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Subject Code

18MEO113T - Design of Experiments

Handled by

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Disclaimer

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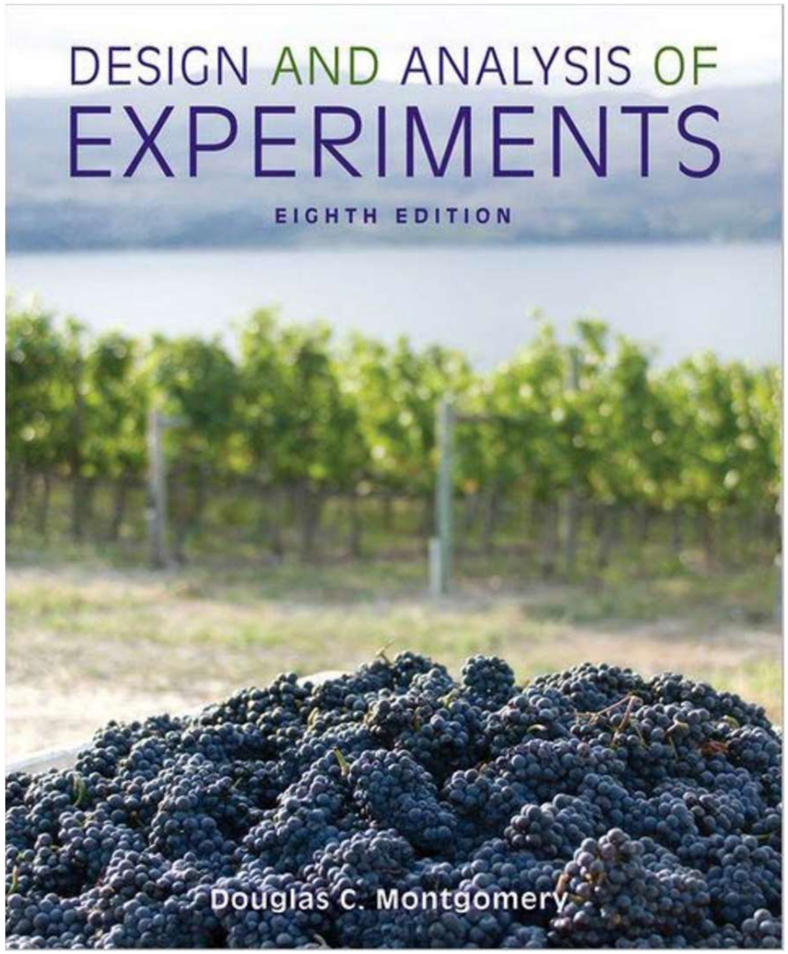
18MEO113T – Design of Experiments

- To learn the fundamentals of design of experiment techniques.
- To familiarize in how to setup experiments and accomplish all analyze tasks using software packages like Minitab, etc.
- After this course, You will be ready to apply the technique confidently to all of your projects .

Course Plan

Duration (hour)		9	9	9	9	9
S-1	SLO-1	Introduction in Design of experiments (DOE)	Need for DOE methodology	Introduction to Robust design, Loss functions	Background of response surface design	Introduction and uses of confounding
S-2	SLO-1	The fundamental and potential practical problems in experimentation	Barriers in the successful application of DOE	Eight steps in Taguchi methodology	Multiple Responses and Contour profile of response surface plot	2^3 factorial experiment with complete confounding
S-3	SLO-1	Statistical thinking and its role within DOE	Practical methodology of DOE and Analytical tools for DOE	Orthogonal array, Selecting the interaction, Linear graphs	Creation of response surface designs	2^3 factorial experiment with partial confounding
S-4	SLO-1	Basic principles of DOE and Degrees of freedom	The confidence interval for the mean response	S/N ratio: Larger-the-better, Smaller-the-better, Nominal-the-best	Central composite designs (Rotatable central composite design)	Confounding in the 2^n series and examples
S-5	SLO-1	Selection of quality characteristics for experiments	Introduction to Screening design	Analyze the data, factor effect diagram	Central composite designs (Rotatable central composite design)	Confounding of 3^n factorial and examples
S-6	SLO-1	Understanding key interaction in processes	Geometric and non-geometric P-B design	Levels of parameters	Box-Behnken design with case studies	ANOVA (One-way and two-way, higher-way ANOVA)
S-7	SLO-1	An alternative method for calculating two-order interaction effect	Introduction of full factorial design, Basic concepts of 2^2, 2^3 and 2^k designs	Confirmation test	Random factor models and its industrial application , Random Effects Models	MANOVA and ANCOVA overview
S-8	SLO-1	Synergistic interaction, Antagonistic interaction	Solving Case studies on Full factorial design with statistics software	Augmented design with simple case studies	Two Factor Factorial with Random Factors	Solving Case studies on ANOVA with statistics software
S-9	SLO-1	Synergistic interaction versus Antagonistic interaction	Solving Case studies on Full factorial design with statistics software	Solving case studies on robust design with statistics software	Two Factor Mixed Models with random factors	Regression Models and Regression Analysis

Course Plan


DESIGN AND ANALYSIS OF
EXPERIMENTS
EIGHTH EDITION
Douglas C. Montgomery

Unit 2

Unit 2

1: Need for DOE methodology.

2: Barriers in the successful application of DOE

3: Practical methodology of DOE and Analytical tools for DOE

4: The confidence interval for the mean response

5: Introduction to Screening design

6: Geometric and non-geometric P-B design

7: Introduction of full factorial design, Basic concepts of 2^2, 2^3 and 2^k designs

8: Solving Case studies on Full factorial design with statistics software

9: Solving case studies on Full factorial design with statistics software

Need for DOE methodology

Experimental design methods are also of fundamental importance in **engineering design activities**, where **new products are developed and existing ones improved**. Some applications of experimental design in engineering design include

1. Evaluation and comparison of basic design configurations

2. Evaluation of material alternatives

3. Selection of design parameters so that the product will work well under a wide variety of field conditions, that is, so that the product is **robust**

4. Determination of key product design parameters that impact product performance

5. Formulation of new products.

Need for DOE methodology

- The use of experimental design in product realization can result in products that are easier to manufacture and that have enhanced field performance and reliability, lower product cost, and shorter product design and development time.
- Designed experiments also have extensive applications in marketing, market research, transactional and service operations, and general business operations.

Need for DOE methodology

- Examples for need
  - Characterizing a process
  - Optimizing a process
  - Designing a product
  - Formulating a product
  - Designing a web page

Need for DOE methodology

- Characterizing a process

EXAMPLE 1.1 **Characterizing a Process**

A flow solder machine is used in the manufacturing process for printed circuit boards. The machine cleans the boards in a flux, preheats the boards, and then moves them along a conveyor through a wave of molten solder. This solder process makes the electrical and mechanical connections for the leaded components on the board.

The process currently operates around the 1 percent defective level. That is, about 1 percent of the solder joints on a board are defective and require manual retouching. However, because the average printed circuit board contains over 2000 solder joints, even a 1 percent defective level results in far too many solder joints requiring rework. The process engineer responsible for this area would like to use a designed experiment to determine which machine parameters are influential in the occurrence of solder defects and which adjustments should be made to those variables to reduce solder defects.

The flow solder machine has several variables that can be controlled. They include

1. Solder temperature
2. Preheat temperature
3. Conveyor speed
4. Flux type
5. Flux specific gravity
6. Solder wave depth
7. Conveyor angle.

In addition to these controllable factors, several other factors cannot be easily controlled during routine manufacturing, although they could be controlled for the purposes of a test. They are

1. Thickness of the printed circuit board
2. Types of components used on the board
3. Layout of the components on the board
4. Operator
5. Production rate.

In this situation, engineers are interested in **characterizing** the flow solder machine; that is, they want to determine which factors (both controllable and uncontrollable) affect the occurrence of defects on the printed circuit boards. To accomplish this, they can design an experiment that will enable them to estimate the magnitude and direction of the factor effects; that is, how much does the response variable (defects per unit) change when each factor is changed, and does changing the factors *together* produce different results than are obtained from individual factor adjustments—that is, do the factors interact? Sometimes we call an experiment such as this a **screening experiment**. Typically, screening or characterization experiments involve using fractional factorial designs, such as in the golf example in Figure 1.8.

The information from this screening or characterization experiment will be used to identify the critical process factors and to determine the direction of adjustment for these factors to reduce further the number of defects per unit. The experiment may also provide information about which factors should be more carefully controlled during routine manufacturing to prevent high defect levels and erratic process performance. Thus, one result of the experiment could be the application of techniques such as control charts to one or more **process variables** (such as solder temperature), in addition to control charts on process output. Over time, if the process is improved enough, it may be possible to base most of the process control plan on controlling process input variables instead of control charting the output.

Need for DOE methodology

- Optimizing a process

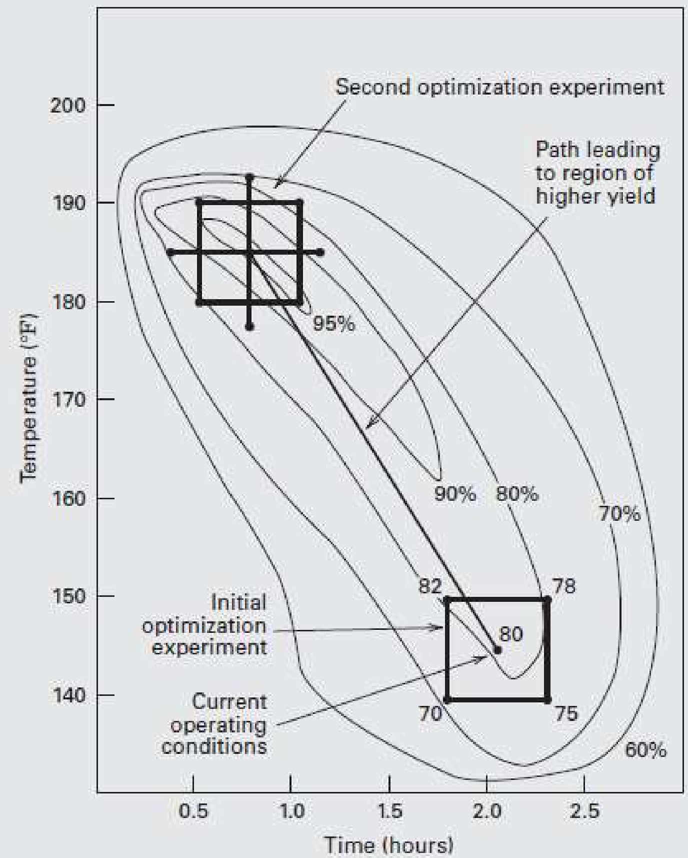
EXAMPLE 1.2 Optimizing a Process

In a characterization experiment, we are usually interested in determining which process variables affect the response. A logical next step is to optimize, that is, to determine the region in the important factors that leads to the best possible response. For example, if the response is yield, we would look for a region of maximum yield, whereas if the response is variability in a critical product dimension, we would seek a region of minimum variability.

Suppose that we are interested in improving the yield of a chemical process. We know from the results of a characterization experiment that the two most important process variables that influence the yield are operating temperature and reaction time. The process currently runs at 145°F and 2.1 hours of reaction time, producing yields of around 80 percent. Figure 1.9 shows a view of the time–temperature region from above. In this graph, the lines of constant yield are connected to form response **contours**, and we have shown the contour lines for yields of 60, 70, 80, 90, and 95 percent. These contours are projections on the time–temperature region of cross sections of the yield surface corresponding to the aforementioned percent yields. This surface is sometimes called a **response surface**. The true response surface in Figure 1.9 is unknown to the process personnel, so experimental methods will be required to optimize the yield with respect to time and temperature.

Need for DOE methodology

- Optimizing a process



■ **FIGURE 1.9 Contour plot of yield as a function of reaction time and reaction temperature, illustrating experimentation to optimize a process**

To locate the optimum, it is necessary to perform an experiment that varies both time and temperature together, that is, a factorial experiment. The results of an initial factorial experiment with both time and temperature run at two levels is shown in Figure 1.9. The responses observed at the four corners of the square indicate that we should move in the general direction of increased temperature and decreased reaction time to increase yield. A few additional runs would be performed in this direction, and this additional experimentation would lead us to the region of maximum yield.

Once we have found the region of the optimum, a second experiment would typically be performed. The objective of this second experiment is to develop an empirical model of the process and to obtain a more precise estimate of the optimum operating conditions for time and temperature. This approach to process optimization is called **response surface methodology**, and it is explored in detail in Chapter 11. The second design illustrated in Figure 1.9 is a **central composite design**, one of the most important experimental designs used in process optimization studies.

Need for DOE methodology

- Designing a Product

EXAMPLE 1.3 **Designing a Product—I**

A biomedical engineer is designing a new pump for the intravenous delivery of a drug. The pump should deliver a constant quantity or dose of the drug over a specified period of time. She must specify a number of variables or design parameters. Among these are the diameter and length of the cylinder, the fit between the cylinder and the plunger, the plunger length, the diameter and wall thickness of the tube connecting the pump and the needle inserted into the patient's vein, the material to use for fabricating both the cylinder and the tube, and the nominal pressure at which the system must operate. The impact of some of these parameters on the design can be evaluated by building prototypes in which these factors can be varied over appropriate ranges. Experiments can then be designed and the prototypes tested to investigate which design parameters are most influential on pump performance. Analysis of this information will assist the engineer in arriving at a design that provides reliable and consistent drug delivery.

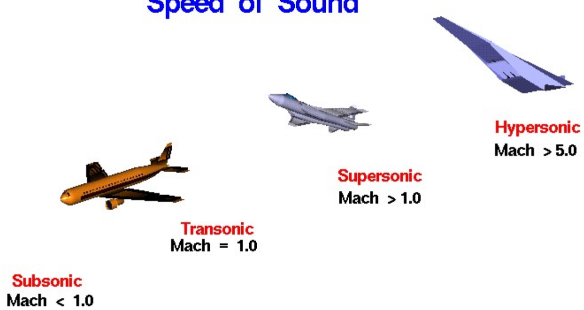
Need for DOE methodology

- Designing a Product

EXAMPLE 1.4 Designing a Product—II

An engineer is designing an aircraft engine. The engine is a commercial turbofan, intended to operate in the cruise configuration at 40,000 ft and 0.8 Mach. The design parameters include inlet flow, fan pressure ratio, overall pressure, stator outlet temperature, and many other factors. The output response variables in this system are specific fuel consumption and engine thrust. In designing this system, it would be prohibitive to build prototypes or actual test articles early in the design process, so the engineers use a **computer model** of the system that allows them to focus on the key design parameters of the engine and to vary them in an effort to optimize the performance of the engine. Designed experiments can be employed with the computer model of the engine to determine the most important design parameters and their optimal settings.

$$\text{ratio} = \frac{\text{Object Speed}}{\text{Speed of Sound}} = \text{Mach Number}$$



Need for DOE methodology

- Formulating a Product

EXAMPLE 1.5 Formulating a Product

A biochemist is formulating a diagnostic product to detect the presence of a certain disease. The product is a mixture of biological materials, chemical reagents, and other materials that when combined with human blood react to provide a diagnostic indication. The type of experiment used here is a **mixture experiment**, because various ingredients that are combined to form the diagnostic make up 100 percent of the mixture composition (on a volume, weight, or mole ratio basis), and the response is a function of the mixture proportions that are present in the product. Mixture experiments are a special type of response surface experiment that we will study in Chapter 11. They are very useful in designing biotechnology products, pharmaceuticals, foods and beverages, paints and coatings, consumer products such as detergents, soaps, and other personal care products, and a wide variety of other products.

Need for DOE methodology

- Designing a Web Page

EXAMPLE 1.6 Designing a Web Page

A lot of business today is conducted via the World Wide Web. Consequently, the design of a business' web page has potentially important economic impact. Suppose that the Web site has the following components: (1) a photoflash image, (2) a main headline, (3) a subheadline, (4) a main text copy, (5) a main image on the right side, (6) a background design, and (7) a footer. We are interested in finding the factors that influence the click-through rate; that is, the number of visitors who click through into the site divided by the total number of visitors to the site. Proper selection of the important factors can lead to an optimal web page design. Suppose that there are four choices for the photoflash image, eight choices for the main headline, six choices for the subheadline, five choices for the main text copy, four choices for the main image, three choices for the background design, and seven choices for the footer. If we use a factorial design, web pages for all possible combinations of these factor levels must be constructed and tested. This is a total of $4 \times 8 \times 6 \times 5 \times 4 \times 3 \times 7 = 80{,}640$ web pages. Obviously, it is not feasible to design and test this many combinations of web pages, so a complete factorial experiment cannot be considered. However, a fractional factorial experiment that uses a small number of the possible web page designs would likely be successful. This experiment would require a fractional factorial where the factors have different numbers of levels. We will discuss how to construct these designs in Chapter 9.

Need for DOE methodology

- Designing a Web Page

Full Factorial Experiment

	Variation	A	B	C
Factor A	1	+1	+1	+1
Factor B	2	+1	+1	-1
Factor C	3	+1	-1	+1
	4	+1	-1	-1
	5	-1	+1	+1
	6	-1	+1	-1
	7	-1	-1	+1
	8	-1	-1	-1

Fractional Factorial Experiment

	Variation	A	B	C
Factor A	1	+1	+1	+1
Factor B	2	-1	-1	+1
Factor C	3	-1	+1	-1
	4	+1	+1	-1

Barriers in the Successful Application of DOE

The ‘effective’ application of DOE by industrial engineers is limited in many manufacturing organisations.

Some noticeable barriers are as follows:

- Educational barriers
- Management barriers
- Cultural barriers
- Communication barriers
- Other barriers

Barriers in the Successful Application of DOE

- Educational barriers
  - The word ‘statistics’ invokes fear in many industrial engineers.
  - The fundamental problem begins with the current statistical education for the engineering community in their academic curriculum. The courses currently available in ‘engineering statistics’ often tend to concentrate on the theory of probability, probability distributions and more mathematical aspects of the subject, rather than practically useful techniques such as DOE, Taguchi method, robust design, gauge capability studies, Statistical Process Control (SPC), etc.

Barriers in the Successful Application of DOE

- Management barriers
  - Managers often don’t understand the importance of DOE in problem solving or don’t appreciate the competitive value it brings into the organisation. In many organisations, managers encourage their engineers to use the so-called ‘home-grown’ solutions for process and quality-related problems.
  - These ‘home-grown’ solutions are consistent with the OVAT approach to experimentation, as managers are always after quick-fix solutions which yield short-term benefits to their organisations.

Barriers in the Successful Application of DOE

- Management barriers
  - Responses from managers with high resistance to change may include the following
    - DOE tells me what I already know.
    - It sounds good, but it is not applicable to my job.
    - I need to make additional effort to prove what I already know.
  - Many managers do not instinctively think statistically, mainly because they are not convinced that statistical thinking adds any value to management and decision-making.
  - Managers in organisations believe that DOE is very demanding of resources.

Barriers in the Successful Application of DOE

- Cultural barriers
  - Cultural barriers are one of the principal reasons why DOE is not commonly used in many organisations.
  - Many organisations are not culturally ready for the introduction and implementation of advanced quality improvement techniques such as DOE and Taguchi.
  - The best way to overcome this barrier is through intensive training programs and by demonstrating the successful application of such techniques by other organisations during the training.

Barriers in the Successful Application of DOE

- Communication barriers
  - Research has indicated that there is very little communication between the academic and industrial worlds.
  - Moreover, the communication among industrial engineers, managers and statisticians in many organisations is limited.
  - For the successful initiative of any quality improvement programme, these communities should work together and make this barrier less formidable.

Barriers in the Successful Application of DOE

- Other barriers
  - Negative experiences with DOE may make companies reluctant to use DOE again. The majority of negative DOE experiences can be classified into two groups. The first relates to technical issues and the second to non-technical issues.
- Technical issues include:
  - choosing unreasonably large or small designs;
  - inadequate or even poor measurement of quality characteristics;
  - not choosing the appropriate levels for the process variables, etc. Non-linearity or curvature effects of process variables should be explored to determine the best operating process conditions;
  - assessing the impact of ‘uncontrolled variables’ which can influence the output of the process. Experimenters should try to understand how the ‘uncontrolled variables’ influence the process behaviour and devise strategies to minimise their impact as much as possible;
  - Lacking awareness of assumptions: data analysis, awareness of different alternatives why they are needed, etc.

Barriers in the Successful Application of DOE

- Non-Technical issues include:
  - lack of experimental planning;
  - executing one-shot experimentation instead of adopting sequential, adaptive and iterative nature of experimentation
  - not choosing the right process variables or design variables for the experiment in the first round of experimentation, etc.

Barriers in the Successful Application of DOE

- Educational barriers
  - Current statistical education for the engineering community in their academic curriculum is not sufficient
- Management barriers
  - Managers encourage their engineers to use ‘home-grown’ solutions for process-and quality-related problems.
- Cultural barriers
  - Principal reason; reluctant and fear of embracing the DOE
- Communication barriers
  - Gap between academia and industry ; lack of knowledge in engineering and statistics
- Other barriers
  - Technical and non-technical issues might bring negative experiences with DOE; makes engineers reluctant to use DOE

Barriers in the Successful Application of DOE

- Commercial software systems and expert systems in DOE provide no guidance whatsoever in classifying and analysing manufacturing process quality-related problems from which a suitable approach (Taguchi, Classical or Shainin’s approach) can be selected.

