Slot: A1 **Set: A**

SRM Institute of Science and Technology
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DEPARTMENT OF MATHEMATICS

SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu

Academic Year: 2021-22 (Even)

Test : **CLAT- III** **Date : 20/06/2022**
Course Code & Title :18MAB302T-Discrete Mathematics for Engineers **Duration:** 2 Periods
Year/ Sem/Branch : II /IV/NWC **Max. Marks:** 50

Answer Key

Q.No	**Part – A(10 x 1 = 10 Marks)**	
	Question	**Answer**
1.	Which of the following is NOT a group? A. $(\mathbb{Z}, \cdot)$ B. $(\mathbb{R},+)$ C. $(\mathbb{Z}_3,+_3)$ D. $(\mathbb{Z},+)$	**A**
2.	Generators of the cyclic group $(Z_5, +_5)$ are A. $\overline{0}$ and $\overline{1}$ B. $\overline{1}$ and $\overline{3}$ C. $\overline{2}$ and $\overline{4}$ D. $\overline{1}, \overline{2}, \overline{3}$ and $\overline{4}$	**D**
3.	If $\circ$ is binary operation on the set of integers defined by $a\circ b=a+b+1$, then the identity element is A. 0 B. 1 C. -1 D. 2	**C**
4.	The order of the generator matrix of the encoding function $e: B^3 \rightarrow B^6$ is A. 3×4 B. 4×3 C. 3×6 D. 6×3	**C**
5.	Find the Hamming distance between these code words $x = 11010$ and $y = 10101$ A. 2 B. 3 C. 4 D. 5	**C**
6.	The chromatic number of the bipartite graph $K_{3,2}$ is A. 2 B. 3 C. 4 D. 5	**A**
7.	A graph in which loops and parallel edges are not allowed is called A. Pseudograph B. Simple graph C. Multigraph D. Weighted graph	**B**
8.	The maximum number of colors is required to color the regions of any map is A. 3 B. 4 C. 5 D. 6	**B**
9.	How many vertices are there in a graph with 16 edges and every vertex has degree 4? A. 4 B. 8 C. 9 D. 16	**B**

10.	A tree _________ A. Is not always a simple graph B. Can have parallel edges C. Is always connected D. Can contains circuits	**C**

Part – B (4x 10 = 40 Marks)
Answer ANY Four

11.

(i) Examine whether the set $G = \{1, -1, i, -i\}$ form a group or not under the operation of multiplication.

$\cdot$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

[2 Marks]

(a) Since all the elements in the composition table belong to the set G, the set G is closed under multiplication.
(b) Here G is a subset of $\mathbb{C}$ (set of complex numbers) and complex numbers obey associative property with respect to multiplication. Hence G satisfies associative property with respect to multiplication.
(c) Here 1 is the identity element.
(d) The inverse of $1, -1, i, and\ -i$ are $1, -1, -i, i$ respectively.

[3 marks]

(ii) Show that a cyclic group is abelian.
Let G be a cyclic group generated by a. Let $p, q \in G$, then there exist two integers r and s such that $p = a^r$and $q = a^s$.

[2 Marks]

$$p \circ q = a^r \circ a^s = a^{r+s} = a^{s+r} = a^s \circ a^r = q \circ p$$

Hence, $p \circ q = q \circ p$, for all $p, q \in G$.
Hence G is abelian.

[3 Marks]

12.

Show that $(\mathbb{Z}, +, \cdot)$ is a commutative ring with identity.
(a) $\mathbb{Z}$ is closed under $+$. As addition of any two integers is again an integer.
(b) Here $\mathbb{Z}$ is a subset of $\mathbb{R}$ (set of real numbers) and real numbers obey associative property with respect to addition. Hence $\mathbb{Z}$ satisfies associative property with respect to addition.
(c) '0' is the identity element.
(d) '$-a$' is the additive inverse for any integer $a \in \mathbb{Z}$.
(e) For any two integers $a, b \in \mathbb{Z}, a + b = b + a$. Hence $\mathbb{Z}$ is commutative under '$+$'.
Therefore $(\mathbb{Z}, +)$ is a commutative group under addition.

[4 Marks]

(f) $\mathbb{Z}$ is closed under multiplication. As multiplication of any two integers is again an integer.
(g) Here $\mathbb{Z}$ is a subset of $\mathbb{R}$ (set of real numbers) and real numbers obey associative property with respect to multiplication. Hence $\mathbb{Z}$ satisfies associative property with respect to multiplication.

<table>
<tr><td></td><td>
Therefore $(\mathbb{Z}, \cdot)$ is a semigroup under multiplication.
[2 Marks]
(h) The distributive laws hold: for all $a, b, c \in \mathbb{Z}$
$$a \cdot (b + c) = a \cdot b + a \cdot c$$
$$(a + b) \cdot c = a \cdot c + b \cdot c$$
Hence $(\mathbb{Z}, +, \cdot)$ is a ring.
[1 Mark]
(i) The ring is commutative with respect to multiplication. As for all $a, b \in \mathbb{Z}$, $a \cdot b = b \cdot a$
(j) 1 is the multiplicative identity
[3 Marks]
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<tr><td>13.</td><td>
<u>Given the generator matrix $\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$ corresponding to the encoding function $e: B^3 \rightarrow B^6$, find the corresponding parity check matrix and use it to decode the following received words
(i)110101 (ii) 001111 (iii) 110001 (iv) 111111</u>

Parity check matrix: $H = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$
[2 Marks]

(i) $H\, r^t = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
The received word in this case is the transmitted word itself.

