ECE ENGINEERING

www.orbitmentor.com GATE ESE 2020 TARGET BE HAPPY DO HARDWORK

GATE ESE PSU's 2019-20

GATEACADEMY

106 EMT SYSTEM_GATEACADEMY

TOTAL PAGE EMT SYSTEM-280 PGAE

LATEST GATEACADEMY CLASSROOM HANDWRITTING

CONTENT COVERED:

1. Theory Notes
2. Explanation
3. Derivation
4. Example
5. Shortcut & Formula Summary
6. Previous year Paper Q. Sol.

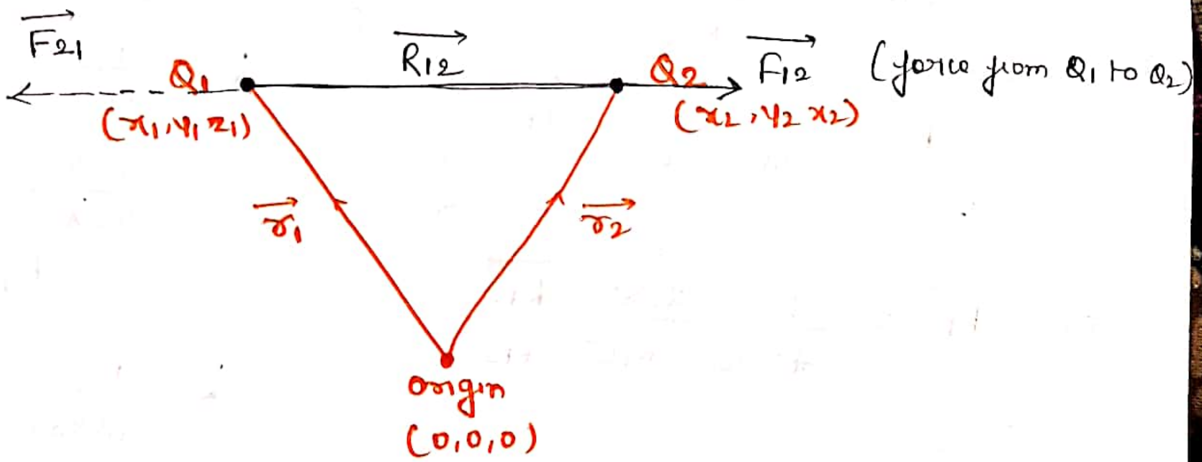
Noted-: Single Source Follow, Revise

Multiple Time Best key of Success

Chapter - 01

Electrostatics & Magnetostatics

* Coulomb's law :- It states that " Force exerted between two point charges is -
  1) Along the line joining between them
  2) Directly proportional to product of point charges
  3) Inversely proportional to square of distance between them



$\vec{r_1} = x_1 \hat{a_x} + y_1 \hat{a_y} + z_1 \hat{a_z}$

$\vec{r_2} = x_2 \hat{a_x} + y_2 \hat{a_y} + z_2 \hat{a_z}$

$\vec{F_{12}}$ = Force on Q_2

$\vec{R_{12}}$ = Displacement Vector between Q_2 & Q_2

Displacement Vector $\longrightarrow$ Displacement from one point to another point

Position Vector $\longrightarrow$ Displacement from origin to a point.

$$|\vec{R_{12}}| = |\vec{r_2} - \vec{r_1}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$\vec{R_{12}} = \vec{r_2} - \vec{r_1} = (x_2-x_1)\hat{a_x} + (y_2-y_1)\hat{a_y} + (z_2-z_1)\hat{a_z}$$

Scanned by CamScanner

Force on charge Q_2 due to Q_1

$$\vec{F_{12}} \propto Q_1 Q_2$$

$$\vec{F_{12}} \propto \frac{1}{|\vec{R_{12}}|^2}$$

So,

$$\vec{F_{12}} \propto \frac{Q_1 Q_2}{R_{12}^2} \hat{a}_{R12}$$

$$\vec{F_{12}} = k \frac{Q_1 Q_2}{R_{12}^2} \hat{a}_{R12}$$

$$\boxed{\vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \hat{a}_{R12}}$$

$\left\{ \begin{array}{l} k = \frac{1}{4\pi\epsilon_0} = 9\times10^9 \\ \epsilon_0 = \frac{10^{-9}}{36\pi} \text{ or } 8.854\times10^{-12} \text{ F/m} \end{array} \right.$

or $$\vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \frac{\vec{R_{12}}}{R_{12}}$$

$$\boxed{\vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2 \vec{R_{12}}}{R_{12}^3}}$$

$\left\{ k = \frac{1}{4\pi\epsilon_0\epsilon_r} \right.$ for dielectric Medium

or $$\vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2 \left[(x_2-x_1)\hat{a_x} + (y_2-y_1)\hat{a_y} + (z_2-z_1)\hat{a_z}\right]}{\left[(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\right]^{3/2}}$$

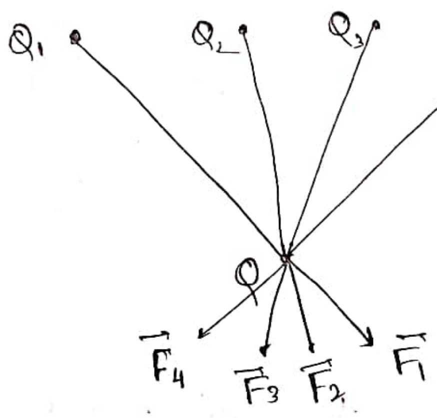
Force on charge Q_1 due to Q_2

$$\vec{F_{21}} = |\vec{F_{21}}| \hat{a}_{R21}$$

$$= |\vec{F_{12}}| (-\hat{a}_{R12})$$

$$= -|\vec{F_{12}}| \hat{a}_{R12}$$

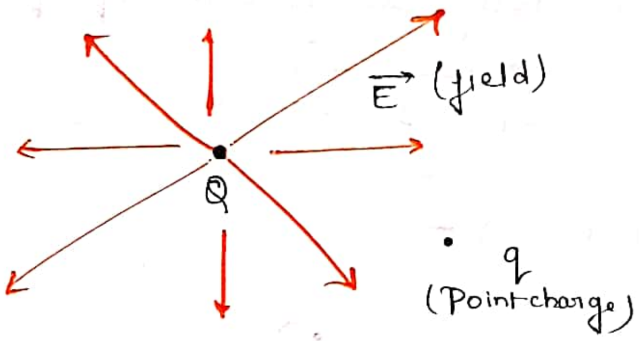
$$\vec{F_{21}} = -\vec{F_{12}}$$



Coulomb's Law obeys Superposition theorem

$$\vec{F_{net}} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

Electric field Intensity



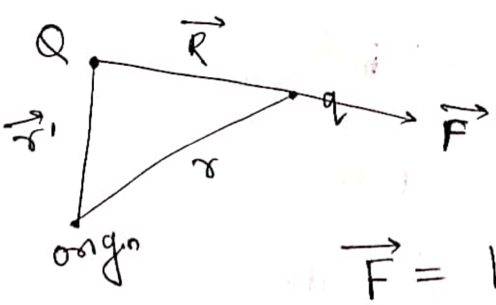
Force exerted on point charge q because of field.

$$\vec{F} = q\vec{E}$$

Electric field Intensity

$$\vec{E} = \frac{\vec{F}}{q} = \text{force per unit charge}$$

$$\frac{\text{Newton}}{\text{Coulomb}} = \frac{\text{Volt}}{\text{m}}$$



$$\vec{F} = k\frac{Qq}{R^2}\hat{a}_R$$

$$\boxed{\vec{E} = \frac{\vec{F}}{q} = \frac{kQ\hat{a}_R}{R^2}}$$

or $$\vec{E} = \frac{kQ}{R^3}\vec{R}$$

$$\boxed{\vec{E} = \frac{kQ}{R^3}(\vec{r} - \vec{r}')}$$ (force direction is also field direction.)

Electric field Intensity also follows superposition theorem

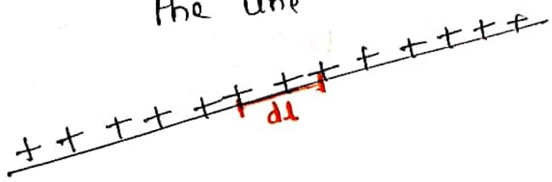
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

* Charge Distribution :-

1) point charge $\cdot Q$

$$\boxed{\vec{E} = \frac{kQ}{R^2}\hat{a}_R}$$

II) line charge :- Charge distributed uniformly along the line



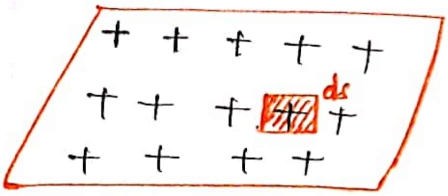
line charge density (ρ_L) in (C/m)

$$dQ = \rho_L \, dl$$

$$Q = \int_l \rho_l \, dl$$

$$\boxed{\vec{E} = \int_l \frac{k \rho_L dl}{R^2} \hat{a}_R}$$

III) Surface charge



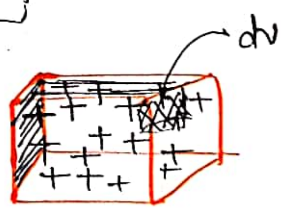
Surface charge density ρ_s in C/m^2

$$dQ = \rho_s \, ds$$

$$Q = \int_s \rho_s \, ds$$

$$\boxed{\vec{E} = \int_s \frac{k \rho_s \, ds}{R^2} \hat{a}_R}$$

IV) Volume charge :-



Volume charge density ρ_v in C/m^3

$$dQ = \rho_v \, dv$$

$$Q = \int_V \rho_v \, dv$$

$$\boxed{\vec{E} = \int_V \frac{k \rho_v \, dv}{R^2} \hat{a}_R}$$

WANT ECE BEST QUALITY-2019-20

LATEST HANDWRITING NOTES

BY

GATE ACADEMY CLICK HERE

Noted-: Above ECE GATE/IES GATEACADEMY 2019-20 CLASSROOM BEST QUALITY Handwriting Notes Unique and Good Handwriting. Above Notes Enough for your Preparation..................

Noted-: Dear Aspirants If you do practice previous year paper 50% your work finished.so Guys daily at least 30minutes give previous year.