Barriers in the Successful Application of DOE

- The selection of a particular approach to experimentation (i.e. Taguchi, Classical or Shainin) is dependent upon a number of criteria:
  - the complexity involved
  - the degree of optimisation required by the experimenter
  - the time required for completion of the experiment
  - cost issues associated with the experiment
  - the allowed response time to report back to management, etc.

A Practical Methodology for DOE

The methodology of DOE is fundamentally divided into four phases.

These are:

1. planning phase

2. designing phase

3. conducting phase

4. analysing phase.

Planning Phase

- Many engineers pay special attention on the statistical details of DOE and very little attention to the non-statistical details.
- Experimental studies may fail not only as a result of lack of technical knowledge of the process under study or wrong use of statistical techniques but also due to lack of planning.
- It is the responsibility of the senior management team in the organisation to create an environment that stimulates a culture of using experimental design techniques for process optimisation problems, product and process development projects, improving process capability through systematically reducing excessive variation in processes, etc.

Planning Phase

The planning phase is made up of the following steps.

- Problem Recognition and Formulation
- Selection of Response or Quality Characteristic
- Selection of Process Variables or Design Parameters
- Classification of Process Variables
- Determining the Levels of Process Variables
- List All the Interactions of Interest

Planning Phase

1. Problem Recognition and Formulation

- A clear and succinct statement of the problem can create a better understanding of what needs to be done. The statement should contain an **objective** that is **specific, measurable** and which can **yield practical value** to the company (Kumar and Tobin, 1990). The creation of a **multidisciplinary team** in order to have a shared understanding of the problem is critical in the planning phase. The multidisciplinary team should be led by someone with good knowledge of the process (a DOE specialist), good communication skills, good interpersonal skills and awareness of team dynamics.
- **Other team members** may include process engineers, a quality engineer/manager, a machine operator, a management representative and manufacturing/production engineers/managers. Sharing experiences and individual knowledge is critical to assure a deeper understanding of the process providing more efficient ways to design experiments (Romeu, 2006).

Planning Phase

1. Problem Recognition and Formulation

- Some manufacturing problems that can be addressed using an experimental approach include
  - development of new products; improvement of existing processes or products;
  - improvement of the process/product performance relative to the needs and demands of customers;
  - reduction of existing process spread, which leads to poor capability.

- The objective of the experiment must be clearly specified and has to be measurable.
- Objectives can be either short term or long term. A short-term objective could be to fix a problem related to a high scrap rate.

Planning Phase

2. Selection of Response or Quality Characteristic

- The selection of a **suitable response** for the experiment is critical to the success of any industrially designed experiment.
- The response can be **variable or attribute** in nature.
- **Variable responses** such as length, thickness, diameter, viscosity, strength, etc. generally provide more information than **attribute responses** such as good/bad, pass/fail or yes/no.
- Moreover, variable characteristics or responses require fewer samples than attributes to achieve the same level of statistical significance.

Planning Phase

2. Selection of Response or Quality Characteristic

- Experimenters should define the measurement system prior to performing the experiment in order to understand what to measure, where to measure and who is doing the measurements, etc. so that various components of variation (measurement system variability, operator variability, part variability, etc.) can be evaluated.
- Defining a measurement system, including human resources, equipments and measurement methods, is a fundamental aspect in planning experimental studies.
- It is important to ensure that equipment exists and is suitable, accessible and calibrated.

Planning Phase

2. Selection of Response or Quality Characteristic

- The quality of a measurement system is usually determined by the statistical properties of the data it generates over a period of time which captures both long- and short-term variation.
- Experimenters should be aware of the repeatability, reproducibility and uncertainty of the measurements prior to the execution of industrial experiments (Launsby and Weese, 1995).
- It is advisable to make sure that the measurement system is capable, stable, robust and insensitive to environmental changes.

Planning Phase

3. Selection of Process Variables or Design Parameters

- Some possible ways to identify potential process variables are the use of engineering knowledge of the process, historical data, cause-and-effect analysis and brainstorming.
- It is a good practice to conduct a screening experiment in the first phase of any experimental investigation to identify the most important design parameters or process variables.

Planning Phase

4. Classification of Process Variables

- After identification of process variables , classify them into controllable and uncontrollable variables
- Control variables are those which can be controlled by a process engineer/production engineer in a production environment.
- Uncontrollable variables (or noise variables) are those which are difficult or expensive to control in actual production environments.

Planning Phase

4. Classification of Process Variables

- Variables such as ambient temperature fluctuations, humidity fluctuations, raw material variations, etc. are examples of noise variables.
- The effect of such nuisance variables can be minimized by the effective application of DOE principles such as blocking, randomization and replication.

Planning Phase

5. Determining the Levels of Process Variables

- A level is the value that a process variable holds in an experiment.
- For example, a car's gas mileage is influenced by such levels as tire pressure, speed, etc.
- The number of levels depends on the nature of the process variable to be studied for the experiment and whether or not the chosen process variable is qualitative (type of catalyst, type of material, etc.) or quantitative (temperature, speed, pressure, etc.).

Planning Phase

5. Determining the Levels of Process Variables

- For quantitative process variables, two levels are generally required in the early stages of experimentation.
- However, for qualitative variables, more than two levels may be required. If a non-linear function is expected by the experimenter, then it is advisable to study variables at three or more levels.
- This would assist in quantifying the nonlinear (or curvature) effect of the process variable on the response function.

Planning Phase

6. List all the Interactions of Interest

- Interaction among variables is quite common in industrial experiments.
- In order to effectively interpret the results of the experiment, it is highly desirable to have a good understanding of the interaction between two process variables (Marilyn, 1993).
- The best way to relate to interaction is to view it as an effect, just like a factor or process variable effect.
- In the context of DOE, we generally study two-order interactions.
- The number of two-order interactions within an experiment can be easily obtained by using a simple equation:

$$N = \frac{n * (n - 1)}{2}$$

Where n is the number of factors.

Planning Phase

6. List all the Interactions of Interest

- For example, if you consider four factors in an experiment, the number of two order interactions can be equal to six.
- The questions to ask include
  - ‘Do we need to study the interactions in the initial phase of experimentation?’
  - ‘How many two-order interactions are of interest to the experimenter?’
- The size of the experiment is dependent on the number of factors to be studied and the number of interactions, which are of great concern to the experimenter.

Designing Phase

- During the design stage, it is quite important to consider the confounding structure and resolution of the design.
  - **Confounding**
    - A concept that basically means that multiple effects are tied together into one parent effect and cannot be separted. For example,
    - Two people fipping two different coins would result in the effect of the person and the effect of the coin to be confounded.
- It is good practice to have the design matrix ready for the team prior to executing the experiment.

Designing Phase

- The design matrix generally reveals all the settings of factors at different levels and the order of running a particular experiment.
- Experimenters are advised to carefully consider the three principles (randomisation, replication and blocking) of experimental design prior to conducting the real experiment.

Conducting Phase

- This is the phase in which the planned experiment is carried out and the results are evaluated.
- Several considerations are recognized as being recommended prior to executing an experiment, such as
  - selection of a suitable location for carrying out the experiment. It is important to ensure that the location is not affected by any external sources of noise (vibration, humidity, etc.);
  - availability of materials/parts, operators, machines, etc. required for carrying out the experiment;

Conducting Phase

- The following steps may be useful while performing the experiment in order to ensure that it is performed according to the prepared experimental design matrix (or layout).
  - The person responsible for the experiment should be present throughout the experiment. In order to reduce the operator-to-operator variability, it is best to use the same operator for the entire experiment.
  - Monitor the experimental trials. This is to find any discrepancies while running the experiment. It is advisable to stop running the experiment if any discrepancies are found.
  - Record the observed response values on the prepared data sheet or directly into the computer.
  - Any experiment deviations and unusual occurrences must be recorded and analysed.

Analysing Phase

- Having performed the experiment, the next phase is to analyze and interpret the results so that valid and sound conclusions can be derived.
- The following are the objectives to be achieved ,
  - Determine the design parameters or process variables that affect the mean process performance.
  - Determine the design parameters or process variables that influence performance variability.
  - Determine the design parameter levels that yield the optimum performance.
  - Determine whether further improvement is possible.

Analysing Phase

- Statistical methods should be used to analyze the data.
- Analysis of variance is widely used to test the statistical significance of the effects through F-Test.
- Confidence interval estimation is also part of the data analysis. empirical models are developed relating the dependent (response) and independent variables (factors).
- Residual analysis and model adequacy checking are also part of the data analysis procedure.
- Statistical analysis of data is a must for academic and scientific purpose.
- Graphical analysis and normal probability plot of the effects may be preferred by Industry

Analytical Tools of DOE

Main Effects Plot

- A main effects plot is a plot of the mean response values at each level of a design parameter or process variable.
- One can use this plot to compare the relative strength of the effects of various factors.
- The sign and magnitude of a main effect would tell us the following:
  - The sign of a main effect tells us of the direction of the effect, that is, whether the average response value increases or decreases.
  - The magnitude tells us of the strength of the effect.

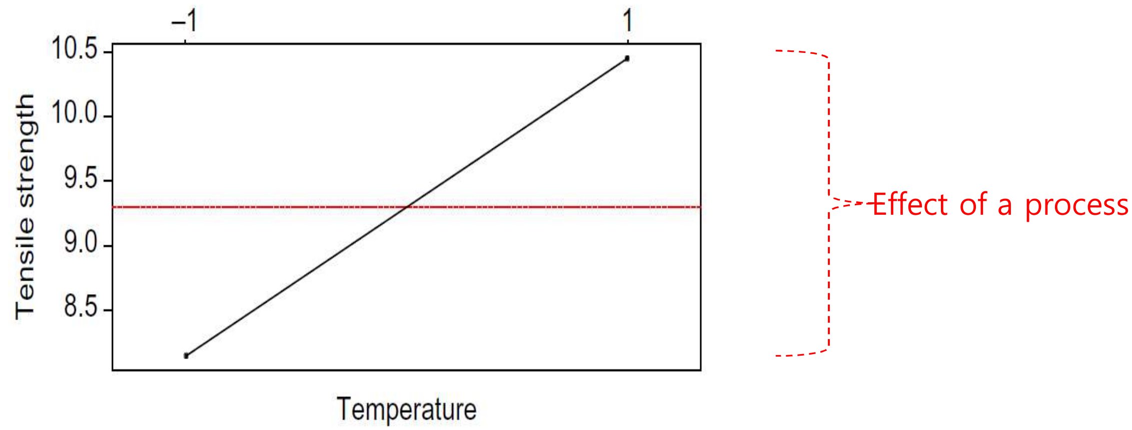
Analytical Tools of DOE

Main Effects Plot

- If the effect of a design or process parameter is positive, it implies that the average response is higher at a high level rather than a low level of the parameter setting.
- In contrast, if the effect is negative, it means that the average response at the low-level setting of the parameter is more than at the high level.

Analytical Tools of DOE

Main Effects Plot



Main effect plot of temperature on tensile strength.

The effect of a process or design parameter (or factor) can be mathematically calculated using the following simple equation:

$$E_f = \overline{F}_{(+1)} - \overline{F}_{(-1)} \tag{4.2}$$

where $\overline{F}_{(+1)}$ = average response at high-level setting of a factor, and $\overline{F}_{(-1)}$ = average response at low-level setting of a factor.

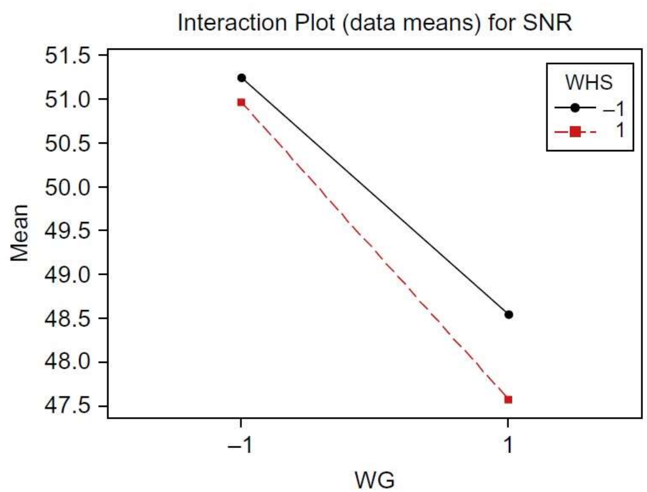
Analytical Tools of DOE

Interactions Plots

- An interactions plot is a powerful graphical tool which plots the mean response of two factors at all possible combinations of their settings.
- If the lines are parallel, this indicates that there is no interaction between the factors.
- Non-parallel lines are an indication of the presence of interaction between the factors.

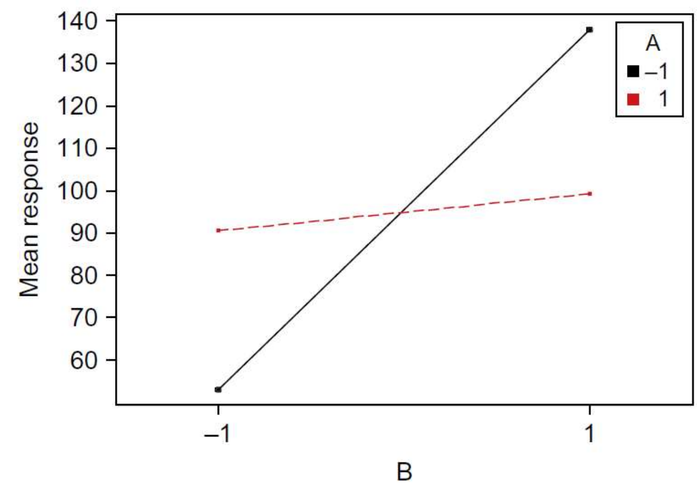
Analytical Tools of DOE

Interactions Plots



Interaction plot between WG and WHS.

Synergistic Interaction : lines nearly parallel – negligible or no interaction exists



Antagonistic interaction between two factors A and B.

Antagonistic Interaction : lines intersect and cross each other– strong interaction exists

Analytical Tools of DOE

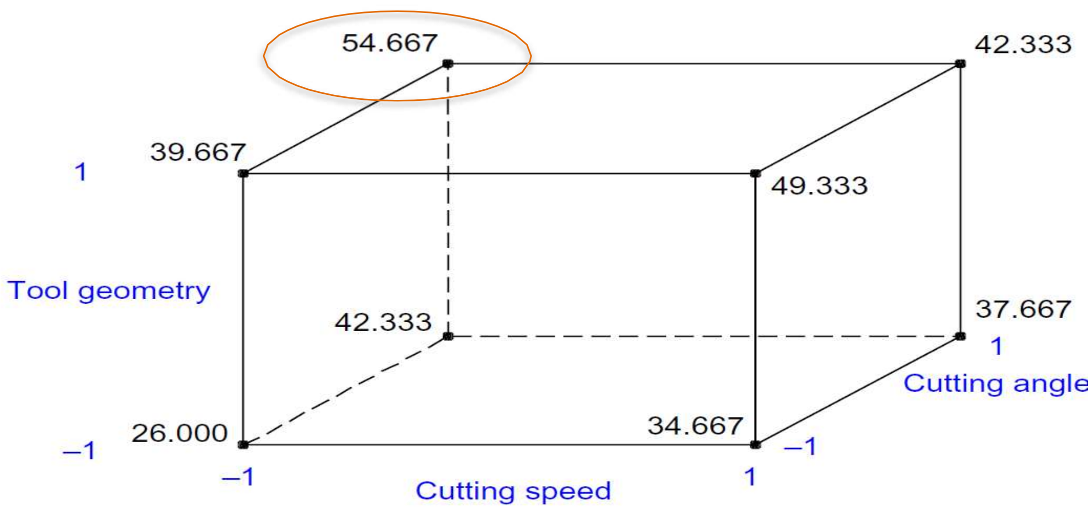
Cube Plots

- Cube plots display the **average response values at all combinations** of process or design parameter settings.
- One can easily determine the best and worst combinations of factor levels for achieving the desired optimum response.
- A cube plot is useful to determine the path of steepest ascent or descent for optimisation problems.

Analytical Tools of DOE

Cube Plots

- Figure below illustrates an example of a cube plot for a cutting tool life optimisation study with three tool parameters: cutting speed, tool geometry and cutting angle.
- The graph indicates that tool life increases when cutting speed is set at low level and cutting angle and tool geometry are set at high levels.
- The worst condition occurs when all factors are set at low levels.



Analytical Tools of DOE

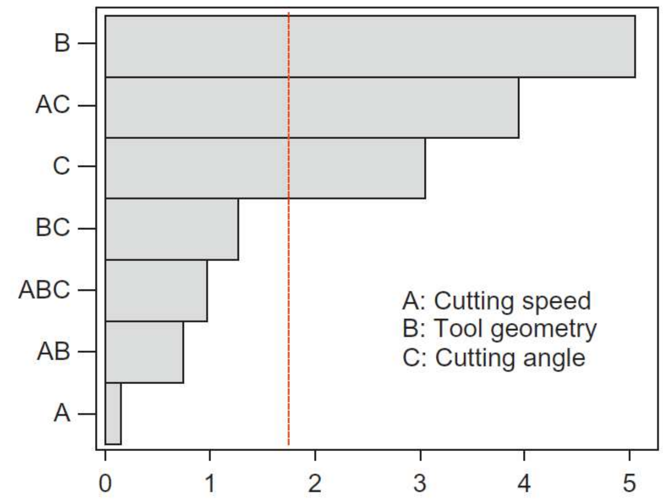
Pareto Plot of Factor Effects

- To detect the **factor and interaction effects** that are most important to the process or design optimisation
- It displays the **absolute values of the effects**, and draws a **reference line** on the chart.
- Any effect that extends past this reference line is **potentially important**.

Analytical Tools of DOE

Pareto Plot of Factor Effects

- The graph shows that factors B and C and interaction AC are most important.
- It is always a good practice to check the findings from a Pareto chart with Normal Probability Plot (NPP) of the estimates of the effects



Pareto plot of the standardised effects.

Analytical Tools of DOE

Pareto Plot of Factor Effects

– Let us take an example, where we need to prepare a chart of feedback analysis for XYZ restaurant, as per the reviews and ratings received from the customers. Here the customers are given a checklist of four points based on which they have to rate the restaurant out of 10. The four points are:

- Taste of the Food
- Quality of the food
- Price
- Presentation

Analytical Tools of DOE

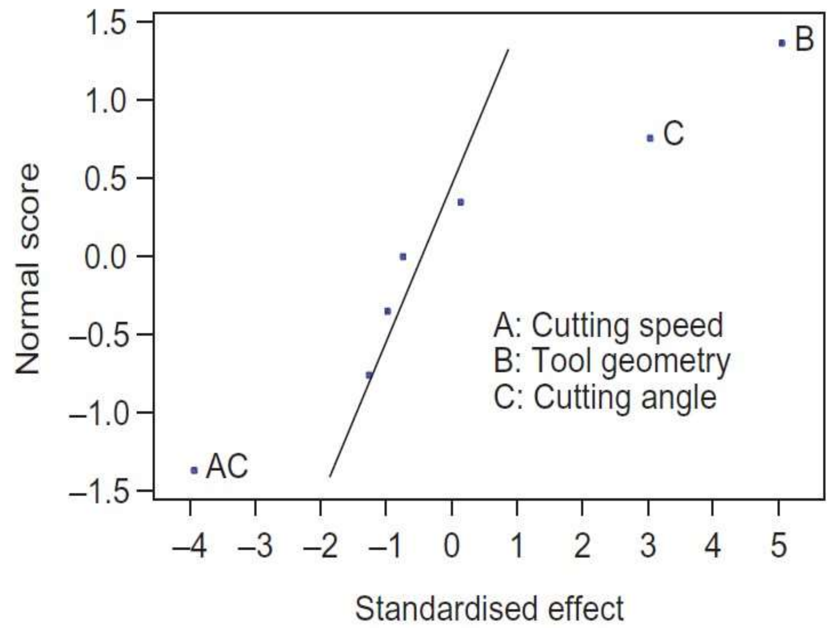
NPP (Normal Probability Plot**) of Factor Effects**

- For NPP, the main and interaction effects of factors or process (or design) parameters should be plotted against **cumulative probability** (%).
- **Inactive main and interaction effects** tend to fall roughly along a straight line, whereas active effects tend to appear as extreme points falling off each end of the straight line (Benski, 1989).

Analytical Tools of DOE

NPP of Factor Effects

- These active effects are judged to be statistically significant.
- The results are absolutely identical to that of a Pareto plot of factor/ interaction effects.



NPP of effects for cutting tool optimisation example.

