Statistics

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performance of class 10th students in govt. schools of India

- Data Distribution
- Statistical values
- Data Normalization
- Missing Value Imputation
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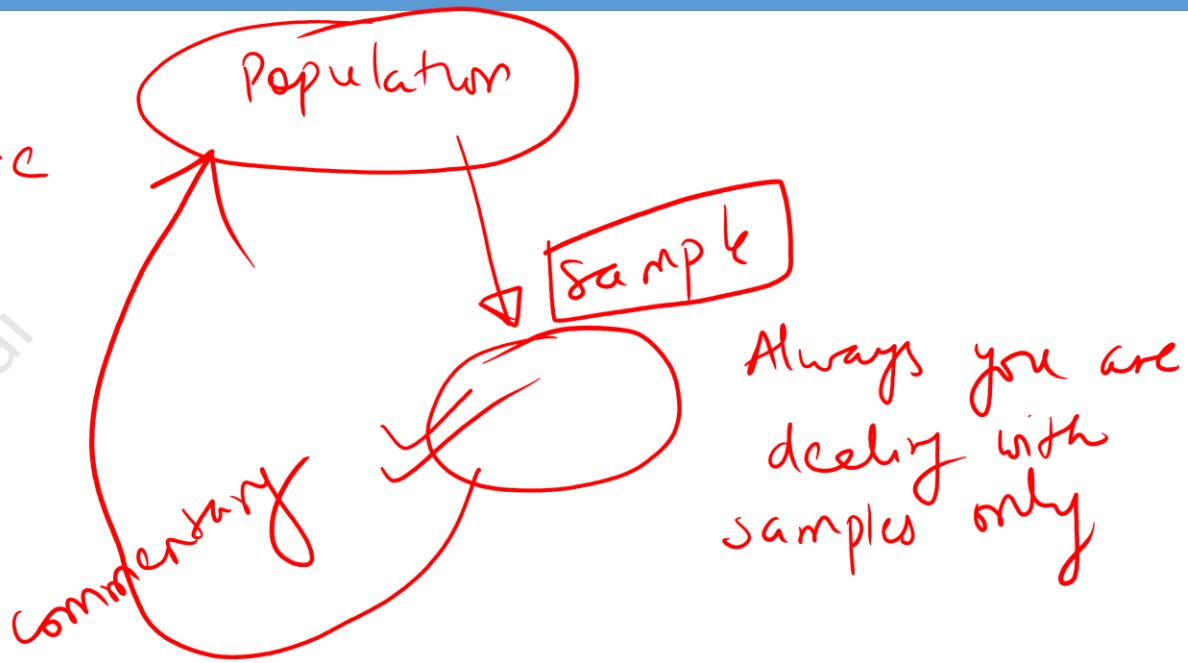
- Correlation and Causation

mean, median, mode

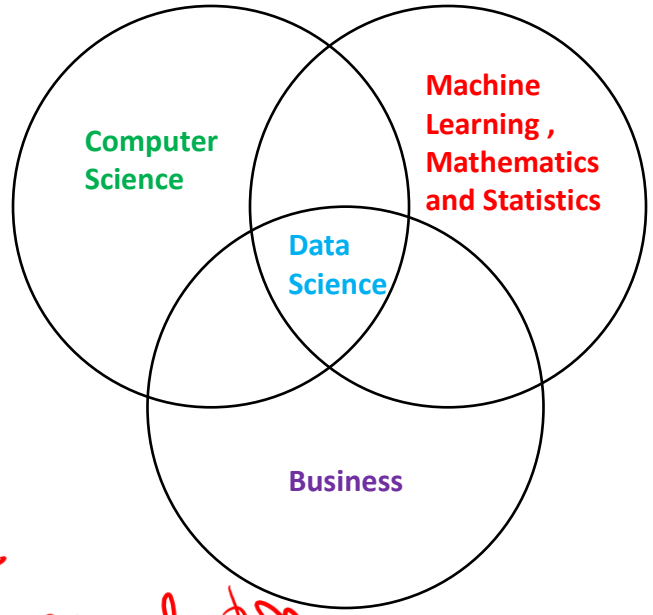


Why should I learn Statistics?

- Statistics is the discipline that covers analysis, interpretation, and presentation of data

- Statistics is useful in data science projects for following tasks
  - Exploratory Data Analysis
  - Analysing relationship between variables
  - Missing value imputation
  - Outlier detection and treatment
  - Hypothesis Testing

Based on the samples we are commenting on the population



Data Distribution

Approaches for calculating the probability
1) MLE (Maximum Likelihood Estimation)
2) Bayesian approach

(1) $P(O_1) = \frac{\text{Number of times you observed } O_1}{\text{Total Number of times you performed the Exp}}$ → ∞

(2) Bayesian
↳ (1) start with starting knowledge about the phenomenon
↳ (2) Experiments
↳ (3) Basic knowledge + Experiment → refine your probabilities

Random Variable

Random variable is a variable whose value is unknown e.g., if we measure height of any person on the street then the value, we will get would be any random value (unknown)

1. **Discrete random variable:**

Fix set of values for outcome e.g., tossing a coin

2. **Continuous random variable:**

It can take an infinite (theoretically) number of possible values e.g., salary of a random person selected from Pune

Probability Distribution

Probability distribution is the mathematical function that gives the probabilities of occurrence of different possible outcomes for an experiment

Suppose X denotes the outcome of a coin toss ("the experiment"),

As X is discrete random variable (as it takes fix set of values) it is sufficient to specify a function representing probability of each outcome

Probability distribution of X would take the value 0.5 for X = heads, and 0.5 for X = tails (for a fair coin).

Experiment

1. Started tossing the coin for 100 ~~coin~~ times
2. rolling the dice 100 times
3. noting down the salaries of X company

} discrete

Different possible outcomes for the random variable

$E_1 = [H, T]$, $E_2 = [1, 2, 3, 4, 5, 1]$

$E_3 = [2\,LPA, 2\,Cr]$

Q1 What is the probability of H occurring? 55%.

Q2 what is the probability distn for tossing this coin

~~55%~~ → H

45% → T

Probability of occurence

E_1 : H → 55 $55/100 = 0.55$

T → 45 $45/100 = 0.45$

1 → 15 → 15/100

2 → 16 → 16/100

3 → 17 → 17/100

4 → 15

5 → 20

6 → 20 → 20/100

Bucket size 0.1 LPA

2 – 2.1 LPA = 3

2.1 – 2.2 LPA = 5

2.2 – 2.3 LPA = 7

10 – 10.1 LPA = 100

20.0 – 20.1 = 1

Continuous random variable

1000 employees

Emp1 → 2 LPA

Emp2 → 2.5 LPA

Emp3 → 2.25 LPA

} 10 LPA

Emp 10 → 5.1 LPA

11 → 5.5 LPA

12 → 5.3 LPA

$P(X = x_0) = 0$ = Not Defined

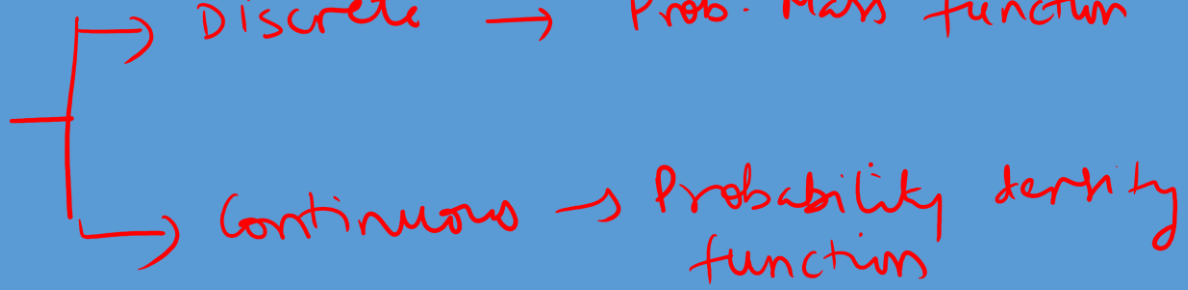
X → salary

x_0 → exact salary

$P(x_0 \leq X \leq x_1)$

$P(2\text{LPA} \leq X \leq 2.1\text{ LPA})$

Probability Distribution

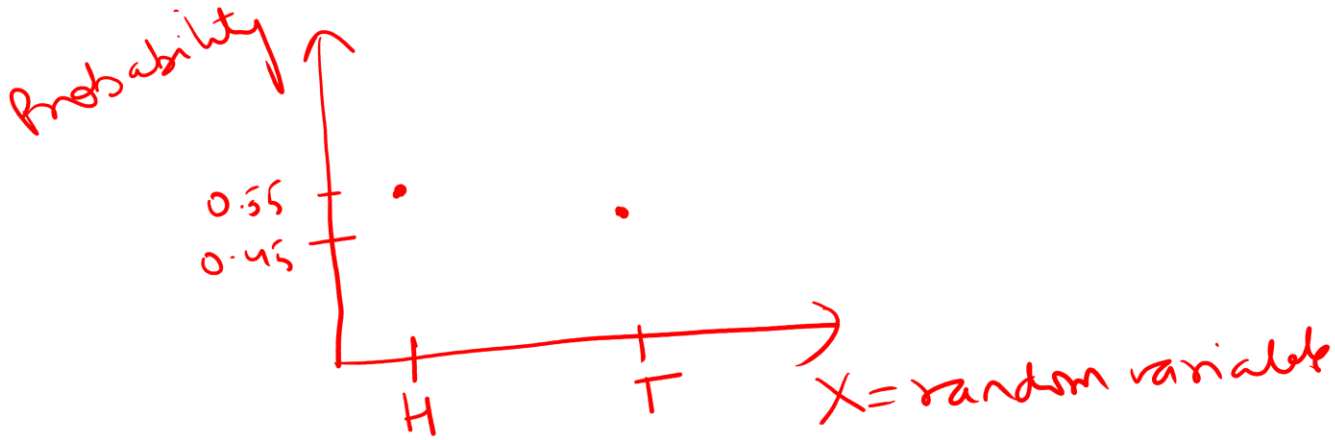


- Probability distribution is represented using probability density function in case of continuous random variable

