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Subject Code

18MEO113T - Design of Experiments

Handled by

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Disclaimer

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18MEO113T – Design of Experiments

- To learn the fundamentals of design of experiment techniques.
- To familiarize in how to setup experiments and accomplish all analyze tasks using software packages like Minitab, etc.
- After this course, You will be ready to apply the technique confidently to all of your projects .

Unit 3

1: Introduction to Robust design, Loss functions.

2: Eight steps in Taguchi methodology

3: Orthogonal array, Selecting the interaction, Linear graphs

4: S/N ratio: Larger-the-better, Smaller-the-better, Nominal-the-best

5: Analyze the data, factor effect diagram

6: Levels of parameters

7: Confirmation test

8: Augmented design with simple case

9: Solving Case studies on robust design with statistics software

Introduction

- **Robust Design method**, also called the **Taguchi Method**, pioneered by **Dr. Genichi Taguchi**, greatly improves engineering productivity.
- **Genichi Taguchi** (1 January 1924 – 2 June 2012) was a statistician and engineer. He obtained his Ph.D. in Statistics and Mathematics Science at Kyushu University University, Japan in 1962.

Introduction

- **Genichi Taguchi** Quotes:

“Cost is more important than quality but quality is the best way to reduce cost.”

“A scientific or technical study always consists of the following three steps:

1. One decides the objective.
2. One considers the method.
3. One evaluates the method in relation to the objective.”

Taguchi Method is quite different:

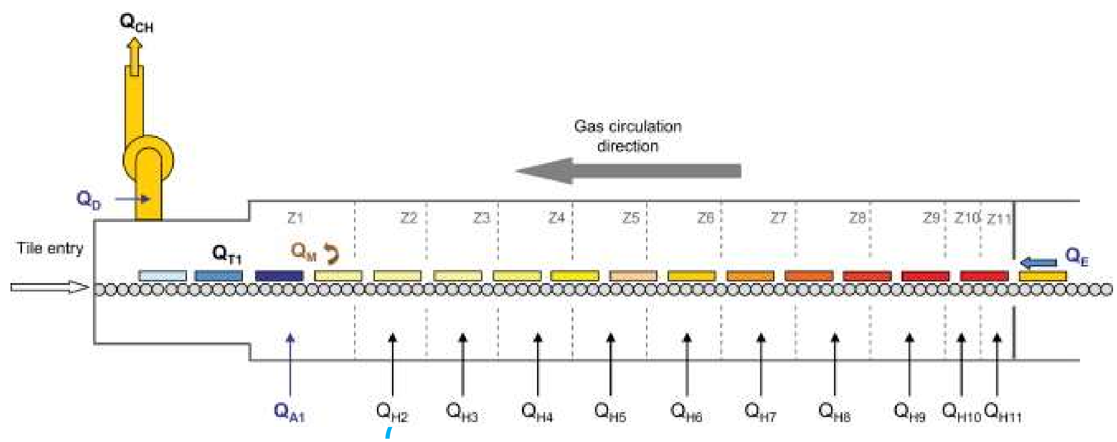
For examples, if seven variables can each be set at two levels, then the total number of experiments possible is given by Level of the power of factors, that is (2^7) = 128.

- It allows us to vary all the factors at a time and still let us evaluate the effect of each individual factor. Also, it allows us to do the limited number of experiments and then provides a method for predicting the other untried combinations.
- For examples, in the cited example, Taguchi method suggests doing only 8 experiments to understand the 128 combinations.

Taguchi Method is quite different:

A well-known example of Taguchi designs is from the **Ina Tile Company** of Japan in the 1950s. The company was manufacturing too many tiles outside specified dimensions.

A quality team discovered that the **temperature** in the kiln used to bake the tiles varied, causing nonuniform tile dimension. They **could not** eliminate the temperature variation because building a new kiln was too costly. Thus, **temperature was a noise factor**. Using Taguchi designed experiments, the team found that by increasing the clay's lime content, a control factor, the tiles became more resistant, or robust, to the temperature variation in the kiln, letting them manufacture more uniform tiles.

