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102 COMMUNICATION SYSTEM_GATEACADEMY

TOTAL PAGE COMMUNICATION SYSTEM-450 PGAE

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CONTENT COVERED:

1. Theory Notes
2. Explanation
3. Derivation
4. Example
5. Shortcut & Formula Summary
6. Previous year Paper Q. Sol.

Noted-: Single Source Follow, Revise

Multiple Time Best key of Success

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Communication

1. Random Process
2. Digital Communication
   - Baseband
   - Band pass
3. Analog communication
   - AM
   - PM, FM
   - Noise
4. Information theory

Unit-1 Random Process

Random Variable :- Random Variable is a function or rule by which we assign real number to each elements of Sample space. It is a mapping from Sample space to real axis.

eg1 Tossing a Coin twice

$$S = \{ \underset{\lambda_1}{HH} \quad \underset{\lambda_2}{TH} \quad \underset{\lambda_3}{HT} \quad \underset{\lambda_4}{TT} \}$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

Mapping of S in real axis

$$X = \{ \underset{x_1}{1} \quad \underset{x_2}{2} \quad \underset{x_3}{3} \quad \underset{x_4}{4} \}$$

$$P(x=3) = \frac{1}{4} = P(x=2) = P(x=1) = P(x=4)$$

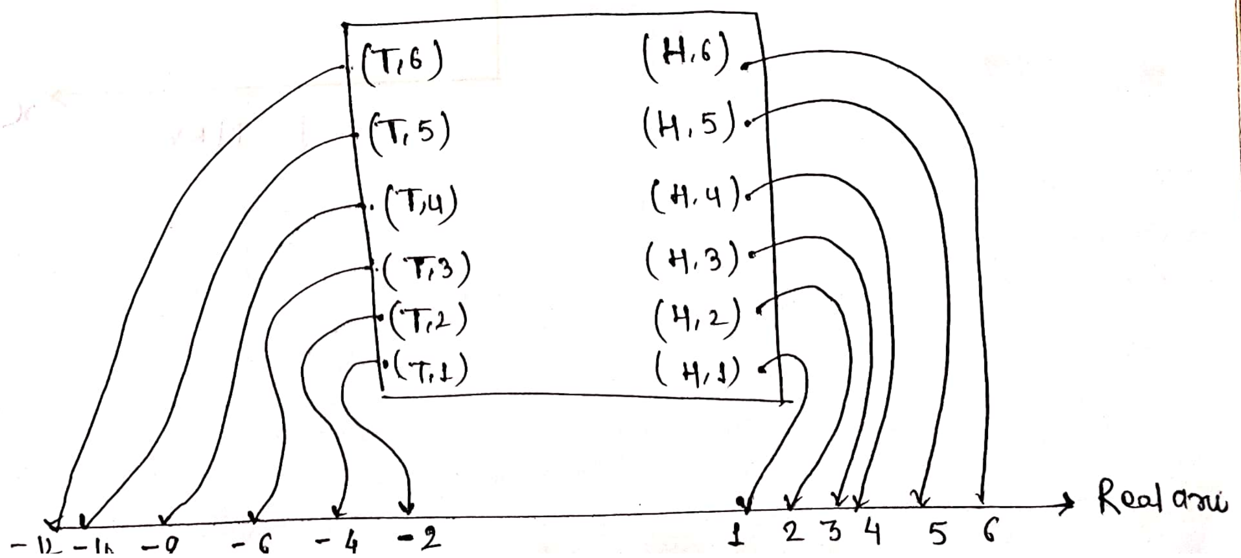
eg2. An experiment Consist of Rolling a dice and flipping a Coin - let the random variable be a function X such that

1) a Coin with head outcome Corresponds to a positive Value of X that are equal to the number that shows up on the dice.

11) a Coin with tail outcome Corresponds to negative Value of X that are equal in Magnitude to twice the number that showsup on the dice. sketch Mapping to real axis

Sol $S = \{ (1H) (1T) (2H)(2T) (3H)(3T) (4H) (4T) (5H)(5T) (6H)(6T) \}$

1)



* Types of Random Variables

1. Continuous Random Variable :- A RV is Said to be Continuous type if its distribution function is Continuous everywhere and Smooth that it can be represented by Integral of Some non-negative function.

2. Discrete Random Variable :- A Random Variable is Said to be discrete type if it has Continuous StairCase distribution function except for finite number of jumped discontinuities.

3. Mixed type RV :- A Random Variable of Mixed type is a random Variable with a distribution function that has jumps on Countable Set of point but also Increases Continuous over atleast one Interval of values of x.

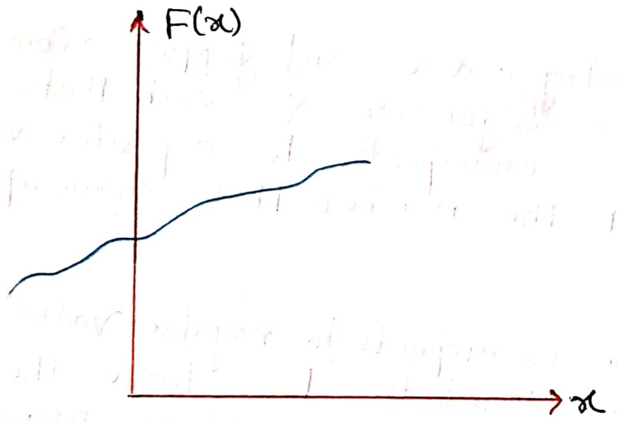


fig. CRV

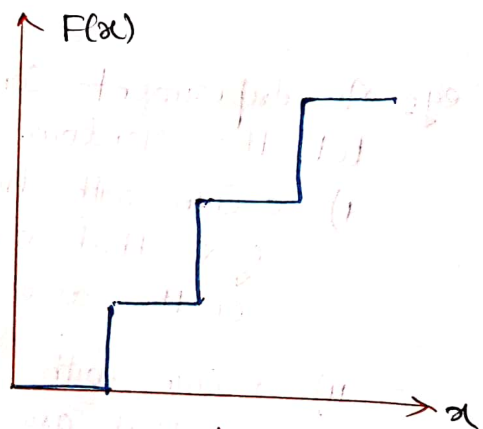


fig. DRV

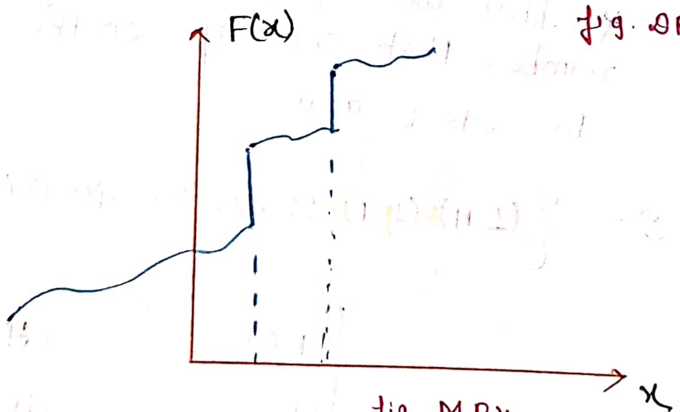


fig. MRV

* Analysis of RV :-

1. Cumulative Distribution function (CDF) :- CDF of Random Variable X is defined as -

$$\boxed{F_X(x) = P[X \leq x]}$$

e.g. $S = \{HH \quad HT \quad TH \quad TT\}$

$X = \{1 \quad 2 \quad 3 \quad 4\}$

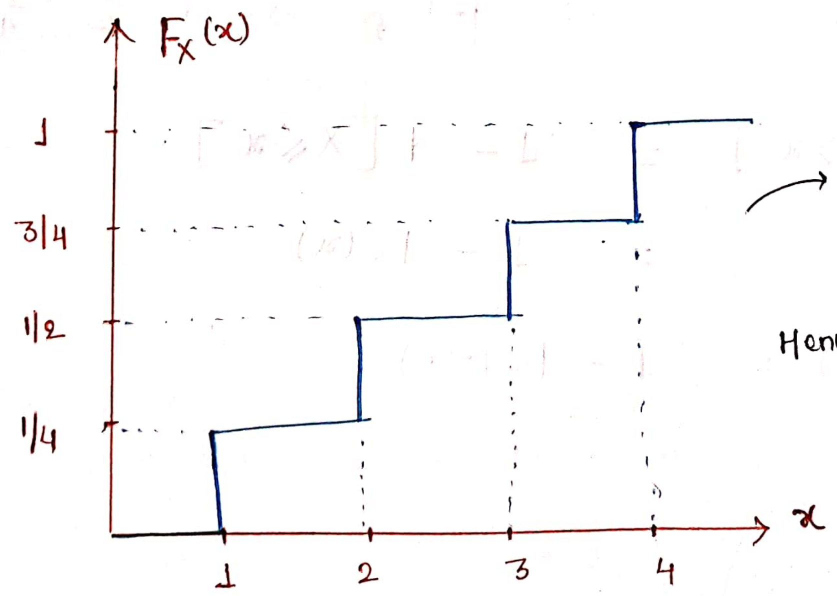
$P(x) = \{\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4}\}$

$$F_X(1) = P[X \leq 1] = P[X<1] + P[X=1]$$
$$= 0 + \frac{1}{4} = \frac{1}{4}$$

$$F_X(2) = P[X \leq 2] = P[X<2] + P[X=2]$$
$$= P[X=1] + P[X=2]$$
$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$F_X(3) = P[X \leq 3] = P[X=1] + P[X=2] + P[X=3]$$
$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$$F_X(4) = P[X \leq 4] = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$$



→ example of discrete Random Var.

Hence DRV must have finite RV

$$F_X(x) = \frac{1}{4}u(x-1) + \frac{1}{4}u(x-2) + \frac{1}{4}u(x-3) + \frac{1}{4}u(x-4)$$

In General for DRV

$$\boxed{F_X(x) = \sum_{i=1}^{4} P(x_i)\, u(x - x_i)}$$

* Properties of CDF :-

1. $F_X(\infty) = 1$

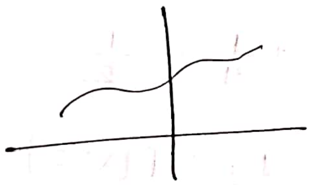
2. $F_X(-\infty) = 0$

3. $0 \leq F_X(x) \leq 1$

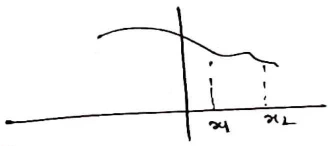
4. $P[x_1 < X \leq x_2] = F_X(x_2) - F_X(x_1)$

5. If $x_2 > x_1$ then, $F_X(x_2) > F_X(x_1)$

Non decreasing function (Increasing fun^n)

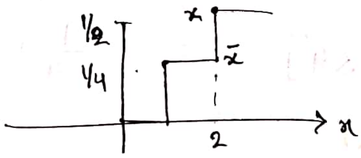


Valid cdf



Invalid CDF

6. $P[X = x] = F_X(x) - F_X(x^-)$ only valid at dis cont. point



$F_X(x) - F_X(x^-)$

$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

7. $P[X > x] = 1 - P[X \leq x]$

$= 1 - F_X(x)$

8. $F_X(x) = 1 - F_X(-x)$

Note:- Property 1, 2 & 5 Must be satisfied for a function to be Valid distribution function.

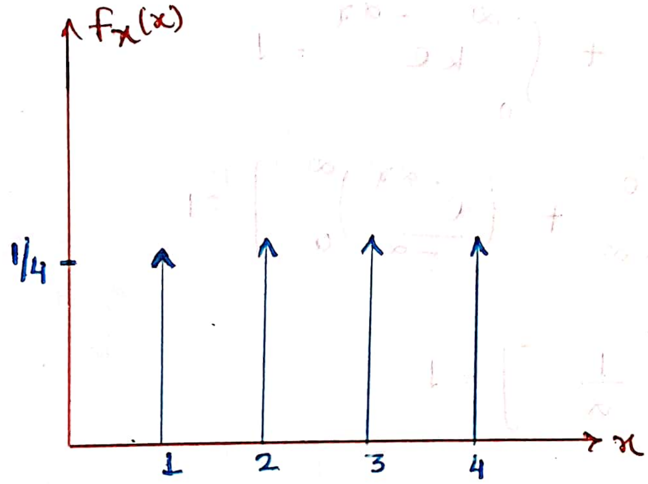
2. Probablity Density function :- A Pdf for Random Variable X is defined as -

$$\boxed{f_X(x) = \frac{d F_X(x)}{dx}}$$

for eg. $S = \{ HH \quad HT \quad TH \quad TT \}$

$X = \{ 1 \quad 2 \quad 3 \quad 4 \}$

$P(x) = \{ \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \}$



$$f_X(x) = \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-3) + \frac{1}{4}\delta(x-4)$$

$$\boxed{f_X(x) = \sum_{i=1}^{N} P(x_i)\, \delta(x - x_i)}$$

Properties of PDF :-

1) $f_X(x) \geq 0$

2) Area Under PDF always equal to 1.

$$\boxed{\int_{-\infty}^{\infty} f_X(x)\, dx = 1}$$

3) $P[x_1 < X \leq x_2] = \int_{x_1}^{x_2} f_X(x)\,dx$

4) $F_X(x) = \int_{-\infty}^{x} f_X(\epsilon)\,d\epsilon$

Que1. A PDF is of the form $P(x) = ke^{-\alpha|x|}$ where $x\,(-\infty, \infty)$ The Value of k is

Sol $\int_{-\infty}^{\infty} ke^{-\alpha|x|}\,dx = 1$

$\Rightarrow \int_{-\infty}^{0} ke^{+\alpha x} + \int_{0}^{\infty} ke^{-\alpha x} = 1$

$\Rightarrow k\left[\left(\frac{e^{\alpha x}}{\alpha}\right)_{-\infty}^{0} + \left(\frac{e^{-\alpha x}}{-\alpha}\right)_{0}^{\infty}\right] = 1$

$\Rightarrow k\left[\frac{1}{\alpha} + \frac{1}{\alpha}\right] = 1$

$k = \frac{\alpha}{2}$

Que2 $P(x) = Me^{-2|x|} + Ne^{-3|x|}$ is PDF for real Random Variable X Over the entire axis M and N are both positive real numbers the eqn relating M & N is

Sol $\frac{M}{1} + \frac{2N}{3} = 1$

By property 2)

$= M + \frac{2N}{3} = 1$

Que 3. A PDF is given by $P(x) = k e^{-x^2/2}$ $x(-\infty, \infty)$
The value of k is