1. Previous year paper 4-5 times practice before final exam.

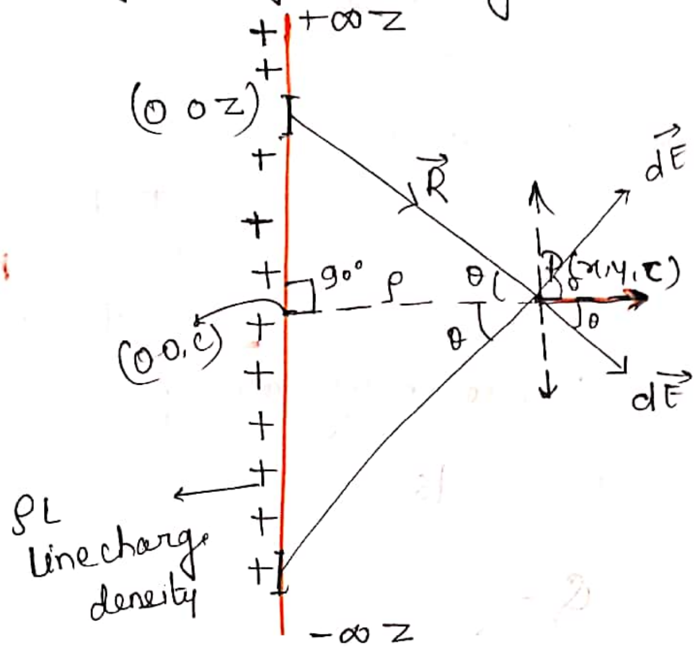
2. Subject wise study reference STD book

3. Test series practice more n more (Try to latest test series 2-3 fully solve then join online test series.)

Noted-: Single Source Follow, Revise Multiple Time Best key of Success

* Electric field Intensity due to Infinite Long line charge

$$\vec{dE} = \frac{k \rho_L \, dl \, \hat{a_R}}{R^2}$$



$$\vec{E} = \int_l \frac{k \rho_L \, dl \, \hat{a_R}}{R^2}$$

$$\Rightarrow \vec{E} = \int_l \frac{k \rho_L \, dl}{R^3} \vec{R}$$

$$\vec{R} = x \hat{a_x} + y \hat{a_y} + (c - z) \hat{a_z}$$

$$R = |\vec{R}| = \sqrt{x^2 + y^2 + (c-z)^2}$$

$$\vec{E} = \int_l \frac{k \rho_L \, dz \left[x \hat{a_x} + y \hat{a_y} + (c-z) \hat{a_z} \right]}{\left(x^2 + y^2 + (c-z)^2\right)^{3/2}}$$

Due to Symmetry, z Component will Cancel each other

$$E = \int_{-\infty}^{\infty} \frac{k \rho_L \, dz \left[x \hat{a_x} + y \hat{a_y} \right]}{\left[x^2 + y^2 + (c-z)^2 \right]^{3/2}}$$

let $(c - z) = \sqrt{x^2 + y^2} \tan\theta$ $\Rightarrow -dz = \sqrt{x^2+y^2} \sec^2\theta \, d\theta$

$$E = -\int_{+\pi/2}^{-\pi/2} \frac{k \rho_L \left[x \hat{a_x} + y \hat{a_y} \right]}{\left[x^2 + y^2 + (x^2+y^2)\tan^2\theta \right]^{3/2}} \sec^2\theta \sqrt{x^2+y^2} \, d\theta$$

$$\Rightarrow -\int_{-\pi/2}^{\pi/2} \frac{k\rho_L\left[x\hat{a_x}+y\hat{a_y}\right]}{\sec\theta\,(x^2+y^2)}\,d\theta$$

$$\Rightarrow -k\rho_L\frac{x\hat{a_x}+y\hat{a_y}}{x^2+y^2}\left(\sin\theta\right)_{-\pi/2}^{\pi/2}$$

$$\Rightarrow 2k\rho_L\frac{x\hat{a_x}+y\hat{a_y}}{x^2+y^2}$$

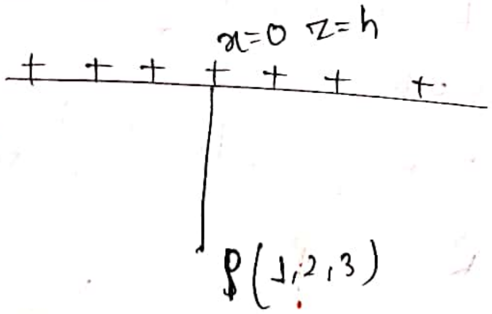
$$\Rightarrow \frac{1}{4\pi\epsilon_0}\times 2\times\rho_L\frac{(x\hat{a_x}+y\hat{a_y})}{x^2+y^2}$$

$$= \frac{\rho_L}{2\pi\epsilon_0}\frac{x\hat{a_x}+y\hat{a_y}}{(x^2+y^2)}$$

$$\left\{\begin{array}{l} \vec{\rho} = x\hat{a_x}+y\hat{a_y}+(c-c)\hat{a_z} \\ \quad = x\hat{a_x}+y\hat{a_y} \\ |\vec{\rho}| = \rho = \sqrt{x^2+y^2} \end{array}\right.$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0\,\rho}\hat{a_\rho}}$$

eg. ρ_L

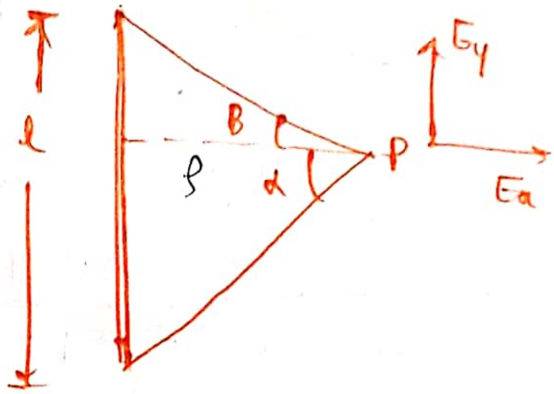


$$\vec{\rho} = (1-0)\hat{a_x}+(2-2)\hat{a_y}+(3-h)\hat{a_z}$$

$$\vec{\rho} = \hat{a_x}+(3-h)\hat{a_z}$$

Scanned by CamScanner

* For finite length

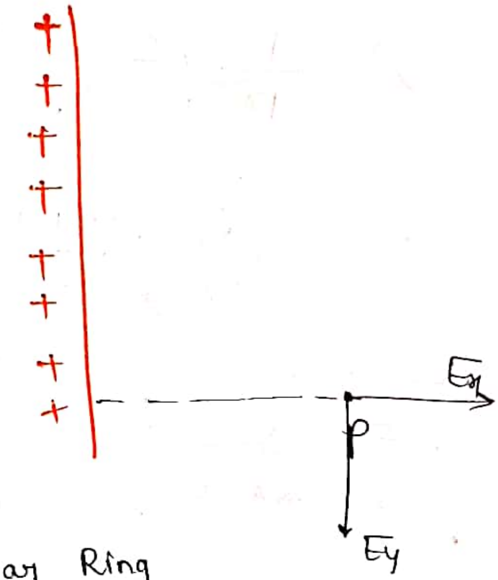


$$E_x = \frac{k\rho_L}{\rho}(\sin\beta + \sin\alpha)$$

$$E_y = \frac{k\rho_L}{\rho}(\cos\beta - \cos\alpha)$$

$$E = \sqrt{E_x^2 + E_y^2}$$

* for Infinite long line with point at $-\infty$

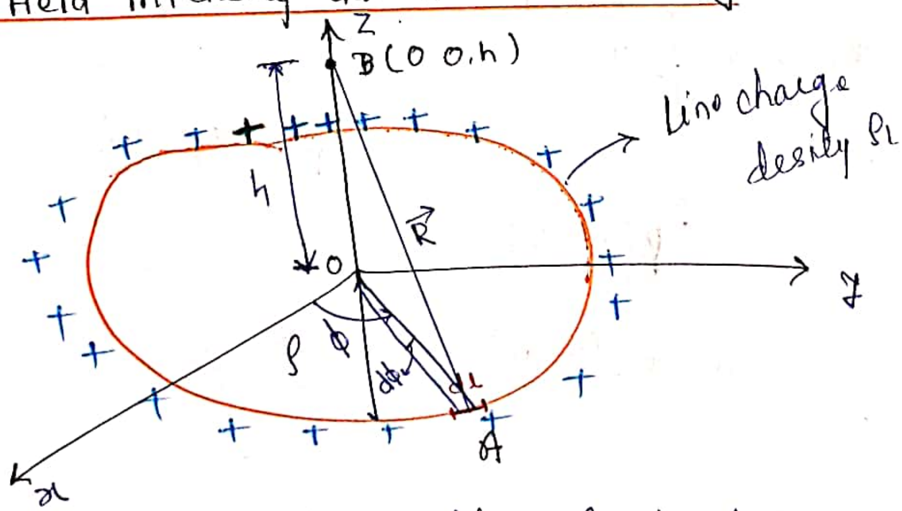


$$|E_x| = |E_y| = \frac{k\rho_L}{\rho}$$

$$\vec{E} = \sqrt{E_x^2 + E_y^2}$$

$$\boxed{E = \frac{\sqrt{2}\, k\rho_L}{\rho}}$$

* Electric Field Intensity due to Circular Ring



ρ = radius of Circular loop.

Chapter-2

Transmission line

- It is used to deliver power from one point to another at high frequency $\lambda = \frac{c}{f}$
- To apply Network theory (KCL or KVL) λ should be very greater than length of xmission line

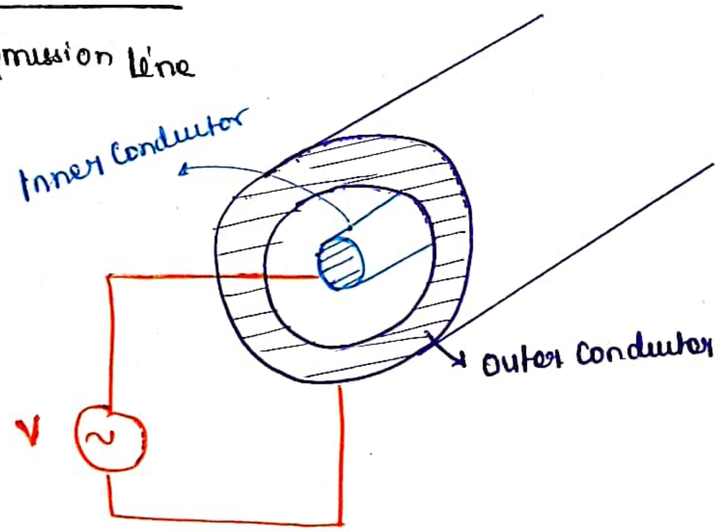
$$\lambda >> l$$

where, λ is wave length of the signal to be propagated.

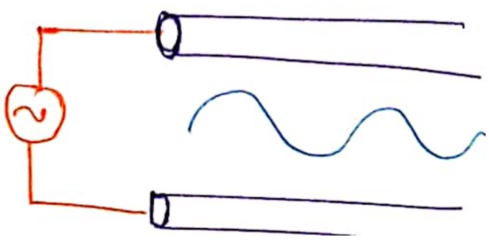
- xmission line supports xverse Electromagnetic mode (TEM).
- In high frequency, xmission line is inefficient to deliver power from source to load. due to losses. At very high frequency we use waveguides to deliver the power.

* Types of xmission line

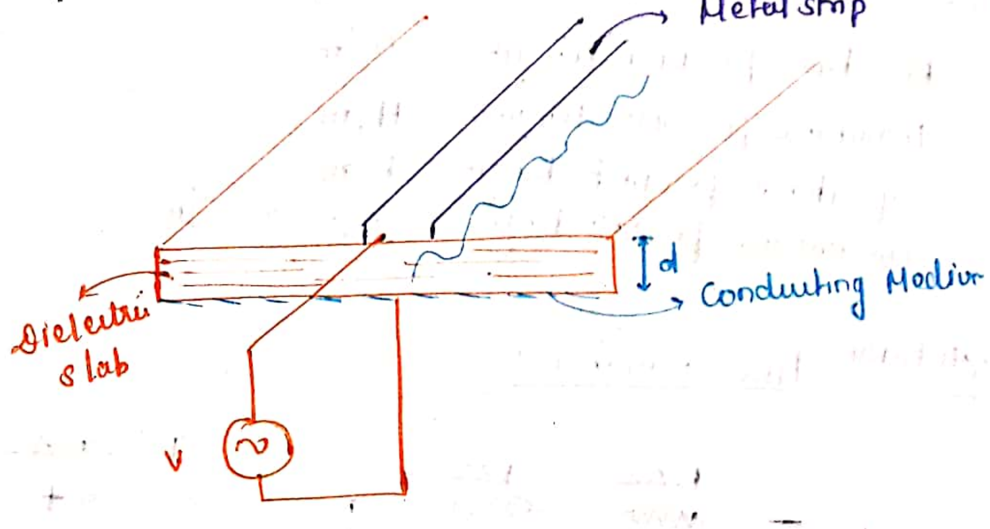
1. Co-axial xmission line



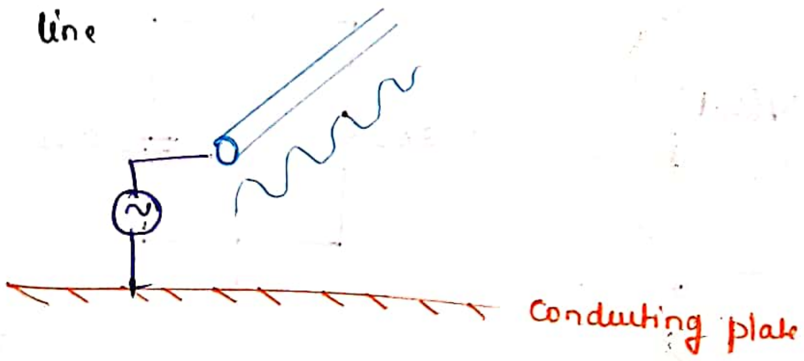
2. Parallel wire xmission line:



