The Bellman-Ford Shortest Path Algorithm

Class Overview

- The shortest path problem
- Differences
- The Bellman-Ford algorithm
- Time complexity

Shortest Path Problem

- Weighted path length (cost): The sum of the weights of all links on the path.

- The single-source shortest path problem: Given a weighted graph G and a source vertex s, find the shortest (minimum cost) path from s to every other vertex in G.

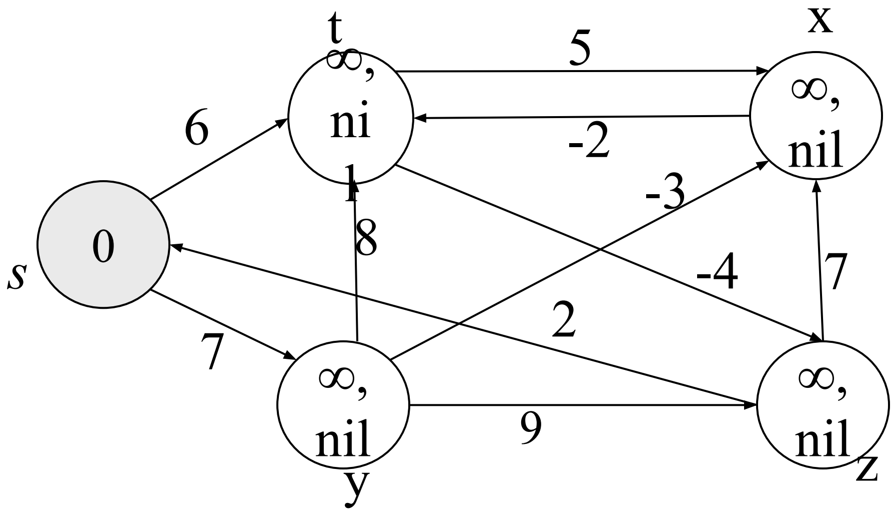
Differences

- Negative link weight: The Bellman-Ford algorithm works; Dijkstra's algorithm doesn't.

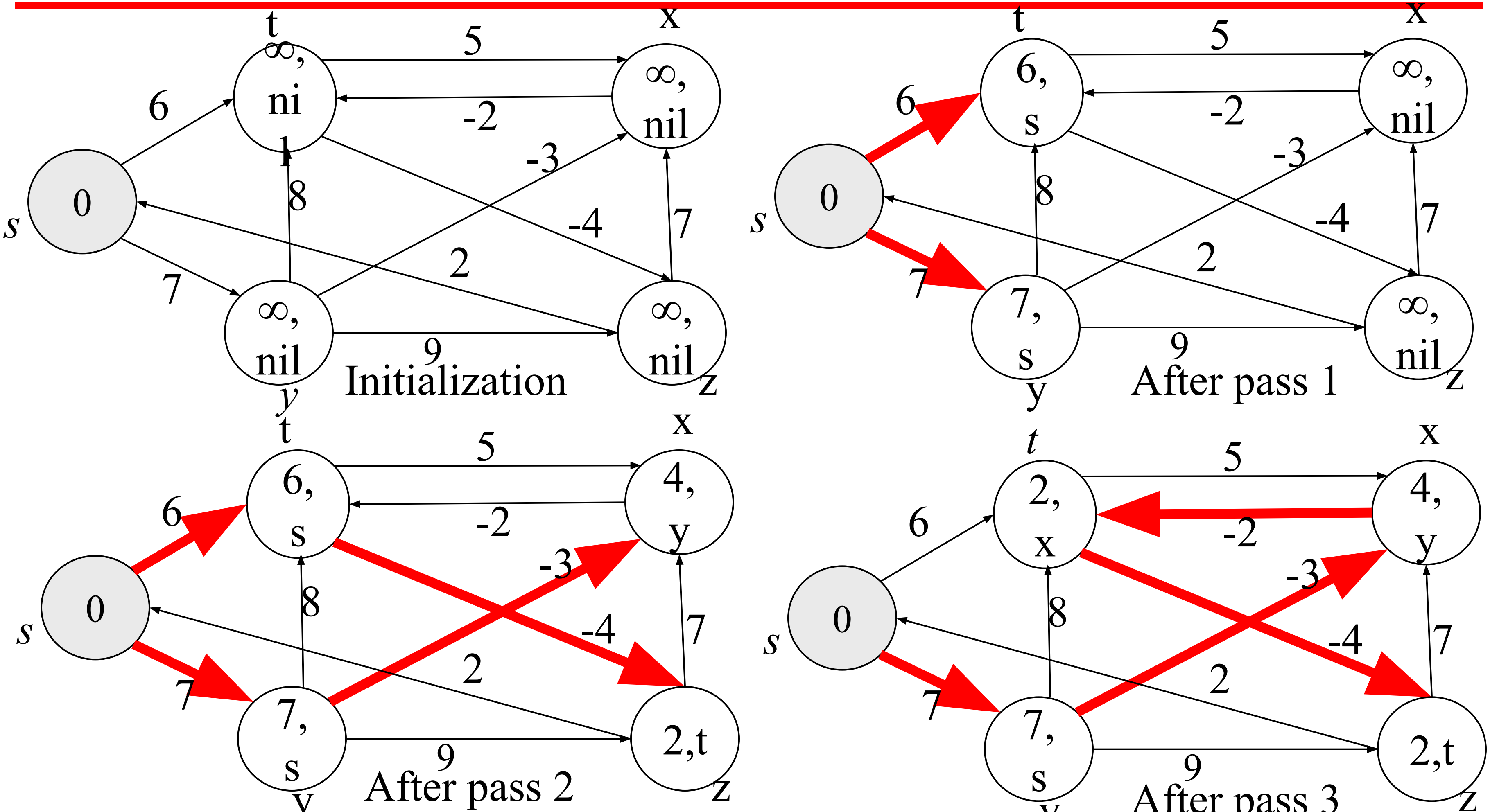
- Distributed implementation: The Bellman-Ford algorithm can be easily implemented in a distributed way. Dijkstra's algorithm cannot.

