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# GATE ESE PSU's 2019-20

# GATEACADEMY

## 110 SIGNAL SYSTEM_GATEACADEMY

TOTAL PAGE SIGNAL SYSTEM-260 PGAE

## LATEST GATEACADEMY CLASSROOM HANDWRITTING

**CONTENT COVERED:**

1. Theory Notes
2. Explanation
3. Derivation
4. Example
5. Shortcut & Formula Summary
6. Previous year Paper Q. Sol.

**Noted-: Single Source Follow, Revise**

**Multiple Time Best key of Success**



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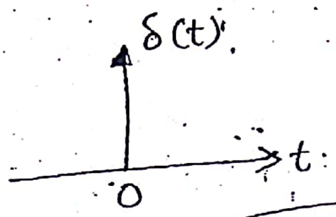
③ **Conti. Impulse or Dirac delta function:-**

$$\delta(t) = \begin{cases} \infty ; & t=0 \\ 0 ; & t \neq 0 \end{cases}$$



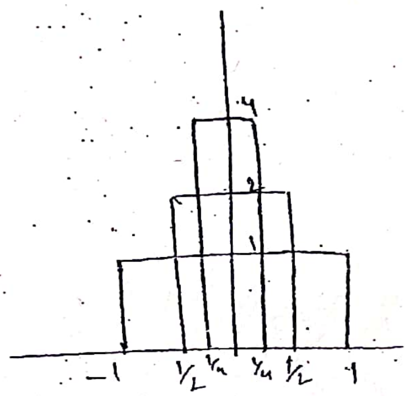

SIFTING Property
⇓
convolution
⇓
filtering.

→ Practically we can't generate this signal.

→ ~~Used~~ By approximation of standard signals.

we are generating impulse function.

Area under unit impulse ⇒ is '1'




(i) $\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = 1$

Area/strength.

(ii) Impulse is an even function of time.

$$\delta(-t) = \delta(t)$$

(iii) Scaling property

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$\int_{-\infty}^{\infty} \delta(at)\, dt = \int_{-\infty}^{\infty} \delta(\tau) \cdot \frac{d\tau}{a}$$

Put $at = \tau$

$a\,dt = d\tau$

$dt = \frac{d\tau}{a}$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

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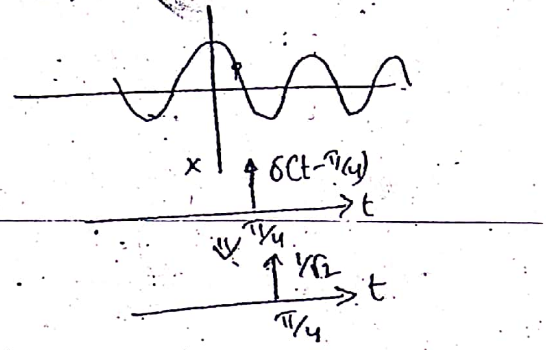
(iv) **Product**

$x(t).\delta(t-t_0) = x(t_0).\delta(t-t_0)$ if $x(t)$ is conti at $t=t_0$.

$t_0 \to$ Time shift



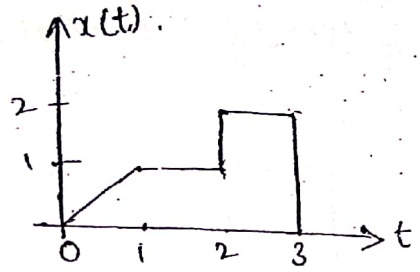

Ex ① ~~$\cos t \cdot \delta(t-\pi/4)$~~

$\Rightarrow \cos(\pi/4)\cdot\delta(t-\pi/4) = \frac{1}{\sqrt{2}}\cdot\delta(t-\pi/4)$.

② $t\cdot\delta(t)$

$t\cdot\delta(t) = t\cdot\delta(t-0)$
$= 0\cdot\delta(t)$
$= 0$

③




$x(t).\delta(t-3) \Rightarrow$ not defined because it is not continuous in time

v) **Sampling**

$$\int_{t_1}^{t_2} x(t).\delta(t-t_0)\,dt = \begin{cases} x(t_0); & t_1 \le t_0 \le t_2 \\ 0; & \text{else where} \end{cases}$$

shift should lies in within the limit

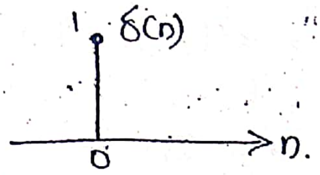
Ex $\int_{-1}^{2} (t+t^2)\,\delta(t-5)\,dt = 0$ ($t_0=5$)

~~$\int_{0}^{2} [t+\cos\pi t]\,\delta(t-1)\,dt$~~

$1+\cos\pi = 0$

## Discrete Impulse or kronecker delta :-

$$\delta(n) = \begin{cases} 1 ; & n = 0 \\ 0 ; & n \neq 0 \end{cases}$$




$$\boxed{\delta(n) = u[n] - u[n-1]}$$

$\delta(t) = \frac{d}{dt} u(t)$ $\quad u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$

$\int \to \Sigma$

$t \to n$

$\tau \to k$

$\delta(n) = u[n] - u[n-1]$

$$\boxed{u(n) = \sum_{k=-\infty}^{n} \delta(k)}$$

Put $n - k = m$

$k = -\infty \Rightarrow m = \infty$

$k = n \Rightarrow m = 0$

$$\boxed{u(n) = \sum_{m=\infty}^{0} \delta[n-m]}$$

$\delta(t) \to$ conti $\to$ Area

$\delta(n) \to$ discrete $\to$ Amplitude.

* $\delta(kn) = \underline{\delta(n)}$

$$\delta(4n) = \begin{cases} 1 ; & 4n = 0 \\ 0 ; & n \neq 0 \end{cases}$$

## TRANSFORMATIONS :-

① Time Scaling :-

$x(at) \mid x[mn]$

② Time shifting :-

Tele call $\Rightarrow$ 540 m sec.

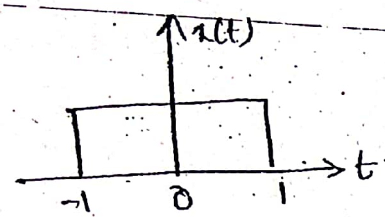
$x(t - t_0) \mid x[n - n_0]$

③ **Time reversal**

$x(-t) \mid x[-n]$

E1

①



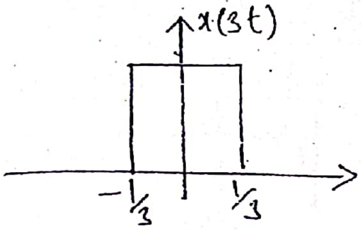
$x(3t)$, $x(t/5) = ?$

$x(3t) = \frac{1}{3} x(t)$

$x(t) = \begin{cases} 1 ; & -1 < t < 1 \end{cases}$

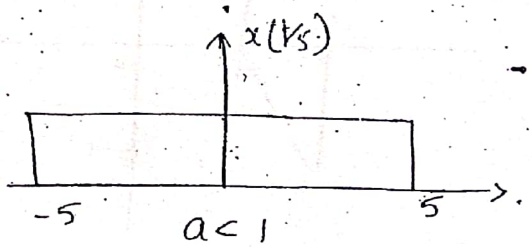
$x(3t) = \begin{cases} 1 ; & -1 < 3t < 1 \\ & \Rightarrow -\frac{1}{3} < t < \frac{1}{3} \end{cases}$

$x(t/5) = \begin{cases} 1 ; & -5 < t < 5 \end{cases}$



$a > 1$

Compression of $x(t)$



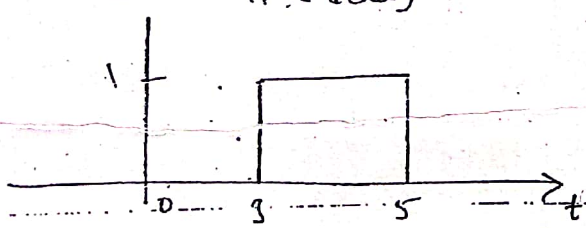
$a < 1$

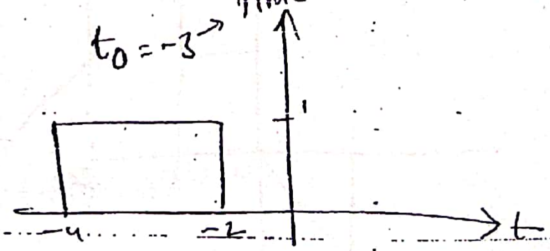
Expansion of $x(t)$.

② $x(t-4)$, $x(t+3)$

$t_0 = 4$ – Time delay

$t_0 = -3 \rightarrow$ Time advance





$t_0 > 0$

$\Downarrow$

Right shift

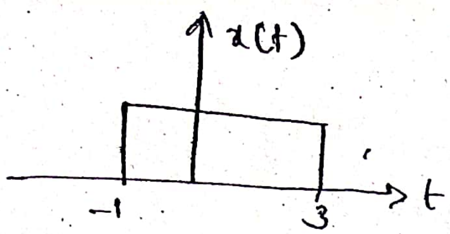
$t_0 < 0$

$\Downarrow$

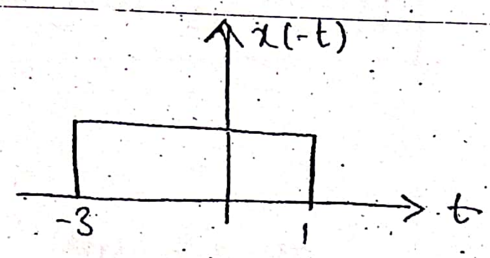
Left shift

Real time processing $x(t+3)$ is not possible. It is off - Unf

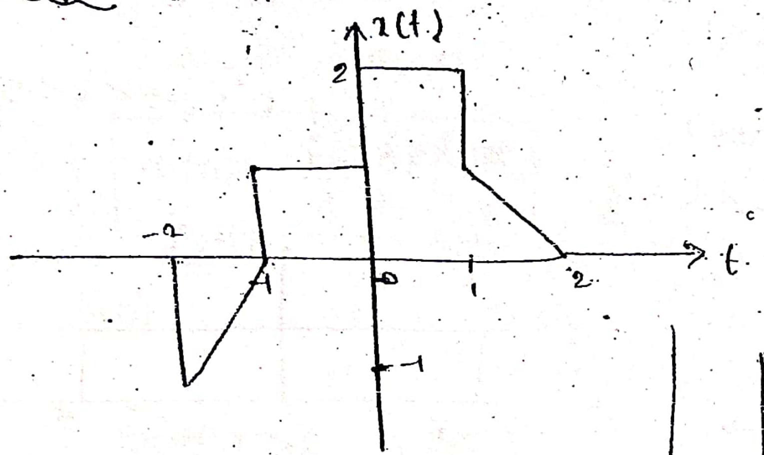
③




$x(-t)$

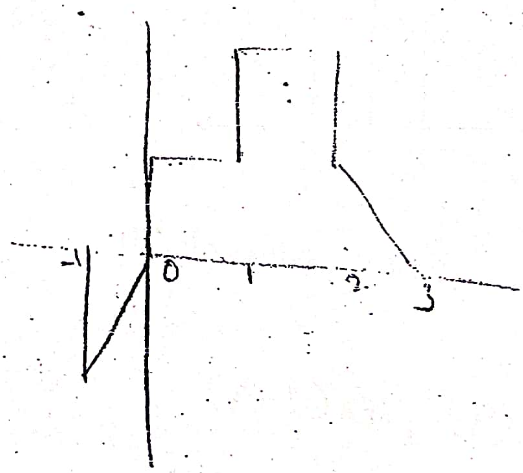


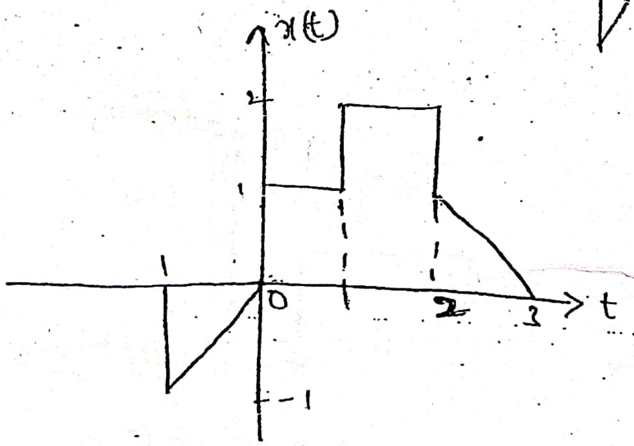

Examples:




a) $x(t-1)$

$t_0 = 1$




b) $x(2t+1)$

$x[2(t+\frac{1}{2})]$

**Method 1:**

$$x(\alpha t+\beta)$$

$$= x\left[\alpha\left[t+\frac{\beta}{\alpha}\right]\right]$$

**Method 2:-**

$$x(t) \longrightarrow x(t+\beta) \xrightarrow{t=\alpha t} x(\alpha t+\beta)$$

---

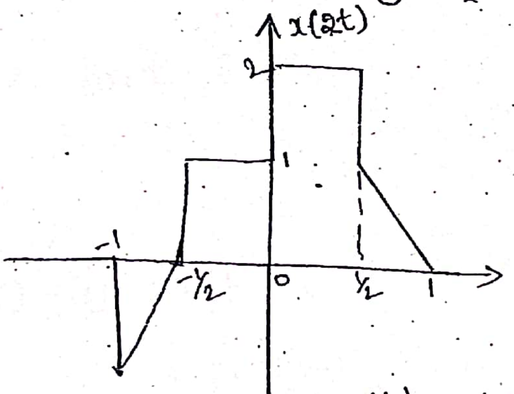
$\Rightarrow x(2t+1)$

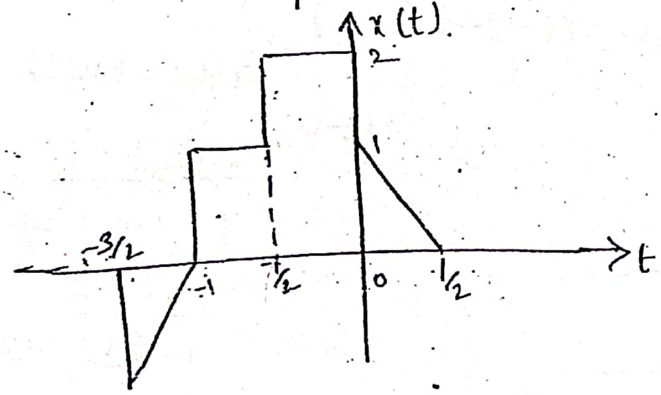
$$= x\left[2\left(t+\tfrac{1}{2}\right)\right]$$

$$t_0 = -\tfrac{1}{2}$$

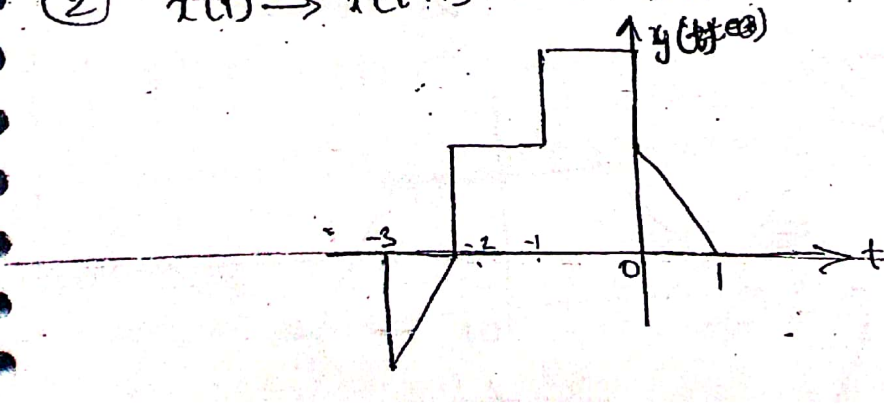
(1) (i) Time Scaling.