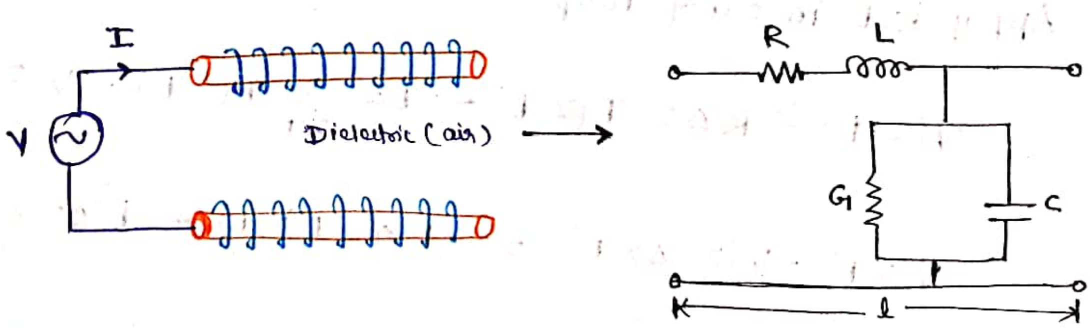
III) Microstrip line



IV) Unbalanced line



* Equivalent Circuit of xmission line



I) Current linked with Magnetic field Creates an Inductor 'L'

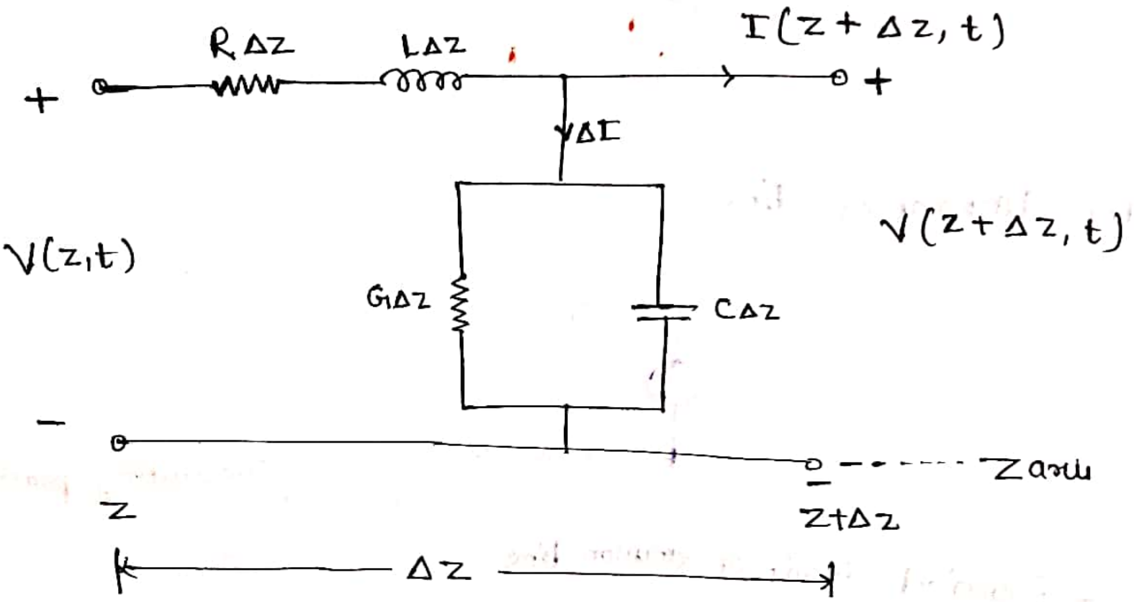
II) Two Conductors are Separated by Some distance filled with dielectric Creates a Capacitor 'C'

III) If Conductor is not perfect then there will be a Conduction loss 'R'

IV) If dielectric is not perfect then there will be a leakage current. Hence there is a dielectric loss 'G'

- R, L, G, C are called primary constant of Xmission line.
- R = Resistance per unit length Ω/m
  L = Inductance per unit length H/m
  C = Capacitance per unit length F/m
  G = Conductance per unit length $\mho/m$ or S/m

* Xmission Line Equation :-



Apply KVL in outer loop

$$V(z,t) - R\Delta z\, I(z,t) - L\Delta z \frac{\partial}{\partial t} I(z,t) - V(z+\Delta z, t) = 0$$

$$\Rightarrow V(z,t) - V(z+\Delta z, t) = R\Delta z\, I(z,t) + L\Delta z \frac{\partial}{\partial t} I(z,t)$$

$$\Rightarrow \frac{V(z,t) - V(z+\Delta z, t)}{\Delta z} = R\cdot I(z,t) + L\frac{\partial}{\partial t} I(z,t) \quad \text{--- (1)}$$

If $\Delta z \longrightarrow 0$

then $$\lim_{\Delta z \to 0} \frac{V(z,t) - V(z+\Delta z, t)}{\Delta z} = \frac{\partial V(z,t)}{\Delta z}$$

eq(1) becomes

$$\frac{\partial V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial}{\partial t} I(z,t)$$

Apply laplace (fourier)

$$\Rightarrow \frac{\partial V(z,t)}{\partial z} = (R + j\omega L)\, I(z,t) \quad \text{--- (2)}$$

Apply kcl at node.

$$I(z,t) = \Delta I + I(z+\Delta z, t)$$

$$\Rightarrow I(z,t) - I(z+\Delta z, t) = \Delta I$$

where,

$$\Delta I = V(z+\Delta z, t)\, G\Delta z + C\Delta z \frac{d}{dt} V(z+\Delta z, t)$$

$$\Rightarrow \frac{I(z,t) - I(z+\Delta z, t)}{\Delta z} = V(z+\Delta z, t)\, G + C\frac{d}{dt} V(z+\Delta z$$

taking limit $\Delta z \to 0$

$$\frac{\partial}{\partial z} I(z,t) = (G + j\omega C)\, V(z,t) \quad \text{--- (3)}$$

from (1) differentiate it wrt z

$$\Rightarrow \frac{\partial}{\partial z}\left(\frac{\partial V(z,t)}{\Delta z}\right) = (R + j\omega L)\frac{\partial}{\partial z} I(z,t)$$

from eq (3) put $\frac{\partial}{\partial z} I(z,t)$

$$\Rightarrow \frac{\partial}{\partial z}\left[\frac{\partial V(z,t)}{\Delta z}\right] = (R + j\omega L)(G + j\omega C)\, V(z,t)$$

$$\Rightarrow \frac{\partial^2 V(z,t)}{\partial z^2} = \gamma^2 V(z,t)$$

$$\Rightarrow \boxed{\frac{\partial^2 V(z,t)}{\partial z^2} - \gamma^2 V(z,t) = 0}$$

Where,

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \text{Propogation Constant } m^{-1}$$

$$\gamma = \alpha + j\beta$$

α = attenuation Constant (Np/m), $1\ Np = 8.686\ dB$

β = phase Constant (rad/m) $= \frac{2\pi}{\lambda}$

Similarly for Current eq^n differentiating eq (3) wrt z

$$\frac{\partial^2 I(z,t)}{\partial z^2} = (R + j\omega L)(G + j\omega C)\, I(z,t)$$

$$\Rightarrow \frac{\partial^2 I(z,t)}{\partial z^2} - (R + j\omega L)(G + j\omega C)\, I(z,t) = 0$$

$$\Rightarrow \boxed{\frac{\partial^2 I(z,t)}{\partial z^2} - \gamma^2 I(z,t) = 0}$$

Similar to plane wave eq^n

$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

Solution of Voltage eq^n

$$V(z,t) = \left[V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \right] e^{j\omega t}$$

$$\boxed{V_s(z) = \underbrace{V_0^+ e^{-\gamma z}}_{\text{Wave travelling in } +z \text{ direction}} + \underbrace{V_0^- e^{\gamma z}}_{\text{wave travelling in } -z \text{ direction}}}$$ Time Suppressed

$$V_s(z,t) = Re\left[V_0^+ e^{-\gamma z} e^{j\omega t} + V_0^- e^{\gamma z} e^{j\omega t} \right]$$

$$V(z,t) = Re\left[V_0^+ e^{-(\alpha+j\beta)z} e^{j\omega t} + V_0^- e^{(\alpha+j\beta)z} e^{j\omega t}\right]$$

$$= Re\left[V_0^+ e^{-\alpha z} e^{j(\omega t-\beta z)} + V_0^- e^{\alpha z} e^{j(\omega t+\beta z)}\right]$$

$$\boxed{V(z,t) = V_0^+ e^{-\alpha z}\cos(\omega t-\beta z) + V_0^- e^{\alpha z}\cos(\omega t+\beta z)}$$

* Characteristic Impedence (Z_0)

- It is defined as ratio of positive travelling voltage wave to the (+ve) current wave at any point of the xmission line.

$$\boxed{Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}}$$

also, $$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

- Secondary constant of xmission lines are $\gamma, \alpha, \beta, Z_0$

Case I :- Lossless xmission line :-

- A xmission line is said to be lossless if perfect conductor is used, i.e $R=0$ and perfect dielectric should be filled in between the conductor

i.e $R = 0$
$G = 0$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$= \sqrt{j\omega L \times j\omega C}$$
$$\gamma = j\omega\sqrt{LC} \quad m^{-1}$$
$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$
$$\alpha = 0, \quad \beta = \omega\sqrt{LC}$$

$$\boxed{\begin{aligned}\alpha &= 0\\ \beta &= \omega\sqrt{LC}\\ Z_0 &= \sqrt{\frac{L}{C}}\end{aligned}}$$

$v_p = \frac{\omega}{\beta}$ Phase Velocity

$$\boxed{v_p = \frac{1}{\sqrt{LC}}} \text{ m/s}$$

Case II Distortionless Xmission line

$$\frac{R}{G} = \frac{L}{C}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{RG\left(1+j\omega \frac{L}{R}\right)\left(1+j\omega\frac{C}{G}\right)}$$

$$= \sqrt{RG\left(1+j\omega\frac{L}{R}\right)^2}$$

$$= \left(1+j\omega\frac{L}{R}\right)\sqrt{RG}$$

we know,

$$\gamma = \alpha + j\beta$$

on comparing

$$\boxed{\alpha = \sqrt{RG}}$$

$$\beta = \omega L\sqrt{\frac{G}{R}}$$

$$= \omega\sqrt{L\left(\frac{LG}{R}\right)}$$

$$\left\{\because \frac{LG}{R} = C\right.$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

Scanned by CamScanner

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$= \sqrt{\frac{R}{G}\left[\frac{1+j\omega L/R}{1+j\omega C/G}\right]}$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}}$$

Calulation of β:

$$v_p = \frac{\omega}{\beta}$$

$$\boxed{v_p = \frac{1}{\sqrt{LC}}}$$

<u>Note:-</u> A lossless line is always a distortionless xmission line but the reverse is not true.

