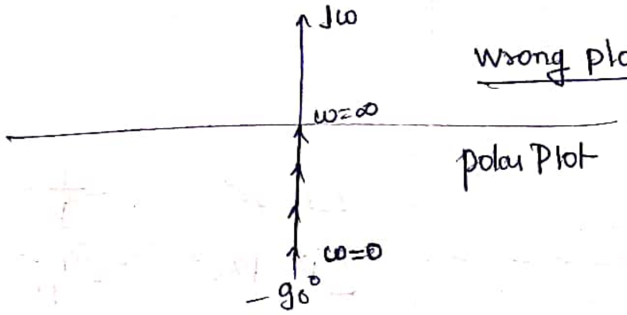
$\angle j^4 = 360^\circ$ $\quad \angle 1/j^4 = -360^\circ$

ω	$\lvert GH(j\omega)\rvert$	$\angle GH(j\omega)$
0	∞	-90°
∞	0	-90°

for $GH(s) = \frac{1}{s}$



GH plane



wrong plot

polar plot is drawn in GH plane not s plane.

ii) $GH = 1/s^2$

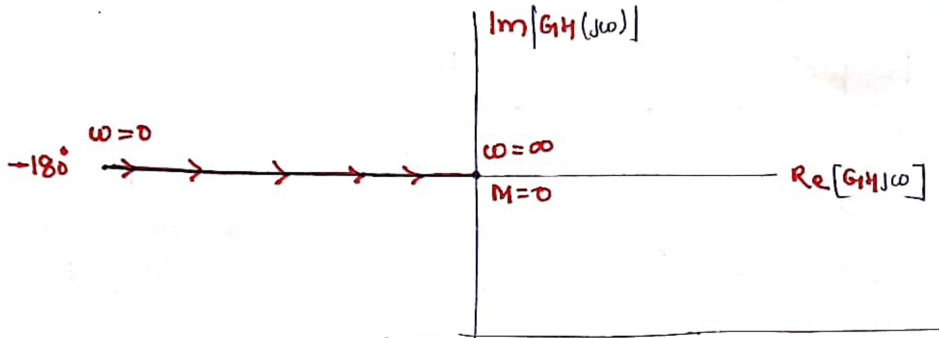
Sol $s \rightarrow j\omega$

$$GH(j\omega) = \frac{1}{(j\omega)^2}$$

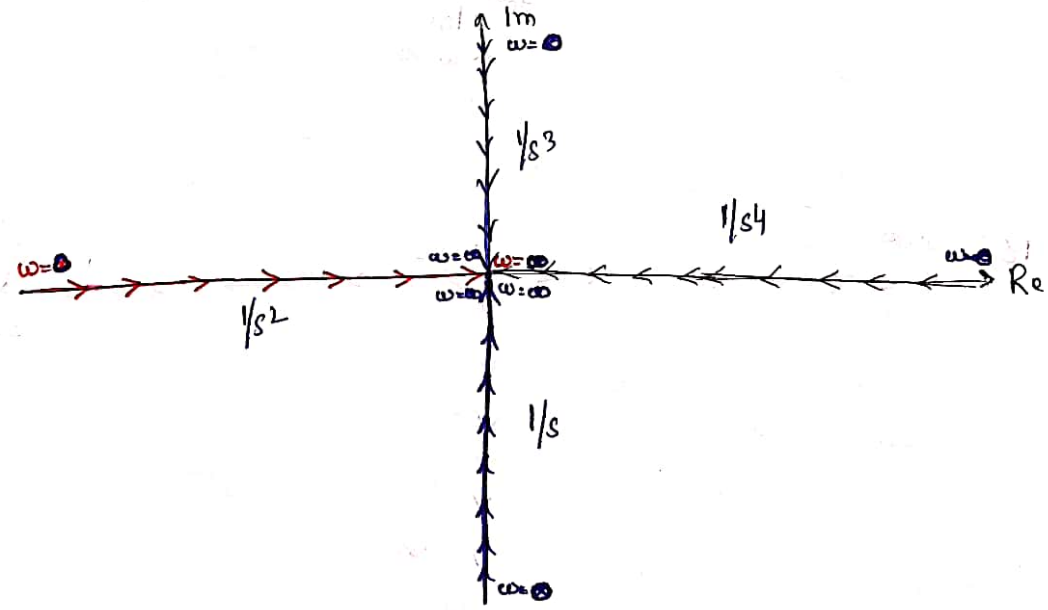
$$|GH(j\omega)| = \frac{1}{\omega^2}$$

$$\angle GH(j\omega) = -180^\circ$$

ω	$\lvert M\rvert$	ϕ
0	∞	-180°
∞	0	-180°



$G(s)H(s)$	$GH(j\omega)$	$\lvert GH(j\omega)\rvert$	$\angle GH(j\omega)$	$\lvert GH(j0)\rvert$	$\lvert GH(j\infty)\rvert$
$\frac{1}{s}$	$\frac{1}{j\omega}$	$\frac{1}{\omega}$	-90°	∞	0.
$\frac{1}{s^2}$	$\frac{1}{(j\omega)^2}$	$\frac{1}{\omega^2}$	-180°	∞	0
$\frac{1}{s^3}$	$\frac{1}{(j\omega)^3}$	$\frac{1}{\omega^3}$	-270°	∞	0
$\frac{1}{s^4}$	$\frac{1}{(j\omega)^4}$	$\frac{1}{\omega^4}$	-360°	∞	0



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Que2 Draw polar plot for following OLTF

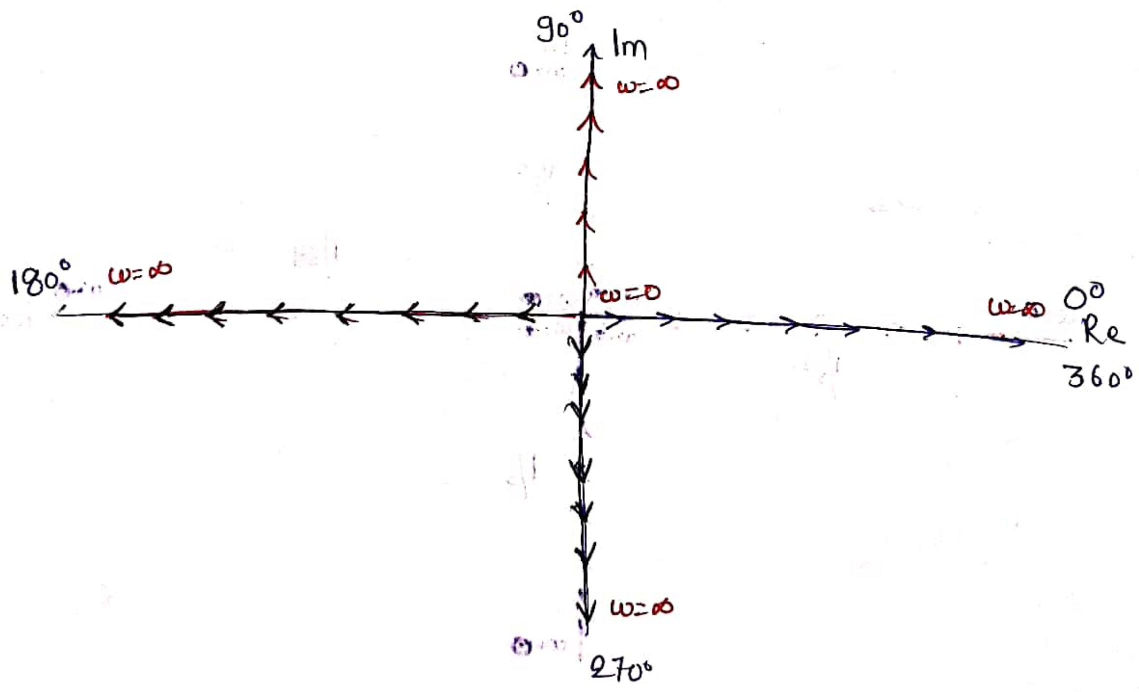
i) s

ii) s^2

iii) s^3

iv) s^4

$GH(s)$	$GH(j\omega)$	$\|GH(j\omega)\|$	$\angle GH(j\omega)$	$\|GH(j0)\|$	$\|GH(j\infty)\|$
s	$j\omega$	ω	90	0	∞
s^2	$(j\omega)^2$	ω^2	180	0	∞
s^3	$(j\omega)^3$	ω^3	270	0	∞
s^4	$(j\omega)^4$	ω^4	360	0	∞



* Polar Plot for Type Zero System

Type zero system represents no poles in origin in OLTF.

