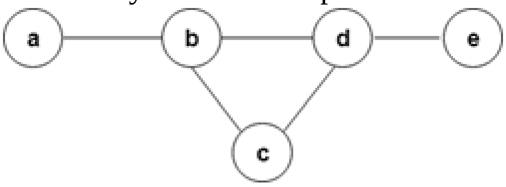
Part - A

1. Backtracking algorithm is implemented by constructing a tree of choices called as?
a) State-space tree
b) State-chart tree
c) Node tree
d) Backtracking tree

2. Of the following given options, which one of the following is a correct option that provides an optimal solution for 4-queens problem?
a) (3,1,4,2)
b) (2,3,1,4)
c) (4,3,2,1)
d) (4,2,3,1)

3. How many Hamiltonian paths does the following graph have?



a) 4
b) 2
c) 1
d) 3

4. Which of the following is not a branch and bound strategy to generate branches?
a) LIFO branch and bound
b) FIFO branch and bound
c) Lowest cost branch and bound
d) Highest cost branch and bound

5. The Data structure used in standard implementation of Breadth First Search is?
a) Stack
b) Queue
c) Tree
d) Linked List

6. What is a randomized Quick Sort?
a) The leftmost element is chosen as the pivot
b) The rightmost element is chosen as the pivot
c) Any element in the array is chosen as the pivot
d) Pivot element can not be chosen

7. What is the purpose of using randomized quick sort over standard quick sort?
a) so as to avoid worst case space complexity
b) so as to avoid worst case time complexity
c) to improve accuracy of output

d) to improve average case time complexity

8. __________ is the class of decision problems that can be solved by non-deterministic polynomial algorithms.
a) NP
b) P
c) Hard
d) Complete

9. What is the basic principle in Rabin Karp algorithm?
a) Hashing
b) Sorting
c) Augmenting
d) Dynamic Programming

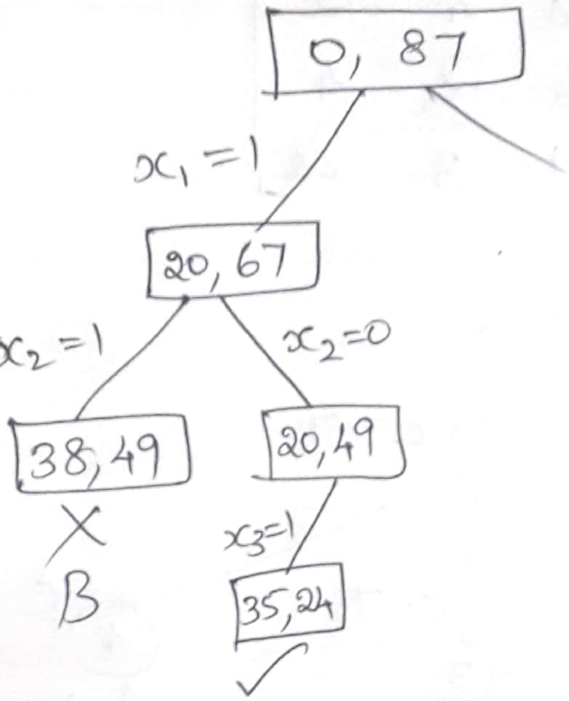
10. Hamiltonian path problem is __________
a) NP class problem
b) P class problem
c) NP complete problem
d) None of the above