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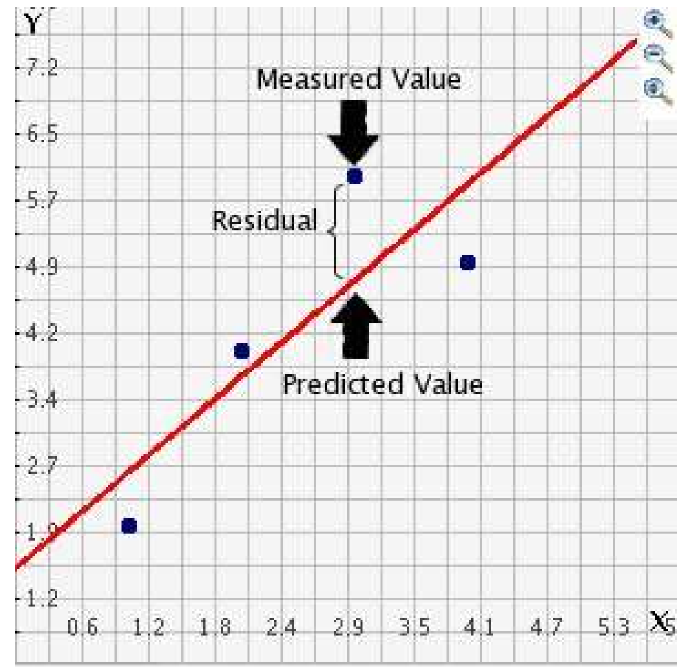
Analytical Tools of DOE

NPP of Residuals

- One of the key assumptions for the statistical analysis of data from industrial experiments is that the data come from a normal distribution.
- In order to check the data for normality, it is best to construct an NPP (Normal Probability Plot) of the residuals.
- NPPs are useful for evaluating the normality of a data set, even when there is a fairly small number of observations.
- Here residual is the mean difference between the observed value (obtained from the experiment) and the predicted or fitted value.



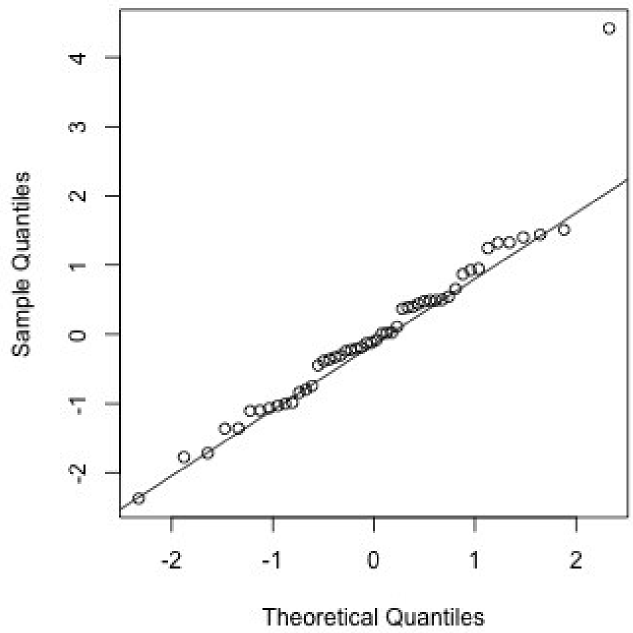
Analytical Tools of DOE

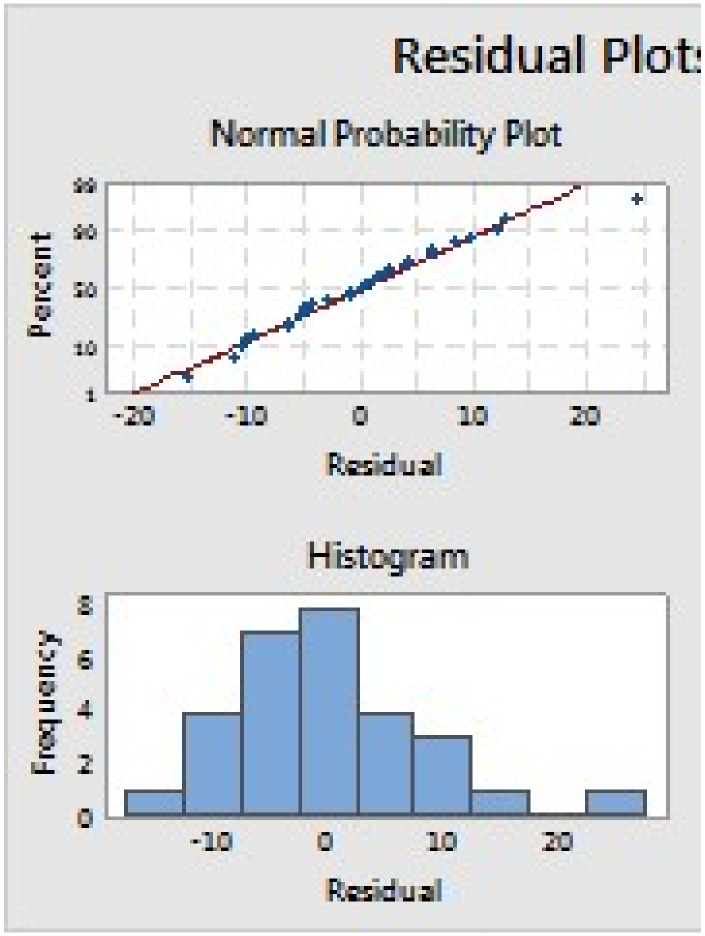
NPP of Residuals

- If the residuals **fall approximately along a straight line**, they are then **normally distributed**.
- **In contrast**, if the residuals **do not fall fairly close to a straight line**, they are then **not normally distributed** and hence the data do not come from a normal population.
- In the graph , points fall fairly close to a straight line, indicating that the data are approximately normal.

Analytical Tools of DOE

NPP of Residuals





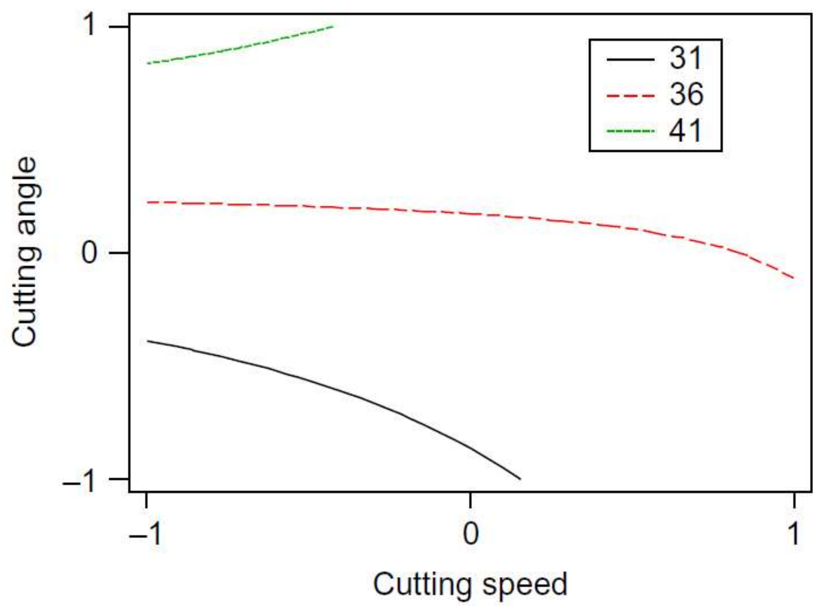
Analytical Tools of DOE

Response Plots

- Response surface plots(display 3D views) such as contour and surface plots are useful for establishing desirable **response values and operating conditions.**
- **In a contour plot**, the response surface is viewed as a two- dimensional plane where all points that have the same response are connected to produce contour lines of constant responses.
- **A surface plot** generally displays a three-dimensional view that may provide a clearer picture of the response.

Analytical Tools of DOE

Response Contour Plots

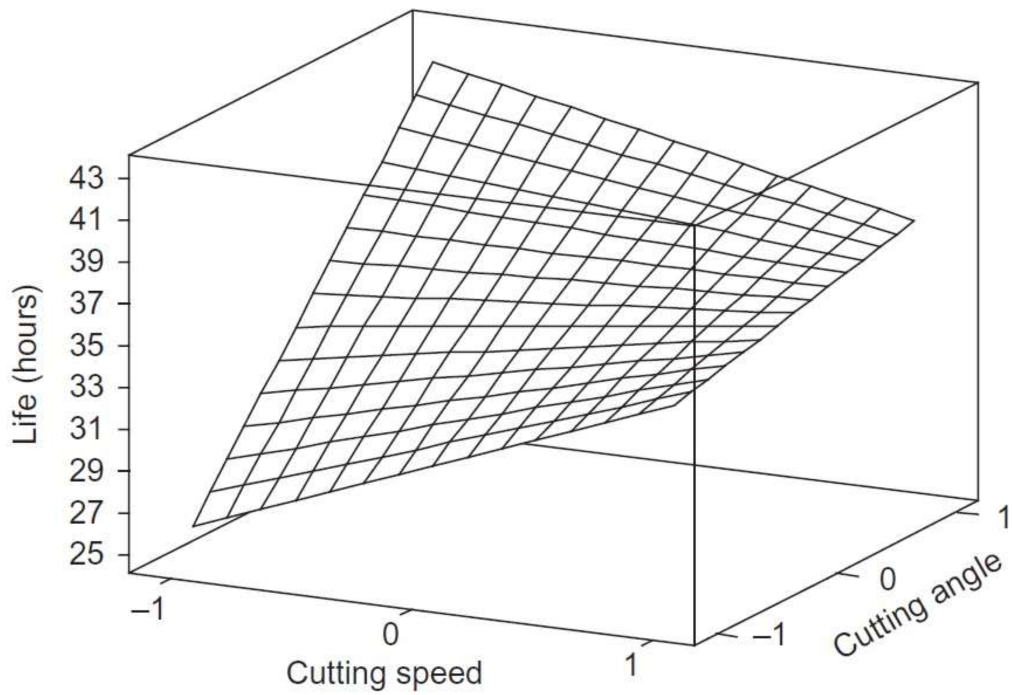


Contour plot of cutting tool life.

- In this Figure, the tool life increases with an increase in cutting angle and a decrease in cutting speed.
- If the regression model (i.e. first-order model) contains only the main effects and no interaction effect, the fitted response surface will be a plane (i.e. contour lines will be straight).
- If the model contains interaction effects, the contour lines will be curved and **not** straight.
- The contours produced by a **second-order model** will be elliptical in nature.

Analytical Tools of DOE

Response Surface Plots



Surface plot of cutting tool life.

Surface plots are diagrams of three-dimensional data. Rather than showing the individual data points, surface plots show a functional relationship between a designated dependent variable (Y), and two independent variables (X and Z). The plot is a companion plot to the contour plot.

Analytical Tools of DOE

Response Surface Plots

- Moreover, we have used a fitted surface in latter figure to find a direction of potential improvement for a process.
- A formal way to seek the direction of improvement in process optimisation problems is called the method of steepest ascent or descent (depending on the nature of the problem at hand, i.e. whether one needs to maximize or minimize the response of interest).

Model for Predicting Response Function

- Use of this **regression model** is **to predict the response** for different combinations of process parameters (or design parameters) at their best levels.
- In order to develop a regression model based on the significant effects (either main or interaction), the first step is to determine the **regression coefficients**.

Regression coefficients are estimates of the unknown population parameters and describe the relationship between a predictor variable and the response. In linear regression, coefficients are the values that multiply the predictor values. Suppose you have the following regression equation: **y = 3X + 5**. In this equation, +3 is the coefficient, X is the predictor, and +5 is the constant.

Model for Predicting Response Function

- **Regression coefficients**
  - The **sign** of each coefficient indicates the **direction of the relationship** between a predictor variable and the response variable.
  - A **positive sign** indicates that as the predictor variable increases, the response variable also increases.
  - A **negative sign** indicates that as the predictor variable increases, the response variable decreases.

Example: The coefficient value represents the mean change in the response given a one unit change in the predictor. For example, if a coefficient is +3, the mean response value increases by 3 for every one unit change in the predictor.

Overview of Linear Models

- An equation can be fit to show the best linear relationship between two variables:

$$\boxed{Y = \beta_0 + \beta_1 X}$$

Where Y is the dependent variable and
X is the independent variable
β_0 is the Y-intercept
β_1 is the slope

Least Squares Regression

- Estimates for coefficients β_0 and β_1 are found using a Least Squares Regression technique

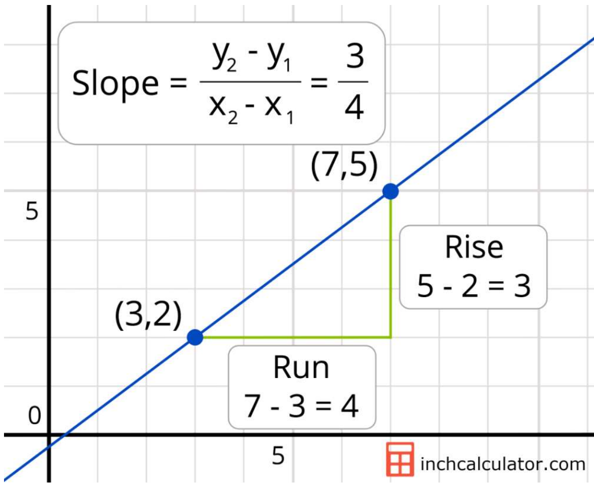
- The least-squares regression line, based on sample data, is

$$\hat{y} = b_0 + b_1 x$$

- Where b_1 is the slope of the line and b_0 is the y-intercept:

$$b_1 = \frac{\text{Cov}(x, y)}{s_x^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$



Introduction to Regression Analysis

- Regression analysis is used to:
  - Predict the value of a dependent variable based on the value of at least one independent variable
  - Explain the impact of changes in an independent variable on the dependent variable

Dependent variable: the variable we wish to explain
(also called the endogenous variable)

Independent variable: the variable used to explain the dependent variable
(also called the exogenous variable)

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Linear Regression Model

- The relationship between X and Y is described by a linear function
- Changes in Y are assumed to be caused by changes in X
- Linear regression population equation model

$$\boxed{Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i}$$

- Where β_0 and β_1 are the population model coefficients and ε is a random error term.

Simple Linear Regression Model

The population regression model:

Dependent Variable

Population Y intercept

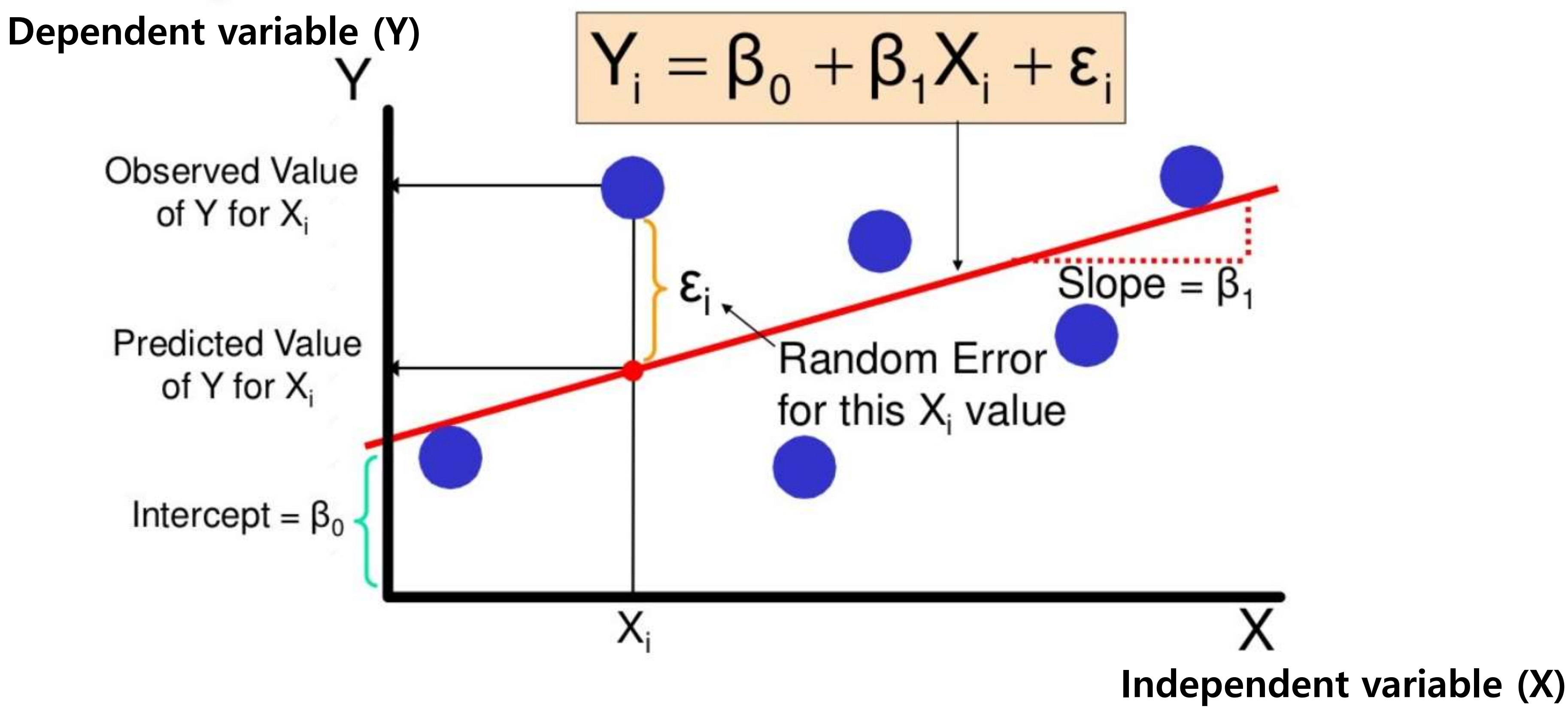
Population Slope Coefficient

Independent Variable

Random Error term

$$Y_i = \underbrace{\beta_0 + \beta_1 X_i}_{\text{Linear component}} + \underbrace{\varepsilon_i}_{\text{Random Error component}}$$

Simple Linear Regression Model



Sum of Squares of Error should be minimal, then only we can say that it is best regression line.

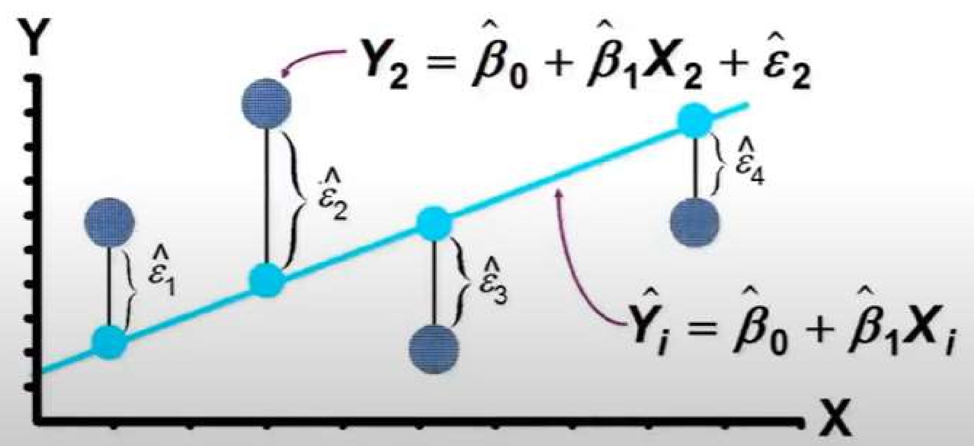
Simple Linear Regression Model

1. 'Best Fit' Means Difference Between Actual Y Values & Predicted Y Values is a Minimum. *But* Positive Differences Off-Set Negative ones. So square errors!

