Unit - IV

Queueing Theory:

Introduction:

There are many situations in daily life when a queue is formed. for example, machines waiting to be repaired, patients waiting in a doctor's room, Cars waiting at a traffic signal and passengers waiting to buy tickets in counters form queues.

Queue is formed if the server required by the customer (machine, patient, car ... etc) is not immediately available, that is if the current demand for a particular service exceeds the capacity to provide the service.

There are many types of Queueing systems, all of them can be classified and described according to the following characteristics.

1. The input (or arrival) pattern

The input describes the manner in which the customers arrive and in the Queueing System. It is not possible to observe and control the actual moment of arrival of a customer for service.

The mode of arrival of customers is expressed by means of the probability distribution of the number of arrivals per unit of time (or) of the inter-arrival time.

Here, the no of arrivals per unit of time has a poisson distribution with mean λ.

Suppose, the time between consecutive arrivals has an exponential distribution with mean $1/\lambda$.

2. The Service Mechanism (or pattern):-

The mode of service is represented by the means of the prob. distribution of the no of customers serviced per unit time or of the inter-service time. we shall mostly deal with only those Queueing systems in which the no of customers serviced per unit of time has a poisson distribution with mean μ (or) equivalently the inter-service time has an exponential distribution with mean $1/\mu$.

3. The Queue discipline:-

The Queue discipline is satisfies the manner in which the customers form the queue (or) equivalently the manner in which they are selected for service, when a queue has been formed.

The mose common discipline is the FCFS (First come first served) (or) FIFO (first in first out) as per which the customers are served in the second order of their arrival.

If the last arrival in the system is served first, we have the LIFO (Last in first out) discipline.

Symbolic Representation of a Queueing Model:-

Usually a queueing model is specified and represented sysbolically in the form $(a|b|c):d|e$

$$\left\{\begin{array}{l} a \to \text{no of arrival per unit time.} \\ b \to \text{the types of distribution of the service time.} \\ c \to \text{the no of servers.} \\ d \to \text{The capacity of the system.} \\ e \to \text{The Queue system.} \end{array}\right.$$

Accordingly, the first four models which we will deal with will be denoted by the symbols $(M|M|1):(\infty|FIFO)$, $(M|M|s):(\infty|FIFO)$, $M|M|1):k|FIFO)$ and $(M|M|s):(k|FIFO)$.

In the above symbols, the letter M stands for "Markov" indicating that the no of arrivals in time t and the no of completed services in time t follow poisson process which is a continuous time markov chain.

Difference equations Related to poisson Queue:

Let $P_n(t)$ be the prob that there are 'n' customers in the system at time t $(n>0)$.
Let λ_n be the average arrival rate when there are n customers in the system, μ_n be the average service rate when there are n customers in the system.

$$\lambda_{n-1} \cdot P_{n-1} - (\lambda_n + \mu_n) \cdot P_n + \mu_{n+1} \cdot P_{n+1} = 0. \rightarrow (1)$$

and $$-\lambda_0 P_0 + \mu_1 P_1 = 0. \rightarrow (2)$$

are the difference equations that, n customers in the poisson system.

Values of P_0 and P_n for Poisson Queue System:-

$$P_n = \frac{\lambda_0 \lambda_1 \lambda_2 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \mu_3 \cdots \mu_n} \cdot P_0, \quad \text{for } n = 1, 2 \ldots$$

$$\rightarrow (3)$$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \left(\frac{\lambda_0 \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n} \right)} \rightarrow (4).$$

eqns (3) and (4) are the important characteristics of Queueing Models.

Type I:- Characteristics of Infinite Capacity

{Single Server Poisson Queue Model I}

$(M / M / 1) : (\infty / FIFO)$

(1)

1. The average number L_s of customers in the system:-

$$L_s = \frac{\lambda}{\mu - \lambda}. \rightarrow (5)$$

2. Average no L_q of customers in the Queue (or) average length of the Queue:-

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \rightarrow (6).$$

3. Average no L_w of customers in nonempty Queues:

$$L_w = \frac{\mu}{\mu-\lambda} \rightarrow (7)$$

4. Probability that the no of customers in the system exceeds k:

$$P(N>k) = \left(\frac{\lambda}{\mu}\right)^{k+1} \rightarrow (8)$$

5. Prob. density function of the waiting time in the system:

Let W_s be the waiting time of a customer in the system.

$$f(\omega) = (\mu-\lambda) \cdot e^{-(\mu-\lambda)\omega} \rightarrow (9)$$ is the p.d.f of an exponential distribution with parameter $\mu-\lambda$.

