Slot: A1 **Set: B**

SRM Institute of Science and Technology

College of Engineering and Technology

DEPARTMENT OF MATHEMATICS

SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu

Academic Year: 2021-22 (Even)

Test : **CLAT- III** **Date : 20/06/2022**
Course Code & Title :18MAB302T-Discrete Mathematics for Engineers **Duration:** 2 Periods
Year/ Sem/Branch : II /IV/NWC **Max. Marks:** 50

Answer Key

Q.No	Part – A(10 x 1 = 10 Marks)	
	Question	**Answer**
1.	In a permutation group S_3 if $p = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, then p^{-1} is A. $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ B. $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ C. $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ D. $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$	**D**
2.	A group is always A. Semigroup B. Cyclic group C. Commutative group D. Abelian group	**A**
3.	If $\circ$ is the binary operation on the set of positive rational numbers defined by $a \circ b = \frac{ab}{2}$, then the identity element is A. 1 B. -1 C. 2 D. 3	**C**
4.	The weight of the word 110101 is A. 3 B. 4 C. 5 D. 6	**B**
5.	The minimum distance between the code words $x = 10110,\ y = 11110,$ $z = 10011$ is A. 1 B. 2 C. 3 D. 4	**A**
6.	A _______ circuit of a graph G is a circuit, which includes every edge of G exactly once. A. Eulerian B. Hamiltonian C. Planar D. Isomorphic	**A**
7.	A graph in which number is assigned to each edge is called A. Pseudograph B. Simple graph C. Multigraph D. Weighted graph	**D**
8.	The chromatic number of a tree with two or more vertices is A. 2 B. 3 C. 4 D. 5	**A**

9.	The number of edges in a complete bipartite graph $K_{5,4}$ is A. 9 B. 15 C. 20 D. 25	**C**
10.	Which of the following statement is false A. A tree with n vertices has $n-1$ edges B. Any connected graph with n vertices and $n-1$ edges is called a tree C. The weight of a minimum spanning tree is unique D. For any graph minimum spanning tree is unique	**D**

Part – B (4x 10 = 40 Marks)
Answer ANY Four

11.

(i) Determine the order of each element of the multiplicative group $\{1, \omega, \omega^2\}$, where ω is the cube root of unity under multiplication.

1 is the identity element of the given group.
Since, $1 \cdot 1 = 1$, $O(1) = 1$.
$\omega \cdot \omega = \omega^2$ And $\omega^2 \cdot \omega = \omega^3 = 1$, $O(\omega) = 3$.
$\omega^2 \cdot \omega^2 = \omega^4 = \omega$, $\omega \cdot \omega^2 = \omega^3 = 1$, $O(\omega^2) = 3$.

[5 Marks]

(ii) Examine whether the identity element of a group $(G,\circ)$ is unique.
Let $(G,\circ)$ be a group. Let e and e' be two identical elements of G.
Then $a \circ e = e \circ a = a$ and $a \circ e' = e' \circ a = a$.

[2 Marks]

Now $e \circ e' = e$ (by the property of e')
Also, $e \circ e' = e'$(by the property of e)
So, $e = e'$. Hence the identity element is unique.

[3 Marks]

12.

Show that $(\mathbb{Z}_7, +_7)$ is a cyclic group. Hence find all of its generators. Is it a commutative group? Justify?

$+_7$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$
$\overline{0}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$
$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	$\overline{0}$
$\overline{2}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	$\overline{0}$	$\overline{1}$
$\overline{3}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	$\overline{0}$	$\overline{1}$	$\overline{2}$
$\overline{4}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$
$\overline{5}$	$\overline{5}$	$\overline{6}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{6}$	$\overline{6}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$

[2 Marks]

(a) Since all the elements in the composition table belong to the set $\mathbb{Z}_7$, the set $\mathbb{Z}_7$ is closed under $+_7$.
(b) $\mathbb{Z}_7$ satisfies associative property with respect to $+_7$. As $\overline{a} + (\overline{b} + \overline{c}) = \overline{a + b + c} = (\overline{a} + \overline{b}) + \overline{c}$, for all $\overline{a}, \overline{b}, \overline{c} \in \mathbb{Z}_7$

(c) Here $\overline{0}$ is the identity element.

(d) The inverse of $\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}$ are $\overline{0}, \overline{6}, \overline{5}, \overline{4}, \overline{3}, \overline{2}, \overline{1}$ respectively.

Hence $(\mathbb{Z}_7, +_7)$ is a group.

[2 Marks]

(e) $\overline{1}$ is a generator for the group. Therefore $Z_7 = <\overline{1}>$ is a cyclic group.

$$\overline{1} = \overline{1}$$
$$\overline{1} + \overline{1} = \overline{2}$$
$$\overline{1} + \overline{1} + \overline{1} = \overline{3}$$
$$\overline{1} + \overline{1} + \overline{1} + \overline{1} = \overline{4}$$
$$\overline{1} + \overline{1} + \overline{1} + \overline{1} + \overline{1} = \overline{5}$$
$$\overline{1} + \overline{1} + \overline{1} + \overline{1} + \overline{1} + \overline{1} = \overline{6}$$
$$\overline{1} + \overline{1} + \overline{1} + \overline{1} + \overline{1} + \overline{1} + \overline{1} = \overline{0}$$

[2 Marks]

$\overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}$ are the generators for the group.

[3 Marks]

The composition table is symmetrical about the main diagonal. Hence the group is commutative.

[1 Mark]

13.

