Unit-iv

Bueueing Theory:

Introduction:

There are many situations in daily life

when a queue is formed for example, trackings woulding to be repaired, padients waiting in a doctor's room, cash waiting at a traffic signal and parsengers waiting to buy lickels in counters form queues.

Queue in formed if the Seaver required by the Customer (machine, podiend, car.edc) is not immediately available, that is if the current demand for a particular Service exceeds the capacity to provide the Service.

There are many types of Queueing Systems, all of them can be Classified and described according to the following characteristics.

1. The input (or curival) pottern

The input describes the manner in which the customers arrive and in the Queueing System. It is not possible to observe and control the adual moment of arrival of a customer for Service.

The mode of arrival of customers is

expressed by means of the probability distribution of the number of arrivals per unit of time (or) of the index-arrival time.

Here, I'm no ay arrivals per unid of time has a poisson divinitudion with mean .

page . (B)

Suppose, the time between consecutive arrivals has an exponential distribution with mean 1/2,

The Service Madanism (or pattern):

The mode of service is represented by the means of the prob distribution of the mood customers serviced per unit time or of the inter-service time. We shall mostly deal with only those auchieng systems in which the mood customers serviced per unit of time has a poisson distribution with mean in correquivalently the inter-service time has an exponential distribution with mean it.

The Queue discipline:

manner in which the customers form the queue con equivalently the manner in which they are selected for service, when a queue has been formed.

The mose common deciptine in the FCFS (First come first Served) for FIFO (First infirst owl) as per which the historness are served in the Served order of their arrival.

It the last arrival in the System is Sowed first, we have the fifth (Last in first out) discipline. Symbolic Representation of a Queueing Model.

Usually a queueing model in specified and represented symbolically in the John (alblidde)

and represented symbolically in the John (alblidde)

a -> two of arrival per unit time.

b -> the lypes of distribution of the Service time.

c-> the capacity of the System.

Accordingly, the first Jour models which we will deal with will be denoted by the Symbols

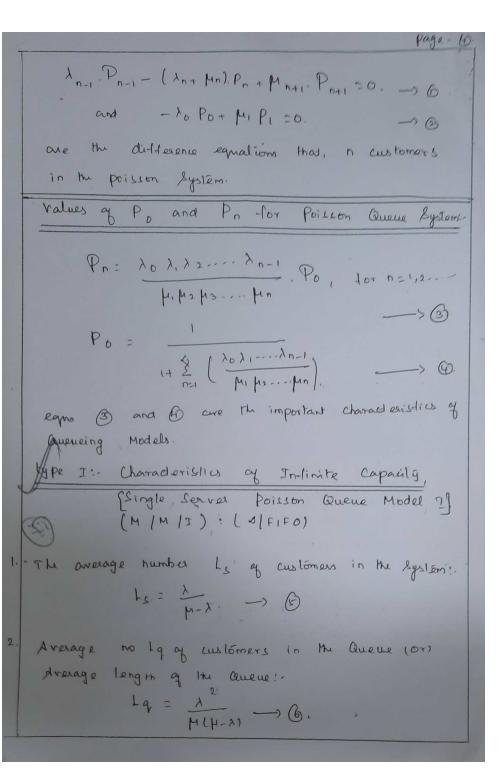
will deal with will be devoted by the symbols [M [M]]: (&[FIFO), (M[M]S): (&[FIFO), M[M]S): (&[FIFO],

In the above symbols, the letter M stands for Markov indicating that the two ag arrivals in time of and the two ag completed services in time of follow poisson process which is a continuous time to asked chain.

Difference equations Related to poisson Queue:

Let Prits be the prob that there are in customers in the system and time t (150).