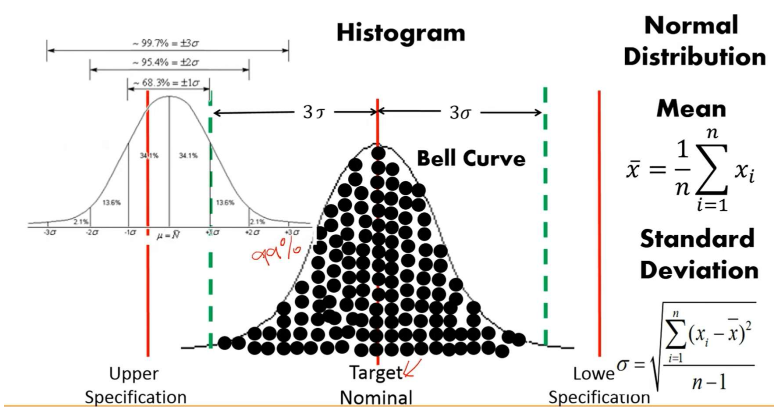


Taguchi Method is quite different:

- In Taguchi designs, **a measure of robustness** used to identify control factors that reduce variability in a product or process by minimizing the effects of uncontrollable factors (noise factors).
- In a Taguchi designed experiment, you **manipulate noise factors** to force variability to occur and from the results, **identify optimal control factor settings** that make the process or product robust, or resistant to variation from the noise factors.

Background of the Taguchi Method

- **Robust Design method (or) Taguchi Method**, pioneered by Dr. Genichi Taguchi, in 1980.
  - Comparable to importance to **Statistical Process Control** (SPC), the **Deming approach** and the **Japanese concept of TQC**.



Total Quality Management (TQM)

Deming's 14 Point Plan for TQM

1	Create constancy of purpose	8	Drive out fear
2	Adopt the new philosophy	9	Eliminate boundaries
3	Cease inspection, require evidence	10	Eliminate the use of slogans
4	Improve the quality of supplies	11	Eliminate numerical standards
5	Continuously improve production	12	Let people be proud of their work
6	Train and educate all employees	13	Encourage self-improvement
7	Supervisors must help people	14	Commit to ever-improving quality

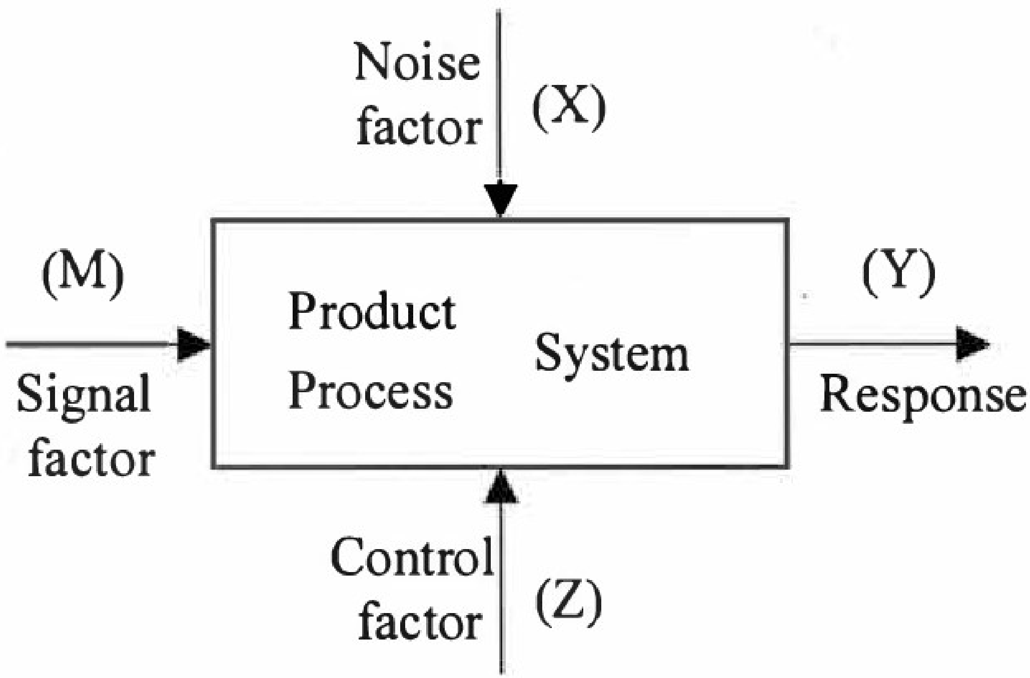
Coach 1000advices.com

TQM expresses about continuous improvement in the processes while QC is about maintaining the quality standards throughout the process.