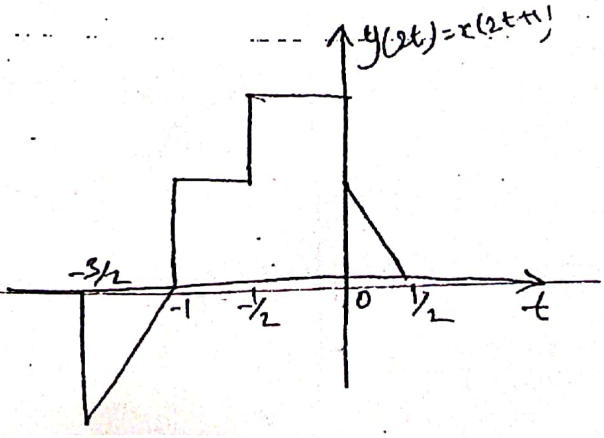
(ii) Shift the $x(2t)$ left by "$-\frac{1}{2}$"



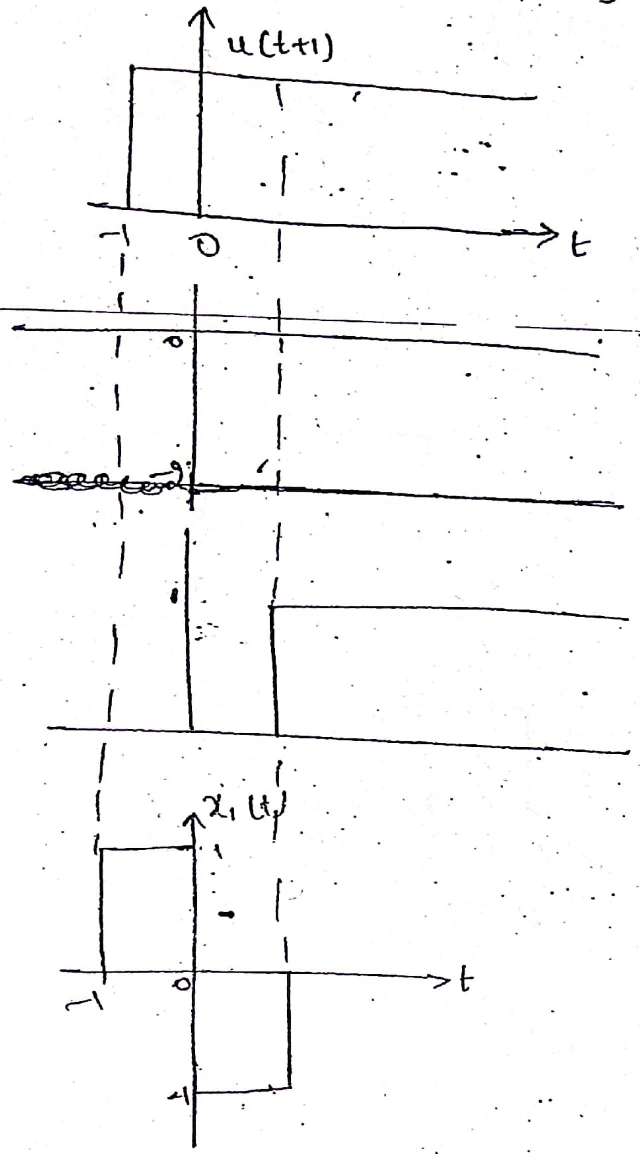




(2) $x(t) \longrightarrow x(t+1) \Rightarrow y(t) \xrightarrow{t=2t} x(2t+1) \Rightarrow y(2t)$

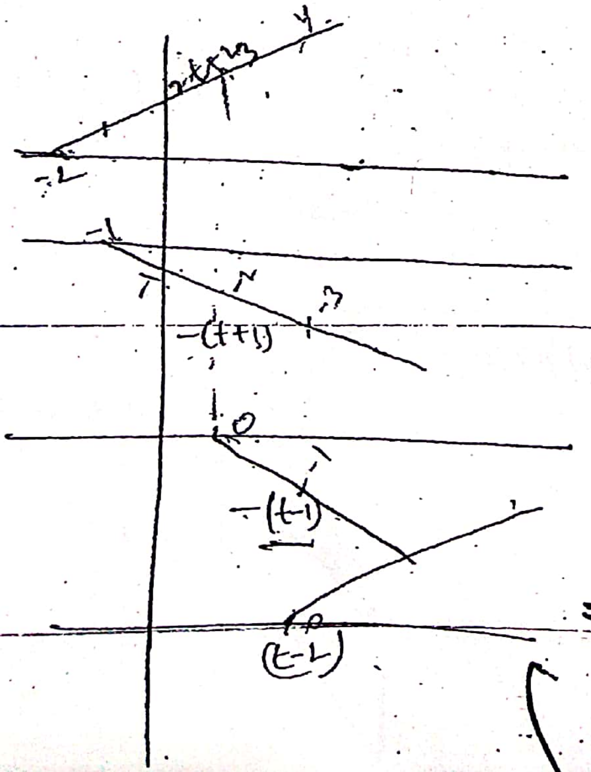


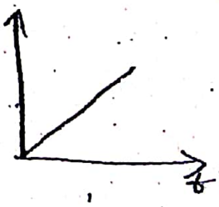
③ a) $x_1(t) = u(t+1) - 2u(t) + u(t-1)$




b) $x_2(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$




$r(t) = t\,u(t)$



$t > 0$

$-1 < t < 1$

$t+2-t-1 = 1$

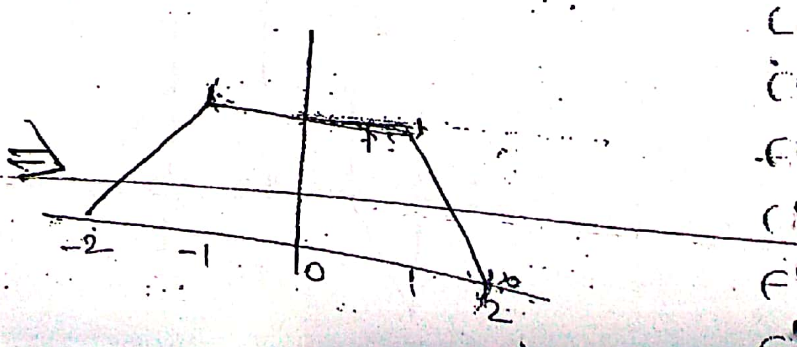
$1 < t < 2$

$1 - t + 1 = (2-t)$

$t > 2$

$2 - t + t - 2 = 0$

$\Rightarrow$



4) Multiplication

$x(\tau)\, h(t-\tau)$, or $x(t-\tau)\, h(\tau)$

5) Integration

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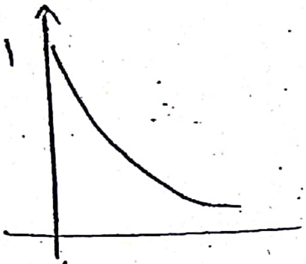
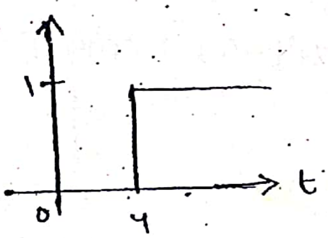
(a) $x(t) = e^{-3t} u(t)$

$h(t) = u(t-4)$

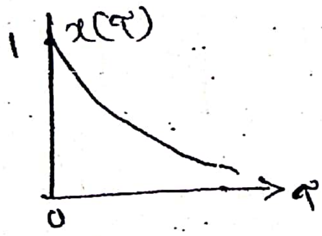
(i)

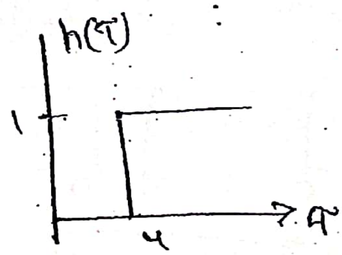


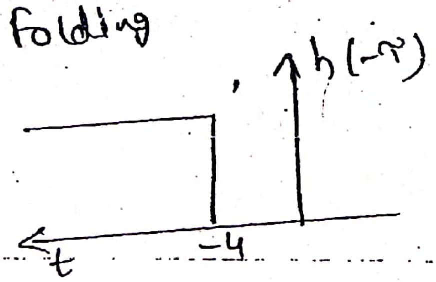

Limits of $y(t)$ : $4 < t < \infty$

(ii) $t \to \tau$




(iii) Folding

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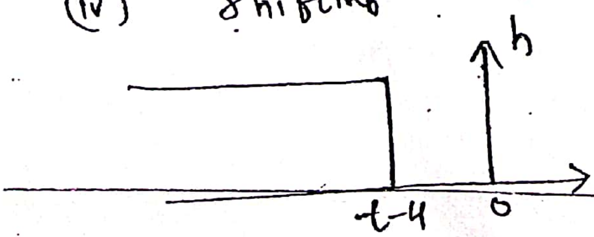
(iv) Shifting

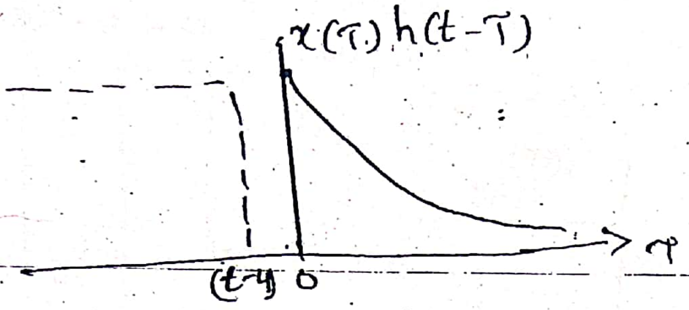



$h(t) = \begin{cases} 1 ; & t > 4 \end{cases}$

$h(t-\tau) = \begin{cases} 1 & ; t-\tau > 4 \\ & t-4 > \tau \end{cases}$

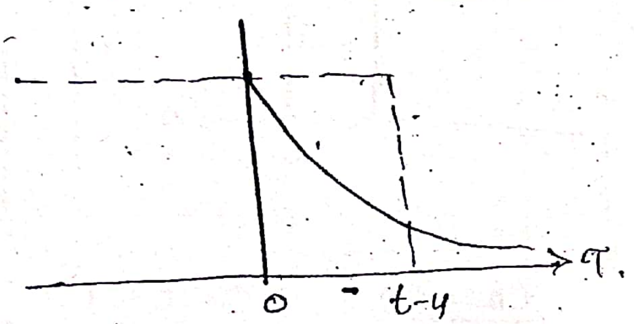
$\tau < t-4$

Case i) :- $t-4<0$




$y(t) = 0 \; ; \; t<4$

Case ii) :-




range of 'τ' for which integrated is defined

$$y(t) = \int_0^{t-4} e^{-3\tau} . u(\tau) (1) . d\tau$$

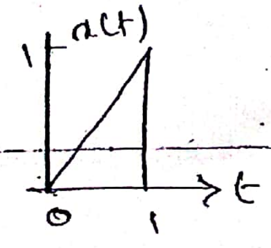
$$= \left(\frac{-e^{-3}}{3}\right)_0^{t-4}$$

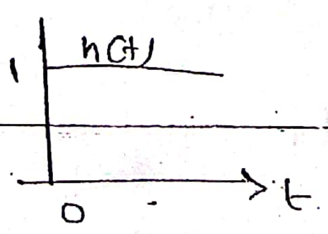
$$= \underline{\qquad} \; ; \; t>4$$

Convolution Two Causal systems are Causal only.

Convolution of Two Non Causal system is anti causal.

(2)

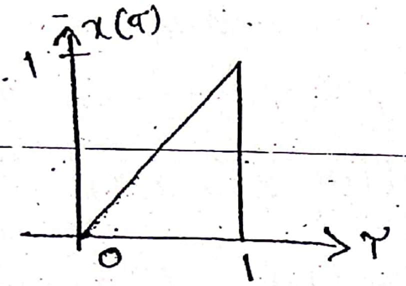


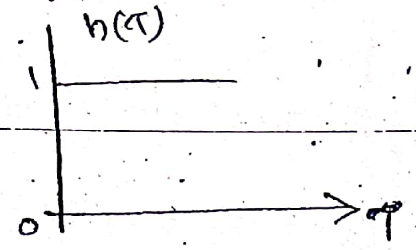



$x(t) = \{ x \;;\; 0 < t < 1$

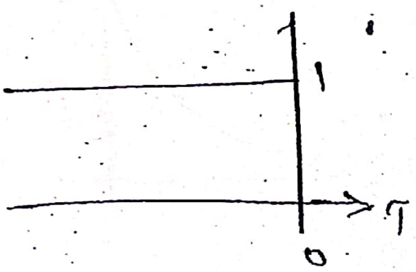
$h(t) = \{ 1 \;;\; t > 0$

limits of $y(t)$ : $0 < t < \infty$

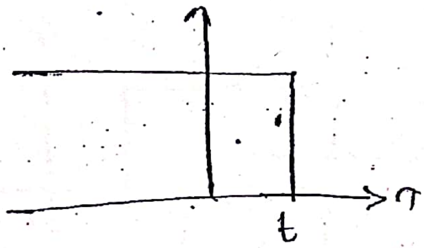


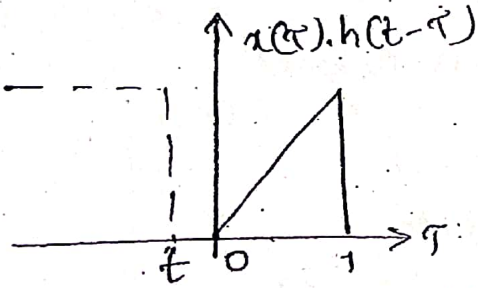

folding




shifting

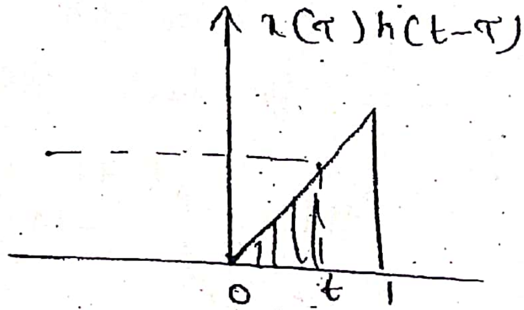



Case (i) :- $t < 0$



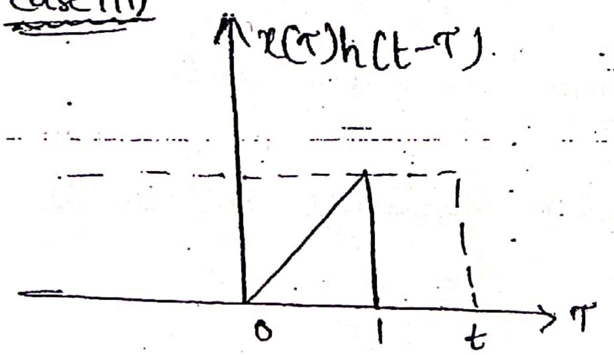

$y(t) = 0 \;;\; t < 0$

Case (ii) :- $0 < t < 1$




$y(t) = \int_0^t (\tau) \cdot d\tau$

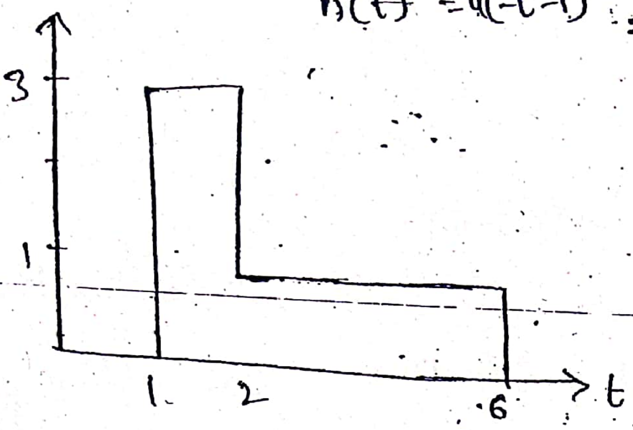
Case (iii)

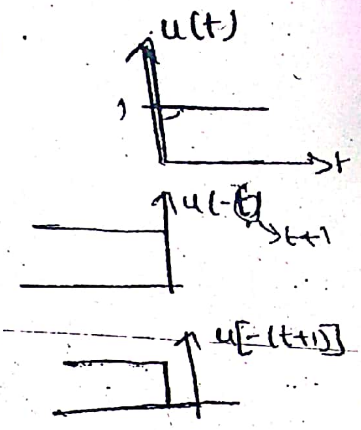



$y(t) = \int_0^1 (\tau)(1) \cdot d\tau$

(8)

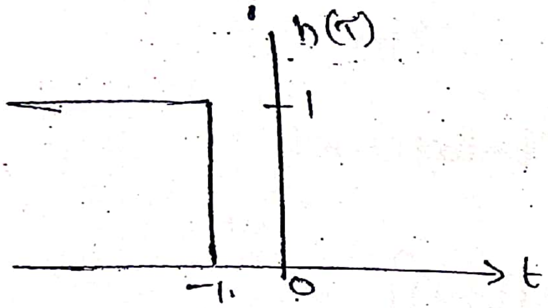
$h(t) = u(-t-1) \Rightarrow u[-(t+1)]$



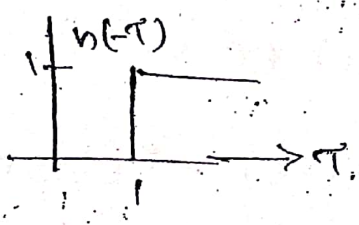




(i)