6. Average waiting time time of a customer in the system:

$$E(W_s) = \frac{1}{\mu-\lambda} \rightarrow (10)$$

7. Probability that the waiting time of a customer in the system exceeds t.

$$P(W_s > t) = e^{-(\mu-\lambda)t} \rightarrow (11)$$

8. Prob. density function of the waiting time (W_q) in the Queue:

$$g(\omega) = \frac{\lambda}{\mu}(\mu-\lambda) \cdot e^{-(\mu-\lambda)\cdot\omega} \rightarrow (12), \quad \omega > 0$$

and $g(\omega) = 1 - \lambda/\mu, \ \omega = 0. \rightarrow (13)$

9. Average waiting time of a customer in the Queue:

$$E(W_q) = \frac{\lambda}{\mu(\mu-\lambda)} \longrightarrow (14).$$

10. Average waiting time of a customer in Queue if he has to wait

$$E(W_q / W_q > 0) = \frac{1}{\mu-\lambda} \longrightarrow (15).$$

11. Relations Among $E(N_s)$, $E(N_q)$, $E(W_s)$ and $E(W_q)$:-

all formulas

(a) $E(N_s) = \dfrac{\lambda}{\mu-\lambda} = \lambda . E(W_s) \longrightarrow (1)$

(b). $E(N_q) = \dfrac{\lambda^2}{\mu(\mu-\lambda)} = \lambda E(W_q). \longrightarrow (2)$

(c). $E(W_s) = E(W_q) + \dfrac{1}{\mu}. \longrightarrow (3)$

(d) $E(N_s) = E(N_q) + \dfrac{\lambda}{\mu} \longrightarrow (4)$

These 4 are the Little's formula.

Problems based Infinite Capacity, Queueing Model I:-

1. Arrivals at a telephone booth are considered to be poisson with an average time of 12 min between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with Mean 4 min.

(a) find the average no of persons waiting in the System.

(b). what is the prob that a person arriving at the booth will have to wait in the Queue?

(c) what is the prob that it will take him more than 10 min, altogether to wait for the phone and complete his call?

(d) Estimate the fraction of the day when the phone will be in use.

(e) The telephone dept will install a second booth when convinced that an arrival has to wait on the average for atleast 3min for phone. By howmuch the flow of arrivals should increase in order to justify a second booth?

(f). what is the average length of the queue that forms from time to time?

Solution:- Mean inter-arrival time $= \frac{1}{\lambda} = 12$ min.

Therefore mean arrival rate $= \lambda = \frac{1}{12}$.

Mean service time $= \frac{1}{\mu} = 4$ min.

$\therefore$ The mean service rate $= \mu = \frac{1}{4}$ per minute.

(a). $E(N) = \frac{\lambda}{\mu - \lambda} = \frac{\frac{1}{12}}{\frac{1}{4} - \frac{1}{12}} = \frac{\frac{1}{12}}{\frac{12-4}{48}} = \frac{1}{\cancel{12}} \times \frac{\cancel{48}^{4}}{8} = \frac{1}{2}$

$\therefore \boxed{E(N) = 0.5}$ customer.

(b). $P(W>0) = 1 - P(W=0)$

$= 1 - P(\text{no of customer in the system})$.

$= 1 - P_0 = 1 - \left(1 - \frac{\lambda}{\mu}\right) = 1 - 1 + \frac{\lambda}{\mu} = \frac{1/12}{1/4} = \frac{1}{3}$

(c). $P(w>10) = e^{-(\mu-\lambda)\times 10}$

$= e^{-\left(\frac{1}{4}-\frac{1}{12}\right)\times 10}$

$= e^{-\left(\frac{8}{48}\times 10\right)}$

$= e^{-\frac{80}{48}}$

$= e^{-5/3} = 0.1889$

(d) P(the phone will be idle) = $P(N=0) = P_0$

$= 1-\frac{\lambda}{\mu}$

$= 1-\frac{1/12}{1/4}$

$= 1-\left[\frac{1}{12}\times\frac{4}{1}\right]$ 3

$= 1-1/3 = 2/3$.

i.e., the fraction of the day, when the phone will be in use $= 1/3$.