<u>Case 3</u> :- Lossy xmission line (Low losses)

$R < j\omega L$ & $G \ll j\omega C$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\gamma = \sqrt{(j\omega L)(j\omega C)\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\sqrt{1+\frac{R}{j\omega L}}\sqrt{1+\frac{G}{j\omega C}}$$

Binomial $(1+x)^n = 1+nx$ where $x \ll 1$

$$\gamma = \sqrt{1+x}\sqrt{1+y} = \left(1+\frac{x}{2}\right)\left(1+\frac{y}{2}\right) = 1+\frac{x}{2}+\frac{y}{2}+\underbrace{\frac{xy}{2}}_{\text{neglect}}$$

$$\gamma = j\omega\sqrt{LC}\left(1 + \frac{R}{2j\omega L} + \frac{G}{2j\omega C}\right)$$

$$\Rightarrow \gamma = \frac{R\sqrt{LC}}{2L} + \frac{G\sqrt{LC}}{2C} + j\omega\sqrt{LC}$$

$$\gamma = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} + j\omega\sqrt{LC}$$

$$\boxed{\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

Calculation of Z_0 :-

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$= \left[\frac{L}{C}\frac{\left(1+\frac{R}{j\omega L}\right)}{\left(1+\frac{G}{j\omega C}\right)}\right]^{1/2}$$

$$\Rightarrow \sqrt{\frac{L}{C}}\left[\frac{1+R/j\omega L}{1+G/j\omega C}\right]^{1/2}$$

$$\Rightarrow \sqrt{\frac{L}{C}}\left(1+\frac{R}{2j\omega L}\right)\left(1-\frac{G}{2j\omega C}\right)$$

$$\Rightarrow \sqrt{\frac{L}{C}}\left[\left(1+\frac{R}{2j\omega L}-\frac{G}{2j\omega C}\right)\right]$$

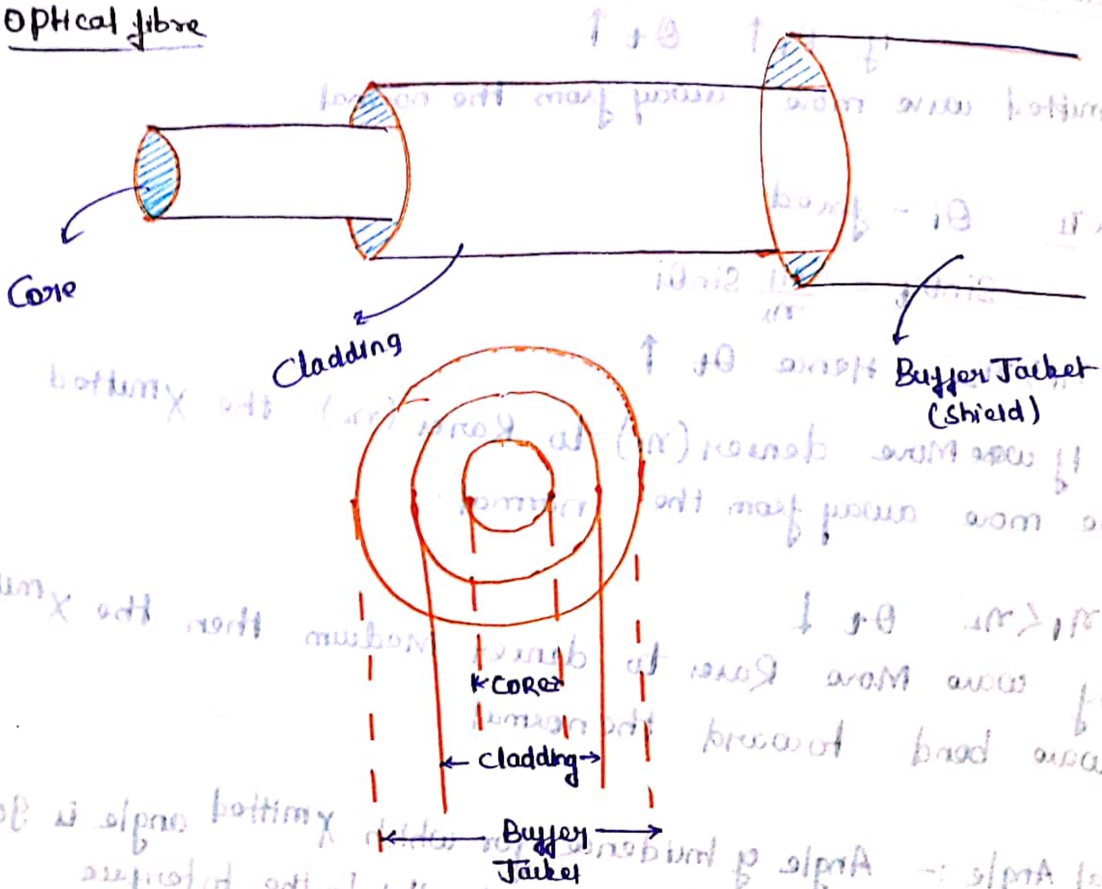
$$Z_0 = \sqrt{\frac{L}{C}} - \frac{jR}{2\omega L}\sqrt{\frac{L}{C}} + \frac{jG}{2\omega C}\sqrt{\frac{L}{C}}$$

$$Z_0 = \underbrace{R_0}_{\text{Resistive}} + j\underbrace{X_0}_{\text{Reactive}}$$

Chapter 3 - Radar & Optical Fibre

Optical fibre

*



Core :- It is made up of glass of very high refractive Index

cladding :- It is also Made of Material same as Core but having Lesser refractive Index than Core.

$$\text{Refractive Index} = \frac{\text{Velocity of light in Vacum}}{\text{Velocity of light in Medium}}$$

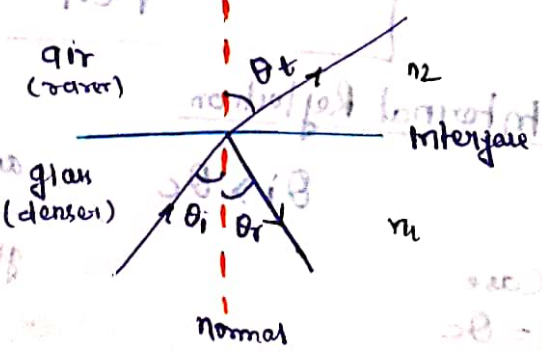
$$\eta = \frac{c}{v_p} = c\sqrt{\mu\epsilon}$$

* Ray Xmission theory

Snell's law.

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\sin\theta_t = \left(\frac{n_1}{n_2}\right)\sin\theta_i$$



Case I n_1 and n_2 are fixed

If $\theta_i \uparrow$ $\theta_t \uparrow$

xmitted wave move away from the normal

Case II θ_i = fixed

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$$

$n_1 > n_2$ Hence $\theta_t \uparrow$

- If wave Move denser (n_1) to Rarer (n_2) the xmitted wave move away from the normal.

$n_1 < n_2$ $\theta_t \downarrow$

- If wave Move Rarer to denser Medium then the xmitted wave bend toward the normal

Critical Angle :- Angle of Incidence for which xmitted angle is 90° or xmitted wave is parallel to the Interface.

$$\theta_t = 90^\circ$$

$$n_1 \sin\theta_c = n_2 \sin 90^\circ$$

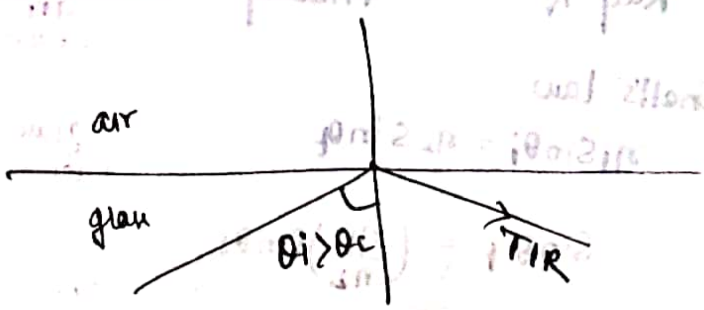
$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

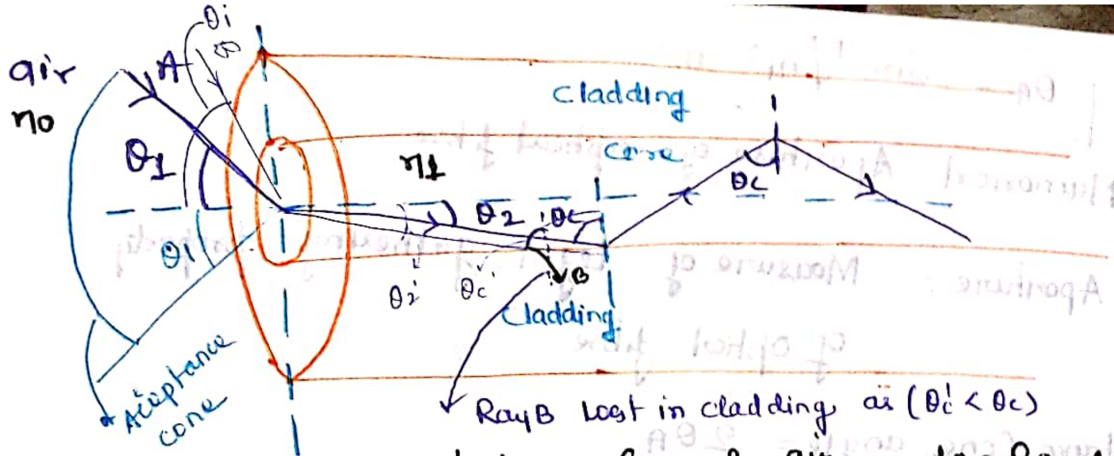
$$\boxed{\theta_c = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right)}$$ for non Magnetic Medium

Total Internal Reflection

$\theta_i > \theta_c$



Worst Case
$\theta_i = \theta_c$



Applying Snell's law between Core & air for Ray A

$$\eta_0 \sin\theta_1 = \eta_1 \sin\theta_2 \quad \text{--- (1)}$$

where $\theta_2 = 90 - \theta_c$

$$\theta_c = 90 - \theta_2 \quad \text{--- (2)}$$

for Ray B.

$$\eta_0 \sin\theta_1' = \eta_1 \sin\theta_2'$$

$$\theta_c' = 90 - \theta_2'$$

$\theta_c' < \theta_c$ Hence no total internal Reflection for wave B it will xmit to cladding and distorted. (does not move further)

$$\eta_0 \sin\theta_1 = \eta_1 \sin\theta_2$$

$$\eta_0 \sin\theta_1 = \eta_1 \sin(90 - \theta_c)$$

$$\eta_0 \sin\theta_1 = \eta_1 \cos\theta_c$$

$$\eta_0 \sin\theta_1 = \eta_1 \sqrt{1 - \sin^2\theta_c}$$

$$= \eta_1 \sqrt{1 - \left(\frac{\eta_2}{\eta_1}\right)^2}$$

θ_1 = Maximum Acceptance Angle

$\theta_1 = \theta_A$

$\eta_0 = 1$ for air

$$\sin\theta_1 = \sqrt{\eta_1^2 - \eta_2^2}$$

$$\sin\theta_A = \sqrt{\eta_1^2 - \eta_2^2}$$

$$\boxed{\theta_A = \sin^{-1}\sqrt{n_1^2 - n_2^2}}$$

$\sin\theta_A$ = Numerical Aperture of optical fibre.

Numerical Aperture :- Measure of light gathering property of optical fibre.