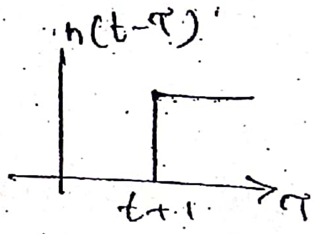
$h(t) = u[-(t+1)]$




limits : $-\infty < t$




$\Rightarrow$



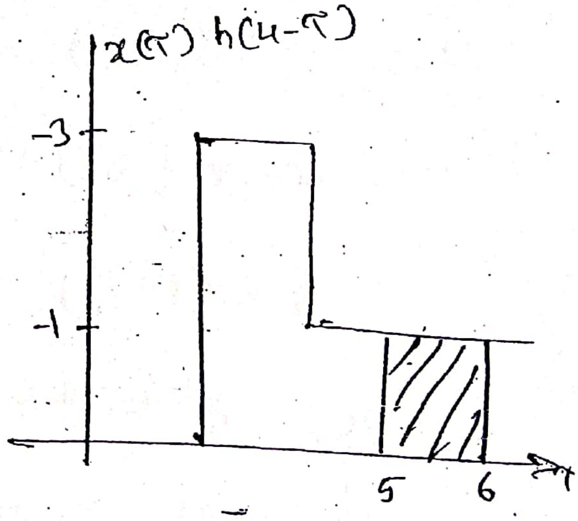

$$y(t) = \int x(\tau).h(t-\tau).d\tau$$

o/p at $t=4$:-

$$y(4) = \int x(\tau).h(4-\tau).d\tau$$

$$= \int_5^t (1)(1).d\tau$$

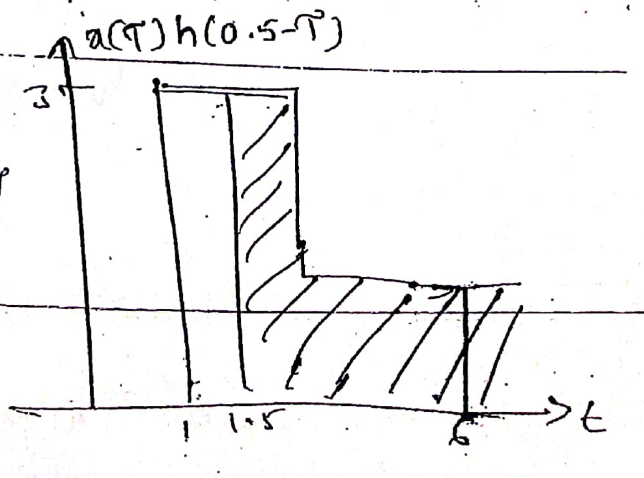
$$= 1$$




$$y(0.5) = \int x(\tau)\ h(0.5-\tau).d\tau$$

$$= \int_{1.5}^{2} (3).(1).d\tau + \int_2^6 (1)(1).d\tau$$

$$= 5.5$$



(4) $z(t) = \int_{-\infty}^{+\infty} x(-\tau+a)\, h(t+\tau)\, d\tau$

$t+\tau = \lambda$

$= \int_{-\infty}^{\infty} x(t-\lambda+a)\ h(\lambda)\, d\tau$

$= \int x(t+a-\lambda)\ h(\lambda)\, d\tau$

$\boxed{z(t) = y(t+a)}$

**Convolution property of impulse**

$x(t) * \delta(t-t_0) = x(t-t_0)$

Ex 1) $x(t+9) * \delta(t-5) = x(t-5+9) = x(t+4)$.

2) $x(t) * \delta(2t+1)$

$* \delta((t+1/2)2)$

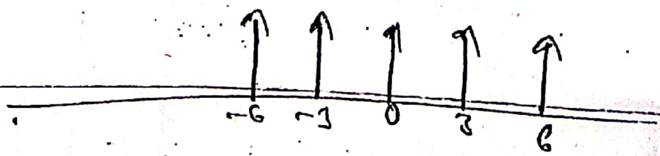
$= x(t) * \frac{1}{2}\delta(t+1/2)$

$= \frac{1}{2} x(t+1/2)$

(6) $y(t) = e^{-t} u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$

$y(t) = A \cdot e^{-t}$ for $0 \le t \le 3$

$e^{-t} u(t) * \sum_{k=-\infty}^{+\infty} \delta(t-3k)$




$= \ldots + e^{-(t+6)} u(t+6) + e^{-(t+3)} u(t+3) + e^{-t} u(t) + e^{-(t-3)} u(t-3)$

$$A \cdot e^{-t} = e^{-t}\left[+ \cdots + e^{-6} + e^{-3} + 1\right]$$

$$= \frac{1}{1-e^{-3}}$$

* Given $x(t) = u(t-2) - u(t-4)$ & $h(t) = e^{t} u(t)$ find

$\frac{d}{dt} x(t) * h(t)$

$$x(t) = u(t-2) - u(t-4)$$

$$h(t) = e^{t} u(t)$$

$$\frac{d}{dt} x(t) * h(t)$$

$$= [\delta(t-2) - \delta(t-4)] * e^{t} u(t)$$

$$= e^{(t-2)} \cdot u(t-2) - e^{(t-4)} u(t-4)$$

* $x^{m}(t) * h^{n}(t) = y^{m+n}(t)$

$m$ & $n$ $\rightarrow$ order of diff eq$^n$s.

* $x(t-a) * h(t-b)$

$$= y(t-a-b).$$

* $x(t) \rightarrow A_x$

$h(t) \rightarrow A_h$

$y(t) \rightarrow A_y = 1$

$$y(t) = \int x(\tau) \cdot h(t-\tau) \cdot d\tau.$$

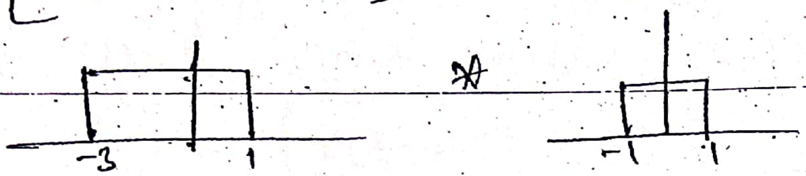
$$\cancel{\int y(t) \cdot dt = \iint x(\tau) h(t-\tau) \cdot d\tau \cdot dt}$$

$$= \int x(\tau) \cdot d\tau \cdot \int h(t-\tau) \cdot dt$$

(10) $x(\alpha t) * h(\alpha t) = \frac{1}{|\alpha|} y(\alpha t)$

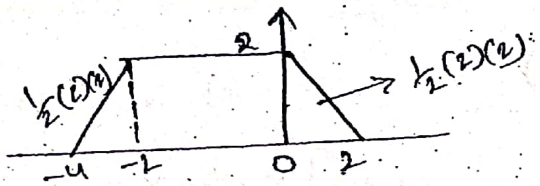
$x(-t) * h(-t) = y(-t)$

(11) $[u(t+3) - u(t-1)] * [u(t+1) - u(t-1)]$



$u(t+3) * u(t+1) - u(t+3) * u(t-1) - u(t-1) * u(t+1) + u(t-1) u(t-1)$

$= r(t+4) - r(t+2) - r(t) + r(t-2)$




Note: Convolution of two unequal duration rectangular functions is Trapezium. If the duration is same then it is Traingle

Conv by diff :-

$\frac{dy(t)}{dt} = \frac{d}{dt} x(t) * h(t)$

or

$x(t) * \frac{d}{dt} h(t)$

$y(t) = \int_{-\infty}^{t} x(t) * \frac{d}{dt} h(t)$.

let $g(t) = u(t) * r(t)$

$\frac{d}{dt} g(t) = \frac{d}{dt} u(t) * r(t)$

$= r(t) = t \cdot u(t)$

$g(t) = \int_{0}^{t} t \cdot dt$

$= \frac{t^2}{2} : t > 0$

Material
(9)

$u(t+1) * r(t-2)$

$= g(t-2+1)$

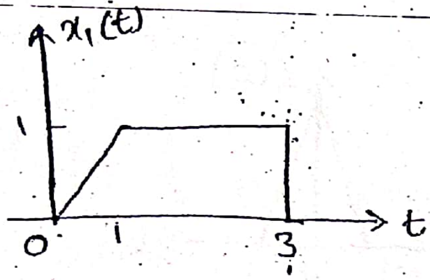
$= g(t-1)$

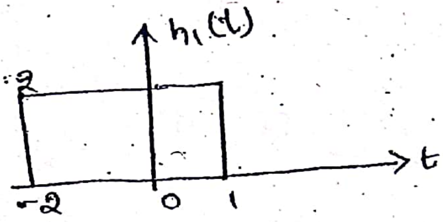
$$= \frac{(t-1)^2}{2} \; ; \; t-1>0$$

$$t>1$$

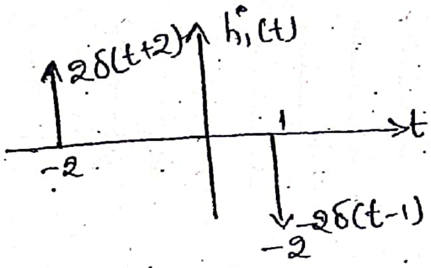
$$= \frac{(t-1)^2}{2} u(t-1)$$

(7)




$$y_1(t) = x_1(t) * h_1(t)$$

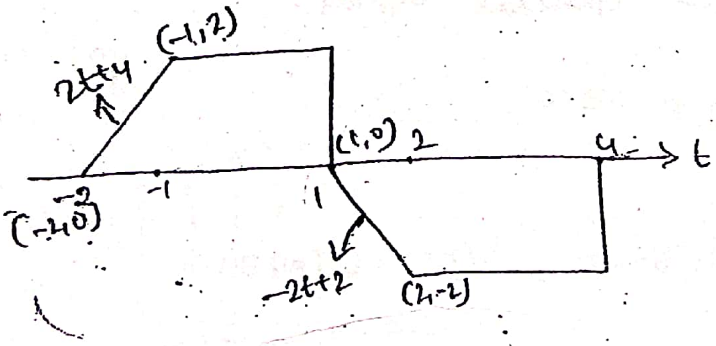



$$\frac{dy_1(t)}{dt} = x_1(t) * \frac{d}{dt} h_1(t).$$

$$y_1(t) = \int_{-\infty}^{t} x_1(t) * \frac{d}{dt} h_1(t)$$

$$\boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$$

$$y_1'(t) = x_1(t) * h_1'(t)$$




Case (i) :- $-2<t<-1$

$$y_1(t) = \int_{-2}^{t} (2t+4)\, dt = t^2+4t+4$$

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0 = \frac{2}{+1}(x+2)$$

$$y = 2x+4$$

Case (ii) : $-1<t<1$

$$y_1(t) = 1 + \int_{-1}^{t} 2\,dt = 2t+3$$

case (iii) :- $1<t<2$

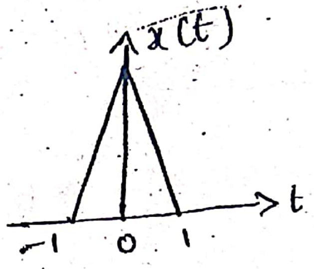
$$y_1(t) = 5t \int_1^t (-2t+2)\,dt = -t^2+2t+4$$

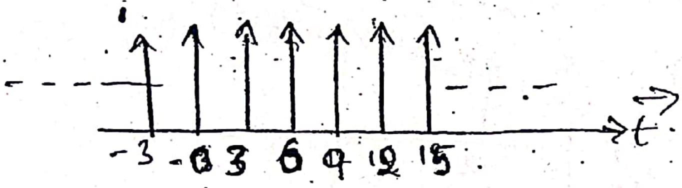
Case (iv) :- $2<t<4$

$$y_1(t) = 4 + \int_2^t -2\,dt = 8-2t$$

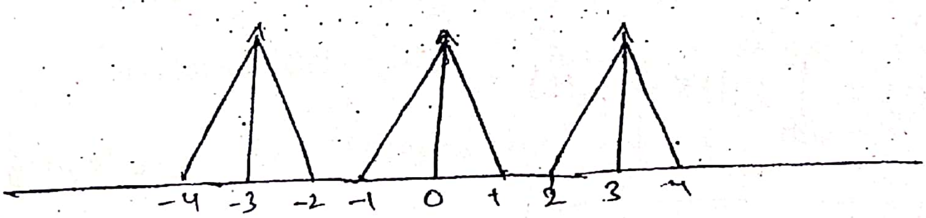
(10)

$$h(t) = \sum_{n=-\infty}^{\infty} \delta(t-3n).$$




→ Impulse train / Dirac comb

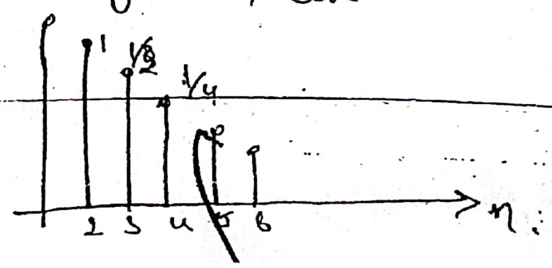



Convolution of a periodic signal with a general signal is periodic repetation of General Signal. It valid when channel / signal b/w is more.

(11)

$$x(n) = \left(\frac{1}{2}\right)^{n-2} u(n-2), \quad h(n) = u(n+2)$$

$$= \begin{cases} \left(\frac{1}{2}\right)^{n-2} & ; n \geq 2 \\ 0 & ; \text{else} \end{cases}$$

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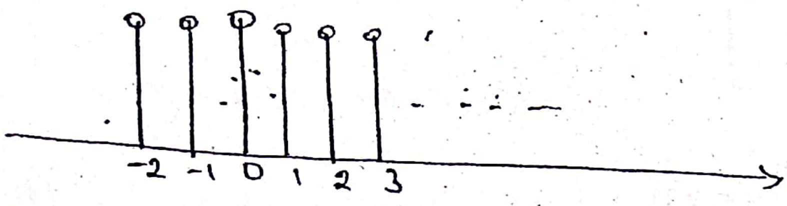
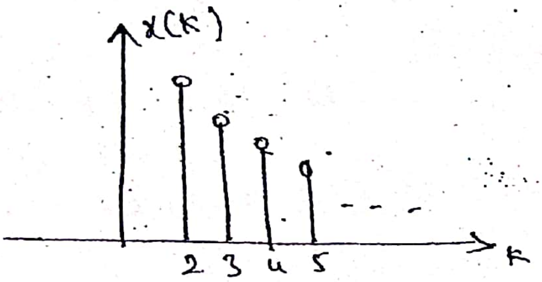
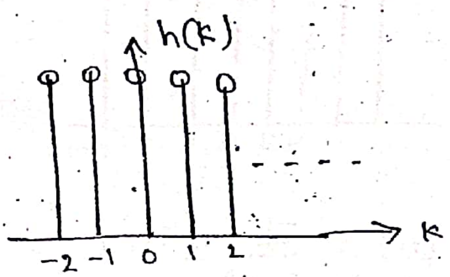
$h(n) = u(n+2)$



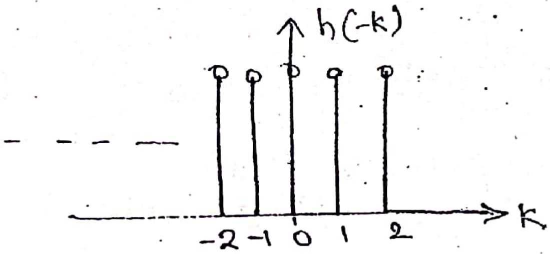

Limits of $y(n)$ : $0 \le n \le \infty$

(ii) Change of axis from 'n' to 'k'.