Sol
$$\int_{-\infty}^{\infty} k e^{-x^2/2} = 1$$

$$\Rightarrow \quad 2 \int_{\infty}^{\infty} k e^{-x^2/2} dx = 1$$

$$\frac{x^2}{2} = t$$

$$\frac{2x}{2} = \frac{dt}{dx}$$

$$x dx = dt$$

$$dx = \frac{dt}{x} = \frac{dt}{\sqrt{2t}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} \frac{k e^{-t}}{\sqrt{t}} dt = 1$$

$$\left\{ \begin{array}{l} \Gamma n = \int_0^{\infty} e^{-t} t^{n-1} dt \\ n-1 = -1/2 \\ n = 1/2 \\ \Gamma 1/2 = \sqrt{\pi} \end{array} \right.$$

$$\Rightarrow \frac{k}{\sqrt{2}} \, 2 \int_{+0}^{\infty} e^{-t} t^{-1/2} = 1$$

$$\Rightarrow \frac{k}{\sqrt{2}} \, 2 \, \Gamma 1/2 = 1$$

$$\Rightarrow k = \frac{1}{\sqrt{2\pi}} \quad //.$$

Que. Which one of the following is a valid distribution function

1) $F_X(x) = \begin{cases} 1 - e^{-x/2} & x \geq 0 \\ 0 & \text{otherw.} \end{cases}$

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DIGITAL COMMUNICATION

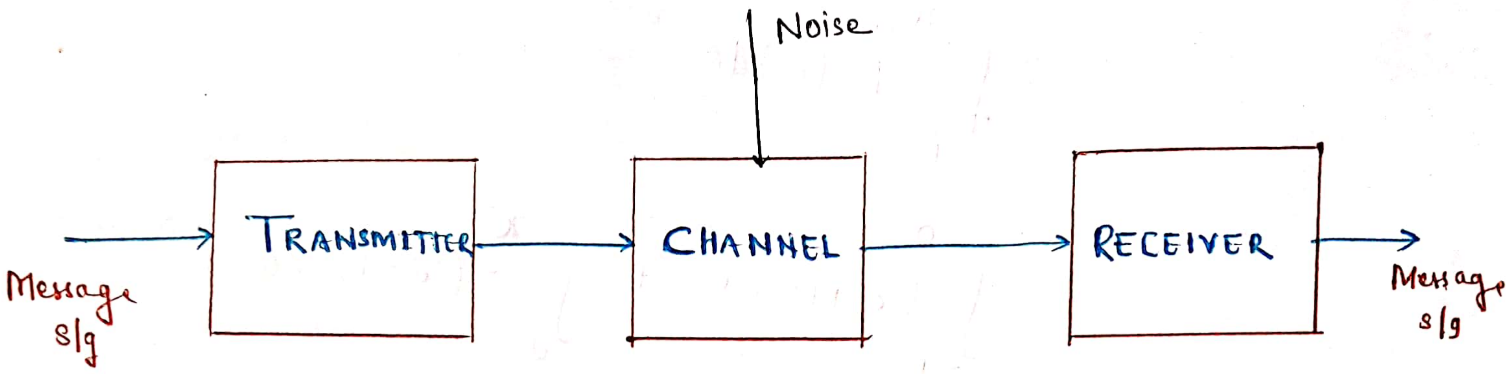
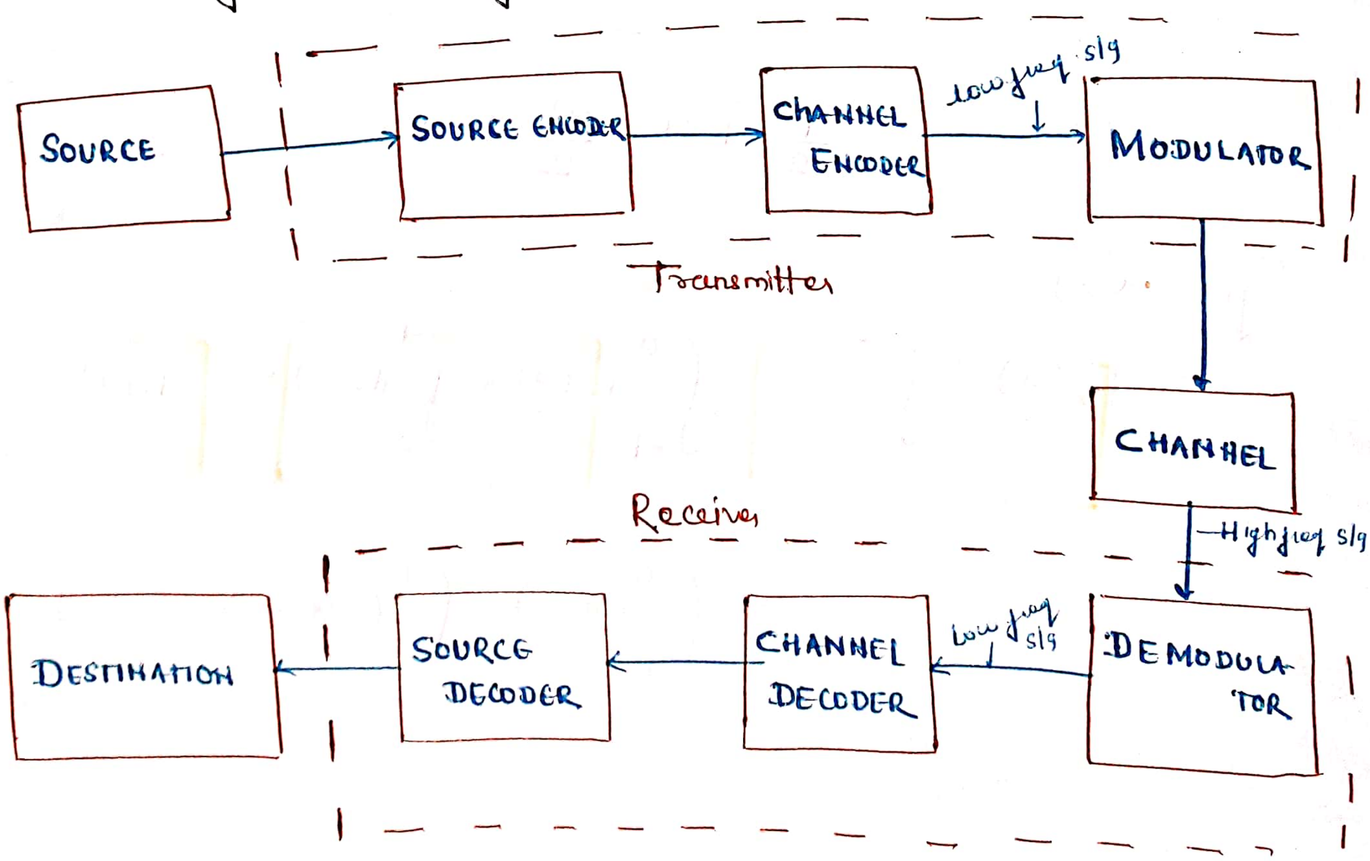


fig. Block Diagram of Commⁿ system

- Voice Signal [300 -3300 Hz]
- Audio Signal [20 Hz to 20kHz]
- Video Signal [upto 5MHz]

* Block Diagram of Digital Communication System :-



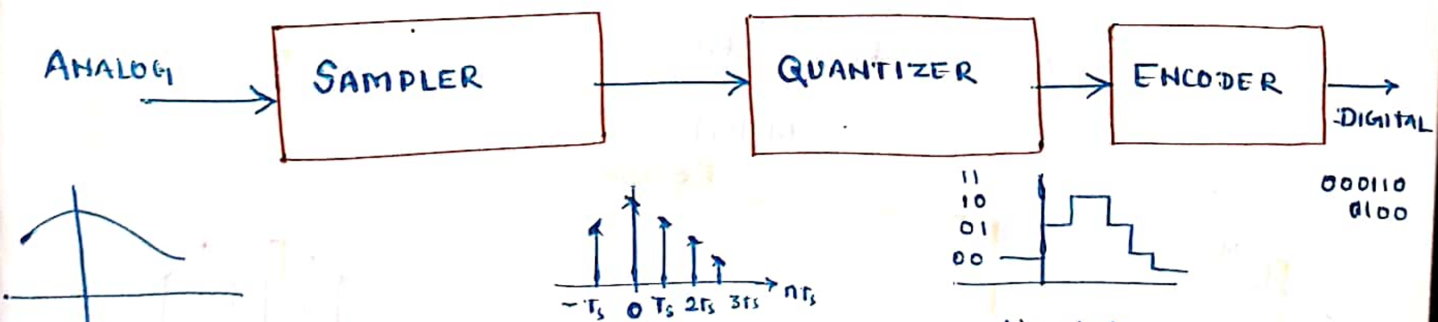
* Advantages of Digital $Comm^n$ System :-

1) Effect of Noise is Less
2) More reliable Communication because of repeaters.
3) Storage is easy & Inexpensive.
4) Hardware Implementation is easy.
5) Error detection & Correction is possible.
6) Error rate Canbe reduced by Some Coding technique.
7) Multiplexing is more efficient & easy.

* Disadvantages of digital $Comm^n$:-

1) Increased Bandwidth requirement.
2) Quantization Error
3) Synchronization is needed.

* Analog to Digital Converter :-



Cont. time
Cont. Ampl.

Cont. Ampl
Discrete Time

discrete Amp.
Discrete Time
(Amplitude Can be fractional Value but Time must be Integer only)

Baseband Transmission

I. Sampling theorem

$$m(t) \xleftrightarrow{FT} M(f)$$

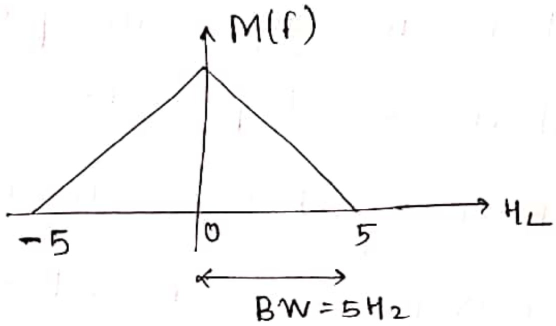
Band limited Signal :-

Max^m freq. is fixed

$BW = f_H - f_L$

$= 5 - 0$

$= 5\ Hz$



OR $BW = 0 - (-5)$

$= 5\ Hz$

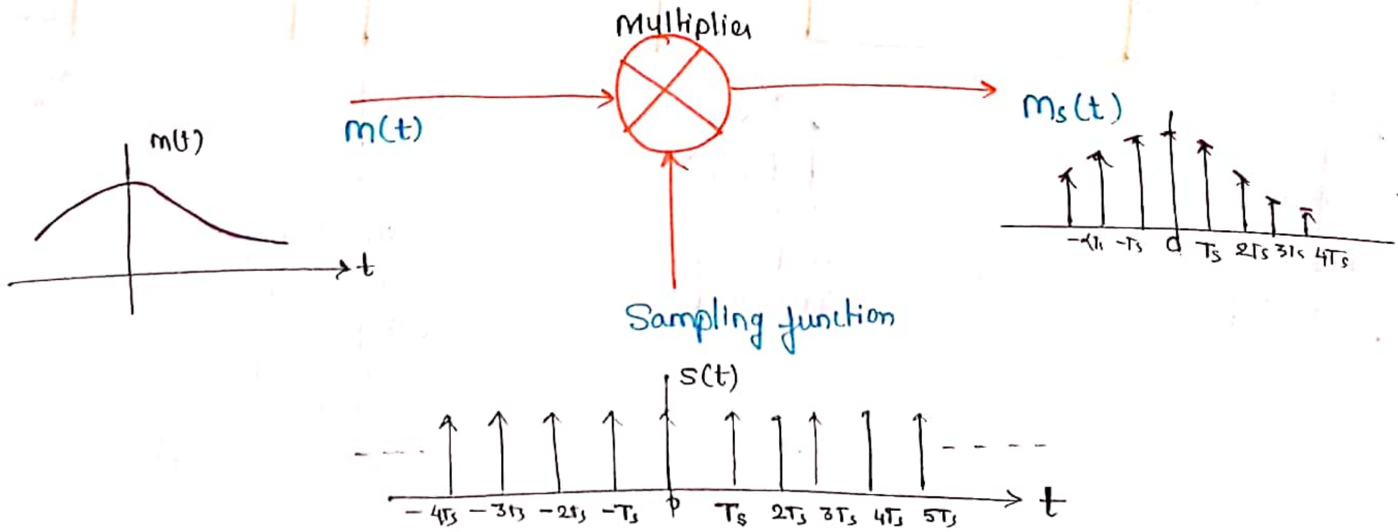
· Bandwidth is Calculated only for one sided S/g.

Theorem :-

→ A Signal which is Bandlimited to f_m Hz i.e it has no frequency component higher than f_m Hz can be Completely recovered from its sample if the samples are taken at the Interval of

$$T_s \leq \frac{1}{2f_m}$$

Or $$f_s \geq 2f_m$$



$$m_s(t) = m(t) \cdot s(t)$$

$$= m(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$\boxed{m_s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t - nT_s)}$$

any $m(t)$ can be made discrete by

$$\boxed{m(n) = m(t)\Big|_{t = nT_s}}$$

fourier Series of periodic s/g $s(t)$

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_s t + b_n \sin n\omega_s t \right)$$

$$a_0 = \frac{2}{T_s} \int_{-T_s/2}^{T_s/2} s(t)\, dt$$

$$= \frac{2}{\cancel{T_s}} \left(\frac{\cancel{T_s}}{\cancel{2}} + \frac{\cancel{T_s}}{\cancel{2}} \right) \qquad \frac{2}{T_s} \times 1$$

$$a_0 = \frac{2}{T_s}$$

$$a_n = \frac{2}{T_s} \int_{-T_s/2}^{T_s/2} s(t) \cos n\omega_s t\, dt$$

$$= \frac{2}{T_s}$$

$$\boxed{s(t) = \frac{1}{T_s} + \sum_{n=1}^{\infty} \frac{2}{T_s} \cos n\omega_s t}$$

$$m_s(t) = \frac{m(t)}{T_s} + \sum_{n=1}^{\infty} \frac{2}{T_s} m(t) \cos n\omega_s t$$

$$= \frac{m(t)}{T_s} + \frac{2}{T_s}\left[m(t)\cos\omega_s t + m(t)\cos 2\omega_s t + \cdots \right]$$