Acceptance Cone angle = $2\theta_A$

* Relative refractive Index difference

$$\Delta = \frac{n_1^2 - n_1^2}{2n_1^2}$$

$$\Delta = \frac{NA^2}{2n_1^2} \quad \Big\{ NA = \sin\theta_A$$

$$NA = n_1\sqrt{2\Delta}$$

Que page 6-18 $n_1 = 1.5$ $n_2 = 1.4$

$\sin\theta_A = \sqrt{n_1^2 - n_2^2}$

$\theta_A = \sin^{-1}(0.53)$

$= 32.58$ //

Que 3

6.13 $\epsilon = \epsilon_r \epsilon_0$

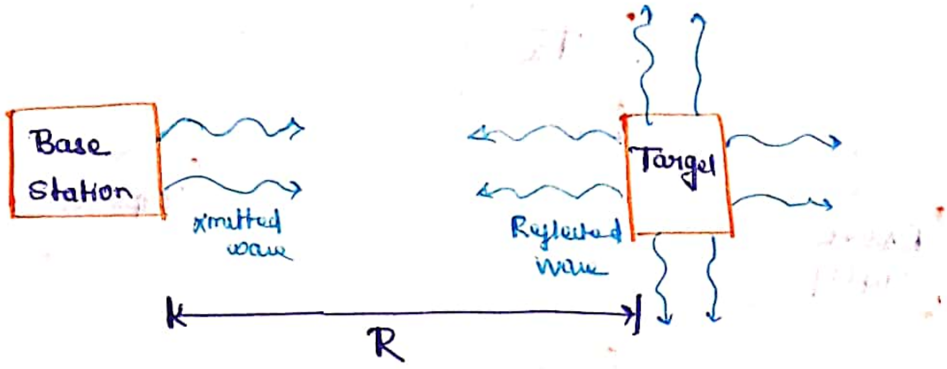
for water $\epsilon = 1.75\,\epsilon_0$

$\tan\theta_c = \frac{5}{d}$

$\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} = \sin^{-1}\sqrt{\frac{1}{1.75}} = 49.10°$

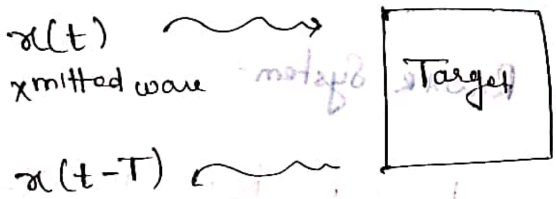
$d = \frac{5}{\tan 49.10°} = 4.33$ //

* Basics of Radar (Radio detection and Ranging)

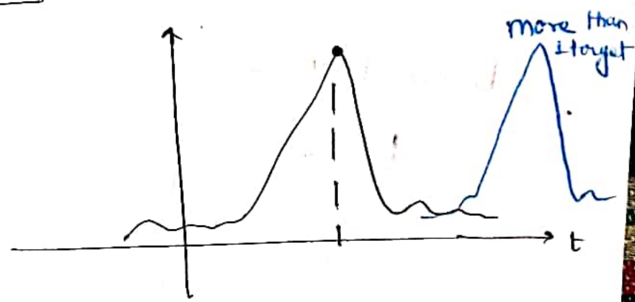


$$T = \frac{2R}{c}$$

$$\boxed{R = \frac{cT}{2}}$$ Range Resolution

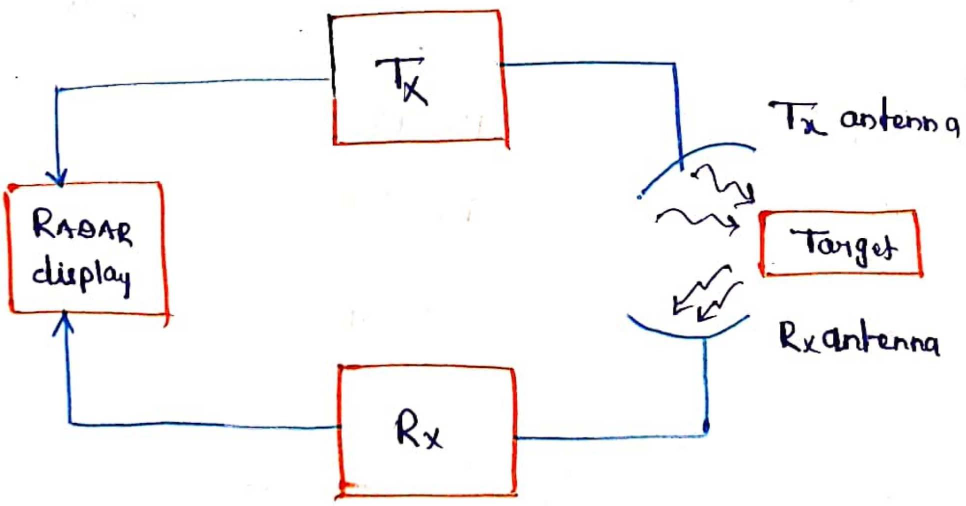


Correlation $[\,x(t)\; x(t-T)\,]$



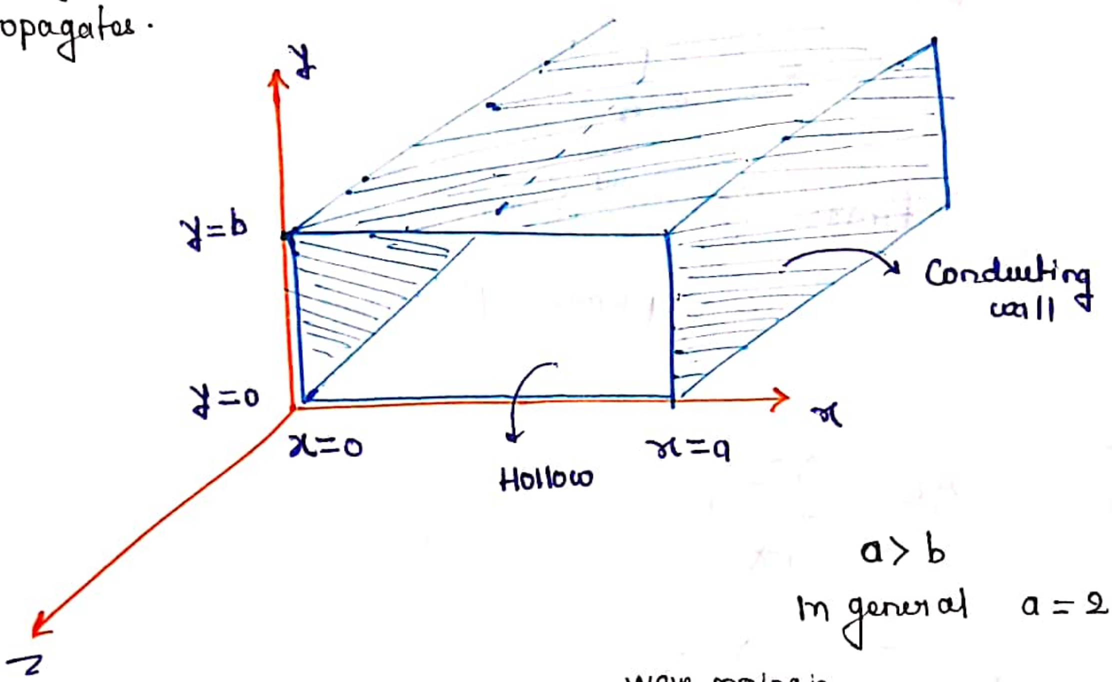
Types of RADAR :-

1. Bistatic RADAR



Chapter-4 Wave Guides

- For xmission of EM wave at very high frequency, wave guide is used
- Wave guide act as a high pass filter (having lowest cutoff frequency and maximum cutoff wavelength)
- Wave guide is a hollow conducting tube through which EM wave propagates.



$a > b$

In general $a = 2b$

Wave equation in wave guide { assumption wave moving in +z direction

$$\nabla^2 \vec{E_s} + k^2 \vec{E_s} = 0 \quad \text{—— (1)}$$

$$\vec{E_s} = (E_{xs}, E_{ys}, E_{zs})$$

$$\nabla^2 \vec{H_s} + k^2 \vec{H_s} = 0 \quad \text{—— (2)}$$

$$\vec{H_s} = (H_{xs}, H_{ys}, H_{zs})$$

$k = \omega\sqrt{\mu\epsilon}$ = wave number

Z component of $\vec{E_s}$

$$\frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0$$

Solution of eqn

$$E_{zs} = (C_1 \cos k_x x + C_2 \sin k_x x)(C_3 \cos k_y y + C_4 \sin k_y y)(C_5 e^{+\gamma z} + C_6 e^{-\gamma z})$$

for +ve direction travelling wave
(forward)

$$E_{zs} = (C_1 \cos k_x x + C_2 \sin k_x x)(C_3 \cos k_y y + C_4 \sin k_y y)\, C_6 e^{-\gamma z} \quad \text{—— (3)}$$

Similarly.

$$H_{zs} = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)\, A_6 e^{-\gamma z} \quad \text{—— (4)}$$

Maxwell's equation,

$$\nabla \times \vec{E_s} = -\frac{\partial \vec{B_s}}{\partial t}$$

$$\nabla \times \vec{E_s} = -j\omega\mu H_s$$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \vec{E_{sx}} & \vec{E_{sy}} & \vec{E_{sz}} \end{vmatrix} = -j\omega\mu \left[H_{xs}\hat{a}_x + H_{ys}\hat{a}_y + H_{zs}\hat{a}_z \right]$$

On Comparing $\hat{a}_x$, $\hat{a}_y$ & $\hat{a}_z$ Components

$$\left[\frac{\partial \vec{E_{zs}}}{\partial y} - \frac{\partial \vec{E_{ys}}}{\partial z} \right] = -j\omega\mu \vec{H_{xs}} \quad \text{—— (5)}$$

$$\left[\frac{\partial \vec{E_{zs}}}{\partial x} - \frac{\partial \vec{E_{xs}}}{\partial z} \right] = j\omega\mu \vec{H_{ys}} \quad \text{—— (6)}$$

$$\left[\frac{\partial \vec{E_{ys}}}{\partial x} - \frac{\partial \vec{E_{xs}}}{\partial y} \right] = -j\omega\mu \vec{H_{zs}} \quad \text{—— (7)}$$

Taking Maxwell eqn

$$\nabla \times \vec{H} = \vec{J_c} + \vec{J_D}$$

$$= \sigma \vec{E_s} + \frac{\partial \vec{D}}{\partial t}$$

for Lossless Medium (Hollow space of waveguide)

$\sigma = 0$

$$\nabla \times \vec{H_s} = j\omega \epsilon \vec{E}$$

$$\begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_{xs} & H_{ys} & H_{zs} \end{vmatrix} = j\omega\epsilon \left[E_{xs}\hat{a_x} + E_{ys}\hat{a_y} + E_{zs}\hat{a_z} \right]$$

$$\left[\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} \right] = j\omega\epsilon E_{xs} \quad \text{——— (8)}$$

$$\left[\frac{\partial H_{zs}}{\partial x} - \frac{\partial H_{xs}}{\partial z} \right] = -j\omega\epsilon E_{ys} \quad \text{——— (9)}$$

$$\left[\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} \right] = j\omega\epsilon E_{zs} \quad \text{——— (10)}$$

from eq (6)

$$H_{ys} = \frac{1}{j\omega\mu} \left[\frac{\partial E_{zs}}{\partial x} - \frac{\partial E_{xs}}{\partial z} \right]$$

put in (8)

$$\frac{\partial H_{zs}}{\partial y} - \frac{1}{j\omega\mu} \left[\frac{\partial^2 E_{zs}}{\partial x \partial z} - \frac{\partial^2}{\partial z^2} E_{xs} \right] = j\omega\epsilon E_{xs}$$

——— (9)

Assuming $E_{zs} = E_{zo} e^{-\gamma z}$ & $E_{xs} = E_{xo} e^{-\gamma z}$

$$\frac{\partial E_{zs}}{\partial z} = -\gamma E_{z0} e^{-\gamma z} \qquad \frac{\partial E_{xs}}{\partial z} = -\gamma E_{x0} e^{-\gamma z}$$

$$\frac{\partial E_{zs}}{\partial z} = -\gamma E_{zs} \qquad \frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 E_{x0} e^{-\gamma z} = \gamma^2 E_{xs}$$

Put in (9)

$$\frac{\partial H_{zs}}{\partial y} - \frac{1}{j\omega\mu}\left[-\gamma \frac{\partial E_{zs}}{\partial x} - \gamma^2 E_{xs}\right] = j\omega\epsilon E_{xs}$$

$$\frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\gamma \frac{\partial E_{zs}}{\partial x} + \frac{\gamma^2 E_{xs}}{j\omega\mu} = j\omega\epsilon E_{xs}$$

$$j\omega\mu \frac{\partial H_{zs}}{\partial y} + \gamma \frac{\partial E_{zs}}{\partial x} + \gamma^2 E_{xs} = -\omega^2\mu\epsilon E_{xs}$$

$$(\gamma^2 + \omega^2\mu\epsilon) E_{xs} = -\gamma \frac{\partial E_{zs}}{\partial x} - j\omega\mu \frac{\partial H_{zs}}{\partial y}$$

$$\boxed{\begin{aligned}
E_{xs} &= \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y} \\
E_{ys} &= \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} \\
H_{xs} &= \frac{-\gamma}{h^2}\frac{\partial H_{zs}}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{\partial y} \\
H_{ys} &= \frac{-\gamma}{h^2}\frac{\partial H_{zs}}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{\partial x}
\end{aligned}}$$