GH(s)	Type	Order
$\frac{1}{s+1}$	0	1
$\frac{1}{(s+1)(s+2)}$	0	2
$\frac{10}{(s+1)(s+2)(s+3)}$	0	3
$\frac{10}{(s+1)(s+2)(s+3)(s+4)}$	0	4

Sol: $GH(s) = \frac{1}{s+1}$

put $s = j\omega$

$$GH(j\omega) = \frac{1}{j\omega+1}$$

$$|GH(j\omega)| = \frac{1}{\sqrt{\omega^2+1}}$$

$$\angle GH(j\omega) = -\tan^{-1}\omega$$

At $\omega = 0$

$$|GH(j0)| = 1$$

$$\angle GH(j0) = 0^\circ$$

At $\omega = 1$

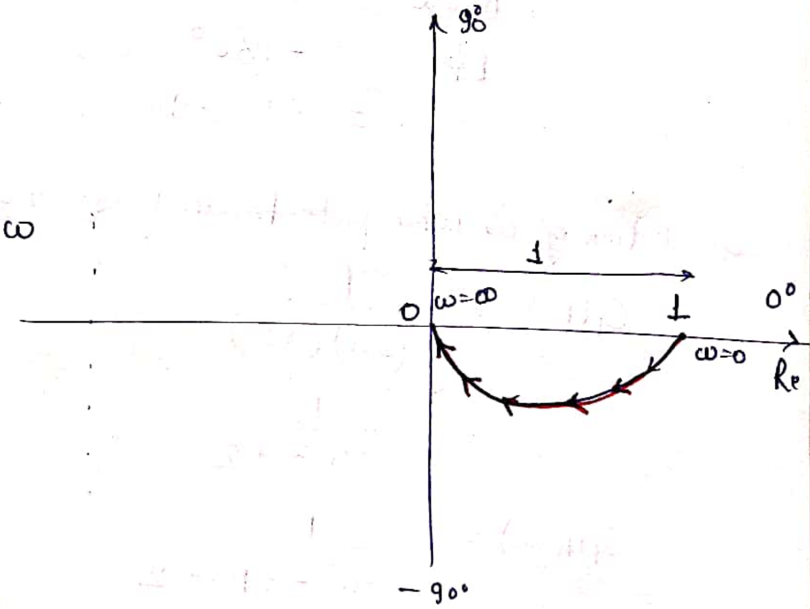
$$|GH(j1)| = 0.707$$

$$\angle GH(j1) = -45^\circ$$

At $\omega = \infty$

$$|GH(j\infty)| = 0$$

$$\angle GH(j\infty) = -90^\circ$$



ω	|GH(jω)|	∠GH(jω)
0	1	0°
1	$1/\sqrt{2}$	−45°
∞	0	−90°

Sol 2 $GH(s) = \dfrac{1}{(s+1)(s+2)}$

$$GH(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}$$

$$|M| = \frac{1}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

$$= \frac{1}{\sqrt{(\omega^2+1)(\omega^2+4)}}$$

$$\angle\phi = -\tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{\omega}{2}$$

ω	$\lvert GH \rvert$	$\angle GH$
0	1/2	0°
∞	0	$-90-90$ $=-180^\circ$

At $\omega = 0$

$$M_{\omega=0} = \frac{1}{2}$$

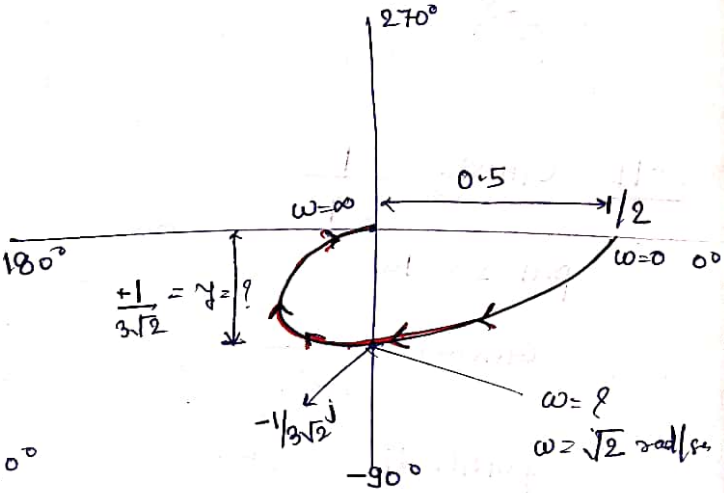
$$\angle\phi_{\omega=0} = 0^\circ$$

At $\omega = \infty$

$$M_{\omega=\infty} = 0$$

$$\angle\phi_{\infty} = -180^\circ$$

$$= -90^\circ - 90^\circ$$



Calculation of ω when plot Intersect -90° axis

$$GH(s) = \frac{1}{(s+1)(s+2)}$$

$$= \frac{1}{s^2+3s+2}$$

$$GH(j\omega) = \frac{1}{-\omega^2 - 3j\omega + 2}$$

Numerator Const. Hence Real part $Re[GH(j\omega)] = 0$

$2 - \omega^2 = 0$ gives Imaginary Value

$\omega = +\sqrt{2}$ always +ve

$$GH(j\omega) = \frac{1}{3j\omega}$$

$$= \frac{1}{3\sqrt{2}j} = -\frac{1}{3\sqrt{2}}j$$

Hence distance from origin

$y = -1/3\sqrt{2}$

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Sol3 $GH = \dfrac{10}{(s+1)(s+2)(s+3)}$

$GH(j\omega) = \dfrac{10}{(j\omega+1)(j\omega+2)(j\omega+3)}$

ω	$\lvert GH \rvert$	$\angle GH$
0	5/3	0°
∞	0	$-90-90-90 = -270^\circ$

$M = \dfrac{10}{\sqrt{(\omega^2+1)(\omega^2+4)(\omega^2+9)}}$

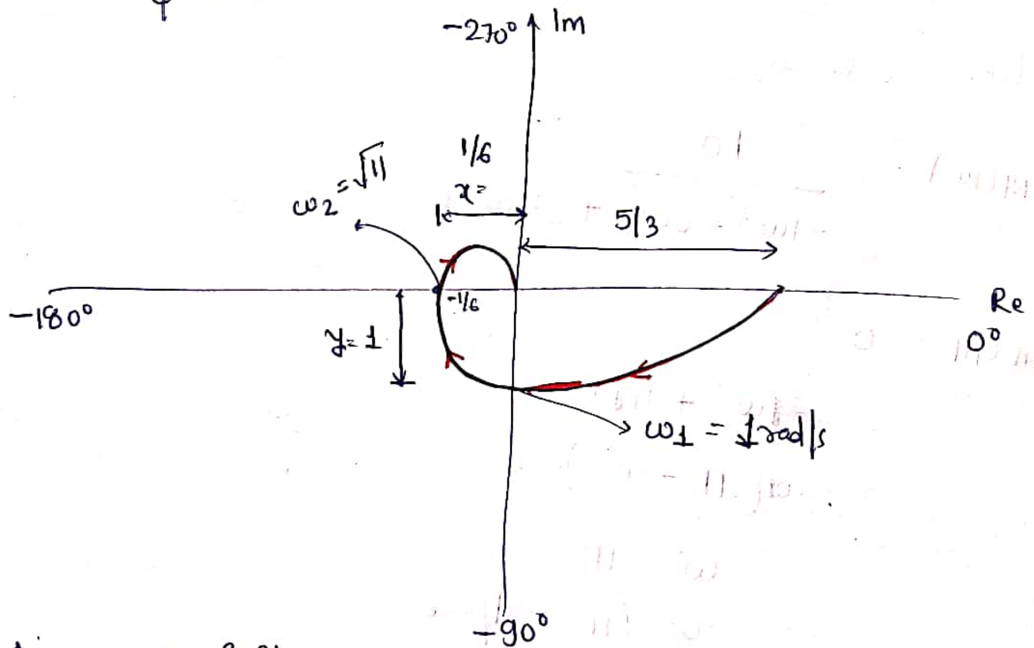
$\phi = 0^\circ - \tan^{-1}\omega - \tan^{-1}2 - \tan^{-1}3$

at $\omega=0$ $\quad M = \dfrac{10}{6} = \dfrac{5}{3}$

$\phi = 0^\circ$

at $\omega=\infty$ $\quad M = 0$

$\phi = -90 - 90 - 90 = -270^\circ$



Calculation: ω_1 & y

$GH(s) = \dfrac{10}{(s+1)(s^2+5s+6)}$

$= \dfrac{10}{s^3+6s^2+11s+6}$

$$GH(j\omega) = \frac{10}{-j\omega^3 - 6\omega^2 + j11\omega + 6}$$

$$Re[GH(j\omega)] = 0$$

$$6\omega^2 - 6 = 0$$

$$\omega^2 = 6/6$$

$$\omega = 1 \text{ rad/sec}$$

$GH(j\omega)$ Imaginary $= \dfrac{10}{-j - 6 + 11j + 6}$

$$= \frac{10}{10j}$$

$$= -j1$$

Calculation x & ω_2

$$GH(j\omega) = \frac{10}{-j\omega^3 - 6\omega^2 + j11\omega + 6}$$

$$Im\ GH = 0$$

$$= -j\omega^3 + 11j\omega = 0$$

$$\Rightarrow \omega(11 - \omega^2) = 0$$

$$\omega^2 = 11$$

$$\omega = \sqrt{11} \text{ rad/sec}$$

Real GH at $\omega = \sqrt{11}$ $= \dfrac{10}{-6 \times 11 + 6}$

$$= \frac{10}{-66 + 6} = \frac{10}{-60} = -1/6$$

$$x = +1/6$$

Nyquist Plot

- In nyquist plot variable is entire s-plane while in polar plot the variable is positive frequency line of s plane.
- In case of nyquist plot both +ve as well as −ve frequency exist while in case of polar plot only +ve frequency exist.
- Nyquist plot describe both absolute & relative stablity while polar plot describe only relative stablity.
- Nyquist plot is used to draw complete frequency response of OLTF.