If $m(t) \xleftrightarrow{FT} M(f)$

So, $$M_s(f) = \frac{M(f)}{T_s} + \frac{2}{2T_s}\left[(M(f-f_s) + M(f+f_s)) + [M(f-2f_s) + M(f+2f_s)] + \cdots \right]$$

$$\Rightarrow M_s(f) = \frac{M(f)}{T_s} + \frac{1}{T_s}\left[M(f-f_s) + M(f-2f_s) + M(f-3f_s) + \cdots + M(f+f_s) + M(f+2f_s) + \cdots \right]$$

$$\Rightarrow M_s(f) = \frac{M(f)}{T_s} + \frac{M(f-f_s)}{T_s} + \frac{M(f+f_s)}{T_s} + \frac{M(f-2f_s)}{T_s} + \frac{M(f+2f_s)}{T_s} + \cdots$$

$$\Rightarrow M_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} M(f-nf_s)$$

$$\Rightarrow \boxed{M_s(f) = f_s \sum_{n=-\infty}^{\infty} M(f-nf_s)}$$

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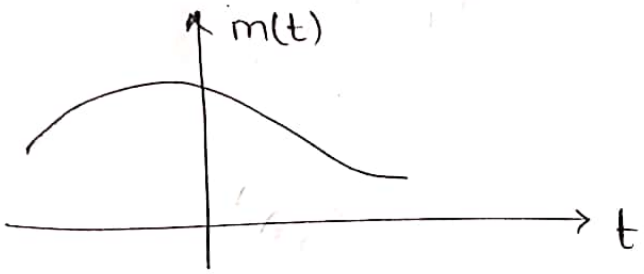
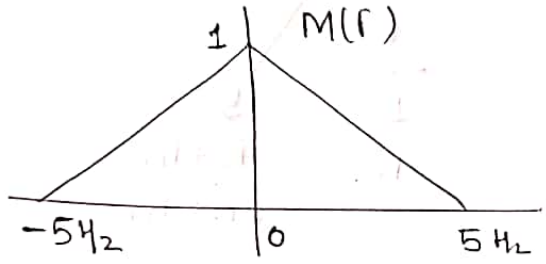
$m(t) \xleftrightarrow{FT} M(f)$



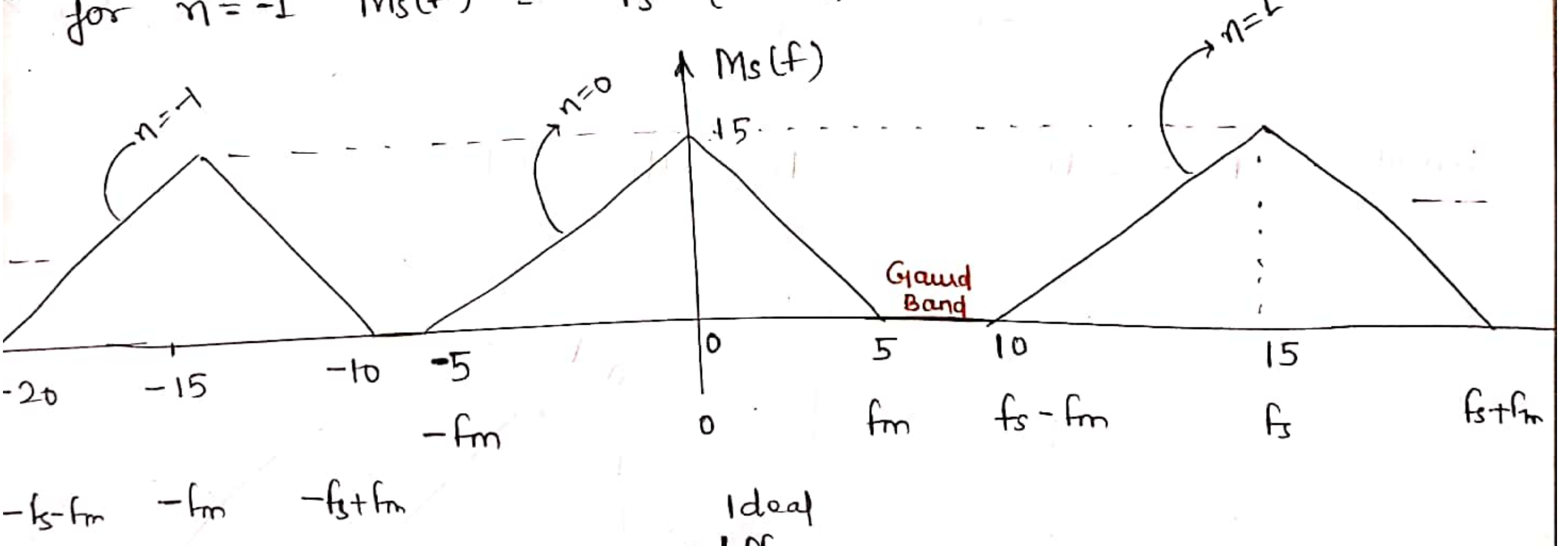
let $f_m = 5\,Hz$ and $f_s = 15\,Hz$

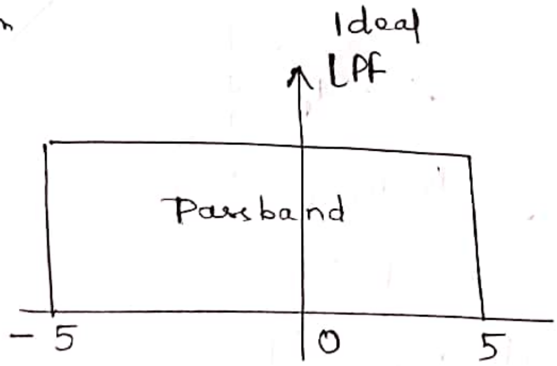


for $n=0$ $M_s(f) = f_s M(f) = 15 M(f)$

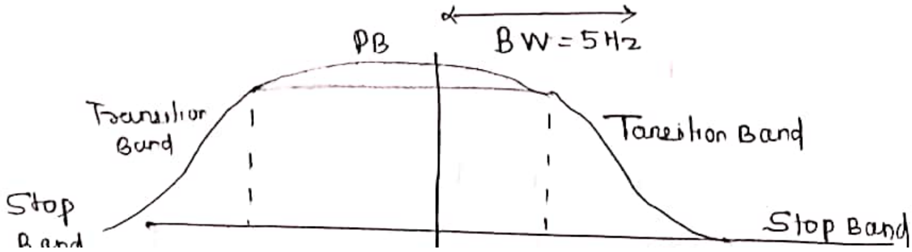
for $n=1$ $M_s(f) = f_s M(f-f_s) = 15 M(f-15)$

for $n=-1$ $M_s(f) = f_s M(f+f_s) = 15 M(f+15)$





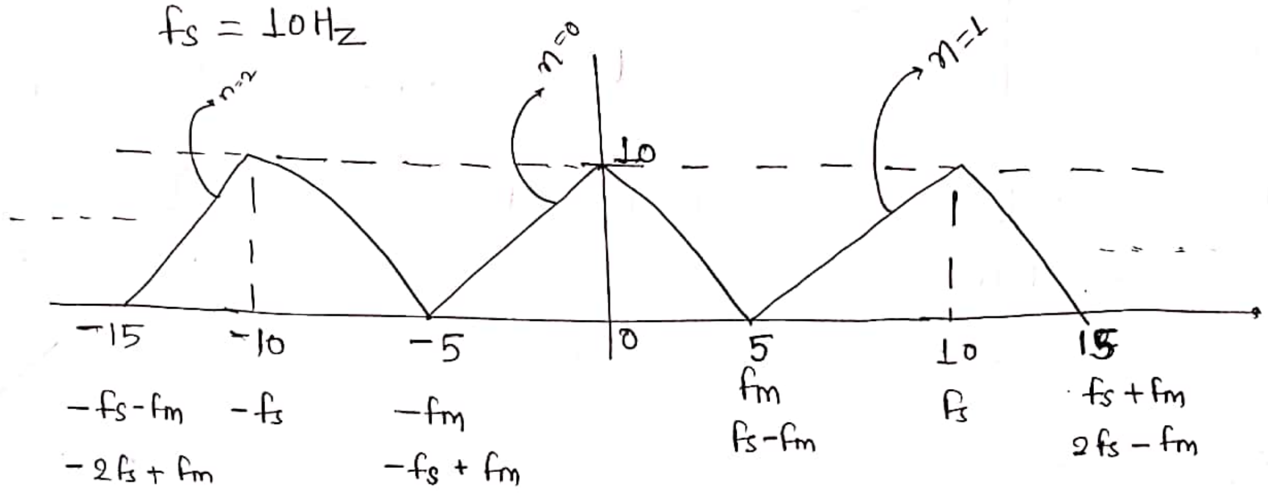
LPF having cut-off freq. 5Hz or having Bandwidth 5Hz (both Same)

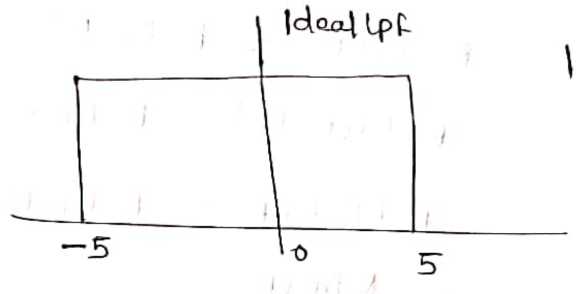


$\boxed{\text{Guard Band} = f_s - 2f_m}$ is used to separate the replica of the slg

Case II $\quad f_s = 2f_m$

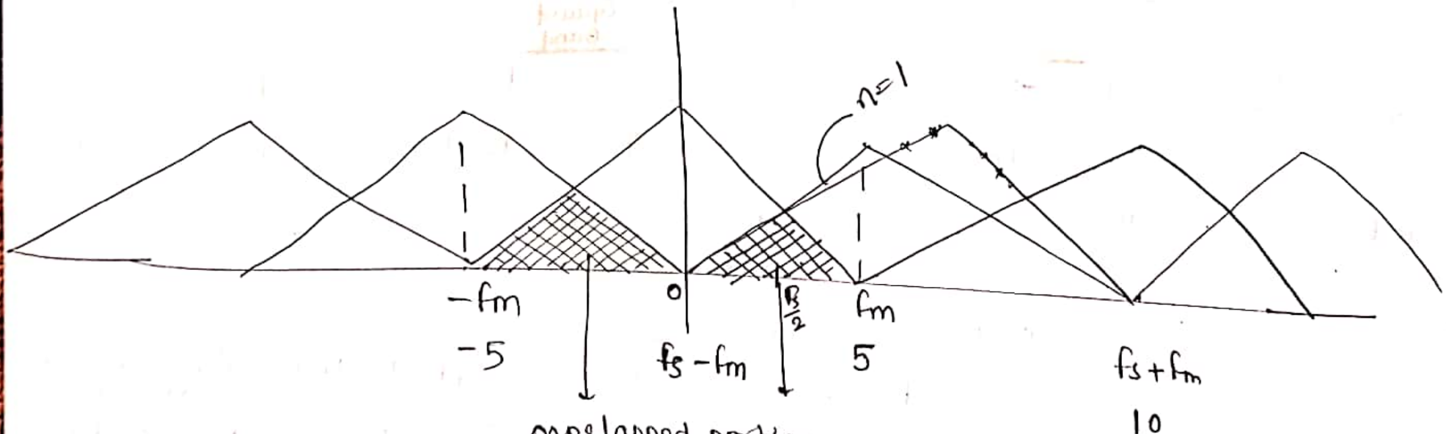
$f_s = 10\,Hz$





Ideal LPF is Must to Reconstruct msg slg In Case $f_s = 2f_m$

Case III $\quad f_s < 2f_m \qquad f_s = 5\,Hz$ lets



overlapped portion
or
Aliasing effect
or
foldover error
or
Spectral folding

No filter can recover original msg slg in this case.

The frequency at which high frequency Component of msg s/g fold itself is called folding frequency

$$\boxed{\text{folding frequency} = \frac{f_s}{2}}$$

Nyquist Rate :- $f_s = 2 f_m$

- The frequency Components present in Spectrum of Sampled s/g

f_m, $f_s \pm f_m$, $2f_s \pm f_m$, $3f_s \pm f_m$ ------

Gate'13

Que. A Band Limited Signal with a Maximum frequency of 5 kHz is to be Sampled. According to Sampling theorem the Sampling frequency which is Invalid is -

i) 12 kHz ii) 15 kHz iii) 5 kHz iv) 20 kHz

Sol $f_s \geq 2 f_m$ Valid

$f_s \geq 2 \times 5$

10 kHz

option c

Que. A Signal $x(t) = 100 \cos(24\pi \times 10^3 t)$ is Ideally Sampled with Sampling Interval of 50 μs and then passed through an Ideal LPF with Cut off freq of 15 kHz which of the following freq are present at filter o/p

Sol $T_s = 50 \mu s$

$$\frac{1}{50 \times 10^{-6}} = 0.2 \times 10^5$$

$$= 20 \text{ kHz}$$

$f_s = 20$ kHz

$f_m = 12$ kHz

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Component present

$f_m, \; f_s \pm f_m, \; 2f_s \pm f_m \; \ldots$ } All right sided

12 , 20 ± 12 , 40 ± 12

$\Rightarrow$ 12 , 32 & 8 , 52 & 28

LPF cut-off = 15 kHz

Hence 8 kHz & 12 kHz will pass

Que. An audio sig is Band Limited to 4 kHz. It is Sampled at 8 kHz what will be the spectrum of sampled sig.