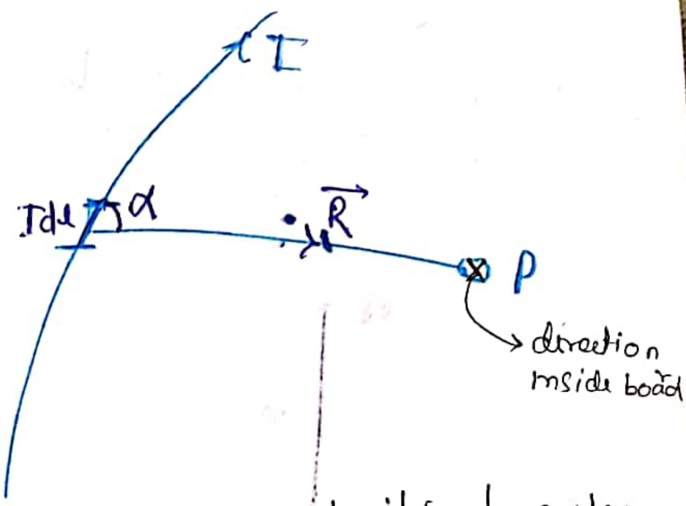
$\{ h^2 = \gamma^2 + \omega^2\mu\epsilon$

(11)

Magnetostatic

* Biot Savart's Law

"It states that Magnetic field Intensity produced at a point p by the differential current element is proportional to product Idl and ~~Sign~~ Sine of angle between the current element and the line joining to the current element and it is inversely proportional to square of distance betn current element and point of interest.



$$dH \propto \frac{I\,dl \sin\alpha}{R^2}$$

$$dH = \frac{1}{4\pi} \frac{I\,dl \sin\alpha}{R^2}$$

$$\vec{dl} \times \hat{a}_R = |\vec{dl}|\,|\hat{a}_R| \sin\alpha$$

$$= dl\,(1) \sin\alpha$$

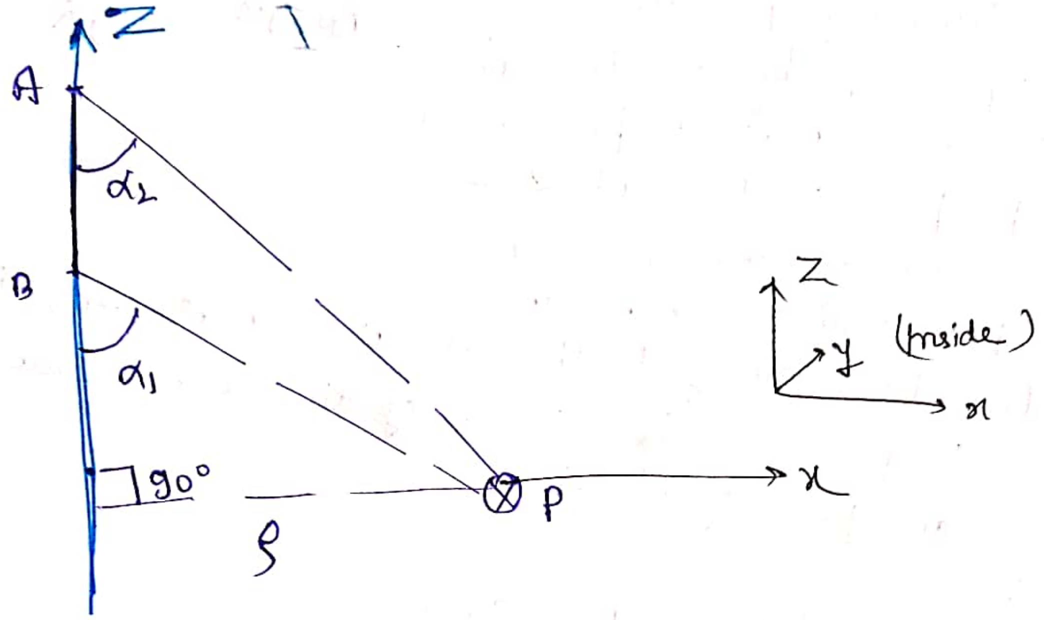
$$= dl \sin\alpha$$

$$dH = \frac{1}{4\pi} \frac{I\,dl \sin\alpha}{R^2} = \frac{I\vec{dl} \times \hat{a}_R}{4\pi R^2} = \frac{I\vec{dl} \times \vec{R}}{4\pi R^3}$$

$$\boxed{\vec{H} = \int_L \frac{I\vec{dl} \times \vec{R}}{4\pi R^3} = \int_L \frac{I\vec{dl} \times \hat{a}_R}{4\pi R^2}}$$

Right hand Rule :- If we point our thumb in the direction of current then finger encircling the wire gives direction of Magnetic field

* Magnetic field due to straight finite length current carrying conductor



$$\vec{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$$

$\rho \rightarrow$ perpendicular distance from conductor to point of Interest

$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$

$\hat{a}_l$ = Unit Vector in the direction of current

$\hat{a}_\rho$ = Unit Vector along perpendicular distance from conductor to point of Interest.

$\Rightarrow \hat{a}_z \times \hat{a}_x = \hat{a}_y$ {Magnetic field}

Case I Semi Infinite Conductor

$\alpha_1 = 90^\circ \quad \alpha_2 = 0$ point B at (0, 0)

$$\boxed{\vec{H} = \frac{I}{4\pi\rho} \hat{a}_\phi}$$

Case II Infinite Conductor

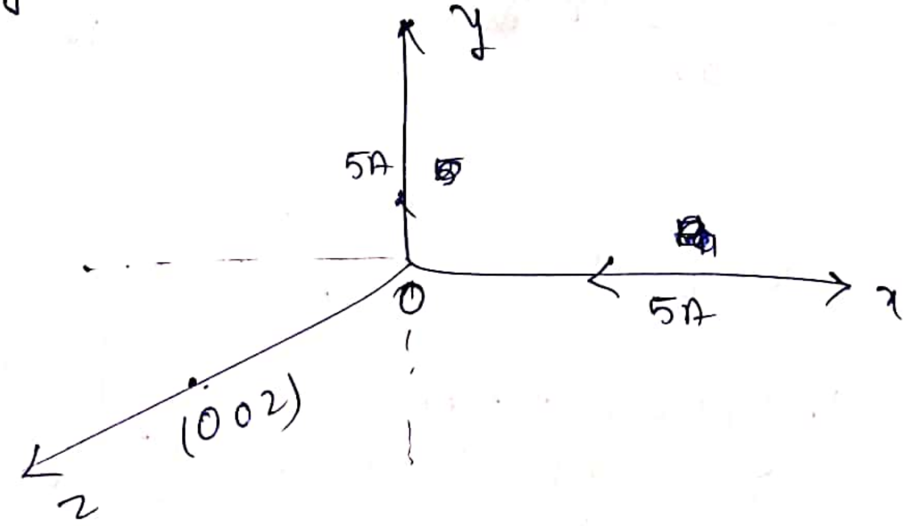
$\alpha_1 = 180^\circ \quad \alpha_2 = 0$

$$\vec{H} = \frac{I}{4\pi\rho}\left[1 + 1\right] \hat{a}_\phi$$

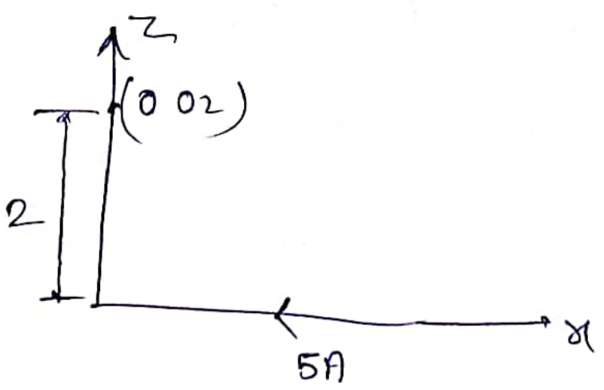
$$\boxed{\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

Que. An Infinite long conductor is bent in L shape if a current of 5A flows find Magnetic field at (0 0 2)

Sol



→ 1st



$$H_1 = \frac{I}{4\pi\rho}\hat{a_\phi}$$

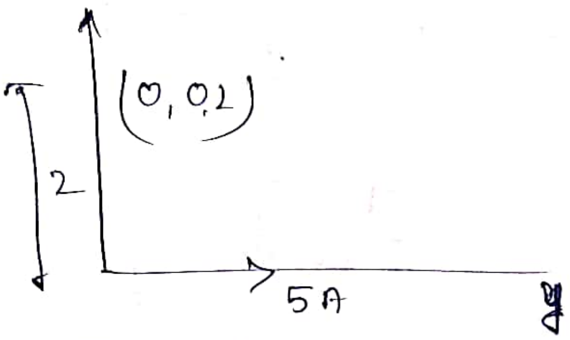
$$= \frac{5}{4\pi 2}\hat{a_\phi}$$

$$\left\{ \hat{a_\phi} = \hat{a_l} \times \hat{a_\rho} \right.$$

$$= \frac{5}{8\pi}\hat{a_y}$$

$$\hat{a_\phi} = -a_x \times a_z$$
$$= \hat{a_y}$$

for 2nd



current flow

$$H_2 = \frac{I}{4\pi\rho}\hat{a_\phi}$$

$$\hat{a_\phi} = \hat{a_l} \times \hat{a_\rho}$$

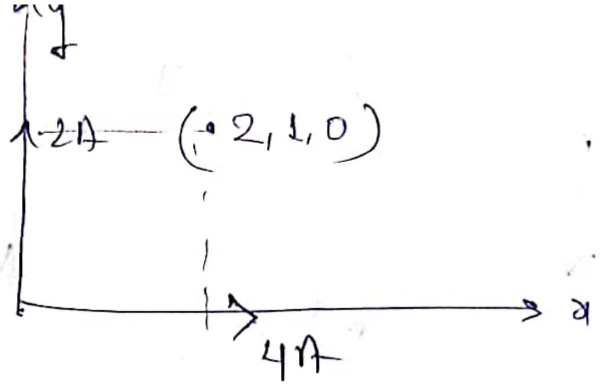
$$= \frac{5}{8\pi}(+\hat{a_x})$$

$$\hat{a_\phi} = a_y \times a_z$$
$$= a_x$$

$$H = \frac{5}{8\pi}\left[\hat{a_y} + \hat{a_x}\right] \quad //$$

Que. An Infinitely extented wire is placed along x axis and carrying current of 4A in + x direction. Another ∞ extended wire is placed along y axis and carry current 2A in +y direction. Magnetic field Intensity at (2,1,0) will be _

Sol



$H_1 = \frac{I}{2\pi 2} (a_\phi)$ (conduct on y)

$a_\phi = a_l \times a_\rho$
$= a_x \times a_y$
$= a_z$

$= \frac{I}{4\pi}(-a_z) = \frac{-a_z}{2\pi}$

$H_2 = \frac{I}{2\pi}(-a_z) = +\frac{2}{2\pi}(+a_z)$

$H_1 + H_2 = \cancel{\frac{I}{4\pi} a_z} - \cancel{\frac{I}{2\pi}(a_z)}$

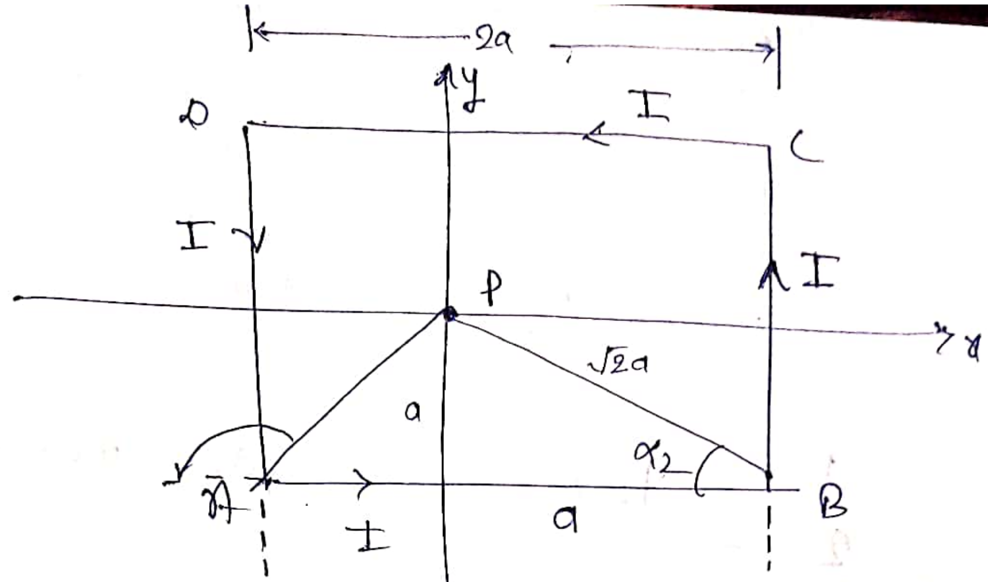
$= \cancel{\frac{-I\hat{a_z}}{4\pi}}$ ~~//~~.