* Nyquist stablity criteria describe closed system stablity from openloop frequency response and open loop poles

- Nyquist plot is used to find number of closed loop poles in the right hand side of s.plane which is not possible in polar plot.
- Nyquist plot is used to find range of 'k' for system stablity.
- Nyquist plot is used to find relative stablity parameter (ω_{pc}, ω_{gc}, GM PM)

- Consider OLTF

$$G(s) = \frac{k N(s)}{D(s)} \quad \text{——— (1)}$$

$$C.E = 1 + GH = 1 + \frac{kN(s)}{D(s)} = \frac{D(s) + N(s)k}{D(s)} \quad \text{——— (2)}$$

$$CLTF = \frac{G(s)}{1 + G(s)} = \frac{k N(s)/D(s)}{1 + kN(s)/D(s)} = \frac{k N(s)}{D(s) + kN(s)} \quad \text{——— (3)}$$

from (1) & (11)

~~Poles of ceq^n~~ (1) & (3)

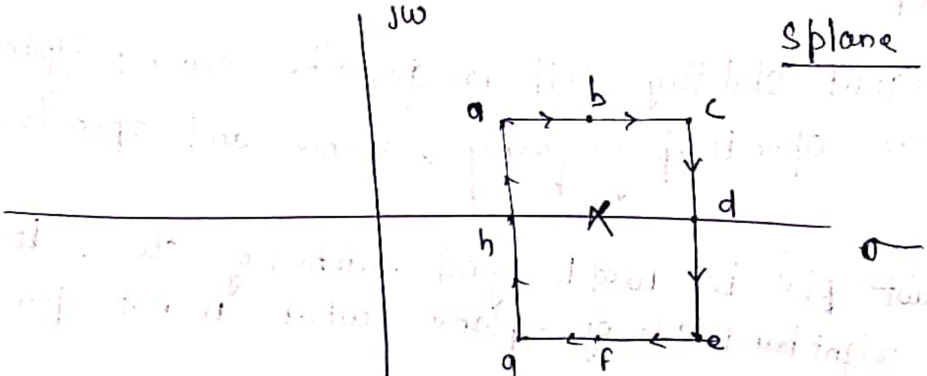
Poles of ceq^n = Poles of OLTF ——— (4)

from (1) & (3)

$\boxed{\text{Zeros of charac. eq}^n = \text{poles of CLTF}}$ ← (5)

ex01

$$G(s) = \frac{1}{s-2}$$



Point	$GH = \frac{1}{s-2}$	$\lvert G(j\omega)\rvert$	$\angle G(s)$
a = 1 + j1	$\frac{1}{j1-1}$	$1/\sqrt{2}$	$-135°$
b = 2 + j1	$\frac{1}{j1}$	1	$-90°$
c = 3 + j1	$\frac{1}{1+j1}$	$1/\sqrt{2}$	$-45°$
d = 3 + j0	$\frac{1}{1}$	1	$0°$
e = 3 − j1	$1/1-j1$	$1/\sqrt{2}$	$45°$
f = 2 − j1	$1/-j1$	1	$90°$
g = 1 − j1	$1/-1-j1$	$1/\sqrt{2}$	$-225°$ (= 135°)
h = 1 − j0	$1/-1$	1	$\pm 180°$

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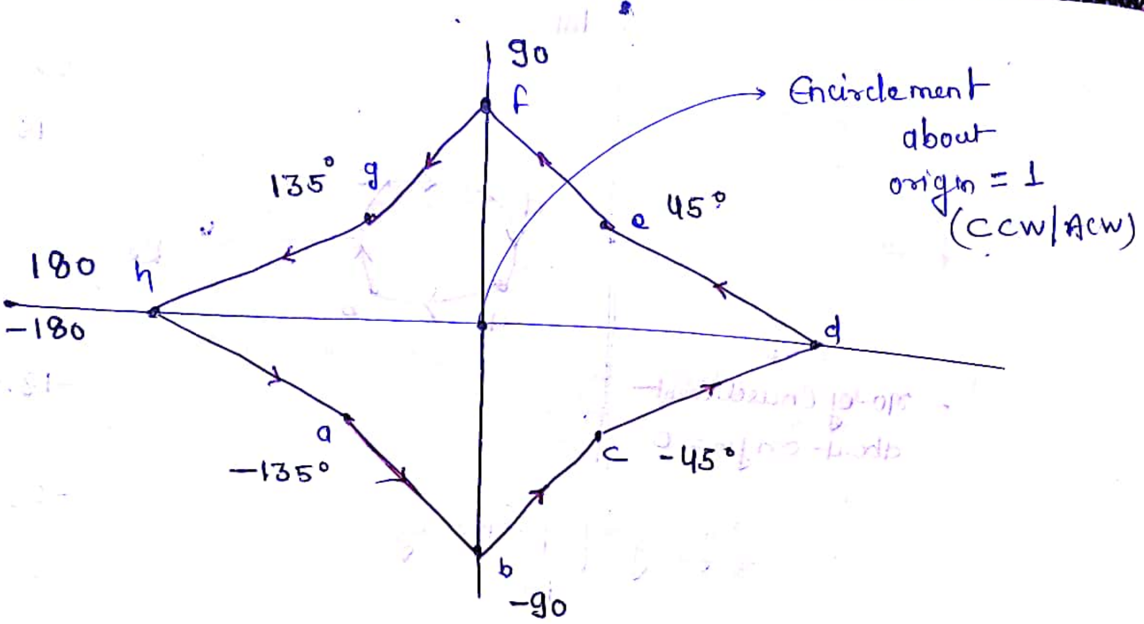
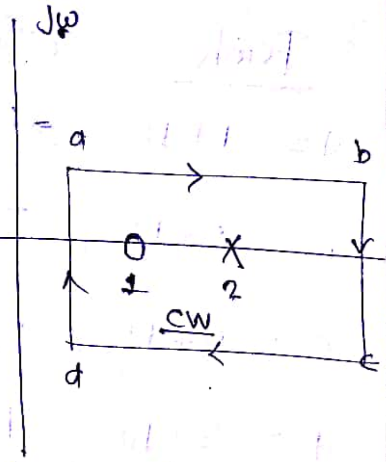


fig. freq.plane of G(s)
polar plot of G(s)

$$C.E = 1 + G(s) = 1 + \frac{1}{s-2} = \frac{s-2+1}{s-2}$$

$$1+G(s) = \frac{s-1}{s-2}$$



Points	$1+G(s) = \frac{s-1}{s-2}$	$\lvert 1+G(s) \rvert$	$\angle(1+G(s))$
$a = 0.5+j1$	$\frac{-0.5+j1}{-1.5+j1}$	0.62	-29.74°
$b = 3+j1$	$\frac{2+j1}{1+j1}$	1.58	-18.43
$c = 3-j1$	$\frac{2-j1}{1-j1}$	1.58	18.45
$d = 0.5-j1$	$\frac{-0.5-j1}{-1.5-j1}$	0.62	29.75

S plane of $1+G(s)$

WANT ECE BEST QUALITY-2019-20

LATEST HANDWRITING NOTES

BY

GATE ACADEMY CLICK HERE

Noted-: Above ECE GATE/IES GATEACADEMY 2019-20 CLASSROOM BEST QUALITY Handwriting Notes Unique and Good Handwriting. Above Notes Enough for your Preparation..................

Noted-: Dear Aspirants If you do practice previous year paper 50% your work finished.so Guys daily at least 30minutes give previous year.

1. Previous year paper 4-5 times practice before final exam.

2. Subject wise study reference STD book

3. Test series practice more n more (Try to latest test series 2-3 fully solve then join online test series.)

Noted-: Single Source Follow, Revise Multiple Time Best key of Success

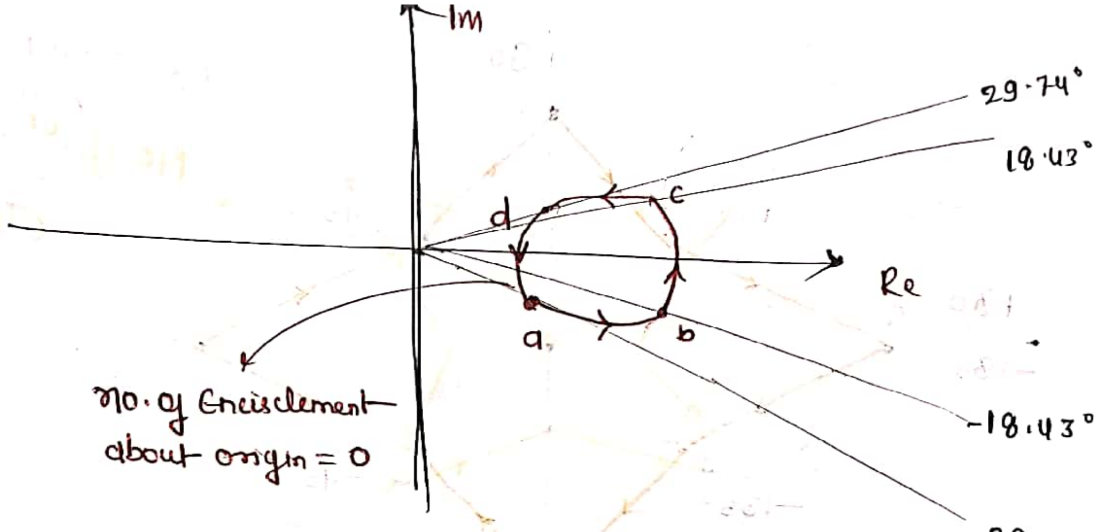
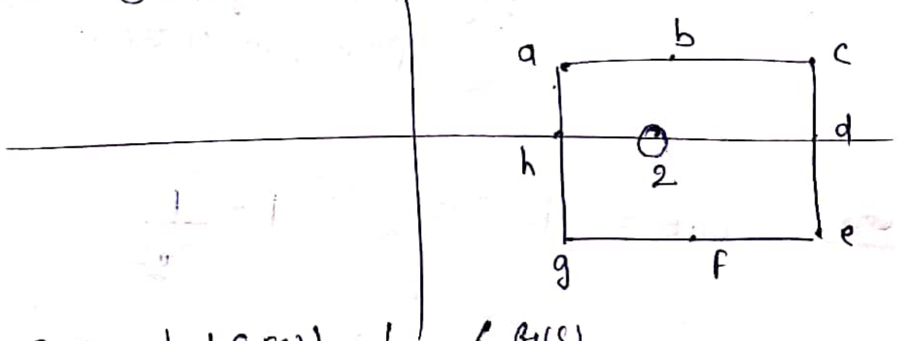


