18MAB204T-Module-III (Mathematical Statistics: Tests of Hypothesis)

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Outline

Population or Universal:
Every statistical investigation aims at collecting information (we normally study or investigate) about some aggregate or collection of individuals rather than the individuals themselves. In statistical language, such a collection is called a Population or Universal.
For example:

1. The population of products turned out by a machine.
2. The population of lives of electric bulbs manufactured by a company etc.

Finite or Infinite Population: A population is finite or infinite, according as the number of elements is finite or infinite.
Sample and Sampling: A finite subset of a population is called a sample and the process of selecting of such samples is called sampling.
Sample Size: The number of individuals in the sample is called the sample size (n).

If $n < 30$ then small sample
If $n \geq 30$ then large sample
The main objective of the theory of sampling is to draw inference about the population using the information of the sample.
For example:
We may arrive at a decision of accepting or rejecting particular items by examining some samples of the items. However, the acceptance or rejection based on the characteristics of the sample or samples gives rise to an error called the sampling error, which is unavoidable and is inherent in such studies.
In our day-to-day life, we use many sampling tests.
Sampling has its advantages, particularly saving of costs and time.

Random Sampling: In random sampling, the samples are selected at random, which excludes the possibility of any biasedness. Thus, each item of the population has equal chance of being included in the samples.

Parameters and Statistics: The statistical measures or constants, like mean μ and standard deviation σ on the basis of population, are called the parameters of the population.

The statistical measures or constants, like mean $\bar{x}$ and standard deviation S on the basis of sample, are called the statistics of the sample.

Sampling distribution of a statistic: A sampling distribution is a probability distribution of a statistic obtained from a larger number of samples drawn from a specific population.

Standard error: The standard deviation of the sampling distribution of a statistic is called the standard error of that quantity. In sampling theory, we shall mainly, be concerned with the following two types of problems.

1. Estimation: Some characteristic or feature of the population in which we are interested may be completely unknown to us and we may like to make a guess about this characteristic entirely on the basis of a random sample drawn from the population. This type of problem is known as the problem of estimation.

2. Testing of Hypothesis: Some information about a characteristic of the population is known. We wish to know whether this information can be accepted. We choose a random sample and obtain information about this characteristic. Based on this information, we conclude whether the available information of the characteristic of the population can be accepted or rejected. We also wish to know that if it can be accepted then, to what degree of confidence it can be accepted. This is called the problem of testing of hypothesis.

Statistical Hypothesis: When we attempt to make decisions about the population on the basis of sample information, we have to make assumptions or guesses about the nature of the population involved or about the value of some parameter of the population. Such assumptions, which may or may not be true, are called statistical hypothesis.

Tests of significance: Let θ be a parameter of the population and θ_0 be the corresponding sample statistic. Since θ_0 is obtained based on a random sample, there will be some deviation (difference) between θ and θ_0, this may be due to the reason that the selection of the sample is not completely random. If this difference is large, then, we say that the difference is significant. If θ_1 is the statistic obtained from a second random sample, we wish to know whether the difference of θ_0 and θ_1 is significant.

The methods that are used to decide whether the difference is significant or not, are called the tests of significance.

Null Hypothesis:

A population is given to us and we wish to have information about a characteristic of the population. We start with the assumption that there is no significant difference between the sample statistic and the corresponding population parameter or between two sample statistics. Such a hypothesis of no significant difference is called a null hypothesis and is denoted by H_0.

Alternative Hypothesis:

A hypothesis that is different from the null hypothesis is called an alternative hypothesis and is denoted by H_1.

Let the null hypothesis be defined as

H_0 : The population has an assured value of mean μ_0, that is $\mu = \mu_0$

Let the alternative hypothesis be defined as any of the following:

(i) $H_1 : \mu \neq \mu_0$ (Two tailed alternative hypothesis)
(ii) $H_1 : \mu > \mu_0$ (Right tailed alternative hypothesis)
(iii) $H_1 : \mu < \mu_0$ (Left tailed alternative hypothesis)

The following example can be generalized:

If θ_0 is a population parameter and θ is the corresponding sample statistics and if we set up the null hypothesis $H_0 : \theta = \theta_0$, then the alternative hypothesis which is complementary to H_0 can be any of the following:

(i) $H_1 : \theta \neq \theta_0$ (Two tailed alternative hypothesis)

(ii) $H_1 : \theta > \theta_0$ (Right tailed alternative hypothesis)

(iii) $H_1 : \theta < \theta_0$ (Left tailed alternative hypothesis)

Errors in Hypothesis Testing

The main objective in sampling theory is to draw valid inferences about the population parameters on the basis of the sample results. In practice we decide to accept or reject the lot after examining the sample from it. As such we have two types of errors.