a) −4 kHz to 4 kHz

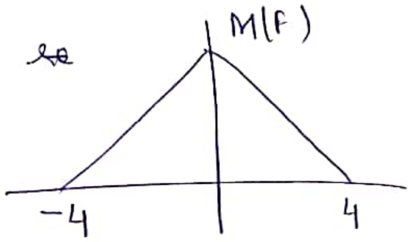
b) −8 kHz to 8 kHz

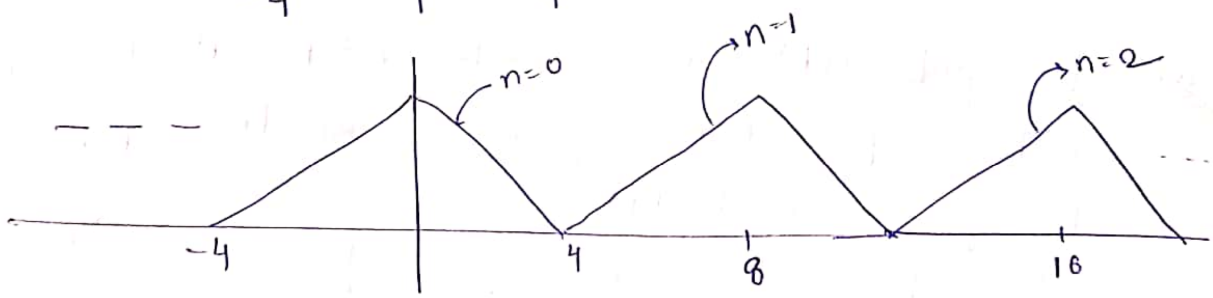
c) every 4n kHz and repeating

d) every 8n kHz & repeating as well as at 0

Sol: Spectrum repeat itself at 8n as well as 0

~~frequency~~ ~~is~~





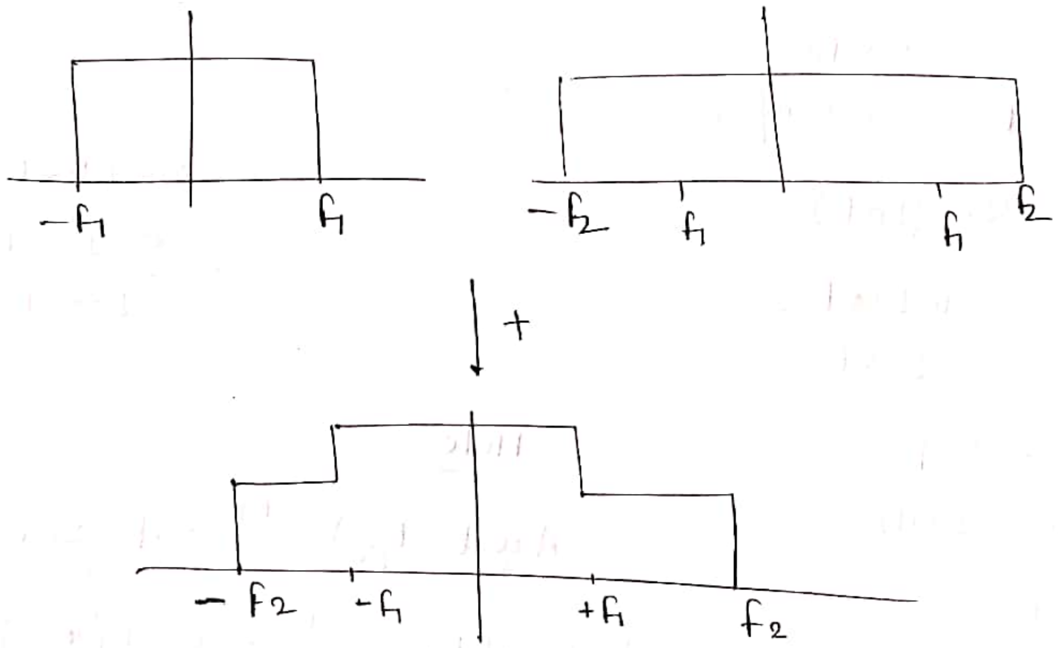
* Sampling frequency for Multitone Signal :-

$$m(t) = \underset{f_1}{m_1(t)} + \underset{f_2}{m_2(t)} + \underset{f_3}{m_3(t)} + \underset{f_4}{m_4(t)} \;---\; \underset{f_N}{m_N(t)}$$

for $N=2$

$$m(t) = \underset{f_1}{m_1(t)} + \underset{f_2}{m_2(t)}$$

$$M(f) = M_1(f) + M_2(f)$$



$$f_s = 2 f_m$$

$$f_s = 2 f_2$$

$$\boxed{f_s = 2 \max[f_1, f_2, f_3 \;----\; f_N]}$$

- Frequency Component present in Sampled s/g of Multitone s/g

$f_1, f_2, f_3 \;---\; f_N$

$f_s \pm f_1 \quad f_s \pm f_2 \;----\; f_s \pm f_N$

$2f_s \pm f_1 \quad 2f_s \pm f_2 \;---\; 2f_s \pm f_N$

$3f_s \pm f_1 \quad 3f_s \pm f_2 \;---\; 3f_s \pm f_N$

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Que 1 $x(t) = \sin(10\pi t)$

Sol:

$$f_m = 5\,Hz$$

$$f_s = 2f_m = 10\,Hz = 10\text{ sample/sec}$$

note: Unit of Samling frequency :- f_s Sample/sec

Que 2 $x(t) = \sin(10\pi t) + \cos(20\pi t)$

Sol

$$f_{m_1} = 5\,Hz \qquad f_{m_2} = 10\,Hz$$

$$f_s = 2\max[f_1, f_2]$$

$$= 2 \times 10$$

$$f_s = 20\text{ sam/sec}$$

Que 3 $x(t) = \text{Sinc}(10t)$

$$= \frac{\sin 10\pi t}{10\pi t}$$

$$\left\{ \begin{array}{l} \text{Sinc}(10\pi t) \\ \Rightarrow \dfrac{\sin 10\pi^2 t}{10\pi^2 t} \end{array} \right.$$

$$f_m = 5\,Hz$$

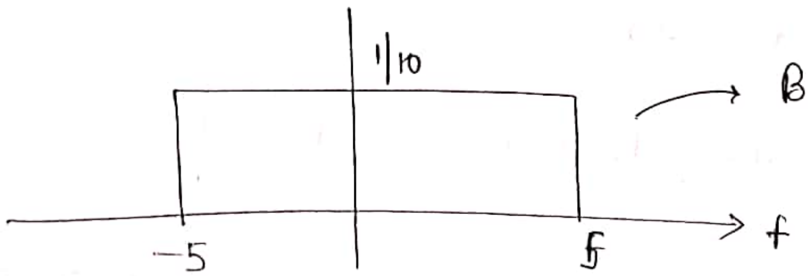
$$\Rightarrow f_s = 10\,Hz$$

note

$$A\,\text{rect}(t/\tau) \xleftrightarrow{FT} A\tau\,\text{Sinc}(f\tau)$$

$$A\tau\,\text{Sinc}(t\tau) \xleftrightarrow{FT} A\,\text{rect}(f/\tau)$$

$$\text{Sinc}(10t) \xleftrightarrow{FT} \frac{1}{10}\text{rect}(f/10)$$



Bandlimited to 5 Hz

Que4. $x(t) = \text{Sinc}^2(10t)$

$\Rightarrow$ $x(t) = \left(\frac{\sin 10\pi t}{10\pi t}\right)^2$

$= \left(\frac{1 - \cos 20\pi t}{2(10\pi t)^2}\right)^{\cancel{2}}$

$= \frac{\overset{\text{I}}{1} - \overset{\text{II}}{\cos 20\pi t}}{200\pi^2 t^2} = \frac{F}{\cancel{200}}$

I. $f_1 = 0$ DC

II $f_2 = 10\text{ Hz}$

$f_s = 2 \times 10 = 20\text{ Hz}$ or 20 sample/sec

OR

$x(t) = x_1(t) \cdot x_2(t)$

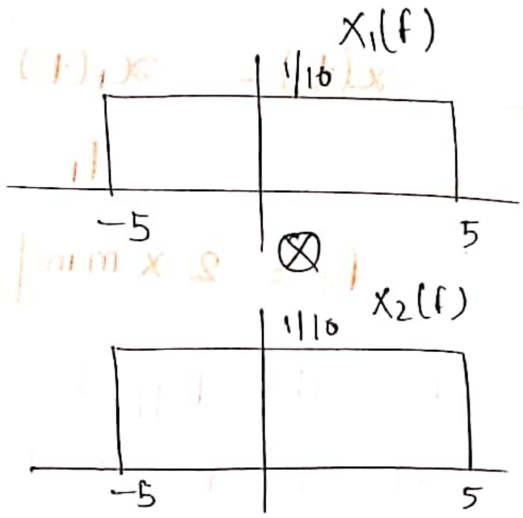
$= \text{Sinc } 10t \cdot \text{Sinc } 10t$

$x(t) \xleftrightarrow{FT} X_1(f) \otimes X_2(f)$

$x_1(t) \longleftrightarrow X_1(f)$

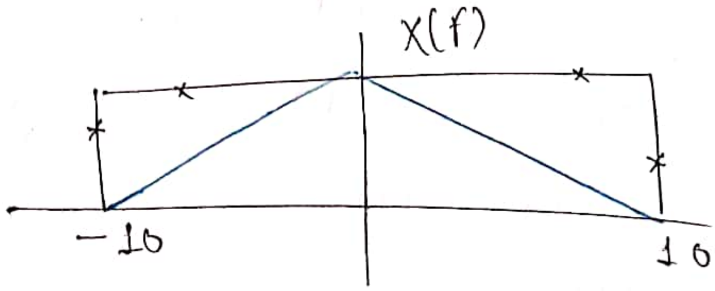
$X_1(f) = \frac{1}{10}\text{rect}\left(\frac{f}{10}\right)$

$X_2(f) = \frac{1}{10}\text{rect}(f/10)$



$f_m = 10\text{Hz}$

$f_s = 20\text{ S/sec}$



Information Theory

- Information theory is quantitative measure of Information contained in message signal and allow us to determine capacity of communication to xfer this Information from source to destination.

- Discrete Memoryless Source :- It can be characterised by ~~least~~ list of symbols the probablity assignment to these symbol and speciyication of rate of generating these symbols by the source.

1) Measure of Information :- Information contained in any message is inversely probablity of occuriance of that message

$$I \propto \frac{1}{P}$$

Consider a Discrete Memoryless Source (DMS) X

$$X = \{ x_1 \; x_2 \; x_3 \; ---- \; x_m \}$$

probablities $P(X) = \{ P(x_1) \; P(x_2) \; P(x_3) \; --- \; P(x_m) \}$

then Information in any symbol

$$I(x_i) \propto \frac{1}{P(x_i)}$$

$$\boxed{I(x_i) = \log_b \left[\frac{1}{P(x_i)} \right]}$$

If $b = 2$ then $I(x_i)$ is in bits

If $b = e$ then $I(x_i)$ is in nat

If $b = 10$ then $I(x_i)$ is in decit or Hartley

Properties of Measure of Information:-

1) If $P(x_i) = 1$ then $I(x_i) = 0$

2) $0 \leqslant P(x_i) \leqslant 1$ then $I(x_i) \geqslant 0$

3) If $P(x_i) > P(x_j)$ then $I(x_j) > I(x_i)$

4) If x_i & x_j are Independent

$$I(x_i x_j) = I(x_i) + I(x_j)$$

Proof:-

$$I(x_i, x_j) = \log_2 \left[\frac{1}{P(x_i x_j)} \right]$$

as x_i & x_j are Independent

Hence, $P(x_i x_j) = P(x_i)\, P(x_j)$

$$I(x_i x_j) = \log_2 \left[\frac{1}{P(x_i)\, P(x_j)} \right]$$

$$= \log_2 \left[\frac{1}{P(x_i)} \right] + \log_2 \left[\frac{1}{P(x_j)} \right]$$

$$I(x_i x_j) = I(x_i) + I(x_j)$$

* Average Information or Entropy :-

- In practical Communication System we xmit long Sequence of Symbol from an Information Source. Thus we are more Interested in average Information produced by the Source.

Average Value i.e Expected Value

$$E[I(x_i)] = \sum_{i=1}^{m} I(x_i) \cdot P(x_i)$$

or

$$\boxed{E[I(x_i)] = \sum_{i=1}^{m} P(x_i) \log \frac{1}{P(x_i)}}$$ bit/symbol

or

$H(X)$

or bit/message or bit/alphabet (a/q to question)

- If we have

$$X = \{x_1\ x_2\ ---\ x_m)$$

$$P(X) = \{P(x_1),\ P(x_2)\ -----\ P(x_m)\}$$

where, $P(x_1) + P(x_2) + --- P(x_m) = 1$

If all the Symbols are equiprobable

$$E[I(x_i)] = H(X) = \sum_{i=1}^{m} P(x_i) \log \frac{1}{P(x_i)}$$

m = no. of symbol generated by Source

$$\Rightarrow \frac{1}{m} \log m + \frac{1}{m} \log m \text{ ----- m times}$$

$$\Rightarrow \frac{m}{m} (\log m\ \cancel{+ \log m}\ --\ \cancel{m\ time})$$

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$$\boxed{H(X) = \log_2 m}$$

Range of Entropy :-

$$\boxed{0 \leq H(X) \leq \log_2 m}$$

Information Rate :- If the rate at which Source x emit Symbol is 'r' Symbol/sec. The Information rate 'R' is given by -

$$R = \frac{\text{Symbol}}{\text{sec}} \times \frac{\text{bit}}{\text{Symbol}} = \frac{\text{bit}}{\text{sec}}$$

$$\boxed{R = r H(X)} \text{ bit/sec}$$

- Extension of a Source :- Consider blocks rather than Individual Symbol with each block Consisting ~~Soo~~ 'n' Successive Source Symbol is Called extension of a Source.

$$X = \{ x_1, x_2, \ldots\ldots x_m \} \quad ; H(X)$$

$$X^n = m^n \quad \text{where } n = \text{order of extension}$$

$$\boxed{H(X^n) = n H(X)}$$

WANT ECE BEST QUALITY-2019-20

LATEST HANDWRITING NOTES

BY

GATE ACADEMY CLICK HERE

Noted-: Above ECE GATE/IES GATEACADEMY 2019-20 CLASSROOM BEST QUALITY Handwriting Notes Unique and Good Handwriting. Above Notes Enough for your Preparation..................

Noted-: Dear Aspirants If you do practice previous year paper 50% your work finished.so Guys daily at least 30minutes give previous year.

1. Previous year paper 4-5 times practice before final exam.
2. Subject wise study reference STD book
3. Test series practice more n more (Try to latest test series 2-3 fully solve then join online test series.)