fig: freq. plane of $1+G(s)$

ex 2 $G(s) = s-2$



Points	$G(s) = s-2$	$\lvert G(s) \rvert$	$\angle G(s)$
a = $1+j1$	$-1+j1$	$\sqrt{2}$	135°
b = $2+j1$	$+j1$	1	90
c = $3+j1$	$1+j1$	$\sqrt{2}$	45
d = $3+j0$	$1+j0$	1	0°
e = $3-j1$	$1-j1$	$\sqrt{2}$	~~135~~° / -45°
f = $2-j1$	$0-j1$	1	-90
g = $1-j1$	$-1-j1$	$\sqrt{2}$	+225°
h = $1-j0$	$-1-j0$	1	-180°

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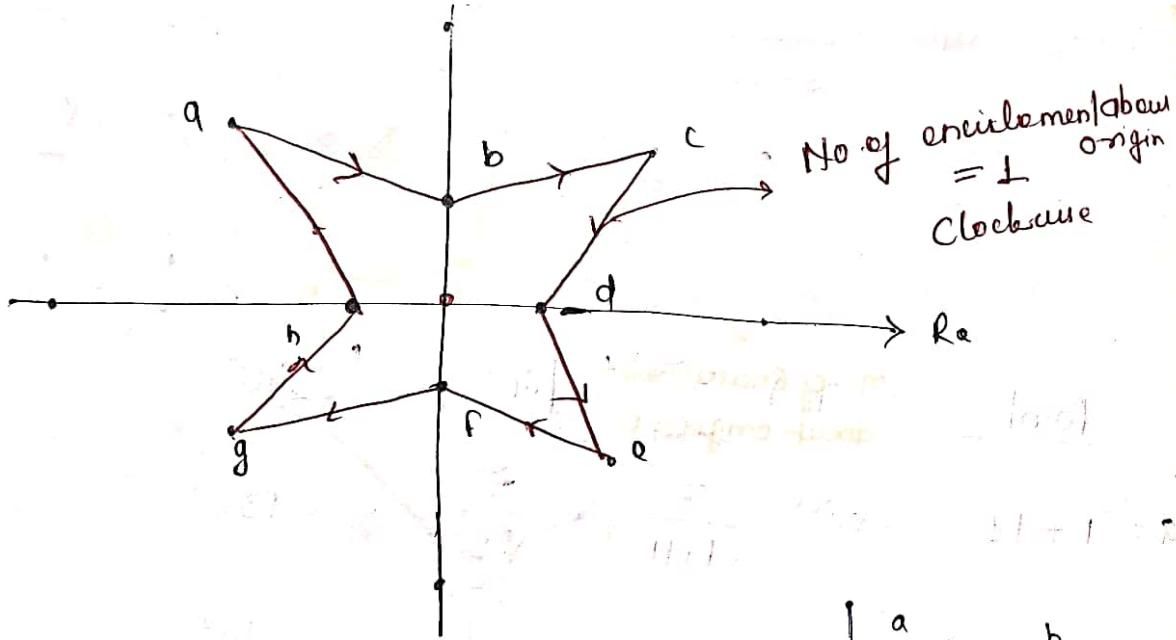
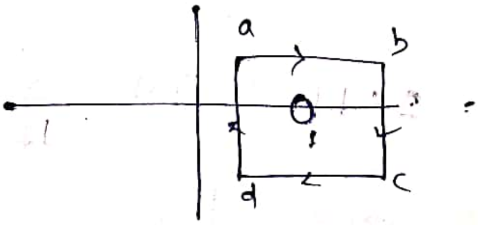


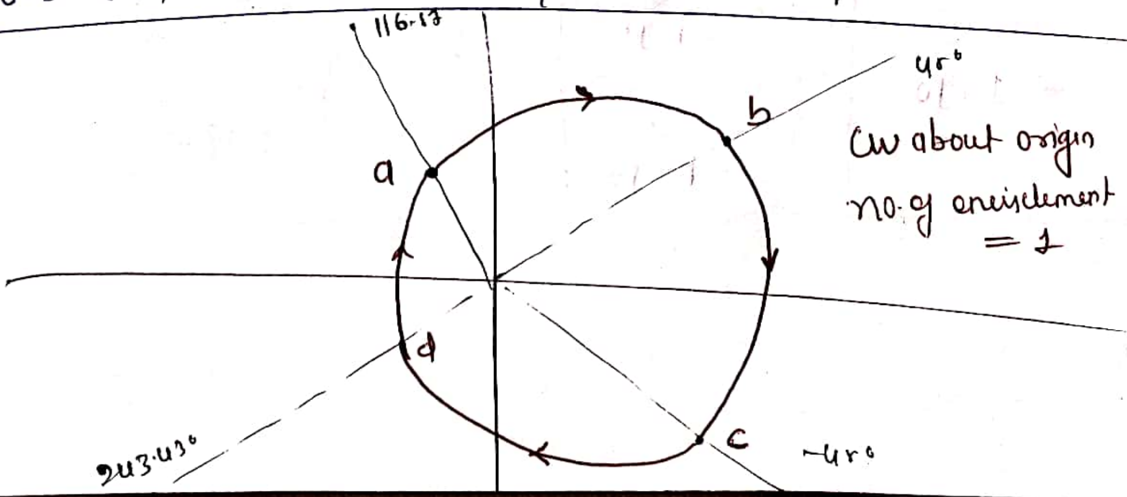


C.E = $1 + G(s)$

$\Rightarrow 1 + s - 2$

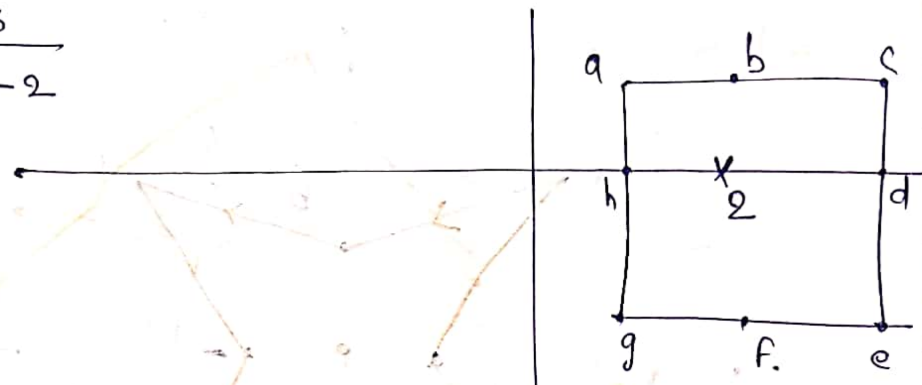
$= s - 1$

Points	$1+G(s) = s-1$	$\lvert 1+G(s) \rvert$	$\angle 1+G(s)$
$a = 0.5 + j1$	$-0.5 + j1$	1.11	116.57°
$b = 2 + j1$	$1 + j1$	$\sqrt{2}$	45°
$c = 2 - j1$	$1 - j1$	$\sqrt{2}$	-45°
$d = 0.5 - j1$	$-0.5 - j1$	1.11	243.43°