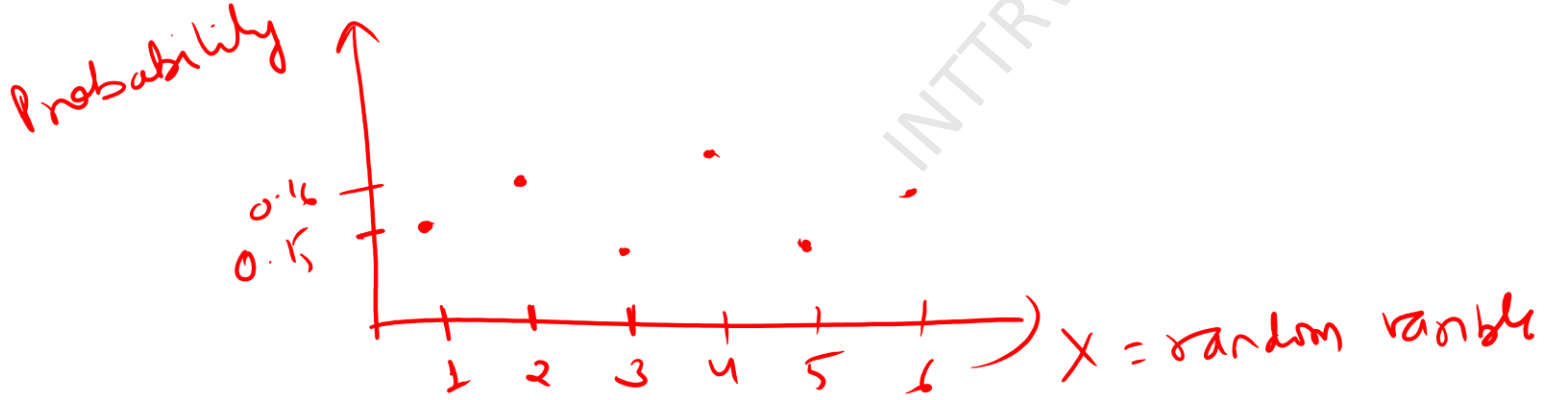
- Examples of continuous random variable distributions:
  - Suppose X denotes the outcome of an experiment where we are randomly sampling heigh of a person from population of city
  - Suppose X denotes the outcome of an experiment where we are randomly sampling average marks obtained by student from all universities

Probability Mass Function for discrete random variables

In PMF; we describe the probabilities for each and every possible outcome



Coin toss



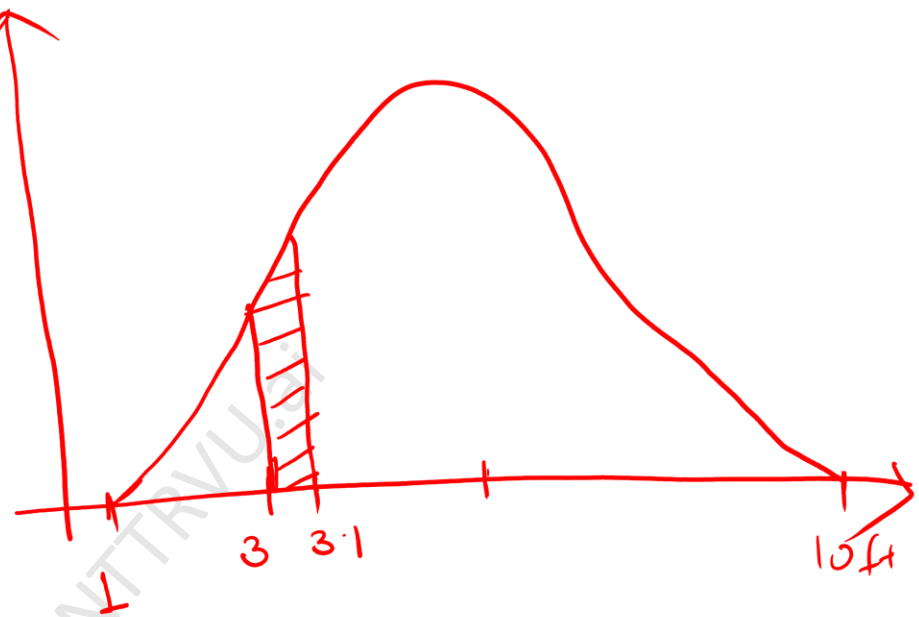
For continuous random variable

we can't use PMF

we always talk about pdf (Probability Density function)

Probability density = density of probability in a given range

$P(3 \le X \le 3.1) = \int_{3}^{3.1} f(x)\,dx$ = probability density



Y axis → we don't have the probabilities but we have probability densities

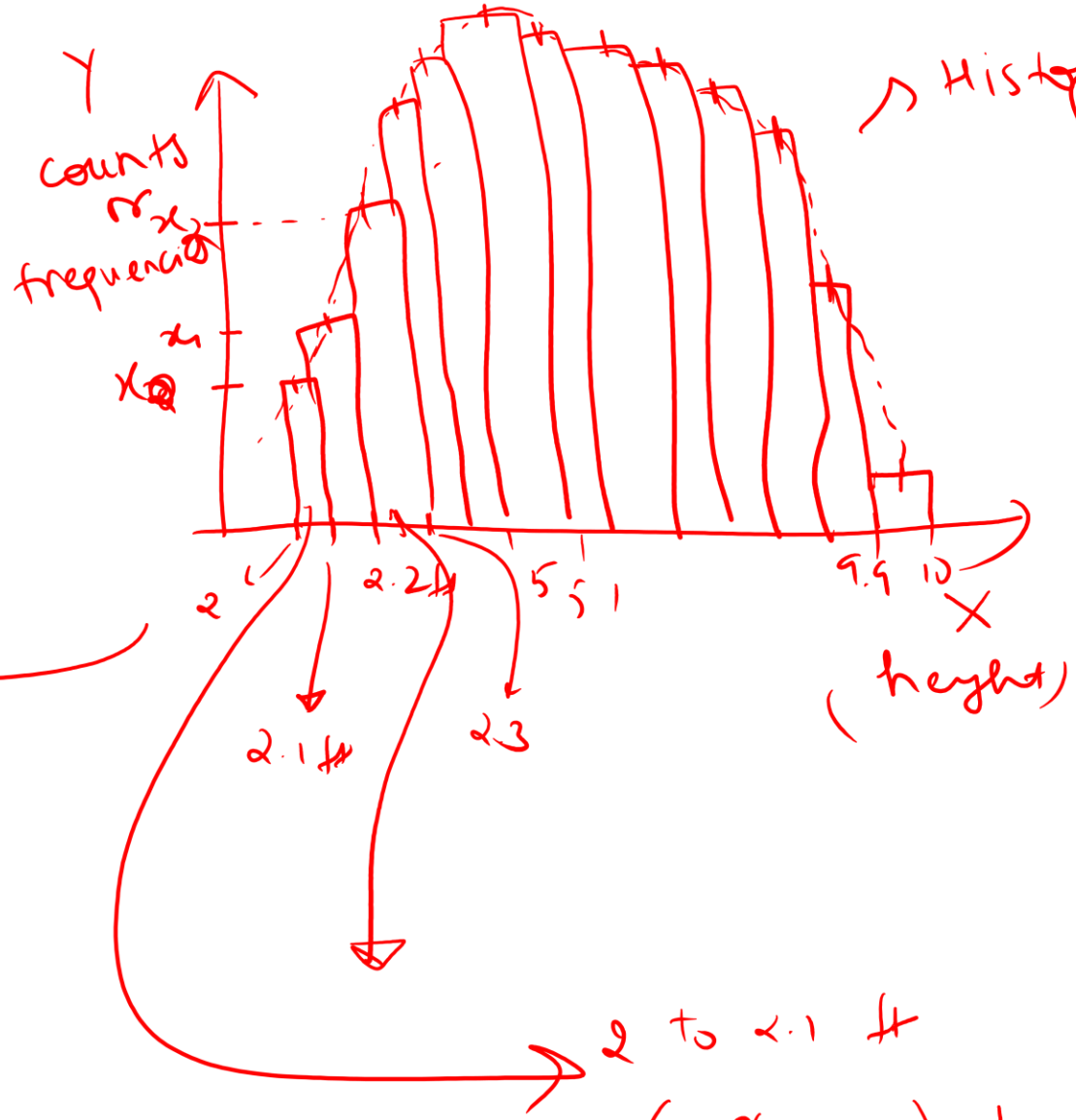
Pdf Probability density function

Observed Height
3 ft
3.2 ft
5 ft
5.4 ft
6 ft
2 ft
2.2 ft

→

Buckets	Count
2 – 2.1	x_0
2.1 – 2.2	x_1
2.2 – 2.3	x_2
⋮	
9.9 – 10 ft	x_n

0.1 ft



→ Histogram

→ reduce the size of each and every bucket 0.01, 0.001 ft

bucket size → 0

2 to 2.1 ft

$$\left(\frac{x_0}{\text{Total Count}}\right)\frac{1}{0.1}$$

bucket = 0.1 ft

bucket size → 0

2 → 2.01

2 → 2.001

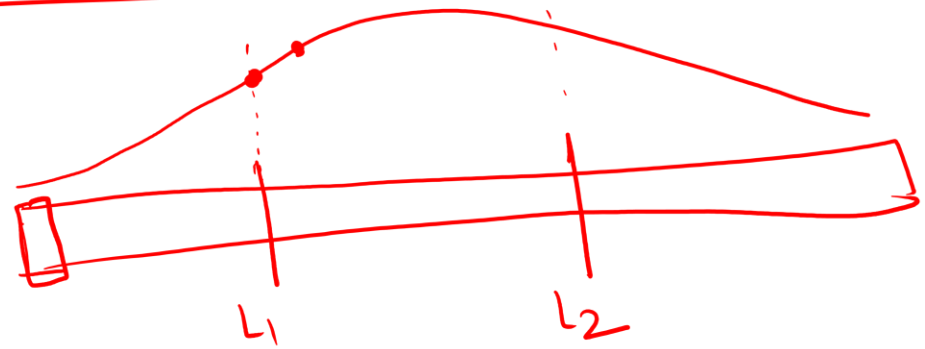
Useless to plot counts or frequencies on Y axis if bucket size → 0 densities Δx