(e). The second phone will be installed.

if $E(W_q) > 3$.

i.e., $\frac{\lambda}{\mu(\mu-\lambda)} > 3$.

$\therefore \frac{\lambda_R}{\frac{1}{4}\left(\frac{1}{4}-\lambda_R\right)} > 3$. ($\lambda_R$ is the required arrival rate).

$\Rightarrow \lambda_R > \frac{3}{4}\left(\frac{1}{4}-\lambda_R\right) \Rightarrow \lambda_R > 3/28$.

∴ The arrival rate should increase by

$3/28 - 1/12 = 1/42$ per minute, to justify a second phone.

(iii) E (Nq / the Queue is always available)

$$= \frac{\mu}{\mu - \lambda} = \frac{1/4}{1/4 - 1/12} = \frac{\frac{1}{4}}{\frac{12-4}{48}}$$

$$= \frac{1}{4} \times \frac{\cancel{48}^{12}}{\cancel{8}_{2}}$$

$$= \frac{12}{8} = \frac{3}{2}$$

$= 1.5$ (persons).

2. Customers arrive at a one-man barber shop according to a poisson process with a mean inter-arrival time of 12 min. customers spend an average of 10 min in the barber's

(a) what is the expected no of customers in the barber shop and in the queue?

(b) calculate the percentage of time an arrival can walk straight into the barber's chair without having a wait.

(c) How much time can a customer expect to spend in the barber's shop? customers waiting time in the shop exceeds 1.25 h how much must the average rate of arrivals increase to warrent a second barber?

(d) Management will provide another chair and hire another barber, when a average rate of arrivals increase to warrent to warrent a second barber?

(e). what is the average time customers spend in the Queue?

(f). what is the prob. that the waiting time in the system is greater than 30 min?

(g) Calculate the percentage of customers who have to wait prior to getting into the barber's chair.

(h) what is the prob. that more than 3 customers are in the system?

Solution:- Given $\frac{1}{\lambda} = 12$. $\therefore \lambda = \frac{1}{12}$ per (Minute)

$\frac{1}{\mu} = 10$ $\therefore \mu = \frac{1}{10}$ per minute.

(a). Expected no of customers in the barber shop and in the Queue

$$L_s = E(N_s) = \frac{\lambda}{\mu - \lambda} = \frac{\frac{1}{12}}{\frac{1}{10} - \frac{1}{12}}$$

$$= \frac{1/12}{\frac{12-10}{120}} = \frac{1}{\cancel{12}} \times \frac{\overset{10}{\cancel{120}}}{2}$$

$$= 5 \text{ customers}$$

$\boxed{E(N_s) = 5}$.

$$\therefore E(n_q) = \frac{\lambda^2}{\mu(\mu-\lambda)} = L_q$$

$$= \frac{1/144}{1/10\left(\frac{1}{10} - 1/12\right)}$$

$$= \frac{\frac{1}{144}}{\frac{1}{10}\left(\frac{12-10}{120}\right)}$$

$$= \frac{1}{\cancel{144}_{\cancel{12}\ \cancel{6}\ 3}} \times \frac{\cancel{10}^{5}}{1} \times \frac{\cancel{120}^{\cancel{10}\ 5}}{2}$$

$$= \frac{22}{6} = 4.17 \text{ customers.}$$

(b) P(a customer straight goes to the barbers chair)

$= P($No customer in the System$)$

$$= P_0 = 1 - \frac{\lambda}{\mu} = 1 - \left[\frac{\frac{1}{12}}{\frac{1}{10}}\right]$$

$$= 1 - \left[\frac{1}{\cancel{12}_{6}} \times \frac{\cancel{10}^{5}}{1}\right]$$

$$= 1 - \frac{5}{6}$$

$$= \frac{6-5}{6} = 1/6.$$

$\therefore$ % of time an arrival need not wait $\Big\} = 16.7$.