QC is used to verify the quality of the output; TQM is the process of managing for quality.

Taguchi Terminology

- A schematic representation of a product / process is shown in a figure. **The response is denoted by y and is called quality characteristics.**



Taguchi Terminology

- The parameters that influence the quality characteristic are classified as follows:
- **Signal factor (M):** These are the parameters set by the user or operator of the product **to express the intended value for the response** of the product based on the knowledge of the product being developed. Sometimes 2 or more signal factors may also be used in combination to express the desired response.

Taguchi Terminology

- **Noise factor (x):** Certain parameters are not controllable by the designer and are known as noise factors. Parameters whose settings (also called levels) are difficult to control in the field or whose levels are expensive to control are also considered noise factors. The levels of the noise factors change from one unit to another, from one environment to another, and from time to time. **The noise factors cause the response y to deviate from the target specified by the signal factor M and lead to quality loss.**
- **Control factor (z):** These are parameters that can be specified by the designer. **Control factor can take multiple values, called levels.**

Control or Noise Factor (Example)

- Most automotive sub-assemblies like the alternator, the ignition coil, and the electronic control module must undergo testing to determine if they are resistant to salt water that may be splashed on them from the road.
- An automotive supplier is testing an ignition coil to determine if it will withstand salt water. The following factors are tested :

Control or Noise Factor?

Factor	Low Level	High Level	Factor Type
Housing Material	Polyethylene ($9/lb)	Polypropylen ($20/lb)	C
Concentration Of Salt	10%	20%	N
Water Temperature	5ºC	15ºC	N
Water Pressure	10 PSI	20 PSI	N
Seal Material	Silicone	Vinyl	C
Seal Thickness	0.02”	0.03”	C
Exposure Time	1 hr.	5 hrs.	N
Type of Salt	Detroit Blue	Chicago Pink	N

Background of the Taguchi Method

- **Unique aspects of the Taguchi Method**
  - The Taguchi definition of quality
  - The concept of Robust Design
  - The Taguchi Quality Loss Function (QLF)

Background of the Taguchi Method

1. The Taguchi definition of quality

   – Ideal quality refers to a target value for determining the quality level

   – Ideal quality is delivered if a product or service tangible performs its intended function throughout its projected life under reasonable operating conditions without harmful side effects

   – Ideal quality is a function of customer perception and satisfaction

   – Service quality is measured in terms of loss to society

What is robust design?

2. The concept of Robust Design

- **Robust design** is an “engineering methodology for improving productivity during research and development so that high-quality products can be produced.
- Robust Design is a technique that **reduces variation** in a product by **reducing the sensitivity of the design of the product to sources of variation rather than** by controlling their sources. The end result is a robust design, a design that has **minimum sensitivity** to variations in uncontrollable factors

What is robust design?

- What is Robust?
  - A system (product or process) is robust if it performs properly in a wide range of conditions.

Robust Products	Products that are not robust
A pen that writes until the ink is empty	Pen that stops writing after a few months
A car that starts at -20 deg	A car that does not start
A vacuum cleaner that maintains suction levels	A vacuum cleaner that loses suction

What is robust design?

- What is Robust?
  - In the design of a new product, any design activity can be called robust, if it leads the product;

1. To have longer life (higher reliability)
2. To be more consistent with use
3. To be more consistent from product to product
4. To perform consistently as temperature and other conditions change

What is robust design?

- Products and services should be designed to be inherently defect free and high quality
  - Meet customer's expectations also under non-ideal conditions
- Disturbances are events that cause the design performance to deviate from its target value
- Taguchi divide disturbances into three categories
  - **External disturbances**: variation in the environment where the product is used
  - **Internal disturbances**: wear and tear inside a specific unit
  - **Disturbances in the production process:** deviation from target values

What is robust design?