Probability density = $\left[\frac{\text{Count}(x_0, x_0 + \Delta x)}{\text{Total count}}\right] \frac{1}{\Delta x}$

Probability : $\int_{4.211}^{4} f(x)\,\Delta x$

$= \int_{4}^{4.211} (\text{probability density function})\, dx$

Mass of a section in a rod



kg/m

density : $\frac{\Delta M}{\Delta L}$

0 to ΔL → M_1

ΔL to $2\Delta L$ → M_2

$2\Delta L$ to $3\Delta L$ → M_3

$\Delta L \to 0$

$$M = \int_{L_1}^{L_2} (\text{density})\, dL$$

$$\int_{L_1}^{L_2} \frac{\Delta M}{\Delta L}\, dL$$

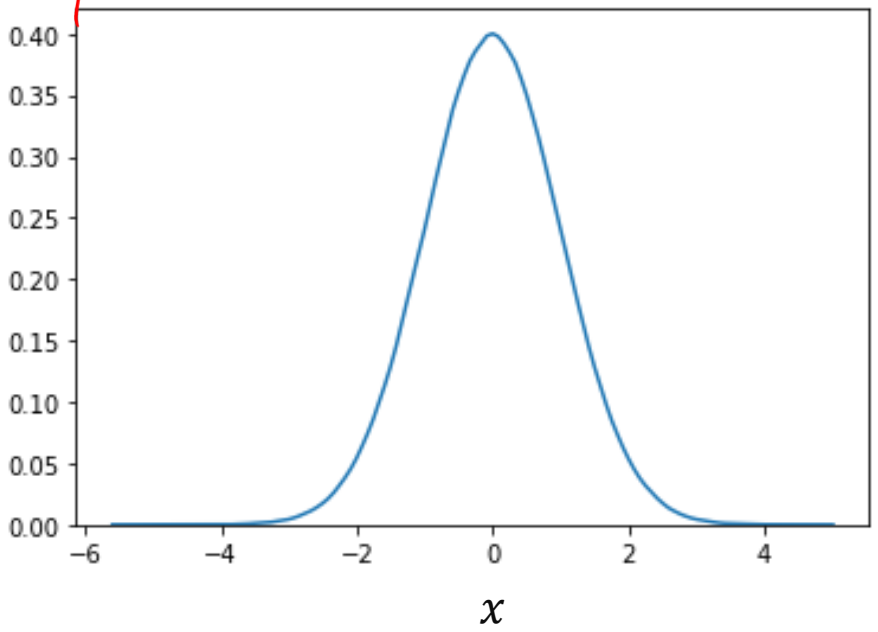
Normal Distribution

In statistics, a normal distribution or Gaussian distribution is used to represent probability distribution of a continuous random variable

As sample size increases the shape of sampling distribution becomes more like a normal distribution

Real word continuous random variables are expected to have normal distribution

- Suppose X denotes the outcome of an experiment where we are randomly sampling average marks obtained by student from all universities



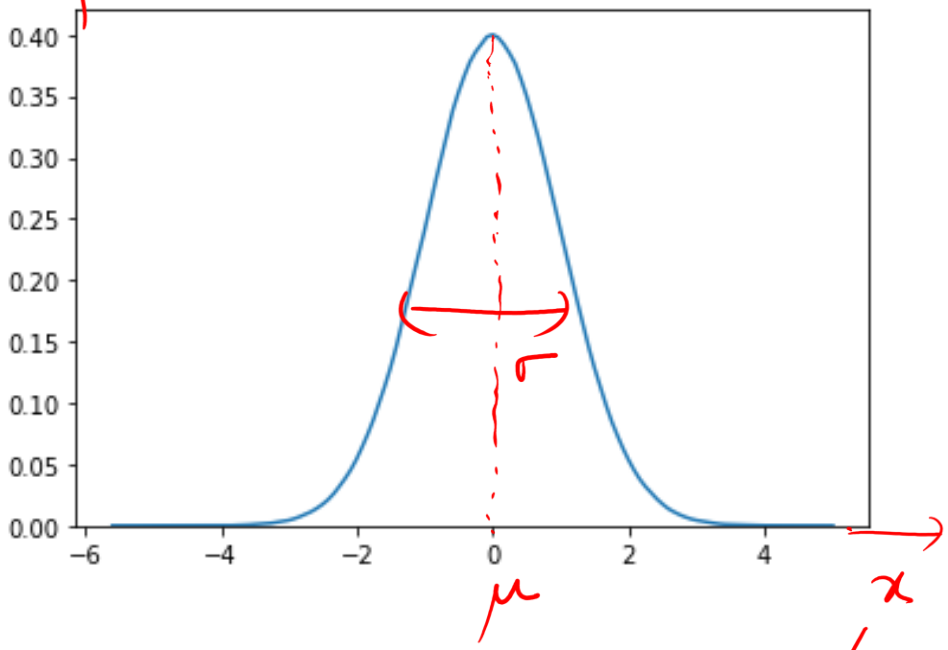
Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\Pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$f(x)$ - probability density function of random variable x

μ is the mean of the distribution

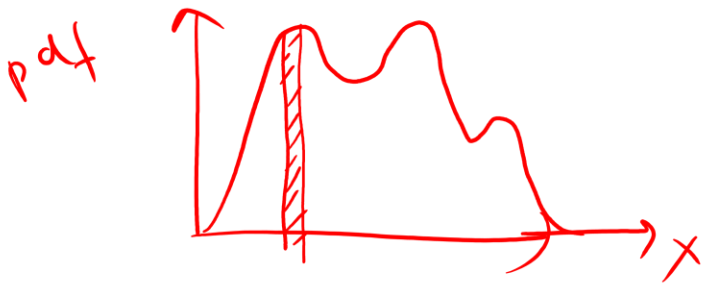
σ is the standard deviation

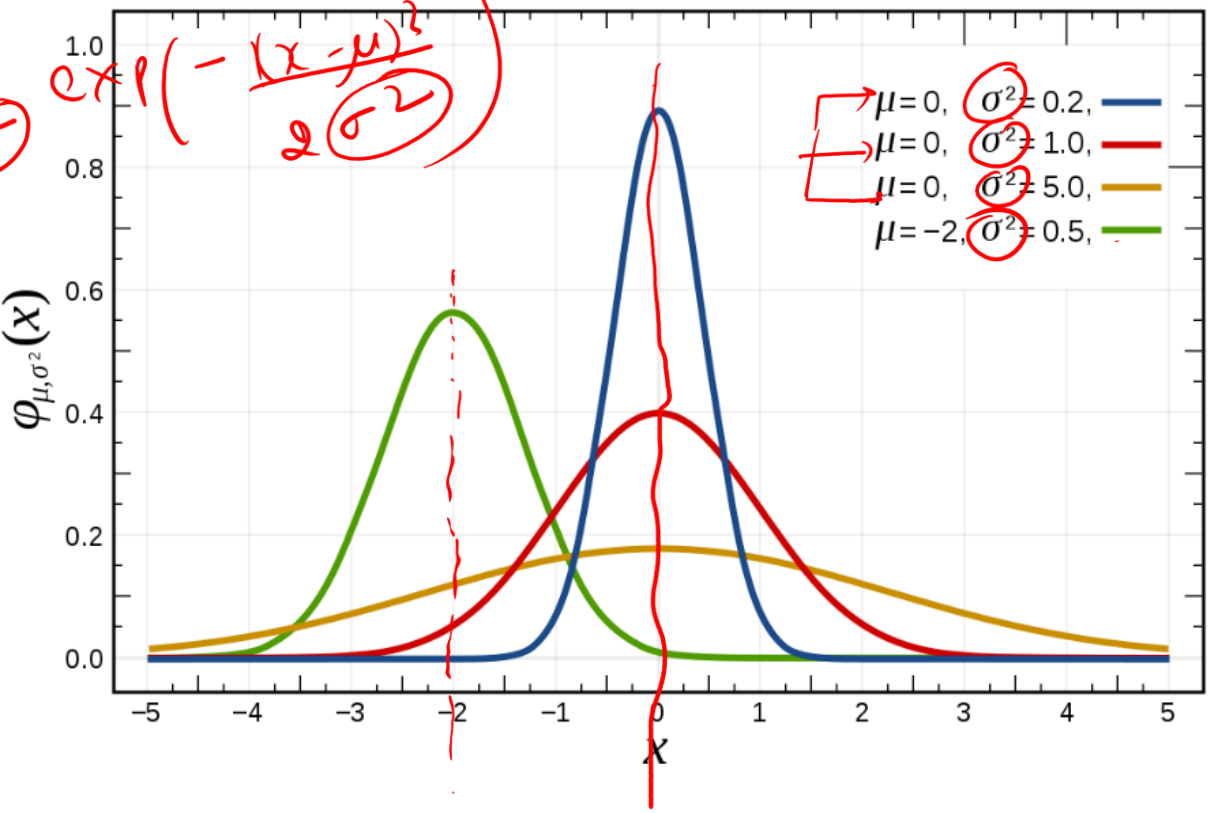


Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\Pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Normal distribution with $\mu = 0$ and $\sigma^2 = 1$ is called as standard normal distribution





Height : $[h_0, h_1, h_2, \ldots, h_{999}]$

descriptive statistic

① μ (ft) $= \dfrac{h_0 + h_1 + h_2 + \ldots + h_{999}}{1000}$

② spread of data points around μ

(ft) $h_0 \rightarrow (h_0 - \mu) = diff_0$ (ft)

(ft) $h_1 \rightarrow h_1 - \mu = diff_1$ (ft)

(ft) $h_2 \rightarrow h_2 - \mu = diff_2$ (ft)

$(ft)^2$ $h_{999} \rightarrow h_{999} - \mu \quad diff_{999}$

variance : $\dfrac{(diff_0)^2 + (diff_1)^2 + \ldots + (diff_{999})^2}{1000}$

$(ft)^2$

standard deviation : $\sqrt{\text{variance}}$

(ft)

Central Limit Theorem

Population

- sample 1 → 300 → μ_1
- sample 2 → μ_2
- ⋮
- sample 100 → μ_{100}
- sample 101
- ⋮
- sample 250

→ pdf → Normal distribution as we keep on increasing sample number

Central limit theorem

The central limit theorem states that if you take sufficiently large number of (>30) samples from a population, the samples mean will be normally distributed, even if the population isn't normally distributed.