Part-B

Sum of subsets

11. $w = \{20, 18, 15, 12, 10, 7, 5\}$

$m = 35.$



x						
1	0	1	0	0	0	0

$20+15=35$

Other solutions

0, 1, 0, 1, 0, 0, 1 $\quad 18+12+5=35$

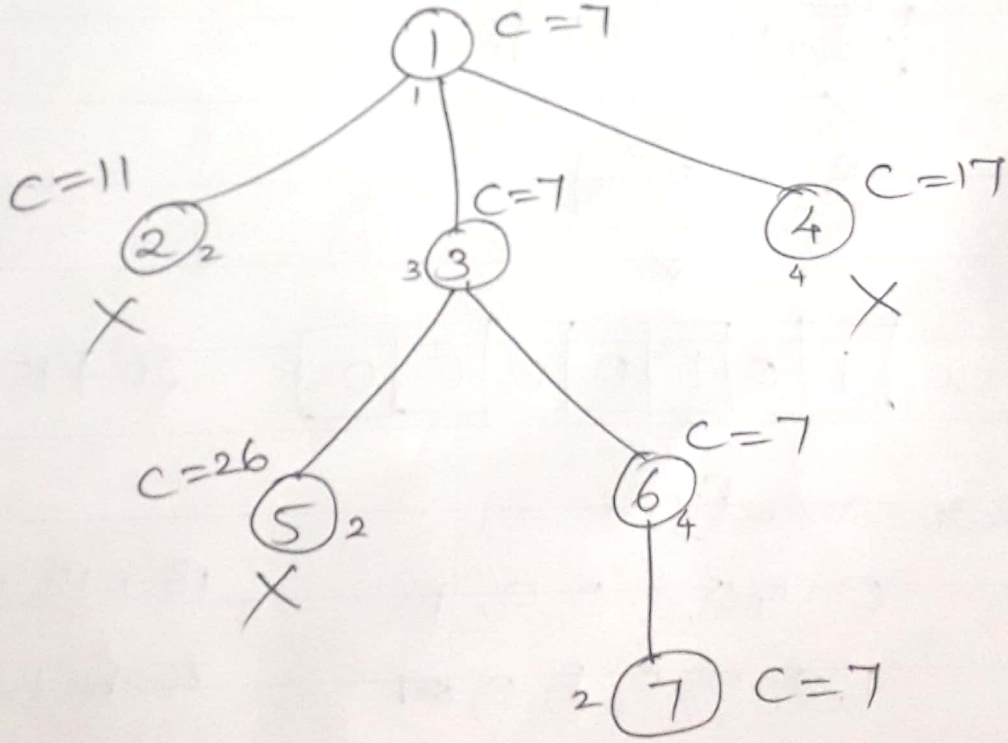
1, 0, 0, 0, 1, 0, 1 $\quad 20+10+5=35$

⋮

12. Travelling sales person Problem - LCBB procedure

$$
\begin{array}{c c}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} &
\begin{bmatrix}
\infty & 5 & 1 & 10 \\
1 & \infty & 4 & 12 \\
3 & 6 & \infty & 4 \\
7 & 1 & 3 & \infty
\end{bmatrix}
\end{array}
$$

State-space Tree



Node 1

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix} \infty & 5 & 1 & 10 \\ 1 & \infty & 4 & 12 \\ 3 & 6 & \infty & 4 \\ 7 & 1 & 3 & \infty \end{bmatrix}
\end{array}
\begin{array}{c} \\ 1 \\ 1 \\ 3 \\ 1 \\ \hline 6 \end{array}
\Rightarrow
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ \\ \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix} \infty & 4 & 0 & 9 \\ 0 & \infty & 3 & 11 \\ 0 & 3 & \infty & 1 \\ 6 & 0 & 2 & \infty \end{bmatrix} \\
\begin{array}{cccc} 0 & 0 & 0 & 1 \end{array} = 1
\end{array}
$$

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix} \infty & 4 & 0 & 8 \\ 0 & \infty & 3 & 10 \\ 0 & 3 & \infty & 0 \\ 6 & 0 & 2 & \infty \end{bmatrix}
\end{array}
$$

Reduced cost $= 6 + 1 = 7$

Node 2

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 7 \\ 0 & \infty & \infty & 0 \\ 4 & \infty & 0 & \infty \end{bmatrix}
\end{array}
$$

$\text{cost}(1,2) + r + \hat{r} = 4 + 7 + 0 = 11$

Node 3

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \infty & \infty & \infty & \infty \\ 2 & 0 & \infty & \infty & 10 \\ 3 & \infty & 3 & \infty & 0 \\ 4 & 6 & 0 & \infty & \infty \end{array} \quad cost(1,3) + r + \hat{r} = 0 + 7 + 0 = 7$$