$\Rightarrow \cancel{\frac{\hat{a_z}}{2\pi}} - \cancel{\frac{2a_z}{2\pi}}$

$\Rightarrow \left(\frac{4}{2\pi} - \frac{1}{2\pi}\right)\hat{a_z}$

$\Rightarrow \frac{3}{2\pi}\hat{a_z}$ //.

Que



Calulate Magnetic field Intensity at point p.

Sol for AB

$$\vec{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a_\phi}$$

$$= \frac{I}{4\pi\rho}\left[\frac{a}{\sqrt{2}a} - \left(\cancel{180} - \frac{a}{\sqrt{2}a}\right)\right]$$

$$= \frac{I}{4\pi a}\left[\frac{1}{\sqrt{2}} + \frac{a}{\sqrt{2}} \quad \cancel{-180}\right]$$

$$\Rightarrow \frac{I}{4\pi a}\left[\sqrt{2}\right]$$

$$\vec{H_1} \Rightarrow \frac{I}{2\pi a\sqrt{2}}\hat{a_z}$$

$$\hat{a_\phi} = \hat{a_x} \times \hat{a_y}$$
$$= \hat{a_z}$$

for BC

$$\vec{H_2} = \frac{I}{4\pi\rho}\left[+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$\vec{H_2} = \frac{I}{2\pi a\sqrt{2}}\,\hat{a_z}$$

for CD :

$$\vec{H_3} = \frac{I}{4\pi\rho}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

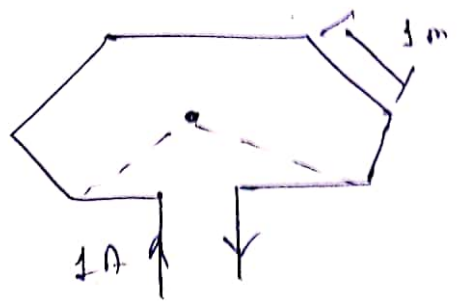
$$\vec{H_3} = \frac{I}{2\pi a\sqrt{2}}\,\hat{a_z}$$

for DA

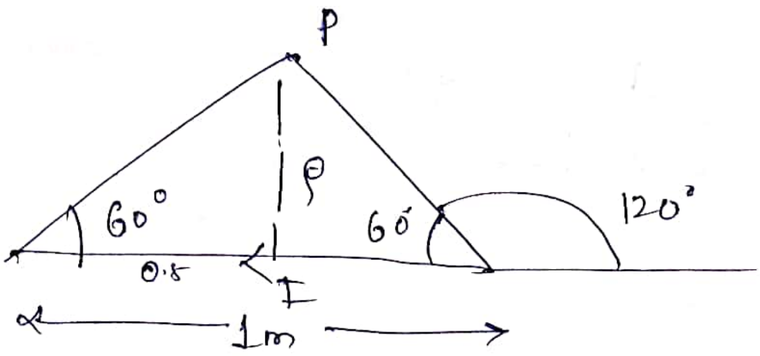
$$H_4 = \frac{I}{2\pi a\sqrt{2}}\,\hat{a_z}$$

$$H = H_1 + H_2 + H_3 + H_4 \,//.$$

Q. The Magnitude of Magnetic field at Centre of loop of a wire wound as a regular Hexagon of side 1m carrying current 1A.



$$\vec{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$$



$$|\vec{H}| = \frac{I}{4\pi\rho}\left[\cos 60 - \cos 120\right]$$

$$\tan 60 = \frac{\rho}{0.5}$$

$$\sqrt{3} = 2\rho$$

$$\rho = \sqrt{3}/2$$

$$\Rightarrow \frac{1}{2\sqrt{3}\pi}\left[\frac{1}{2} + \frac{1}{2}\right]$$

$$|\vec{H}| = \frac{1}{2\sqrt{3}\pi}$$

$$H_{net} = 6|\vec{H}| = 6 \times \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{\pi} = 0.559 \ A/m$$

* Ampere's law:-

It states that " Line Integral of Magnetic field Intensity around a closed Loop is equal to Current enclosed by that Closed loop"

$$\oint_L \vec{H}\cdot\vec{dl} = I_{enc} \quad \text{—①}$$

$$\oint_S \vec{J}\cdot\vec{ds} = I \quad \text{—②}$$

from (I) & (II)

$$\boxed{\oint_L \vec{H}\cdot\vec{dl} = \int_S \vec{J}\cdot\vec{ds}}$$

Maxwell third equation in Integral form

According to stokes theorem

$$\oint_L \vec{H}\cdot\vec{dl} = \int_S (\nabla\times\vec{H})\cdot\vec{ds}$$

$$\therefore \int_S (\nabla\times\vec{H})\cdot\vec{ds} = \int_S \vec{J}\cdot\vec{ds}$$

$$\boxed{\nabla\times\vec{H} = \vec{J}}\ A/m^2$$

Maxwell third eqⁿ in Differential form or Point form

For time Varying field

$$\boxed{\nabla\times\vec{H} = \vec{J} + \vec{J}_D}$$

Unit - 5

Antenna

- An Antenna is a Source or radiator of EM wave.
- An Antenna is a xducer which Converter Voltage or Current wave Into EM wave. or Vice Versa.
- An Antenna is an Impedence Matching device between xmission line and Surrounding Medium.
- An Antenna may be a piece of Conducting Material in the form of wire, Rod or any other shape with excitation.
- The basic equation of radiation is given by

$$L\frac{di}{dt} = Q\frac{dv}{dt} \quad (1)$$

where, L = Length of Antenna or length of wire

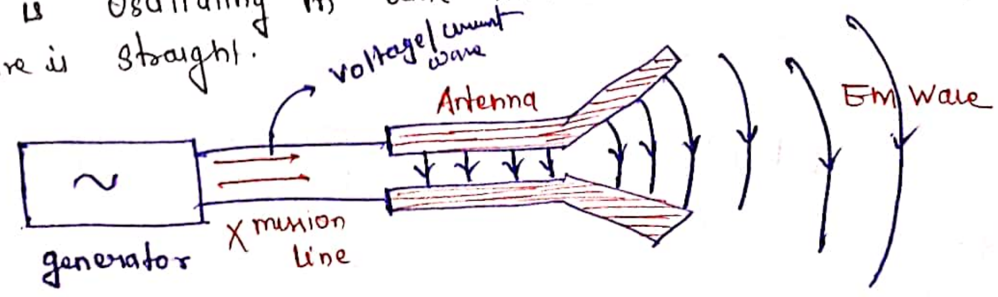
$\frac{di}{dt}$ = change in Current

$\frac{dv}{dt}$ = change in Velocity or acelaration

Q = charge

eq (1) shows change in Current and acelarated charge radiates

- If a charge is not moving, Current is not created and there is no radiation.
- If a charge is moving with a Uniform Velocity there is no radiation if the wire is straight and Infinitely extented.
- If charge is moving with uniform Velocity there is radiation if the wire is Curved bent discontinuous or terminated
- If charge is Oscillating in time motion it radiates even if the wire is straight.



Types of Antenna

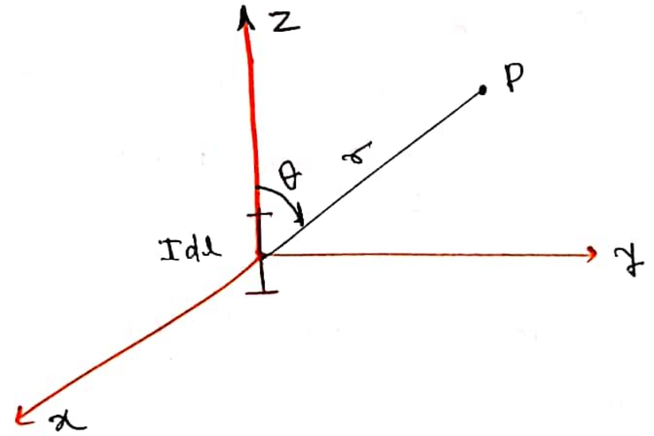
1) Hertzian Dipole
2) Halfwave Dipole
3) Quarter wave monopole
4) Small loop antenna

To determine Radiation field Regardless of Types of Antenna take following steps –

1) Select an appropriate Coordinate System and determine Magnetic Vector potential.
2) find $\vec{H}$ Magnetic field Intensity by $\vec{H} = \frac{B}{\mu}$
   $\vec{B} = \nabla \times A$ (curl of Magnetic Vector potential)
3) Determine Electric field using $\nabla \times \vec{H} = \vec{J_c} + \vec{J_D}$
   $= \sigma E + \frac{\partial D}{\partial t} = \sigma E + j\omega\epsilon E$
4) find the far field and determine the time average power radiated
   $$P_{rad} = \int \vec{P}_{avg} \cdot \vec{ds} \quad \left(\frac{W}{m^2} \times m^2 = \text{Watt}\right)$$
   $$P_{avg} = \frac{1}{2} \text{Re}\left[\vec{E} \times \vec{H}^*\right]$$

I: Hertzian Dipole

By Hertzian dipole we mean Very small Current element Idl where $dl < \lambda/10$



$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{Idl}{r} \hat{a_z}$$

$[I]$ = retarded Current

$$[I] = I_0 \cos\omega\left(t - \frac{r}{c}\right)$$

$\left\{ \frac{\text{Distance}(r)}{\text{velocity}(c)} = t \right.$

$\frac{r}{c}$ = propagation Delay

$$[I] = I_0 \cos(\omega t - \frac{\omega}{c} r) = I_0 \cos(\omega t - \beta r)$$

$$= I_0 Re\left[e^{j(\omega t - \beta r)}\right]$$

Suppressing the time factor

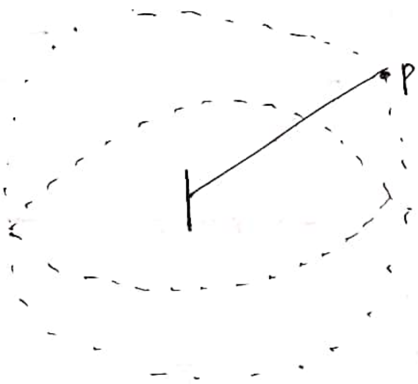
$$\vec{A}_{zs} = \frac{\mu_0}{4\pi} \frac{e^{-j\beta r} I_0 dl}{r} \hat{a}_z$$

Step 2. $\nabla \times \vec{A} = B$

$\vec{A}(A_{xs}, A_{ys}, A_{zs})$

$\downarrow$

$\vec{A}(A_{rs}, A_{\theta s}, A_{\phi s})$



Antenna Radiates outward in all direction (spherically.