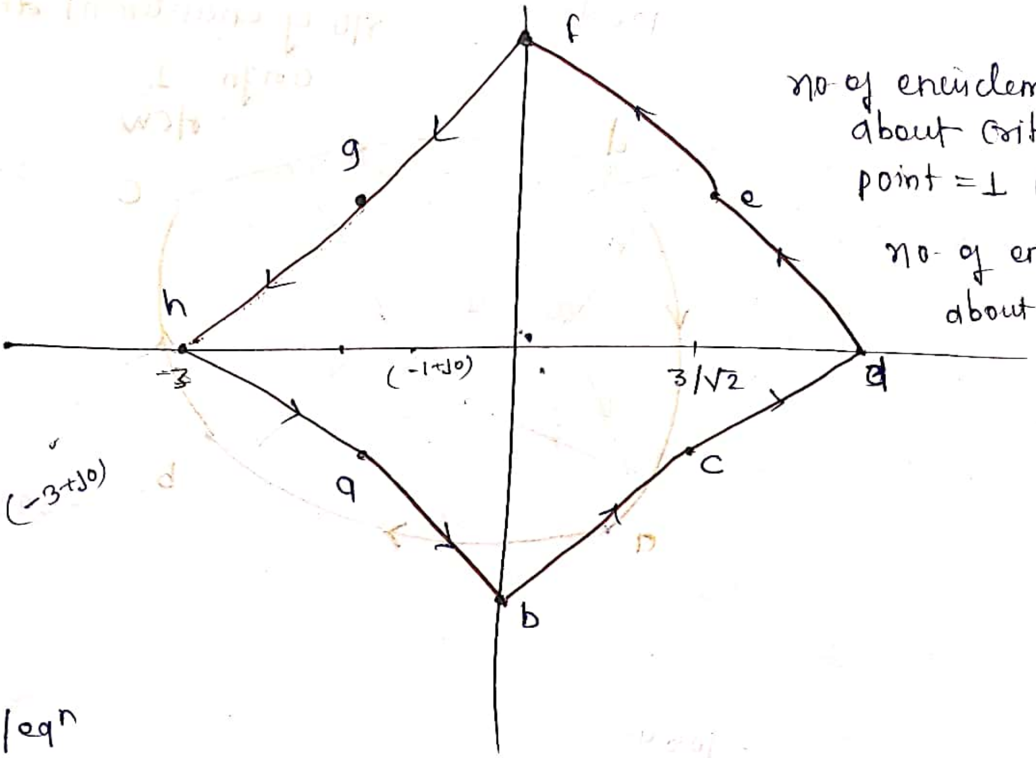


Q.3

$$G(s) = \frac{3}{s-2}$$



Points	$G(s) = \frac{3}{s-2}$	$\lvert G(s) \rvert$	$\angle G(s)$
$a = 1 + j1$	$G(s) = \frac{3}{-1+j1}$	$\frac{3}{\sqrt{2}}$	-135°
$b = 2 + j1$	$G(s) = \frac{3}{j1}$	3	-90°
$c = 3 + j1$	$G(s) = \frac{3}{1+j1}$	$\frac{3}{\sqrt{2}}$	-45°
$d = 3 + j0$	$G(s) = \frac{3}{1+j0}$	$\frac{3}{1} = 3$	0°
$e = 3 - j1$	$G(s) = \frac{3}{1-j1}$	$\frac{3}{\sqrt{2}}$	45°
$f = 2 - j1$	$G(s) = \frac{3}{0-j1}$	3	90°
$g = 1 - j1$	$G(s) = \frac{3}{-1-j1}$	$\frac{3}{\sqrt{2}}$	-225° $(= 135^\circ)$
$h = 1 + j0$	$G(s) = \frac{3}{-1-j0}$	3	$\pm 180^\circ$



no of encirclement about critical point = 1 (CCW)

no. of encirclement about origin = 1 (CCW)

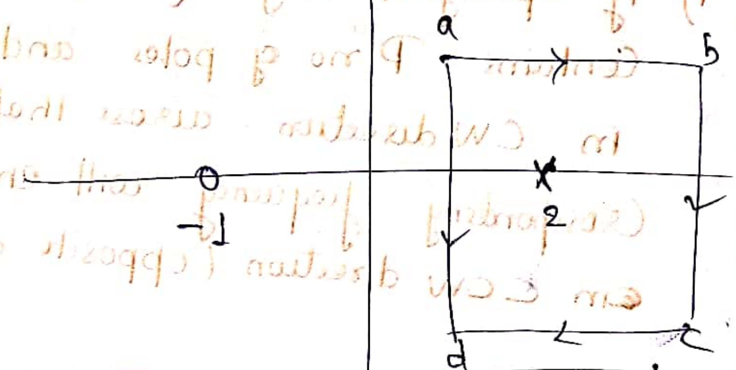
C/eqⁿ

$1 + G(s)$

$= 1 + \frac{3}{s-2}$

$= \frac{s-2+3}{s-2}$

$= \frac{s+1}{s-2}$



points	$1+G(s) = \frac{s+1}{s-2}$	$\lvert 1+G(s) \rvert$	$\angle 1+G(s)$
$a = 1+j1$	$\frac{2+j1}{-1+j1}$	$\sqrt{5}/\sqrt{2} = 1.58$	-108.43
$b = 3+j1$	$\frac{4+j1}{1+j1}$	2.91	-30.96
$c = 3-j1$	$\frac{4-j1}{1-j1}$	2.91	30.96
$d = 1-j1$	$\frac{2-j1}{-1-j1}$	$\sqrt{5}/\sqrt{2} = 1.58$	108.43

Bode Plot

- Used to determine frequency response of OLTF
- Determines the stability of CLTF by freq. response of OLTF
- Used to find relative stability parameters (ω_{pc}, ω_{gc}, GM PM)
- Addition of poles and Zeros can easily be analysed with the help of Bodeplot.
- ✱ Bode plot is analysed only for MPS (stable OLTF)
- Bode plot is Combination of two Individual plot, Magnitude & phase plot.
- It is plotted on Semi Log paper.
- In Bode plot, Bode Magnitude plot is a logarithmic plot. (Magnitude in dB)
- In Bode Magnitude plot log 0 is invalid quantity So it is impossible to calculate Magnitude at $\omega = 0$.
- In Bode phase plot, Phase at $\omega = 0$ is Valid quantity So it is possible to calculate phase angle at $\omega = 0$. but Magnitude and phase plot is drawn for common axis. So if $\omega = 0$ is invalid for Magnitude plot it is also invalid for phase plot.

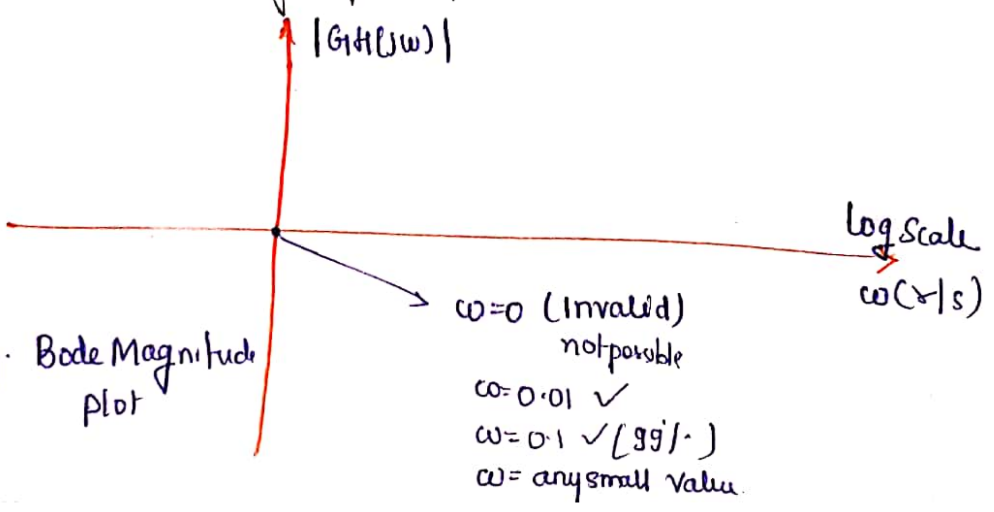


fig. Bode Magnitude plot

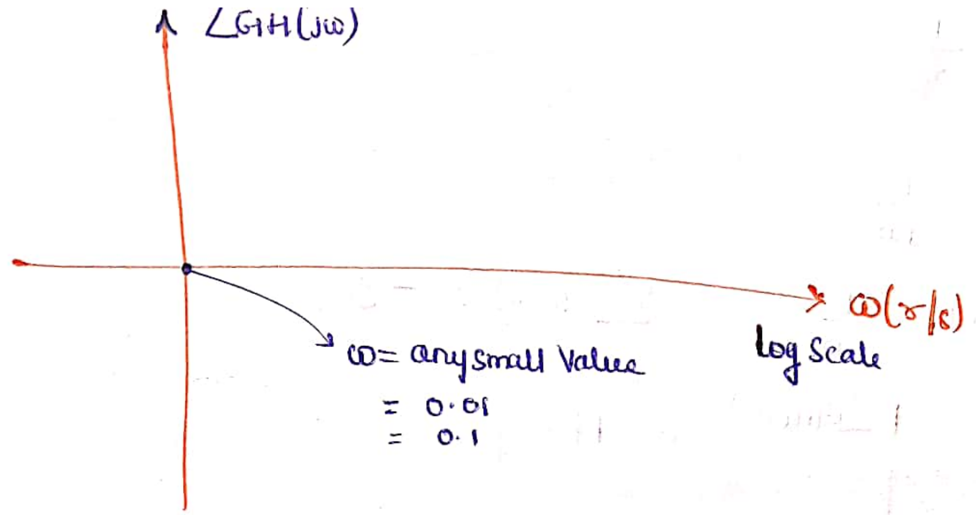


fig. Bode phase plot

The Concept of Minimum & Non Minimum phase System.

1. In Case of MPS all poles and Zeros should be Located in the LHS of S-plane.
2. MPS is always stable system.
3. In Case of NMPS, it Contain at least one pole or Zero in the right hand Side of splane.
4. NMPS may be unstable (in case of RH pole).
5. Mostly RH zero system is reffered as NMPS.