(c). $E(w) = \frac{1}{\mu - \lambda} = \frac{1}{\frac{1}{10} - \frac{1}{12}} = \frac{1}{\frac{12-10}{120}} = \frac{120}{2} = 60$ min (or) 1.h

w_s ↗

(d). $E(w) > 75$ if $\frac{1}{\mu - \lambda} > 75$.

if $\lambda_r > \mu - \frac{1}{75}$.

$\lambda_r > \frac{1}{10} - \frac{1}{75}$, if $\lambda_r > \frac{13}{150}$.

Hence, to warrant a second barber, the average arrival rate must increase by $\frac{13}{150} - \frac{1}{12} = \frac{1}{300}$ per minute.

(e) $E(W_q) = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{1/12}{1/10(1/10 - 1/12)}$

W_q

$= \frac{1/12}{1/10\left(\frac{12-10}{120}\right)} = \frac{1}{\cancel{12}} \times \frac{\cancel{10}^{5}}{1} \times \frac{\cancel{120}^{10}}{\cancel{2}}$

$= 050$ min.

(f). $P(w > t) = e^{-(\mu-\lambda)t}$

(from formula) $= e^{-(1/10 - 1/12) \times 30}$

$= e^{-\left(\frac{12-10}{120}\right) \times 30}$

$= e^{-\left(\frac{2}{\cancel{120}_{4}} \times \cancel{30}\right)}$

$= e^{-1/2} = e^{-0.5} = 0.6065$

(g) P(a customer has to wait) $= P(w > 0)$.

$= 1 - P(w = 0)$

$= 1 - P(N = 0)$

$= 1 - P_0 = \frac{\lambda}{\mu} = \frac{1/12}{1/10} = \frac{1}{12} \times \frac{10}{1} = \frac{5}{6}$.

∴ Percentage of customers who have to wait $= \frac{5}{6} \times 100 = 83.33$

(ii) $P(N > 3) = P_4 + P_5 + P_6 + \cdots$

$$= 1 - [P_0 + P_1 + P_2 + P_3]$$

$$= 1 - \left[1 - \frac{\lambda}{\mu}\right] \cdot \left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \left(\frac{\lambda}{\mu}\right)^3\right]$$

$\left\{ \because P_n = \left(\frac{\lambda}{\mu}\right)^n \cdot \left(1 - \frac{\lambda}{\mu}\right) \right\}, n \geq 0.$ {from Model I}

$$= \left(\frac{\lambda}{\mu}\right)^4 = \left(5/6\right)^4 = 0.4823.$$

3. If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and it its takes exactly 1.5 min to reach the correct seat after purchasing the ticket,

(a) Can he expect to be seated for the start of the picture?

(b) What is the prob that he will be seated for the start of the picture?

(c) How early must he arrive in order to be 99% sure of being seated for the start of the picture?

Solution:- $\lambda = 6/\text{min}$, $\mu = 8/\text{min}$.

(a) $E(W) = \frac{1}{\mu - \lambda} = \frac{1}{8-6} = \frac{1}{2}$ min.

$\therefore$ E (total time required to purchase the ticket and to reach the seat)

$$= \frac{1}{2} + 1 \cdot \frac{1}{2} = 2 \text{ min}.$$

Hence, he can just be sealed for the start of the picture.

(b) $P(\text{total time} < 2\text{ min}) = P(w < 1/2)$

$$= 1 - P(w > 1/2)$$

$$= 1 - e^{-\mu\left(1-\frac{\lambda}{\mu}\right) \times 1/2}$$

$$= 1 - e^{-8\left(1 - 6/8\right) \times 1/2}$$

$$= 1 - e^{-8\left(2/8\right) \times 1/2}$$

$$= 1 - e^{-1} = 0.63$$

(c) $P(w < t) = 0.99$

i.e., $P(w > t) = 0.01$

i.e., $e^{-(\mu - \lambda)t} = 0.01$

$$e^{-(8-6)t} = 0.01$$

$$e^{-(2t)} = 0.01$$

$$-2t = \log(0.01) = 2.3$$

$$-2t = 2.3 \quad \boxed{t = 1.15 \text{ min}}$$

i.e., $P(\text{ticket purchasing time} < 1.15) = 0.99$

$\therefore$ $P(\text{total time to get the ticket and to go to the seat}) < (1.15 + 1.5) = 0.99$

$\therefore$ The person must arrive at least 2.65 min early so as to be 99% sure of seeing the start of the picture.