Line of best fit: Best fit means that the sum of the squares of the vertical distance from each point to the line is at minimum.

$$\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n}\hat{\varepsilon}_i^2$$

LS minimizes $\sum_{i=1}^{n}\hat{\varepsilon}_i^2 = \hat{\varepsilon}_1^2 + \hat{\varepsilon}_2^2 + \hat{\varepsilon}_3^2 + \hat{\varepsilon}_4^2$



Simple Linear Regression Model

- Prediction equation

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

- Sample slope

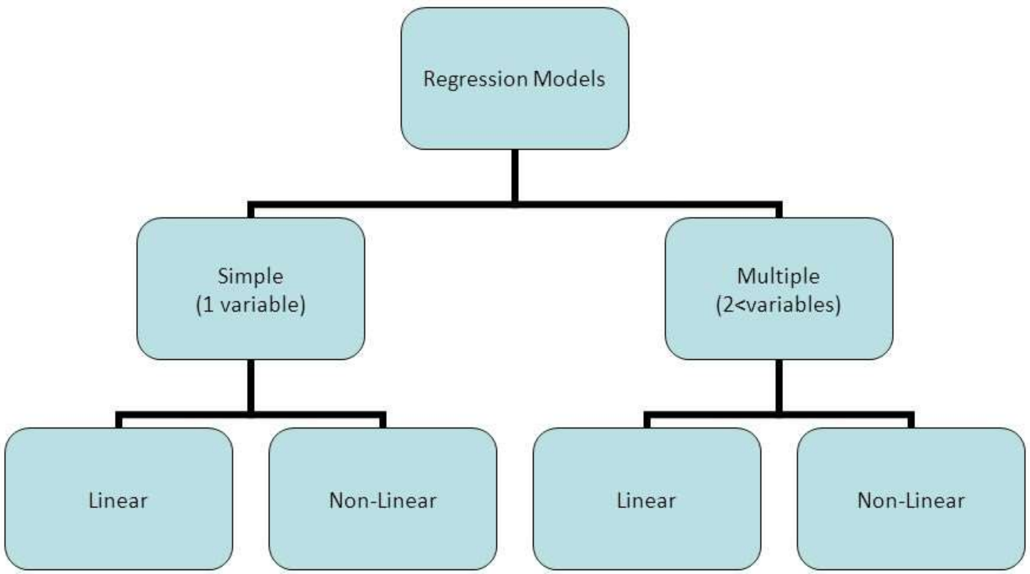
$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

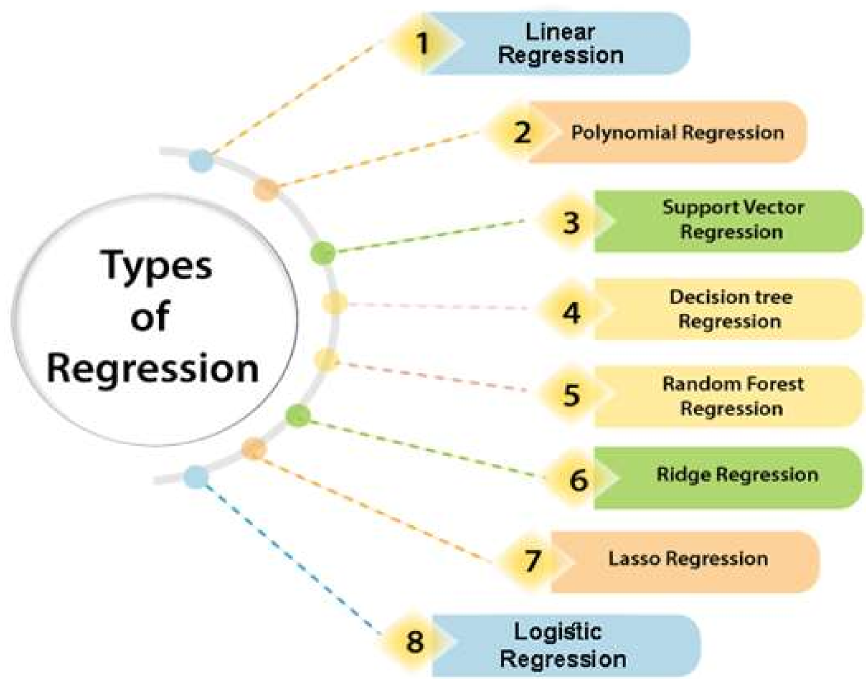
- Sample Y - intercept

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Regression Model

Types of Regression Model

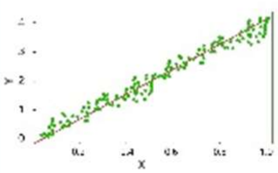




Regression Model

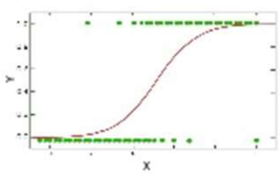
Linear Regression

- When there is a linear relationship between independent and dependent variables.



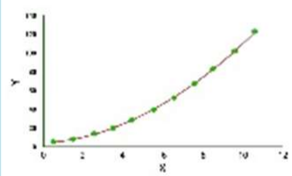
Logistic Regression

- When the dependent variable is categorical (0/ 1, True/ False, Yes/ No, A/B/C) in nature.



Polynomial Regression

- When the power of independent variable is more than 1.



Univariate Linear Regression	$y = m_1x_1 + c$
Multivariate Linear Regression	$y = m_1x_1 + m_2x_2 + m_3x_3 + \ldots + m_nx_n + c$
Polynomial Linear Regression	$y = m_1x_1 + m_2x_1^2 + m_3x_1^3 + \ldots + m_nx_1^n + c$

Simple Linear Regression

$$y = b_0 + b_1{}^*x_1$$

Multiple Linear Regression

Dependent variable (DV) Independent variables (IVs)

$$y = b_0 + b_1{}^*x_1 + b_2{}^*x_2 + \ldots + b_n{}^*x_n$$

Constant Coefficients

Model for Predicting Response Function

- **For factors at 2-levels, the regression coefficients are obtained by dividing the estimates of effects by 2.**
- A regression model for factors at 2-levels is usually of the form

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \cdots + \varepsilon$$

- where β_1 and β_2 are the regression coefficients and β_0 is the average response in a factorial experiment.
- The term 'ε' the random error component which is approximately normal and independently distributed with mean zero and constant variance σ_2.
- The regression coefficient β_{12} corresponds to the interaction between the process parameters *x1* and *x2*

Model for Predicting Response Function

- For example, the regression model for the cutting tool life optimisation study is given by

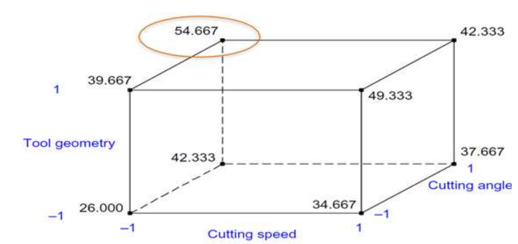
$$\hat{y} = 40.833 + 5.667(B) + 3.417(C) - 4.417(AC)$$

- The response values obtained from above Eq. are called predicted values and the actual response values obtained from the experiment are called observed values.

Model for Predicting Response Function

- Residuals can be obtained by taking the difference of observed and predicted (or fitted) values.
- We can predict the cutting tool life for various combinations of these tool parameters.
- For instance, if all the cutting tool life parameters are kept at low level settings, the predicted tool life then would be

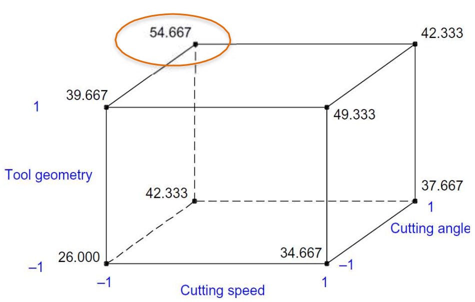
$$\begin{aligned}\hat{y} &= 40.833 + 5.667(B) + 3.417(C) - 4.417(AC) \\ &= 40.833 + 5.667(-1) + 3.417(-1) - 4.417(-1) \times (-1) \\ &= 27.332\end{aligned}$$



Model for Predicting Response Function

- The observed value of tool life (refer to cube plot) is 26 h.
- The difference between the observed value and predicted value (i.e. residual) is − 1.332.
- Similarly, if all the cutting tool life parameters are kept at the optimal condition (i.e. cutting speed = low, tool geometry = high and cutting angle = high), the predicted tool life would then be

$$\begin{aligned}\hat{y} &= 40.883 + 5.667(+1) + 3.417(+1) - \{4.417(-1) \times (+1)\} \\ &= 54.384\end{aligned}$$

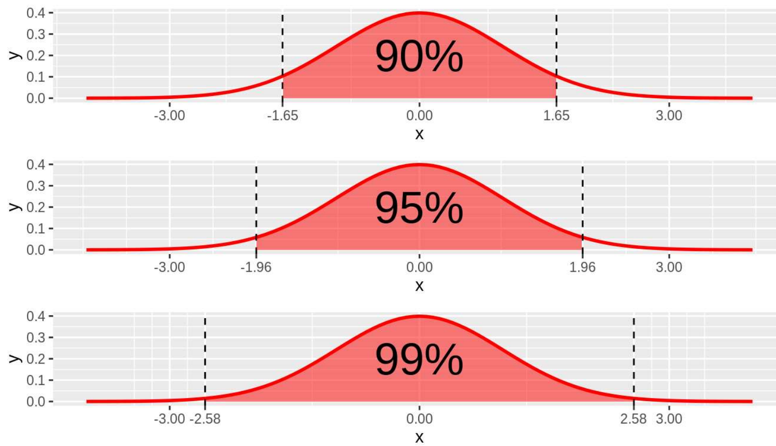


Model for Predicting Response Function

- Once the statistical analysis is performed on the experimental data, it is important to verify the results by means of **confirmatory experiments** or trials.
- The number of confirmatory runs at the optimal settings can vary from **4 to 20** (4 runs if expensive, 20 runs if cheap).

Confidence Interval for the Mean Response

- The **statistical confidence interval (CI)** (at 99% confidence limit) for the mean response can be computed using the equation



$$\mathrm{CI} = \bar{y} \pm 3\left\{\frac{\mathrm{SD}}{\sqrt{n}}\right\}$$

where

y = mean response obtained from confirmation trials or runs

SD = standard deviation of response obtained from confirmation trials

n = number of samples (or confirmation runs).

Confidence Interval for the Mean Response

- For the cutting tool life example, five samples were collected from the process at the optimal condition (i.e. cutting speed = low, tool geometry = high and cutting angle = high).
- Confirmation trials

Results from Confirmation Trials
53.48
52.69
53.88
54.12
54.36

Y(bar) = 53.71h SD = 0.654 h

Standard Deviation Formula	
Population	Sample
$\sigma = \sqrt{\frac{\sum(X-\mu)^2}{N}}$	$s = \sqrt{\frac{\sum(X-\bar{x})^2}{n-1}}$
X – The Value in the data distribution μ – The population Mean N – Total Number of Observations	X – The Value in the data distribution $\bar{x}$ – The Sample Mean n - Total Number of Observations

Standard Deviation, s: **0.65435464390497**

Count, N: 5
Sum, Σx: 268.53
Mean, x̄: 53.706
Variance, s^2: 0.42818

Steps

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\overline{x})^2},$$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{N - 1}$$

$$= \frac{(53.48 - 53.706)^2 + ... + (54.36 - 53.706)^2}{5 - 1}$$

$$= \frac{1.71272}{4}$$

$$= 0.42818$$

$$s = \sqrt{0.42818}$$

$$= 0.65435464390497$$

Confidence Interval for the Mean Response

- 99% CI for the mean response is given by:

$$\text{CI} = 53.71 \pm 3\left\{\frac{0.654}{\sqrt{5}}\right\}$$
$$= 53.71 \pm 0.877 = (54.55, 52.83)$$

- As the predicted value (54.384) based on the regression model falls within the statistical CI, we will consider our model good.

Confidence Interval for the Mean Response

- If the results from the confirmation trials or runs fall outside the statistical CI, possible causes must be identified. Some of the possible causes may be
  - incorrect choice of experimental design for the problem at hand
  - improper choice of response(s) for the experiment
  - inadequate control of noise factors, which cause excessive variation
  - omission of some important process or design parameters in the first rounds of experimentation
  - measurement error
  - wrong assumptions regarding interactions
  - errors in conducting the experiment, etc.

Confidence Interval for the Mean Response

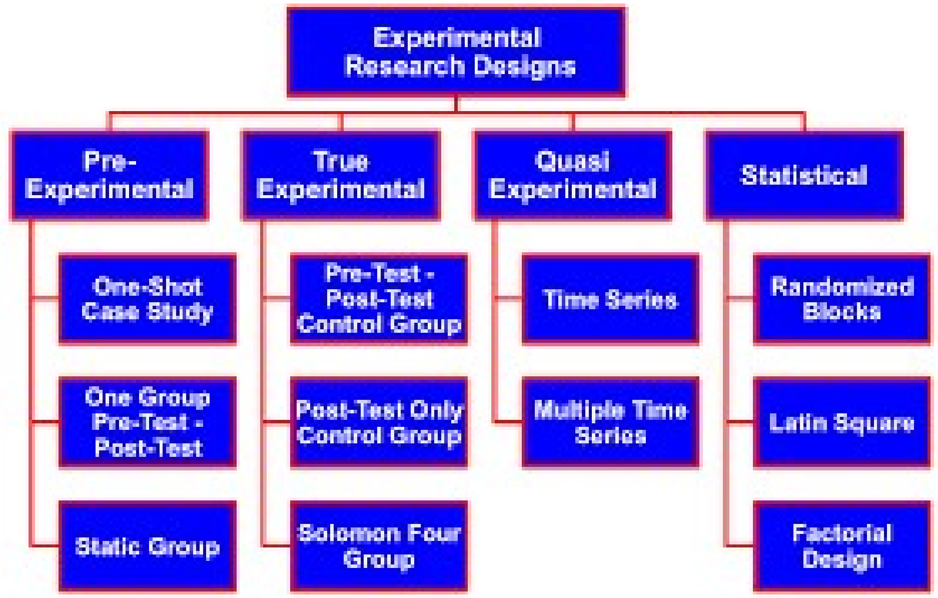
- If the results from the confirmatory trials or runs are **within the CI**, then improvement action on the process is recommended.
- The new process or design parameters should be implemented with the involvement of top management.
- After the solution has been implemented, control charts on the response(s) or key process parameters should be constructed for constantly monitoring, analyzing, managing and improving the process performance.

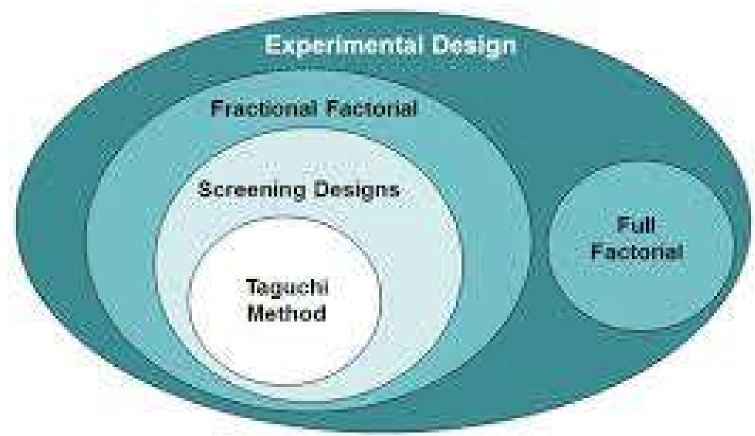
Screening Designs

- In many process development and manufacturing applications, the number of potential process or design variables or parameters (or factors) is **large**.
- **Screening reduces the number of process or design parameters (or factors) by identifying the key factors affecting product quality or process performance.**
- This reduction allows one to focus process improvement efforts on the few really important factors, or the ‘vital few’.

Types of Factorial Design

1. Full Factorial Design(FD)
a. Two Levels Full FD
b. Three level Full FD
2. Fractional Factorial Design
a. Homogenous fractional
b. Mixed level fractional
c. Box-Hunter
d. Plackett - Burman
e. Taguchi
f. Latin square





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Full Factorial Design

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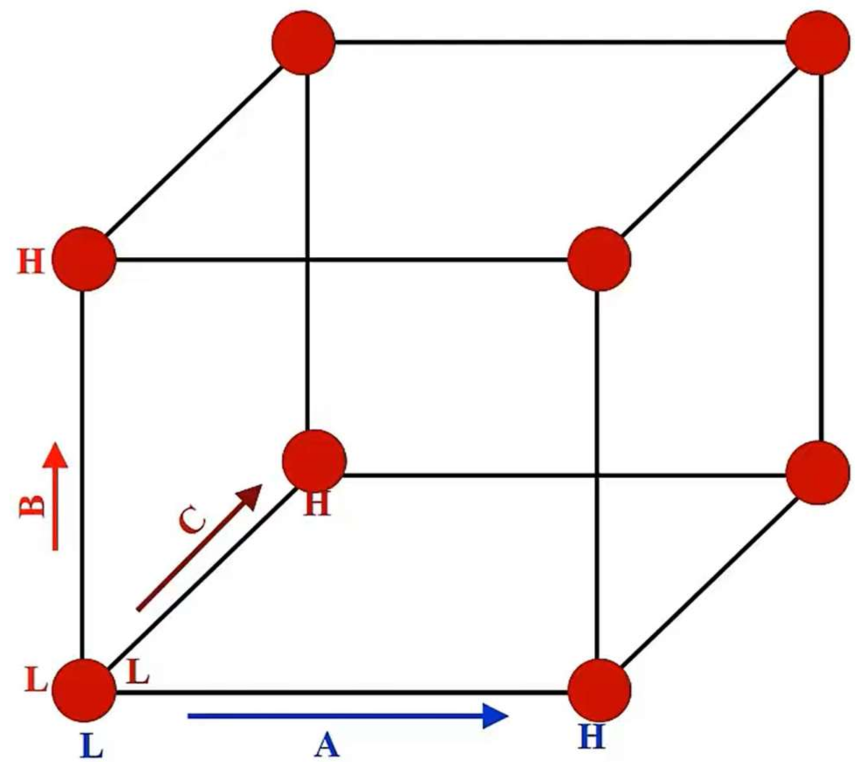
The Full Factorial Designs

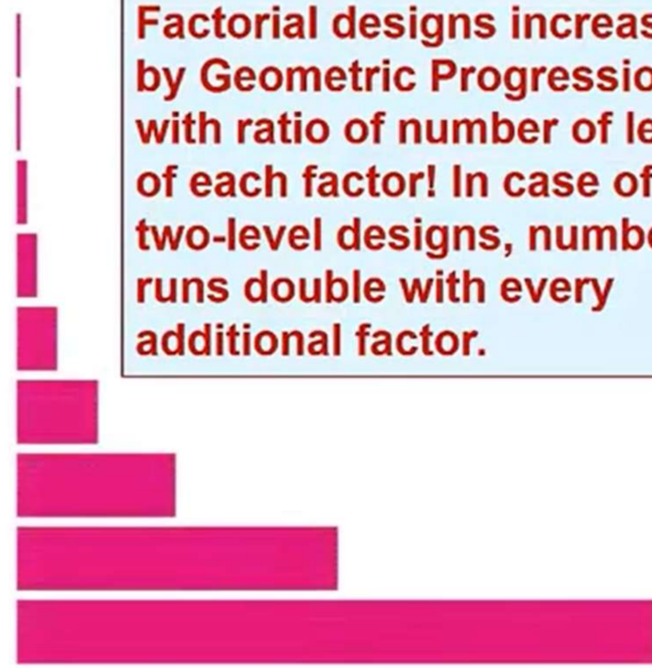
When we run trials with *each possible treatment*, it is a Full Factorial Experiment!



Full Factorial Design

Number of runs in Two level Full Factorial Designs

Factors	Levels	Runs
2	2	4
3	2	8
4	2	16
5	2	32
6	2	64
7	2	128
8	2	256
9	2	512
10	2	1024



The number of runs in Full Factorial designs increase by Geometric Progression with ratio of number of levels of each factor! In case of two-level designs, number of runs double with every additional factor.

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Full Factorial Design

Number of runs in Three-Level Full Factorial Designs

Factors	Levels	Runs
2	3	9
3	3	27
4	3	81
5	3	243
6	3	729
7	3	2187
8	3	6561
9	3	19683
10	3	59049

In case of three-level designs, number of runs treble with every additional factor!

Fractional Factorial Design

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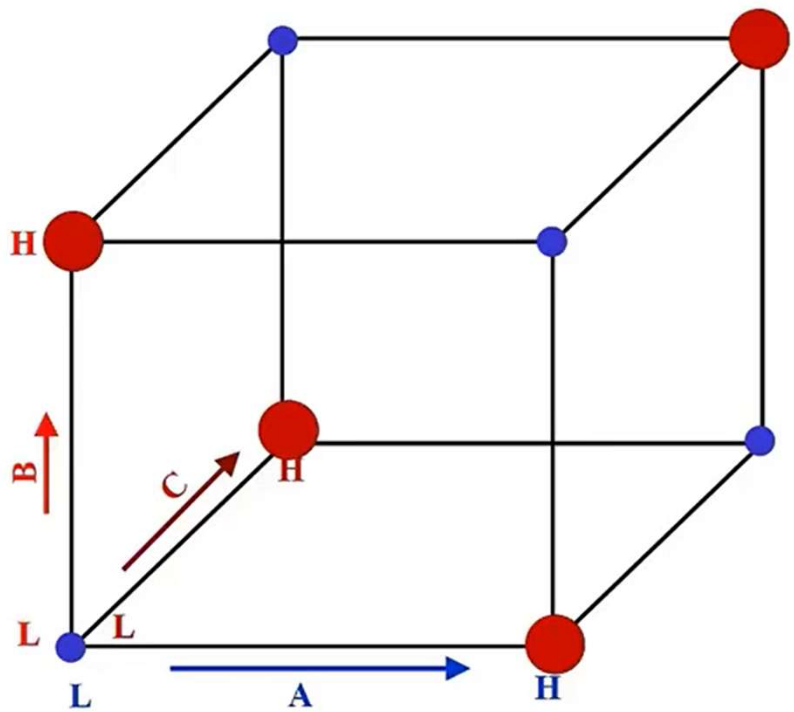
Statisticians have designed Fractional Factorial Experiments (FFE) to reduce number of runs or trials. Only selected treatment combinations are tried instead of all combinations.



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Consider a Full Factorial Design 2^3

Create design in coded units in XL → **Add Interaction columns by algebraic multiplication**

A	B	C	AB	BC	AC	ABC
-1	-1	-1	1	1	1	-1
1	-1	-1	-1	1	-1	1
-1	1	-1	-1	-1	1	1
1	1	-1	1	-1	-1	-1
-1	-1	1	1	-1	-1	1
1	-1	1	-1	-1	1	-1
-1	1	1	-1	1	-1	-1
1	1	1	1	1	1	1

Let us create a half fractional design with only four runs from this design! Experimenters have found that higher order interactions of three and more factors tend to be small and can be ignored often. Thus we decide to omit three-factor i.e. ABC interaction from the analysis!

Select only rows with ABC=+1

As we want to omit ABC interaction, we will use only those rows with ABC=1 marked red in the table and exclude rows with ABC = -1.