Ques Draw Bodeplot for following OLTF.

i) $GH(s) = \frac{1}{s}$

ii) $GH(s) = \frac{1}{s^2}$

iii) $GH(s) = \frac{1}{s^3}$

iv) $GH(s) = \frac{1}{s^4}$

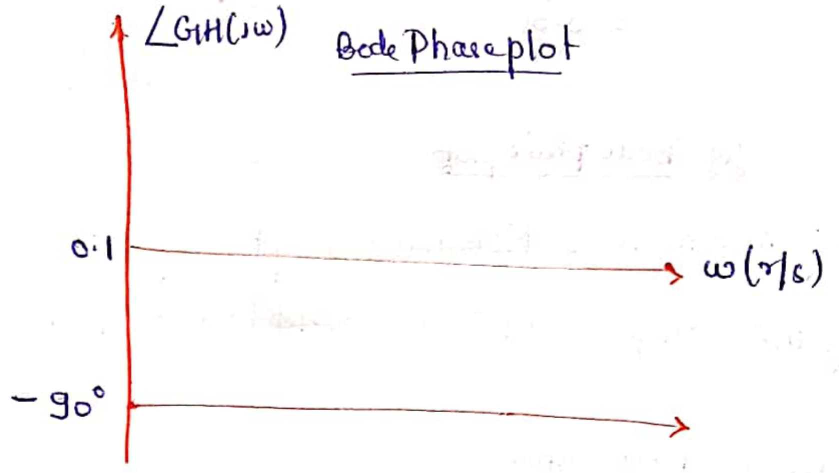
Sol 1 $GH(s) = \frac{1}{s}$

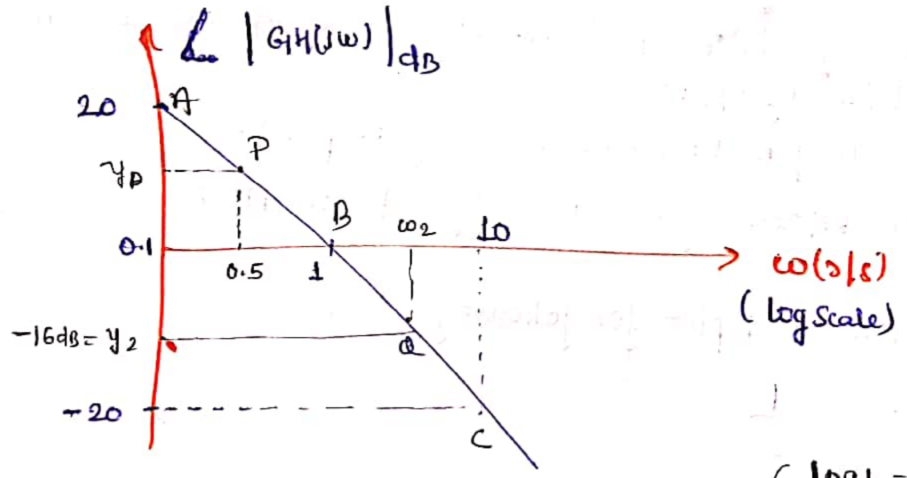
$s = j\omega$

$GH(j\omega) = \frac{1}{j\omega}$

$|GH(j\omega)| = \frac{1}{\omega}$ $\quad\quad \angle GH(j\omega) = -90^\circ$

Bode Phase plot





$|GH(j\omega)| = \frac{1}{\omega}$

$= -20 \log \omega$

ω	$\|GH(j\omega)\|_{dB}$
0.1	$-20 \log 0.1 = 20$ dB
1.0	$-20 \log 1 = 0$ dB
10	$-20 \log 10 = -20$ dB

$\log 1 = 0$
$\log 10 = 1$
$\log 10^2 = 2$
$\log 1000 = 3$
$\log 0.1 = -1$
$\log 0.01 = -2$
$\log 0.001 = -3$

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$\left.\begin{aligned}\omega_2 &= 10\,\omega_1 \\ \omega_3 &= 10\,\omega_2\end{aligned}\right]$ Decade frequency

$\omega_2 = 10\omega_1$

$\frac{\omega_2}{\omega_1} = 10$

$20\log\frac{\omega_2}{\omega_1} = 20\log 10 = 20\,dB/dec$

$\left.\begin{aligned}\omega_2 &= 2\omega_1 \\ \omega_3 &= 2\omega_2\end{aligned}\right]$ Octave freq.

$\frac{\omega_2}{\omega_1} = 2$

$20\log\frac{\omega_2}{\omega_1} = 20\log 2$
$= 6\,dB/oct$

$6\,dB/oct = 20\,dB/dec$

$12\,dB/oct = 40\,dB/dec$

$18\,dB/oct = 60\,dB/dec$

$-24\,dB/oct = -80\,dB/dec$

$-30\,dB/oct = -100\,dB/dec$

$$\text{Slope}_{AB} = \frac{y_B - y_A}{\log\omega_B - \log\omega_A} = \frac{0-20}{\log\left(\frac{\omega_B}{\omega_A}\right)} = \frac{-20}{\log\left(\frac{1}{0.1}\right)} = -20\,dB/dec$$

$$\text{slope}_{BC} = \frac{y_C - y_B}{\log\omega_C - \log\omega_B} = \frac{-20-0}{\log\left(\frac{\omega_C}{\omega_B}\right)} = \frac{-20}{\log\left(\frac{10}{1}\right)} = -20\,dB/dec$$

$$\text{Slope}_{AC} = \frac{y_C - y_A}{\log\omega_C - \log\omega_A} = \frac{-20-20}{\log\left(\frac{10}{0.1}\right)} = \frac{-40}{\log 100} = -20\,dB/dec$$

Magnitude at point P :

Slope $AP = -20\,dB/decade$

$$\frac{y_P - y_A}{\log\left(\frac{\omega_P}{\omega_A}\right)} = -20$$

$\Rightarrow \frac{y_P - 20}{\log 0.5/0.1} = -20 \Rightarrow y_P = -20\log 5 + 20$
$= 6.02\,dB$

ω_2 :

$$\text{Slope } BQ = \frac{y_Q - y_B}{\log(Q/B)}$$

$$-20 = \frac{-16 - 0}{\log\left(\frac{\omega_2}{0.1}\right)}$$

$$\log \cancel{10}\,\omega_2 = \frac{16}{20}$$

$$\cancel{\log 10 +} \log \omega_2 = \frac{16}{20}$$

$$\log \omega_2 = \frac{16}{20}$$

$$\cancel{\omega_2 = 0.6309}$$

$$\cancel{10\omega_2} = 6.3 \; //.$$

$$\text{Slope}_{AQ} = \frac{y_Q - y_A}{\log \frac{\omega_Q}{\omega_A}}$$

$$-20 = \frac{-10-26}{\log \omega_2/0.1}$$

$$\log\left(\frac{\omega_2}{0.1}\right) = \frac{-36}{-20} = 1.8$$

$$\frac{\omega_2}{0.1} = 10^{1.8} = 63.09$$

$$\omega_2 = 6.309 \text{ r/s}$$

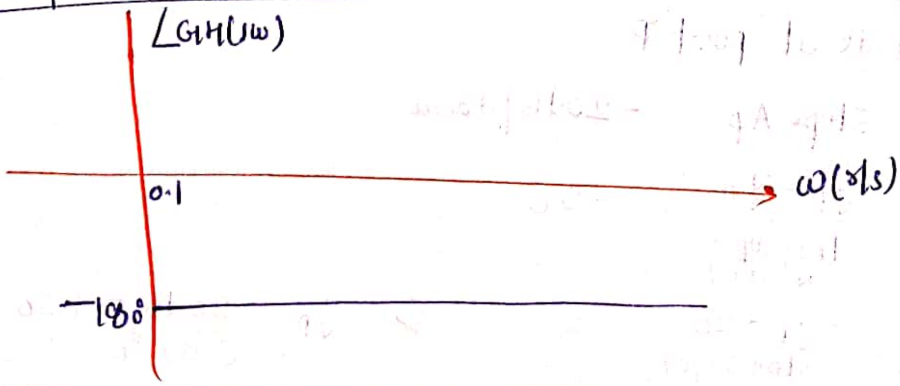
Q2

$$GH(s) = \frac{1}{s^2}$$

$$|GH(j\omega)| = \frac{1}{\omega^2}$$

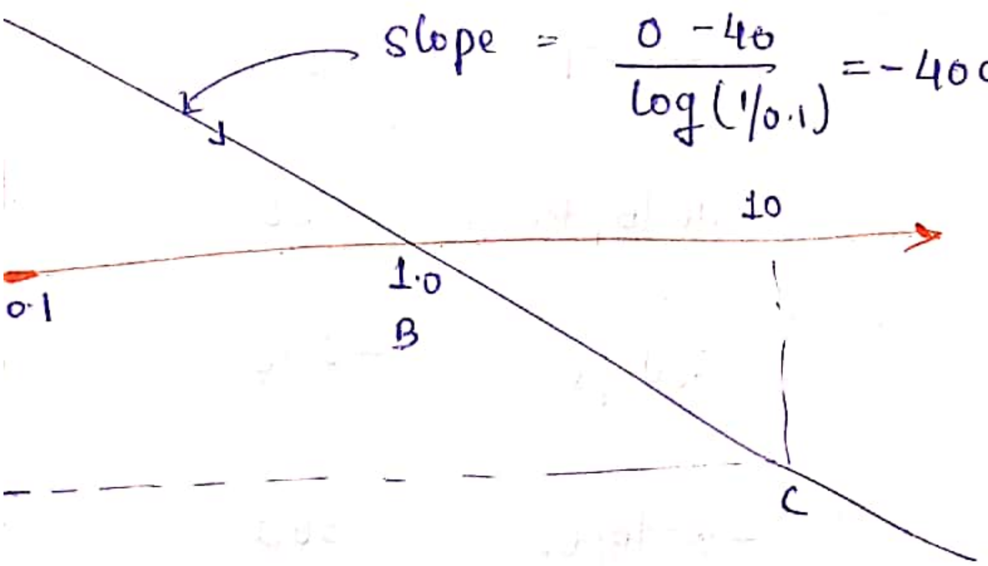
$$\angle GH(j\omega) = -180^\circ$$

ω	$\lvert GH(j\omega)\rvert$	$\angle GH(j\omega)$
0.1	100	-180°
1	1	-180°
10	0.01	-180°



