RA2011028010120
Harshit Tripathi
L1

1

09/06/22

ASSIGNMENT-2

Part-A

1. Using truth table show that $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$

→

p	q	r	$q \vee r$	$p \to q \vee r$	$p \to q \equiv a$	$p \to r \equiv b$	$a \vee b$	
T	T	T	T	T	T	T	T	
T	T	F	T	T	T	F	T	
T	F	T	T	T	F	T	T	
T	F	F	F	F	F	F	F	
F	T	T	T	T	T	T	T	
F	T	F	T	T	T	T	T	
F	F	T	T	T	T	T	T	
F	F	F	F	T	T	T	T	

2. Write the negation of each of the following statement

i) He swims if and only if the water is warm.

→ He swims if and only if water is not warm.

ii) This computer program is correct if and only if it produces the correct answer for all possible sets of input data.

→ This computer program is correct if and only if it produces the wrong answer for all possible sets of input data.

3. Prove the validity of the following argument

→ Let p:→ I get the job

q:→ I work hard

r:→ I'll get promoted

2

$p \wedge q \rightarrow r$ for the 1^{st} argument

now

t:→ I'll be happy

So for 2^{nd} argument it becomes $r \rightarrow t$

now we have 3^{rd} argument as $\sim t$

so from modus tollens

$r \rightarrow t$

$\sim t$

$\therefore \sim r$

Now combining arguments

$p \wedge q \rightarrow r$ and $\sim r$

from modus tollens, we get

$\sim(p \wedge q) = \sim p$ OR $\sim q$

Hence proved either I will not get the job or I will not work hard.

4. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

⇒ Let us assume that $\sqrt{2}$ is a rational no. with p and q as co-prime integers and $q \neq 0$

$\Rightarrow \sqrt{2} = p/q$

On squaring both sides we get,

$\Rightarrow 2q^2 = p^2$

$\Rightarrow p^2$ is an ~~eq~~. even no. that divides q^2. $\therefore$, p is an even no. that divides q.

Let $p = 2x$ where x is a whole no.

By substituting this value of p in $2q^2 = p^2$, we get

$\Rightarrow 2q^2 = (2x)^2$

$\Rightarrow 2q^2 = 4x^2$

$\Rightarrow q^2 = 2x^2$

$\Rightarrow q^2$ is an even no. that divides x^2. $\therefore$ q is an even no. that divides x.

Since p and q both are even no. with 2 as a common multiple which means that p and q are not co-prime numbers as their HCF is 2.

This leads to the contradiction that $\sqrt{2}$ is a rational no. in the form of p/q with p and q both co-prime no. and $q \neq 0$.

Thus $\sqrt{2}$ is an irrational no. by the contradiction method.

5. Using mathematical induction, prove that $3^{2n} - 8n - 1$ is divisible by 64.

$\rightarrow$ $P(n)$: $3^{2n} - 8n - 1$ is divisible by 64

$P(1)$: $3^{2\times1} - 8\times1 - 1 = 9 - 8 - 1 = 0$ which is divisible by 64.

$\therefore$ $P(1)$ is true.

Now let if it is true for $n = m$

$P(m) = 3^{2m} - 8m - 1$ is divisible by 64

$3^{2m} - 8m - 1 = 64k \; (k \in N)$

$\Rightarrow 3^{2m} = 64k + 8m + 1$

$P(m+1) = 3^{2(m+1)} - 8(m+1) - 1$

$= 3^{2m+2} - 8m - 8 - 1$

$= 3^{2m} \cdot 3^2 - 8m - 9$

$= 9(64k + 8m + 1) - 8m - 9$

$= 9 \times 64k + 72m + \cancel{9} - 8m - \cancel{9}$

$= 9 \times 64k + 64m = 64(9k + m)$

So $P(m+1)$ is true

By using mathematical induction

$\therefore P(n)$ is also true i.e $3^{2n} - 8n - 1$ is divisible by 64.

Part-B

1. How many integers are between 1 and 200 which are divisible by any one of the integers 2, 3 and 5?

→ A no. divisible by 2 $= \frac{200}{2} = 100$

B " " " 3 $= \frac{200}{3} = 66$

C " " " 5 $= \frac{200}{5} = 40$

Counting twice

AB No. divisible by 6 : $\frac{200}{6} = 33$

AC) " " " 10 : $\frac{200}{10} = 20$

BC) " " " 15 : $\frac{200}{15} = 13$

Counting 3 times

ABC) No. divisible by 30 : $\frac{200}{30} = 6$

Total no $= A + B + C - AB - AC - BC + ABC$

$= 100 + 66 + 40 - 33 - 20 - 13 + 6$

$\boxed{= 146}$

⑤

2. A sample of 80 people have revealed that 24 like cinema and 62 like T.V programmes. Find the no. of people who like both cinema and TV programmes

→ Sample set $= 80$

People who like cinema $= 24$

People who like TV $= 62$

People who like boath $= 62 + 24 - 80$

$= 86 - 80$

$= 6$

$\therefore$ 6 people are those who like both cinema and TV programmes.

3. Use division algorithm to prove that the square of an odd integer is of the form $8k+1$, where k is an integer.

→ By Euclid division algorithm

$a = bq + r$, where $0 \leq r \leq b$

Put $b = 4$

$a = 4q + r$, where $0 \leq r \leq 4$

If $r = 0$, then $a = 4q$ even

If $r = 1$, then $a = 4q + 1$ odd

If $r = 2$, then $a = 4q + 2$ even

If $r = 3$, then $a = 4q + 3$ odd

Now $(4q+1)^2 = (4q)^2 + 2(4q)(1) + (1)^2$

$= 16q^2 + 8q + 1$

$= 8(2q^2 + q) + 1$

$= 8m + 1$ where m is some integer

Hence the square of an odd integer is of the form $8q + 1$, for some integer q.

4. Calculate gcd [567, 315] and express gcd [567, 315] as $567x + 315y$, where x, y are integers.

→ We know that

Euclid's divison algorithm ⇒ $a = bq + r$

$\therefore$ $567 = 1 \times 315 + 252$

$315 = 252 \times 1 + 63$

$252 = 63 \times 4 + 0$

$\therefore$ gcd [567, 315] = 63

Now = $63 = 315 - 252 \times 1$

$63 = 315 - (567 - 315 \times 1)$

$63 = 315 - 567 + 315$

$63 = 315 \times 2 - 567$

$\therefore x = -1$

and $y = 2$

5. Using prime factorisation, find gcd and lcm of 1300, 3575. Also verify that gcd (a, b) · lcm (a, b) = ab

→

2	1300
2	650
5	325
5	65
13	13
	1

5	3575
5	715
11	143
13	13
	1

$\therefore 1300 = 2 \times 2 \times 5 \times 5 \times 13$

$= 2^2 \times 5^2 \times 13^1$

$3575 = 5 \times 5 \times 11 \times 13$

$= 5^2 \times 11^1 \times 13^1$

$\therefore \text{lcm}(1300, 3575) = 2^2 \times 5^2 \times 11^1 \times 13^1$

$= 4 \times 25 \times 143$

$= 14300$

$\text{gcd}(1300, 3575) = 5^2 \times 13^1$

$= 25 \times 13$

$= 325$

Now $\text{gcd}(1300, 3575) \times \text{lcm}(1300, 3575) = 1300 \times 3575$

L.H.S $325 \times 14300 = 4647500$ $\qquad = 46,47,500$

~~R.H.S~~

$\therefore$ L.H.S = R.H.S

Hence $\text{gcd}(a, b) \cdot \text{lcm}(a, b) = ab$ is true.