A	B	C	AB	BC	AC	ABC
-1	-1	-1	1	1	1	-1
1	-1	-1	-1	1	-1	1
-1	1	-1	-1	-1	1	1
1	1	-1	1	-1	-1	-1
-1	-1	1	1	-1	-1	1
1	-1	1	-1	-1	1	-1
-1	1	1	-1	1	-1	-1
1	1	1	1	1	1	1

We will exclude these four rows with ABC = -1

Select only rows with ABC=+1

As we want to omit ABC interaction, we will use only those rows with ABC=1 marked red in the table and exclude rows with ABC = -1.

A	B	C	AB	BC	AC	ABC
1	-1	-1	-1	1	-1	1
-1	1	-1	-1	-1	1	1
-1	-1	1	1	-1	-1	1
1	1	1	1	1	1	1

We now get the Half Fraction of the 2^3 design. ABC=+1 is the Identity of the design.

We will exclude these four rows with ABC = -1

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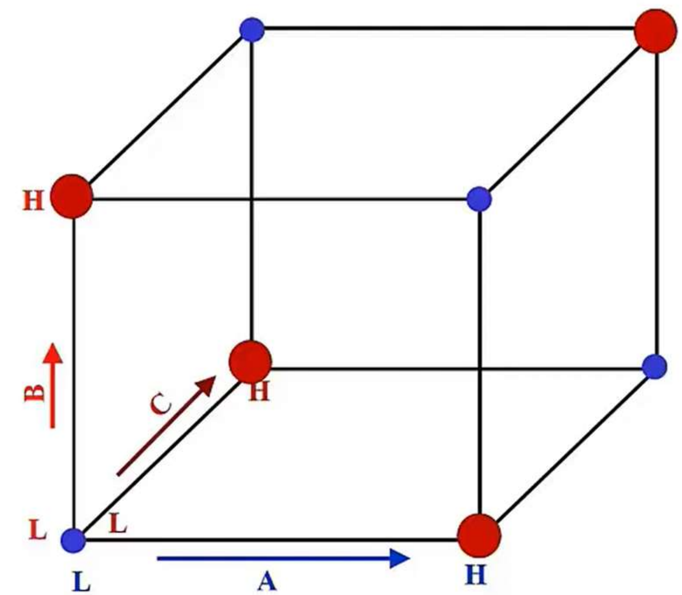
Representing the rows on the cube

These four runs with ABC=+1 can be represented on the cube as shown.

A	B	C	AB	BC	AC	ABC
1	-1	-1	-1	1	-1	1
-1	1	-1	-1	-1	1	1
-1	-1	1	1	-1	-1	1
1	1	1	1	1	1	1



The confounding effect

Now observe whether any of the columns have same sign sequence! Ready? What did you observe?

Did you notice that columns C and AB have same sign sequence?

And columns B and AC have same sign sequence?

And columns A and BC have same sign sequence?

A	B	C	AB	BC	AC	ABC
1	-1	-1	-1	1	-1	1
-1	1	-1	-1	-1	1	1
-1	-1	1	1	-1	-1	1
1	1	1	1	1	1	1

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The confounding effect

Understand the consequence of same sign sequence!

For example, when columns C and AB have same sign sequence, the calculation of effect also would be identical as effect is calculated as difference between Average response at high level and low level of factor. Thus we will be unable to distinguish between effect of C and AB. This means effects of C and AB are 'confounded' and C and AB are called aliases.

A	B	C	AB	BC	AC	ABC
1	-1	-1	-1	1	-1	1
-1	1	-1	-1	-1	1	1
-1	-1	1	1	-1	-1	1
1	1	1	1	1	1	1

Similarly, we will be unable to distinguish between effect of B and AC. Thus effects of B and AC are 'confounded'. And with the same logic, effects of A and BC are also confounded!

Resolution Design

Resolution III Screening Designs

For example, resolution III design is represented with three fingers!

A single finger represents main effects and two fingers represent two factor interactions! These are confounded or aliased in a resolution three design. Thus resolution is 2+1=3.

Resolution IV design

Resolution IV is represented with four fingers!

Two fingers represents two factor interactions! Thus some two-factor interactions are aliased with some other two factor interactions!

Three fingers represents three factor interactions and one finger represents main effects! Thus some three-factor interactions are aliased with main effects. Thus resolution is 3+1=4.

Resolution Design

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Resolution V design

Resolution V design is represented with five fingers!

One finger represents main effects and four fingers represents four-factor interactions and one finger represents main effects! Thus main effects are aliased with four-factor interactions! Resolution of the design is therefore 4+1=5.

Three fingers represents three-factor interactions and two fingers represents two-factor interactions! Thus some two-factor interactions are aliased with some three factor interactions!

Selection of Designs

Following table shows number of runs in various designs along with their resolution codes. The top row shows number of factors and the left column shows number of runs. These designs are known as classical designs.

Runs	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	Full	III												
8		Full	IV	III	III	III								
16			Full	V	IV	IV	IV	III	III	III	III	III	III	III
32				Full	VI	IV	IV	IV	IV	IV	IV	IV	IV	IV
64					Full	VII	V	IV	IV	IV	IV	IV	IV	IV
128						Full	VIII	VI	V	V	IV	IV	IV	IV

Designs of Resolution V and higher are shown in Green colour

Resolution III designs are shown in red colour

Resolution IV designs are shown in Yellow colour.

For example, if a team has shortlisted 9 factors, a screening design of resolution III will have 16 runs.

(Source Minitab)

Screening Designs

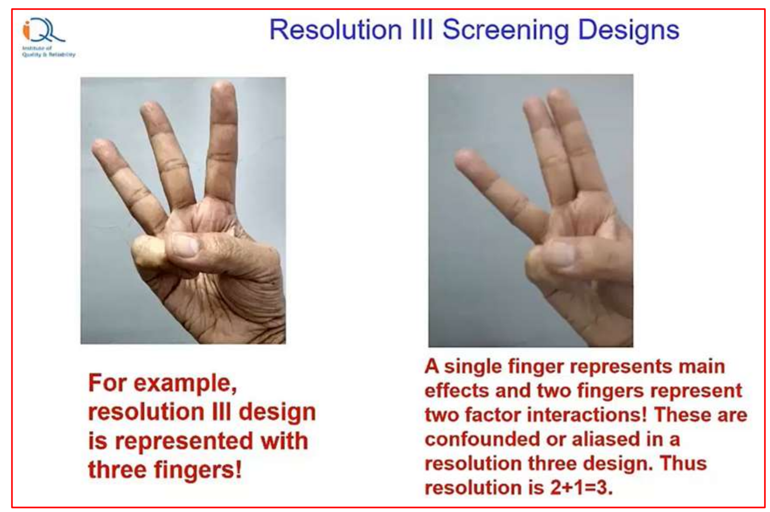
- Screening designs provide an effective way to consider a large number of process or design parameters (or factors) in a minimum number of experimental runs or trials (i.e. with minimum resources and budget).
- The purpose of screening designs is **to identify and separate** out those factors that demand further investigation.
- **For screening designs, experimenters are generally not interested in investigating the nature of interactions among the factors (Antony, 2002).**

P-B design

- Focus on - Screening Designs expounded by R.L. Plackett and J.P. Burman in 1946 – hence the name **Plackett–Burman designs (P–B designs).**
- P–B designs are based on Hadamard matrices in which the number of experimental runs or trials is a **multiple of four**, i.e. N = 4, 8, 12, 16 and so on, where N is the number of trials/runs (Plackett and Burmann, 1946).
- **P–B designs are suitable for studying up to $k = (N-1)/(L-1)$ factors, where L is the number of levels and k is the number of factors.**
- For instance, using a 12-run experiment, it is possible to study up to 11 process or design parameters at 2-levels.

P-B design

- **One of the interesting properties of P–B designs is that all main effects are estimated with the same precision.** This implies that **one does not have to anticipate** which factors are most likely to be important when setting up the study.
- **Resolution III Design**→ No main effects are **confounded** (aliased) with any other main effect, but main effects are confounded with two-factor interactions.



P-B design

- The aim of P-B design is to study **as many factors** as possible in a **minimum number** of trials and to identify those that need to be studied in further rounds of experimentation in which interactions can be more thoroughly assessed.
- Thus, they can determine which factors are important. i.e., the main effects are interested (not interaction effects)
- However, they have the **advantage of requiring fewer experimental** runs.
- P-B design is equivalent to fractional factorial design (are multiples of four but are not powers of two.)
- It uses standard orthogonal arrays.
- Here, k = N-1, there will be no degrees of freedom available for error to be estimated.

P-B Designs

Advantages:

- Limited number of runs to evaluate large number of factors.
- Important main effects can be selected for more in- depth study

Disadvantage of P-B Designs

- Their disadvantage is their complexity.
- As P-B designs can't be represented as cubes, they are sometimes called nongeometric designs.
- Assumption required to evaluate up to $k = (N - 1)$ factors in N runs.
- One can study fewer than $(N - 1)$ factors in N runs.
- **Geometric P–B designs are resolution III designs and therefore these designs can be folded over to achieve a design resolution IV.**

P-B Design vs. Fractional factorial

Number of factors	Plackett-Burman	Fractional factorial of resolution III
$4 \leq n \leq 7$	8	8
$8 \leq n \leq 11$	12	16
$12 \leq n \leq 15$	16	16
$16 \leq n \leq 19$	20	32
$20 \leq n \leq 23$	24	32
$24 \leq n \leq 27$	28	32
$28 \leq n \leq 31$	32	32
$32 \leq n \leq 35$	36	64
$\vdots$	$\vdots$	$\vdots$
$64 \leq n \leq 67$	68	128

P-B Designs

- Geometric
  - 2^N (N = 4, 8, 16, 32, etc.)
  - Geometric designs are identical to fractional factorial designs in which one may be able to study the interactions between factors.
  - If the process is suspected to be highly interactive
- Non-geometric
  - are multiples of four but are not powers of two. E.g. have runs of 12, 20, 24, 28, etc.
  - These designs do not have complete confounding of effects.
  - if interactions are of no concern to the experimenter

P-B Designs

Table 5.2 Design Matrix for a Four-Run Geometric P–B Design

A	B	C
−1	+1	+1
+1	−1	+1
+1	+1	−1
−1	−1	−1

P-B Designs

Table 2.16. Generators for Plackett-Burman designs

# Factors	# Runs	Generator
4-7	8	+ + + − + − −
8-11	12	+ + − + + + − − − + −
12-15	16	+ + + + − + − + + − − + − − −
16-19	20	+ + − − + + + + − + − + − − − − + + −
20-23	24	+ + + + + − + − + + − − + + − − + − + − − − −
32-35	36	− + − + + + − − − + + + + + − + + + − − + − − − − + − + − + + − − + −

P-B Designs

# Factors	# Runs	Generator
4-7	8	+ + + − + − −

- Step 1: Assign the levels for a first factor as per P-B design generator table
- Step 2: Keep the last run/trail at a low level for all factors.
- Step 3: To make levels for consecutive factors follow the cyclic manner as represented.

Table 5.1 An Eight-Run Geometric P–B Design

A	B	C	D	E	F	G
+1	−1	−1	+1	−1	+1	+1
+1	+1	−1	−1	+1	−1	+1
+1	+1	+1	−1	−1	+1	−1
−1	+1	+1	+1	−1	−1	+1
+1	−1	+1	+1	+1	−1	−1
−1	+1	−1	+1	+1	+1	−1
−1	−1	+1	−1	+1	+1	+1
−1	−1	−1	−1	−1	−1	−1

1

2

Generator vectors

P-B Designs

# Factors	# Runs	Generator
4-7	8	+ + + − + − −

- Step 1: Assign the levels for a first factor as per P-B design generator table
- Step 2: Keep the last run/trail at a low level for all factors.
- Step 3: To make levels for consecutive factors follow the cyclic manner as represented.

Table 5.1 An Eight-Run Geometric P–B Design

A	B	C	D	E	F	G
+1	−1	−1	+1	−1	+1	+1
+1	+1	−1	−1	+1	−1	+1
+1	+1	+1	−1	−1	+1	−1
−1	+1	+1	+1	−1	−1	+1
+1	−1	+1	+1	+1	−1	−1
−1	+1	−1	+1	+1	+1	−1
−1	−1	+1	−1	+1	+1	+1
−1	−1	−1	−1	−1	−1	−1

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P-B Designs

Table 5.3 A 12-Run Non-geometric P–B Design

A	B	C	D	E	F	G	H	I	J	K
+1	−1	+1	−1	−1	−1	+1	+1	+1	−1	+1
+1	+1	−1	+1	−1	−1	−1	+1	+1	+1	−1
−1	+1	+1	−1	+1	−1	−1	−1	+1	+1	+1
+1	−1	+1	+1	−1	+1	−1	−1	−1	+1	+1
+1	+1	−1	+1	+1	−1	+1	−1	−1	−1	+1
+1	+1	+1	−1	+1	+1	−1	+1	−1	−1	−1
−1	+1	+1	+1	−1	+1	+1	−1	+1	−1	−1
−1	−1	+1	+1	+1	−1	+1	+1	−1	+1	−1
−1	−1	−1	+1	+1	+1	−1	+1	+1	−1	+1
+1	−1	−1	−1	+1	+1	+1	−1	+1	+1	−1
−1	+1	−1	−1	−1	+1	+1	+1	−1	+1	+1
−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1

Ex 1. Paperboard product

- This example is based on the manufacturing process of a paperboard product.
- The objective of the experiment was to increase the puncture resistance of this paperboard product.
- The **response or quality** characteristic of interest to the team conducting the experiment was the **force required to penetrate the material.**
- The objective was to **maximize the mean force** required to penetrate the material.

Ex 1. Paperboard product

Table 5.4 List of Factors and Their Levels for the Experiment

Factors	Labels	Low-Level Setting	High-Level Setting
Paste temperature	A	130°F	160°F
Amount of additive	B	0.2%	0.5%
Press roll pressure	C	40 psi	80 psi
Paper moisture	D	Low	High
Paste type	E	No clay	With clay
Cure time	F	10 days	5 days
Machine speed	G	120 fpm	200 fpm

Number of factors: 7
Number of levels: 2
Hence, it is 2^7 designs.

Experimental design

2^7 = 128 runs

P-D = 8 runs

Ex 1. Paperboard product

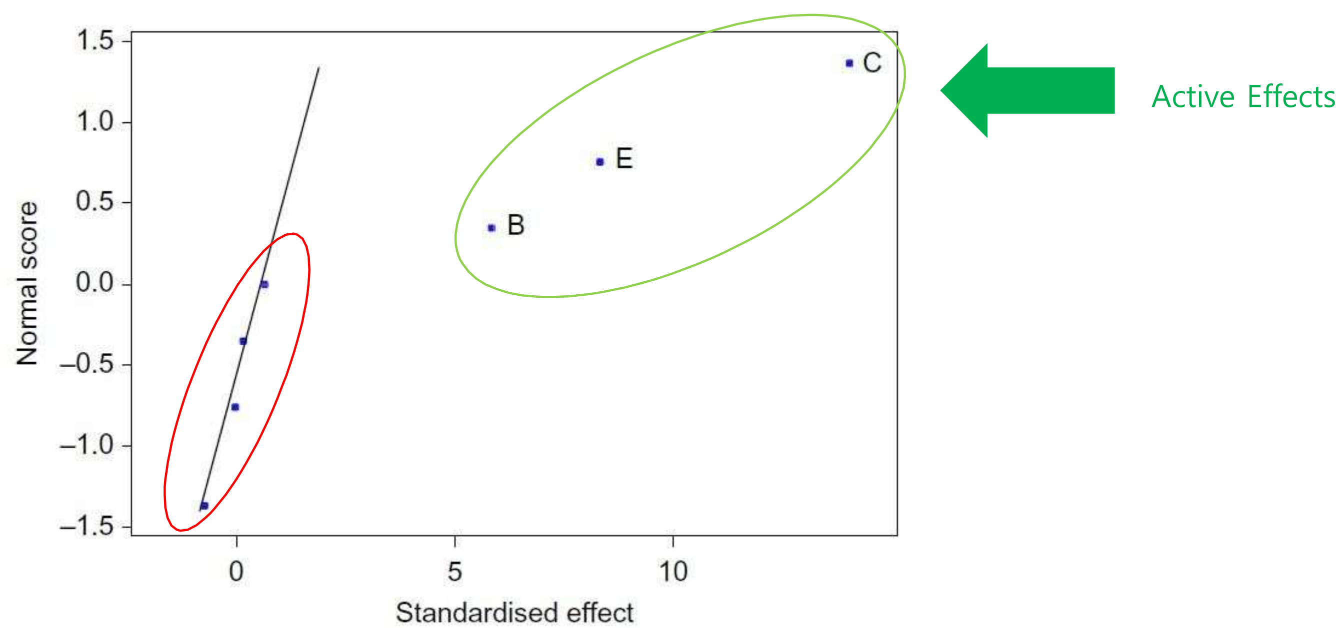
Table 5.5 Design Matrix of an Eight-Run Geometric P–B Design for the Experiment

A	B	C	D	E	F	G	R1	R2
+1	−1	−1	+1	−1	+1	+1	12.5	16.84
+1	+1	−1	−1	+1	−1	+1	42.44	39.29
+1	+1	+1	−1	−1	+1	−1	55.08	47.57
−1	+1	+1	+1	−1	−1	+1	49.37	47.69
+1	−1	+1	+1	+1	−1	−1	55.43	52.80
−1	+1	−1	+1	+1	+1	−1	42.51	35.02
−1	−1	+1	−1	+1	+1	+1	51.13	57.92
−1	−1	−1	−1	−1	−1	−1	15.61	13.65

Ex 1. Paperboard product

Finding key main effects

- Minitab software – used for plotting
- Toidentify the key main effects that were most influential on the response (i.e. force).



NPP of standardised effects.

Ex 1. Paperboard product

Pareto chart –Substantiation

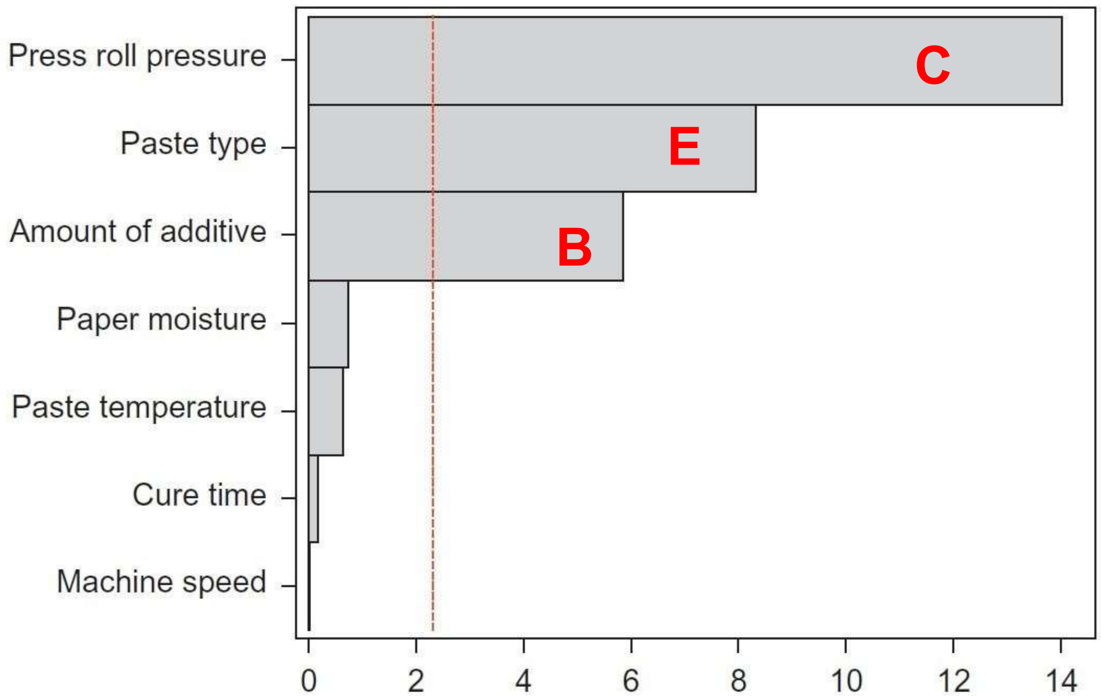


Figure 5.2 Pareto plot of the effects for the experiment.

Ex 1. Paperboard product

Main effects

Conclusion: Main effects C (press roll pressure), E (paste type) and B (amount of additive) are found to have significant impact on the mean puncture resistance (i.e. the force required to penetrate the paper board).

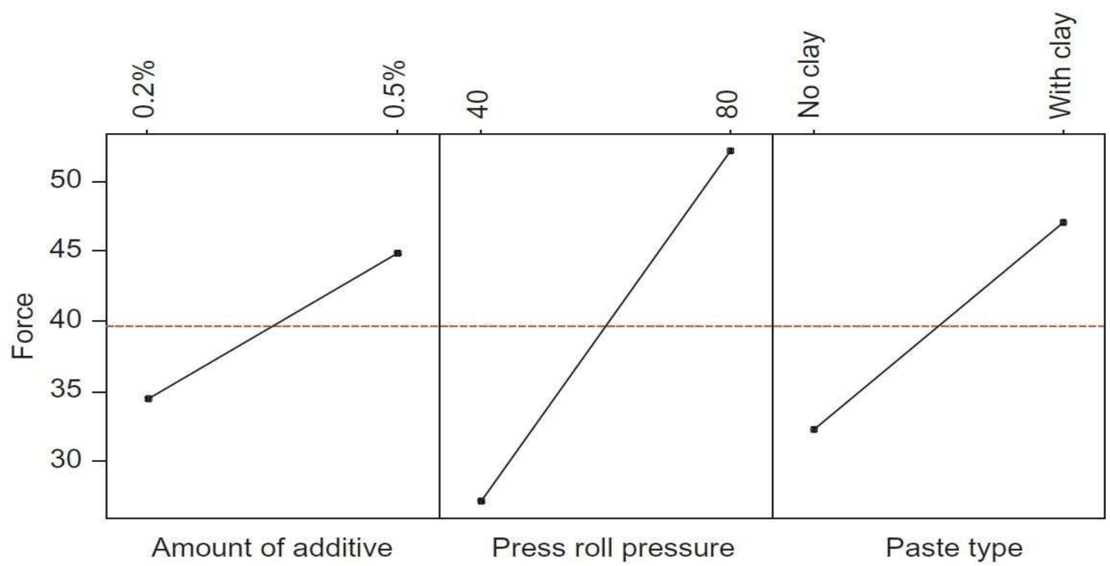


Figure 5.3 Main effects plot of the significant effects.