E.g., Sample salary of 1000 people living in Pune and plot its distribution

$[h_0, h_1, h_2, \ldots, h_{9999}]$ → 10000 height values

$\mu = \frac{h_0 + h_1 + h_2 + \cdots + h_{9999}}{10000}$, median step 1 sort the data

$[h'_0, h'_1, h'_2, \ldots, h'_{9999}]$

Statistical Values

Dycriptive Statistics

Mode

most occuring data point ; data point with highest probability

[1, 1, 2, 3, 3, 3, 4, 5]

1 → 2
2 → 1
3 → 3 → mode
4 → 1
5 → 1

step 2 To identify the middle data point

[3, 1, 2] median

step 1

[1, 2, 3]

[4, 1, 3, 2]

[1, 2, 3, 4]

$\frac{2+3}{2} = 2.5$

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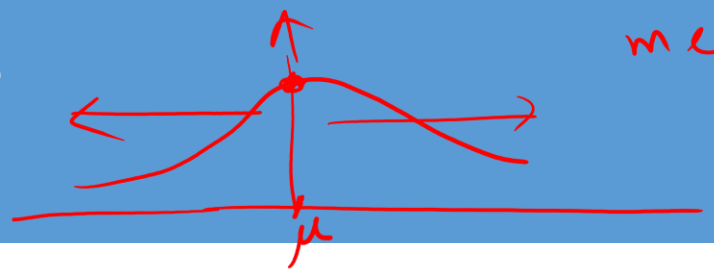
Statistical values

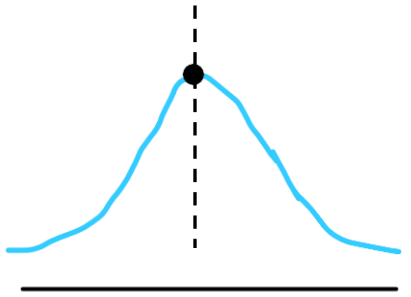


Mean (μ): It is the arithmetic average value of the dataset

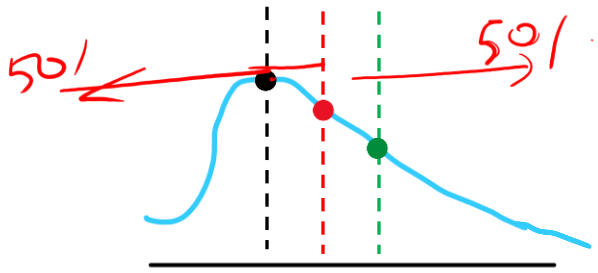
Median: Median is the middle value in sorted dataset. It is calculated by sorting the data in ascending order and then calculating the value at middle index

Mode: It is the most occurring value in the dataset

Mean, Median, Mode



Mode, Median, Mean



Right skewed distribution

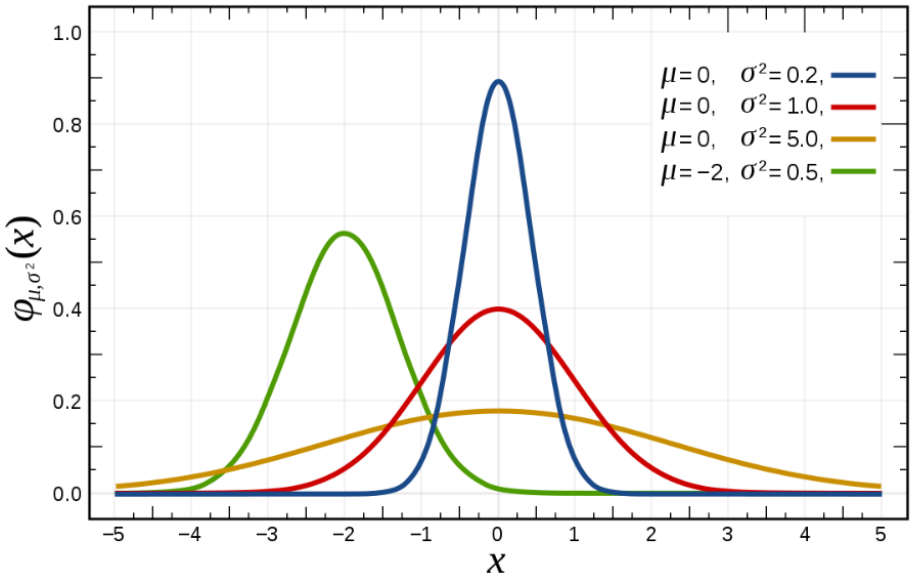
Statistical values

Deviation: It is the difference between the observation and mean of the sample population

Deviation = $x_i - \mu$

Variance: It is mean of squares of deviation

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$



Standard Deviation (σ) : It is the square root of the variance

Height	deviation		
h_0	$h_0 - \mu = d_0$	$\lvert d_0 \rvert$	d_0^2
h_1	$h_1 - \mu = d_1$	$\lvert d_1 \rvert$	d_1^2
h_2	$h_2 - \mu = d_2$	$\lvert d_2 \rvert$	d_2^2
⋮	⋮	⋮	⋮
h_{999}	$h_{999} - \mu = d_{999}$	$\lvert d_{999} \rvert$	d_{999}^2

$$\text{Variance} = \frac{d_0^2 + d_1^2 + d_2^2 + \cdots + d_{999}^2}{1000} \qquad \mu = \frac{\sum h_i}{1000}$$

↓

spread in the data

$[-1, 0, 1] \rightarrow$ 0

$[-5, 0, 5] \rightarrow$ 0

I have 1000 deviation value, how do I summarize them? spread of the data

$$\frac{d_0 + d_1 + d_2 + \cdots + d_{999}}{1000}$$

-1	$-1 - 0 = d_0 = -1$
0	$0 - 0 = d_1 = 0$
1	$1 - 0 = d_2 = 1$

$$\frac{d_0 + d_1 + d_2}{3} = 0$$

Variance (Dataset 1)

Data = [-2, -1, 0, 1, 2]

step 1 $\mu = \frac{(-2) + (-1) + 0 + 1 + 2}{5} = 0$

step 2

Data	Deviations
-2	$d_0 = (-2) - \mu = -2$
-1	$d_1 = (-1) - \mu = -1$
0	$d_2 = 0 - \mu = 0$
1	$d_3 = 1 - \mu = 1$
2	$d_4 = 2 - \mu = 2$

$$\text{variance} = \frac{d_0^2 + d_1^2 + d_2^2 + d_3^2 + d_4^2}{5}$$

$$= \frac{(-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2}{5}$$

$$= \frac{4 + 1 + 0 + 1 + 4}{5} = \frac{10}{5} = 2$$

standard deviation = $\sqrt{\text{variance}}$

$= \sqrt{2}$

Variance Dataset2

Data = $[-10, -5, 0, 5, 10]$

$\mu = 0 = \frac{(-10) + (-5) + 0 + 5 + 10}{5}$

Data	deviation
-10	$-10 - 0 = -10$
-5	-5
0	0
5	5
10	10

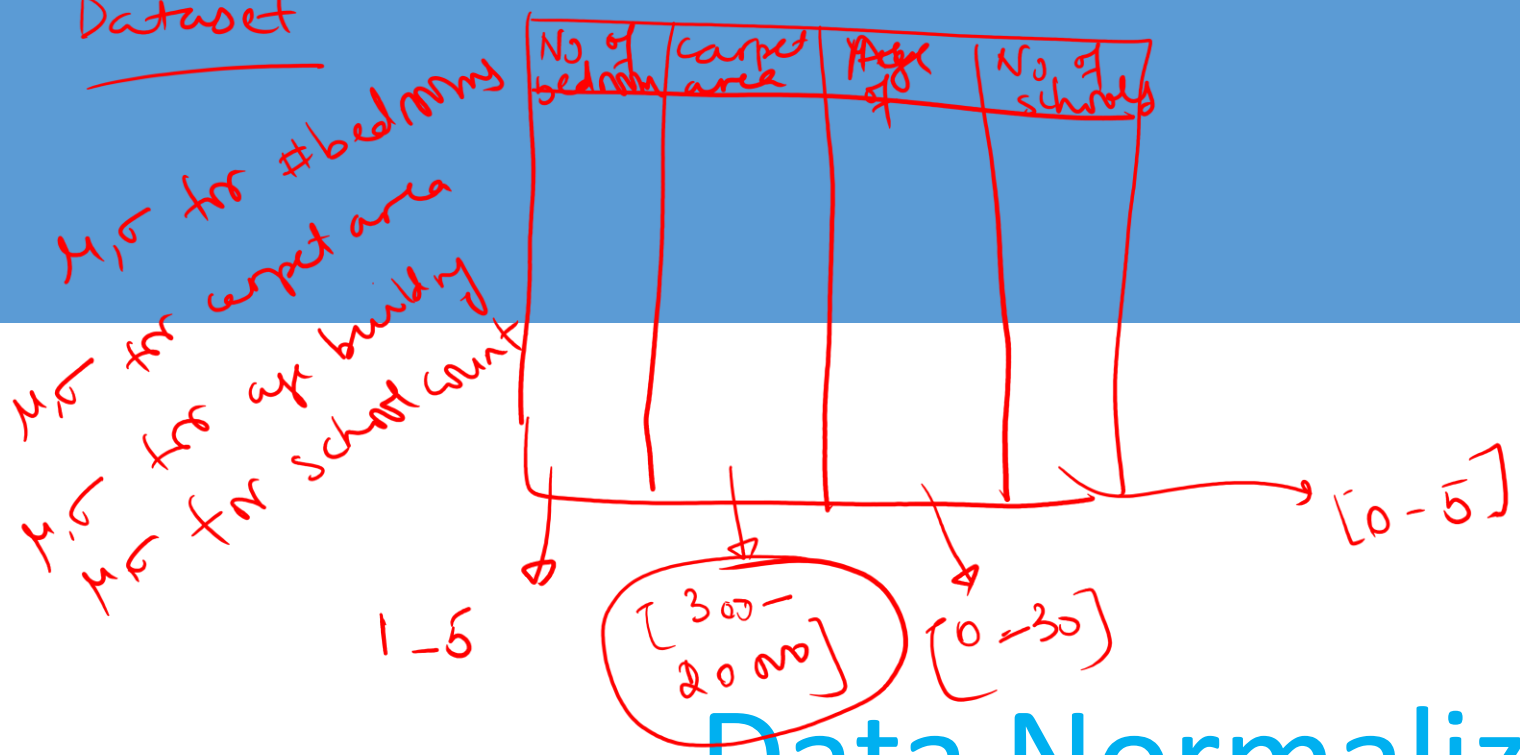
$$\text{variance} = \frac{1}{5}\left[(-10)^2 + (-5)^2 + 0^2 + 5^2 + 10^2\right]$$

$$= \frac{1}{5}\left[100 + 25 + 0 + 25 + 100\right]$$

$$= \frac{250}{5} = 50$$

$$\text{std} = \sqrt{50}$$

Dataset



house price prediction data.