Noted-: Single Source Follow, Revise Multiple Time Best key of Success

eg. $X = \{ x_1 \quad x_2 \quad x_3 \}$

$P(X) = \{ \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{2} \}$

$H(X) = \frac{1}{4} \log 4 + \frac{1}{4} \log 4 + \frac{1}{2} \log 2$ $\quad \{ \because \log_2 2 = 1$

$\Rightarrow \frac{1}{4} \times 2 + \frac{2}{4} + \frac{1}{2}$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{3}{2}$ bits/symbol

now let Second order extention

$X^2 = \{ x_1x_1, x_1x_2, x_1x_3, x_2x_1, x_2x_2, x_2x_3, x_3x_1, x_3x_2 \; x_3x_3 \}$

$m^n = 3^2$ where, m is no. of Symbol

$= 9$ elements

$P(X^2) = \{ \frac{1}{4} \times \frac{1}{4}, \frac{1}{4} \times \frac{1}{4}, \frac{1}{4} \times \frac{1}{2}, \frac{1}{4} \times \frac{1}{4}, \frac{1}{4} \times \frac{1}{2}, \frac{1}{4} \times \frac{1}{4}, \frac{1}{4} \times \frac{1}{2}, \frac{1}{2} \times \frac{1}{4}, \frac{1}{4} \times \frac{1}{2}, \frac{1}{2} \times \frac{1}{2} \}$

$= \{ \frac{1}{16}, \frac{1}{16}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{4} \}$

$H(X^2) = 4 \times \frac{1}{16} \log 16 + 4 \times \frac{1}{8} \log 8 + \frac{1}{4} \log_2 4$

$= \frac{1}{4} \times 4 + \frac{4}{8} \times 3 + \frac{2}{4}$

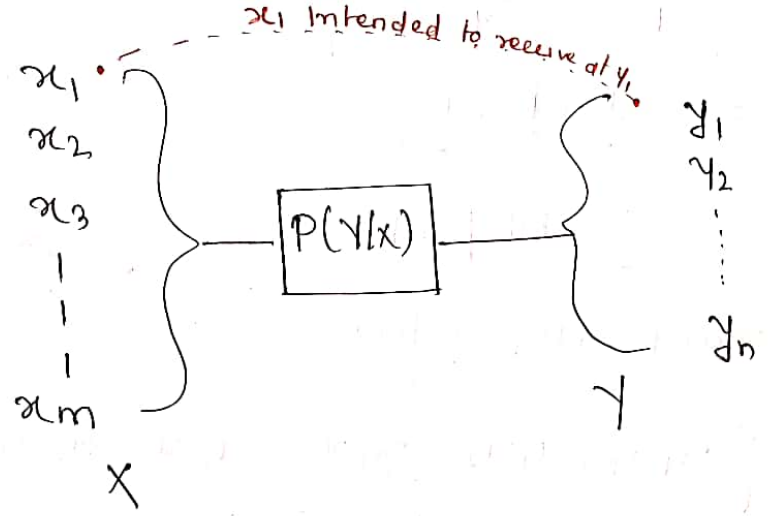
$= 1 + \frac{3}{2} + \frac{1}{2} = 3$ bit/symbol

By $H(X^n) = n H(X)$

$$= 2 \times \frac{3}{2}$$

$$H(X^n) = 3 \text{ bits/symbol}$$

* Channel Representation of Discrete Memoryless channel :- (DMC)



$P(Y|X) \rightarrow$ Probability of receiving Y when X is xmitted

A diagram of DMC with 'm' Inputs and 'n' outputs is shown in figure. The Input 'X' consist of input Symbols $x_1\ x_2$ ---- x_m with probabilities $P(x_1)\ P(x_2)$ ---- $P(x_m)$. The o/p consist of Symbols $y_1\ y_2$ ---- y_n. Each possible Input to o/p path is Indicated along with Conditional probability $P(Y|X)$ which is also known as channel Xsition probability.

$$P[Y|X] = \begin{array}{c|cccc} & \text{o/p} \rightarrow y_1 & y_2 & & y_n \\ \text{Input} \downarrow x_1 & P(y_1/x_1) & P(y_2/x_1) & \text{-------} & P(y_n/x_1) \\ x_2 & P(y_1/x_2) & P(y_2/x_2) & \text{-------} & P(y_n/x_2) \\ & \vdots & & & \\ x_m & P(y_1/x_m) & P(y_2/x_m) & \text{-------} & P(y_n/x_m) \end{array}$$

$m \times n$

Analog Communication

* Amplitude Modulation

Modulation :- It is the process in which low frequency signal is translated to high frequency signal

OR

Modulation is the process in which characteristic of one waveform is varied in accordance with instantaneous value of another waveform

msg signal

Voice – 300 – 3400 Hz

Audio – 20 Hz – 20 kHz

Video – 0 – 5 MHz

Suppose $f_m = 10\ kHz$

Length of antenna $L = \frac{\lambda}{10}$ (say)

$$\lambda = \frac{c}{f_m} = \frac{3 \times 10^8}{10 \times 10^3} = 10^4 \times 3\ m$$

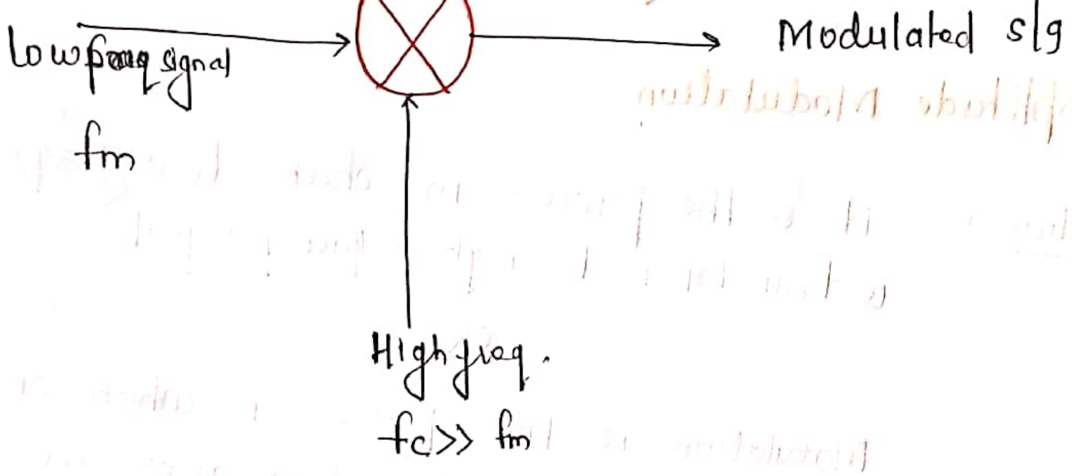
$$= 30\ km$$

$$L = \frac{\lambda}{10} = 3\ km$$ which is not possible

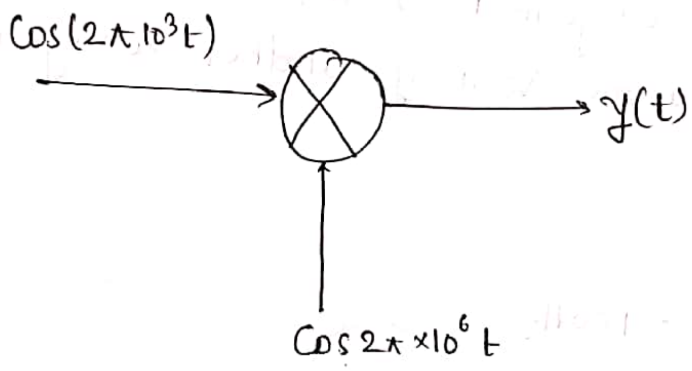
Hence frequency need to be increase

$$f_m \uparrow \Rightarrow \lambda \downarrow \Rightarrow L \downarrow$$

1. Purpose of Modulation is to decrease Antenna Length.



eq.



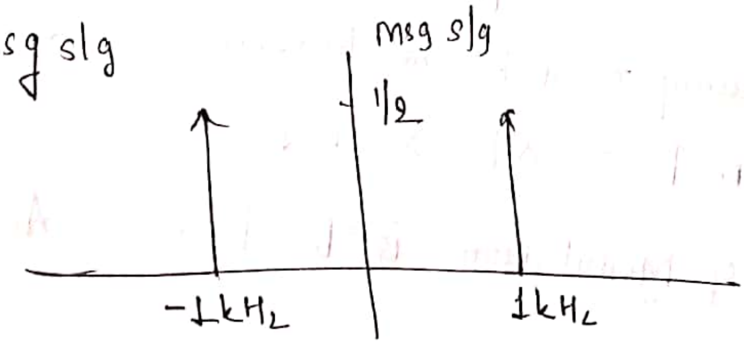
$$y(t) = \cos 2000\pi t \, \cos 2000\pi \times 10^3 \, t$$

$$= \frac{1}{2}\left[\cos 2002\pi \times 10^3 \, t + \cos(1998\,\pi \times 10^3)\right]$$

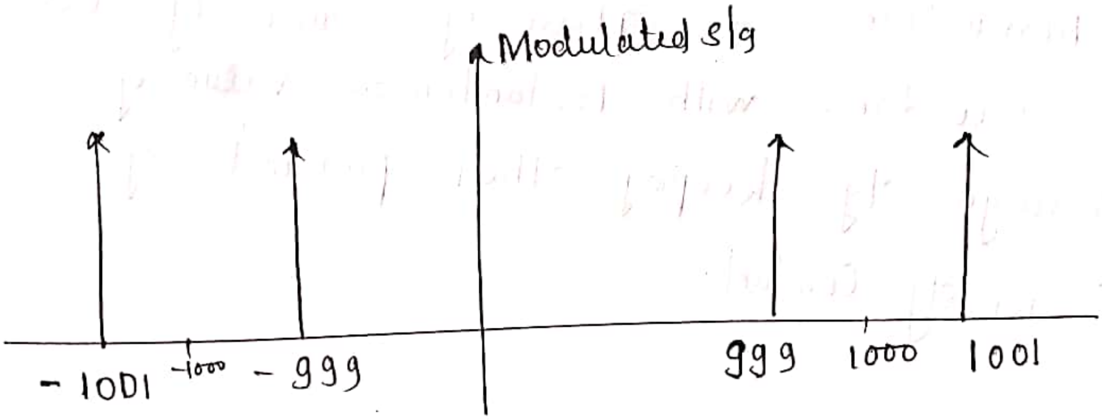
$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{1001\times 10^3} \approx 3\times 10^2 = 300\text{ m}$$

$$L = \frac{\lambda}{10} = \frac{300}{10} = 30\text{ m}$$ {practically feasible}

Spectrum of msg s/g



Spectrum of Carrier.



2. Modulation is used for effective FDM.

* Analog Communication Inputs

- msg s/g or Modulating s/g or Baseband s/g or Unmodulated s/g
- Carrier s/g

$$A_c \cos[2\pi f_c t + \phi]$$

A_c → Amplitude of Carrier

f_c → frequency of Carrier

ϕ → Phase of Carrier

1) Amplitude Modulation → Amplitude of Carrier s/g is Varied in accordance to instantaneous value of message Signal keeping phase & frequency of Carrier unchanged

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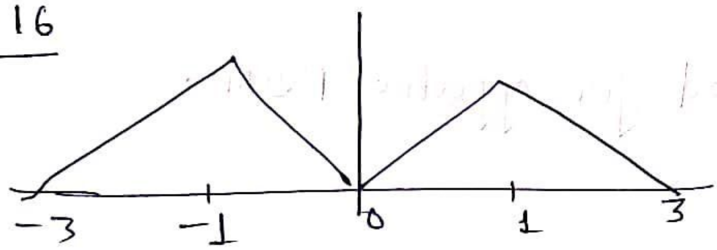
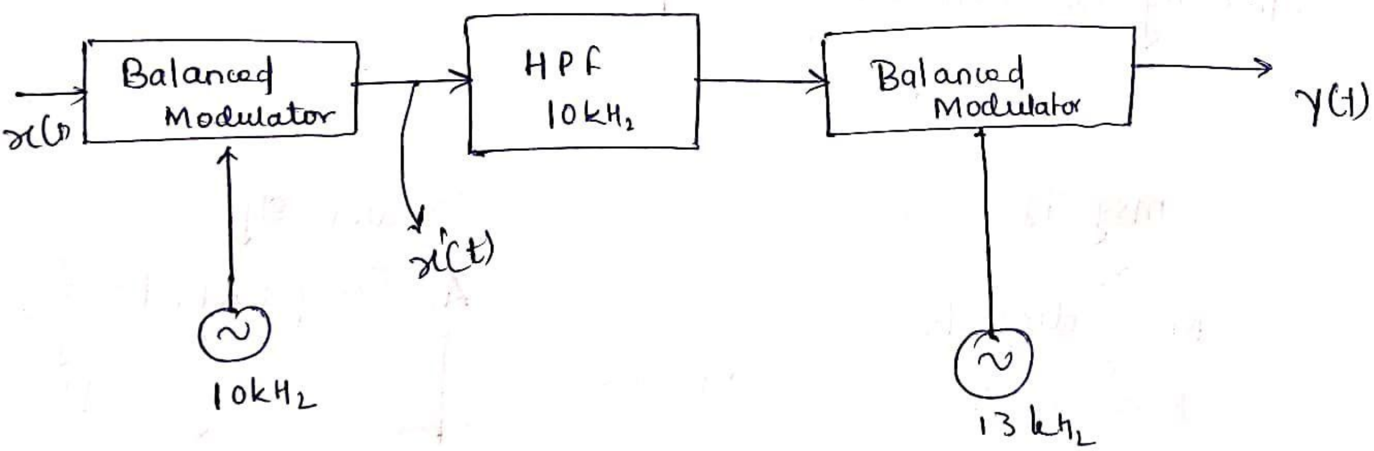
ii) Frequency Modulation ⟶ Frequency of Carrier Signal is Varried in accordance with instantaneous Value of message Signal keeping other parameters of Carrier slg Constant.

iii) Phase Modulation ⟶ phase of Carrier slg is Varried in accordance with Instantaneous Value of message slg keeping other parameter of Carrier slg Constant.

P-34

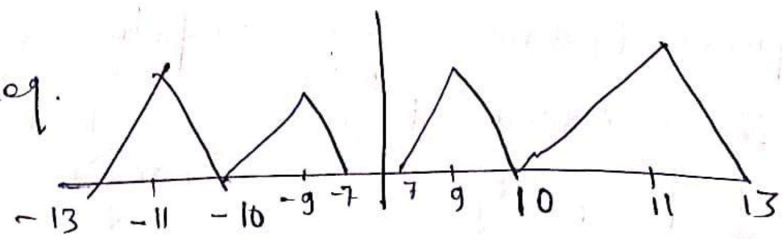
Que 16



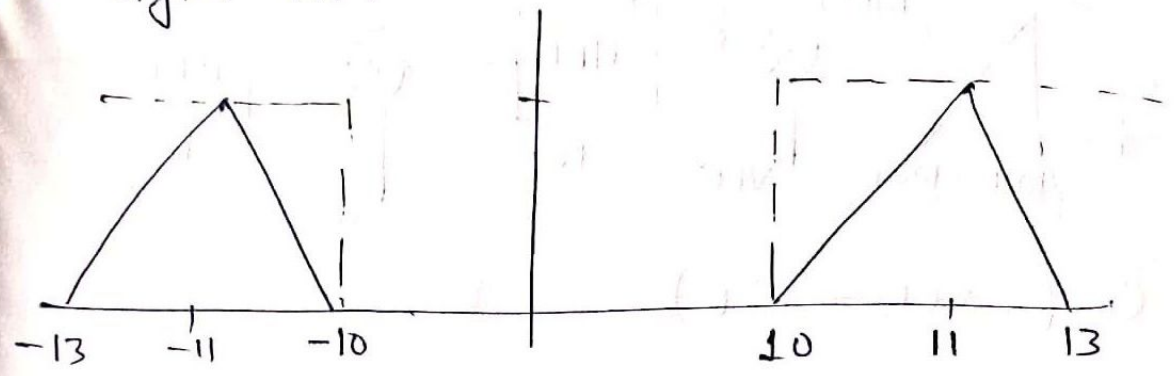


The positive freq. where $Y(f)$ has spectral peaks are -

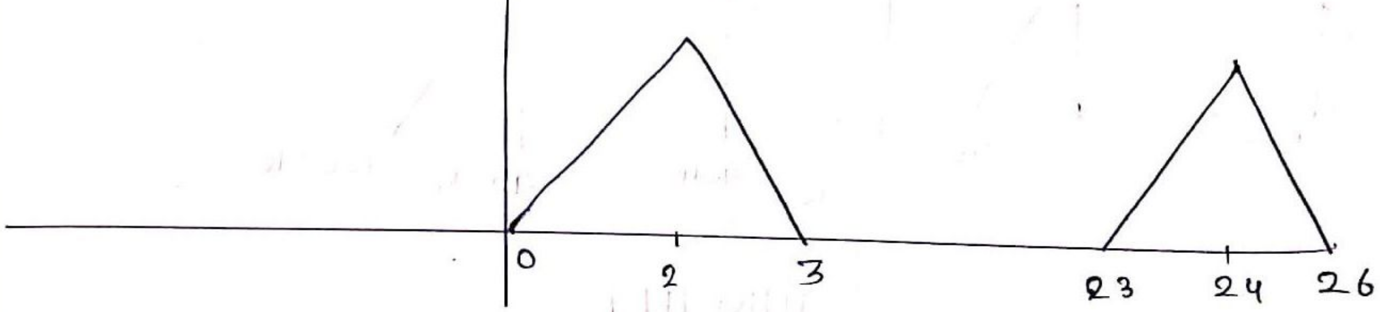
Sol $x'(t)$ has freq.



