Set B

SRM Institute of Science and Technology
College of Engineering and Technology
Department of Mechanical Engineering

Academic Year: 2022-23 Even **Semester: VI** **Mark: 50**

Subject Code: 18MEO113T **Title: Design of Experiments**

Duration: 90 min **Type of Test: CLA III**

Q. No	Part A Question
1	A factor with a range of settings, that is controlled by the user during use is called as – a) random factor b) robust factor c) nominal factor d) signal factor
2	In CCD, Experiments +- alpha on the variable axes is used to estimate ____________. a) Main effect b) Interaction effect c) Experimental error d) Quadratic terms
3	The features of Box-Behnken design a) -5 Levels b) -No Corner points c) +Corner points d) -more tests
4	Box-Behnken designs usually have ____ design points than central composite design, thus, they are less expensive. a) Less b) More c) Equal d) Not sufficient
5	In Central Composite Design, for spherical CCD the choice of α =__? a) $(k)^{1/2}$ b) 1 c) 0.5 d) $(n_f)^{1/4}$
6	What is the appropriate statistical test for a factorial design? a) ANOVA b) t-test c) the Modes test d) chi-square

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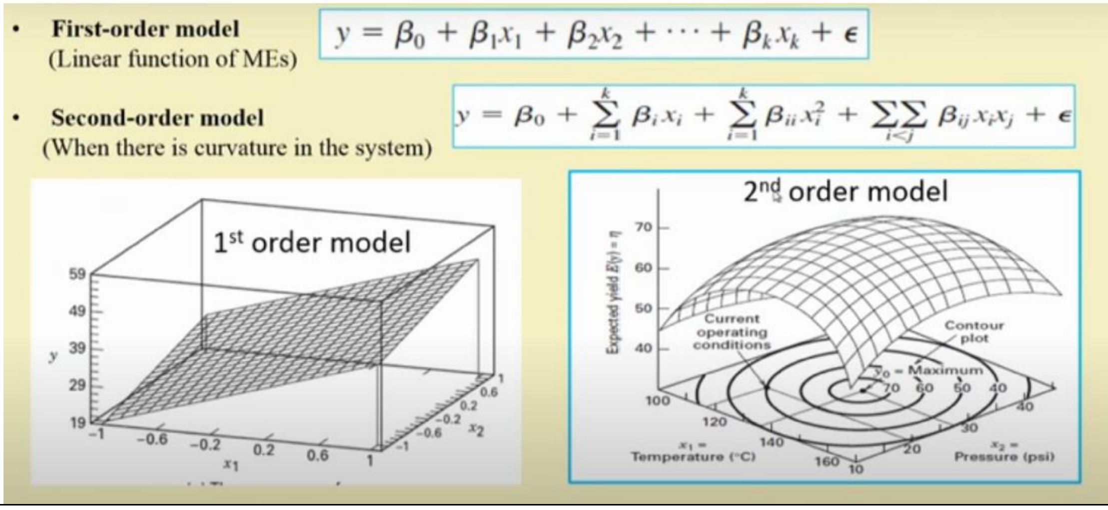
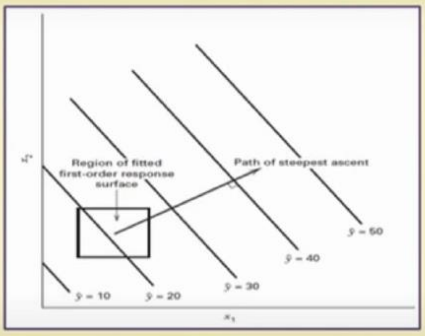
Set B

7	When conducting an ANOVA, F_{DATA} will always be within what range? a) Between negative infinity and infinity b) Between 0 to 1 c) Between 0 to infinity d) Between 1 to infinity
8	An analysis of variance comparing three treatment conditions produces df total = 24. For this ANOVA, what is the value of df Within? a) 2 b) 21 c) 3 d) 22
9	What is the primary purpose of ANOVA? a) To compare means of two or more groups b) To test for a normal distribution c) To evaluate the overall significance of a regression model d) To determine the correlation between two variables
10	In two-way ANOVA with a=5, b=4, then the total degrees of freedom is_____. a) 20 b) 21 c) 19 d) 18
	Part B Question
	Answer any TWO (Unit 4)
11	**Differentiate first-order and second-order model with graph.** 
12	**Discuss about steepest ascent method.**