range of the values of different features becomes a hinderance to the model

Data Normalization

No of bedrooms [0,1]

carpet area [0,1]

Age [0,1]

No of schools [0,1]

DP1 → (2, 1000, 5, 2)

DP2 → (4, 2000, 10, 4)

$\sqrt{2^2 + 1000^2 + 5^2 + 2^2}$

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Data Normalization

- Normalization is used to scale the data column (feature) such that all features fall in the same range e.g., 0.0 to 1.0 or to transform the data into distribution with zero mean and standard deviation/variance 1

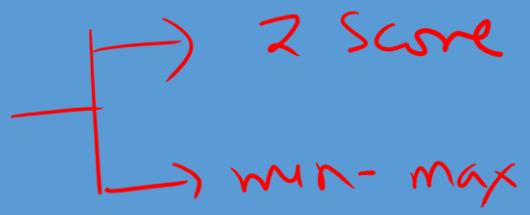
- It is a recommended step for some machine learning algorithms to get correct results e.g., k-means clustering

$DP_1 \rightarrow [0.5, 0.6, 0.2, 0.3]$

$DP_2 = [0.52, 0.8, 0.3, 0.5]$

$\sqrt{(0.2)^2 + 0.2^2 + 0.1^2 + 0.2^2}$

Data Normalization



Z-score normalization:

This method transforms data such that mean will be zero and standard deviation will be 1 for transformed data

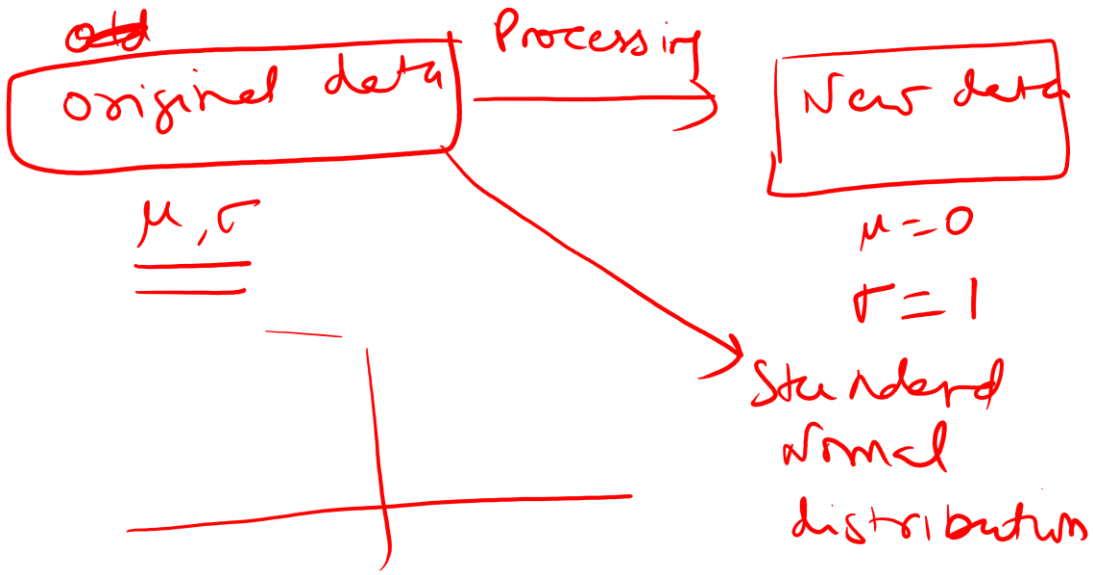
$$z = \frac{x-\mu}{\sigma}$$

z is the new value of feature after applying normalization

x is original value of the feature

μ is mean value

σ is the standard deviation



step 1 μ, σ for original data

step 2 $z_i = \frac{x_i - \mu}{\sigma}$ $\quad x_i$ = i^{th} data point

μ, σ for height

height original

height	Normalized height
h_0	$\frac{h_0 - \mu}{\sigma}$
h_1	$\frac{h_1 - \mu}{\sigma}$
h_2	$\frac{h_2 - \mu}{\sigma}$
⋮	
h_M	$\frac{h_M - \mu}{\sigma}$

Normalised column

Goal was that new column should have new $\mu = 0$ and new $\sigma = 1$

$$\text{new } \mu = \frac{\frac{h_0 - \mu}{\sigma} + \frac{h_1 - \mu}{\sigma} + \frac{h_2 - \mu}{\sigma} + \cdots + \frac{h_M - \mu}{\sigma}}{M+1}$$

$$= \frac{(h_0 - \mu) + (h_1 - \mu) + (h_2 - \mu) + \cdots + (h_M - \mu)}{\sigma \cdot (M+1)}$$

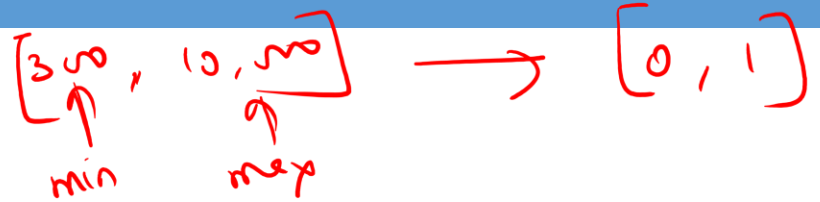
$$= \frac{(h_0 + h_1 + h_2 + \cdots + h_M) - (M+1)\mu}{\sigma(M+1)}$$

$$= \frac{1}{\sigma}\left[\frac{h_0 + h_1 + h_2 + \cdots + h_M}{M+1} - \mu\right]$$

Data Normalization

z score normalization → $\mu = 0$, $\sigma = 1$

Min-Max normalization:



This method transforms data to the range of 0 to 1 (minimum value be 0 and maximum value will be 1 after transformation)

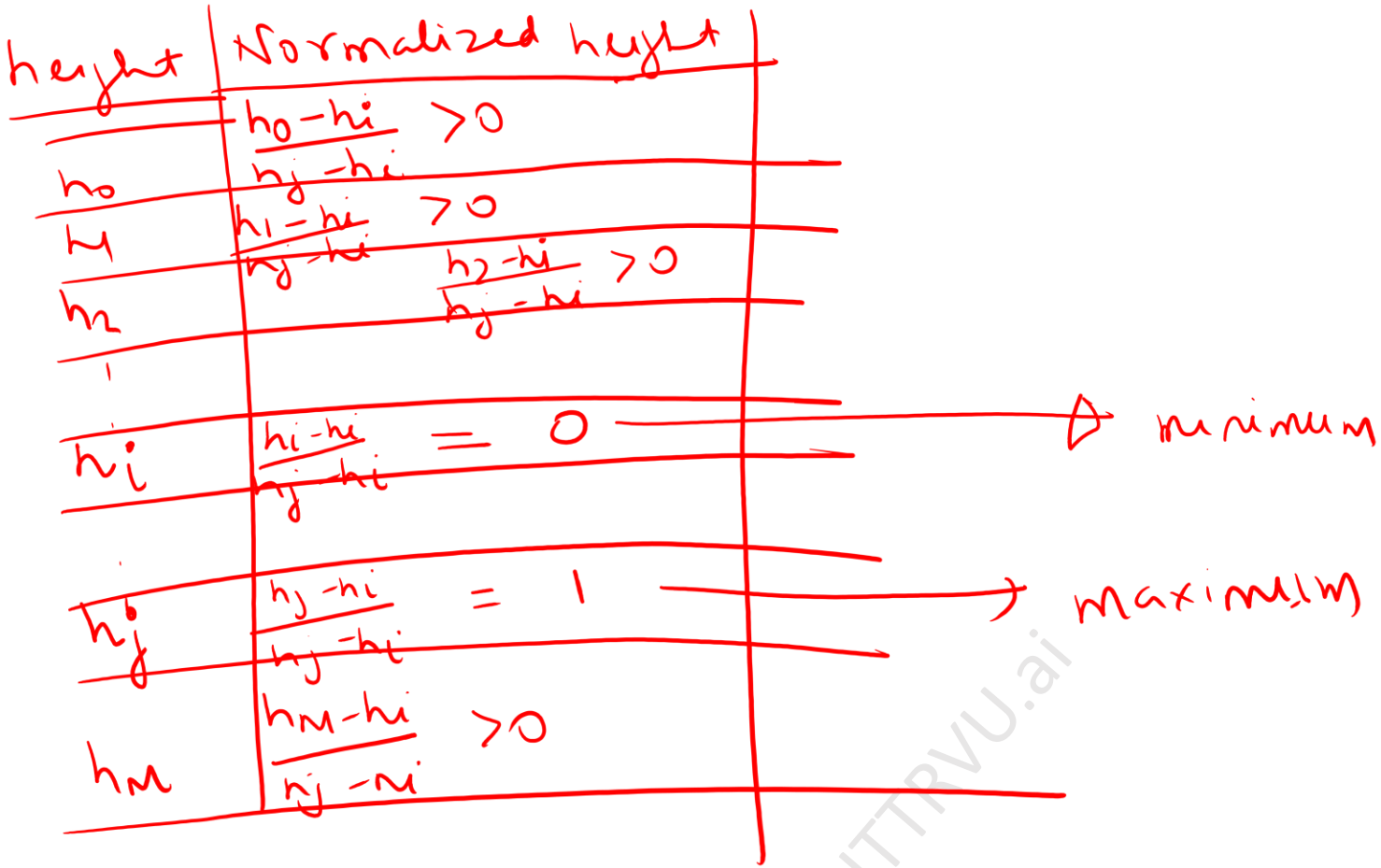
$$z_i = \frac{x_i - \min(x)}{\max(x) - \min(x)}$$

z_i is the new value of feature after applying normalization

x_i is original value of the feature

$\min(x)$ is the min value of feature

$max(x)$ is the max value of feature



Step 1 find min height and max height

$h_i \rightarrow$ minimum height

$h_j \rightarrow$ maximum height

$[0, 1]$

new minimum

new maximum

Data = $[-5, -1, 0, 1, 5, 6, 10]$

$$z_i = \frac{x_i - \min}{\max - \min}$$

min = -5, max = 10

-5	$\frac{-5-(-5)}{10-(-5)} = 0$
-1	$\frac{-1-(-5)}{10-(-5)} = \frac{4}{15}$
0	$\frac{0-(-5)}{10-(-5)} = \frac{5}{15}$
1	$\frac{1-(-5)}{10-(-5)} = \frac{6}{15}$
5	$\frac{5-(-5)}{10-(-5)} = \frac{10}{15}$
6	$\frac{6-(-5)}{10-(-5)} = \frac{11}{15}$
10	$\frac{10-(-5)}{10-(-5)} = \frac{15}{15} = 1$

