Slot: A2 **Set: B**

SRM Institute of Science and Technology
College of Engineering and Technology
DEPARTMENT OF MATHEMATICS

SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu

Academic Year: 2021-22 (Even)

Test : **CLAT- III** **Date : 20/06/2022**
Course Code & Title :**18MAB302T-Discrete Mathematics for Engineers** **Duration:** 2 Periods
Year/ Sem/Branch : II /IV/NWC **Max. Marks:** 50

Q.No	**Part – A(10 x 1 = 10 Marks)**	
	Question	**Answer**
1.	Generators of the cyclic group $(\mathbb{Z}_{6,} +_6)$ are A. $\overline{1}$ and $\overline{3}$ B. $\overline{3}$ and $\overline{5}$ C. $\overline{1}$ and $\overline{5}$ D. $\overline{1}$, $\overline{3}$ and $\overline{5}$	**C**
2.	If $\circ$ is binary operation on the set of integers defined by $a \circ b = a+b+2$, then the identity element is A. 0 B. 1 C. -1 D. -2	**D**
3.	A cyclic group is always A. Abelian B. Non-commutative C. Non-abelian D. Permutation group	**A**
4.	The weight of the word 101011 is A. 2 B. 3 C. 4 D. 5	**C**
5.	The minimum distance between these code words 0000, 0110, 1011, 1100 is A. 1 B. 2 C. 3 D. 4	**B**
6.	The chromatic number of a complete graph of 5 vertices is A. 2 B. 3 C. 4 D. 5	**D**
7.	A vertex, which is adjacent to exactly one vertex in a graph is called _________ vertex. A. Isolated B. Pendant C. Incident D. Simple	**B**
8.	The degree of an isolated vertex is A. 0 B. 1 C. 2 D. 3	**A**
9.	A __________ path of a graph G is a path, which includes every edges of G exactly once. A. Eulerian B. Hamiltonian C. Planar D. Isomorphic	**A**

10.	If two graphs are isomorphic then which statement is false? A. Number of vertices of two graphs are equal B. Number of edges of two graphs are equal C. The degree sequence of the vertices of the graph are same D. Corresponding vertices may not have the same degree	**D**

Part – B (4x 10 = 40 Marks)
Answer ANY Four

11.

(i) If a is a generator of the group G then a^{-1} is also a generator.
$G =< a >$. So any element $b \in G, b = a^r$, where r is an integer.

[2 Marks]

Now, $b = a^r = (a^{-1})^{-r}$.
Therefore, $< a^{-1} >= G$
Hence a^{-1} is also a generator.

[3 Marks]

(ii) Show that group (G, ∘), $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$ for all $a, b \in$ G.

$$(a \circ b) \circ (b^{-1} \circ a^{-1}) = a \circ (b \circ b^{-1}) \circ a^{-1} = a \circ e \circ a^{-1} = a \circ a^{-1} = e,$$

[2 Marks]

$$(b^{-1} \circ a^{-1}) \circ (a \circ b) = b^{-1} \circ (a^{-1} \circ a) \circ b = b^{-1} \circ e \circ b = b^{-1} \circ b = e.$$

[2 Marks]

Hence by definition of inverse, $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$ for all $a, b \in$ G.

[1 Mark]

12.

Show that the set $G = \{1,2,3,4,5,6\}$ form a cyclic group under the operation multiplication modulo 7. Find all generators of this group. Is it a commutative group? Justify?

Modulo 7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

(a) Since all the elements in the composition table belong to the set G, the set G is closed under multiplication.
(b) Here G is a subset of ℝ (set of real numbers) and real numbers obey associative property with respect to multiplication. Hence G satisfies associative property with respect to multiplication.
(c) Here 1 is the identity element.
(d) The inverse of $1, 2, 3, 4, 5, and$ 6 are $1, 4, 5, 2, 3, 6$ respectively.
Hence G is a group under the operation multiplication modulo 7.

[4 Marks]

Now, '3' is the generator of the group G. Hence G is a cyclic group.

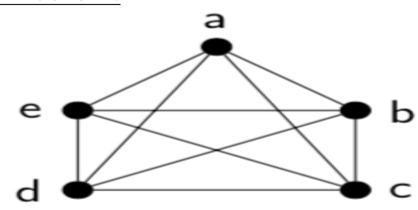
[2 Marks]

The elements 3 and 5 are the generators of the group G.