(For example, in a shop we assess the quality of rice or any other commodity by taking handful of it from the bag and decide to purchase it or not.)

Type I error: Reject the null hypothesis when it is really true but it is rejecting by the customer. This is similar to a good product being rejected by the customer and hence it is known as producer's risk.

The probability of type I error is denoted by α.

Type II error: Accept the null hypothesis when it is really wrong but it is accepting by the customer. This is similar to a bad product being accepted by the customer and hence it is known as consumer's risk.

The probability of type II error is denoted by β.

Critical Region: Let the sample statistic t lie in the certain region R in the sample space. If we decide that the difference between the population parameter and the sample statistic is significant (that is, the null hypothesis is rejected), then, the region R is called the critical region or region of rejection.

The complementary region $\bar{R}$ is called the region of acceptance.

Test statistic: In case of large samples, the sampling distributions of many statistics tend to become normal distribution. If t is the statistics large samples, then t follows a normal distribution. The corresponding population parameter is mean$= E(t)$ and standard deviation equals $SE(t)$ [the standard deviation of the sampling distribution of a statistic t]. That is, for large samples,

$$Z = \frac{t - E(t)}{SE(t)} \sim N(0,1),$$

In the two tailed or single tailed tests, the critical region is given by the portion of the area under the probability curve of the sampling distribution of the test statistic Z. In the case of two tailed tests, the critical region is given by the portion of the area lying at both the ends (symmetrically) of the probability curve. In the case of the left tailed tests, the critical region is in the left tail under the probability curve while in the right tailed test, the critical region is in the right tail under the probability curve.

Level of significance:

Let $t = (X_1,\ X_2, \dots\ X_n)$ be the value of statistic obtained using a random sample size n. Let R be the critical region (region of rejection) and $\bar{R}$ be the region of acceptance. Define

$$P(t \in R/H_0) = \alpha,\ P(t \in R/H_1) = \beta.$$

[P(Reject a consignment of terms when they are good) = P(Reject $H_0/H_0 = \alpha$) and P(Reject a consignment of terms when they are not good) = P(Accept $H_0/H_1 = \beta$)]

The probability α that a random value of the statistic lies in the region R is known as the level of significance.

We note that level of significance is always fixed in advanced before the characteristic of the random sample is studied. The level of significance is usually expressed as a percentage. That is, the total area of the critical region is written as $\alpha\%$ level of significance.

Critical values or Significant values: Let a level of significance α be prescribed. The value of the test statistics Z for which the critical region and accepted region are separated is called the critical value or the significant value of Z and denoted by Z_α.

For three tests, we have the following results:

(i) Two tailed tests: In general, the critical value Z_α for the level of significance α is given by the equation $P(|Z| > Z_\alpha) = \alpha$ for a two tailed test.
The total area of the critical region under the probability curve is α. Because of symmetry of the probability curve, we get $P(Z > Z_\alpha) = P(Z < -Z_\alpha)$
$\therefore P(|Z| > Z_\alpha) = \alpha$
$\implies P(Z > Z_\alpha) + P(Z < -Z_\alpha) = \alpha$
$\implies P(Z > Z_\alpha) + P(Z > Z_\alpha) = \alpha$
$\implies P(Z > Z_\alpha) = \frac{\alpha}{2}$.

This implies that the area under each tail is $\frac{\alpha}{2}$. We call this value $Z = Z_\alpha$ as the upper critical value and $Z = -Z_\alpha$ as the lower critical value. The acceptance region is given by $(-Z_\alpha, Z_\alpha)$.

(ii) Right tailed test: In general, the critical value Z_α for the level of significance α is given by the equation $P(Z > Z_\alpha) = \alpha$ for a right tailed test.