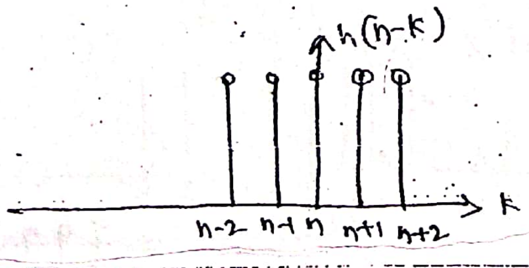


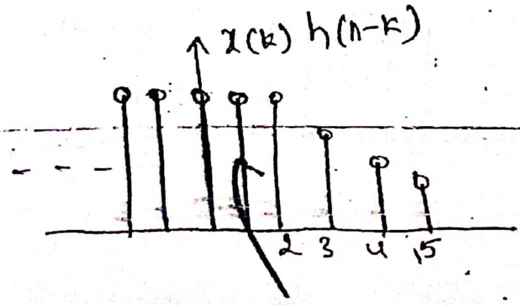

(iii) Folding $\Rightarrow h(-k)$



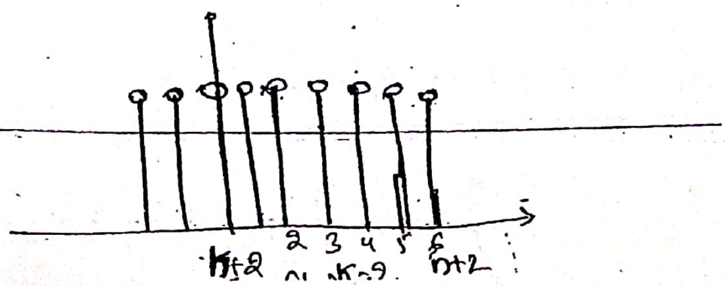

(iv) Shifting $h(n-k)$




V) Case (i) :- $n+2<2$




Case (ii) $n+2 \ge 2$



①

$e^{4t}u(t) \leftrightarrow \frac{1}{4-j\omega}$

Q $\frac{1}{4-jt} \xleftrightarrow{\text{Dual}} 2\pi \cdot e^{4(-\omega)} \cdot u(\omega)$

②

$e^{-|t|} \leftrightarrow \frac{2}{\omega^2+1}$

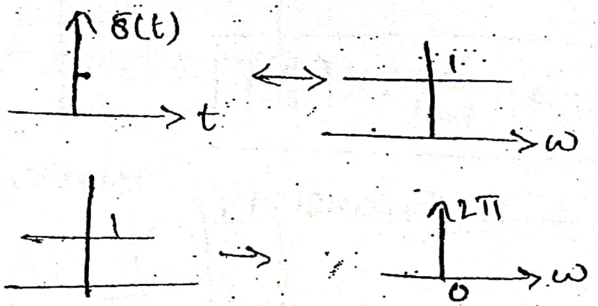
Q: $\frac{2}{t^2+1} \leftrightarrow 2\pi \cdot e^{-|-\omega|}$

③

$\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\,\delta(-\omega)$

$2\pi\,\delta(\omega)$




④

$sgn(t) \leftrightarrow \frac{2}{j\omega}$

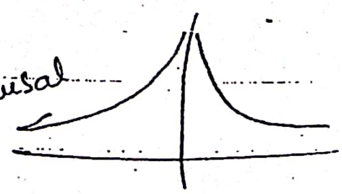
$\boxed{\frac{2}{j\omega} \to \frac{2}{j(t)}}$ $\quad \frac{\cancel{2}}{jt} \leftrightarrow \cancel{2}\pi\, sgn(-\omega)$

$\frac{1}{jt} \leftrightarrow -\pi\, sgn(\omega)$

$\boxed{\frac{1}{\pi t} \leftrightarrow -j\, sgn(\omega)}$ ⇒ Non Causal

Impulse response of Hilbert T.F.

$\left|\frac{1}{\pi t}\right|$



because two sided signal

$sgn(t) = 2u(t) - 1$

$u(t) = \frac{1+sgn(t)}{2}$

$u(t) \leftrightarrow 2\pi\delta(\omega) + 2/j\omega \leftrightarrow \frac{1}{\cdot} + \pi\delta(\omega)$

① I.F.T. of $u(\omega) = ?$ :

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} \cdot d\omega}$$

$$\frac{1}{jt} + \pi\delta(t) \longleftrightarrow 2\pi . u(-\omega).$$

Apply Time reversal $-t \to -\omega$

$$\frac{1}{-jt} + \pi\delta(-t) \longleftrightarrow 2\pi u(\omega)$$

$$\left[\frac{1}{-jt} + \pi\delta(-t)\right]\frac{1}{2\pi} \longleftrightarrow u(\omega).$$

③ Time Scaling :-

$$\boxed{x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|} \cdot X\left(\frac{\omega}{\alpha}\right)}$$

Compression $\longleftrightarrow$ Expansion.

$$F\{x(\alpha t)\} = \int_{-\infty}^{\infty} x(\alpha t) \cdot e^{-j\omega t} \cdot dt.$$

put $\alpha t = \tau$

$\alpha dt = d\tau$

$dt = \frac{d\tau}{\alpha}$

$$= \int_{-\infty}^{+\infty} x(\tau) \cdot e^{-j\omega\frac{\tau}{\alpha}} \cdot \frac{d\tau}{\alpha}$$

$$= \frac{1}{\alpha}\int_{-\infty}^{+\infty} x(\tau) \cdot e^{-j\left(\frac{\omega}{\alpha}\right)\tau} \cdot d\tau$$

$$= \frac{1}{|\alpha|} \cdot X\left(\frac{\omega}{\alpha}\right).$$

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(Jhu.edu)

* $x_1(t) = A\, rect\left(\frac{2t}{T}\right)$. Assume $A\,rect(t/T) \longleftrightarrow AT\,Sa\left(\frac{\omega T}{2}\right)$

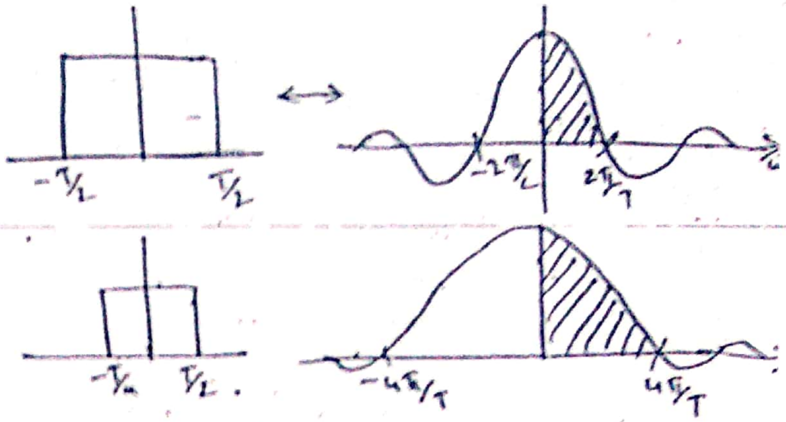
$= x(2t)$

$\alpha = 2.$

$X_1(\omega) = \frac{1}{2} X(\omega/2)$

$= \frac{AT}{2} \cdot Sa\left(\frac{\frac{\omega}{2} \cdot T}{2}\right)$

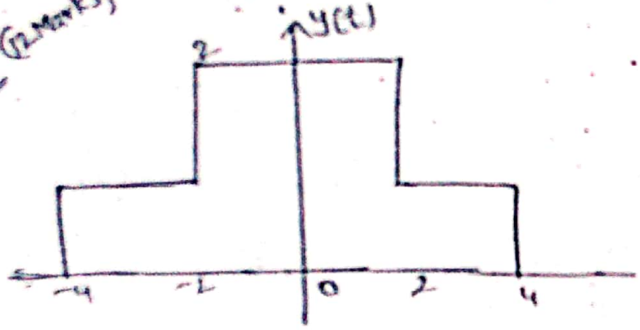
$= \frac{AT}{2} \cdot Sa\left(\frac{\omega T}{4}\right).$




$\frac{\omega T}{4} = \pm n\pi$

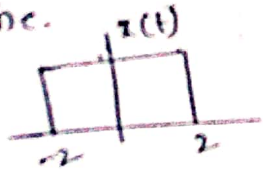
$\boxed{\omega = \pm \frac{4n\pi}{T}}$

(4) * IES (12 Marks)



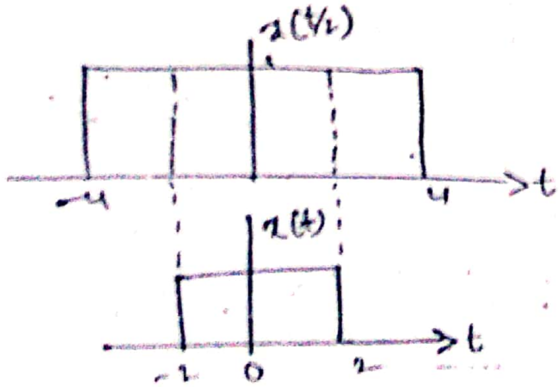

Assume.

A=1
T=4




$x(\omega)$.

$\longleftrightarrow 4\,Sa\left(\frac{4\omega}{2}\right)$




$y(t) = x(t) + x(t/2)$

$\Big\downarrow$ F.T $\qquad \alpha = 1/2$

$Y(\omega) = X(\omega) + \frac{1}{|1/2|} \cdot X\left(\frac{\omega}{(1/2)}\right)$,, ① A signal $x(t)$ having spectrum $X(\omega)$, which satisfies the relation $\ln X(\omega) = -|\omega|$. Find the signal if

a) $x(t)$ is real & even

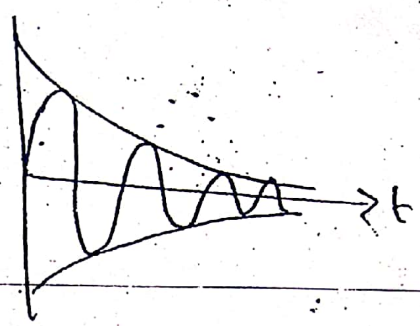
b) $x(t)$ is real & odd.

2) $y(t) = e^{-at}\sin\omega_c t\, u(t) \rightarrow$ damped sinusoidal

$= x(t)\cdot\left[\frac{e^{j\omega_c t}-e^{-j\omega_c t}}{2j}\right]$

F.T

$Y(\omega) = \frac{X(\omega-\omega_c)-X(\omega+\omega_c)}{2j}$



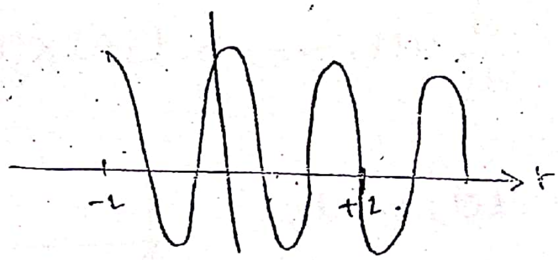

$x(t)e^{-at} = \frac{1}{a+j\omega}$

$= \frac{1}{2j}\left[X(\omega-\omega_c)-X(\omega+\omega_c)\right]$

$= \frac{1}{2j}\left[\frac{1}{a+j(\omega-\omega_c)} - \frac{1}{a+j(\omega+\omega_c)}\right]$

3) $y_1(t) = rect(t/4)\cos 10t$

$= x(t)\cdot\left[\frac{e^{10t}+e^{-10t}}{2}\right]$




$Y_1(\omega) = \frac{X(\omega-10)+X(\omega+10)}{2}$

Let $x(t) = rect(t/4)$ $A=1$, $T=4$

$X(\omega) = 4Sa\left(\frac{4\omega}{2}\right)$

$= \frac{1}{2}\left[4Sa\left(\frac{4(\omega-10)}{2}\right) + 4Sa\left(\frac{4(\omega+10)}{2}\right)\right]$

4) $x(t) = \frac{\cos 3t}{t^2+1}$

$x(t) \leftrightarrow X(\omega)$

$= x(t)\cdot\left[\frac{e^{j3t}+e^{-j3t}}{2}\right]$

$\frac{2}{t^2+1} \leftrightarrow 2\pi\cdot e^{-|\omega|}$

$Z(\omega) = \frac{X(\omega-3)+X(\omega+3)}{2}$

$= \pi\cdot e^{-|\omega-3|} + \pi\cdot e^{-|\omega+3|}$

⑤ I.F.T. of $X(3\omega+5)$

$$X(3\omega+5) \rightarrow X\left[3\left(\omega+\tfrac{5}{3}\right)\right]$$

$$Z\left(\omega+\tfrac{5}{3}\right).$$

$$Z(\omega) = X(3\omega)$$

$$= Z\left(\omega+\tfrac{5}{3}\right) \qquad = \frac{1}{3}\cdot\frac{1}{\left|\frac{1}{3}\right|}\cdot X\left(\frac{\omega}{\left(\frac{1}{3}\right)}\right)$$

$$\Leftrightarrow z(t)\cdot e^{j\left(-\frac{5}{3}\right)t} \qquad z(t) = \frac{1}{3}\cdot x\left(\tfrac{t}{3}\right)$$

$$X(3\omega+5) = \frac{1}{3}x\left(\tfrac{t}{3}\right)\cdot e^{+j\left(-\frac{5}{3}\right)t}$$

**Diff. in time:**

$$x(t) \longleftrightarrow X(\omega)$$

$$\frac{d}{dt}x(t) \longleftrightarrow (j\omega)X(\omega)$$

$$X(\omega) \neq \frac{F\left\{\frac{d}{dt}x(t)\right\}}{j\omega}$$

Ex ①

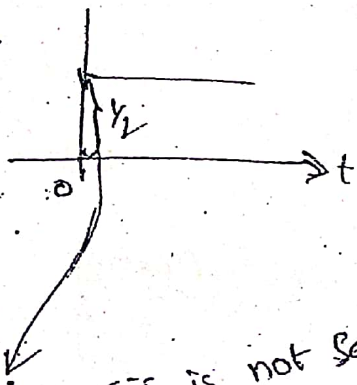
$$x(t) = u(t)$$

$$\frac{d}{dt}x(t) = \delta(t)$$

$\downarrow$ F.T.

$$j\omega X(\omega) = 1$$

$$X(\omega) = \frac{1}{j\omega} + (?)$$




This is not satisfying by the formula $X(\omega) \neq \frac{F\left\{\frac{d}{dt}x(t)\right\}}{j\omega}$

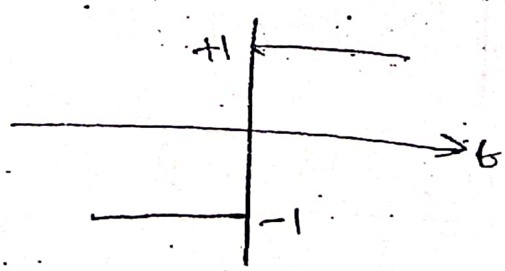
Ex ②

$$x(t) = \operatorname{sgn}(t) = 2u(t) - 1$$

$$\frac{d}{dt}x(t) = 2\delta(t)$$

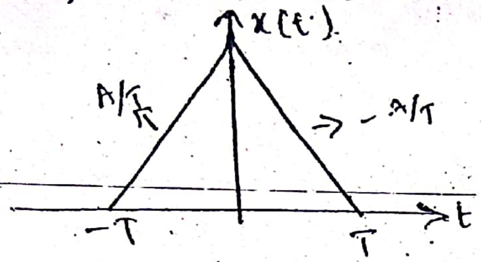
$$j\omega X(\omega) = 2$$

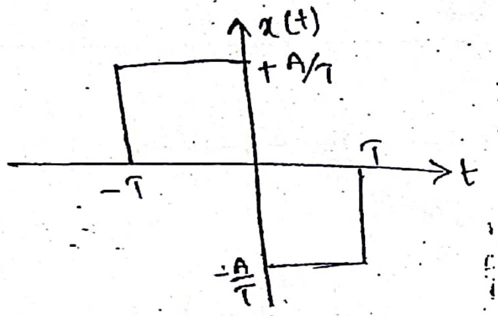
$$X(\omega) = \frac{2}{j\omega}$$

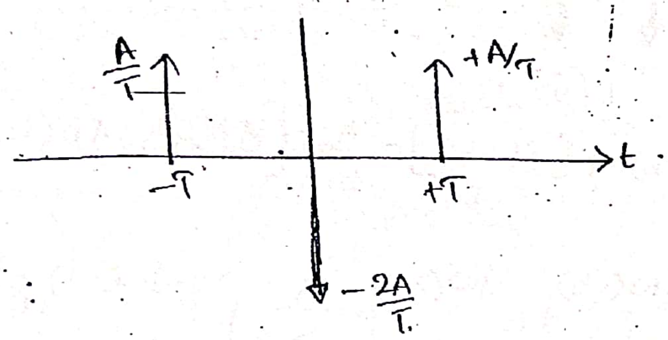


IES

(13) By using differentiation property find the F.T. of Triangula function shown below?




