Slot: A2 **Set: A**

SRM Institute of Science and Technology
College of Engineering and Technology
DEPARTMENT OF MATHEMATICS

SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu

Academic Year: 2021-22 (Even)

Test : **CLAT- III** — **Date : 20/06/2022**

Course Code & Title :18MAB302T-Discrete Mathematics for Engineers — **Duration:** 2 Periods

Year/ Sem/Branch : II /IV/NWC — **Max. Marks:** 50

Q.No	Part – A(10 x 1 = 10 Marks) Instructions: Answer all Questions Question	Answer
1.	Which of the following is a group? A. $(\mathbb{N},\ \cdot)$ B. $(\mathbb{N}, -)$ C. $(\mathbb{Z}, +)$ D. $(\mathbb{R}, -)$	**C**
2.	The order of the permutation group S_3 is A. 3 B. 6 C. 9 D. 12	**B**
3.	In a permutation group S_3 if $p = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$, then p^{-1} is A. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ B. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$ C. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$ D. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$	**D**
4.	The Hamming distance between the code words $x = 111001$ and $y = 100110$ is A. 3 B. 4 C. 5 D. 6	**C**
5.	The order of the generator matrix of the encoding function $e: B^2 \to B^5$ is A. 2×5 B. 5×2 C. 3×5 D. 5×3	**A**
6.	The chromatic number of a circuit of length 8 is A. 2 B. 3 C. 4 D. 8	**B**
7.	Which of the following graph is connected and has no circuits? A. Cyclic graph B. Regular graph C. Tree D. Complete graph	**C**
8.	The maximum number of edges in a bipartite graph of 14 vertices is A. 24 B. 49 C. 98 D. 196	**B**
9.	A __________ path of a graph G is a path, which includes every vertex of G exactly once. A. Eulerian B. Hamiltonian C. Planar D. Isomorphic	**B**

10.	Length of the path of a graph is defined by the A. Number of vertices in the graph B. Number of edges in the graph C. Number of vertices in the path D. Number of edges in the path	**D**

Part – B (4x 10 = 40 Marks)
Answer ANY Four

<table>
<tr><td>11.</td><td>(i) Show that a group (G, ∘) is abelian iff $(a \circ b)^2 = a^2 \circ b^2$, for all $a, b \in$ G
First assume that (G, ∘) is abelian, then $a \circ b = b \circ a\ \forall a, b \in G$
$(a \circ b)^2 = (a \circ b) \circ (a \circ b) = a \circ (b \circ a) \circ a = a \circ (a \circ b) \circ b = (a \circ a) \circ (b \circ b) = a^2 \circ b^2\ \forall a, b \in G$
[2 Marks]
Conversely, let $(a \circ b)^2 = a^2 \circ b^2 \Rightarrow (a \circ b) \circ (a \circ b) = (a \circ a) \circ (b \circ b)$
$\Rightarrow a \circ (b \circ a) \circ b = a \circ (a \circ b) \circ b$
$\Rightarrow (b \circ a) = (a \circ b)$ [by both side calculation law]
Hence G is abelian.
[3 Marks]
(ii) Show that in a group (G, ∘) inverse of each element is unique.
Let $a \in G$. If possible a', a'' be two inverses of a then we have $a \circ a' = a' \circ a = e$ and $a \circ a'' = a'' \circ a = e$
[2 Marks]
Now by associative law, $(a' \circ a) \circ a'' = a' \circ (a \circ a'')$
$\Rightarrow e \circ a'' = a' \circ e$
$\Rightarrow a'' = a'$
Hence inverse is unique.
[3 Marks]</td></tr>
<tr><td>12.</td><td>Find the order of each of the elements of the permutation group $(S_3,\circ)$. Is $(S_3,\circ)$ abelian? Justify?
$$O\left(\begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix}\right) = 1, O\left(\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}\right) = 3, O\left(\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}\right) = 3,$$
$$O\left(\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix}\right) = 2, O\left(\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}\right) = 2, O\left(\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix}\right) = 2,$$
[6 Marks]
$(S_3,\circ)$ is not abelian. As $\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} \neq \begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} \circ \begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}$
[4 Marks]</td></tr>
</table>

13.

List the code word generated by the encoding function $e: B^3 \rightarrow B^6$ with respect to the parity check matrix

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Generator matrix: $G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$

[3 Marks]

$B^2 = \{(0,0,0), (0,0,1), (0,1,0), (1,0,0), (0,1,1), (1,0,1), (1,1,0), (1,1,1)\}$

For encoding code $e(\omega) = \omega G$

[2 Marks]

$$e(0\,0\,0) = (0\,0\,0)\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} = (0\ 0\ 0\ 0\ 0\ 0)$$

$$e(0\,0\,1) = (0\,0\,1)\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} = (0\ 0\ 1\ 0\ 1\ 1)$$

$$e(0\,1\,0) = (0\,1\,0)\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} = (0\ 1\ 0\ 1\ 0\ 1)$$

$$e(1\,0\,0) = (1\,0\,0)\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} = (1\ 0\ 0\ 1\ 1\ 0)$$