(iiii) Left tailed test: In general, the critical value Z_α for the level of significance α is given by the equation $P(Z < -Z_\alpha) = \alpha$ for a left tailed test.

Now, let Z_α be the critical value of Z corresponding to the level of significance α in the right tailed test, that is $P(Z > Z_\alpha) = \alpha$. Due to symmetry, we have

$P(|Z| > Z_\alpha) = P(Z > Z_\alpha) + P(Z < -Z_\alpha) = P(Z > Z_\alpha) + P(Z > Z_\alpha) = \alpha + \alpha = 2\alpha$

Therefore, the critical value of Z for a single tailed test at level of significance α is same as the critical value of Z for a two tailed test at level of significance 2α.

Confidence interval (Interval estimation of population parameter):

We would like to determine an interval in which the population parameter is supposed to lie. The procedure to determine this interval is called interval estimation and the interval is called the confidence interval for that population parameter. The end points of this interval are called the confidence limits.

The critical values for some standard LOS (level of significance) are given in the following table both for two tailed and one tailed tests.

LOS / Nature of test	1%(0.01)	2%(0.02)	5%(0.05)	10%(0.1)
Two tailed	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 2.33$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Right tailed	$Z_\alpha = 2.33$	$Z_\alpha = 2.055$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left tailed	$Z_\alpha = -2.33$	$Z_\alpha = -2.055$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

Procedure for testing of hypothesis:

(1) Set up the null hypothesis H_0.

(2) Set up the alternative hypothesis H_1.

(3) Choose the appropriate LOS.

(4) Compute the test hypothesis $Z = \frac{t - E(t)}{SE(t)}$.

(5) Compare between between $|Z|$ and Z_α

If $|Z| < Z_\alpha$, H_0 is accepted or H_1 is rejected, i.e., it is concluded that the difference between t and $E(t)$ is not significant at $\alpha\%$ LOS.

If $|Z| > Z_\alpha$, H_0 is rejected or H_1 is accepted, i.e., it is concluded that the difference between t and $E(t)$ is significant at $\alpha\%$ LOS.

Assumptions for large samples:

(1) The sampling distribution of a statistic is approximately normal.

(2) Sample statistics are sufficiently close to the corresponding population parameters.

Test 1: Test of significance of the difference between sample proportion and population proportion.

Let X be the number of successes in n independent Bernoulli trials in which the probability of success for each trial is a constant =P(say). Then it is known that X follows a binomial distribution with mean $E(X) = nP$ and variance $V(X) = nPQ$.

When n is large, X follows $N(nP\sqrt{nPQ})$, where $Q = 1 - P$.

$\therefore$ $\frac{X}{n}$ follows $N\left(\frac{nP}{n}, \frac{\sqrt{nPQ}}{n}\right)$.

Now, $\frac{X}{n}$ is the proportion of success in the sample consisting of n trials, i.e., denoted by p. Thus, the sample proportion p follows $N\left(P, \frac{\sqrt{PQ}}{n}\right)$.

Test statistic $z = \frac{p-P}{n}$.

If $|z| \leq z_\alpha$, the difference between the sample proportion p and the population proportion P is not significant at $\alpha\%$ LOS.

Test 2: Test of significance of the difference between two sample proportions.

Let p_1 and p_2 are the proportion os successes in two large samples of size n_1 and n_2 respectively drawn from the same proportion or from two population with the same proportion P.

Then p_1 follows $N\left(P, \frac{\sqrt{PQ}}{n_1}\right)$ and p_2 follows $N\left(P, \frac{\sqrt{PQ}}{n_2}\right)$.

Therefore $p_1 - p_2$, which is a linear combination of two normal variables, also follows a normal distribution

Now, $E(p_1 - p_2) = E(p_1) - E(p_2) = P - P = 0$
$V(p_1 - p_2) = V(p_1) + V(p_2) = PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$
as$[V(aX + bY)^2 = aV(X)^2 + bV(Y)]$
$\therefore$ $p_1 - p_2$ follows $N\left(0, \sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\right)$.
$\therefore$ the test statistic $z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$.