$$\frac{d^2}{dt^2}x(t) = \frac{A}{T}\left[\delta(t+T) + \delta(t-T)\right] - \frac{2A}{T}\delta(t)$$

$$\delta(t-t_0) \xleftrightarrow{F.T} e^{-j\omega t_0} \quad (1)$$

$\downarrow$ F.T =

$$(j\omega)^2 X(\omega) = \frac{2A}{T}\left[\frac{e^{-j\omega(-T)} + e^{-j\omega(+T)}}{2}\right] - \frac{2A}{T}$$

$$= \frac{2A}{T}\left[\cos\omega T - 1\right]$$

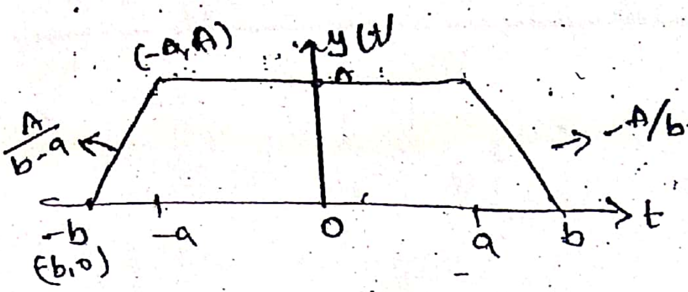
$$1 - \cos\theta = 2\sin^2\theta/2$$

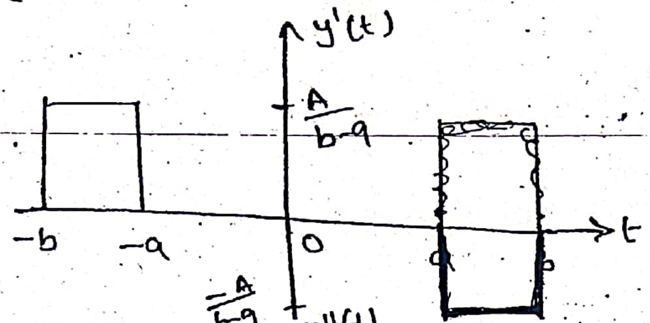
$$+\omega^2 X(\omega) = \frac{+2A}{T}\left[1 - \cos\underbrace{\omega T}_{\theta}\right]$$

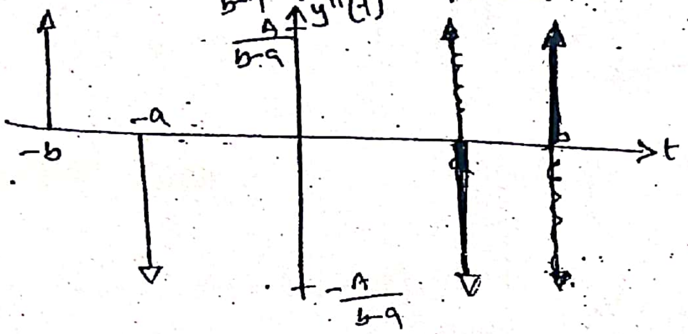
$$\omega^2 X(\omega) = \frac{2A}{T}\left[2\sin^2\left(\frac{\omega T}{2}\right)\right]$$

$$X(\omega) = \frac{4A\sin^2(\omega T/2)}{\omega^2 T} = \frac{4A \cdot \sin^2\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)^2 \cdot \frac{4}{T}}$$

(4)




$$\frac{d^2}{dt^2}y(t) = \frac{A}{b-a}\left[\delta(t+b)+\delta(t-b)\right] - \frac{A}{b-a}\left[\delta(t+a)+\delta(t-a)\right]$$

$$(j\omega)^2 Y(\omega) = \frac{A}{b-a}\left[e^{-j\omega(-b)} + e^{-j\omega(b)}\right] - \frac{A}{b-a}\left[e^{-j\omega(-a)} + e^{-j\omega(a)}\right]$$

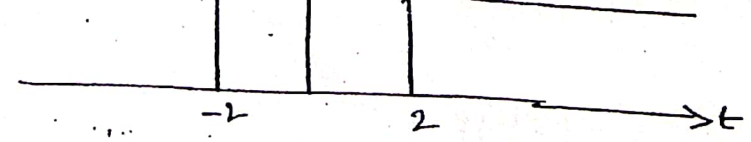
$$-\omega^2 Y(\omega) = \frac{2A}{b-a}\left[\cos\omega b - \cos\omega a\right]$$

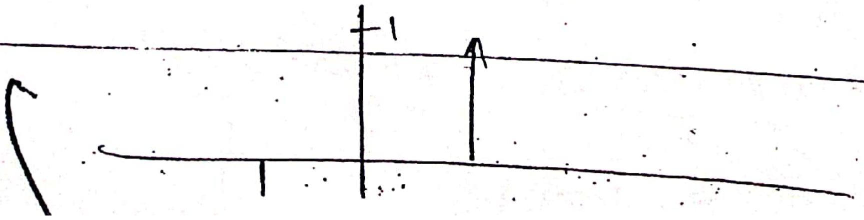
$$Y(\omega) = \frac{2A}{(a-b)\omega^2}\left[\cos\omega b - \cos\omega a\right]$$

tutorial

(6)

$$y(t) = \frac{d}{dt}\left[u(t-2) + u(-t-2)\right]$$



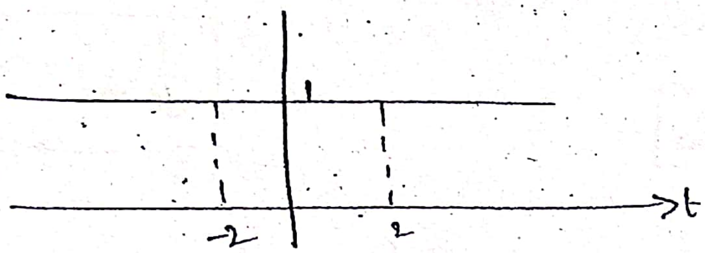



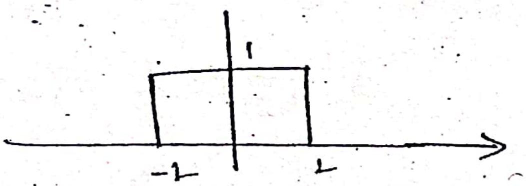
$$y(t) = -\delta(t+2) + \delta(t-2)$$

$$\Big\downarrow F.T.$$

$$Y(\omega) = -e^{-j\omega(-2)} + e^{-j\omega(2)}$$

Alitive





$$\frac{d}{dt}(1) = 0$$

$$j\omega\, 2\pi\, \delta(\omega) = 0$$

$$j(0)2\pi\,\delta(\omega) = 0.$$

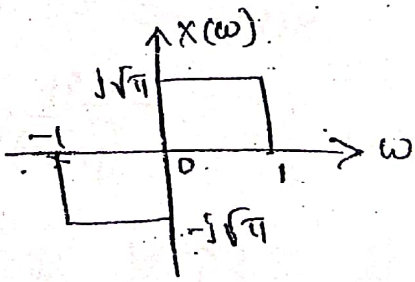
$$y(t) = \frac{d}{dt}\left[1 - \text{rect}\left(t/4\right)\right]$$

$$= -\frac{d}{dt}\,\text{rect}\left(t/4\right)$$

=

17

$$\frac{d}{dt} x(t)\Big|_{t=0} = ?$$



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\cdot e^{j\omega t}\, d\omega$$

$$\frac{d}{dt} x(t)\Big|_{t=0} = \frac{1}{2\pi}\int_{-1}^{1} (j\omega)\, X(\omega)\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-1}^{0} (-j\sqrt{\pi})(j\omega)\, d\omega + \int_{0}^{1} (j\sqrt{\pi})\, j\omega \cdot d\omega\right]$$

$$\frac{d}{dt} x(t)\Big|_{t=0} = -\frac{1}{2\sqrt{\pi}}$$

## Frequency differentiation :-

$$-jt\,x(t) \longleftrightarrow \frac{d}{d\omega}X(\omega)$$

$\frac{d}{dt} \Leftrightarrow j\omega$

$\frac{d}{d\omega} \Leftrightarrow -jt$

(18) $y(t) = t\cdot e^{-at}u(t)$

$= t\cdot x(t)$

F.T

$$Y(\omega) = j\cdot\frac{d}{d\omega}\left[\frac{1}{a+j\omega}\right]$$

$\boxed{\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}}$

$$= \frac{1}{(a+j\omega)^2}$$

(19) $= t\cdot e^{-|t|}$

$$t\cdot e^{-|t|} \longleftrightarrow j\,\frac{d}{d\omega}\left[\frac{2}{\omega^2+1}\right]$$

$\frac{1}{x}\,dx$

$-\frac{1}{x^2}$

$$\longleftrightarrow \frac{-4j\omega}{(1+\omega^2)^2}$$

Duality

$$x(t) \longleftrightarrow 2\pi\,x(-\omega)$$

$$\frac{-4jt}{(1+t^2)^2} \longleftrightarrow 2\pi\,(-\omega)\cdot e^{-|\omega|}$$

* Find the I.F.T of $X(\omega) = \frac{d}{d\omega}\left[\frac{e^{j2\omega}}{1+j\omega/3}\right]$

$$= 3\frac{d}{d\omega}\left[\frac{e^{j2\omega}}{3+j\omega}\right]$$

$y(t-t_0) \leftrightarrow e^{-j\omega t_0}\cdot Y(\omega)$

$t_0 = -2$

$$= +3\left[-jt\cdot e^{-3(t+2)}\cdot u(t+2)\right]$$

$$\cancel{= -3jt\cdot e^{-3(t+2)}\cdot u(t+2)}$$

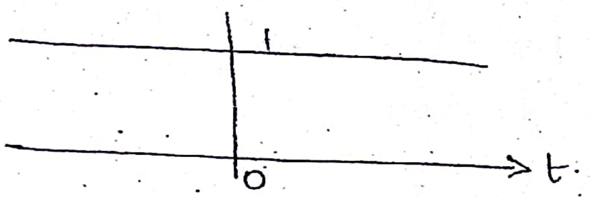
③ $x_3(t) = e^{-t}u(-t) + e^{5t}u(t)$

$$e^{-t}u(-t) \leftrightarrow \frac{-1}{s+1} : \sigma < -1 \quad , \quad e^{5t}u(t) \leftrightarrow \frac{1}{s-5} : \sigma > 5$$

No common R.O.C , No laplace

---

④ $x(t) = 1 \ \forall t$



$x(t) = u(t) + u(-t)$

$$u(t) \leftrightarrow \frac{1}{s} ; \sigma > 0 \qquad u(-t) \leftrightarrow \frac{-1}{s} : \sigma < 0$$

No L.T.

⑤ $x(t) = \mathrm{sgn}(t)$

$= u(t) - u(-t)$

No L.T $\quad \frac{1}{s} - \left(-\frac{1}{s}\right) = \cancel{\frac{2}{s}}$ $\quad$ only stable system we can put $s = j\omega$

3) $x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$ $\quad$ ROC: $Re\{s\} > -3$

$$e^{-5t}u(t) \rightarrow \frac{1}{s+5} : \sigma > 5 \qquad e^{-\beta t}u(t) \rightarrow \frac{1}{s+\beta} ; \sigma > Re\{-\beta\}$$

---

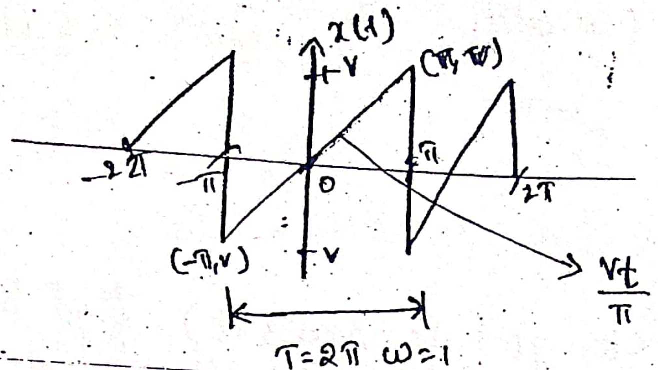
$\therefore Re\{\beta\} = 3$

Real part $= 3$

$Img\{\beta\}$ = arbitary

---

① 




real & odd signal means phase ⇒ ±90.
real & even phase = 0° or ±180

odd symmetry ⇒ $a_n = a_0 = 0$

$$x(t) = \left\{ \frac{Vt}{\pi} \; ; -\pi < t < \pi \right.$$

$$= \frac{1}{T}\int_0^T (\text{odd})(\text{odd}) \cdot dt$$

$$= \frac{2}{T}\int_0^{T/2} (\text{even}) \cdot dt$$

$$b_n = \frac{2}{T}\int_0^T \frac{Vt}{\pi} \cdot \sin(n\omega_0)t \cdot dt$$

$T = 2\pi$
$\omega_0 = 1$

$$= \frac{4}{2\pi}\int_0^{\pi} \frac{Vt}{\pi} \sin n\omega_0 t \, dt$$

$$= \frac{4V}{2\pi^2}\int_0^{\pi} t \cdot \sin nt \cdot dt$$

$$= \frac{4V}{2\pi^2}\left[-t \cdot \frac{\cos nt}{n} + \frac{\sin nt}{n^2}\right]_0^{\pi}$$

$$= \frac{4V}{2\pi^2}\left[\frac{-\pi(-1)^n}{n}\right]$$

$$= \frac{2V}{\pi}\left[\frac{(-1)^{n+1}}{n}\right]$$

Magnitude:

$d_0 = a_0 = 0$

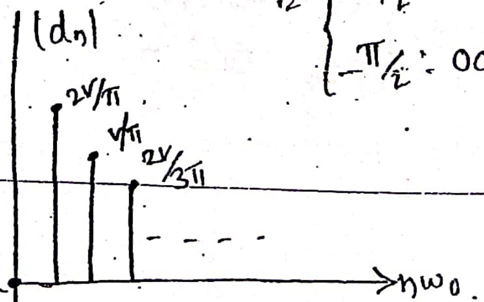
$$|d_n| = \sqrt{a_n^2 + b_n^2} = |b_n| = \frac{2V}{\pi n}$$

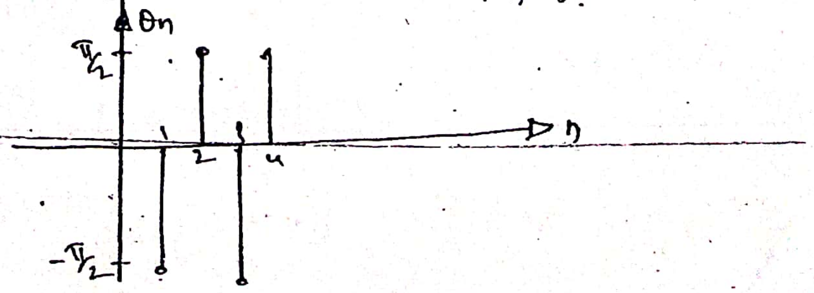
phase:

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

$$= \begin{cases} \pi/2 \; : \text{even } n \\ -\pi/2 \; : \text{odd } n . \end{cases}$$

$$b_n = \begin{cases} \frac{-2V}{\pi n} \; : \text{even } n \\ \frac{2V}{\pi n} \; : \text{odd } n \end{cases}$$



1) The E.F.S representation for a periodic signal is

$$g(t) = \sum_{n=-\infty}^{+\infty} \frac{3}{4+(n\pi)^2} e^{jn\pi t}.$$

(i) find 'T'

(ii) One of the component of $g(t)$ is $A\cos(3\pi t)$ find the value of 'A'.

$$g(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{jn\omega_0 t}$$

$\omega_0 = \pi$

$\frac{2\pi}{T} = \pi$ $\boxed{T = 2}$

$A\cos(3\pi t) \rightarrow n=3$

$$A = \frac{3}{4+(-3\pi)^2} + \frac{3}{4+(3\pi)^2} = \frac{3(2)}{4+(3\pi)^2}$$

(2) $x(t) - x(t-T/2) \rightarrow$ F.S

$\rightarrow C_n - e^{-jn(\frac{2\pi}{T})(\frac{T}{2})} C_n$

$= C_n[1-(-1)^n]$

$$= \begin{cases} 0 ; & \text{even } n \\ 2C_n ; & \text{odd } n. \end{cases}$$

## Time - Shifting:

$x(t) \longleftrightarrow X(s)$ : ROC $= R$

$x(t-t_0) \longleftrightarrow e^{-st_0} \cdot X(s)$ ; ROC $= R$.

① $y_1(t) = u(-t+4)$ ROC:

$= u[-(t-4)]$

$\big\downarrow$ L.T

$Y_1(s) = e^{-4s}\left[\frac{-1}{s}\right]$ : $\sigma < 0$

② $Y(s) = \dfrac{e^{-3s}}{(s+1)(s+2)}$ : $\sigma > -1$

$Y(s) = e^{-3s} X(s)$

$\big\downarrow$ I.L.T

$y(t) = x(t-3)$

$y(t) = e^{-(t-3)} u(t-3) - e^{-2(t-3)} \cdot u(t-3)$

let $X(s) = \dfrac{1}{(s+1)(s+2)}$

$\big\downarrow$ $= \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$x(t) = e^{-t}u(t) - e^{-2t}u(t)$

⑧ $x(t) = e^{-5t} u(t-1)$

a) $= e^{-5(t-1+1)} u(t-1)$

$= e^{-5} \cdot e^{-5(t-1)} \cdot u(t-1)$

$X(s) = e^{-5}\left[\dfrac{e^{-s(1)}}{s+5}\right]$ : $\sigma > -5$

b) $g(t) = A \cdot e^{-5t} u(-t-t_0)$.