$|GH(j\omega)|$

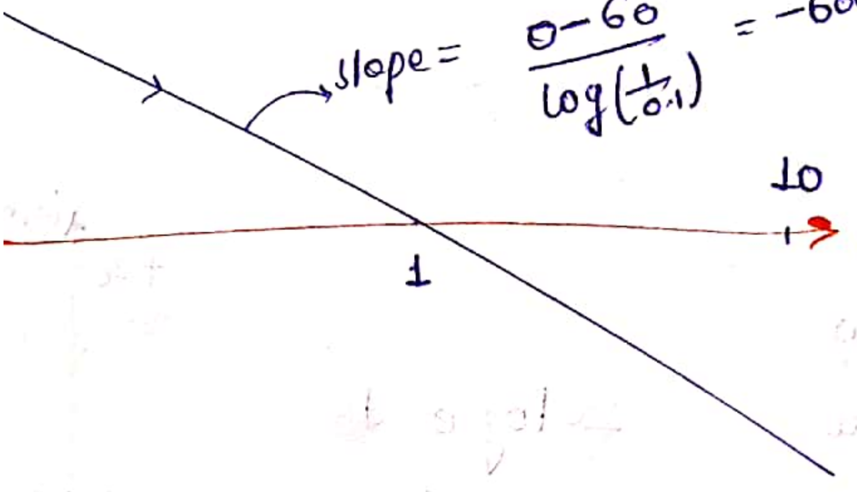
slope $= \dfrac{0-40}{\log(1/0.1)} = -40\,dB/de$



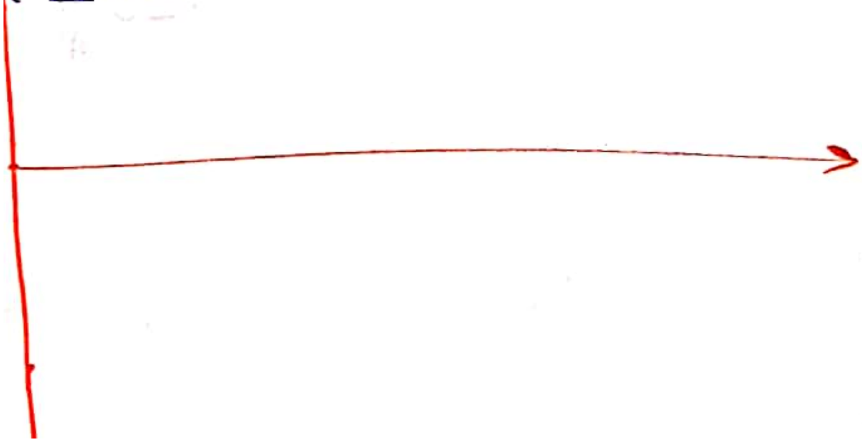
$\dfrac{1}{S^3}$

$|GH(j\omega)|$

slope $= \dfrac{0-60}{\log(\frac{1}{0.1})} = -60\,dB/dec$



$\angle GH(j\omega)$



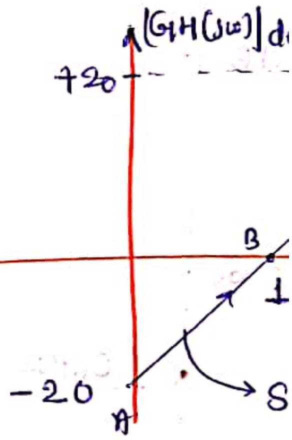
GH(s)	Slope	Angle
$\frac{1}{s}$	-20 dB/dec	-90
$\frac{1}{s^2}$	-40 dB/dec	-180
$\frac{1}{s^3}$	-60 dB/dec	-270
$\frac{1}{s^4}$	-80 dB/dec	-360
$\frac{1}{s^n}$	$-20n$ dB/dec	$-90n^\circ$

Q. $GH(s) = S$

$GH(j\omega) = j\omega$

$|GH(j\omega)| = \omega \quad = 20\log\omega \text{ dB}$

$\angle GH(j\omega) = 90^\circ$



GH(s)	Slope	Angle
s	20 dB/dec	90
s^2	40 dB/dec	180
s^3	60 dB/dec	270
s^4	80 dB/dec	360
s^n	20n dB/dec	90n

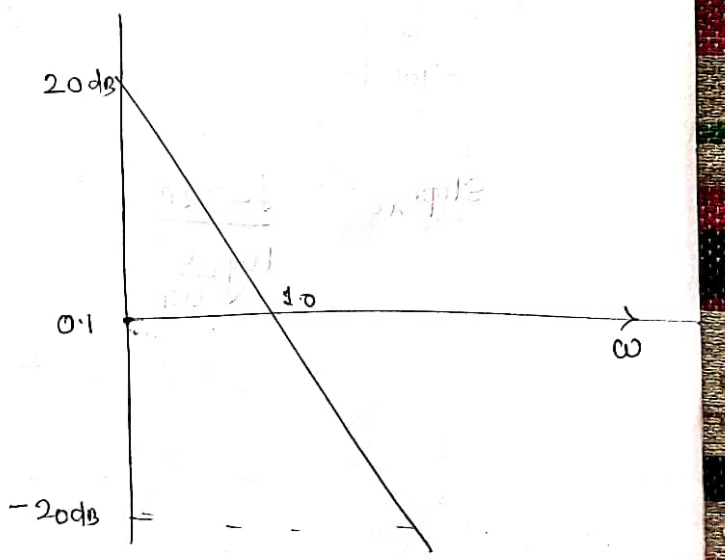
Que. $GH(s) = \frac{k}{s}$

Sol:- Cases $k = 1.0$

$GH(s) = \frac{1}{s}$

Case 1:-

$k = 0.1 \quad (k < 1)$



$GH(s) = \frac{0.1}{s}$

$GH(j\omega) = \frac{0.1}{j\omega}$

$|GH(j\omega)| = \frac{0.1}{\omega}$

$\angle GH(j\omega) = -90^\circ$

$|GH(j\omega)|_{dB} = 20 \log \left(\frac{0.1}{\omega}\right)$

$= 20 \log 0.1 - 20 \log \omega$

Unit-9 State Space Analysis

System Analysis Method

- Classical Method or X^{fer} function based Method
  - RL
  - PP
  - NP
  - BP
- State Variable Method

*** Limitation of classical Method**

i) It is valid only for LTI System.

ii) It is valid for SISO (Single Input Single o/p) System.

iii) This Methods doesnot describe the Internal state of the System (Initial Value of the System).

iv) This Method gives only Zero state Response.

*** Advantages of State Variable Method**

i) It is Valid for linear, Nonlinear, Time Invariant & Time Variant System

ii) It is Valid for both SISO & MIMO System
Multiple i/p Multiple o/p

iii) State Variable Method describes the Internal state of the System (Initial Value of the System).

iv) This Method gives both ZeroInput response (ZIR) & Zero state response (ZSR)

* Representation of State Variable Method :-

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

$$\boxed{\begin{aligned} [\dot{X}]_{n\times 1} &= [A]_{n\times n}[X]_{n\times 1} + [B]_{n\times m}[U]_{m\times 1} \quad \text{--- (1)} \\ [Y]_{p\times 1} &= [C]_{p\times n}[X]_{n\times 1} + [D]_{p\times m}[U]_{m\times 1} \quad \text{--- (2)} \end{aligned}}$$

where,

(1)

$[X]_{n\times 1}$ = State Vector

n → no. of State Variable

State Vector is a Collection of state Variable

No. of state Variale n := No. of effective storing element
= No. of time Constant
= No. of Initial Value
= order of x fer function

e.g 1 $n = 1 \quad (x)$

$$[X]_{1\times1} = [x]$$

eg 2 $n = 2 \quad (x_1, x_2)$

$$[X]_{2\times1} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times1}$$ Colum Matrix or Vertical Matrix

eg 3 $n = 3 \quad (x_1 \; x_2 \; x_3)$

$$[X]_{3\times1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1}$$

② $[\dot{X}]_{n\times1}$ = Differential State Vector

eg 1 $n = 2 \quad (x_1, x_2)$

$$[\dot{X}]_{2\times1} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}_{2\times1}$$

③ $[U]_{m\times1}$ = Input Vector

m = number of Input (or Input Variable)

Input Vector is Collection of Input Variables.

eg. If $m=1$ i.e Single Input

$m=1 \quad [u]$

$$[U]_{1\times1} = [u]_{1\times1}$$

eg 2 $m = 2$ $[u_1, u_2]$ Multiple Input

$$[U]_{m\times 1} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{m\times 1}$$