(ii) $H\, r^t = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$
Since the syndrome is the fifth column of H, the element in the fifth position of r is changed. Therefore the decode word is 001101.

(iii) $H\, r^t = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
Since the syndrome is the fourth column of H, the element in the fourth position of r is changed. Therefore the decode word is 110101.
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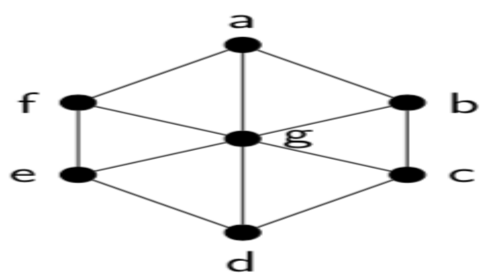
(iv) $H\,r^t = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Since the syndrome is not identical to any column of H, the received word cannot be decode uniquely.

[8 Marks]

14.

(i) Find the number of vertices, the number of edges and degree of each vertices. Also verify the handshaking theorem.



Number of vertices: 7

Number of edges (e): 12

$$deg(a) = deg(b) = deg(c) = deg(d) = deg(e) = deg(f) = 3$$

and $deg(g) = 6$

[3 Marks]

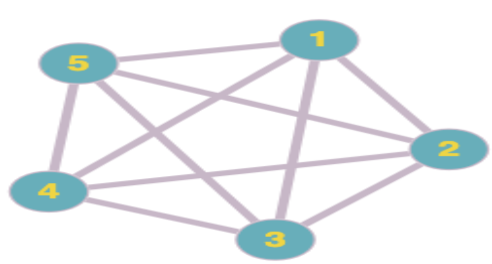
$$\sum(\text{degrees of 7 vertices}) = 24 = 2 \cdot 12 = 2e$$

Hence handshaking theorem verified.

[2 Marks]

(ii) Draw the graph represented by the following adjacency matrix.

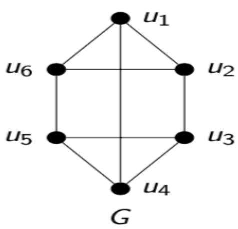
$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

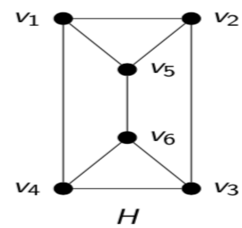


[5 Marks]

15.

Check whether the following two graphs G and H are isomorphic





Graphs	Number of vertices	Number of edges	Degree sequences
G	6	9	3,3,3,3,3,3
H	6	9	3,3,3,3,3,3

[3 Marks]

Bijective mapping:

$$u_1 \longrightarrow v_5$$
$$u_2 \longrightarrow v_2$$
$$u_3 \longrightarrow v_3$$
$$u_4 \longrightarrow v_6$$
$$u_5 \longrightarrow v_4$$
$$u_6 \longrightarrow v_1$$

[3 Marks]

Adjacency matrices:

$$A_G = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}; \quad A_H = \begin{array}{c} \\ v_5 \\ v_2 \\ v_3 \\ v_6 \\ v_4 \\ v_1 \end{array} \begin{array}{c} \begin{array}{cccccc} v_5 & v_2 & v_3 & v_6 & v_4 & v_1 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

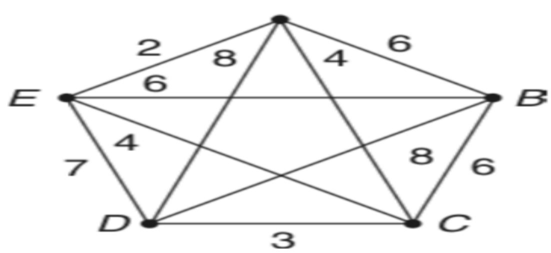
[3 Marks]

Therefore $A_G = A_H$.
Hence G and H are isomorphic graphs.

[1 Mark]

16.

Write down the Kruskal's algorithm for finding the minimum spanning trees. Find a minimum spanning tree using Kruskal's algorithm for the graph given below:

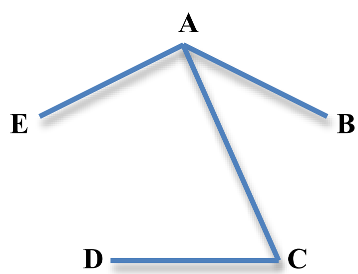


Kruskal's algorithm:
(a) The edges of the given graph G are arranged in the order of increasing weights.
(b) An edge G with minimum weight is selected as an edge of the required spanning tree.
(c) Edges with minimum weight that do not from a circuit with the already selected edges are successively added.
(d) The procedure is stopped after $(n-1)$ edges have been selected.

[3 Marks]

Edge	Weight	Included or not	If not then why
AE	2	Yes	–
CD	3	Yes	–
AC	4	Yes	–
CE	4	No	$A-E-C-A$
AB	6	Yes	–
BC	6	No	$A-B-C-A$
BE	6	–	–
DE	7	–	–
AD	8	–	–
BD	8	–	–

[5 Marks]



[2 Marks]