$= A \cdot e^{-5t} u[-(t+t_0)]$

$= A \cdot e^{-5(t+t_0-t_0)} \cdot u[-(t+t_0)]$

$= A \cdot e^{+5t_0} \cdot e^{-5(t+t_0)} \cdot u[-(t+t_0)]$.

$$\xleftrightarrow{L.T} \quad A\cdot e^{5t_0}\cdot\left[\frac{-e^{-s(t_0)}}{s+5}\right] : \sigma < -5.$$

$$\therefore A=-1,\ t_0=-1$$

(9)

$$= u(t)-u(t-1)+2\left[u(t-1)-u(t-2)\right]$$

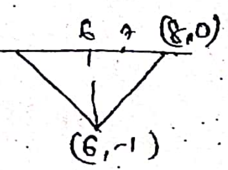
$$= u(t)+u(t-1)-2u(t-2).$$

$$\xleftrightarrow{L.T} \frac{1}{s}+\frac{e^{-s}}{s}-\frac{2\cdot e^{-2s}}{s}.$$

(10)

$$= \frac{1}{s^2}\left(1+A\cdot e^{-s}+B\cdot e^{-4s}+C\cdot e^{-6s}+D\cdot e^{-8s}\right)$$

$$\Rightarrow \left[\frac{t}{2}-4\right]u\left[-(t-6)+u(t-8)\right]$$




$$(y+1) = \frac{0+1}{8-6}(x-6)$$

$$y+1=\frac{1}{2}(x-6)$$

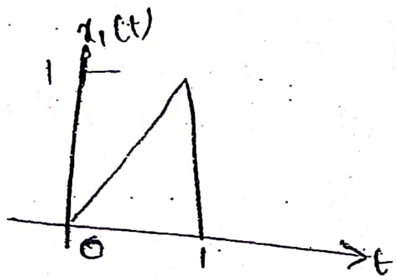
$$2y+2=x-6$$

$$y=\frac{x}{2}-4$$

$$y=\frac{t}{2}-4$$

(11)

a)




$$x_1(t) = t\left[u(t)-u(t-1)\right]$$

$$= t\,u(t)-(t-1+1)u(t-1)$$

$$= t\cdot u(t)-(t-1)u(t-1)-u(t-1)$$

$$= \frac{1}{s^2}-\frac{e^{-s}}{s^2}-\frac{e^{-s}}{s}$$

$$\sin\omega_0 t\,u(t) \longleftrightarrow \frac{\omega_0}{s^2+\omega_0^2} : \sigma>0$$

(c)

$$x_3(t) = \sin\left(\omega_0(t-T)\right)u(t-T)$$

$$\xleftrightarrow{L.T} \quad e^{-sT}\left[\frac{\omega_0}{s^2+\omega_0^2}\right] : \sigma>0$$

d) $\sin[\omega_0(t-T)]u(t)$.

$$L[\sin\omega_0 t\cdot\cos\omega_0 T\cdot u(t) - \cos\omega_0 t\,\sin\omega_0 T u(t)]$$

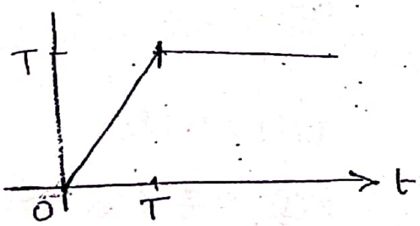
$$\xleftrightarrow{L.T} \frac{\omega_0\cos\omega_0 T - s\sin\omega_0 T}{s^2+\omega_0^2}$$

e) $\sin\omega_0 t\, u(t-T) = \sin\omega_0[\underbrace{t-T}_{A}+\underbrace{T}_{B}]\,u(t-T)$

$$\sin\omega_0(t-T)u(t-T).\cos\omega_0 T + \cos\omega_0(t-T).u(t-T).\sin\omega_0 T.$$

$$\xleftrightarrow{L.T} \frac{e^{-sT}\omega_0}{s^2+\omega_0^2}.\cos\omega_0 T + \frac{e^{-sT}s}{s^2+\omega_0^2}.\sin\omega_0 T$$

f) $x_6(t) = \begin{cases} t : 0\le t\le T \\ T : t>T \\ 0 : \text{else where.} \end{cases}$



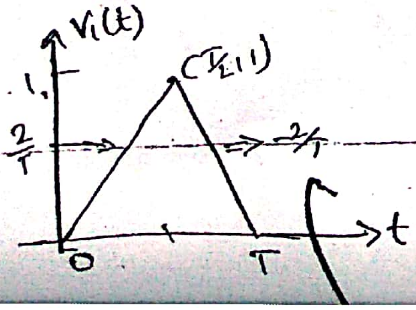

$$x_6(t) = t[u(t)-u(t-T)]+Tu(t-T)$$

$$= t\,u(t) - (t+T-T)u(t-T) + Tu(t-T)$$

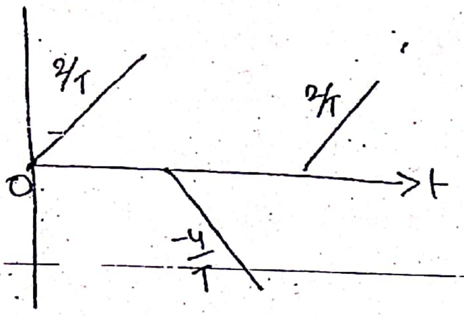
$$= t.u(t) - (t-T)u(t-T) \cancel{-Tu(t-T)} \cancel{+Tu(t-T)}$$

$$= r(t) - r(t-T)$$

(15)



$$\frac{2}{T}r(t) - \frac{4}{T}r(t-T/2) + \frac{2}{T}r(t-T)$$




$$\longleftrightarrow \frac{2}{Ts^2} - \frac{4}{Ts^2}\cdot e^{-sT/2} + \frac{2}{Ts^2}\cdot e^{-sT}$$

$$\longleftrightarrow \frac{2}{Ts^2}\left[1 - 2\cdot e^{-sT/2} + e^{-sT}\right]$$

$$\longleftrightarrow \frac{2}{Ts^2}\left[1 - e^{-sT/2}\right]^2$$

→ **Shift in S-domain:-**

$x(t) \longleftrightarrow X(s)$ $\quad$ ROC = R

$x(t)\cdot e^{s_0 t} \longleftrightarrow X(s-s_0)$

ROC = R + $Re\{s_0\}$

(2)
a) $x_1(t) = \cos\omega_0 t\, u(t)$

$$= \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] u(t)$$

L.T

$x(t) = u(t)$

$X(s) = \frac{1}{s}$ ; $\sigma > 0$

$$Y(s) = \frac{X(s-j\omega_0) + X(s+j\omega_0)}{2}$$

$$= \frac{1}{2}\left[\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right]$$

$$= \frac{s}{s^2+\omega_0^2} \;;\; \sigma > 0.$$

b) $x(t) = e^{-\alpha t}\cdot\cos\omega_0 t\, u(t) \longleftrightarrow \frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$ ; $\sigma > 0-\alpha$

'$\alpha$' is real $\qquad\qquad \sigma > -\alpha$

(3) $y(t) = t \cdot e^{-at} u(t)$ 'a' is real

$\downarrow L.T = x(t) \cdot e^{-at}$

$Y(s) = X(s+a)$

$= \frac{1}{(s+a)^2} ; \sigma > -a$

$x(t) = t\,u(t)$

$X(s) = \frac{1}{s^2} ; \sigma > 0$

$$\boxed{\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \longleftrightarrow \frac{1}{(s+a)^n}}$$

(13) $s = -1$, & $s = -3$ , $g(t) = e^{2t} x(t)$.

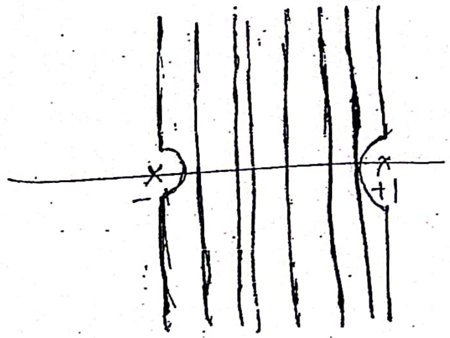
$X(s) = \frac{1}{(s+1)(s+3)}$

$g(t) = e^{2t} x(t)$

$\Downarrow$

$G(\omega)$ converges

$\Rightarrow$ F.T is defined




$G(s) = X(s-2)$

$= \frac{1}{(s-1)(s+1)}$

(14) a) $X_1(s) = \frac{3s^2+22s+27}{(s+1)(s+2)(s^2+2s+5)}$

$= \frac{2}{s+1} + \frac{1}{s+2} + \frac{Cs+D}{(s+1)^2+(2)^2}$

Put $s = 0$

$s = 1$

Solve C & D

(17) $\frac{dx(t)}{dt} = -2y(t) + \delta(t)$ $\frac{dy(t)}{dt} = 2x(t)$

$\downarrow$

$s \cdot X(s) = -2Y(s) + 1$ $sY(s) = \frac{2}{s}X(s)$

$sX(s) = \frac{-4}{s}X(s) + 1$ $Y(s) = \frac{2}{s}X(s)$

$X(s) = \frac{s}{s^2+4} : a > 0$

$X(s) \cdot \left[s + \frac{1}{4}\right] = 1$

(18) a) $x_1(t) = \frac{d^2}{dt^2}\left[e^{-3(t-2)} \cdot u(t-2)\right]$

b) $x_2(t) = e^{-t} \cdot \frac{d}{dt}\left[e^{-(t+1)} \cdot u(t+1)\right]$

$\downarrow$ $\rightarrow$

$e^{-t}y(t)$ $= \frac{s \cdot e^{-s(-1)}}{(s+1)} : a > -1$

$e^{-t}y(t) \xleftrightarrow{L.T} Y(s+1)$

$s_0 = -1$

$= \frac{(s+1) \cdot e^{s+1}}{(s+2)} : a > -2$

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(13) $y(t) = t \cdot e^{-at} u(t)$ 'a' is real

$$\xrightarrow{L.T} = x(t) \cdot e^{-at}$$

$$Y(s) = X(s+a)$$

$$= \frac{1}{(s+a)^2} ; \sigma > -a$$

$x(t) = t u(t)$

$X(s) = \frac{1}{s^2} ; \sigma > 0$

$$\boxed{\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \longleftrightarrow \frac{1}{(s+a)^n}}$$

(13) $s = -1$, & $s = -3$ , $g(t) = e^{2t} x(t)$.

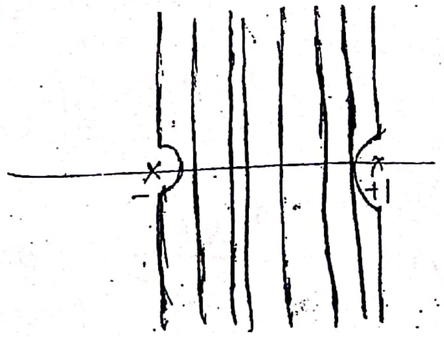
$$X(s) = \frac{1}{(s+1)(s+3)}$$

$$g(t) = e^{2t} x(t)$$

$$\Downarrow$$

$G(\omega)$ converges

$\Rightarrow$ F.T is defined




$$G(s) = X(s-2)$$

$$= \frac{1}{(s-1)(s+1)}$$

(14) a) $X_1(s) = \dfrac{3s^2+22s+27}{(s+1)(s+2)(s^2+2s+5)}$

$$= \frac{2}{s+1} + \frac{1}{s+2} + \frac{Cs+D}{(s+1)^2+(2)^2}$$

Put $s=0$

$s=1$

Solve C & D

⑥ $z = e^{j\pi/2}$ ; $x(1) = 1$

$$X(z) = \frac{z^2}{(z-e^{j\pi/2})(z-e^{-j\pi/2})} = \frac{z^2}{z^2 - z(e^{j\pi/2}+e^{-j\pi/2})+1}$$

$$= \frac{2z^2}{z^2+1} \; ; \; |z| > |\pm j|$$

$$|z| > 1$$

---

**Time-Shifting:**

$$x[n-n_0] \longleftrightarrow z^{-n_0}X(z) \; ; \; ROC = R \text{ except } z=0 / z=\infty$$

$$e^{-j\omega n_0} X(e^{j\omega})$$

⑦ $y(n) = \left(-\frac{1}{3}\right)^n u(-n-2)$

$$= \frac{3z}{1+\left(\frac{1}{3}\right)z^{-1}}$$

$$x(n) = a^n u[-(n+1)]$$

$$x(n+1) = a^{n+1} u[-(n+2)]$$

$$= \left(-\frac{1}{3}\right)^{n+1-1} u[-(n+2)]$$

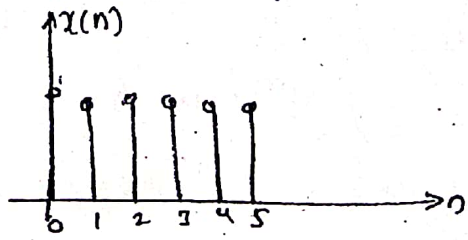
$$= \left(-\frac{1}{3}\right)^{-1} \underbrace{\left(\frac{1}{3}\right)^{n+1} u[-(n+2)]}$$

$$= -3\left[\frac{-z^{-(-1)}}{1+\frac{1}{3}z^{-1}}\right]$$

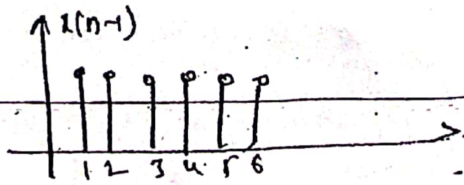
$$Y(z) = \frac{3z}{1+\left(\frac{1}{3}\right)z^{-1}}$$

⑧ $$x(n) = \begin{cases} 1 \; ; & 0 \le n \le 5 \\ 0 \; ; & \text{else} \end{cases}$$

$g(n) = x(n) - x(n-1)$ ; Find $G(z)$ with R.O.C.




$$g(n) = x(n) - x(n-1)$$
$$= \delta(n) - \delta(n-6)$$
$$G(z) = 1 - z^{-6} \; ; \; |z| > 0$$

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$X(z) = \frac{z^3 - 2z}{z-2}$  $x(n)$ is ~~left sided~~ (improper z-Tf)

$= \frac{z^3}{z-2} - 2\left(\frac{z}{z-2}\right) = z^2\left(\frac{z}{z-2}\right) - 2\left(\frac{z}{z-2}\right).$

$= z^2 Y(z) - 2Y(z)$  Let $Y(z) = \frac{z}{z-2}$

$= 2^n u[-(n-1)]$

$\downarrow$

$x(n) = y(n+2) - 2y(n).$

## Exponential multiplication (or) Scaling in z-domain:-

$a^n x[n] \longleftrightarrow X(z/a)$

$ROC = |a| R$

$$Z\{a^n x(n)\} = \sum_{n=-\infty}^{+\infty} a^n x(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} x[n] \cdot (a z^{-1})^n$$

$$= \sum x[n] \cdot (a^{-1} z)^{-n}$$

$$= X(a^{-1} z). \text{ by def}$$

(10) $y(n) = \cos\omega_0 n \, u(n)$

let $x(n) = u(n)$

$= \left[\frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}\right] x(n)$

$X(z) = \frac{1}{1-z^{-1}}; |z| > 1$

$Y(z) = \frac{X(z/e^{j\omega_0}) + X(z/e^{-j\omega_0})}{2} = \frac{1}{2}\left[\frac{1}{1-z^{-1}e^{j\omega_0}} + \frac{1}{1-z^{-1}e^{-j\omega_0}}\right]$

$= \frac{1}{2}\left[\frac{2 - z^{-1}(e^{j\omega_0} + e^{-j\omega_0})}{1 - z^{-1}(e^{j\omega_0} + e^{-j\omega_0}) + z^{-2}}\right]$

$= \frac{1 - z^{-1}\cos\omega_0}{1 - 2z^{-1}\cdot\cos\omega_0 + z^{-2}}$ ; $|z| > (1) |e^{\pm j\omega_0}|$

$|z| > (1)(1)$

$\sin\omega_0 n \, u(n) \longleftrightarrow \frac{z^{-1}\cdot\sin\omega_0}{1 - 2z^{-1}\cos\omega_0 + z^{-2}}$ ; $|z| > 1$

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* $a^n \cdot \cos w_0 n \, u(n)$

$$\longleftrightarrow \frac{1 - az^{-1}\cos w_0}{1 - 2az^{-1}\cos w_0 + a^2 z^{-2}} \; ; \; |z| > (1)|a|$$

(ii) $y(n) = (\tfrac{1}{2})^n x(n)$

$$X(z) = \frac{(z+j)(z-j)}{z - \tfrac{1}{2}}$$

$y(n) = (\tfrac{1}{2})^n x(n)$

$a = \tfrac{1}{2}$

$Y(z) = X(z/\tfrac{1}{2}) = X(2z)$.