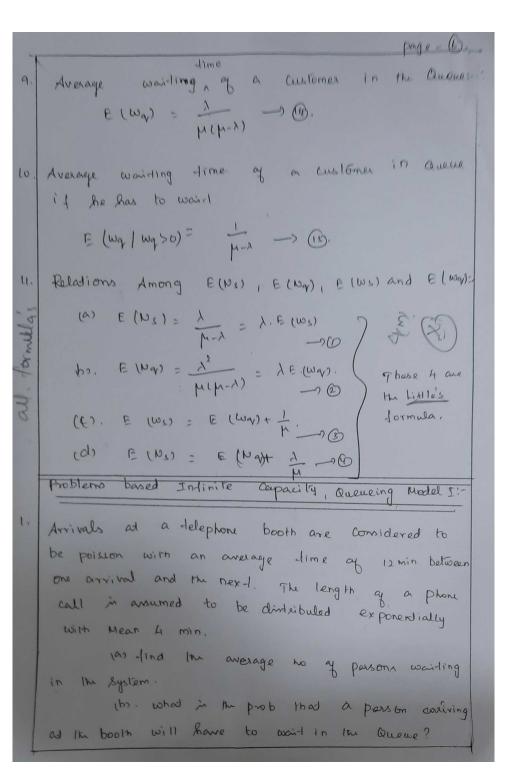
Let An be the average against rate when there are in constoners in the system; Min be the average Service rate when there are in customers in the system.



E (ws) = 1 - 10

7. Probability that the waiting time of a customer in the Bystem exceeds t.

Hu Queue:
9 (w) = 1 (\mu - 1) . w - 1 (\mu -



there is mis, altogether to would for the phone and longlish his cast?

Ido Estimals the Fraction of the day when the Thomas will be in use.

When servined their an arrival has to wait on the amongs for attend 3min for phone. By howmuch the flow of arrivals should increase in order to justify a second booth?

How downs from time to line?

Solution: Mean inter-actival time = $\frac{1}{\lambda}$ = 12 min. Therefore mean actival rate = λ = $\frac{1}{12}$. Mean Service time = $\frac{1}{\mu}$ = 4 min.

.. The Mean Service rate = $\mu = \frac{1}{H}$ presminute.

.. | E (1) = 0.5 | customer.

60. Plubol: 1- p(N=0)

= 1 - Para of customes in the System). = 1 - Po = 1 - (1 - fr) = 1 - 1 + 1 = 1/12 = 1/3

(d) P(the phone will be idle) = P(1) = 0.18

to fraction of the day, when the phone will be in use = 1/3.

(e). The Second Phone will be installed.

14 E (wg)>3.

ie, 2 >3.

Page - (9) is the arrival rate Should increase by Blas 1/10: 1/42 per minute, to gusting a Second phone. VI) EL Ny | the Queue is always available) Customers arrive ad a come-man barber shop according to a poisson process with a mean interactival time 12 min. customers Spend an average of 10 min in the barbers. (as what in the expected no a austomers in the bouber shop and in the queue? in calculate the periendage of time an arrival can walk straight into the barber's chair without having a waid. (c) How much time can a costoma expect to spent in the barba's shop? customers waiting time in the Shop exceeds 1.25 & how much must the average rati orrivals increase to warrend a second backer?

Lire another barber, when a average sale of assimble increase to women to warrend a second barber?

(e). what in the average time customers

Spend in In Queue?

H). what is the prob that I'm waiting -lime

in the System in greaterthan 30 min?

Go Calculate the parentage of customers who have to ward prior to gotting into the bar beils chair.

who what in the prob. It and more than 3 customers are in the system?

Johnson: Given = 12. .. $\lambda = \frac{1}{12}$ per (minute) $\frac{1}{\mu} > 10$.: $\mu = \frac{1}{10}$ per minute.

shop and in the Queue

hs: = (N1) = 1 = 12 /10 - 1/2

$$\frac{1}{12D} = \frac{1}{12D} \times \frac{10}{12D}$$

$$= 5 \text{ customers}$$

[E (Us)=5 |

b) Pla customer straight goes to the barbers char)

$$= \frac{1}{10} \frac{1}{120}$$

$$= \frac{2}{6} = 4.17 \text{ customers.}$$

b) Pla customer straight goes to the barbers char)

$$= \frac{1}{10} \frac{1}{120} = \frac{1}{120} =$$

(d). E(w)>75 id 1/4-2>75. 1-> 10- 75 11 1x> 13

Hence, to warrend a second backer, the average arrows Palt must in crease by 13 150 - 12 - 1 300 per (minuly).

E) E(wq 1: 1/2 / 1/2)

| 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 |

20 50 min.