Node 4

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \infty & \infty & \infty & \infty \\ 2 & 0 & \infty & 3 & \infty \\ 3 & 0 & 3 & \infty & \infty \\ 4 & \infty & 0 & 2 & \infty \\ \hline & 0 & 0 & 2 & \end{array} = 2 \Rightarrow \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \infty & \infty & \infty & \infty \\ 2 & 0 & \infty & 1 & \infty \\ 3 & 0 & 3 & \infty & \infty \\ 4 & \infty & 0 & 0 & \infty \end{array}$$

$$Cost(1,4) + r + \hat{r} = 8 + 7 + 2 = 17$$

Node 5

$$\begin{array}{c|cccc|c} & 1 & 2 & 3 & 4 & \\ \hline 1 & \infty & \infty & \infty & \infty & \\ 2 & \infty & \infty & \infty & 10 & 10 \\ 3 & \infty & \infty & \infty & \infty & \\ 4 & 6 & \infty & \infty & \infty & 6 \\ \hline & & & & & 16 \end{array} \Rightarrow \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & 0 \\ 3 & \infty & \infty & \infty & \infty \\ 4 & 0 & \infty & \infty & \infty \end{array}$$

$$Cost(3,2) + C(3) + \hat{r} = 3 + 7 + 16 = 26$$

Node 6

$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & \infty & \infty & \infty & \infty \\
2 & 0 & \infty & \infty & \infty \\
3 & \infty & \infty & \infty & \infty \\
4 & \infty & 0 & \infty & \infty
\end{array}
$$

$\text{Cost}(3,4) + \text{cost}(3) + \hat{r} = 0 + 7 + 0 = 7$

Node 7

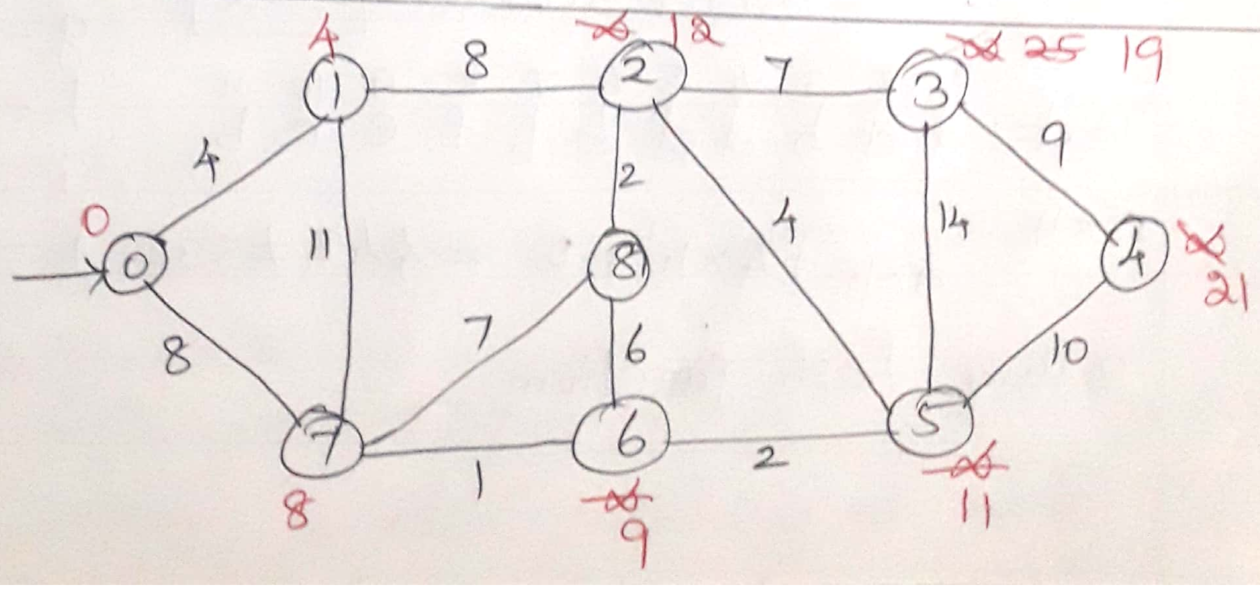
$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & \infty & \infty & \infty & \infty \\
2 & \infty & \infty & \infty & \infty \\
3 & \infty & \infty & \infty & \infty \\
4 & \infty & \infty & \infty & \infty
\end{array}
$$