## Time Scaling:

$x[\tfrac{n}{k}] \longleftrightarrow X(z^k) \quad X(e^{jwk})$

"n is integer multiplication of k"

## Time - Reversal:-

$x(-n) \longleftrightarrow X(z^{-1})$ ; $ROC = 1/R$

$$Z[x(-n)] = \sum_{n=-\infty}^{+\infty} x[-n] \cdot z^{-n}, \qquad -n = m$$

$$= \sum_{m=+\infty}^{-\infty} x[m] \cdot z^{m} = \sum_{m=-\infty}^{+\infty} x[m] (z^{-1})^{-m}$$

$= X(z^{-1})$. By def

* $y(n) = 3^n u(-n)$

$= (\tfrac{1}{3})^{-n} u(-n)$     Let $x(n) = (\tfrac{1}{3})^n u(n)$

$= x(-n)$     $X(z) = \frac{1}{1 - \frac{1}{3} z^{-1}}$ ; $|z| > \tfrac{1}{3}$

$\downarrow$

~~$Y(z) = X(z^{-1}) = \frac{1}{1 - \frac{z}{3}}$ ; $|z| < \frac{1}{1/3}$~~

$|z| < 3$

## Diff in z-domain :

$$n x(n) \longleftrightarrow -z \cdot \frac{d}{dz} X(z) \quad , \text{ROC} = R$$

$-jn \leftrightarrow \frac{d}{d\omega}$

$z = e^{j\omega}$

$dz = j \cdot e^{j\omega} d\omega$

$d\omega = \frac{dz}{jz}$

* $y(n) = n \cdot a^n u(n-1)$

$= n \cdot a^{n+1-1} \cdot u(n-1)$

$= a \cdot \underbrace{n a^{n-1}} \cdot u(n-1)$

$= a \cdot -\cancel{z} \frac{d}{dz}\left[\frac{z^{-1}}{1-az^{-1}}\right] \; ; \; |z| > |a|$

12/11/11

* $X(z) = \log(1+az^{-1}) \; ; \; |z| > |a|$

$\frac{d}{dz} x(z) = \frac{1 \{-az^{-2}\}}{1+az^{-1}}$

$-z \cdot \frac{d}{dz} x(z) = \frac{az^{-1}}{1+az^{-1}}$

$y(n-n_0) \Leftrightarrow z^{-n_0} Y(z)$

$\downarrow$ I.Z.T

$$\boxed{n x(n) = a(-a)^{n-1} u(n-1)}$$

* $X(z) = \log\left(\frac{1}{1-a^{-1}z}\right) \; ; \; |z| < |a|$

$x(z) = -\log(1-a^{-1}z)$

$\frac{d}{dz} x(z) = -\left(\frac{1(-a^{-1})}{1-a^{-1}z}\right)$

$-z \frac{d}{dz} x(z) = \frac{-a^{-1}z}{1-a^{-1}z}$ or $\frac{z}{z-a}$

$\downarrow$

$n x(n) = a^n[-u(-n-1)]$

## Convolution :

$x[n] \longleftrightarrow X(z) \; ; \; \text{ROC} = R_1$

$\underset{\text{I.R}}{h(n)} \longleftrightarrow \underset{\text{T.F}}{H(z)} \; ; \; \text{ROC} = R_2$

$\ldots H(z) \; ; \; \text{ROC} = R_1 \cap R_2$

$$X(z) = \sum_{n=-\infty}^{+\infty} x(n) \cdot z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{+\infty} x(n) * h(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} x(k) \cdot h(n-k) \cdot z^{-n}$$ $n-k=m$

$$= \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{\infty} x(k) \cdot h(m) \cdot z^{-(m+k)}$$

$$= \sum_{k=-\infty}^{\infty} x(k) \cdot z^{-k} \cdot \sum_{m=-\infty}^{\infty} h(m) \cdot z^{-m}$$

$$= X(z) \cdot H(z). \checkmark$$

15) $x(n) \leftrightarrow X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$

$y(n) = x[n] * h[n]$

$Y(z) = X(z) H(z)$

$= [z^4 + z^2 - 2z + 2 - 3z^{-4}] [2z^{-3}]$

$= 2z + 2z^{-1} - 4z^{-2} + 4z^{-3} - 6z^{-7}$

o/p at n = 4 is 0

o/p at n = 7 ⇒ −6

o/p at n = 3 ⇒ 4

16) $y_1(n) = x_1(n+3) * x_2(-n+1)$

$x_1(n) = (\frac{1}{2})^n u(n) \longrightarrow$ ROC : $|z| > \frac{1}{2}$

$x_2(n) = (\frac{1}{3})^n u(n) \longrightarrow$ ROC : $|z| > \frac{1}{3}$

$Y_1(z) = z^{-(-3)} \cdot X_1(z) \cdot z^{-1} X_2(z^{-1})$. ROC $X_2(z^{-1})$ : $|z| < 3$

$\frac{1}{2} < |z| < 3$

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(17) $y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$ , $x(n) = \delta(n) - \frac{1}{3}\delta(n-1)$ ;

$\downarrow$

$Y(z) = \frac{5}{6} z^{-1} Y(z) - \frac{1}{6} z^{-2} Y(z) + X(z)$ $X(z) = 1 - \frac{1}{3} z^{-1}$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}} = \frac{1}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$Y(z) = X(z) H(z)$

$$= \frac{1}{(1-\frac{1}{2}z^{-1})}$$

No condition given in problem means its rightsided signal.

$y(n) = (\frac{1}{2})^n u(n).$

(18) $y(n) - \frac{1}{3} y(n-1) = x(n)$ , $x(n) = (\frac{1}{2})^n u(n)$

$Y(z) - \frac{1}{3} Y(z) z^{-1} = X(z)$ $X(z) = \frac{1}{1 - \frac{1}{2} z^{-1}}$

$$\frac{Y(z)}{X(z)} = \frac{1}{(1-\frac{1}{3}z^{-1})}$$

$Y(z) = X(z) H(z)$

$$= \left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{1}{3}z^{-1}}\right]$$

$$= \frac{3}{1-\frac{1}{2}z^{-1}} - \frac{2}{1-\frac{1}{3}z^{-1}}$$

I.Z.T $\downarrow$

$y(n) = 3 (\frac{1}{2})^n u(n) - 2 (\frac{1}{3})^n u(n)$

(26) $y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n).$

$Y(z) - \frac{3}{4} Y(z) z^{-1} + \frac{1}{8} Y(z) z^{-2} = X(z)$

$$\text{T.f} = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{-1}{(1-\frac{1}{4}z^{-1})} + \frac{2}{(1-\frac{1}{2}z^{-1})}$$

$\downarrow$

$h(n) = -1 (\frac{1}{4})^n u(n) + 2 (\frac{1}{2})^n u(n)$

* **step response:**

$x(n) = u(n) \Rightarrow y(n) = ?$

$Y(z) = X(z) H(z)$

$$= \frac{1}{(1-z^{-1})(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

---

$$Y(z) = \frac{\frac{8}{3}}{(1-z^{-1})} + \frac{\frac{1}{3}}{1-\frac{1}{4}z^{-1}} + \frac{(-2)}{1-\frac{1}{2}z^{-1}}$$

$$y(n) = \frac{8}{3}u(n) + \frac{1}{3}\left(\frac{1}{4}\right)u(n) - 2\left(\frac{1}{2}\right)u(n)$$

(25.) $x(n) = n\left(\frac{-1}{2}\right)^n u(n) * \left(\frac{1}{3}\right)^{-n} u(-n)$.

$\left(\frac{1}{3}\right)^n u(n) \leftrightarrow \frac{1}{1-\frac{1}{3}z^{-1}}, |z| > \frac{1}{3}$

$\left(\frac{1}{3}\right)^{-n} u(-n) \leftrightarrow \frac{1}{1-\frac{z}{3}}; |z| < 3$

$\downarrow$ Z.T

$$= -z \cdot \frac{d}{dz}\left[\frac{1}{1+\frac{1}{2}z^{-1}}\right] \cdot \left[\frac{1}{1-\frac{z}{3}}\right]$$

$\left[\frac{1}{1-\frac{z}{3}}\right] \rightarrow |z| < 3$

$\left[\frac{1}{1+\frac{1}{2}z^{-1}}\right] \rightarrow |z| > \frac{1}{2}$

$\Rightarrow$ ROC: $\frac{1}{2} < |z| < 3$

(24) $h(n) = 3\left(\frac{1}{4}\right)^n u(n-1)$

$$= 3\left(\frac{1}{4}\right)^{n-1+1} u(n-1)$$

$$h(n) = \frac{3}{4}\left(\frac{1}{4}\right)^{n-1} u(n-1)$$

$\downarrow$ Z.T

$$H(z) = \frac{3}{4}\left[\frac{z^{-1}}{1-\frac{1}{4}z^{-1}}\right] = \frac{Y(z)}{X(z)}$$

$$\frac{3}{4}z^{-1}X(z) = Y(z) - \frac{1}{4}z^{-1}Y(z)$$

$$\frac{3}{4} \cdot x(n-1) = y(n) - \frac{1}{4}y(n-1)$$

---

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(19) $G(z) = \alpha z^{-1} + \beta z^{-3}$.

$\rightarrow$ (digital L.P.F)

$y(n) = x(n) + x(n-1) \rightarrow$ L.P.F

$y(n) = x(n) - x(n-1) \rightarrow$ H.P.F

T.F $\rightarrow$ F.R

$z \rightarrow e^{j\omega}$

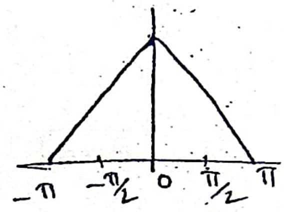
$G(e^{j\omega}) = \alpha \cdot e^{-j\omega} + \beta \cdot e^{-3j\omega}$

---

Assume $\alpha = \beta$

$G(e^{j\omega}) = \alpha \cdot e^{-j2\omega} \left[\underbrace{e^{j\omega} + e^{-j\omega}}_{2\cos\omega}\right]$

$\theta(\omega) = -2\omega$




(21)

$h(2) = 1$ & $h(3) = -1$

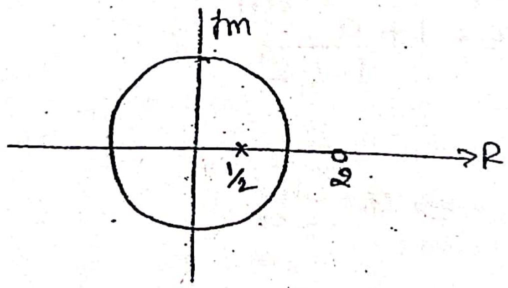
$h(k) = 0$ else

$H(z) = \sum_{n=-\infty}^{+\infty} h(n) z^{-n} = h[2] z^{-2} + h[3] z^{-3}$

$= z^{-2} - z^{-3}$ $\Rightarrow$ change in to timedomain we can conclude easily.

$\boxed{y(n-2) - y(n-3)}$

$= z^{-2}[1 - z^{-1}] \rightarrow$ HPF




Note: In the Z-domain if poles and zeros are Reciprocal ret represents a All pass filter or the Numerator polinomial is mirror image of denominator polinomial.

Ex: $H(z) = \dfrac{\alpha + z^{-1}}{1 + \alpha z^{-1}}$

$z = e^{j\omega}$ All Pass filter

$|z| = 1$

Let $Q(z) = 1+\alpha z^{-1}$

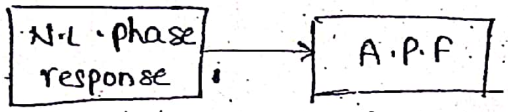
$Q(z^{-1}) = 1+\alpha z$

$z^{-1}[Q(z^{-1})] = z^{-1}+\alpha$

$$H(z) = \frac{z^{-1}[Q(z^{-1})]}{Q(z)} \quad [\because \text{1st order A.P.F}]$$

---

$$H_N(z) = \frac{z^{-N}[Q_N(z^{-1})]}{Q_N(z)} \quad [\because N^{th} \text{ order A.P.F}]$$




$\underbrace{\hspace{3cm}}_{\text{delay Equaliser}}$

(20) $x(n) = (-2)^n \ \forall n$ then $y(n) = 0 \ \forall n$

(i) $x(n) = (\frac{1}{2})^n u(n) \ \forall n$, then $y(n) = \delta(n) + a(\frac{1}{4})^n u(n)$

$x(n) = z^n \Rightarrow y[n] = z^n H(z)$

Sol

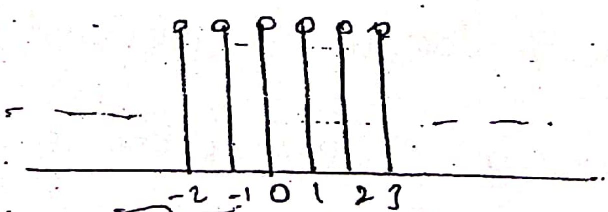
$(-2)^n \Rightarrow 0 = (-2)^n H(z)$

$$H(z)\Big|_{z=-2} = 0$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \qquad Y(z) = 1 + \frac{a}{1-\frac{1}{4}z^{-1}}$$

$$\frac{Y(z)}{X(z)} = \frac{1+\frac{a}{1-\frac{1}{4}z^{-1}}}{\left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]} \Rightarrow a = -\frac{9}{8}$$

(ii) $x[n] = 1 \ \forall n$




$x(n) = z^n \Rightarrow y(n) = z^n H(z)$

$= (1)^n \qquad = (1)^n H(z)\Big|_{z=1}$

$u(-n-1) + u(n)$

$|z|<1 \qquad |z|>1$

(23)

$f_s = 9kHz$

$H(z) = 1 - z^{-6}$

$H(e^{j\omega}) = 1 - z^{j6\omega}$

$= 1 - e^{-j6 \frac{2\pi}{9k}(1.5k)}$

$= 1 - e^{-j2\pi}$

$\omega = \frac{2\pi f}{f_s}$

$\omega = \frac{2\pi}{9k}(f)$

## Accumulation:

$$\sum_{k=-\infty}^{n} x[k] \longleftrightarrow \frac{X(z)}{1-z^{-1}}$$

ROC: $R \cap |z| > 1$

$\frac{d}{dt} x(t) \longleftrightarrow sX(s)$

$x(n) - x(n-1)$

$\longrightarrow Z.T \Rightarrow X(z) - z^{-1}X(s)$

$(1-z^{-1})X(s)$

$s \Rightarrow (1-z^{-1})$

$\frac{1}{s} \Rightarrow \frac{1}{(1-z^{-1})}$

Eg $x[n] = u[n]$

$X(z) = \frac{1}{1-z^{-1}}$ ; $|z| > 1$

$$\sum_{k=-\infty}^{n} u(k) \longleftrightarrow \frac{\frac{1}{1-z^{-1}}}{(1-z^{-1})}$$

$$\sum_{k=0}^{n} (1) \longleftrightarrow \frac{1}{(1-z^{-1})^2}$$

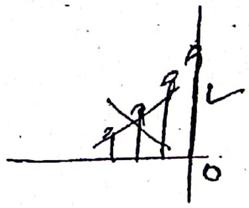
$\underbrace{(n+1)\, u(n)}$

$u(n) * u(n)$

Note: The following properties are valid for unilateral Z-T.F.