Objective : real world may have missing data points

ML models can handle complete dataset only

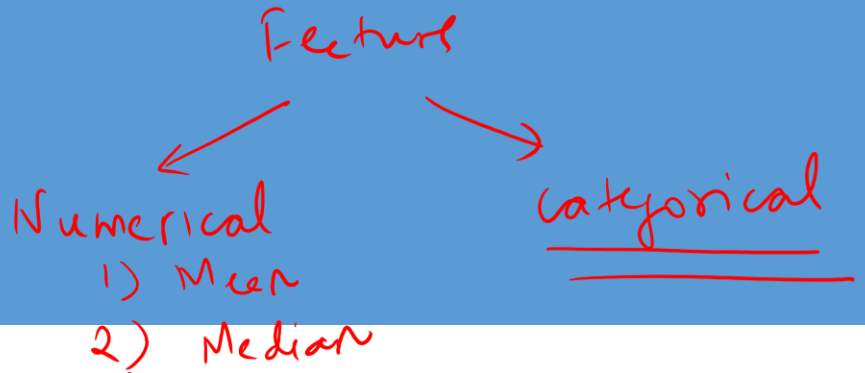
put "some value" in the place of missing value

"NV"

Missing Value Imputation

"informed value" → we will decide what to put in missing value with the help of existing data

Missing value Imputation



- In real world dataset we might get features with missing values
- To use those features in ML models we have to impute the missing values

- **Missing value imputation for numerical feature**
  - Mean value of feature – mean is sensitive to outliers
  - Median value of feature

Temperature
23
25
21
25
26
29
24
27
28

Missing value

Impute by mean: 25.3

- **Missing value imputation for categorical feature**
  - Mode value of feature

Mean method of missing value imputation

X_0	X_1	X_2

X_0
10
20
30
15
λ
40
45

$$\lambda = \frac{10+20+30+15+40+45}{6}$$

Median method

λ = median(10, 15, 20, 30, 40, 45) $= \frac{20+30}{2} = \underline{25}$

Mean vs Median

(1) Mean is affected by outliers

1, 2, 3, 4, 5, 10000

$$\mu = \frac{1+2+3+4+5+10000}{6}$$

(2) Data distribution

Plot the histogram for X_0

X_0 — Numerical

10.1
10.2
9.89
5.5

$P(X = x_0)$ = Not Defined

$P(x_1 \leq X \leq x_2)$ = 

median = (1, 2, 3, 4, 5, 10000)

$\frac{3+4}{2} = 3.5$

Missing value imputation for categorical features

X_1	
'S'	0
'M'	1
'L'	2
'XL'	3
'XXL'	4
'S'	0
'M'	1
'M'	1
'M'	1
'M'	1
~	~

⇒ Mode is 'M'

⇒ we will impute the missing value with mode

0 1 2 3 4

'S', 'M', 'L', 'XL', 'XXL' → categorical

'M', 'F'

0 1

1 > 0

'F' > 'M' → makes no sense

0 - 2 = 2

0.5 0.6

Mean = $\frac{0+1+2+3+4+0+1+1+1+1}{10}$

Median

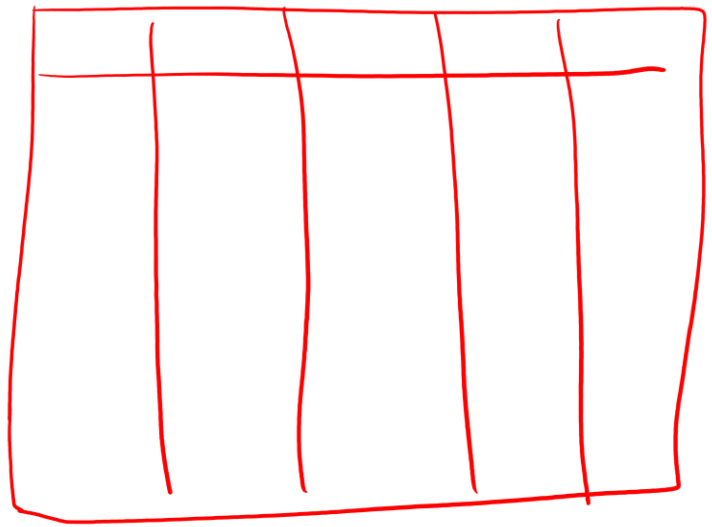
14/10 = 1.4

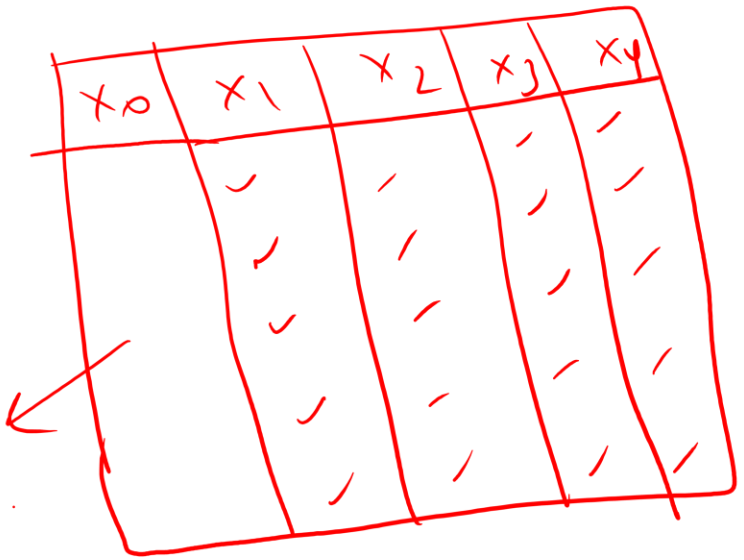
0 + 1 = 1

'S' + 'M' ✗ 'M'

'L' - 'M' = 'M'

Missing value imputation





100,000 total number of rows

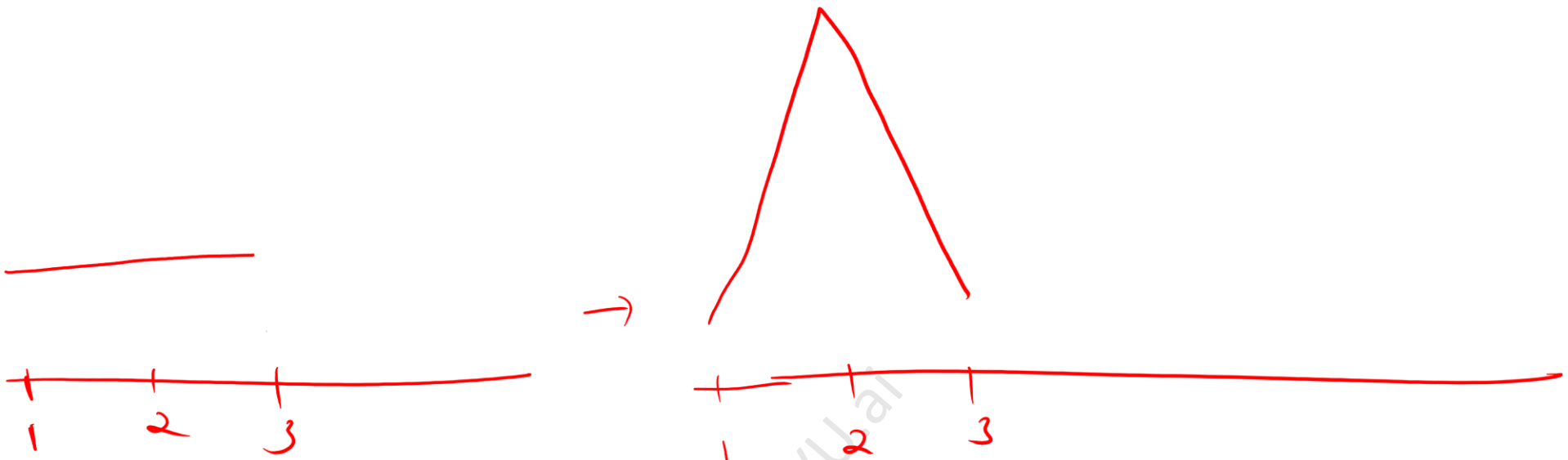
100 rows have missing data

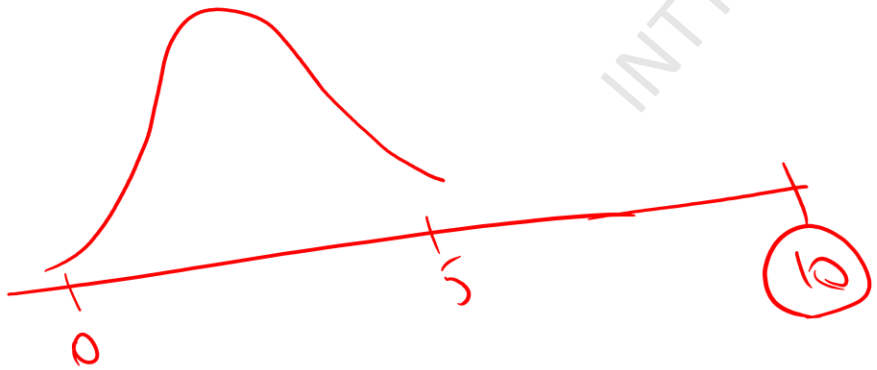
It is easy to change the data distribution