After HPF



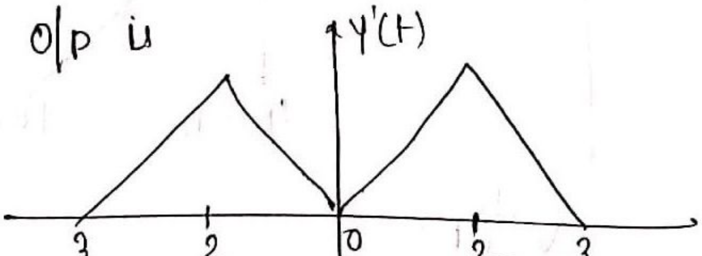
now y(t) (positive side)



Hence Spectral peak at 2kHz & 24kHz //.

Note:- If an LPF of 3kHz is used after y(t) then

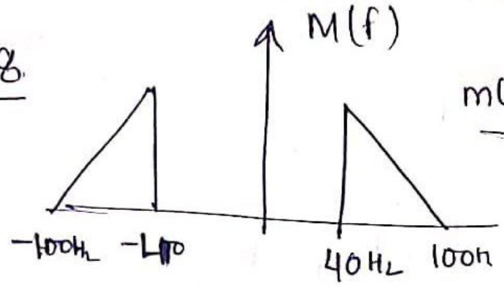
o/p is

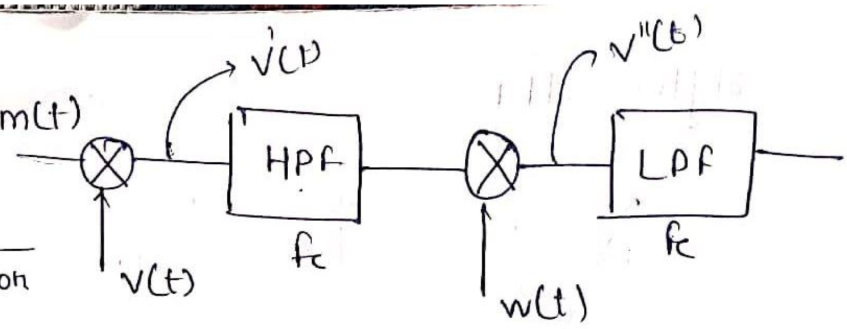


Hence each o/p waveform is folded version of input in its own place

This is an Scrambler ckt used in telephone line for Security.

Q28.

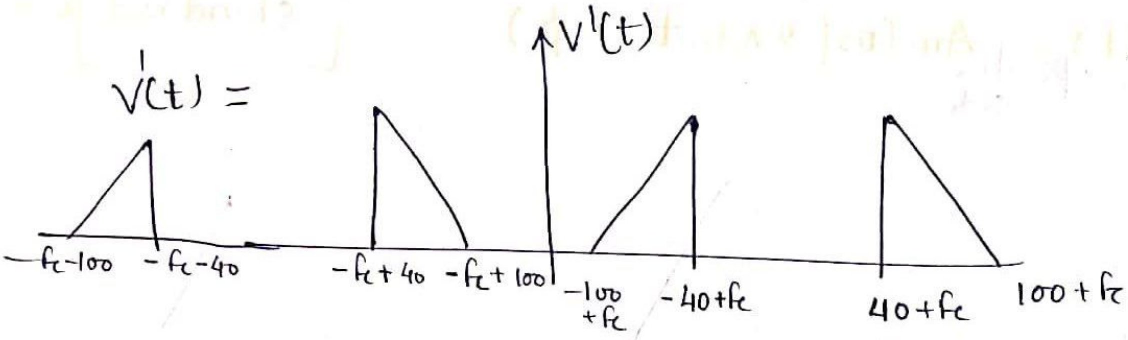




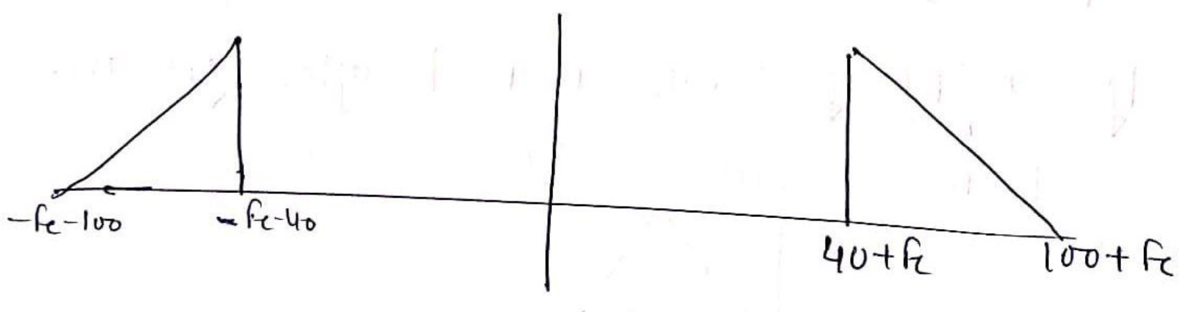
$$w(t) = \cos(2\pi(f_c + \Delta)t)$$

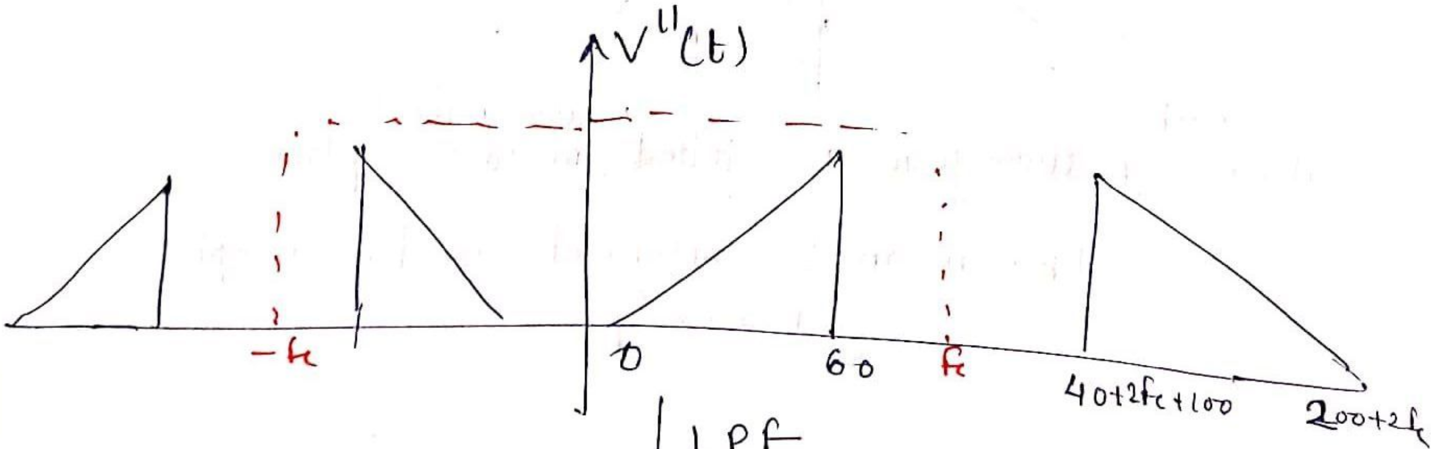
$$v(t) = \cos(2\pi f_c t)$$

$V'(t) =$

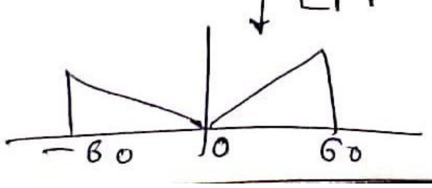


After HPF



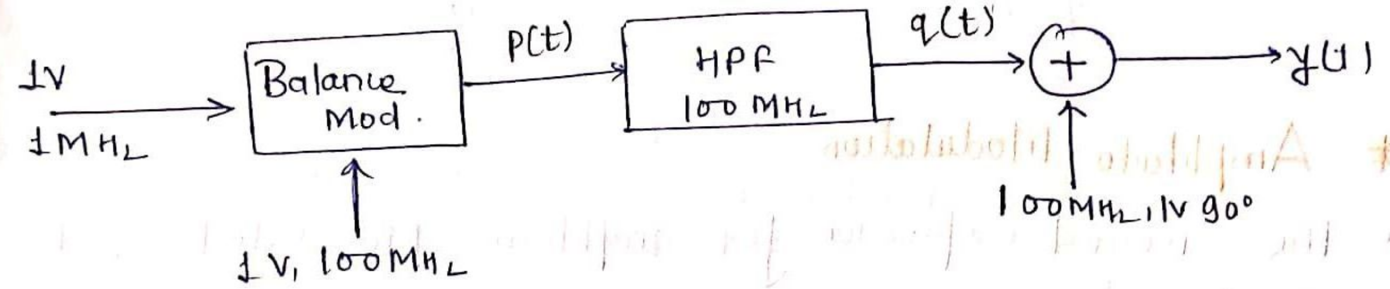


LPF



BW = 60 Hz //.

Que 15



The Envelope of $y(t)$ is

Sol $m(t) = A_m \cos(2\pi f_m t + \phi)$ { standard form

$$P(t) = \frac{1}{2}\left[\cos 2\pi(101\times10^6)t + \cos(2\pi\, 99\times10^6)t\right]$$

$$q(t) = \frac{1}{2}\cos 2\pi\,(101\times10^6)\,t$$

$$y(t) = q(t) + \cos 2\pi\,(100\times10^6 t + 90^\circ)$$

$$= \frac{1}{2}\cos 2\pi(101\times10^6 t) + \cos 2\pi\,(100\times10^6 t + 90^\circ)$$

$$= \frac{1}{2}\cos(2\pi 100\times10^6 t)\cos(2\pi\times10^6 t) -$$

$$\frac{1}{2}\sin(2\pi\,100\times10^6 t)\sin(2\pi\times10^6 t)$$

$$\mp \sin\left[2\pi(100\times10^6 t)\right]$$

$$\text{Envelope} = \sqrt{\frac{1}{4}\cos^2(2\pi\times10^6)t + \frac{1}{4}\sin^2(2\pi\times10^6 t) \pm 2\times1\times\frac{1}{2}\sin(2\pi\times10^6)t}$$

Band Pass Xmission

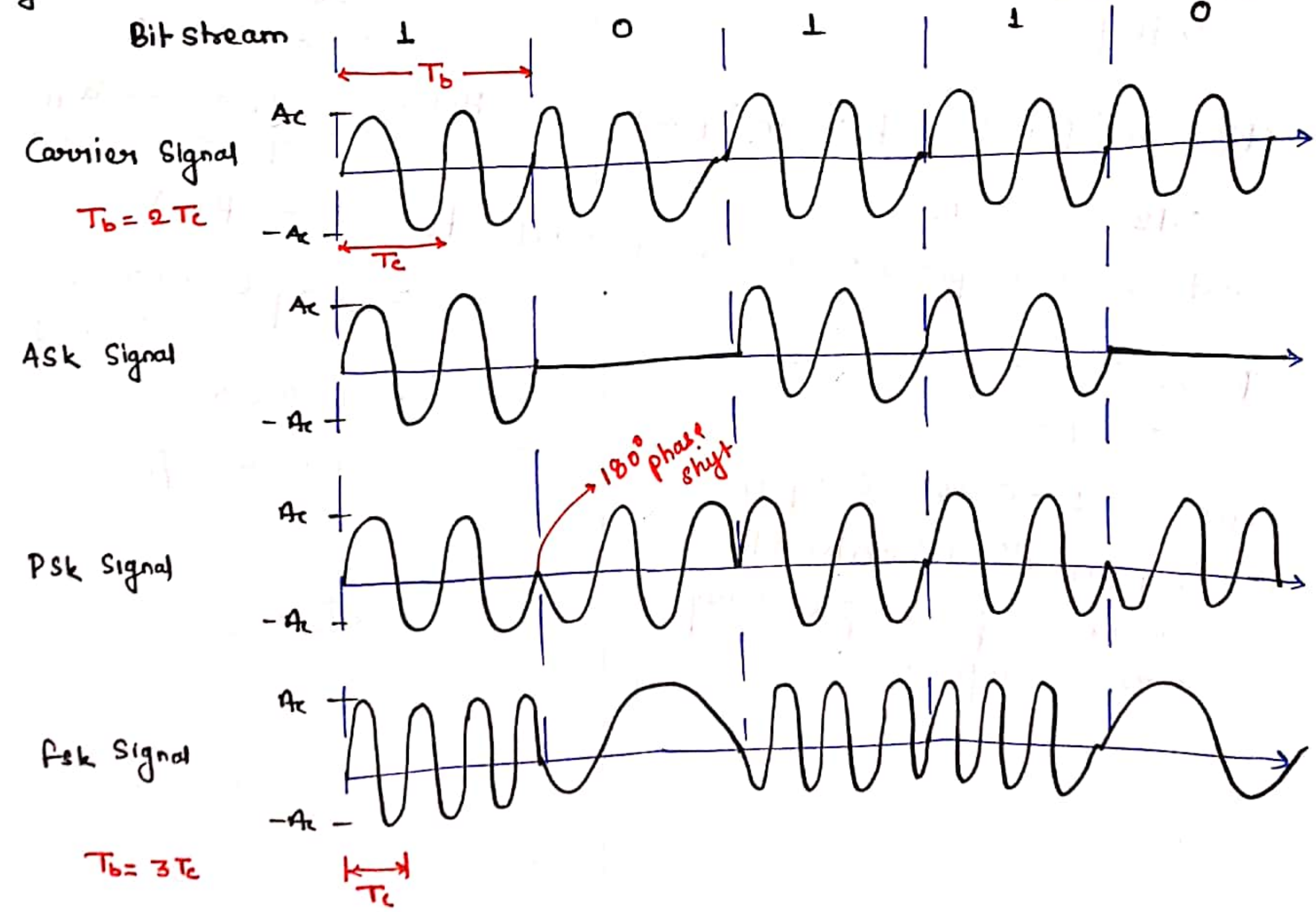
+ Requirements for Bandpass xmission :-

1. Maximum data rate for a given bandwidth.
2. Minimum probablity of error.
3. xmitter power should be minimum.
4. Circuit complexity must be minimum.
5. Inter Symbol Interference (ISI) should be minimum.