Ex 1. Paperboard product

Design Matrix

In order to analyze the factors affecting variability in force, we need to calculate the SD of observations at each experimental design point.

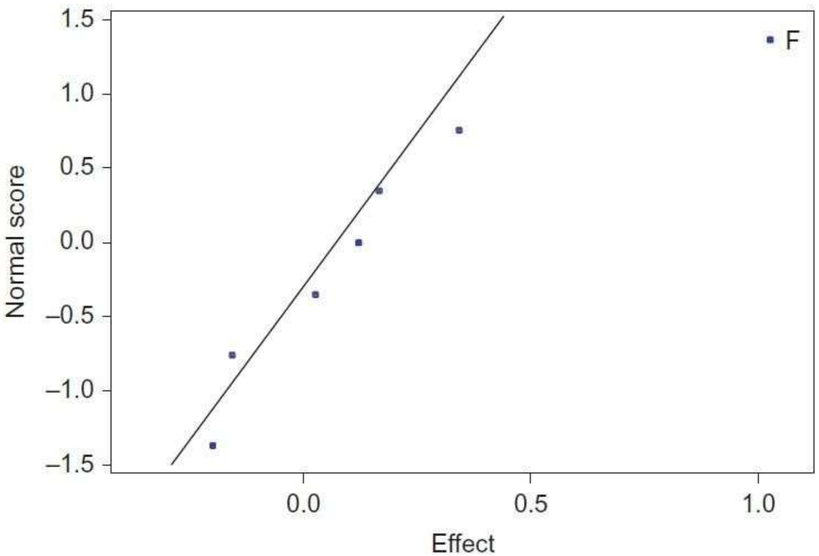
Table 5.6 Design Matrix of an Eight-Run Geometric P-B Design with Standard Deviation Values

A	B	C	D	E	F	G	s	ln(SD)
+1	−1	−1	+1	−1	+1	+1	3.07	1.122
+1	+1	−1	−1	+1	−1	+1	2.23	0.802
+1	+1	+1	−1	−1	+1	−1	5.31	1.670
−1	+1	+1	+1	−1	−1	+1	1.18	0.166
+1	−1	+1	+1	+1	−1	−1	1.86	0.621
−1	+1	−1	+1	+1	+1	−1	5.30	1.668
−1	−1	+1	−1	+1	+1	+1	4.80	1.569
−1	−1	−1	−1	−1	−1	−1	1.39	0.329

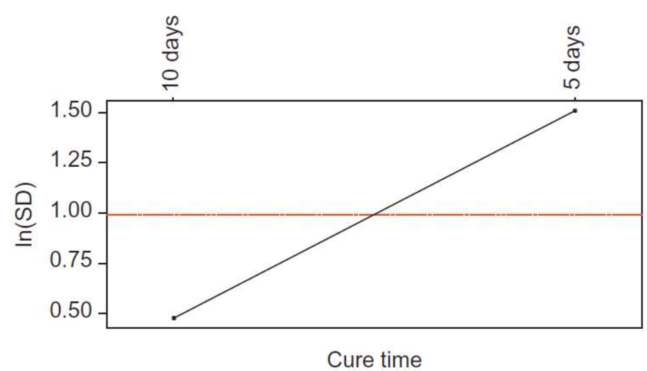
Ex 1. Paperboard product

Normal plot: Effects on SD

The normal plot indicates that only factor F (cure time) influenced the variation in the puncture resistance (i.e. force). Further analysis of factor F has revealed that variability is maximum when cure time is set at high level (i.e. 5 days).



Normal plot of effects affecting variability in puncture resistance.



Main effects plot for ln(SD).

Ex 1. Paperboard product

Conclusion

- Factors C, B and E have a significant impact on process average, whereas factor F has a significant impact on process variability.
- Other factors such as A, D and G can be set at their economic levels since they do not appear to influence either the process average or the process variability.
- The next stage of the experimentation would be to consider the interaction among the factors and select the optimal settings from the experiment that yields maximum force with minimum variability. [Full or fractional factorial with resolution 4]

Ex 2. Plastic Extrusion Process

- Problem : Affection of porosity of plastic parts.
- Eight process parameters might have some impact on porosity.
- Each factor was studied at 2-levels.
- As the total degrees of freedom for studying eight factors at 2-levels is equal to 8, it was decided to Choose a non-geometric 12-run P–B design with 11 degrees of freedom.
- The extra 3 degrees of freedom can be used to estimate experimental error.

Ex 2. Plastic Extrusion Process

- Design Matrix

Table 5.8 Experimental Layout for 12-Run P–B Design with Response Values

Run	A	B	C	D	E	F	G	H	Porosity (%)
1 (6)	+1	+1	−1	+1	+1	+1	−1	−1	44.8
2 (11)	+1	−1	+1	+1	+1	−1	−1	−1	37.2
3 (9)	−1	+1	+1	+1	−1	−1	−1	+1	36.0
4 (7)	+1	+1	+1	−1	−1	−1	+1	−1	34.8
5 (2)	+1	+1	−1	−1	−1	+1	−1	+1	46.4
6 (1)	+1	−1	−1	−1	+1	−1	+1	+1	24.8
7 (5)	−1	−1	−1	+1	−1	+1	+1	−1	43.6
8 (12)	−1	−1	+1	−1	+1	+1	−1	+1	44.8
9 (3)	−1	+1	−1	+1	+1	−1	+1	+1	24.0
10 (8)	+1	−1	+1	+1	−1	+1	+1	+1	34.4
11 (4)	−1	+1	+1	−1	+1	+1	+1	−1	27.2
12 (10)	−1	−1	−1	−1	−1	−1	−1	−1	49.6

Note: Numbers in parentheses represent the random order of experimental runs or trials.

Ex 2. Plastic Extrusion Process

- Effects of the experiment

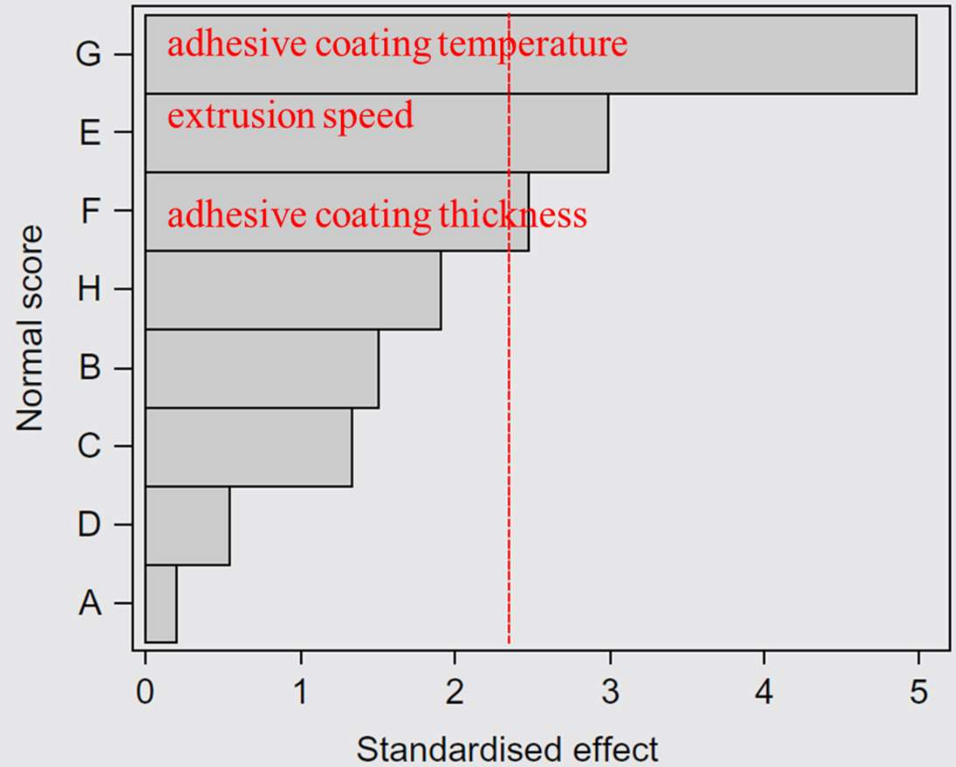


Figure 5.6 Standardised Pareto plot of Effects for the plastic foam extrusion process.

Ex 2. Plastic Extrusion Process

- Main Effects

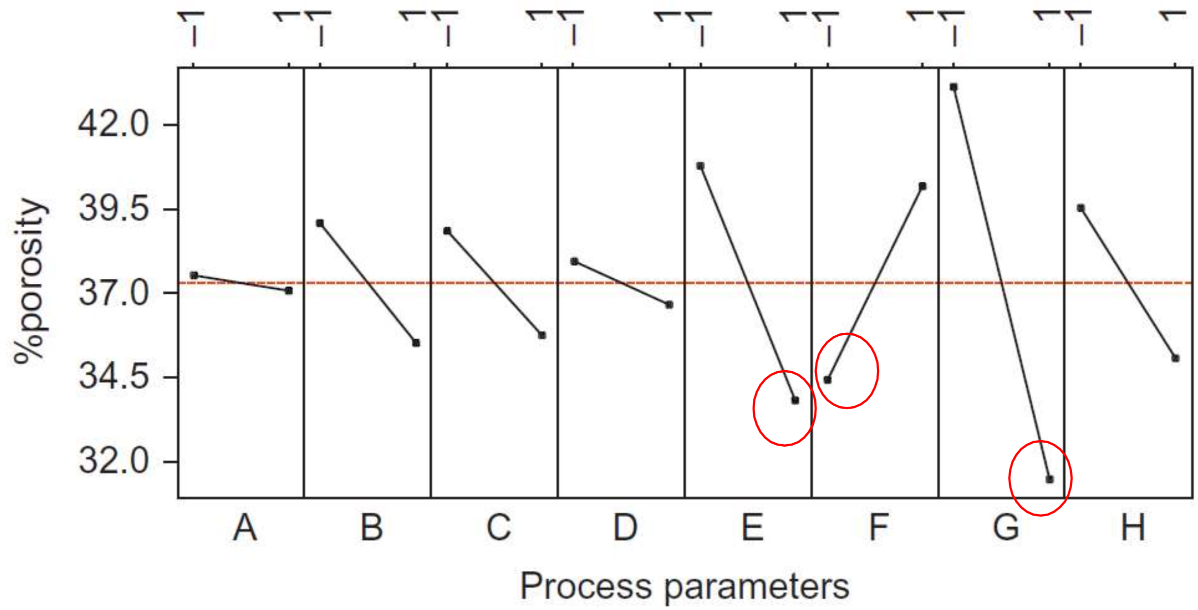


Figure 5.7 Main effects plot for the experiment.

Introduction of full factorial design, Basic concepts of 2^2, 2^3 and 2^k designs

- It is widely accepted that the most commonly used experimental designs in manufacturing companies are **full and fractional factorial designs at 2-levels and 3-levels**.

- Factorial designs would enable an experimenter to study the **joint effect of the factors** (or process/design parameters) on a response.

- A **full factorial design** consists of all possible factor combinations in a test, and, most importantly, varies the factors simultaneously rather than one factor at a time.
- Using this approach, the tester can examine both main effects (effect of the independent variables on the dependent variable) and interactions (effect of the interaction between independent variables on the dependent variable) associated with both categorical and continuous factors.

2-Level Factorial Designs

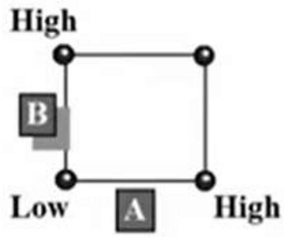
1-Factor

$2^1 = 2$ Runs

		Factors
Runs	tc	**A**
1	(1)	**-1**
2	a	**+1**

2-Factor

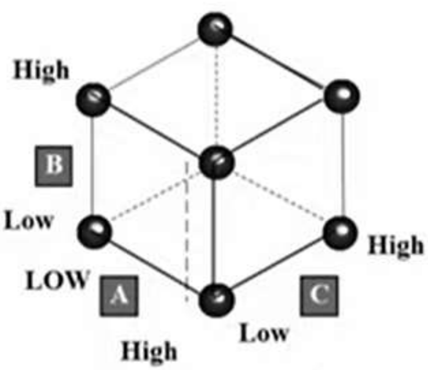
$2^2 = 4$ Runs



		Factors		Interaction
Runs	tc	**A**	**B**	AB
1	(1)	**-1**	**-1**	+1
2	a	**+1**	**-1**	-1
3	b	**-1**	**+1**	-1
4	ab	**+1**	**+1**	+1

3-Factor

$2^3 = 8$ Runs



		Factors			Interactions			
Runs	tc	**A**	**B**	**C**	AB	AC	BC	ABC
1	(1)	**-1**	**-1**	**-1**	+1	+1	+1	-1
2	a	**+1**	**-1**	**-1**	-1	-1	+1	+1
3	b	**-1**	**+1**	**-1**	-1	+1	-1	+1
4	ab	**+1**	**+1**	**-1**	+1	-1	-1	-1
5	c	**-1**	**-1**	**+1**	+1	-1	-1	+1
6	ac	**+1**	**-1**	**+1**	-1	+1	-1	-1
7	bc	**-1**	**+1**	**+1**	-1	-1	+1	-1
8	abc	**+1**	**+1**	**+1**	+1	+1	+1	+1

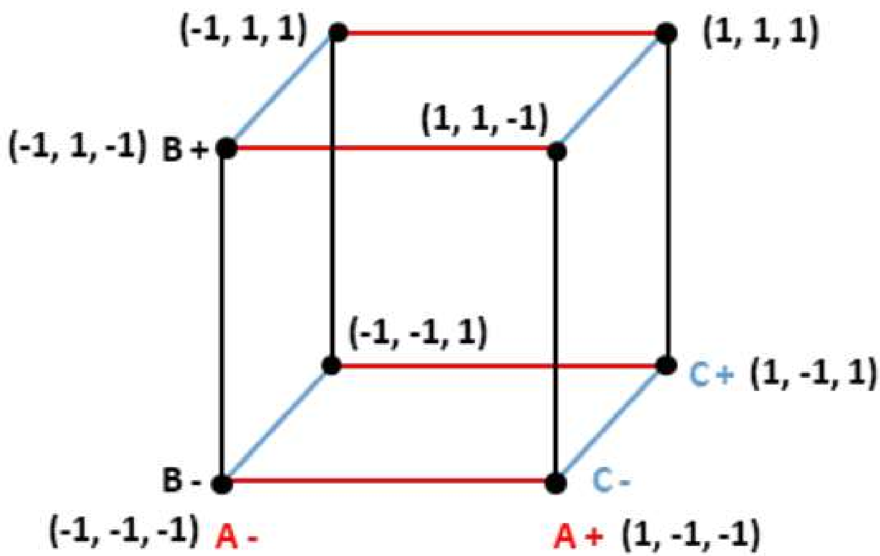


Figure 2: 2^3 Full Factorial Design

Balance and Orthogonality

X

Run	Order	A	B	AB
1	(1)	-1	-1	1
2	a	1	-1	-1
3	b	-1	1	-1
4	ab	1	1	1

Balanced $\sum X_i = 0$ for each factor sum

This feature helps to simplify the analysis

Orthogonal $\sum X_i X_j = 0$ for all dot product pairs

This feature ensures the effects are independent

(Adapted from Mikel J. Harry. The Vision of Six Sigma. 1994. Page 18.9)

All the experimental design should be balanced and orthogonal.

Balanced: in terms of factors
Orthogonal: in terms of interactions.

Factorial Design always balanced and orthogonal.

Also, it should be Unbiased.

Introduction of full factorial design, Basic concepts of 2^2, 2^3 and 2^k designs

The properties of the 2^k factorial design matrix are:

- Every column has an equal number of - and + signs (low and high settings)
- The sum of the product of signs in any two columns is zero (called the orthogonality property)
- Multiplying any column by 1 leaves that column unchanged (identity property)
- Product of any two columns yields a column that can calculate the effect of another term (for example, $A \times B = AB$)

Drawback of Full Factorial Designs

- The factorial approach is an efficient approach to experimentation.
- the number of experimental runs is 2^k.
- can result in a large number of runs,

Number of Runs Required for a Two Level Full Factorial DOE:

1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024
⋮	⋮
15	32,768
⋮	⋮
20	1,048,576

(Adapted from Six Sigma Black Belt Training - *Improve*, October 1996, Pages 3.1 & 3.2)

Full Factorials

Number Factors	Main Effects	Order of Interactions 2	3	4	5	6	7	8	9	10
2	2	1								
3	3	3	1							
4	4	6	4	1						
5	5	10	10	5	1					
6	6	15	20	15	6	1				
7	7	21	35	35	21	7	1			
8	8	28	56	70	56	28	8	1		
9	9	36	84	126	126	84	36	9	1	
10	10	45	120	210	252	210	120	45	10	1

Box et al. (1978) "There tends to be a redundancy in [full factorial designs] – redundancy in terms of an excess number of interactions that can be estimated … Fractional factorial designs exploit this redundancy …"

2^2 factorial designs

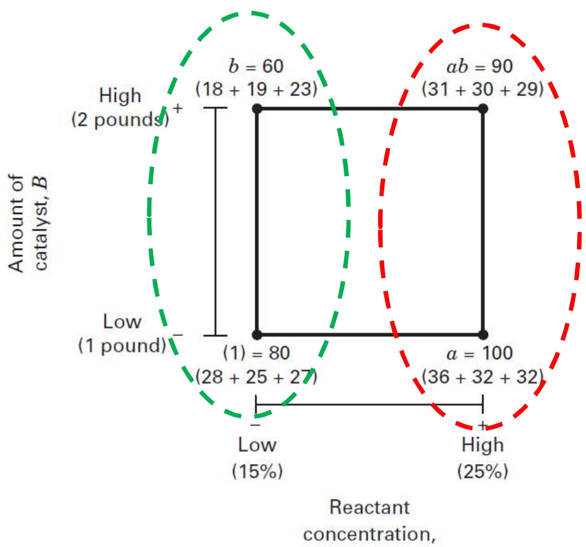
The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SS_A	$a - 1$	$MS_A = \dfrac{SS_A}{a - 1}$	$F_0 = \dfrac{MS_A}{MS_E}$
B treatments	SS_B	$b - 1$	$MS_B = \dfrac{SS_B}{b - 1}$	$F_0 = \dfrac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a - 1)(b - 1)$	$MS_{AB} = \dfrac{SS_{AB}}{(a - 1)(b - 1)}$	$F_0 = \dfrac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n - 1)$	$MS_E = \dfrac{SS_E}{ab(n - 1)}$	
Total	SS_T	$abn - 1$		

F value = variance of the group means (Mean Square Between) / mean of the within group variances (Mean Squared Error)

Example 2^2 design

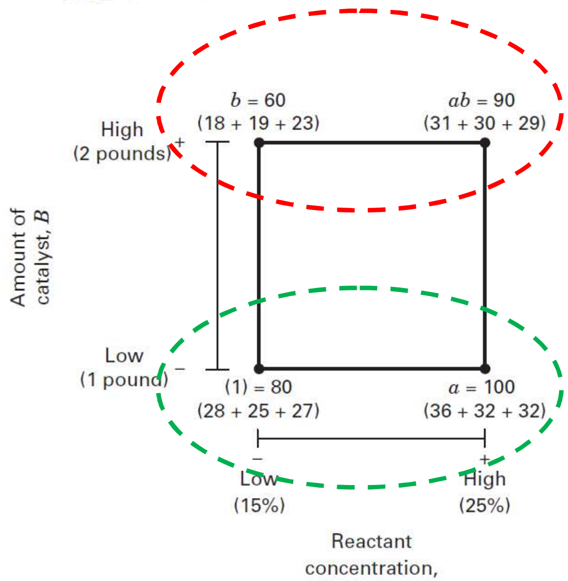
The formulas for the effects of A, B, and AB may be derived by another method. The effect of A can be found as the difference in the average response of the two treatment combinations on the right-hand side of the square in Figure 6.1 (call this average $\bar{y}_{A^+}$ because it is the average response at the treatment combinations where A is at the high level) and the two treatment combinations on the left-hand side (or $\bar{y}_{A^-}$). That is,



$$
\begin{aligned}
A &= \bar{y}_{A^+} - \bar{y}_{A^-} \\
&= \frac{ab + a}{2n} - \frac{b + (1)}{2n} \\
&= \frac{1}{2n}[ab + a - b - (1)]
\end{aligned}
$$