- Time complexity: The Bellman-Ford algorithm is higher than Dijkstra's algorithm.

The Bellman-Ford Algorithm



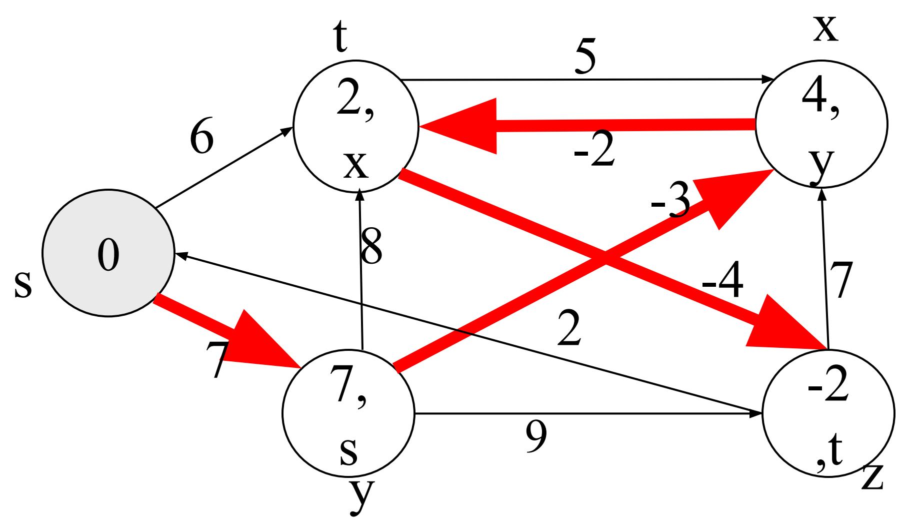
The Bellman-Ford Algorithm



The order of edges examined in each pass:

(t, x), (t, z), (x, t), (y, x), (y, t), (y, z), (z, x), (z, s), (s, t), (s, y)

The Bellman-Ford Algorithm



After pass 4

The order of edges examined in each pass:

(t, x), (t, z), (x, t), (y, x), (y, t), (y, z), (z, x), (z, s), (s, t), (s, y)

The Bellman-Ford Algorithm

```
Bellman-Ford(G, w, s)
    Initialize-Single-Source(G, s)
    for i := 1 to |V| - 1 do
        for each edge (u, v) ∈ E do
            Relax(u, v, w)
    for each vertex v ∈ u.adj do
        if d[v] > d[u] + w(u, v)
            then return False    // there is a negative cycle
    return True
```

```
Relax(u, v, w)
if d[v] > d[u] + w(u, v)
   then  d[v] := d[u] + w(u, v)
         parent[v] := u
```

Time Complexity

Bellman-Ford(G, w, s)

Initialize-Single-Source(G, s) $\longrightarrow$ O(|V|)
for i := 1 to |V| - 1 **do**
    for each edge (u, v) ∈ E **do**
        Relax(u, v, w) $\longrightarrow$ O(|V||E|)
for each vertex v ∈ u.adj **do** $\longrightarrow$ O(|E|)
    if d[v] > d[u] + w(u, v)
        then return False // there is a negative cycle
return True

Time complexity: O(|V||E|)