



## Chomsky Normal Form-

A context free grammar is said to be in chomsky normal form (CNF)

if all its productions are of the form-

$A \rightarrow BC$  or  $A \rightarrow a$

where A, B, C are non-terminals and a is a terminal.

## Steps for converting CFG into CNF

1. Eliminate Start symbol from RHS

eg:

If the start symbol T is at the RHS of any production, create a new production as:  $S_1 \rightarrow S$   
Where  $S_1$  is the new start symbol.

2. Reduce the grammar completely by-

Eliminating  $\epsilon$  productions  
Eliminating unit productions  
Eliminating useless productions

3. Eliminate terminals from the RHS of the production if they exist with other non-terminals or terminals.

production  $S \xrightarrow{R} aA$  can be decomposed as:  
 $S \rightarrow RA$   $R \rightarrow a$

4. Eliminate RHS with more than two non-terminals.

For example,  $S \xrightarrow{R} ASB$  can be decomposed as:  
 $S \rightarrow RS$   $R \rightarrow AS$

construct the following set of production and convert them to be in CNF

$S \rightarrow ABC$   $A \rightarrow aAb$   $A \rightarrow \epsilon$   $B \rightarrow b$   $C \rightarrow aC$   $C \rightarrow a$   $C \rightarrow \epsilon$

rule 2: eliminate  $\epsilon$

step 1:

removing  $\epsilon$  from A,C

$S \rightarrow ABC$   $S \rightarrow BC$   $S \rightarrow AB$   $S \rightarrow B$

$A \rightarrow aAb$   $A \rightarrow ab$

$C \rightarrow aC$   $C \rightarrow a$

$B \rightarrow b$

step 2 : Removing unit production

remove  $S \rightarrow B$  so

$S \rightarrow ABC$   $S \rightarrow BC$   $S \rightarrow AB$   
 $A \rightarrow aAb$   $A \rightarrow ab$   $S \rightarrow b$   
 $C \rightarrow aC$   $C \rightarrow a$   
 $B \rightarrow b$

step 3:

to replace combined terminals  
 new production  $X1 \rightarrow a$   
 $X2 \rightarrow b$

$S \rightarrow ABC$

$C \rightarrow a$

$S \rightarrow BC$

$A \rightarrow X1AX2$

$S \rightarrow AB$

$A \rightarrow X1X2$

$S \rightarrow b$

$C \rightarrow X1C$

STEP 4: EITHER A SINGLE  
 STRING OR  
 2 NON NON TERMINALS

REPLACE

$AC \rightarrow X3$

$X1 A \rightarrow X4$

$S \rightarrow BX3$

$C \rightarrow a$

$S \rightarrow BC$

$A \rightarrow X4X2$

$S \rightarrow AB$

$A \rightarrow X1X2$

$S \rightarrow b$

$C \rightarrow X1C$



convert CFG to CNF. Consider the given grammar G1:

$S \rightarrow ASB$   
 $A \rightarrow aAS|a|\epsilon$   
 $B \rightarrow SbS|A|bb$

START SYMBOL APPEAR

ELIMINATE

$S_0 \rightarrow S$

$S \rightarrow ASB$   
 $A \rightarrow aAS|a|\epsilon$   
 $B \rightarrow SbS|A|bb$

eliminate A from  $\Gamma$

$S \rightarrow ASB$   
 $S \rightarrow SB$   
 $A \rightarrow aAS$   
 $A \rightarrow aS$   
 $A \rightarrow a$   
 $B \rightarrow SbS$   
 $B \rightarrow A$   
 $B \rightarrow bb$   
 $S_0 \rightarrow S$

B become

$S \rightarrow ASB \mid SB \mid S \mid AS$

$A \rightarrow aAS \mid as \mid a$

$B \rightarrow SbS \mid A \mid bb$

$S_0 \rightarrow S$  remove unit prd  
 $B \rightarrow A, S_0 \rightarrow S, S \rightarrow S$

$S_0 \rightarrow AS \mid ASB \mid SB$

$S \rightarrow AS \mid ASB \mid SB$

$A \rightarrow aAS \mid aS \mid a$

$B \rightarrow SbS \mid bb \mid a \mid aAs \mid aS$

step 3: apply production rule

$a \rightarrow A_1$

$b \rightarrow B_1$

$A \rightarrow A_1AS \mid A_1S \mid A_1$

$B \rightarrow SB_1S \mid B_1B_1 \mid A_1AS \mid A_1S$

$S_0 \rightarrow AS \mid ASB \mid SB$

$S \rightarrow AS \mid ASB \mid SB$

$A \rightarrow a \quad B \rightarrow b$

replace

$A_1A \rightarrow X$

$SB_1 \rightarrow Y$

$AB \rightarrow Z$

$A \rightarrow XS \quad A_1S \quad A_1$

$B \rightarrow YS \quad B_1B_1 \quad XS \quad A_1S$

$S_0 \rightarrow AS \quad ZS \quad SB$

$S \rightarrow AS \quad ZS \quad SB$

$A \rightarrow a \quad x_1 \rightarrow$

$B \rightarrow b$

## Practise Problems

convert the following grammer to CNF

$S \rightarrow bA \mid aB$

$A \rightarrow bAA \mid aS \mid a$

$B \rightarrow aBB \mid bs \mid b$

Convert the given CFG to CNF. Consider the given grammar G1:

$S \rightarrow a \mid aA \mid B \quad A \rightarrow aBB \mid \epsilon \quad B \rightarrow Aa \mid b$

## Greibach Normal Form (GNF)

GNF stands for Greibach normal form. A CFG(context free grammar) is in GNF(Greibach normal form) if all the production rules satisfy one of the following conditions:

A start symbol generating  $\epsilon$ . For example,  $S \rightarrow \epsilon$ .