If P is not known, an unbiased estimate of P based on both samples, given by $\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$, is used in the place of P.
If $|Z| \leq Z_\alpha$, the difference between the two sample proportions P and P_2 is not significant at $\alpha\%$ LOS.

Test 3: Test of significance of the difference between sample mean and population mean.

Let $X_1, X_2, ...X_n$ be the sample observations in a sample of size n, drawn from a population that is $N(\mu, \sigma)$.

Then each X_i follows $N(\mu, \sigma)$. It is known as $X_i (i = 1, 2, ...n)$ are independent normal variates with mean μ_i and variance ${\sigma_i}^2$, then $\sum c_i X_i$ is a normal variate with mean $\mu = \sum c_i \mu_i$ and variance $\sigma^2 = \sum c_i^2 \sigma_i^2$.

Now, putting $c_i = \frac{1}{n}, \mu_i = \mu$ and $\sigma_i = \sigma$, we get

$\sum c_i X_i = \frac{1}{n} \sum X_i = \bar{X}, \; \sum c_i \mu_i = \frac{n\mu}{n} = \mu$ and $\sum c_i^2 \sigma_i^2 = \frac{n\sigma^2}{n^2} =$.

Thus, if X_i are n independent normal variates with the same mean μ and same variance σ^2, then their mean $\bar{X}$ follows a $N(\mu, \frac{\sigma}{\sqrt{n}})$.

Even if the population, from which the sample is drawn, is non-normal, it is known(from central limit theorem) that the above result holds good, provided n large.

$\therefore$ the test statistic $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$.

As usual if $|Z| \leq Z_\alpha$, the difference between the sample mean $\bar{X}$ and the population mean μ is not significant at $\alpha\%$ LOS.

Test 4: Test of significance of the difference between the means of two samples.

Let $\bar{X}_1$ and $\bar{X}_2$ be the means of two large samples of sizes n_1 and n_2 drawn from two populations (normal or non-normal) with the same mean μ and variances σ_1^2 and σ_2^2 respectively. Then $\bar{X}_1$ follows a $N(\mu, \frac{\sigma_1}{\sqrt{n_1}})$ and $\bar{X}_2$ follows a $N(\mu, \frac{\sigma_2}{\sqrt{n_2}})$ either exactly or approximately. $\therefore$ $\bar{X}_1 - \bar{X}_2$ also follows a normal distribution.
$E(\bar{X}_1 - \bar{X}_2) = E(\bar{X}_1) - E(\bar{X}_2) = \mu - \mu = 0$ and
$V(\bar{X}_1 - \bar{X}_2) = V(\bar{X}_1) + V(\bar{X}_2) = \frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}$ [As, $\bar{X}_1$ and $\bar{X}_2$ are independent due to independency of samples].

Thus, $(\bar{X}_1 - \bar{X}_2)$ follows a $N(0, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}})$.

$\therefore$ the test statistic $Z = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$.

If $|Z| \leq Z_\alpha$, the difference between $(\bar{X}_1 - \bar{X}_2)$ and 0 or the difference between $\bar{X}_1$ and $\bar{X}_2$ is not significant at $\alpha\%$ LOS.

Example 1: The fatality rate of typhoid patients is believed to be 17.26%. In a certain year 640 patients suffering from typhoid were treated in a metropolitan hospital and only 63 patients died. Can you consider the hospital efficient ?

Given $P = 17.26\% = 0.1726, Q = 1 - P = 0.8274, p = \frac{63}{640} = 0.0984, n = 640$.

Let $H_0 : p = P$ and $H_1 : p < P$ (left tailed test). Let LOS $\alpha\% = 1\%$.

We know that the test statistic $z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$.

$\therefore z = -4.96$.

From z-table, $z_{0.01} = -2.33$.

Here $|z| = 4.96 > z_{0.01} = -2.33$, Hence H_0 is rejected and so that the hospital is efficient.

Example 2: In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant ?