No missing value imputation

=> we simply drop the column

[1, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2]





Outlier → value which is different from the remaining values in dataset

[1, 2, 3, 4, 5, 10000]

[1, 2, 3, 4, 5, 10]

Why do we care about outlier? it changes the underlying data distribution (metrics)

Outlier Detection

~~ centimeters → error

min 1.3

max 2.4

260

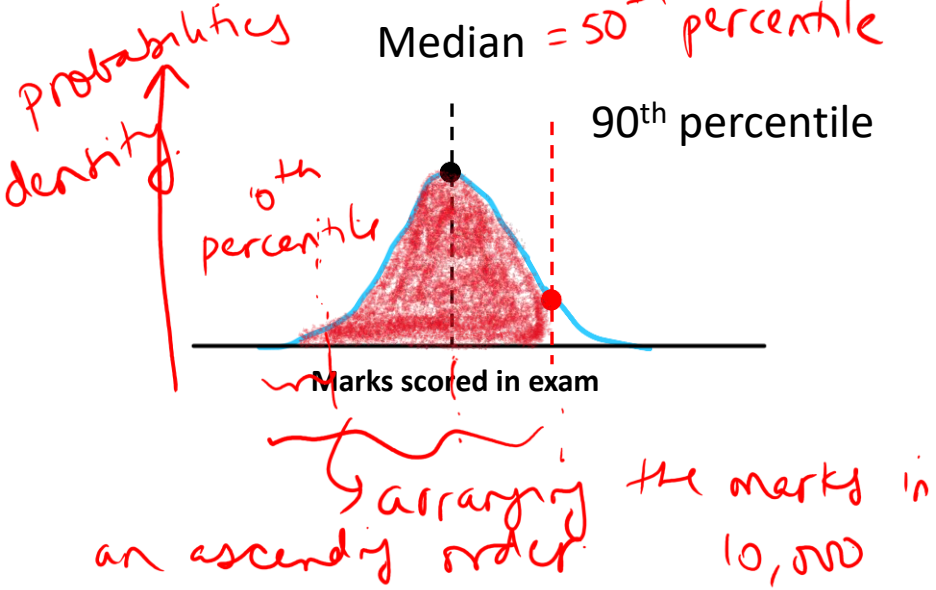
Percentiles

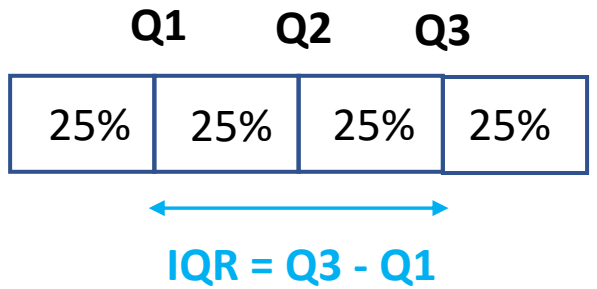
Percentile: It is the value at j% index in data sorted in ascending order

Lower Quartile (Q1): It is the value at 25% index in data sorted in ascending order

Upper Quartile (Q3): It is the value at 75% index in data sorted in ascending order

Inter Quartile Range: IQR = Q3 – Q1





Array 1 [3, 8, 10, 20, 30, 40, 45, 50, 55, 70] electrical engineering

0 1 2 3 4 5 6 7 8 9

50%.

$\frac{5}{10} = 50\%$.

$\frac{10}{10} = 100\%$.

Array 2 [8, 10, 30, 35, 85] mechanical engineering

0 1 2 3 4

$\frac{4}{5} = 80\%$.

Percentiles are useful when we try to compare the ranks

Percentiles are percentages for array indices

Percentiles are "percentages of array indices"

Step 1 Take the array and sort it

Input array → [0, 8, 2, 4, 6, 10, 20, 1, 30, 15]

50th percentile

10 × 0.5 = 5

[0, 1, 2, 4, 6, 8, 10, 15, 20, 30]

0 1 2 3 4 5 6 7 8 9 → indices

25% percentile

10 × 0.25 = 2.5 ← interpolation method

↳ Answer will be number present at 2.5th index

2nd number → 1st index
or
3rd number → 2nd index

Outlier Detection

Q_1 , Q_3

$IQR = Q_3 - Q_1$

① threshold 1 = Lower threshold = $Q_1 - 1.5^* IQR$

② Upper threshold = $Q_3 + 1.5^* IQR$

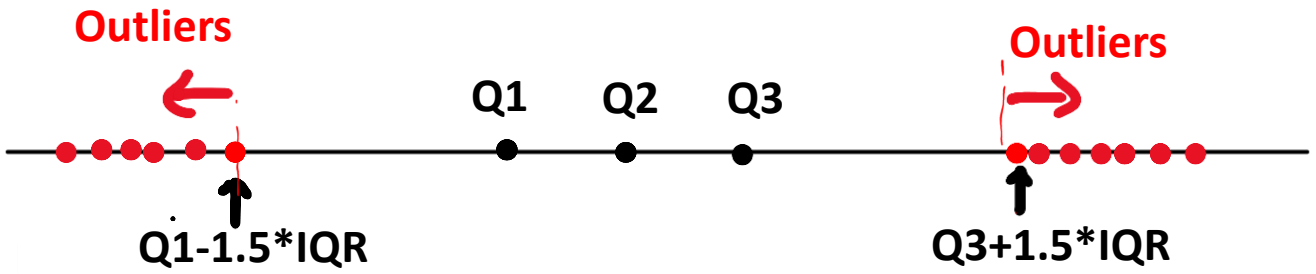
Outlier is an observation which lies at an abnormal value from other values in sample population

IQR is useful in identifying the outliers

good values

$[Q_1 - 1.5^* IQR, \quad Q_3 + 1.5^* IQR]$

Potential outliers lie outside the range: [Q1-1.5*IQR , Q3+1.5*IQR]

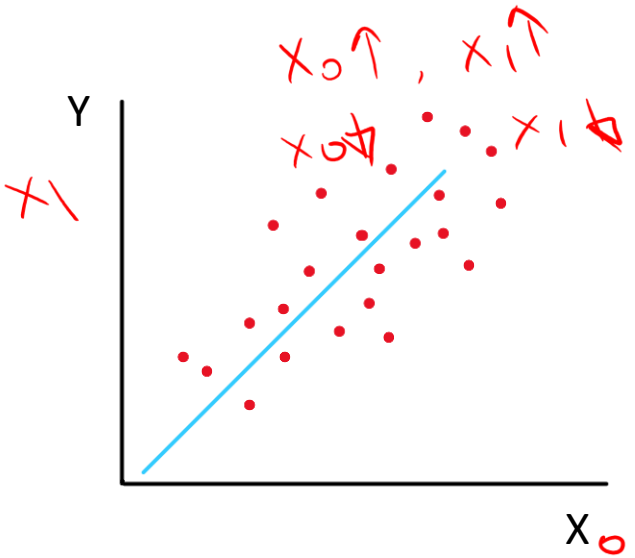


Potential outliers
↳ Bad outliers
↳ Good outliers

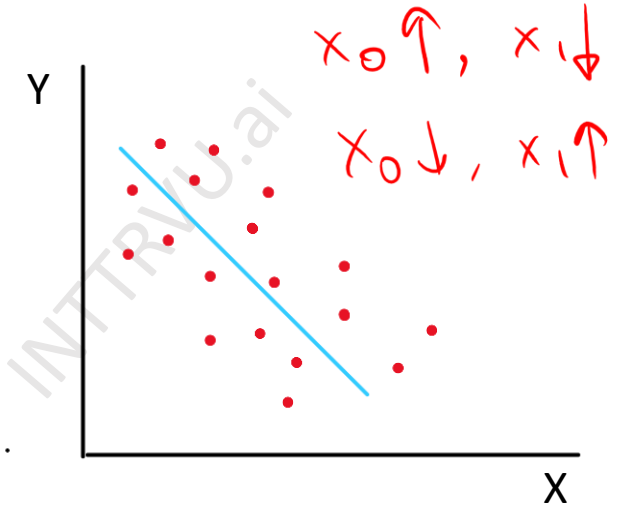
Correlation and Causation

Correlation

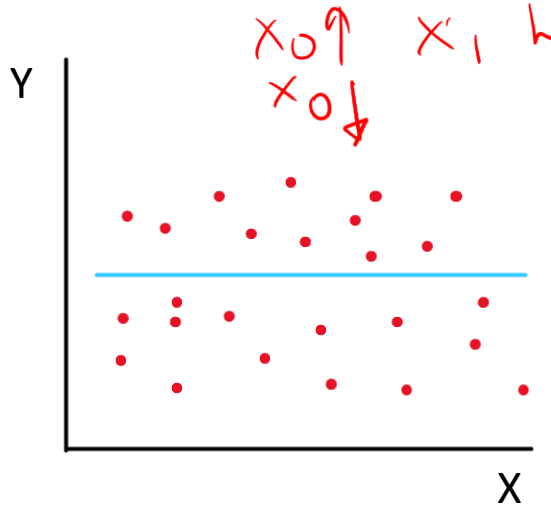
Correlation indicates the degree to which a pair of variables are **linearly** related



Positive Correlation



Negative Correlation



No Correlation

1 Pearson's Correlation Coefficient [-1 to 1]

It is calculated by dividing the covariance of the two variables by the product of their standard deviations.