List the code word generated by the encoding function $e: B^2 \rightarrow B^5$ with respect to the parity check matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Generator matrix: $G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

[3 Marks]

$B^2 = \{(0,0), (0,1), (1,0), (1,1)\}$

For encoding code $e(\omega) = \omega G$

[2 Marks]

$e(0\ 0) = (0\ 0)\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} = (0\ \ 0\ \ 0\ \ 0\ \ 0)$

$e(0\ 1) = (0\ 1)\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} = (0\ \ 1\ \ 1\ \ 0\ \ 1)$

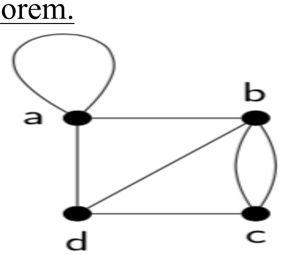
$e(1\ 0) = (1\ 0)\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} = (1\ \ 0\ \ 1\ \ 0\ \ 1)$

$e(1\ 1) = (1\ 1)\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} = (1\ \ 1\ \ 0\ \ 0\ \ 0)$

[5 Marks]

14.

(i) Find the number of vertices, the number of edges and degree of each vertices. Also verify the handshaking theorem.



Number of vertices: 4
Number of edges: 7

$$deg(a) = deg(b) = 4, deg(c) = deg(d) = 3$$

[3 Marks]

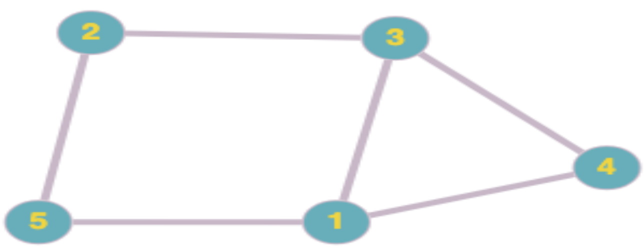
$$\sum(\text{degrees of 4 vertices}) = 14 = 2 \cdot 7 = 2e$$

Hence handshaking theorem verified.

[2 Marks]

(ii) Draw the graph represented by the following adjacency matrix.

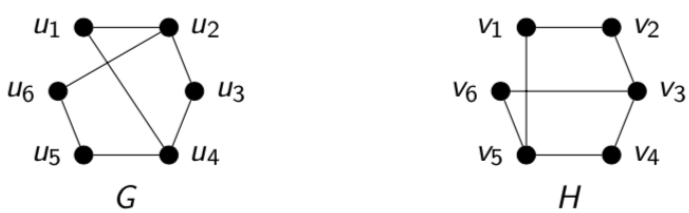
$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$



[5 Marks]

15.

Check whether the following two graphs G and H are isomorphic



Graphs	Number of vertices	Number of edges	Degree sequences
G	6	7	2,2,2,2,3,3
H	6	7	2,2,2,2,3,3

[3 Marks]

Bijective mapping:

$$u_1 \longrightarrow v_6$$
$$u_2 \longrightarrow v_3$$
$$u_3 \longrightarrow v_4$$
$$u_4 \longrightarrow v_5$$
$$u_5 \longrightarrow v_1$$
$$u_6 \longrightarrow v_2$$

[3 Marks]

Adjacency matrices:

$$A_G = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array}\begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array} ; \quad A_H = \begin{array}{c} \\ v_6 \\ v_3 \\ v_4 \\ v_5 \\ v_1 \\ v_2 \end{array}\begin{array}{c} \begin{array}{cccccc} v_6 & v_3 & v_4 & v_5 & v_1 & v_2 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

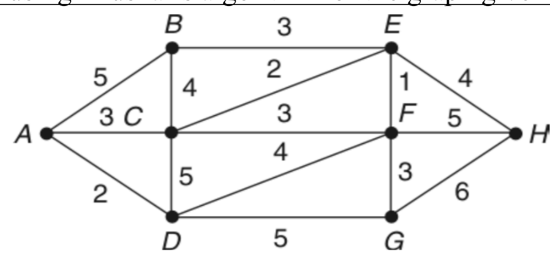
[3 Marks]

Therefore $A_G = A_H$.
Hence G and H are isomorphic graphs.

[1 Mark]

16.

Write down the Kruskal's algorithm for finding the minimum spanning trees. Find a minimum spanning tree using Kruskal's algorithm for the graph given below:

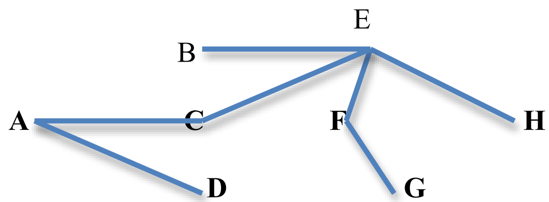


Kruskal's algorithm:
(a) The edges of the given graph G are arranged in the order of increasing weights.
(b) An edge G with minimum weight is selected as an edge of the required spanning tree.
(c) Edges with minimum weight that do not from a circuit with the already selected edges are successively added.
(d) The procedure is stopped after $(n-1)$ edges have been selected.

[3 Marks]

Edge	Weight	Included or not	If not then why
EF	1	Yes	–
CE	2	Yes	–
AD	2	Yes	–
BE	3	Yes	–
AC	3	Yes	–
CF	3	No	$C-E-F-C$
FG	3	Yes	–
EH	4	Yes	–
DF	4	–	–
BC	4	–	–
AB	5	–	–
DG	5	–	–
FH	5	–	–
CD	5	–	–
GH	6	–	–

[5 Marks]

	 [2 Marks]