Type-II :-

Characteristics of Finite Capacity, Single server poisson Queue Model III.

$[(M/M/1) : k / FIFO \text{ Model I.}]$ - notation.

1. The values of P_0 and P_n :-

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \begin{cases} \dfrac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} & \text{if } \lambda \neq \mu \\ \dfrac{1}{k+1} & \text{if } \lambda = \mu \end{cases}$$

$$P_0 = \begin{cases} \dfrac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} & \text{if } \lambda \neq \mu \rightarrow (1) \\ \dfrac{1}{k+1} & \text{if } \lambda = \mu \rightarrow (2). \end{cases}$$

2. Average no of customers in the System :-

$$E(N) = \begin{cases} \dfrac{\lambda}{\mu-\lambda} - \left[\dfrac{(k+1)\cdot\left(\frac{\lambda}{\mu}\right)^{k+1}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}\right], & \text{if } \lambda \neq \mu \rightarrow (3) \\ = \dfrac{k}{2}, & \text{if } \lambda = \mu \rightarrow (4). \end{cases}$$

3. Average no of customers in the Queue :-

$$E(N_q) = E(N) - (1-P_0).$$

As per little formula,

$$E(N_q) = E(N) - \frac{\lambda'}{\mu} \Rightarrow \frac{\lambda'}{\mu} = 1-P_0 \text{ (or) } \lambda' = \mu(1-P_0)$$

$$\therefore \boxed{E(N_q) = E(N) - \frac{\lambda'}{\mu}}$$

$$E(N_q) = E(N) - \frac{\lambda'}{\mu} \longrightarrow (5).$$

4. Average waiting times in the system and in the Queue:-

By the Modified Little's formula,

$$E(W_s) = \frac{1}{\lambda'} E(N). \longrightarrow (6)$$

$$E(W_q) = \frac{1}{\lambda'} E(N_q). \longrightarrow (7).$$

λ' is the effective rate

Problems based on finite Capacity, Single Server:-

1. The local one-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customers arrive according to a Poisson distribution with Mean 5 per hour. The barber cuts hair at an average rate of 4 per hour. (Exponential Service time).

(a) what percentage of time is the barber idle?

(b) what fraction of the potential customers waiting for a hair-cut?

(c) what is the expected no of customers waiting for a hair-cut?

(d) how much time can a customer expect to spend in the barber shop?

Solution:- $\lambda = 5$, $\mu = 4$, $k = 5$.

(a) P(the barber is idle) = $P(N=0)$.

$$P_0 = \frac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}$$

$$P_0 = \frac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} = \frac{1-5/4}{1-(5/4)^{5+1}} = 0.0888.$$

$\therefore$ % of time when the barber is idle = 9.

(b) P(a customer is turned away) = P(N > 5).

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \cdot \left[\frac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}\right], \text{ if } \lambda \neq \mu.$$

$$= \left(\frac{5}{4}\right)^5 \cdot \left[\frac{1-5/4}{1-(5/4)^6}\right].$$

$$= \frac{3125}{11529} = 0.2711$$

$\therefore$ prob of a customer is turned away is 0.2711

(c). The expected no of customers waiting for a hair-cut $E(N_q) = E(N) - (1-P_0)$

$$= \frac{\lambda}{\mu-\lambda} - \frac{(k+1)\cdot\left(\frac{\lambda}{\mu}\right)^{k+1}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} - (1-P_0)$$

$$= -5 - \frac{6 \times \left(\frac{5}{4}\right)^6}{1-(5/4)^6} - (1-0.0888)$$

$$= -6 \times \frac{\frac{15625}{4096}}{\frac{11529}{4096}} - 5.9112 = 2.2$$

= 2 customers.