+ Digital Modulation techniques :-

1. Amplitude shift keying (ASK) or ON-OFF keying (OOK)
2. phase shift keying (PSK)
3. frequency shift keying (FSK)

eg.



$$\boxed{T_b = nT_c}$$

I. Binary Amplitude shift keying (BASK):

Binary '1' $\quad S_1(t) = A_c \cos\omega_c t$

Binary '0' $\quad S_2(t) = 0$

$$\boxed{S_{BASK}(t) = b(t)\, A_c \cos\omega_c t}$$

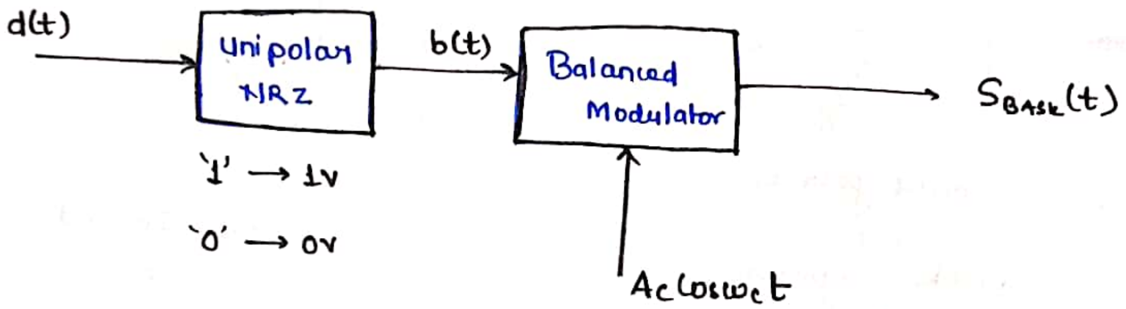


fig. Generation of BASK slg.

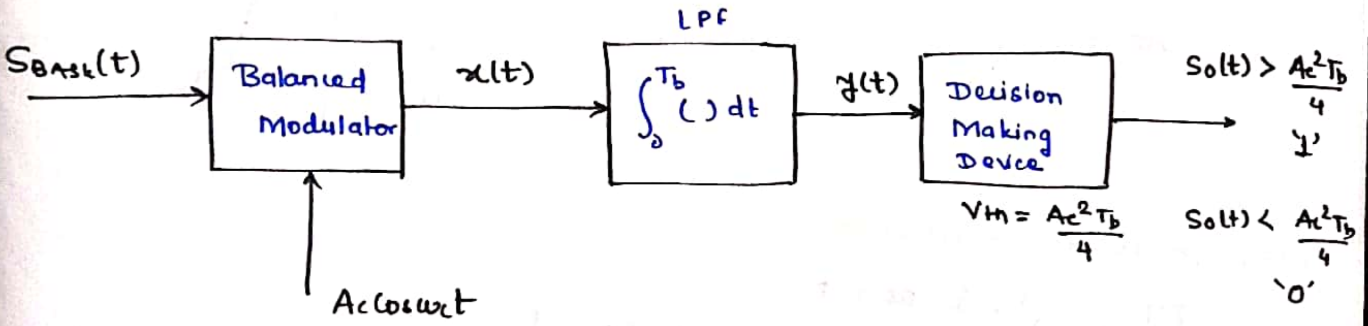


fig. Detection of BASK slg

$$x(t) = [b(t)\, A_c \cos\omega_c t]\, A_c \cos\omega_c t$$

$$x(t) = \frac{b(t)\, A_c^2}{2}\left[1 + \cos 2\omega_c t\right]$$

$$y(t) = \int_0^{T_b} \frac{b(t)\, A_c^2}{2}\left[1 + \cos 2\omega_c t\right] dt$$

$$y(t) = \frac{b(t)\, A_c^2 T_b}{2} + \cancel{\frac{\sin 2\omega_c T_b}{2\omega_c}} \to 0$$

$$\boxed{y(t) = \frac{b(t)\, A_c^2\, T_b}{2}}$$

{ $b(t)$ is Constant in T_b

{ $T_b = n T_c$

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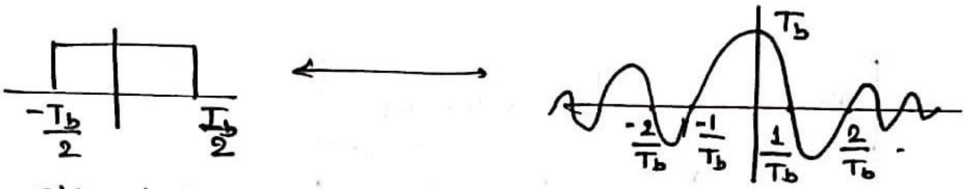
* Bandwidth Requirement of BASK :-

$$S_{bask}(t) = b(t)\, A_c \cos \omega_c t$$

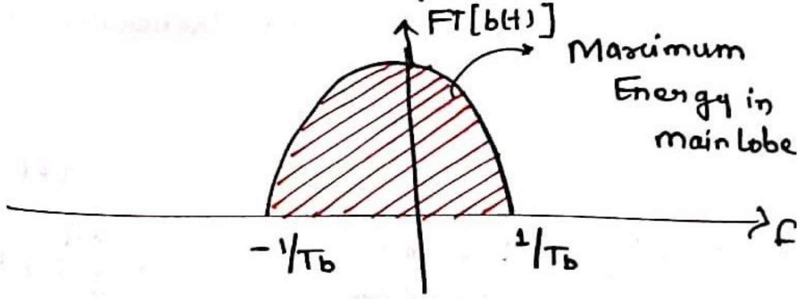
b(t) :-

fourier xform



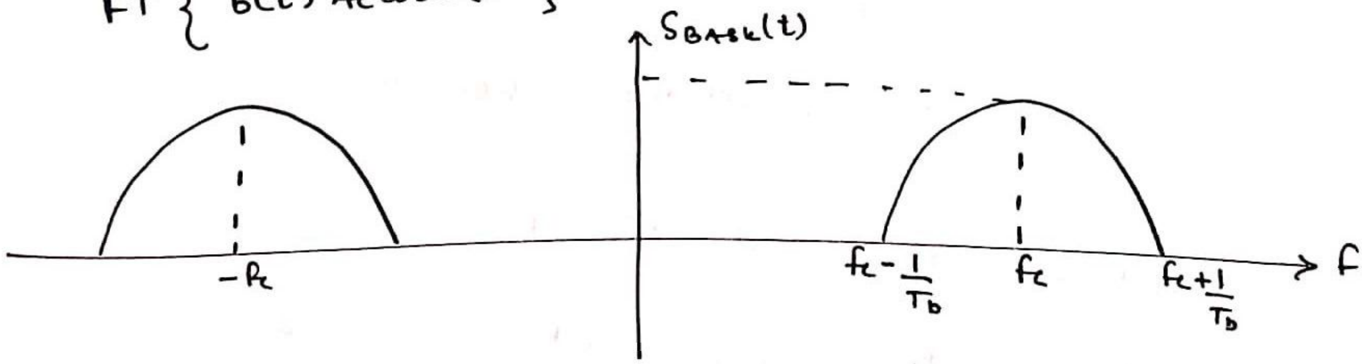
Shifted form of b(t) (Same magnitude) as b(t)

approximated to



now, to obtain Bandwidth :-

$$FT\left\{ b(t)\, A_c \cos \omega_c t \right\}$$



$$BW = f_H - f_L$$

$$= f_c + \frac{1}{T_b} - f_c + \frac{1}{T_b}$$

$$= \frac{2}{T_b}$$

$$\boxed{B = 2F_b} \qquad \left\{ F_b = \frac{1}{T_b} \text{ is bit rate} \right.$$

For Raised Cosine filter { Zero ISI }

$f_b \longrightarrow \frac{R_b}{2}(1+\alpha)$ { R_b & f_b are same bitrate

where, α = Roll off factor

$\therefore \quad BW = 2 f_b$

$= 2 \frac{R_b}{2}(1+\alpha)$

$$\boxed{BW = R_b(1+\alpha)}$$

for Min BW $\quad \alpha = 0$

$$\boxed{BW = R_b}$$

* Energy per bit :-

Binary '1' $\qquad S_1(t) = A_c \cos\omega_c t$

$$E_b = \int_0^{T_b} S_1^2(t)\,dt = \int_0^{T_b} A_c^2 \cos^2\omega_c t$$

$$= \int_0^{T_b} \frac{A_c^2}{2}(1+\cos 2\omega_c t)$$

$$\boxed{E_b = \frac{A_c^2 T_b}{2}}$$

OR $\quad \boxed{A_c = \sqrt{\frac{2E_b}{T_b}}}$

OR $\quad \boxed{A_c = \sqrt{2P_s}}$ $\qquad$ { $\frac{E_b}{T_b} = P_s$ Power

for Binary '0'

$E_b = 0$

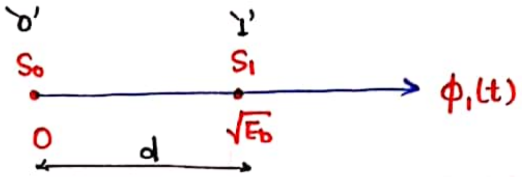
* Constellation Diagram (Signal Space Diagram)

$$S_{BASK}(t) = b(t)\, A_c \cos \omega_c t$$

$$= b(t) \sqrt{\frac{2E_b}{T_b}} \cos \omega_c t$$

$$= \sqrt{E_b}\, b(t) \underbrace{\left(\sqrt{\frac{2}{T_b}} \cos \omega_c t\right)}_{\text{Basis function } \phi(t)}$$

$$\therefore \boxed{S_{BASK}(t) = \sqrt{E_b}\, b(t)\, \phi_1(t)}$$

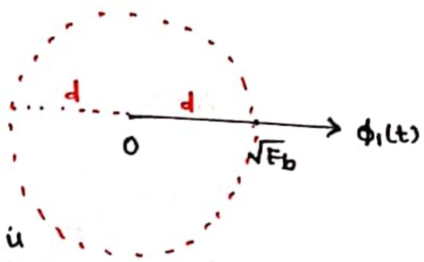


Here S_0, S_1 are symbol and they are made of single bit '0' & '1' resp.

Hence Here $\boxed{E_s = E_b}$ Symbol Energy is bit energy

d = distance between two symbols known as Euclidean distance.

Probablity of error $P_e \propto \frac{1}{d}$



- Symbol energy = $(\text{radius})^2$

- Symbol 1 is having amplitude $\sqrt{2}$ & Symbol 2 is having amp 0

Constellation diagram

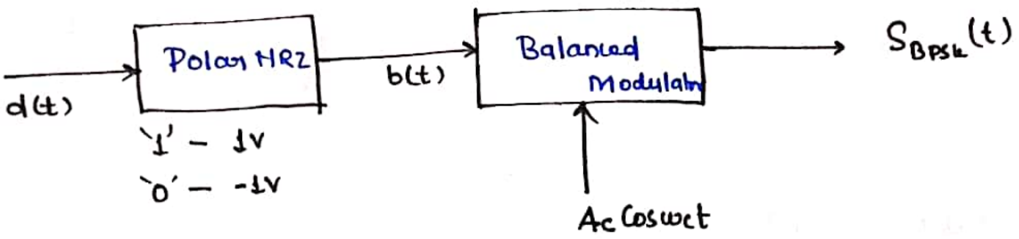
II. Binary Phase shift keying (BPSK) :-

Binary '1' $\quad S_1(t) = A_c \cos \omega_c t$

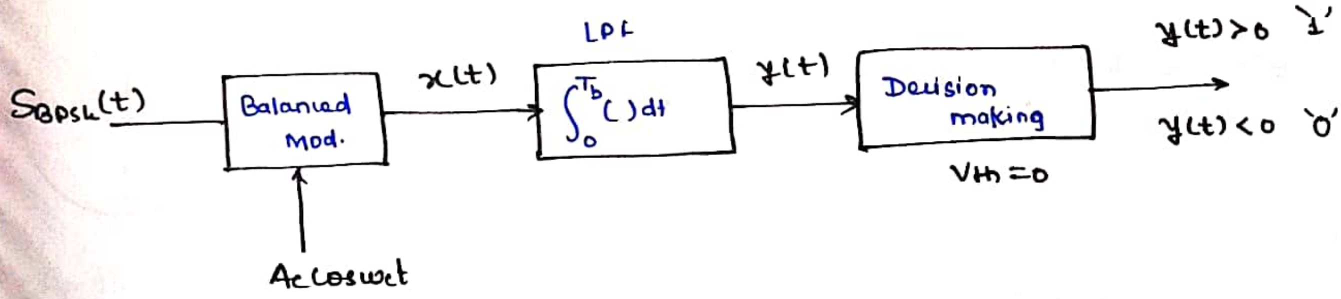
Binary '0' $\quad S_2(t) = -A_c \cos \omega_c t$

$$\boxed{S_{BPSK}(t) = b(t)\, A_c \cos \omega_c t}$$

Generation of BPSK :-



Detection of BPSK



$$x(t) = \left(b(t)\, A_c \cos w_c t\right) A_c \cos w_c t$$

$$= b(t)\, A_c^2 \cos^2 w_c t$$

$$y(t) = \frac{b(t)\, A_c^2\, T_b}{2}$$

Error in BPSK :- Because of channel Noise if we receive $-b(t) A_c \cos w_c t$ then at o/p of decision making device we get '0' ~~when~~ when we sent $b(t) = 1$

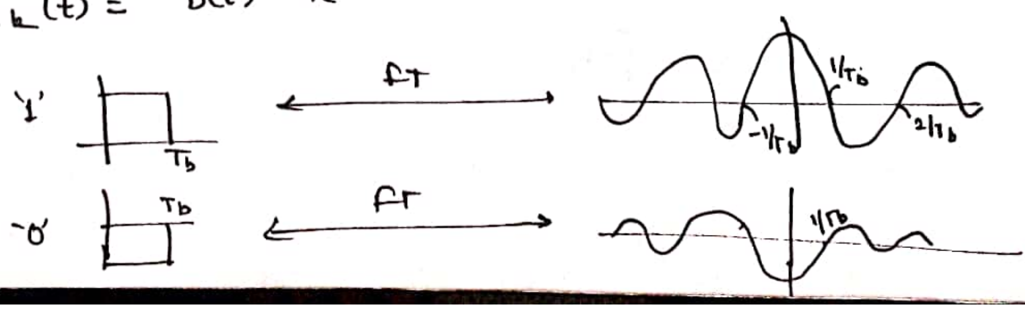
$$y(t) = -\frac{A_c^2\, T_b}{2} \quad \text{i.e} < 0 \Rightarrow \text{'0'}$$