4) $[Y]_{p\times 1}$ = Output Vector

p = no. of output Variables (or simply output)

eg 1. $P = 1$ (Single o/p y)

$$[Y]_{p\times 1} = [y]_{p\times 1}$$ { 99% question

eg 2 $P = 2$ (Multiple o/p y_1, y_2)

$$[Y]_{2\times 1} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$ { 1% Question

5) $[A]_{n\times n}$ = System Matrix (square Matrix)

eg $n = 2$

$$[A]_{2\times 2} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$n = 3$

$$[A]_{3\times 3} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}_{3\times 3}$$

⑥ $[B]_{n \times m}$ = Input Matrix

m = no. of Input Variable

n = no. of state Variable

eg. $n = 2 \quad m = 1$

$$[B]_{2\times1} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

eg. $n = 2, \quad m = 2$

$$[B]_{2\times2} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}_{2\times2}$$

eg. $n = 3 \quad m = 1$

$$[B]_{3\times1} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

eg $n = 3 \quad m = 2$

$$[B]_{3\times2} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix}_{3\times2}$$

⑦ $[C]_{p\times n}$ = Output Matrix

eg: $n = 2, \quad p = 1$

$$[C]_{1\times2} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}_{1\times2}$$

eg. $n = 3, \quad p = 1$

$$[C]_{1\times3} = \begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix}$$

eg. $n = 2 \quad p = 2$

$$[C]_{2\times2} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}_{2\times2}$$

⑧ $[D]_{p \times m}$ = data transmission Matrix.

└→ no. of input Variable
└→ no. of o/p Variable

eg. $m=1,\ p=1$

$$[D]_{1\times1} = [D]$$

$m=2,\ p=2$

$$[D]_{2\times2} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

{ 99% $[D]=0$

eg. $n=2 \quad (x_1, x_2)$
$m=1 \quad (u)$
$p=1 \quad (y)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times1} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}_{2\times1} \begin{bmatrix} U \end{bmatrix}_{1\times1}$$

$$\begin{bmatrix} y \end{bmatrix}_{1\times1} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}_{1\times2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times1} + [D]_{1\times1} \begin{bmatrix} U \end{bmatrix}$$

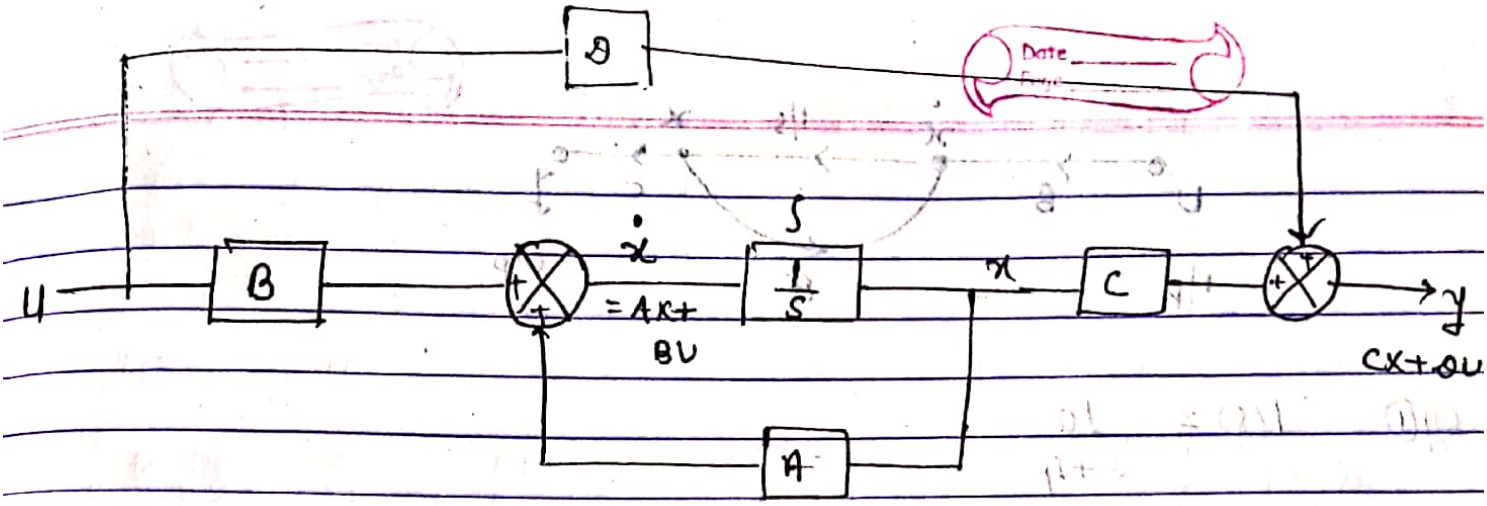


fig. Block Diag. of State ~~Variable~~ eqⁿ

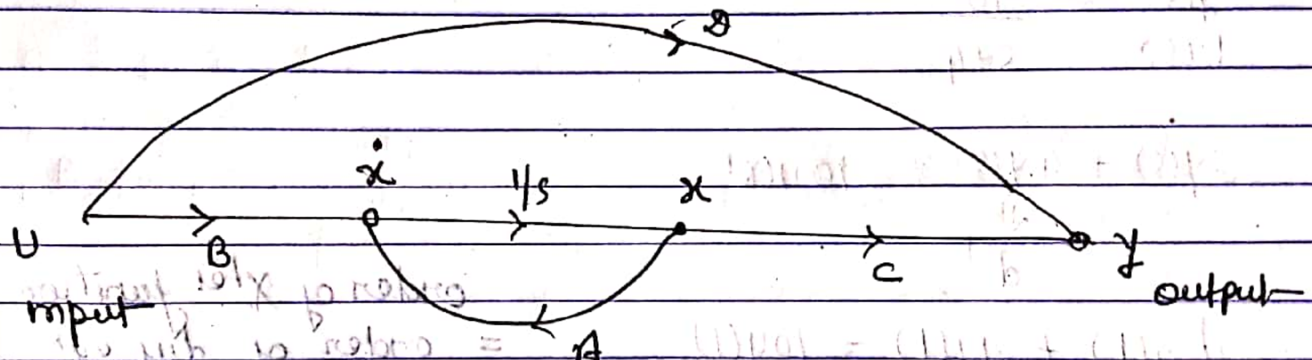
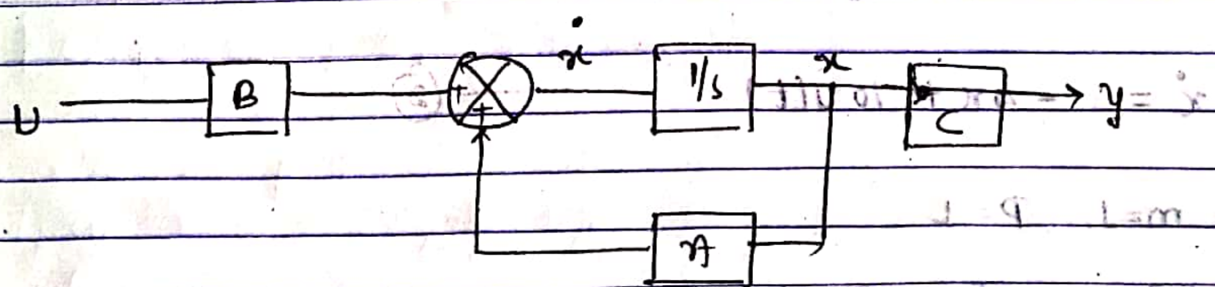


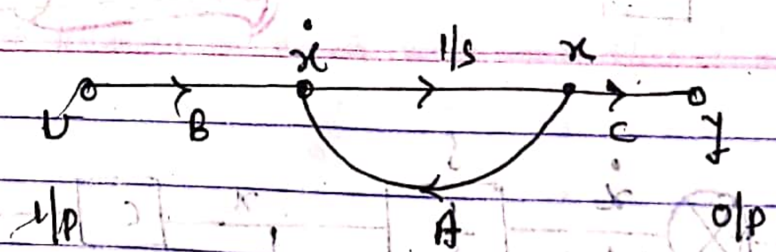
fig. ~~Block Di.~~ SFG of State eqⁿ

If $D = 0$

$\dot{x} = Ax + Bu$

$y = Cx$





eg① $T(s) = \frac{10}{s+4}$

$\frac{Y(s)}{U(s)} = \frac{10}{s+4}$

$\Rightarrow$ $sY(s) + 4Y(s) = 10U(s)$

$\Downarrow$

d

$\Rightarrow$ $\frac{d}{dt}y(t) + 4y(t) = 10u(t)$

order of x^{fer} function
= order of diff-eqn
= no. of state variable
$n=1$

let $y(t) = x$ (state Variable)

$\frac{dy(t)}{dt} = \dot{x}$

$\Rightarrow$ $\dot{x} + 4x = 10u(t)$

$\dot{x} = -4x + 10u(t)$ ②

$n=1, \ m=1, \ P=1$

$[\dot{x}]_{1\times1} = [A]_{1\times1}[x]_{1\times1} + [B]_{1\times1}[U]_{1\times1}$

$[Y]_{1\times1} = [C]_{1\times1}[x]_{1\times1} + [0]_{1\times1}[U]_{1\times1}$