Example 2^2 design

This is exactly the same result as in Equation 6.1. The effect of B, Equation 6.2, is found as the difference between the average of the two treatment combinations on the top of the square ($\bar{y}_{B^+}$) and the average of the two treatment combinations on the bottom ($\bar{y}_{B^-}$), or



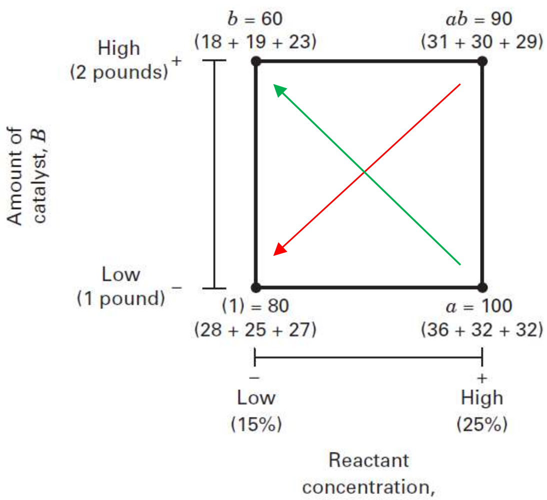
$$B = \bar{y}_{B^+} - \bar{y}_{B^-}$$

$$= \frac{ab + b}{2n} - \frac{a + (1)}{2n}$$

$$= \frac{1}{2n}[ab + b - a - (1)]$$

Example 2^2 design

Finally, the interaction effect AB is the average of the right-to-left diagonal treatment combinations in the square [ab and (1)] minus the average of the left-to-right diagonal treatment combinations (a and b), or



$$AB = \frac{ab + (1)}{2n} - \frac{a + b}{2n}$$

$$= \frac{1}{2n}[ab + (1) - a - b]$$

2^2 factorial designs

$$A = \frac{1}{2n}\{[ab - b] + [a - (1)]\}$$
$$= \frac{1}{2n}[ab + a - b - (1)]$$

$$B = \frac{1}{2n}\{[ab - a] + [b - (1)]\}$$
$$= \frac{1}{2n}[ab + b - a - (1)]$$

$$AB = \frac{1}{2n}\{[ab - b] - [a - (1)]\}$$
$$= \frac{1}{2n}[ab + (1) - a - b]$$

$$SS_A = \frac{[a + ab - b - (1)]^2}{4n}$$
$$SS_B = \frac{[b + ab - a - (1)]^2}{4n}$$
$$SS_{AB} = \frac{[ab + (1) - a - b]^2}{4n}$$

$$SS_T = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{4n}$$

$$SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

2^2 factorial designs

$$SS_T = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{4n}$$

Replicate			
I	II	III	Total
28	25	27	80
36	32	32	100
18	19	23	60
31	30	29	90

Example:

$SS_T = [28^2 + 36^2 + 18^2 + 31^2 + 25^2 + 32^2 + 19^2 + 30^2 + 27^2 + 32^2 + 23^2 + 29^2] - [80+100+60+90]^2/4n$

F-Distribution Table (alpha = 0.05) for Critical Value

	DF1	α = 0.05																	
DF2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	Inf
1	161.45	199.5	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.1	251.14	252.2	253.25	254.31
2	18.513	19	19.164	19.247	19.296	19.33	19.353	19.371	19.385	19.396	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.487	19.496
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.572	8.5494	8.5264
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.041	5.9988	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.717	5.6877	5.6581	5.6281
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.365
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.099	4.06	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.866	3.787	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	5.3177	4.459	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10	4.9646	4.1028	3.7083	3.478	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.913	2.845	2.774	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.948	2.8962	2.8536	2.7876	2.7186	2.6464	2.609	2.5705	2.5309	2.4901	2.448	2.4045
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.341	2.2962
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.671	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.463	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	4.494	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	4.4513	3.5915	3.1968	2.9647	2.81	2.6987	2.6143	2.548	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.104	2.0584	2.0107	1.9604
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.308	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.878
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.599	2.514	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.366	2.321	2.2504	2.1757	2.096	2.054	2.0102	1.9645	1.9165	1.8657	1.8117
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.938	1.8894	1.838	1.7831
23	4.2793	3.4221	3.028	2.7955	2.64	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	2.005	1.9605	1.9139	1.8648	1.8128	1.757
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.939	1.892	1.8424	1.7896	1.733
25	4.2417	3.3852	2.9912	2.7587	2.603	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.711
26	4.2252	3.369	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.901	1.8533	1.8027	1.7488	1.6906
27	4.21	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	4.196	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.236	2.19	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	4.183	3.3277	2.934	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	4.0847	3.2317	2.8387	2.606	2.4495	2.3359	2.249	2.1802	2.124	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.097	2.0401	1.9926	1.9174	1.8364	1.748	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.175	2.0868	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.429	1.3519	1.2539
Inf	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.394	1.318	1.2214	1

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F-Distribution Table (alpha = 0.025) for Critical Value

	DF1	α = 0.025																	
DF2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	Inf
1	647.79	799.5	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63	976.71	984.87	993.1	997.25	1001.4	1005.6	1009.8	1014	1018.3
2	38.506	39	39.166	39.248	39.298	39.332	39.355	39.373	39.387	39.398	39.415	39.431	39.448	39.456	39.465	39.473	39.481	39.49	39.498
3	17.443	16.044	15.439	15.101	14.885	14.735	14.624	14.54	14.473	14.419	14.337	14.253	14.167	14.124	14.081	14.037	13.992	13.947	13.902
4	12.218	10.649	9.9792	9.6045	9.3645	9.1973	9.0741	8.9796	8.9047	8.8439	8.7512	8.6565	8.5599	8.5109	8.461	8.411	8.36	8.309	8.257
5	10.007	8.4336	7.7636	7.3879	7.1464	6.9777	6.8531	6.7572	6.6811	6.6192	6.5245	6.4277	6.3286	6.278	6.227	6.175	6.123	6.069	6.015
6	8.8131	7.2599	6.5988	6.2272	5.9876	5.8198	5.6955	5.5996	5.5234	5.4613	5.3662	5.2687	5.1684	5.1172	5.065	5.012	4.959	4.904	4.849
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.9949	4.8993	4.8232	4.7611	4.6658	4.5678	4.4667	4.415	4.362	4.309	4.254	4.199	4.142
8	7.5709	6.0595	5.416	5.0526	4.8173	4.6517	4.5286	4.4333	4.3572	4.2951	4.1997	4.1012	3.9995	3.9472	3.894	3.84	3.784	3.728	3.67
9	7.2093	5.7147	5.0781	4.7181	4.4844	4.3197	4.197	4.102	4.026	3.9639	3.8682	3.7694	3.6669	3.6142	3.56	3.505	3.449	3.392	3.333
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.9498	3.8549	3.779	3.7168	3.6209	3.5217	3.4185	3.3654	3.311	3.255	3.198	3.14	3.08
11	6.7241	5.2559	4.63	4.2751	4.044	3.8807	3.7586	3.6638	3.5879	3.5257	3.4296	3.3299	3.2261	3.1725	3.118	3.061	3.004	2.944	2.883
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7283	3.6065	3.5118	3.4358	3.3736	3.2773	3.1772	3.0728	3.0187	2.963	2.906	2.848	2.787	2.725
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.4827	3.388	3.312	3.2497	3.1532	3.0527	2.9477	2.8932	2.837	2.78	2.72	2.659	2.595
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.3799	3.2853	3.2093	3.1469	3.0502	2.9493	2.8437	2.7888	2.732	2.674	2.614	2.552	2.487
15	6.1995	4.765	4.1528	3.8043	3.5764	3.4147	3.2934	3.1987	3.1227	3.0602	2.9633	2.8621	2.7559	2.7006	2.644	2.585	2.524	2.461	2.395
16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.2194	3.1248	3.0488	2.9862	2.889	2.7875	2.6808	2.6252	2.568	2.509	2.447	2.383	2.316
17	6.042	4.6189	4.0112	3.6648	3.4379	3.2767	3.1556	3.061	2.9849	2.9222	2.8249	2.723	2.6158	2.5598	2.502	2.442	2.38	2.315	2.247
18	5.9781	4.5597	3.9539	3.6083	3.382	3.2209	3.0999	3.0053	2.9291	2.8664	2.7689	2.6667	2.559	2.5027	2.445	2.384	2.321	2.256	2.187
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	3.0509	2.9563	2.8801	2.8172	2.7196	2.6171	2.5089	2.4523	2.394	2.333	2.27	2.203	2.133
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	3.0074	2.9128	2.8365	2.7737	2.6758	2.5731	2.4645	2.4076	2.349	2.287	2.223	2.156	2.085
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.9686	2.874	2.7977	2.7348	2.6368	2.5338	2.4247	2.3675	2.308	2.246	2.182	2.114	2.042
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.9338	2.8392	2.7628	2.6998	2.6017	2.4984	2.389	2.3315	2.272	2.21	2.145	2.076	2.003
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.9023	2.8077	2.7313	2.6682	2.5699	2.4665	2.3567	2.2989	2.239	2.176	2.111	2.041	1.968
24	5.7166	4.3187	3.7211	3.3794	3.1548	2.9946	2.8738	2.7791	2.7027	2.6396	2.5411	2.4374	2.3273	2.2693	2.209	2.146	2.08	2.01	1.935
25	5.6864	4.2909	3.6943	3.353	3.1287	2.9685	2.8478	2.7531	2.6766	2.6135	2.5149	2.411	2.3005	2.2422	2.182	2.118	2.052	1.981	1.906
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.824	2.7293	2.6528	2.5896	2.4908	2.3867	2.2759	2.2174	2.157	2.093	2.026	1.954	1.878
27	5.6331	4.2421	3.6472	3.3067	3.0828	2.9228	2.8021	2.7074	2.6309	2.5676	2.4688	2.3644	2.2533	2.1946	2.133	2.069	2.002	1.93	1.853
28	5.6096	4.2205	3.6264	3.2863	3.0626	2.9027	2.782	2.6872	2.6106	2.5473	2.4484	2.3438	2.2324	2.1735	2.112	2.048	1.98	1.907	1.829
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.884	2.7633	2.6686	2.5919	2.5286	2.4295	2.3248	2.2131	2.154	2.092	2.028	1.959	1.886	1.807
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.746	2.6513	2.5746	2.5112	2.412	2.3072	2.1952	2.1359	2.074	2.009	1.94	1.866	1.787
40	5.4239	4.051	3.4633	3.1261	2.9037	2.7444	2.6238	2.5289	2.4519	2.3882	2.2882	2.1819	2.0677	2.0069	1.943	1.875	1.803	1.724	1.637
60	5.2856	3.9253	3.3425	3.0077	2.7863	2.6274	2.5068	2.4117	2.3344	2.2702	2.1692	2.0613	1.9445	1.8817	1.815	1.744	1.667	1.581	1.482
120	5.1523	3.8046	3.2269	2.8943	2.674	2.5154	2.3948	2.2994	2.2217	2.157	2.0548	1.945	1.8249	1.7597	1.69	1.614	1.53	1.433	1.31
Inf	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.2875	2.1918	2.1136	2.0483	1.9447	1.8326	1.7085	1.6402	1.566	1.484	1.388	1.268	1

F-Distribution Table (alpha = 0.1) for Critical Value

	DF1	α = 0.10																	
DF2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**15**	**20**	**24**	**30**	**40**	**60**	**120**	**Inf**
1	39.863	49.5	53.593	55.833	57.24	58.204	58.906	59.439	59.858	60.195	60.705	61.22	61.74	62.002	62.265	62.529	62.794	63.061	63.328
2	8.5263	9	9.1618	9.2434	9.2926	9.3255	9.3491	9.3668	9.3805	9.3916	9.4081	9.4247	9.4413	9.4496	9.4579	9.4662	9.4746	9.4829	9.4912
3	5.5383	5.4624	5.3908	5.3426	5.3092	5.2847	5.2662	5.2517	5.24	5.2304	5.2156	5.2003	5.1845	5.1764	5.1681	5.1597	5.1512	5.1425	5.1337
4	4.5448	4.3246	4.1909	4.1073	4.0506	4.0098	3.979	3.9549	3.9357	3.9199	3.8955	3.8704	3.8443	3.831	3.8174	3.8036	3.7896	3.7753	3.7607
5	4.0604	3.7797	3.6195	3.5202	3.453	3.4045	3.3679	3.3393	3.3163	3.2974	3.2682	3.238	3.2067	3.1905	3.1741	3.1573	3.1402	3.1228	3.105
6	3.776	3.4633	3.2888	3.1808	3.1075	3.0546	3.0145	2.983	2.9577	2.9369	2.9047	2.8712	2.8363	2.8183	2.8	2.7812	2.762	2.7423	2.7222
7	3.5894	3.2574	3.0741	2.9605	2.8833	2.8274	2.7849	2.7516	2.7247	2.7025	2.6681	2.6322	2.5947	2.5753	2.5555	2.5351	2.5142	2.4928	2.4708
8	3.4579	3.1131	2.9238	2.8064	2.7265	2.6683	2.6241	2.5894	2.5612	2.538	2.502	2.4642	2.4246	2.4041	2.383	2.3614	2.3391	2.3162	2.2926
9	3.3603	3.0065	2.8129	2.6927	2.6106	2.5509	2.5053	2.4694	2.4403	2.4163	2.3789	2.3396	2.2983	2.2768	2.2547	2.232	2.2085	2.1843	2.1592
10	3.285	2.9245	2.7277	2.6053	2.5216	2.4606	2.414	2.3772	2.3473	2.3226	2.2841	2.2435	2.2007	2.1784	2.1554	2.1317	2.1072	2.0818	2.0554
11	3.2252	2.8595	2.6602	2.5362	2.4512	2.3891	2.3416	2.304	2.2735	2.2482	2.2087	2.1671	2.1231	2.1	2.0762	2.0516	2.0261	1.9997	1.9721
12	3.1766	2.8068	2.6055	2.4801	2.394	2.331	2.2828	2.2446	2.2135	2.1878	2.1474	2.1049	2.0597	2.036	2.0115	1.9861	1.9597	1.9323	1.9036
13	3.1362	2.7632	2.5603	2.4337	2.3467	2.283	2.2341	2.1954	2.1638	2.1376	2.0966	2.0532	2.007	1.9827	1.9576	1.9315	1.9043	1.8759	1.8462
14	3.1022	2.7265	2.5222	2.3947	2.3069	2.2426	2.1931	2.1539	2.122	2.0954	2.0537	2.0095	1.9625	1.9377	1.9119	1.8852	1.8572	1.828	1.7973
15	3.0732	2.6952	2.4898	2.3614	2.273	2.2081	2.1582	2.1185	2.0862	2.0593	2.0171	1.9722	1.9243	1.899	1.8728	1.8454	1.8168	1.7867	1.7551
16	3.0481	2.6682	2.4618	2.3327	2.2438	2.1783	2.128	2.088	2.0553	2.0282	1.9854	1.9399	1.8913	1.8656	1.8388	1.8108	1.7816	1.7508	1.7182
17	3.0262	2.6446	2.4374	2.3078	2.2183	2.1524	2.1017	2.0613	2.0284	2.0009	1.9577	1.9117	1.8624	1.8362	1.809	1.7805	1.7506	1.7191	1.6856
18	3.007	2.624	2.416	2.2858	2.1958	2.1296	2.0785	2.0379	2.0047	1.977	1.9333	1.8868	1.8369	1.8104	1.7827	1.7537	1.7232	1.691	1.6567
19	2.9899	2.6056	2.397	2.2663	2.176	2.1094	2.058	2.0171	1.9836	1.9557	1.9117	1.8647	1.8142	1.7873	1.7592	1.7298	1.6988	1.6659	1.6308
20	2.9747	2.5893	2.3801	2.2489	2.1582	2.0913	2.0397	1.9985	1.9649	1.9367	1.8924	1.8449	1.7938	1.7667	1.7382	1.7083	1.6768	1.6433	1.6074
21	2.961	2.5746	2.3649	2.2333	2.1423	2.0751	2.0233	1.9819	1.948	1.9197	1.875	1.8272	1.7756	1.7481	1.7193	1.689	1.6569	1.6228	1.5862
22	2.9486	2.5613	2.3512	2.2193	2.1279	2.0605	2.0084	1.9668	1.9327	1.9043	1.8593	1.8111	1.759	1.7312	1.7021	1.6714	1.6389	1.6042	1.5668
23	2.9374	2.5493	2.3387	2.2065	2.1149	2.0472	1.9949	1.9531	1.9189	1.8903	1.845	1.7964	1.7439	1.7159	1.6864	1.6554	1.6224	1.5871	1.549
24	2.9271	2.5383	2.3274	2.1949	2.103	2.0351	1.9826	1.9407	1.9063	1.8775	1.8319	1.7831	1.7302	1.7019	1.6721	1.6407	1.6073	1.5715	1.5327
25	2.9177	2.5283	2.317	2.1842	2.0922	2.0241	1.9714	1.9293	1.8947	1.8658	1.82	1.7708	1.7175	1.689	1.659	1.6272	1.5934	1.557	1.5176
26	2.9091	2.5191	2.3075	2.1745	2.0822	2.0139	1.961	1.9188	1.8841	1.855	1.809	1.7596	1.7059	1.6771	1.6468	1.6147	1.5805	1.5437	1.5036
27	2.9012	2.5106	2.2987	2.1655	2.073	2.0045	1.9515	1.9091	1.8743	1.8451	1.7989	1.7492	1.6951	1.6662	1.6356	1.6032	1.5686	1.5313	1.4906
28	2.8939	2.5028	2.2906	2.1571	2.0645	1.9959	1.9427	1.9001	1.8652	1.8359	1.7895	1.7395	1.6852	1.656	1.6252	1.5925	1.5575	1.5198	1.4784
29	2.887	2.4955	2.2831	2.1494	2.0566	1.9878	1.9345	1.8918	1.8568	1.8274	1.7808	1.7306	1.6759	1.6466	1.6155	1.5825	1.5472	1.509	1.467
30	2.8807	2.4887	2.2761	2.1422	2.0493	1.9803	1.9269	1.8841	1.849	1.8195	1.7727	1.7223	1.6673	1.6377	1.6065	1.5732	1.5376	1.4989	1.4564
40	2.8354	2.4404	2.2261	2.091	1.9968	1.9269	1.8725	1.8289	1.7929	1.7627	1.7146	1.6624	1.6052	1.5741	1.5411	1.5056	1.4672	1.4248	1.3769
60	2.7911	2.3933	2.1774	2.041	1.9457	1.8747	1.8194	1.7748	1.738	1.707	1.6574	1.6034	1.5435	1.5107	1.4755	1.4373	1.3952	1.3476	1.2915
120	2.7478	2.3473	2.13	1.9923	1.8959	1.8238	1.7675	1.722	1.6843	1.6524	1.6012	1.545	1.4821	1.4472	1.4094	1.3676	1.3203	1.2646	1.1926
Inf	2.7055	2.3026	2.0838	1.9449	1.8473	1.7741	1.7167	1.6702	1.6315	1.5987	1.5458	1.4871	1.4206	1.3832	1.3419	1.2951	1.24	1.1686	1

Example 2^2 design

- As an example, consider an investigation into the effect of the concentration of the reactant and the amount of the catalyst on the conversion (yield) in a chemical process. The objective of the experiment was to determine if adjustments to either of these two factors would increase the yield. Let the reactant concentration be factor A and let the two levels of interest be **15 and 25 percent**. The catalyst is factor B, with the **high level** denoting the use of **2 pounds** of the catalyst and the **low level** denoting the use of only **1 pound**. The experiment is replicated three times, so there are 12 runs.

Factor *A*	Factor *B*	Treatment Combination	Replicate I	Replicate II	Replicate III	Total
−	−	*A* low, *B* low	28	25	27	80
+	−	*A* high, *B* low	36	32	32	100
−	+	*A* low, *B* high	18	19	23	60
+	+	*A* high, *B* high	31	30	29	90

Example 2^2 design

Factor			Replicate			
A	*B*	Treatment Combination	I	II	III	Total
−	−	*A* low, *B* low	28	25	27	80
+	−	*A* high, *B* low	36	32	32	100
−	+	*A* low, *B* high	18	19	23	60
+	+	*A* high, *B* high	31	30	29	90

$$A = \frac{1}{2n}\{[ab - b] + [a - (1)]\}$$
$$= \frac{1}{2n}[ab + a - b - (1)]$$

$$B = \frac{1}{2n}\{[ab - a] + [b - (1)]\}$$
$$= \frac{1}{2n}[ab + b - a - (1)]$$

$$AB = \frac{1}{2n}\{[ab - b] - [a - (1)]\}$$
$$= \frac{1}{2n}[ab + (1) - a - b]$$

$$A = \frac{1}{2(3)}(90 + 100 - 60 - 80) = 8.33$$

$$B = \frac{1}{2(3)}(90 + 60 - 100 - 80) = -5.00$$

$$AB = \frac{1}{2(3)}(90 + 80 - 100 - 60) = 1.67$$

Example 2^2 design

$$A = \frac{1}{2(3)}(90 + 100 - 60 - 80) = 8.33$$

$$B = \frac{1}{2(3)}(90 + 60 - 100 - 80) = -5.00$$

$$AB = \frac{1}{2(3)}(90 + 80 - 100 - 60) = 1.67$$

The effect of A (reactant concentration) is positive; this suggests that increasing A from the low level (15%) to the high level (25%) will increase the yield. The effect of B (catalyst) is negative; this suggests that increasing the amount of catalyst added to the process will decrease the yield. The interaction effect appears to be small relative to the two main effects.