(d). $E(W) = \frac{1}{\lambda'} \cdot E(N)$.

$$= \frac{1}{\mu(1-P_0)} \times E(N)$$

$$= \frac{3.1317}{3.6448} = 0.8592 \text{ hr}$$

(or) 51.5 min.

2. At a railway station, only one train is handled at a time. The railway yard is sufficient only for 2 trains to wait, while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 6 per hour. Assuming Poisson arrivals and exponential service distribution. find the prob for the numbers of trains in the system. Also find the average waiting time of a new train coming into the yard. If the handling rate is doubled, how will the above results get modified?

Solution:- $\lambda = 6$ per hr, $\mu = 6$ per/hr. $k = 2+1 = 3$.

Since, $\lambda = \mu$, $P_0 = \frac{1}{k+1}$

$$= \frac{1}{3+1} = \frac{1}{4}.$$

$$P_n = \frac{1}{k+1} = \frac{1}{4}.$$

$$E(N) = \frac{k}{2} = 1.5 \text{ trains}.$$

$$E(W) = \frac{1}{\lambda'} E(N)$$

$$= \frac{1.5}{\mu(1-P_0)} = \frac{1.5}{6 \times (1-\frac{1}{4})} = \frac{1.5}{\cancel{6}^{3} \times (\frac{3}{\cancel{4}_{2}})}$$

$$= \frac{1.5 \times 2}{9}$$

$$= \frac{3}{9} = \frac{1}{3} h \text{ (or)}$$

$$= 20 \text{ min}$$

(ii) $\lambda = 6, \mu = 12, k = 3.$

Since $\lambda \neq \mu$,

$$P_0 = \frac{1-\frac{\lambda}{\mu}}{1-(\frac{\lambda}{\mu})^{k+1}} = \frac{1-\frac{6}{12}}{1-(\frac{1}{2})^4} = \frac{1-\frac{1}{2}}{1-\frac{1}{16}}$$

$$= \frac{\frac{1}{2}}{\frac{16-1}{16}}$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{16}^{8}}{15}$$

$$\boxed{P_0 = \frac{8}{15}} = 0.533.$$

$$\therefore P_n = (\frac{\lambda}{\mu})^n \left[\frac{1-\frac{\lambda}{\mu}}{1-(\frac{\lambda}{\mu})^{k+1}} \right]$$

$$= (\frac{8}{15}) \cdot (\frac{1}{2})^n, \quad n = 1, 2, 3 \ldots$$

(or).

$$(\frac{\lambda}{\mu})^3 \cdot E(N).$$

$$= (\frac{1}{2})^3 \times 0.533$$

$$= 0.0666$$

$$\therefore E(N) = \frac{\lambda}{\mu-\lambda} - \left[\frac{(k+1)\cdot\left(\frac{\lambda}{\mu}\right)^{k+1}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} \right]$$

$$= \frac{6}{12-6} - \frac{(k+1)\cdot\left(\frac{\lambda}{\mu}\right)^{k+1}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}$$

$$= \frac{6}{6} - \frac{(3+1)\cdot(1/2)^4}{1-(1/2)^4}$$

$$= 1 - \left[\frac{\cancel{4} \times \frac{1}{\cancel{16}\,4}}{1-\frac{1}{16}} \right] = 1 - \left[\frac{\frac{1}{4}}{15/16} \right]$$

$$= 1 - \left[\frac{1}{\cancel{4}} \times \frac{\cancel{16}^{\,4}}{15} \right]$$

$$= 1 - \frac{4}{15} = \frac{11}{15}$$

$$= 0.73 \text{ train.}$$

$$\therefore E(W) = \frac{1}{\lambda'} \cdot E(N).$$

$$= \frac{1}{\mu(1-P_0)} \times E(N)$$

$$= \frac{\frac{11}{15}}{12(1-8/15)} = \frac{\frac{11}{15}}{\cancel{12}_{4}\left(\frac{7}{\cancel{15}_{5}}\right)} = \frac{11}{\cancel{15}_{3}} \times \frac{\cancel{5}}{28}.$$

$$= \frac{11}{3 \times 28}$$

$$= \frac{11}{84} \text{ h}$$

$$= 7.9 \text{ min}$$