$\text{Cost}(4,2) + c(6) + \hat{r} = 0 + 7 + 0 = 7$

cost of the tour = 7

Tour is 1 – 3 – 4 – 2 – 1

3)



Shortest paths

0-1 is 4

0-2 is 12

0-3 is 19

0-4 is 21

0-5 is 11

0-6 is 9

0-7 is 8

0-8 is 14

14. Rabin Karp string Matching Algorithm.

Text: d d b d d b b f e c b

Pattern: e c b

a-1
b-2
c-3
d-4
e-5
f-6
g-7
h-8
i-9
j-10

Pattern: $\overset{1}{e}\ \overset{2}{c}\ \overset{3}{b}$

m=3

$5\times10^2+3\times10^1+2\times10^0$

$500+30+2=\boxed{532}$

Text: $\overset{1}{d}\ \overset{2}{d}\ \overset{3}{b}\ \overset{4}{d}\ \overset{5}{d}\ \overset{6}{b}\ \overset{7}{b}\ \overset{8}{f}\ \overset{9}{e}\ \overset{10}{c}\ \overset{11}{b}$

n=11

$4\times10^2+4\times10^1+2=442\neq532$

Rolling hash function

$\Rightarrow$ d b d

$442 - 400 = 42 \times 10 = 420 + 4 = 424$

$424 \neq 532$

$\Rightarrow$ b d d

$424 - 400 = 24 \times 10 = 240 + 4 = 244 \neq 532$

$\Rightarrow$ d d b

$244 - 200 = 44 \times 10 = 440 + 2 = 442 \neq 532$

$\Rightarrow$ d b b

$442 - 400 = 42 \times 10 = 420 + 2 = 422 \neq 532$

$\Rightarrow$ b b f

$422 - 400 = 22 \times 10 = 220 + 6 = 226 \neq 532$

$\Rightarrow$ b f e

$226 - 200 = 26 \times 10 = 260 + 5 = 265 \neq 532$

$\Rightarrow$ f e c

$265 - 200 = 65 \times 10 = 650 + 3 = 653 \neq 532$

$\Rightarrow$ e c b

$653 - 600 = 53 \times 10 = 530 + 2 = 532 = 532$

Pattern is also matching. at indicos 9, 10, 11.

class P

- set of deterministic algorithms ~~the~~ which are taking polynomial time.

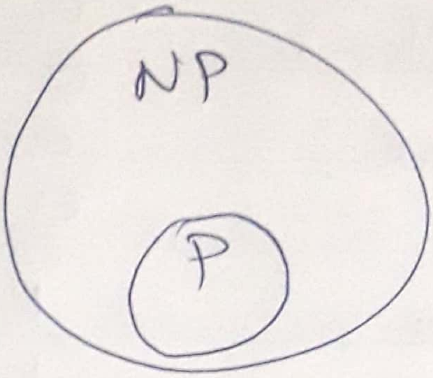
  Ex: Linear search, Binary search, MST, Huffman coding, etc.

class NP

- set of non-deterministic algs taking polynomial time.

Ex: (~~Exponential algs~~)

For exponential time algs, we try to make polynomial. But we don't know how to make. So we can leave the stmts as ~~d~~ non-deterministic. (Ex. Previous one). So alg. becomes non-deterministic.



P is a subset of NP.
The deterministic alg. today were the part of non-deter. alg.