- A three step method for achieving robust design (Taguchi)
  1. **Concept design**
  2. **Parameter design**
  3. **Tolerance design**

1. Concept Design / System Design

- – The process of examining competing technologies for producing a product – Includes choices of technology and process design.
- – A prototype design that can be produced and meets customers needs under ideal conditions without disturbances

What is robust design?

2. Parameter Design

- The selection of control factors (parameters) and their optimal levels
  - The objective is to make the design Robust!
- Control factors are those process variables in a process that management can influence
  - Ex. The procedures used and the type and amount of training
  - Often a complex (non-linear) relationship between the control factors and product / design performance
- The optimal parameter levels can be determined through experimentation.

What is robust design?

3. Tolerance Design

- Tolerance design deals with developing specification limits.
    - Necessary because there will always be some variation in the production process
    - **Taguchi fiercely advocates aiming for the target value not just settle for inside the specification limits.**
- Tolerance design occurs after parameter design has been used to reduce variation and the resulting improvement has been insufficient.
- Often results in increased production costs
    - More expensive input material might have to be used to meet specifications.
- Interchangeability.

What is robust design?

- What is interchangeability?
  - For example, assembly of a shaft and a part with a hole.
  - The two mating parts are produced in bulk, say 1000 each. By interchangeable assembly any shaft chosen randomly should assemble with any part with a hole selected at random, providing the desired fit.

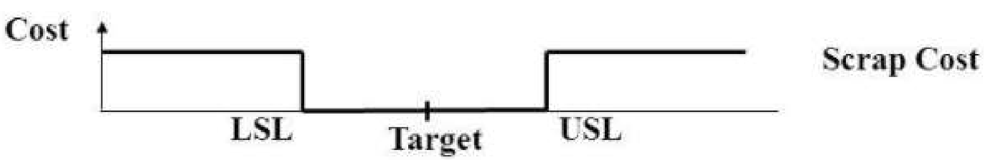
<hole>

<Shaft>

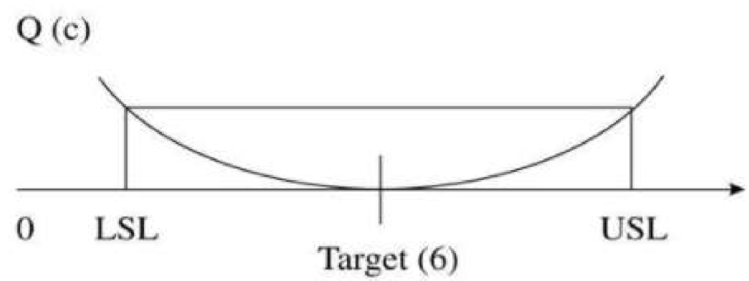
Background of the Taguchi Method

3. Taguchi Quality/Quadratic Loss Function (QLF)

- Taguchi defines Quality as "the loss imparted to the society by the product from the time the product is designed to the time it is shipped to the customer"
- LOSS = Cost to operate, Failure to function, maintenance and repair cost, customer satisfaction, poor design.