- **The Method of Steepest Ascent**

Let the fitted first-order model is

$$\hat{y} = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i$$



- **Path of steepest ascent:** The line through the center of the region of interest and normal to the fitted surface

- The steps along the path are proportional to the regression coefficients.
- The actual step size is determined by the experimenter based on process knowledge or other practical considerations.
- Experiments are conducted along the path of steepest ascent until no further increase in response is observed.

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Write three parts in central composite designs.

Three parts:

1.	2.	3.
Factorial or reduced factorial design	At least one center point experiment	Experiments $\pm\alpha$ on the variable axes
Estimate of main effects and interactions	Possible to reveal curvature and estimate the experimental error	Possible to estimate quadratic terms
		$\alpha = \sqrt[4]{N_{FD}}$

Answer any TWO (Unit 5)

14

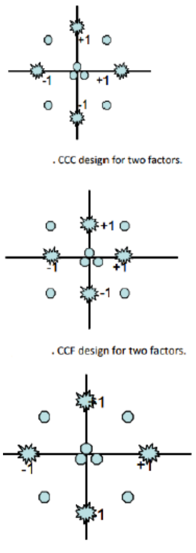
Write the ANOVA techniques.

- ANOVA Technique
  - Obtain the mean of each sample
  - Work out the mean of the sample means
  - Calculate sum of squares for variance between the samples (or SS between)
  - Obtain variance or mean square (MS) between samples
  - Calculate sum of squares for variance within samples (or SS within)
  - Obtain the variance or mean square (MS) within samples
  - Find sum of squares of deviation for total variance
  - Finally, find F-ratio.

15	**Differentiate two way ANOVA with replication and without replication with an example** • Two way ANOVA without replication – We are testing one set of individual before and after they take a medication to see if it works or not. • Two way ANOVA with replication – Two groups, and the members of those groups are doing more than one thing. – For example, two groups of patients from different hospitals trying two different therapies.
16	**Briefly discuss various types of ANOVA.** • ANOVA is two types: – One Way ANOVA: Only one factor is investigated • One independent variable (With 2 levels) • Analysis of Variance could have one independent variable – Two Way ANOVA: Investigate two factors at the same time • Two independent variables (can have multiple levels) • Analysis of Variance could have two independent variables – Two way ANOVA without replication – Two way ANOVA with replication
	Part C (Unit 4)
17	Explain briefly about the central composite designs (CCD) with necessary diagram.

Central Composite designs (CCD)

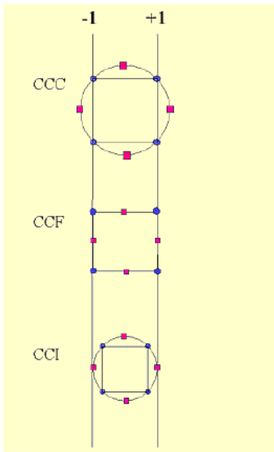
- A central composite design is the most commonly used response surface designed experiment. **Central composite designs are a factorial or fractional factorial design with center points, augmented with a group of axial points (also called star points) that let you estimate curvature.** You can use a central composite design to:
  - Efficiently estimate first- and second-order terms.
  - Model a response variable with curvature by adding center and axial points to a previously-done factorial design.
- Central composite designs are especially useful in sequential experiments because you can often build on previous factorial experiments by adding axial and center points.



. CCC design for two factors.

. CCF design for two factors.