6. p(w>t) = [(H-A)6]

(1) (1 orange) = 0 (1/6 - 1/2) × 30 $= \left(\begin{array}{c} 2 \\ +2 \phi \end{array} \times 8 \phi \right)$ = e = e = 0.6065

(9) Pla customer has to wait) = plasso)

$$= 1 - P(w_{20})$$

$$= 1 - P_{0} = \frac{\lambda}{\mu} = \frac{1}{2} \frac{1}{2} = \frac{1}{2} \frac{1}{2} \times \frac{10}{1} = \frac{1}{2} \frac{1}{6}$$

-: Perendage of customers who have to wast = 5/2×100 = 83.33 [iRs $P(NSS) = P_{4} + P_{5} + P_{6} + \dots$] $= 1 - [P_{6} + P_{1} + P_{2} + P_{3}]$ $= 1 - [1 - \frac{\lambda}{\mu}] \cdot [1 + \frac{\lambda}{\mu} + (\frac{\lambda}{\mu})^{2} \cdot (\frac{\lambda}{\mu})^{3}].$ $\int_{-1}^{1} P_{n} = (\frac{\lambda}{\mu})^{n} \cdot (1 - \frac{\lambda}{\mu})^{3} \cdot n \cdot so. \quad \text{{f from Modell}}.$

= 1/2 (5/6)4=0.4823. (x)

overage rate of 6 per minute, it takes an average of the Seconds to purchase a tracket. It a Person arrives 2 min before the picture starts and it its takes exactly 1-5 min to reach the correct seat after purchasing the ticket.

(a) Can he expect to be seaded for the stand of the picture?

(b) what in the prob that he will be sealed for the Start of the picture?

(C) How early must he arrive in order to be 99 x

Sure of being secuted for the start of the picture?

Solution: $\lambda = 6 \mid \min, \quad \mu = 8 \mid \min$

(a) $E(w) = \frac{1}{8-6} = \frac{1}{2} min.$

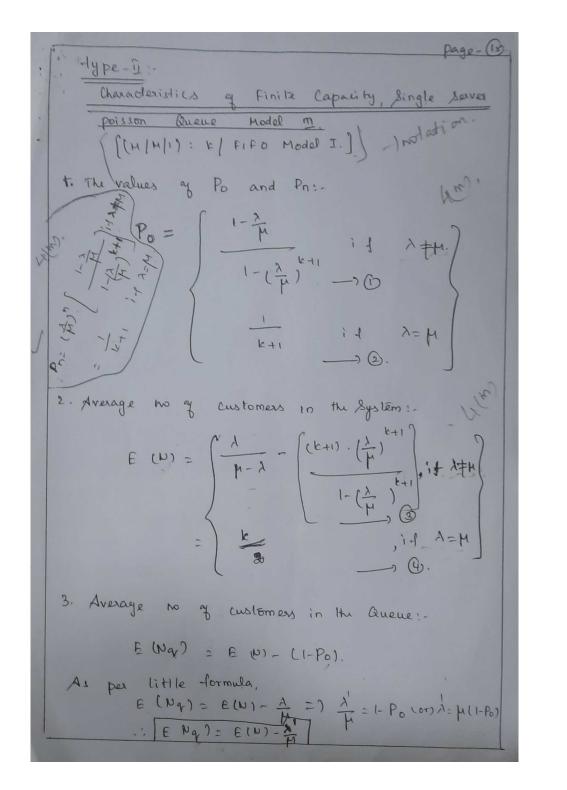
is E (total time required to purchase the dicket and to reach the lead)

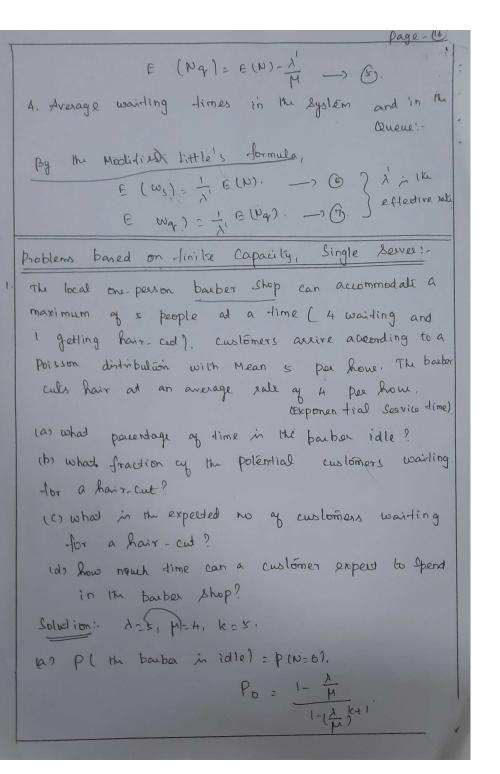
= \frac{1}{2} + 1. \frac{1}{2} = 2 \text{min.}