Given $p_1 = 20\% = 0.2, p_2 = 18.5\% = 0.185, n_1 = 900, n_2 = 1600$.
Let $H_0 : p_1 = p_2$ and $H_1 : p_1 \neq p_2$ (two tailed test). Let LOS $\alpha\% = 5\%$.
We know that the test statistic $z = \frac{p_1 - p_2}{\sqrt{PQ(\frac{1}{n_1} + \frac{1}{n_2})}}$.
But P is not given, we estimate it as
$\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.1904, \hat{Q} = 1 - \hat{P} = 0.8096$.
$\therefore z = 0.92$.
From z-table, $z_{0.05} = 1.96$.
Here $|z| = 0.92 < z_{0.05} = 1.96$, Hence H_0 is accepted and so that the difference between the proportions is not significant at 5% LOS.

Example 3: A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm, and the SD is 10 cm ?

Given $\bar{x} = 160, n = 100, \mu = 165$ and $\sigma = 10$.

Let $H_0 : \bar{x} = \mu$ and $H_1 : \bar{x} \neq \mu$ (two tailed test). Let LOS $\alpha\% = 1\%$.

We know that the test statistic $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$.

$\therefore z = -5$.

From z-table, $z_{0.01} = 2.58$.

Here $|z| = 5 > z_{0.01} = 2.58$, Hence H_0 is rejected and so that it is not statistically correct to assume $\mu = 165$.

Example 4: In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the samples have been drawn from the same population with SD 4 ?

Given $\bar{x_1} = 20, n_1 = 500, \bar{x_2} = 15, n_2 = 400, \sigma = 4$.
Let $H_0 : \bar{x_1} = \bar{x_2}$ and $H_1 : \bar{x_1} \neq \bar{x_2}$ (two tailed test). Let LOS $\alpha\% = 1\%$.
We know that the test statistic $z = \frac{\bar{x_1} - \bar{x_2}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$.
$\therefore z = 18.6$.
From z-table, $z_{0.01} = 2.58$.
Here $|z| = 18.6 > z_{0.01} = 2.58$, Hence H_0 is rejected and so that the samples could not have been drawn from the same population.

The sampling distribution of many statistics are not normal, even though the parent populations may be normal.

Students t-Distribution: A RV T is said to follow Students t-Distribution or simply t-Distribution, if its pdf is given by

$$f(t) = \frac{1}{\sqrt{v}\beta(v/2, 1/2)}(1 + t^2/v)^{-(\frac{v+1}{2})}, -\infty < t < \infty$$

where v is called the number of degrees of freedom of the t-distribution.

Properties t-Distribution:

1. The probability curve of the t-distribution is similar to the standard normal curve and is symmetric about $t = 0$.
2. For sufficiently large n, the t-distribution tends to the standard normal distribution.
3. The mean of the t-distribution is 0.
4. The variance of the t-distribution is $\frac{v}{v-2} > 1$ if $n > 2$ but it tends to 1 as $v \to \infty$.

Test 1: Test of significance of the difference between sample mean and population mean.

Let $\overline{x}$ is the mean of a sample of size n, drawn from a population $N(\mu, \sigma)$. If σ is not known then we estimate it using the sample SD S.

The test statistic (t-distribution) is given by $t = \frac{\overline{x}}{\frac{s}{\sqrt{n-1}}}$ with degree of freedom $v = n - 1$.

If $|t| < t_v(\alpha)$ (critical value), then null hypothesis is accepted at LOS α. Otherwise it is rejected.

Test 2: Test of significance of the difference between means of two samples drawn from the same population.

Let $\overline{x_1}$ and $\overline{x_2}$ are the means of two samples of sizes n_1 and n_2 with SD S_1 and S_2, respectively, drawn from normal population.

The test statistic (t-distribution) is given by

$t = \frac{\overline{x_1}-\overline{x_2}}{\sqrt{\left(\frac{n_1S_1^2+n_2S_2^2}{n_1+n_2-2}\right)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ with degree of freedom $v = n_1 + n_2 - 2$.

If $|t| < t_v(\alpha)$ (critical value), then null hypothesis is accepted at LOS α. Otherwise it is rejected.

Note: If $n_1 = n_2 = n$ and the pairs of values x and y are associated in some way (or correlated).

In this case, we shall assume that $H_0 : \bar{d}(\bar{x} - \bar{y}) = 0$ and test the significance of the difference between $\bar{d}$ and 0. The test statistic

$t = \frac{\bar{d}}{\frac{s}{\sqrt{n-1}}}$ with degree of freedom $v = n - 1$, where

$d_i = x_i - y_i (i = 1, 2, 3, ..., n), \bar{d} = \bar{x} - \bar{y}$ and $s =$ SD of

$d = \frac{1}{n}\sum(d_i - \bar{d})^2$.