To remove this error we use DPSK differential phaseshift keying

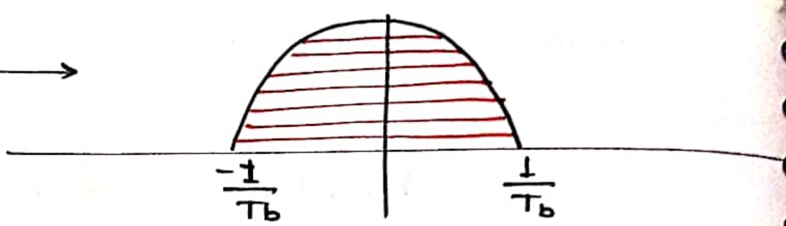
Bandwidth :-

$$S_{BPSK}(t) = b(t)\, A_c \cos w_c t$$

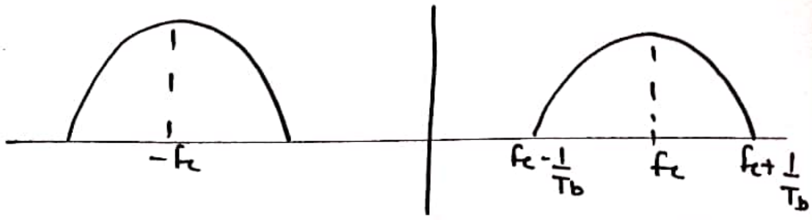
$FT\{b(t)\}$



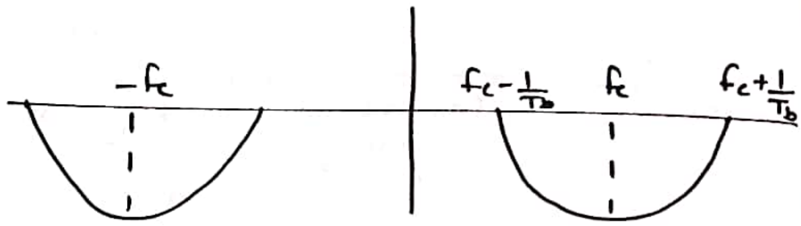
$b(t)\ A_c \cos\omega_c t \xleftrightarrow{FT}$



$b(t)\ A_c \cos\omega_c t \xleftrightarrow{FT}$
'1'



$b(t)\ A_c \cos\omega_c t \longleftrightarrow$
'0'



$\boxed{BW = 2 f_b}$

and for Raised Cosine $f_b \rightarrow \frac{f_b}{2}(1+\alpha)$

$BW = f_b(1+\alpha)$

$BW_{min} = f_b$

* Energy per bit :

$$E_b = \int_0^{T_b} (A_c \cos\omega_c t)^2 dt \qquad \text{for '1'}$$

$$E_b = \frac{A_c^2 T_b}{2}$$

$$A_c = \sqrt{\frac{2E_b}{T_b}} = \sqrt{2P_s} \qquad \left\{ \frac{E_b}{T_b} = P_s \right.$$

for '0' $\quad S_2(t) = -A_c \cos\omega_c t$

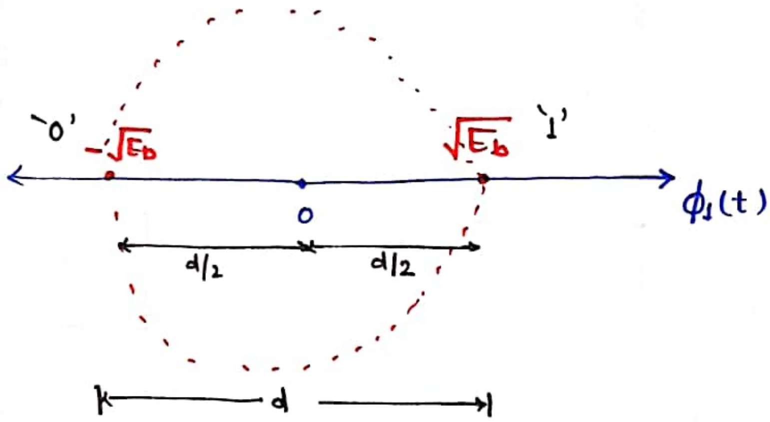
$$E_b = \frac{A_c^2 T_b}{2}$$

$$A_c = \sqrt{2P_s}$$

$S_1(t) = A_c\, b(t) \cos\omega_c t$

$S_1(t) = b(t)\sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos\omega_c t$ for '1'

also, $S_2(t) = -b(t)\sqrt{E_b}\,\underbrace{\sqrt{\frac{2}{T_b}}\cos\omega_c t}_{\text{Basis function } \phi_1(t)}$ for '0'

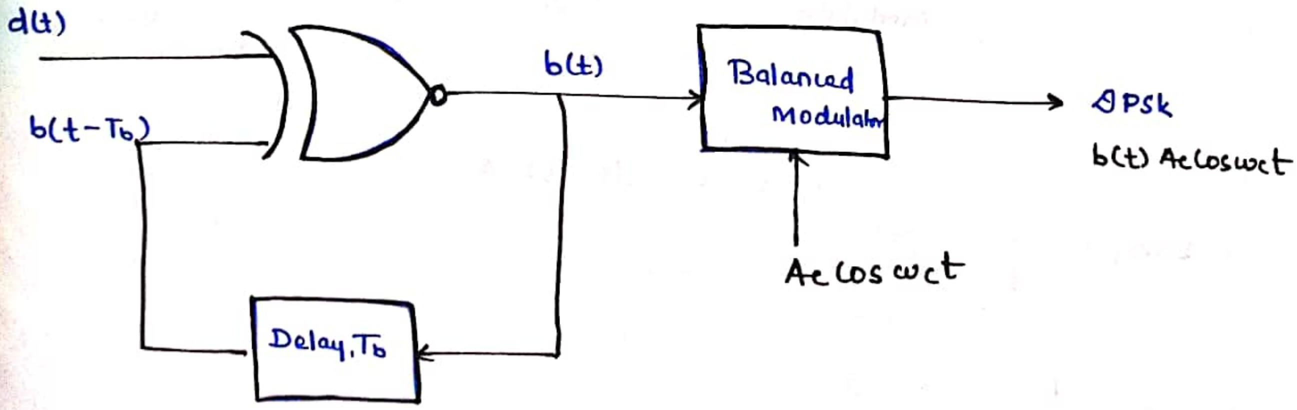


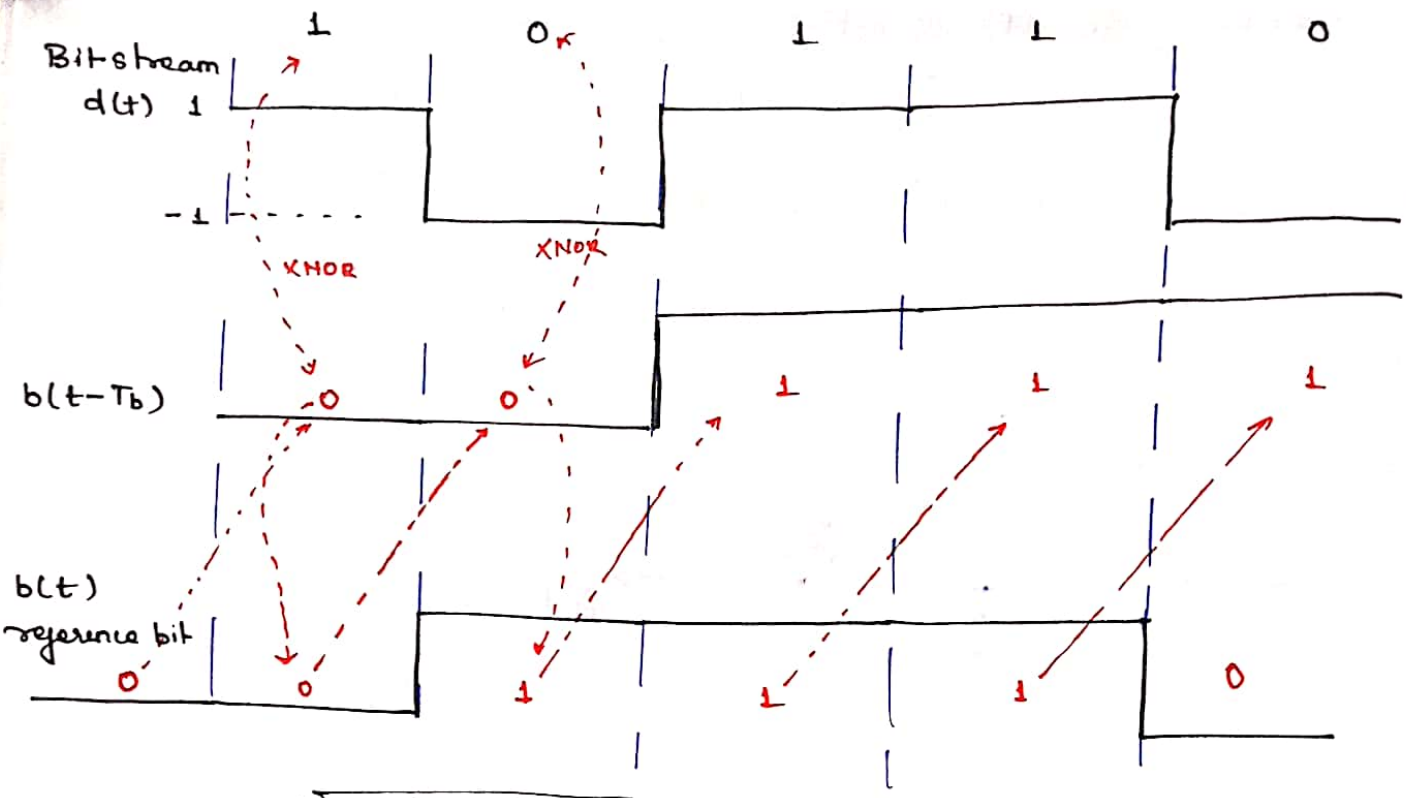
Amplitude is Constant as both Symbol lie on Same Circle

$\because \quad d_{BPSK} > d_{BASK}$

$\therefore \quad P_{e\,BASK} > P_{e\,BPSK}$ { Probablity of error in BASK is more than in BPSK just double. }

* Differential phase shift keying :- (DPSK)



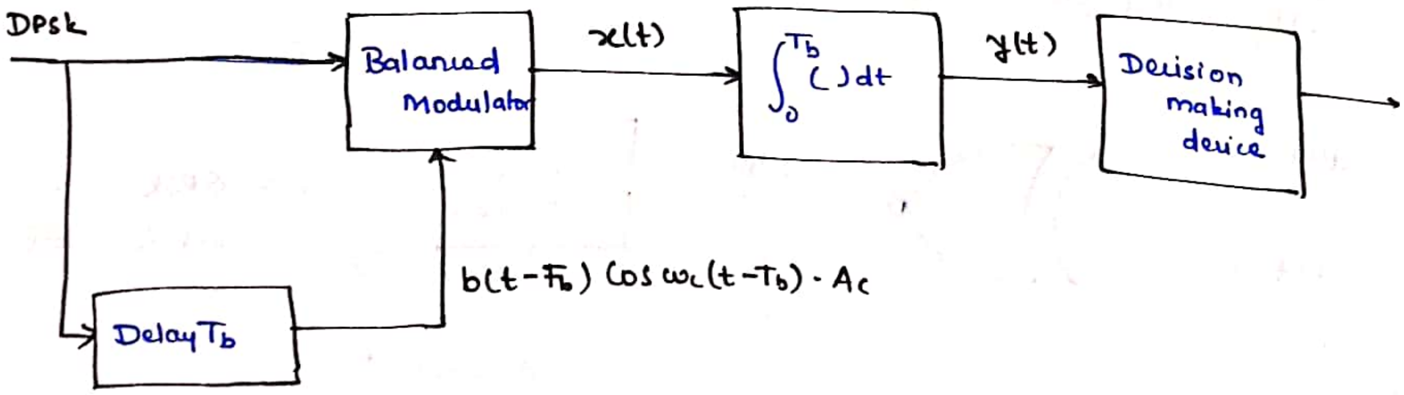


XNOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Receiver / Detection of DPSK :-

(non coherent detector)



$$DPSK = b(t)\, A_c \cos \omega_c t$$

$$x(t) = A_c^2\, b(t)\, b(t-T_b) \cos \omega_c t \cos \omega_c (t-T_b)$$

$$x(t) = A_c^2\, b(t)\, b(t-T_b) \left[\cos(\omega_c t + \omega_c t - \omega_c T_b) + \cos \omega_c T_b \right]$$

$$y(t) = \int_0^{T_b} \frac{A_c^2 \, b(t)\, b(t-T_b)}{2} \left[\cancel{\cos(2\omega_c t - \omega_c T_b)}^{\,0} + \cos \omega_c T_b \right]$$

$$y(t) = \frac{A_c^2 \, b(t)\, b(t-T_b)}{2} \left[\frac{\sin \omega_c T_b + \sin \omega_c T_b}{2\omega_c} + \sin \omega_c T_b - \frac{\sin \omega_c T_b}{\omega_c} \right]$$

$$y(t) = \frac{A_c^2 \, b(t)\, b(t-T_b)}{2} \quad \frac{\sin \omega_c T_b}{\cancel{\omega_c}} \quad [t]_0^{T_b}$$

$$y(t) = \frac{A_c^2 \, b(t)\, b(t-T_0)}{2} \quad \cancel{\cos \frac{2\pi}{T_c} \times nT_c}^{\,1} \quad T_b$$

$$y(t) = A_c^2 \, b(t)\, b(t-T_0) \quad T_b$$

$y(t) \propto b(t)\, b(t-T_b)$ { If both are same then o/p is '1'

$y(t) = 1$ binary '1'

$= -1$ binary '0'

* M-ary PSK

$$d = \sqrt{4 E_b N \sin^2\left(\frac{\pi}{M}\right)}$$

$$\boxed{BW = \frac{2 f_b}{N}}$$

$$\text{Spectral efficiency} = \frac{\text{Bit rate}}{\text{Bandwidth}} \qquad \boxed{\rho = N/2}$$

$$= \frac{f_b}{2 f_b / N} = N/2$$

$$\text{Baud rate} = \frac{\text{Bit rate}}{\log_2 M}$$

{ N = no. of bits
M = no. of symbol

$$\boxed{E_s = N E_b}$$