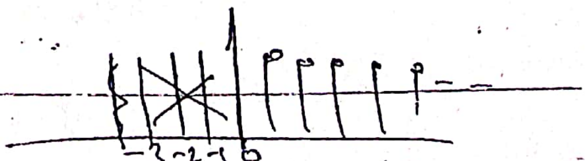
unilateral Z.T $\Rightarrow X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$

→ U.Z.T of $2^n u[-n]$ is $\underline{1}$. $\Rightarrow 2^n ; n \leq 0$



→ U.Z.T of $(1/3)^n u(-n-2)$ is $\underline{0}$

U.Z.T of $u(n+3)$ is $\frac{1}{1-z^{-1}}$

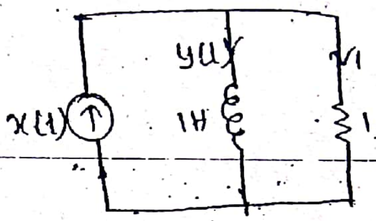


* Consider the RL ckt shown in figure $x(t)$ is the i/p current, & $y(t)$ is the o/p current flowing through inductor

a). Find Z.S.R (Forced response) if the i/p current $x(t) = e^{-2t}u(t)$.

b) find Z.I.R if $y(0^-) = 1$

c) Find the total o/p




$$\frac{dy(t)}{dt} + y(t) = x(t)$$

**Z.S.R:-** All initial conditions are zero. o/p is applied.

$$sY(s) - y(0) + Y(s) = X(s)$$

$$Y(s)[s+1] = \frac{1}{s+2}$$

$$Y(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2} \quad y^{f}(t) = e^{-t}u(t) - e^{-2t}u(t)$$

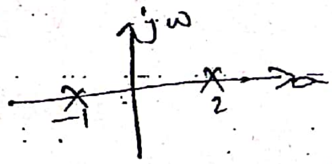
$\downarrow$ Forced

**Z.I.R:** $Y(s)[s+1] - 1 = 0$

$$Y(s) = \frac{1}{s+1} \rightarrow y^{n}(t) = e^{-t}u(t) \rightarrow \text{Natural}$$

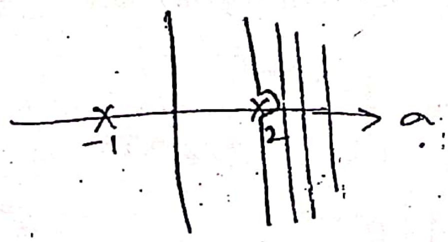
$$\text{Total} = y^{(f)}(t) + y^{n}(t) = 2e^{-t}u(t) - e^{-2t}u(t).$$

* $H(s) = \dfrac{1}{s^2 - s - 2} = \dfrac{1}{(s-2)(s+1)} = \dfrac{\frac{1}{3}}{s-2} - \dfrac{\frac{1}{3}}{s+1}$



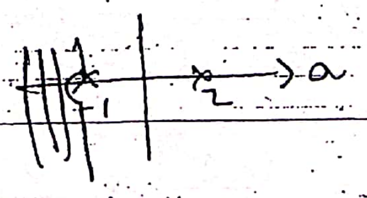

**Causal:** $a > 2$

$$h(t) = \frac{1}{3}e^{+2t}u(t) - \frac{1}{3}e^{-t}u(t)$$



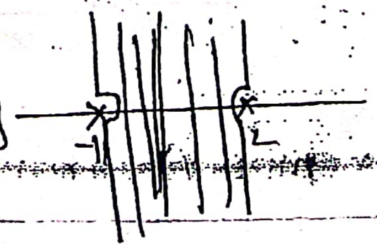

**anti-Causal:-** $a < -1$

$$h(t) = \frac{1}{3}e^{2t}[-u(-t)] - \frac{1}{3}e^{-t}[-u(-t)]$$




**Non Causal & Stable)** $-1 < a < 2$

$$h(t) = \frac{1}{3}e^{2t}[-u(-t)] - \frac{1}{3}e^{-t}[u(t)]$$



Z.T.

① Using the Method indicated find the signal for each of the following Z-Transforms given?

a) P.F expansion $X(z) = \dfrac{1-2z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}}$ & $x(n)$ is absolutely summable

$$= \frac{(1-2z^{-1})}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$$ ⇒ ROC Must include & unit circle $|z|>\frac{1}{2}$

$x(n) = (\frac{1}{2})^n u(n)$.

b) Long division $X(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}}$, $x(n)$ is right-sided

$$1+\tfrac{1}{2}z^{-1}\,\big)\; 1-\tfrac{1}{2}z^{-1}\; \big(1-z^{-1}+\tfrac{1}{2}z^{-2}+\ldots$$
$$1+\tfrac{1}{2}z^{-1}$$
$$-z^{-1}$$
$$-z^{-1}-\tfrac{1}{2}z^{-2}$$
$$\tfrac{1}{2}z^{-2}+\ldots$$

$= 1-z^{-1}+\frac{1}{2}z^{-2}+\ldots$

I.Z.T $= \delta(n) - \delta(n-1) + \frac{1}{2}\delta(n-2) + \ldots$

Ⓒ P.F. expansion $x(z) = \dfrac{3}{z-\frac{1}{4}-\frac{1}{8}z^{-1}} = \dfrac{3z^{-1}}{1-\frac{z^{-1}}{4}-\frac{1}{8}z^{-2}}$

$\Rightarrow \dfrac{A}{(1-\frac{1}{2}z^{-1})} + \dfrac{B}{(1+\frac{1}{4}z^{-1})} = \dfrac{3z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$

Stability $|z| > \frac{1}{2}$ $x(n) = A(\frac{1}{2})^n u(n) + B(-\frac{1}{4})^n u(n)$.

* Find the inverse Z-Transform of $X(z) = \dfrac{1-e^{-\alpha T}}{(z-1)(z-e^{-\alpha T})}$

$$X(z) = \frac{1-e^{-\alpha T}}{(z-1)(z-e^{-\alpha T})}$$

Take $\dfrac{X(z)}{z} = \dfrac{1-e^{-\alpha T}}{z(z-1)(z-e^{-\alpha T})} = \dfrac{\frac{1-e^{-\alpha T}}{e^{-\alpha T}}}{z} + \dfrac{1}{z-1} - \dfrac{e^{-\alpha T}}{z-e^{-\alpha T}}$

$$X(z) = \frac{1-e^{\alpha T}}{e^{\alpha T}} + \frac{z}{z-1} - e^{-\alpha T}\left[\frac{z}{z-e^{-\alpha T}}\right]$$

$x(n) = (\quad)\delta(n) + u(n) - e^{-\alpha T}(e^{-\alpha T})^n u(n)$

* Find the convolution of $x_1(n) = \left(\frac{1}{4}\right)^n u(n-1)$ & $x_2(n)$

$$x_1(n) = \left(\tfrac{1}{4}\right)\left(\tfrac{1}{4}\right)^{n-1} u(n-1) \qquad x_2(n) = u(n)$$

$$X_1(z) \cdot X_2(z) = \left[\frac{\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}}\right]\left[\frac{1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}\right]$$

$$= \frac{\frac{1}{4}z^{-1}\left[2-\frac{3}{2}z^{-1}\right]}{(1-\frac{1}{4}z^{-1})(1-z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$= \frac{A}{(1-\frac{1}{4}z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{C}{(1-\frac{1}{2}z^{-1})}$$

$\downarrow$ I.Z.T

$$\Rightarrow A\left(\tfrac{1}{4}\right)^n u(n) + B\,u(n) + C\left(\tfrac{1}{2}\right)^n u(n)$$

* A Causal L.T.I system is described a diff eqn.

$$y(n) = y(n-1) + y(n-2) + x(n-1)$$

a) Find T.F, R.O.C, Impulse Response

b) Find the impulse response if the system stable

$$Y(z) = z^{-1}Y(z) + z^{-2}Y(z) + z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1-z^{-1}-z^{-2}} = \frac{z}{z^2-z-1}$$

$$z = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm 2.236}{2} = 1.618,\ -0.618$$

$$\frac{H(z)}{z} = \frac{1}{z^2-z-1}$$

$$\frac{H(z)}{z} = \frac{1}{(z-1.618)(z+0.618)}$$

$$H(z) = \frac{Az}{z-1.618} + \frac{Bz}{z+0.618}$$

Causal I.R $h(n) = A(1.618)^n u(n) + B(-0.618)^n u(n)$.

Stable I.R: $0.618 < |z| < 1.618$

$$h(n) = B(-0.618)^n u(n) + A(1.618)^n[-u(-n-1)]$$

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## Articles:-

দ → The boy → Consonant Sound

দি → The Umberella → Vowel Sound

a/an → Indefinite Article

The → definite Article

a/an → single/one.

↳ Singular Countable noun

weather → singular/un Countable/noun → ~~a~~ weather

boy → Singular/countable/noun → a boy

a good boy → singular/countable/noun ✓

an umbrella → " / " / " ✓

M/em

L/el

1. He is <u>an</u> M.a. Student
2. He is <u>an</u> LLB student
3. He is <u>a</u> European
4. He is <u>an</u> honest man
5. It is <u>a</u> university
6. He is <u>a</u> one eyed man
7. It is ~~the~~ <u>a</u> one way street
8. He is <u>an</u> BBA student

The:

you began with a sentence with ~~indefinate article~~ & you reffered back to the indefinate it will become definite does = he/she/it → singular

does + I form

did = I, we, you, you, he/she/it, they → single/plural

did + I form

had had

have/has

have = I, we, you, you, they → plural

have + III form

has = he, she, it = singular

has + III form

had = I, we, you, you, he/she/it, they → S/pl

had + III form

be = Is, am, are, was, were (been) + III form + ing

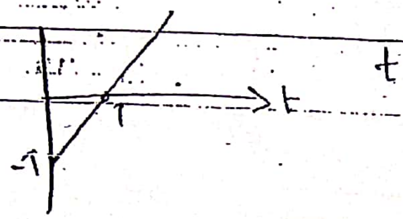
Can could
may might
shall should
will would
must (had to)
ought to
need
} + Ist form

* Draw the signals & find ...

i) $t\,u(t)$ 2) $(t-T)\,u(t-T)$ 3) $(t-T)\,u(t)$ 4) $t\,u(t-T)$

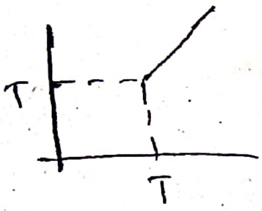
1) 
$t\,u(t) \longrightarrow -\frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2}$.

2) 
$(t-T)\,u(t-T) \Rightarrow \frac{e^{-sT}}{s^2}$.

3) 

$t\,u(t) - T\,u(t) \to \frac{1}{s^2} - \frac{T}{s}$.

$(t-T+T)\,u(t-T) \Rightarrow (t-T)\,u(t-T) + T\,u(t-T)$.

4) 

$= t;\ t > T$

$\frac{e^{-sT}}{s^2} + T\cdot\frac{e^{-sT}}{s}$

* Show that a) If $x(t)$ even $\Rightarrow X(s) = X(-s)$

b) If $x(t)$ odd $\Rightarrow X(s) = -X(-s)$

c) Determine which of the following pole-zero plots corresponds to even function of Time.

$x_1(s) = \frac{ks}{s^2-1}$ $x_2(s) = \frac{k(s^2-1)}{s}$ $x_3(s) = \frac{k(s^2+1)}{s^2-1}$ ✓ $x_4(s) = \frac{s-1}{s+1}$

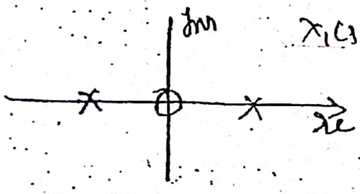


fig ①

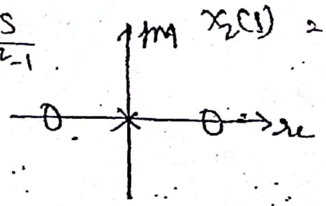


fig ②

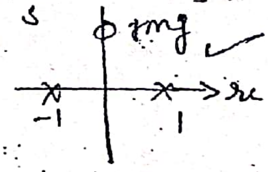


fig ③

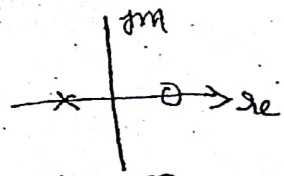


fig ④

a) $x(-t) \xleftrightarrow{L.T} X(-s)$

ROC = $-R$

$L\{x(-t)\} = \int_{-\infty}^{+\infty} x(-t)\, e^{-st}\, dt$

put $-t = \lambda$
$dt = -d\lambda$

$= \int_{+\infty}^{-\infty} x(\lambda)\, e^{s\lambda}\,(-d\lambda)$

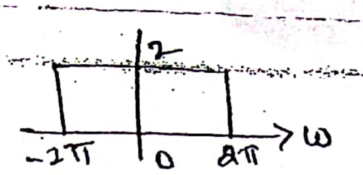
$= \int_{-\infty}^{+\infty} x(\lambda)\, e^{-(-s)\lambda}\, d\lambda$

$= X(-s)$

b) Even: $x(t) = x(-t)$
$\downarrow$ L.T
$X(s) = X(-s)$

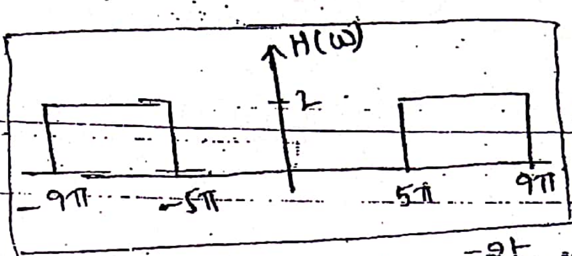
ODD: $x(t) = -x(-t)$
$X(s) = -X(-s)$

* An LTI system has the impulse response $h(t) = 2\frac{\sin 2\pi t}{\pi t} \cdot \cos(7\pi t)$, use F.T to find the o/p if the input is $x(t) = \cos 2\pi t + \sin 6\pi t$




$$h(t) = 2\frac{\sin 2\pi t}{\pi t} \cdot \cos 7\pi t$$

$$i/p \quad x(t) = \cos 2\pi t + \sin 6\pi t$$




* The o/p of An LTI system to the i/p $x(t) = e^{-2t}u(t)$ is $y(t) = e^{-t}$ find the frequency response and impulse response.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2+j\omega}{1+j\omega} = 1 + \frac{1}{1+j\omega}$$

$$1+j\omega \overline{) 2+j\omega (} 1$$

$$= \delta(t) + e^{-t}u(t)$$

I.L.T

$$X(s) = F\{x(t) \cdot e^{-at}\}$$

$$x(t) \cdot e^{-at} = F^{-1}\{X(s)\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(s)\, e^{j\omega t} \cdot d\omega$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(s)\, e^{at} e^{j\omega t} \cdot d\omega$$

$$s = a + j\omega$$

$$ds = j \cdot d\omega$$

$$d\omega = \frac{ds}{j}$$

$$x(t) = \frac{1}{2\pi j}\int_{a-j\infty}^{a+j\infty} X(s) \cdot e^{st} ds$$

I.Z.T $\quad z = r \cdot e^{j\omega}$

$$X(z) = F\{x(n) \cdot r^{-n}\}$$

$$x(n)\, r^{-n} = F^{-1}\{X(z)\}$$

$$x(n) \cdot r^{-n} = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(z) \cdot e^{j\omega n} \cdot d\omega$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(z) \cdot (r \cdot e^{j\omega})^n d\omega$$

$$z = r \cdot e^{j\omega} \Rightarrow dz = r \cdot e^{j\omega} j\, d\omega$$

$$= jz\, d\omega$$

$$d\omega = \frac{dz}{jz}$$

$$x(n) = \frac{1}{2\pi j}\oint X(z)\, z^n \frac{dz}{z}$$

$$\boxed{x(n) = \frac{1}{2\pi j}\oint X(z) \cdot z^{n-1} dz}$$

1727

# Electro Magnetics:

Vector Calculus → Schaum Series

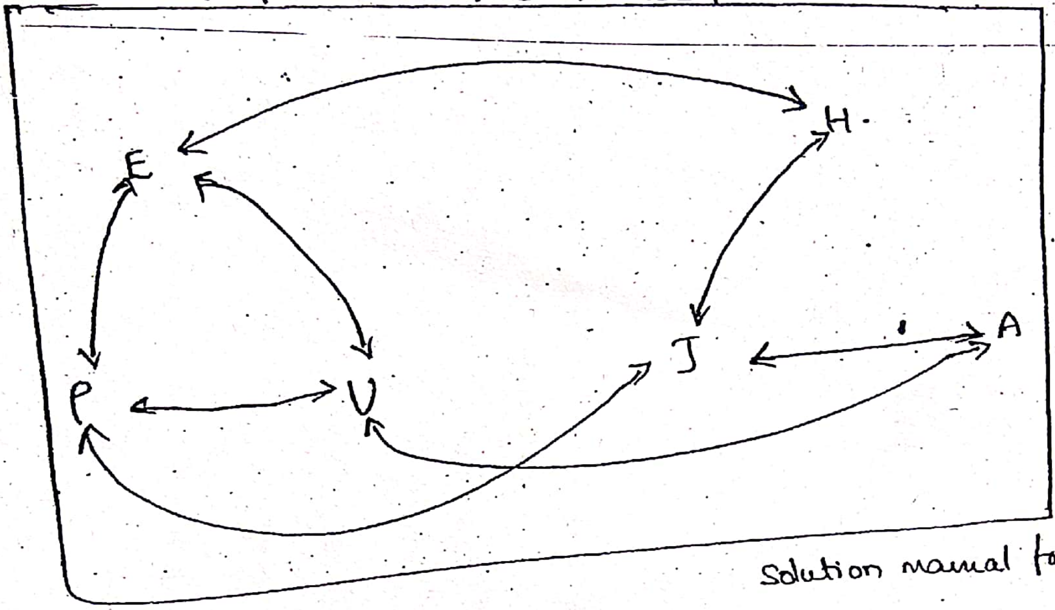
EMT – Johrden & Balmen

Elements of Engg EMT – N.N. rao (website)
also (Pearson Publisher)
— M.N.O Sadiku

Maxwell's proposed 17 eqn's in 1854.

11

6




Solution manual for Hyat & buck.