Hence, he can just be sealed for the start of the picture.

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to plantal ame Lamin) = Plus 1/6)
                       = 1- P(00 > /s)
                       = 1-e + (1-2) ×1/2
                       = 1-8(1-6/8) × /2
                       = 1-e 2/9)×1/2
                        21-0 20.63
er Pl wet 1:0.99
        ie., P(w>t) 20.01
         ien = (4-2) = 0.01
               -(8-6) b
               [2t]
              -2+ = log(0.01) = 2.3
              -2 t = 2.3 | t = 1.15 min
ie., Pl ticked puckasing time L1.15) =0.99
    -> P ( total time to get the ticked and to go to
the seal ) L (1.15+1.5) =0.99
      .. The poeson must arrive alleast 2.61 min leady
     so as to be 99%. Sure of Seeing the stant of
     the pidue.
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-: Y. of time when the barber in idle: 9.

(b) Pla customer is turned away) = p(1) 5 51.

$$P_{n} = \left(\frac{\lambda}{\mu}\right)^{n} \cdot \left(\frac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)}\right)^{n} \cdot \left(\frac{\lambda+\mu}{1-\left(\frac{\lambda}{\mu}\right)}\right)^{n} \cdot \left(\frac{\lambda+\mu}{1-\left(\frac{\lambda}{\mu}\right)}\right)^{n$$

in probay a customer in Juned away in 0.2711

(c). The expended no of customers wai-ling for a

$$=\frac{\lambda}{\mu-\lambda}-\frac{(b+1)\cdot\left(\frac{\lambda}{\mu}\right)^{b+1}}{1-\left(\frac{\lambda}{\mu}\right)}$$

(ds. E(W)= 1, E(W).

= 1 xE(W)

M(1-Po)

= 3.1317 = 0.8592 h)

3.6448

(or) \$1.5 min.

At a railway station, only one train in handled at a lime. The vailway yard in sufficient only for 2 trains to wait, while the other in given Signal to leave the Station. Trans arrive at the Station at an average sate of 6 per how and the Railway Station can handle them on an average of 6 per how. Assuming Poisson arrivals and exponerdial service dintribution. Find the prob for the numbers of dains in the system. Also find the average waiting time of a new train coming into the food. It the handling gate in doubled, how will the above Results get modi-fied? Solution: 1/26 par hr, p/26 per lhr. 6=2+1=3. Since, do M, Po= 1 1x+1

|x+1| $=\frac{1}{3+1}=\frac{1}{4}$ $|x-1|=\frac{1}{4}$

F(N) = & = 1-5 draim.

$$F(w) : \frac{1}{A} = F(w)$$

$$= \frac{1 \cdot S}{M(1-P_0)} = \frac{1 \cdot S}{b \times (1-\frac{1}{4})} = \frac{1 \cdot S}{b \times (2 \times 4)}$$

$$= \frac{3}{9} : \frac{1}{3} A \text{ form}$$

$$= \frac{3}{9} : \frac{1}{3} A \text{ form}$$

$$= \frac{3}{9} : \frac{1}{3} A \text{ form}$$

$$= 20 \text{ min}$$

$$P_0 : \frac{A}{\mu}$$

$$= \frac{1}{1-(\frac{1}{4})^{k+1}} = \frac{1-\frac{k}{1}}{1-(\frac{1}{4})^{k}} = \frac{1-\frac{1}{4}}{1-\frac{1}{4}}$$

$$= \frac{1}{1-\frac{1}{4}}$$

$$= \frac{1}{1-$$

$$\frac{1}{|x-\lambda|} = \frac{1}{|x-\lambda|} =$$