It is in the range of -1 (perfect negative linear relationship) to 1 (perfect positive linear relationship)
Values near 0 indicate that there is no correlation between the variables

$$p_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

-1 to 1

Covariance is a measure of the joint variability of two random variables (e.g., Higher values of X generally correspond to higher values of Y)

High covariance will give high correlation coefficient as well

$$\rho_{xy} = \frac{\mathrm{Cov}(X,Y)}{\sigma_x \sigma_y} = \frac{E\left[(X-\mu_x)(Y-\mu_y)\right]}{\sigma_x \sigma_y}$$

σ_x, σ_y = standard deviations of X and Y columns

Temperature in °C X	Units of ice cream sold Y
20	100
30	150
40	200
50	400
45	350
35	175

Expectation / mean

$E\left[(X-\mu_x)(Y-\mu_y)\right]$

values in Y column

values in X column

mean of X column

mean of Y column

Co-varying

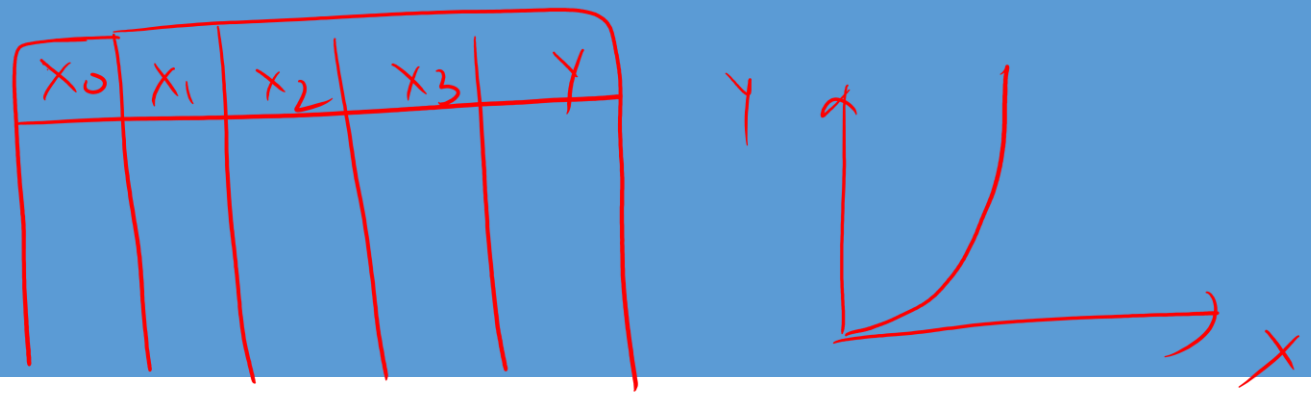
co-variance = variation together

$$E(X) = \frac{20+30+40+50+45+35}{6}$$

Temp x	Units of ice cream sold y	$x-\mu_x$	$y-\mu_y$	$(x-\mu_x)(y-\mu_y)$
20	150	-16.67	-129.16	$16.67 * 129.16$
30	150	-6.67	-79.16	$6.67 * 79.16$
40	200	3.33	-29.16	$-3.33 * 29.16$
50	450	13.33	170.84	$13.33 * 170.84$
45	350	8.33	120.84	$8.33 * 120.84$
35	175	-1.67	-54.16	$16.7 * 54.16$

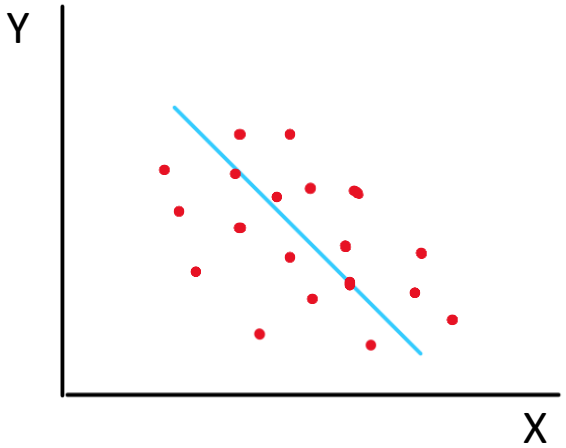
$\mu_x = 36.67$ $\quad \mu_y = 229.16$ $\quad E[(x-\mu_x)(y-\mu_y)]$ = Covariance

Correlation



Correlation for feature selection:

- Correlation is useful in finding numerical features with predictive power which can be used in regression model
- Low correlation does not mean that feature is not useful as correlation covers only linear relationship
- While building machine learning model it is recommended to use the feature in model and evaluate its importance if the feature is useful in that business domain



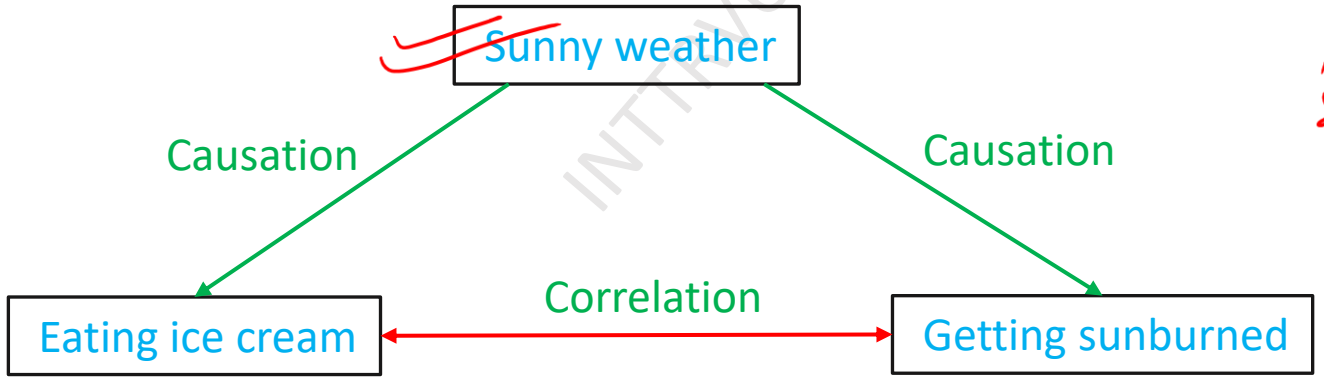
Negative Correlation

Causation

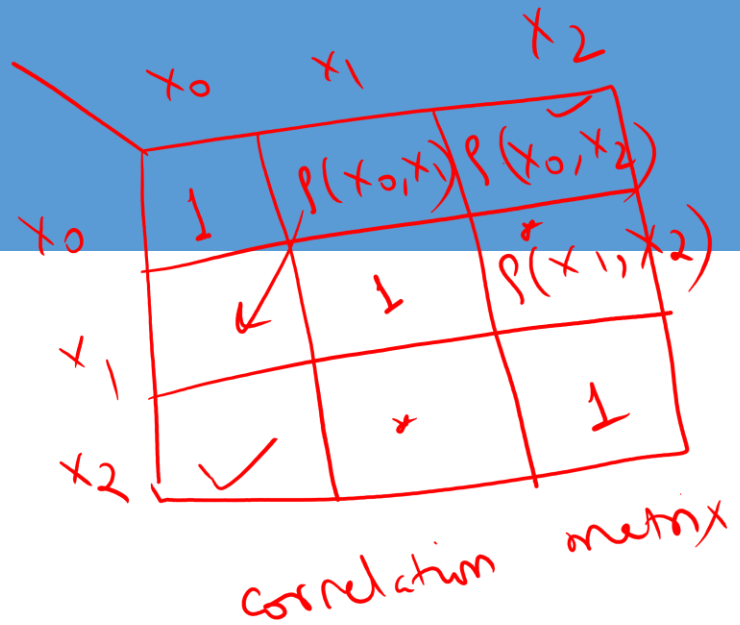
While correlation and causation can exist at same time, correlation does not imply causation

Causation means one thing causes another — e.g., action A causes outcome B.

On the other hand, correlation is simply a relationship where action A relates to action B—but one event doesn't necessarily cause the other event to happen.



There is correlation between eating ice cream and getting sunburned because the two events are related. But neither event causes the other. Instead, both events are caused by something else—sunny weather



heatmap

x_0 = Weight
x_1 = BP
x_2 = Cholesterol

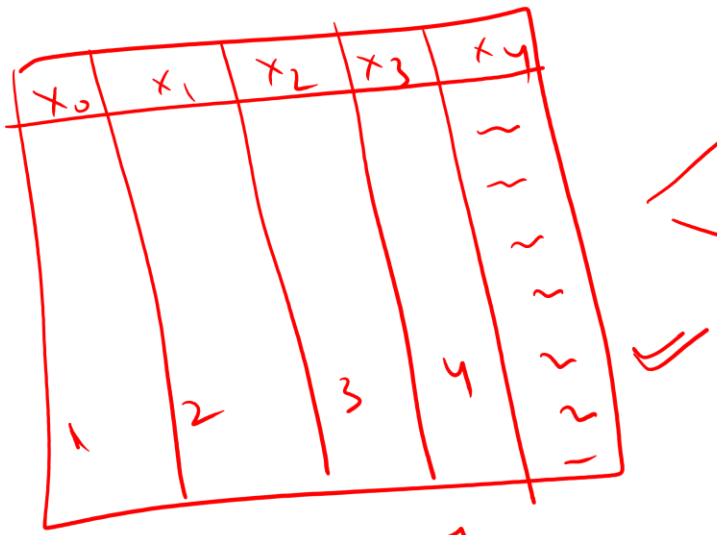
1 feature from row
1 feature from column

Questions?

color mapping with the numbers

INTTRVU

Using ML model for missing value imputation



Data 1 having no missing values (400 rows) ⇒ train a ML model x_0, x_1, x_2, x_3 as features and x_4 as the predict column

Data 2 with missing values of x_4 (100 rows)

(1, 2, 3, y) → Trained ML model → predicted x_4 value

Normalization

sklearn → MinMaxScaler

X	X_transformed
1	(1-1) / (10-1) = 0
2	(2-1) / (10-1) = 1/9
3	(3-1) / (10-1) = 2/9
4	
5	
6	
7	
8	
9	1
10	

x_i → $\frac{x_i - min}{max - min}$ = z_i

min = 1
max = 10

fit "method"

$z_i = \frac{x_i - min}{max - min}$ → transform "method"

fit_transform = fit + transform

Standard Scaler

$$z = \frac{x - \mu}{\sigma}$$

μ, σ → fit

$\frac{x - \mu}{\sigma}$ → transform

X
1
2
3
4
5
6
7
8
9
10