	[3 Marks] The composition table is symmetric about the main diagonal. Hence the group is commutative. [1 Mark]
13.	Given the generator matrix $\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$ corresponding to the encoding function $e: B^3 \to B^6$, find the corresponding parity check matrix and use it to decode the following received words (i) 111101 (ii) 100100 (iii) 111100 (iv) 010100 Parity check matrix: $H = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$ [2 Marks] (i) $H\,r^t = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ Since the syndrome is the third column of H, the element in the third position of r is changed. Therefore the decode word is 110101. (ii) $H\,r^t = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ Since the syndrome is the fifth column of H, the element in the fifth position of r is changed. Therefore the decode word is 100110. (iii) $H\,r^t = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ Since the syndrome is the fourth column of H, the element in the fourth position of r is changed. Therefore the decode word is 111000. (iv) $H\,r^t = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ Since the syndrome is not identical to any column of H, the received word cannot be decode uniquely. [8 Marks]

14.

(i) Find the number of vertices, the number of edges and degree of each vertices. Also verify the handshaking theorem.



Number of vertices: 5

Number of edges (e): 10

$$deg(a) = deg(b) = deg(c) = deg(d) = deg(e) = 4$$

[3 Marks]

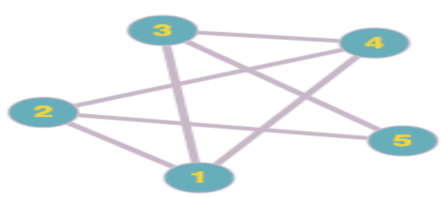
$$\sum(\text{degrees of 5 vertices}) = 20 = 2 \cdot 10 = 2e$$

Hence handshaking theorem verified.

[2 Marks]

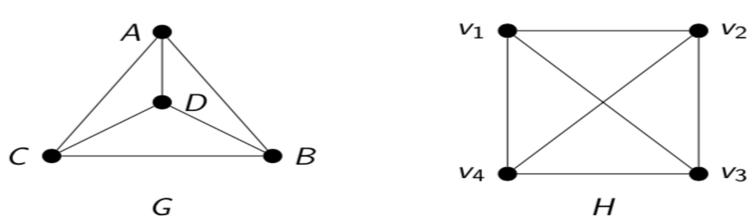
(ii) Draw the graph represented by the following adjacency matrix.

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$



15.

Check whether the following two graphs G and H are isomorphic



Graphs	Number of vertices	Number of edges	Degree sequences
G	4	6	3,3,3,3
H	4	6	3,3,3,3

[3 Marks]

Bijective mapping:

$$A \longrightarrow v_1$$
$$B \longrightarrow v_2$$
$$C \longrightarrow v_3$$
$$D \longrightarrow v_4$$

[3 Marks]

Adjacency matrices:

$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array}; \quad A_H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array};$$

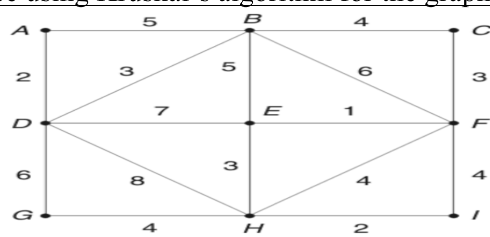
[3 Marks]

Therefore $A_G = A_H$.

Hence G and H are isomorphic graphs. [1 Mark]

16.

Write down the Kruskal's algorithm for finding the minimum spanning trees. Find a minimum spanning tree using Kruskal's algorithm for the graph given below:



Kruskal's algorithm:

(a) The edges of the given graph G are arranged in the order of increasing weights.

(b) An edge G with minimum weight is selected as an edge of the required spanning tree.

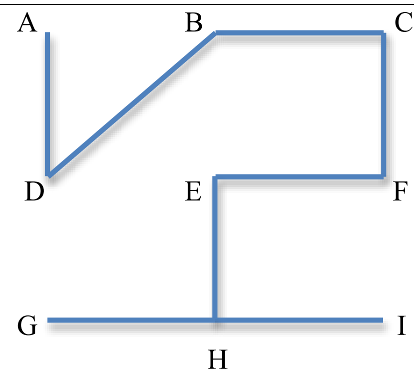
(c) Edges with minimum weight that do not from a circuit with the already selected edges are successively added.

(d) The procedure is stopped after $(n-1)$ edges have been selected.

[3 Marks]

Edge	Weight	Included or not	If not then why
EF	1	Yes	–
AD	2	Yes	–
HI	2	Yes	–
CF	3	Yes	–
BD	3	Yes	–
EH	3	Yes	–
BC	4	Yes	–
GH	4	Yes	–
FI	4	No	$F-I-H-E$
HF	4	–	–
AB	5	–	–
BE	5	–	–
DG	6	–	–
BF	6	–	–
DE	7	–	–
DH	8	–	–

[5 Marks]

	 [2 Marks]

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