$$\begin{bmatrix} A_{xs} \\ A_{ys} \\ A_{zs} \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_{rs} \\ A_{\theta s} \\ A_{\phi s} \end{bmatrix}$$

$$\begin{bmatrix} A_{rs} \\ A_{\theta s} \\ A_{\phi s} \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_{xs} \\ A_{ys} \\ A_{zs} \end{bmatrix}$$

$A_{xs} = 0$

$A_{ys} = 0$

$$\vec{A}_{zs} = \frac{\mu_0}{4\pi} e^{-j\beta r} \frac{I_0 dl}{r} \hat{a}_z$$

$A_{rs} = A_{zs} \cos\theta$

$A_{\theta s} = -\sin\theta A_{zs}$

$A_{\phi s} = 0$

Scanned by CamScanner

$$\nabla \times \vec{A} = B = \mu \vec{H} \quad \text{— (1)}$$

where, $\vec{H} = H_{rs}\hat{a}_r + H_{\theta s}\hat{a}_\theta + H_{\phi s}\hat{a}_\phi$

$$\Rightarrow \nabla \times \vec{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a_r} & r\hat{a}_\theta & r\sin\theta\, \hat{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_{zs}\cos\theta & -r\sin\theta A_{zs} & 0 \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[\hat{a_r}(0) - r\hat{a}_\theta(0) + r\sin\theta\, \hat{a}_\phi \left(-\sin\theta \cancel{A_{zs}} + A_{zs}\cancel{\sin\theta}\right)\right] + \cos\theta\left(\frac{\mu_0}{4\pi}\frac{A_{zs}}{r}\right)$$

$$\Rightarrow \frac{\hat{a}_\phi}{r^2\sin\theta}\left(-\sin\theta A_{zs}' + \sin\theta A_{zs} + \cos\theta \frac{A_{zs}}{r}\right)$$

$$= \frac{\hat{a}_\phi}{r^2\sin\theta}\left[\cos\theta \frac{\bar{A}_{zs}}{r} + \bar{A}_{zs}(-j\beta r)\right]$$

$$\Rightarrow \frac{\hat{a}_\phi}{r^3} A_{zs}\cot\theta - \frac{\beta r\, \hat{a}_\phi \vec{A}_{zs}}{r^2}$$

~~On Comparing LHS & RHS of eq (1)~~

$$\nabla \times \vec{A} = \frac{1}{r^2\sin\theta}\, r\sin\theta \left[\frac{\partial}{\partial r}\left(-r\frac{\mu_0}{4\pi r} e^{-j\beta r} I_0\, dl \sin\theta\right) - \frac{d}{d\theta}\left(\frac{\mu_0}{4\pi r} e^{-j\beta r} I_0\, dl\cos\theta\right)\right]$$

$$= \mu_0 H_{\phi s}$$

$$\Rightarrow \frac{I_0 dl \sin\theta \, e^{-j\beta r}}{4\pi r}\left[j\beta + \frac{1}{r}\right] = H_{\phi s}$$

$$\therefore \boxed{H_{\phi s} = \frac{I_0 dl \sin\theta \, e^{-j\beta r}}{4\pi}\left[\frac{j\beta}{r} + \frac{1}{r^2}\right]}$$

$$\nabla \times \vec{H} = j\omega \epsilon_0 \vec{E}$$

$$\nabla \times \vec{H} = j\omega\epsilon_0 \left[E_{rs}\hat{a}_r + E_{\theta s}\hat{a}_\theta + E_{\phi s}\hat{a}_\phi\right]$$

$$\frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ H_{rs} & rH_{\theta s} & r\sin\theta\, H_{\phi s} \end{vmatrix}$$

$$= j\omega\epsilon_0\left[E_{rs}\hat{a}_r + E_{\theta s}\hat{a}_\theta + E_{\phi s}\hat{a}_\phi\right]$$

$$H_{rs} = H_{\theta s} = 0$$

$$\Rightarrow \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 0 & 0 & r\sin\theta\, H_{\phi s} \end{vmatrix} = j\omega\epsilon_0\left[E_{rs}\hat{a}_r + E_{\theta s}\hat{a}_\theta + E_{\phi s}\hat{a}_\phi\right]$$

$$\boxed{E_{\phi s} = 0}$$

Scanned by CamScanner

$$j\omega\epsilon_0 E_{rs} = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[r\sin\theta\,\frac{I_0\,dl\,e^{-j\beta r}\sin\theta}{4\pi}\left\{\frac{j\beta}{r}+\frac{1}{r^2}\right\}\right]$$

$$j\omega\epsilon_0 E_{rs} = \frac{1}{r^2\sin\theta}\left[\frac{r\,I_0\,dl}{4\pi}e^{-j\beta r}\left(\frac{j\beta}{r}+\frac{1}{r^2}\right)\left(\cos\theta\sin\theta+\cos\theta\sin\theta\right)\right]$$

$$j\omega\epsilon_0 E_{rs} = \frac{1}{r}\frac{I_0\,dl\,e^{-j\beta r}}{4\pi}\left(\frac{j\beta}{r}+\frac{1}{r^2}\right)2\cos\theta$$

$$E_{rs} = \frac{I_0\,dl\,e^{-j\beta r}\cos\theta}{2\pi\epsilon_0\,\omega}\left[\frac{\beta}{r}+\frac{j}{r^2}\right]$$

$$\boxed{E_{rs} = \frac{I_0\,dl\,e^{-j\beta r}\cos\theta}{2\pi\epsilon_0\,\omega}\left[\frac{j\beta}{r^2}+\frac{1}{r^3}\right]}$$

also,

$$j\omega\epsilon_0 E_{\theta s} = \frac{r}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\,\frac{r\sin\theta\,I_0\,dl\,\sin\theta\,e^{-j\beta r}}{4\pi}\left\{\frac{j\beta}{r}+\frac{1}{r^2}\right\}\right]$$

$$= \frac{1}{r^2\sin\theta}\,r\,\frac{\sin^2\theta\,I_0\,dl}{4\pi}\left[\frac{d}{dr}\left\{re^{-j\beta r}\left(\frac{j\beta}{r}+\frac{1}{r^2}\right)\right\}\right]$$

$$= \frac{I_0\,dl\sin\theta}{4\pi r}\frac{\partial}{\partial r}\left(j\beta e^{-j\beta r}+\frac{e^{-j\beta r}}{r}\right)$$

Scanned by CamScanner

$$= \frac{I_o dl \sin\theta}{4\pi r}\left[-(j\beta)^2 e^{-j\beta r} + e^{-j\beta r}\left(-\frac{1}{r^2}\right) - \frac{j\beta e^{-j\beta r}}{r}\right]$$

$$\Rightarrow -\frac{I_o dl \sin\theta}{4\pi r}\left[-\beta^2 e^{-j\beta r} + \frac{e^{-j\beta r}}{r^2} + \frac{j\beta e^{-j\beta r}}{r}\right]$$

$$\Rightarrow E_{\theta s} = \frac{I_o dl \sin\theta \, e^{-j\beta r}}{4\pi r j\omega\varepsilon_0}\left[-\beta^2 + \frac{1}{r^2} + \frac{j\beta}{r}\right]$$

$$\Rightarrow E_{\theta s} = \frac{I_o dl \sin\theta \, e^{-j\beta r} \beta}{4\pi r \omega\varepsilon_0}\left[-\frac{\beta}{j} + \frac{1}{j\beta r^2} + \frac{1}{r}\right]$$

$$E_{\theta s} = \frac{I_o dl \sin\theta \, e^{-j\beta r}}{4\pi r}\frac{\beta}{\omega\varepsilon_0}\left[j\beta - \frac{j}{\beta r^2} + \frac{1}{r}\right]$$

$$\Rightarrow \boxed{E_{\theta s} = \frac{I_o dl \sin\theta \, e^{-j\beta r}}{4\pi}\frac{\beta}{\omega\varepsilon_0}\left[\frac{j\beta}{r} - \frac{j}{\beta r^3} + \frac{1}{r^3}\right]}$$

$$\because \frac{\beta}{\omega\varepsilon} = \eta$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

Scanned by CamScanner

$\frac{1}{r}$ term is Called Radiation field or far field because it is the only term that remain at far zone. i.e at a point very far from the current element.

$\frac{1}{r^2}$ term is Called Induction field or near field

$\frac{1}{r^3}$ term is Called Electrostatic field

• The Distance at which Induction (near) field and radiation (far) field or far field and near field has equal amplitude is $\lambda/6$.

$$\therefore \frac{\beta}{r} = \frac{1}{r^2}$$

$$\Rightarrow r = \frac{1}{\beta}$$

$$r = \frac{\lambda}{2\pi}$$

$$\boxed{r \approx \frac{\lambda}{6}}$$

• The Boundary between the near & far Zone is defined by

$$\boxed{r = \frac{2d^2}{\lambda}}$$

$r > \frac{2d^2}{\lambda}$ far field

$r \leq \frac{2d^2}{\lambda}$ near field

d = largest Dimension of Antenna

Scanned by CamScanner

• At far field $\frac{1}{r^2}$ & $\frac{1}{r^3}$ can be neglected

So. ~~E_θ, E_r, E_ϕ~~

$E_{rs} = 0$

$$E_{\theta s} = \eta \frac{I_o dl \sin\theta \, e^{-j\beta r}}{4\pi r} (j\beta) \quad \hat{a_\theta}$$

$E_{\phi s} = 0$

$H_{rs} = 0$

$$H_{\phi s} = \frac{I_o dl \sin\theta \, e^{-j\beta r} (j\beta)}{4\pi r} \quad \hat{a_\phi}$$

$H_{\theta s} = 0$

when $\eta = \frac{E_{\theta s}}{H_{\phi s}}$

$$\vec{P_{avg}} = \frac{1}{2} \quad Re\left[\vec{E_{\theta s}} \times \vec{H_{\phi s}^{*}} \right]$$

where $= \frac{E_{\theta s}}{H_{\phi s}} = \eta$

$$\vec{P_{avg}} = \frac{1}{2} Re \left[E_{\theta s} \hat{a_\theta} \times \vec{H_{\phi s}^{*}} \right]$$

$$\frac{1}{2} Re \left[\eta H_{\phi s} \hat{a_\theta} \times H_{\phi s}^{*} \right]$$

$$P_{avg} \Rightarrow \frac{1}{2} Re \, |H_{\phi s}|^2 \; \hat{a_r}$$

$$P_{rad} = \int_s \vec{P_{avg}} \cdot \vec{ds}$$

$$P_{rad} = \int_s \frac{1}{2} \eta |H_{\phi s}|^2 \hat{a_r}\, r^2 \sin\theta\, d\theta\, d\phi\, \hat{a_r}$$

$$= \int_s \frac{1}{2} \eta |H_{\phi s}|^2 r^2 \sin\theta\, d\theta\, d\phi$$

$$\Rightarrow \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{1}{2} \eta \left(\frac{I_o\, dl \sin\theta}{4\pi r} \beta \right)^2 r^2 \sin\theta\, d\theta\, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi} \eta \frac{1}{2} \frac{(I_o\, dl)^2}{(4\pi)^2} \beta^2 \sin^3\theta\, d\theta\, \cancel{d\phi}$$

$$\Rightarrow \int_0^{2\pi} d\phi\, \eta \frac{1}{2} \frac{I_o dl}{(4\pi)^2} \beta^2 \left[\int_0^{\pi} \frac{3\sin\theta - \sin 3\theta}{4} \right] d\theta$$

$$\Rightarrow \int_0^{2\pi} d\phi\, \eta \frac{1}{2} \frac{I_o dl}{(4\pi)^2} \beta^2 \left(-\frac{3}{4} \cos\theta + \frac{\cos 3\theta}{12} \right)_0^{\pi}$$

$$\Rightarrow \left[\frac{2\pi \eta I_o dl}{2 (4\pi)^2} \beta^2 \right] \left[\frac{3}{4} - \frac{1}{12} + \frac{3}{4} - \frac{1}{12} \right]$$

$$\Downarrow$$

$$\frac{3}{2} - \frac{1}{6}$$

$$\frac{9 - 1}{6} = 4/3$$