Example 1: A machinist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter 1.85 cm with a SD of 0.1 cm. On the basis of this sample, would you say that the work of the machinist is inferior ?

Given $\bar{x} = 1.85, S = 0.1, n = 10, \mu = 1.75$.

Let $H_0 : \bar{x} = \mu$ and $H_1 : \bar{x} \neq \mu$ (two tailed test). Let LOS $\alpha\% = 5\%$.

We know that the test statistic $t = \frac{\overline{x}}{\frac{s}{\sqrt{n-1}}}$ with degree of freedom $v = n - 1$.

$\therefore t = 3$ with $v = 9$.

From t-table, $t_{0.05} = 2.262$.

Here $|t| = 3 > t_{0.05} = 2.26$, Hence H_0 is rejected and so that the work of the machinist to be inferior.

Example 2: The biological values of protein from cow's milk and buffalo's milk at a certain level are given as

$Cow'smilk(x_1):$ 1.82 2.02 1.88 1.61 1.81 1.54

$Buffalo'smilk(x_2):$ 2.00 1.83 1.86 2.03 2.19 1.88

Examine if the average values of protein in the two samples significantly differ.

Given $n = n_1 = n_2 = 6$. Now

$\bar{x_1} = \frac{1.82+2.02+1.88+1.61+1.81+1.54}{6} = 1.78$ and

$\bar{x_2} = \frac{2.00+1.83+1.86+2.03+2.19+1.88}{6} = 1.965.$

Again $S_1^2 = \frac{\sum x_1^2}{6} - (\bar{x_1})^2 = 0.0261$ and $S_2^2 = \frac{\sum x_2^2}{6} - (\bar{x_2})^2 = 0.0154.$

Let $H_0 : \bar{x_1} = \bar{x_2}$ and $H_1 : \bar{x_1} \neq \bar{x_2}$ (two tailed test). Let LOS $\alpha\% = 5\%$.

We know that the test statistic $t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\left(\frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ with degree of freedom $v = n_1 + n_2 - 2$.

$\therefore t = -2.03$ with $v = 10$.

From t-table, $t_{0.05} = 2.23$.
Here $|t| = 2.03 < t_{0.05} = 2.23$, Hence H_0 is accepted and so that the difference between the mean protein values of two samples is not significant at 5% LOS.

Example 3: The following data relate to the marks obtained by 11 students in 2 tests, one held at the beginning of a year and the other at the end of the year after intensive coaching.

$Test1:$	19	23	16	24	17	18	20	18	21	19	20
$Test2:$	17	24	20	24	20	22	20	20	18	22	19

Do the data indicate that the students have benefited by coaching ?

The given data relate to the marks obtained in 2 tests by the same set of students. Hence the marks in the 2 tests can be regarded as correlated and so the t-test for paired values should be used.
Let $d = x - y$, where x, y denote the marks in the 2 tests. Thus the values of d are $2, -1, -4, 0, -3, -4, 0, -2, 3, -3, 1$.

Now $\sum d = -11$ and $\sum d^2 = 69$.
$\therefore \bar{d} = \frac{1}{n}\sum d = -1, s^2 = \frac{1}{n}\sum d^2 - (\bar{d})^2 = 5.27 \Rightarrow s = 2.296$.
Let $H_0 : \bar{d} = 0$ (the students have not benefitted by coaching) and $H_1 : \bar{d} < 0 (x < y)$ (one tailed test). Let LOS $\alpha\% = 5\%$.
We know that the test statistic $t = \frac{\bar{d}}{\frac{s}{\sqrt{n-1}}}$ with degree of freedom $v = n - 1$.
$\therefore t = -1.38$ with $v = 10$. From t-table, $t_{0.05} = 1.81$.
Here $|t| = 1.38 < t_{0.05} = 1.81$, Hence H_0 is accepted and so that there is no significant difference between the two sets of marks.
That is, the students have not benefitted by coaching.

Thank You.