. CCI design for two factors



Comparison of the Three Types of

Central Composite Design (CCD) has three different design points: edge points as in two- level designs (±1), star points at ±α; Iα 1≥I Ithat take care of quadratic effects and centre points,

Three variants exist:

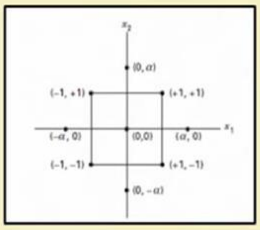
1. Circumscribed (CCC)
2. Face centered (CCF)
3. Inscribed (CCI)

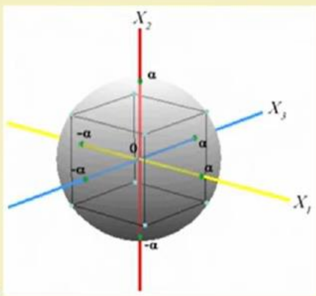
Design for fitting the second-order model

CCD for k=3

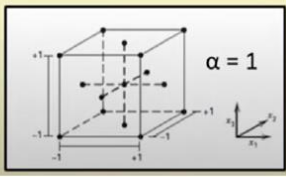
- Central composite design or CCD
- Box-Behnken Design

CCD for k=2



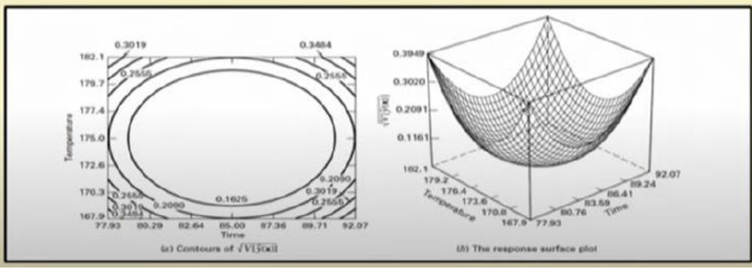


- **CCD** consists of a 2^k factorial (or 2^{k-p} with resolution V) with n_F factorial runs, $2k$ axial or star runs, and n_C center runs
- Choice of α
  - Spherical CCD: α = $(k)^{1/2}$
  - Rotatable CCD: α = $(n_F)^{1/4}$
  - Face-centered CCD: α = 1



CCD (Contd.)

- **Rotatability**: An experimental design is said to be rotatable if the variance of the predicted response at any point is a function of the distance from the centre point alone
- The variance of the predicted response at some point x is $V[\hat{y}(x)] = \sigma^2 x'(X'X)^{-1}x$
- This variance is the same at all points x that are at the same distance from the design centre



OR

18 The table below shows the height, x, in inches and the pulse rate, y, per minute, for 6 people. Find the correlation coefficient and interpret your result.

x	5	7	4	15	12	9
y	8	9	12	26	16	13

Kindly refer **SET C** answer key

Part C (Unit 5)

19 An education researcher is comparing four different algebra curricula. Eighth grade students are randomly assigned to one of the four groups. Their state achievement test scores are compared at the end of the year. Use the appropriate statistical procedure to determine whether the curricula differ with respect to math achievement. An alpha criterion of .05 should be used for the test. $F_{critical}$ (3,196)=2.65

	n	mean	sd
curriculum 1	50	170.5	14.5
curriculum 2	50	168.3	12.8
curriculum 3	50	167.6	17.7
curriculum 4	50	172.8	16.8

Solution

$Var_1 = 14.5^2 = 210.25$

$Var_2 = 12.8^2 = 163.84$

$Var_3 = 17.7^2 = 313.29$

$Var_3 = 16.8^2 = 282.24$

$MS_{error} = \frac{210.25+163.84+313.29}{4} = 242.41$ *Note: this is just the average within-group variance; it is not sensitive to group mean differences!*

Calculating the remaining *error* (or *within*) terms for the ANOVA table:

$df_{error} = 200 - 4 = 196$

$SS_{error} = (242.41)(200-4) = 47511.38$

Intermediate steps in calculating the variance of the sample means:

Grand mean ($\bar{x}_{grand}$) = $\frac{170.5+168.3+167.6}{3} = 169.8$

```
group mean grand mean deviations sq deviations
     170.5      169.8        0.7         0.49
     168.3      169.8       -1.5         2.25
     167.6      169.8       -2.2         4.84
     172.8      169.8        3.0         9.00
```