Ex: Merge sort found recently. It becomes deterministic. Previously it was non-deterministic

Non-deter (Yesterday) - Deter (Today)
Non-determ (Today) - Determ (Tomorrow)

CNF- Satisfiability.

$$x_i = \{x_1, x_2, x_3\}$$

$$CNF = \underbrace{(x_1 \vee \bar{x}_2 \vee x_3)}_{C_1} \wedge \underbrace{(\bar{x} \vee x_2 \vee \bar{x}_3)}_{C_2}$$

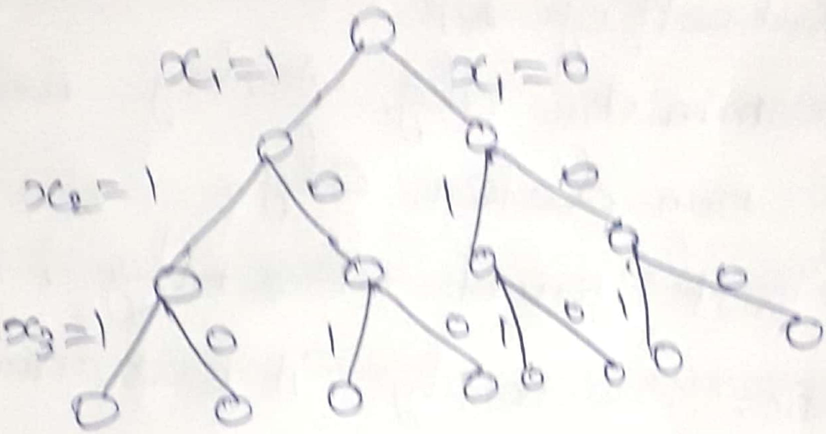
Satis problem is to find, for what values of x_i this formula is true.

Time = $2^3 \Rightarrow 2^n$

Exponential Time.

x_1	x_2	x_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

State space Tree



Parse from Root to leaves give solution

Ex:

0/1 Knapsack

$P = \{10, 8, 12\}$ $n=3$

$W = \{5, 4, 3\}$ $m=8$

$x_i = \{0/1, 0/1, 0/1\}$

x_1	x_2	x_3
0	0	0
0	0	1
0	1	0
⋮		

This is similar to the satisfiabily pblm.
so if we get soln for this, we get for that also.

NP hard

Satisfiability is a base pblm for converting exp → poly time.

It is NP hard. If sat is reduced to 0/1 Knapsack, then 0/1 knap is also NP hard

$$\underset{\text{NP hard}}{\text{Sat}} \propto \underset{\text{NP hard}}{\text{0/1 Knapsack}}$$

$I_1 \Rightarrow$ Instance

$$I_1 \propto I_2$$

$$\underset{\text{NP hard}}{\text{Sat}} \propto \underset{\text{NP hard}}{L_1} \propto \underset{\text{NP hard}}{L_2}$$

(Transitivity Property)

NP Complete

we have a non-deterministic polynomial time alg for satisfiability pblm. So it is NP-complete.

$$\underset{\text{NP hard}}{\text{Sat}} \propto \underset{\text{NP hard}}{L}$$

Non-det poly time alg (NP)

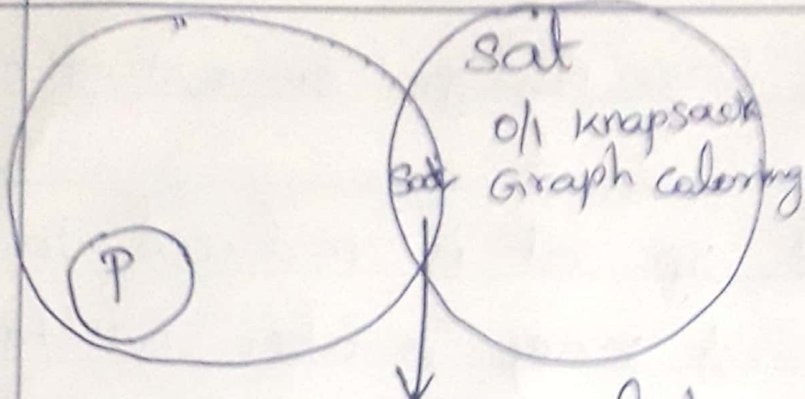
NP-complete.

On any problem L we need to show that it is directly/indirectly related to sat. & write non-deter poly time alg.

NP (write non-deter alg)

NP-Hard

NP- Non determin alg.



NP-complete.

$P \subseteq NP$

Able to prove $P = NP$

Cook's has proved that, if satisfiability is lying in P, ~~iff~~ if and only if $P = NP$.