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$$e(1\,0\,1) = (1\,0\,1)\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} = (1\ 0\ 1\ 1\ 0\ 1)$$

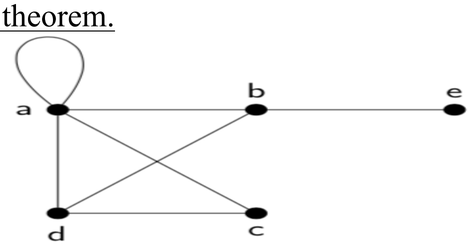
$$e(1\,1\,0) = (1\,1\,0)\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} = (1\ 1\ 0\ 0\ 1\ 1)$$

$$e(1\,1\,1) = (1\,1\,1)\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} = (1\ 1\ 1\ 0\ 0\ 0)$$

[5 Marks]

14.

(i) Find the number of vertices, the number of edges and degree of each vertices. Also verify the handshaking theorem.



Number of vertices: 5

Number of edges (e): 7

$$deg(a) = 5, deg(b) = 3, deg(c) = 2, deg(d) = 3, deg(e) = 1$$

[3 Marks]

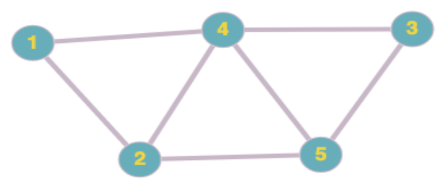
$$\sum(\text{degrees of 7 vertices}) = 14 = 2 \cdot 7 = 2e$$

Hence handshaking theorem verified.

[2 Marks]

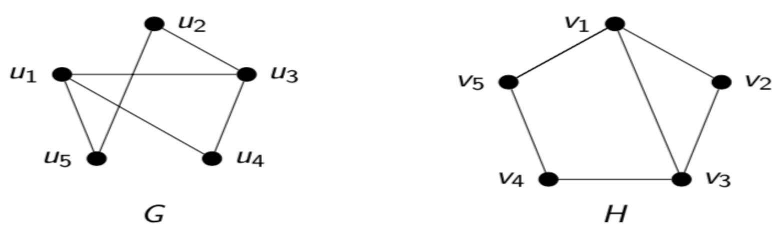
(ii) Draw the graph represented by the following adjacency matrix.

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$



[5 Marks]

15. Check whether the following two graphs G and H are isomorphic



Graphs	Number of vertices	Number of edges	Degree sequences
G	5	6	2,2,2,3,3
H	5	6	2,2,2,3,3

[3 Marks]

Bijective mapping:

$$u_1 \longrightarrow v_3$$
$$u_2 \longrightarrow v_5$$
$$u_3 \longrightarrow v_1$$
$$u_4 \longrightarrow v_2$$
$$u_5 \longrightarrow v_4$$

[3 Marks]

Adjacency matrices:

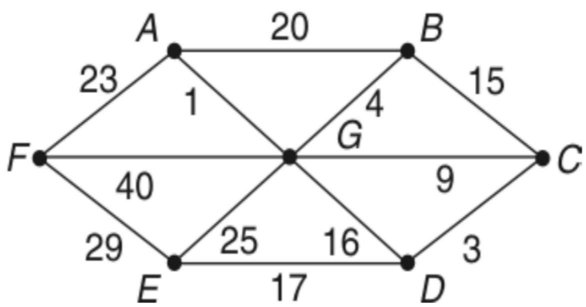
$$A_G = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array} ; \quad A_H = \begin{array}{c} \\ v_3 \\ v_5 \\ v_1 \\ v_2 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} v_3 & v_5 & v_1 & v_2 & v_4 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

[3 Marks]

Therefore $A_G = A_H$.
Hence G and H are isomorphic graphs.

[1 Mark]

16. Write down the Kruskal's algorithm for finding the minimum spanning trees. Find a minimum spanning tree using Kruskal's algorithm for the graph given below:

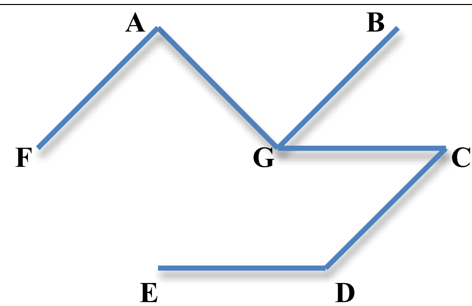


Kruskal's algorithm:
(a) The edges of the given graph G are arranged in the order of increasing weights.
(b) An edge G with minimum weight is selected as an edge of the required spanning tree.
(c) Edges with minimum weight that do not from a circuit with the already selected edges are successively added.
(d) The procedure is stopped after $(n-1)$ edges have been selected.

[3 Marks]

Edge	Weight	Included or not	If not then why
AG	1	Yes	–
CD	3	Yes	–
BG	4	Yes	–
CG	9	Yes	–
BC	15	No	$B-C-G-B$
GD	16	No	$G-D-C-G$
ED	17	Yes	–
AB	20	No	$A-B-G-A$
AF	23	Yes	–
EG	25	–	–
EF	29	–	–
FG	40	–	–

[5 Marks]

	 [2 Marks]