Sum of squares (SS_{means}) $= 16.58$

$Var_{means} = \frac{16.58}{4-1} = 5.53$

$MS_{between} = (5.53)(50) = 276.33$ *Note: This method of estimating the variance IS sensitive to group mean differences!*

Calculating the remaining *between* (or *group*) terms of the ANOVA table:

$df_{groups} = 4 - 1 = 3$

$SS_{group} = (276.33)(4-1) = 829$

Test statistic and critical value

$F = \frac{276.33}{242.41} = 1.14$

$F_{critical}(3, 196) = 2.65$

$\boxed{\text{Decision: fail to reject H0}}$

ANOVA table

source	SS	df	MS	F
group	829	3	276.33	1.14
error	47511.38	196	242.41	
total	48340.38			

OR

20 Solve using Two-way ANOVA method

Obser vation	A	B	C	D	E	F
1	1200	1000	980	900	750	800
2	1000	1100	700	800	500	700
3	890	650	1100	900	400	350

Solution:

Given problem

Observation	A	B	C	D	E	F
1	1200	1000	980	900	750	800
2	1000	1100	700	800	500	700
3	890	650	1100	900	400	350

	A	B	C	D	E	F	Row total (x_r)
1	1200	1000	980	900	750	800	**5630**
2	1000	1100	700	800	500	700	**4800**
3	890	650	1100	900	400	350	**4290**
Col total (x_c)	**3090**	**2750**	**2780**	**2600**	**1650**	**1850**	**14720**

$$\sum x^2 = 13010000 \rightarrow (A)$$

$$\frac{\sum x_c^2}{r} = \frac{1}{3}\left(3090^2 + 2750^2 + 2780^2 + 2600^2 + 1650^2 + 1850^2\right)$$

$$= \frac{1}{3}(9548100 + 7562500 + 7728400 + 6760000 + 2722500 + 3422500)$$

$$= \frac{1}{3}(37744000)$$

$$= 12581333.3333 \longrightarrow (B)$$

$$\frac{\sum x_r^2}{c} = \frac{1}{6}\left(5630^2 + 4800^2 + 4290^2\right)$$

$$= \frac{1}{6}(31696900 + 23040000 + 18404100)$$

$$= \frac{1}{6}(73141000)$$

$$= 12190166.6667 \longrightarrow (C)$$

$$\frac{(\sum x)^2}{n} = \frac{(14720)^2}{18}$$

$$= \frac{216678400}{18}$$

$$= 12037688.8889 \longrightarrow (D)$$

Sum of squares total

$$SS_T = \sum x^2 - \frac{(\sum x)^2}{n} = (A) - (D)$$

$= 13010000 - 12037688.8889$

$= 972311.1111$

Sum of squares between rows

$$SS_R = \frac{\sum x_r^2}{c} - \frac{(\sum x)^2}{n} = (C) - (D)$$

$= 12190166.6667 - 12037688.8889$

$= 152477.7778$

Sum of squares between columns

$$SS_C = \frac{\sum x_c^2}{r} - \frac{(\sum x)^2}{n} = (B) - (D)$$

$= 12581333.3333 - 12037688.8889$

$= 543644.4444$

Sum of squares Error (residual)

$$SS_E = SS_T - SS_R - SS_C$$

$= 972311.1111 - 152477.7778 - 543644.4444$

$= 276188.8889$

ANOVA table

Source of Variation	Sums of Squares SS	Degrees of freedom DF	Mean Squares MS	F
Between rows	$SS_R = 152477.7778$	$r - 1 = 2$	$MS_R = \frac{152477.7778}{2} = 76238.8889$	$\frac{76238.8889}{27618.8889} = 2.7604$
Between columns	$SS_C = 543644.4444$	$c - 1 = 5$	$MS_C = \frac{543644.4444}{5} = 108728.8889$	$\frac{108728.8889}{27618.8889} = 3.9368$
Error (residual)	$SS_E = 276188.8889$	$(r - 1)(c - 1) = 10$	$MS_E = \frac{276188.8889}{10} = 27618.8889$	
Total	$SS_T = 972311.1111$	$rc - 1 = 17$		