Example 2^2 design

$$SS_A = \frac{(50)^2}{4(3)} = 208.33$$

$$SS_B = \frac{(-30)^2}{4(3)} = 75.00$$

and

$$SS_{AB} = \frac{(10)^2}{4(3)} = 8.33$$

The total sum of squares is found in the usual way, that is,

$$SS_T = \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{4n}$$

$$SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

For the experiment in Figure 6.1, we obtain

$$\begin{aligned} SS_T &= \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{3} y_{ijk}^2 - \frac{y_{...}^2}{4(3)} \\ &= 9398.00 - 9075.00 = 323.00 \end{aligned}$$

and

$$\begin{aligned} SS_E &= SS_T - SS_A - SS_B - SS_{AB} \\ &= 323.00 - 208.33 - 75.00 - 8.33 \\ &= 31.34 \end{aligned}$$

2^2 factorial designs

The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SS_A	$a-1$	$MS_A = \dfrac{SS_A}{a-1}$	$F_0 = \dfrac{MS_A}{MS_E}$
B treatments	SS_B	$b-1$	$MS_B = \dfrac{SS_B}{b-1}$	$F_0 = \dfrac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = \dfrac{SS_{AB}}{(a-1)(b-1)}$	$F_0 = \dfrac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n-1)$	$MS_E = \dfrac{SS_E}{ab(n-1)}$	
Total	SS_T	$abn-1$		

F value = variance of the group means (Mean Square Between) / mean of the within group variances (Mean Squared Error)

F-Distribution Table (alpha = 0.05) for Critical Value

	DF1	α = 0.05																	
DF2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	Inf
1	161.45	199.5	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.1	251.14	252.2	253.25	254.31
2	18.513	19	19.164	19.247	19.296	19.33	19.353	19.371	19.385	19.396	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.487	19.496
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.572	8.5494	8.5264
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.041	5.9988	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.717	5.6877	5.6581	5.6281
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.365
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.099	4.06	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.866	3.787	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	5.3177	4.459	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10	4.9646	4.1028	3.7083	3.478	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.913	2.845	2.774	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.948	2.8962	2.8536	2.7876	2.7186	2.6464	2.609	2.5705	2.5309	2.4901	2.448	2.4045
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.341	2.2962
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.671	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.463	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	4.494	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	4.4513	3.5915	3.1968	2.9647	2.81	2.6987	2.6143	2.548	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.104	2.0584	2.0107	1.9604
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.308	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.878
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.599	2.514	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.366	2.321	2.2504	2.1757	2.096	2.054	2.0102	1.9645	1.9165	1.8657	1.8117
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.938	1.8894	1.838	1.7831
23	4.2793	3.4221	3.028	2.7955	2.64	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	2.005	1.9605	1.9139	1.8648	1.8128	1.757
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.939	1.892	1.8424	1.7896	1.733
25	4.2417	3.3852	2.9912	2.7587	2.603	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.711
26	4.2252	3.369	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.901	1.8533	1.8027	1.7488	1.6906
27	4.21	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	4.196	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.236	2.19	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	4.183	3.3277	2.934	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	4.0847	3.2317	2.8387	2.606	2.4495	2.3359	2.249	2.1802	2.124	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.097	2.0401	1.9926	1.9174	1.8364	1.748	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.175	2.0868	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.429	1.3519	1.2539
Inf	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.394	1.318	1.2214	1

Example 2^2 design

Factor A: F Critical Value (1,8) = 5.3177
Factor B: F Critical Value (1,8) = 5.3177
Factor AB: F Critical Value (1,8) = 5.3177

■ TABLE 6.1

Analysis of Variance for the Experiment in Figure 6.1

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
A	208.33	1	208.33	53.15	0.0001
B	75.00	1	75.00	19.13	0.0024
AB	8.33	1	8.33	2.13	0.1826
Error	31.34	8	3.92		
Total	323.00	11			

Factor A: F Value > F- Critical Value, P<0.05
Factor B: F Value > F- Critical Value, P<0.05
Factor AB: F Value < F- Critical Value, P>0.05

If F value is greater than the F- Critical Value, then significant difference exists

Example (2) 2^2 design

An article in the AT&T Technical Journal describes the application of **two-level factorial designs** to integrated circuit manufacturing. **A basic processing step in this industry is to grow an epitaxial layer on polished silicon wafers**. The wafers are mounted on a susceptor and positioned inside a bell jar. Chemical vapors are introduced through nozzles near the top of the jar. The susceptor is rotated, and heat is applied.

A deposition time and B arsenic flow rate.

two levels of deposition time are short (-) and long (+)

two levels of arsenic flow rate are 55% (-) and 59%(+)

n=4 replications

Table 14-13 The 2^2 Design for the Epitaxial Process Experiment

Treatment Combination	Design Factors			Thickness (μm)				Thickness (μm)	
	A	B	AB					Total	Average
(1)	−	−	+	14.037	14.165	13.972	13.907	56.081	14.020
a	+	−	−	14.821	14.757	14.843	14.878	59.299	14.825
b	−	+	−	13.880	13.860	14.032	13.914	55.686	13.922
ab	+	+	+	14.888	14.921	14.415	14.932	59.156	14.789

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Example (2) 2^2 design

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b	−	+	−	13.880	13.860	14.032	13.914	55.686	13.922
ab	+	+	+	14.888	14.921	14.415	14.932	59.156	14.789

Example (2) 2^2 design

Effect of A, B, AB..?

$$A = \frac{1}{2n}[a + ab - b - (1)]$$

$$= \frac{1}{2(4)}[59.299 + 59.156 - 55.686 - 56.081] = 0.836$$

$$B = \frac{1}{2n}[b + ab - a - (1)]$$

$$= \frac{1}{2(4)}[55.686 + 59.156 - 59.299 - 56.081] = 0.067$$

$$AB = \frac{1}{2n}[ab + (1) - a - b]$$

$$AB = \frac{1}{2(4)}[59.156 + 56.081 - 59.299 - 55.686] = 0.032$$

SS of A, B, AB..?

$$SS_A = \frac{[a + ab - b - (1)]^2}{16} = \frac{[6.688]^2}{16} = 2.7956$$

$$SS_B = \frac{[b + ab - a - (1)]^2}{16} = \frac{[-0.538]^2}{16} = 0.0181$$

$$SS_{AB} = \frac{[ab + (1) - a - b]^2}{16} = \frac{[0.252]^2}{16} = 0.0040$$

$$SS_T = 14.037^2 + \cdots + 14.932^2 - \frac{(56.081 + \cdots + 59.156)^2}{16}$$

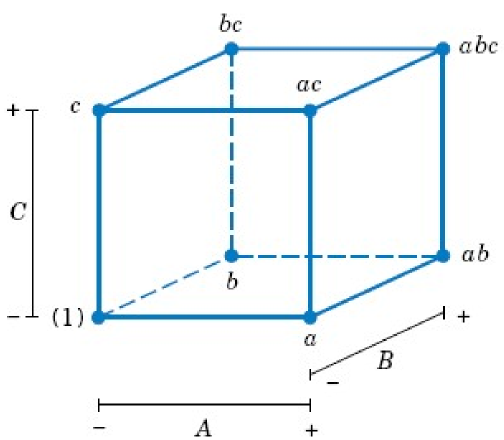
$$= 3.0672$$

Table 14-14 Analysis of Variance for the Epitaxial Process Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	f_0	P-Value
A (deposition time)	2.7956	1	2.7956	134.40	7.07 E-8
B (arsenic flow)	0.0181	1	0.0181	0.87	0.38
AB	0.0040	1	0.0040	0.19	0.67
Error	0.2495	12	0.0208		
Total	3.0672	15			

2^3 factorial design

- The experiment consists of k factors, each factor consists of 2 level (+,-)
- For example k=3;



(a) Geometric view

Run	A	B	C
1	−	−	−
2	+	−	−
3	−	+	−
4	+	+	−
5	−	−	+
6	+	−	+
7	−	+	+
8	+	+	+

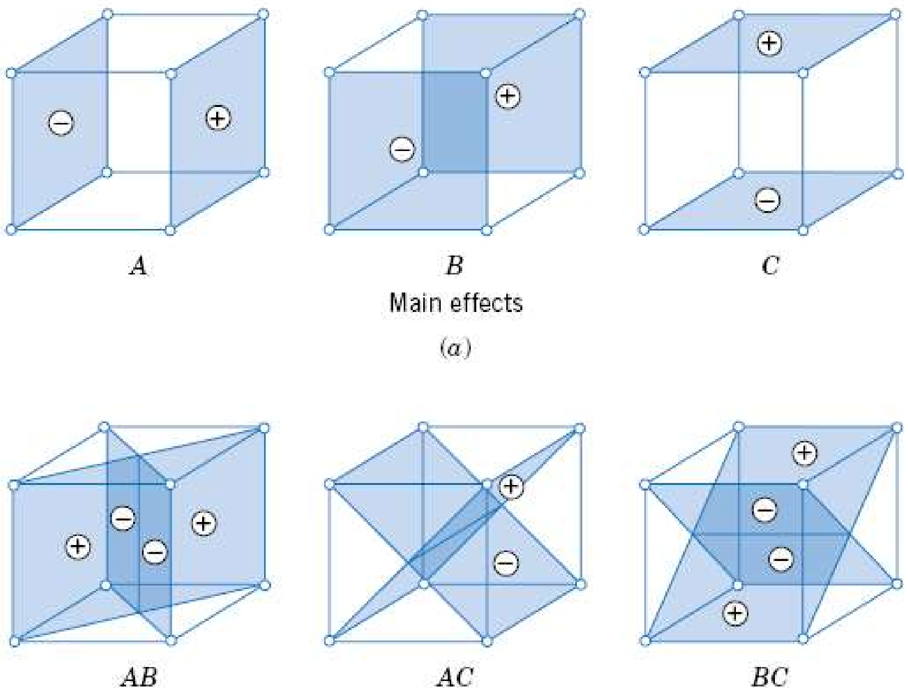
(b) The 2^3 design matrix

Algebraic Signs for Calculating Effects in the 2^3 Design

Treatment Combination	Factorial Effect							
	I	A	B	AB	C	AC	BC	ABC
(1)	+	−	−	+	−	+	+	−
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
ab	+	+	+	+	−	−	−	−
c	+	−	−	+	+	−	−	+
ac	+	+	−	−	+	+	−	−
bc	+	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+	+

2^3 factorial design

- Main and Interaction Effects

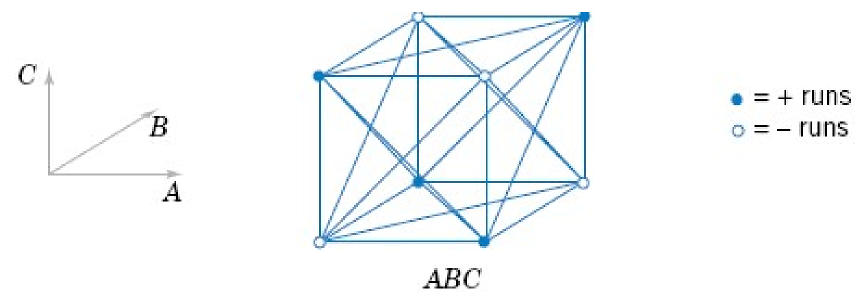


Main effects

(a)

Two-factor interactions

(b)



Three-factor interaction

2^3 factorial design

- Main and Interaction Effects

$$A = \frac{1}{4n}[a + ab + ac + abc - (1) - b - c - bc]$$

$$\begin{aligned} B &= \bar{y}_{B+} - \bar{y}_{B-} \\ &= \frac{1}{4n}[b + ab + bc + abc - (1) - a - c - ac] \end{aligned}$$

$$\begin{aligned} C &= \bar{y}_{C+} - \bar{y}_{C-} \\ &= \frac{1}{4n}[c + ac + bc + abc - (1) - a - b - ab] \end{aligned}$$

$$AB = \frac{1}{4n}[abc - bc + ab - b - ac + c - a + (1)]$$

$$AC = \frac{1}{4n}[(1) - a + b - ab - c + ac - bc + abc]$$

$$BC = \frac{1}{4n}[(1) + a - b - ab - c - ac + bc + abc]$$

$$ABC = \frac{1}{4n}[abc - bc - ac + c - ab + b + a - (1)]$$

The value in the brackets are "Contrast"

$$\text{Effect} = \frac{\text{Contrast}}{n2^{k-1}}$$

$$SS = \frac{(\text{Contrast})^2}{n2^k}$$

2^3 factorial design

Anova table for the 3 factor Experiment

Source	SS	*df*	MS	*F*	*p* -value
A	SS_A	$a - 1$	MS_A	MS_A/MS_{Error}	
B	SS_B	$b - 1$	MS_B	MS_B/MS_{Error}	
C	SS_C	$c - 1$	MS_C	MS_C/MS_{Error}	
AB	SS_{AB}	$(a - 1)(b - 1)$	MS_{AB}	MS_{AB}/MS_{Error}	
AC	SS_{AC}	$(a - 1)(c - 1)$	MS_{AC}	MS_{AC}/MS_{Error}	
BC	SS_{BC}	$(b - 1)(c - 1)$	MS_{BC}	MS_{BC}/MS_{Error}	
ABC	SS_{ABC}	$(a - 1)(b - 1)(c - 1)$	MS_{ABC}	MS_{ABC}/MS_{Error}	
$Error$	SS_{Error}	$abc(n - 1)$	MS_{Error}		

Example: 2^3 Factorial design

A 2^3 factorial design was used to develop a nitride etch process on a single-wafer plasma etching tool. The design factors are the gap between the electrodes, the gas flow (C_2F_6 is used as the reactant gas), and the RF power applied to the cathode (see Figure 3.1 for a schematic of the plasma etch tool). Each factor is run at two levels, and the design is replicated twice. The response variable is the etch rate for silicon nitride (Å/m). The etch rate data are shown in Table 6.4, and the design is shown geometrically in Figure 6.6.

Using the totals under the treatment combinations shown in Table 6.4, we may estimate the factor effects as follows:

$$A = \frac{1}{4n}[a - (1) + ab - b + ac - c + abc - bc]$$

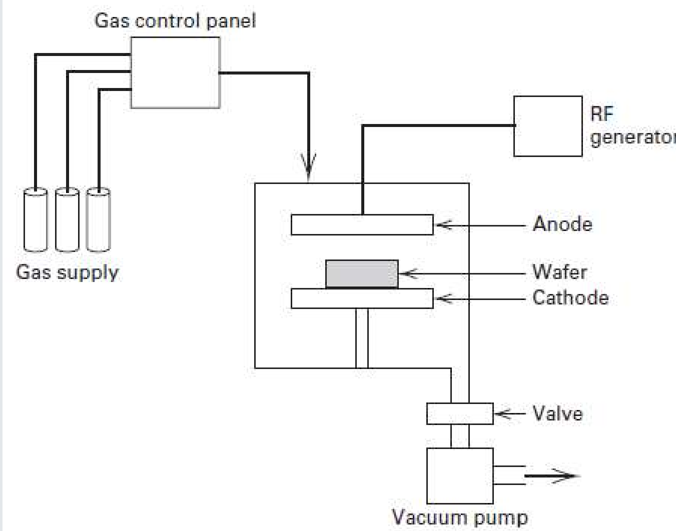
$$= \frac{1}{8}[1319 - 1154 + 1277 - 1234 + 1617 - 2089 + 1589 - 2138]$$

$$= \frac{1}{8}[-813] = -101.625$$

■ TABLE 6.4

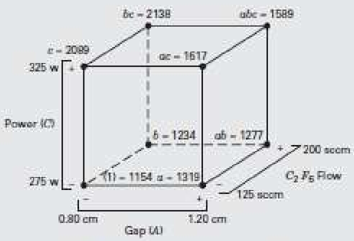
The Plasma Etch Experiment, Example 6.1

	Coded Factors			Etch Rate			Factor Levels		
Run	A	B	C	Replicate 1	Replicate 2	Total		Low (−1)	High (+1)
1	−1	−1	−1	550	604	(1) = 1154	A (Gap, cm)	0.80	1.20
2	1	−1	−1	669	650	a = 1319	B (C_2F_6 flow, SCCM)	125	200
3	−1	1	−1	633	601	b = 1234	C (Power, W)	275	325
4	1	1	−1	642	635	ab = 1277			
5	−1	−1	1	1037	1052	c = 2089			
6	1	−1	1	749	868	ac = 1617			
7	−1	1	1	1075	1063	bc = 2138			
8	1	1	1	729	860	abc = 1589			



■ FIGURE 3.1 A single-wafer plasma etching tool

Example: 2^3 Factorial design



■ **FIGURE 6.6** **The 2^3 design for the plasma etch experiment for Example 6.1**

$$B = \frac{1}{4n}[b + ab + bc + abc - (1) - a - c - ac]$$
$$= \frac{1}{8}[1234 + 1277 + 2138 + 1589 - 1154 - 1319 - 2089 - 1617]$$
$$= \frac{1}{8}[59] = 7.375$$

$$C = \frac{1}{4n}[c + ac + bc + abc - (1) - a - b - ab]$$
$$= \frac{1}{8}[2089 + 1617 + 2138 + 1589 - 1154 - 1319 - 1234 - 1277]$$
$$= \frac{1}{8}[2449] = 306.125$$

$$AB = \frac{1}{4n}[ab - a - b + (1) + abc - bc - ac + c]$$
$$= \frac{1}{8}[1277 - 1319 - 1234 + 1154 + 1589 - 2138 - 1617 + 2089]$$
$$= \frac{1}{8}[-199] = -24.875$$

$$AC = \frac{1}{4n}[(1) - a + b - ab - c + ac - bc + abc]$$
$$= \frac{1}{8}[1154 - 1319 + 1234 - 1277 - 2089 + 1617 - 2138 + 1589]$$
$$= \frac{1}{8}[-1229] = -153.625$$

$$BC = \frac{1}{4n}[(1) + a - b - ab - c - ac + bc + abc]$$
$$= \frac{1}{8}[1154 + 1319 - 1234 - 1277 - 2089 - 1617 + 2138 + 1589]$$
$$= \frac{1}{8}[-17] = -2.125$$

and

$$ABC = \frac{1}{4n}[abc - bc - ac + c - ab + b + a - (1)]$$
$$= \frac{1}{8}[1589 - 2138 - 1617 + 2089 - 1277 + 1234 + 1319 - 1154]$$
$$= \frac{1}{8}[45] = 5.625$$

The largest effects are for power ($C = 306.125$), gap ($A = -101.625$), and the power–gap interaction ($AC = -153.625$).

The sums of squares are calculated from Equation 6.18 as follows:

$$SS_A = \frac{(-813)^2}{16} = 41,310.5625$$
$$SS_B = \frac{(59)^2}{16} = 217.5625$$
$$SS_C = \frac{(2449)^2}{16} = 374,850.0625$$
$$SS_{AB} = \frac{(-199)^2}{16} = 2475.0625$$
$$SS_{AC} = \frac{(-1229)^2}{16} = 94,402.5625$$
$$SS_{BC} = \frac{(-17)^2}{16} = 18.0625$$

and

$$SS_{ABC} = \frac{(45)^2}{16} = 126.5625$$

The total sum of squares is $SS_T = 531,420.9375$ and by subtraction $SS_E = 18,020.50$. Table 6.5 summarizes the effect estimates and sums of squares. The column labeled "percent contribution" measures the percentage contribution of each model term relative to the total sum of squares. The percentage contribution is often a rough but effective guide to the relative importance of each model term. Note that the main effect of C (Power) really dominates this process, accounting for over 70 percent of the total variability, whereas the main effect of A (Gap) and the AC interaction account for about 8 and 18 percent, respectively.

The ANOVA in Table 6.6 may be used to confirm the magnitude of these effects. We note from Table 6.6 that the main effects of Gap and Power are highly significant (both have very small P-values). The AC interaction is also highly significant; thus, there is a strong interaction between Gap and Power.

■ **TABLE 6.5**
Effect Estimate Summary for Example 6.1

Factor	Effect Estimate	Sum of Squares	Percent Contribution
A	−101.625	41,310.5625	7.7736
B	7.375	217.5625	0.0409
C	306.125	374,850.0625	70.5373
AB	−24.875	2475.0625	0.4657
AC	−153.625	94,402.5625	17.7642
BC	−2.125	18.0625	0.0034
ABC	5.625	126.5625	0.0238

■ **TABLE 6.6**
Analysis of Variance for the Plasma Etching Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Gap (A)	41,310.5625	1	41,310.5625	18.34	0.0027
Gas flow (B)	217.5625	1	217.5625	0.10	0.7639
Power (C)	374,850.0625	1	374,850.0625	166.41	0.0001
AB	2475.0625	1	2475.0625	1.10	0.3252
AC	94,402.5625	1	94,402.5625	41.91	0.0002
BC	18.0625	1	18.0625	0.01	0.9308
ABC	126.5625	1	126.5625	0.06	0.8186
Error	18,020.5000	8	2252.5625		
Total	531,420.9375	15			

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Thank you