A non-terminal generating a terminal. For example,  $A \rightarrow a$ .

A non-terminal generating a terminal which is followed by any number of non-terminals.

For example,  $S \rightarrow \underline{a}ASB$ .

check this :

$$G1 = \{S \rightarrow aAB \mid aB, A \rightarrow aA \mid a, B \rightarrow bB \mid b\}$$

$$G2 = \{S \rightarrow aAB \mid aB, A \rightarrow aA \mid \epsilon, B \rightarrow bB \mid \epsilon\}$$

g2 is GNF

# Steps for converting CFG into GNF

1. Eliminating  $\epsilon$  productions  
Eliminating unit productions
2. RHS of production in Grammar G is restricted to be either a terminal or a terminal followed by one or more non terminal **eg :  $\rightarrow aABC$**
3. if the RHS of a production does not begin with a terminal, the first non terminal in the production is replaced by its RHS with a terminal



convert the CFG with the following  
to GNF

$S \rightarrow ABA$   $A \rightarrow aA \mid \epsilon$   
 $B \rightarrow bB \mid \epsilon$

Eliminating  $\epsilon$  productions  
Eliminating unit productions

RHS of production in Grammar G is restricted to be either a terminal or  
a terminal followed by one or more non terminal

if the RHS of a production does not begin with a terminal, the first non terminal  
in the production is replaced by its RHS with a terminal

step 1: remove production

$S \rightarrow ABA \mid AB \mid BA \mid AA \mid A \mid B$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

step 2: remove unit production  
 $A_2 \rightarrow b$   
 $S \rightarrow A \mid S \rightarrow B$

$A_3 \rightarrow a$   
 $S \rightarrow ABA \mid AB \mid BA \mid AA \mid a \mid b \mid aA \mid bB$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

step 3:

$S \rightarrow a b aA bB$

$A \rightarrow aA \mid a$

$b \rightarrow bB \mid b$

$S \rightarrow aABA \mid aBA \mid aAB \mid aB$   
 $bBA \mid bA \mid aA \mid aAA$





CFG to GNF

$$S \rightarrow AA_1a$$

$$A \rightarrow SS/b$$

$$A_1 \rightarrow A_2 A_2 / a \quad \checkmark$$

$$A_2 \rightarrow A_1 A_1 / b$$

$$A_2 \rightarrow A_2 A_2 A_1 / a A_1 / b$$

$$A_2 \rightarrow \underbrace{A_2}_{\alpha} \underbrace{A_2}_{\alpha} \underbrace{A_1}_{\beta} / \underbrace{a A_1}_{\beta} / b$$

$$A \rightarrow A\alpha / \beta$$

$$\alpha = A_2 A_1, \quad \beta = a A_1 / b$$

$$A \rightarrow BA'$$

$$A' \rightarrow \alpha A' / \epsilon$$

$$A_2 \rightarrow a A_1 z / b z$$

$$z \rightarrow A_2 A_1 z / \epsilon$$

Eliminate  $\epsilon$  production:

$$A_2 \rightarrow a A_1 z / b z / a A_1 / b \quad \text{in GNF}$$

$$z \rightarrow A_2 A_1 z / A_2 A_1$$

Sub.  $A_2$  in  $z \times A_1$ .

$$z \rightarrow a A_1 z A_1 z / b A_1 z / a A_1 A_1 z / b A_1 z /$$

$$a A_1 z A_1 / b A_1 / a A_1 A_1 / b A_1$$

$$A_1 \rightarrow a A_1 z A_2 / b z A_2 / a A_1 A_2 / b A_2 / a$$

$$A_2 \rightarrow a A_1 z / b z / a A_1 / b$$

3 Remove unit productions

$$S \rightarrow B$$

$$S \rightarrow Aa/a/b$$

New  $S \rightarrow a/a/a/a/Aa/b$

$$A \rightarrow aBB$$

$$B \rightarrow Aa/a/b$$

4) Find out production that has more variables

$$A \rightarrow aBB$$

$$A \rightarrow ax$$

$$x \rightarrow BB$$

New  $S$

$$S \rightarrow a/a/a/a/Aa/b$$

$$A \rightarrow ax$$

$$B \rightarrow Aa/a/b$$

$$y \rightarrow a$$

$$x \rightarrow BB$$

5) No check  $S \rightarrow aA, A \rightarrow ax$

$$S \rightarrow yA, A \rightarrow yx \quad y \rightarrow a$$

$$S \rightarrow Ay, B \rightarrow Ay$$

$$S \rightarrow Ay, B \rightarrow Ay$$

Final grammar is:

$$S \rightarrow a/BA/a/BB/b$$

$$A \rightarrow BX$$

$$B \rightarrow AB/a/b$$

$$x \rightarrow BB$$

$$S \rightarrow a/yA/xAy/b$$

$$S \rightarrow a/yA/yAy/b$$

$$A \rightarrow yx$$

$$B \rightarrow Ay/a/b$$

$$x \rightarrow BB \quad y \rightarrow a$$