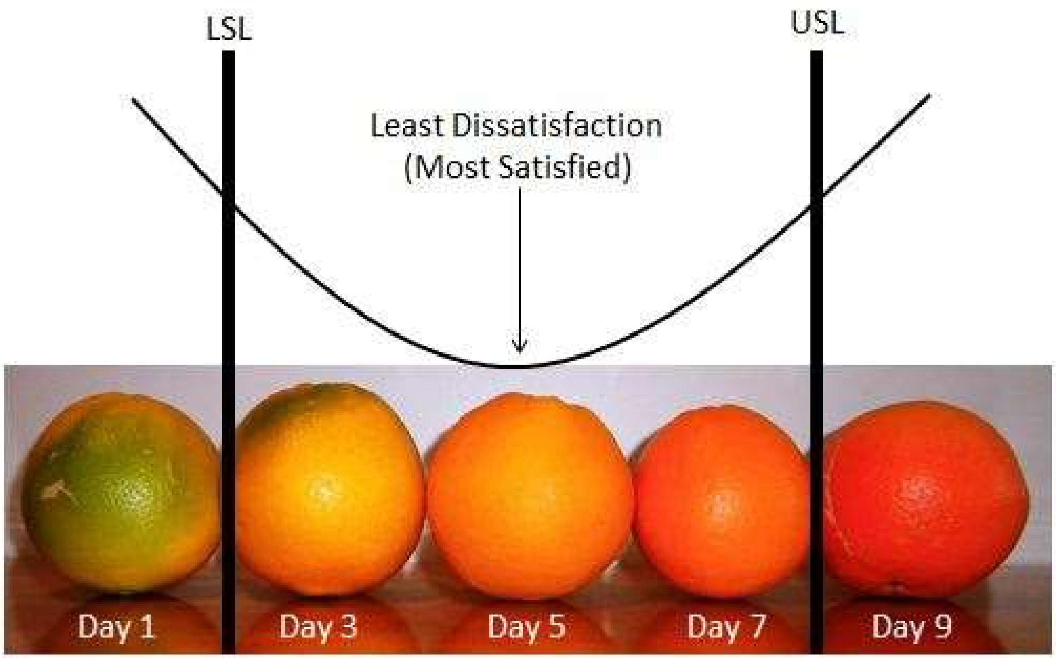


<Traditional approach>



<Taguchi approach>

The Taguchi Quality/ Quadratic Loss Function (QLF)

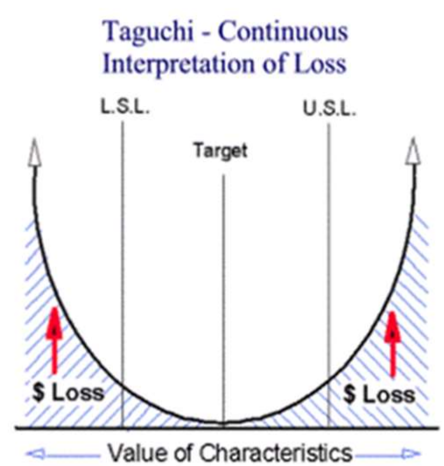


<Taguchi approach>

Figure 5

<Traditional approach>

Figure 6



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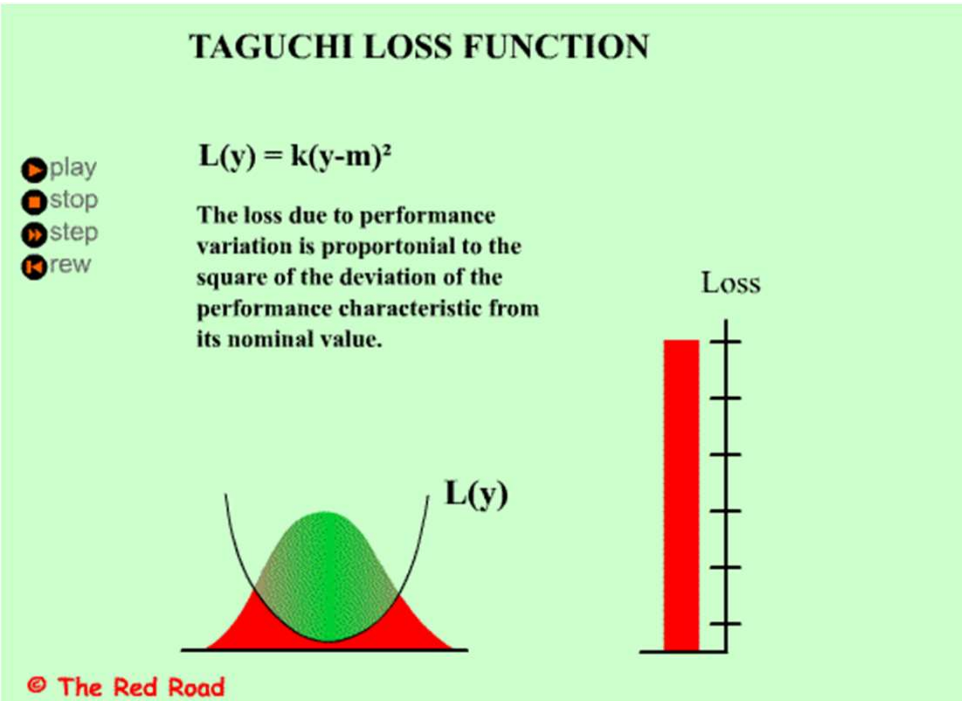
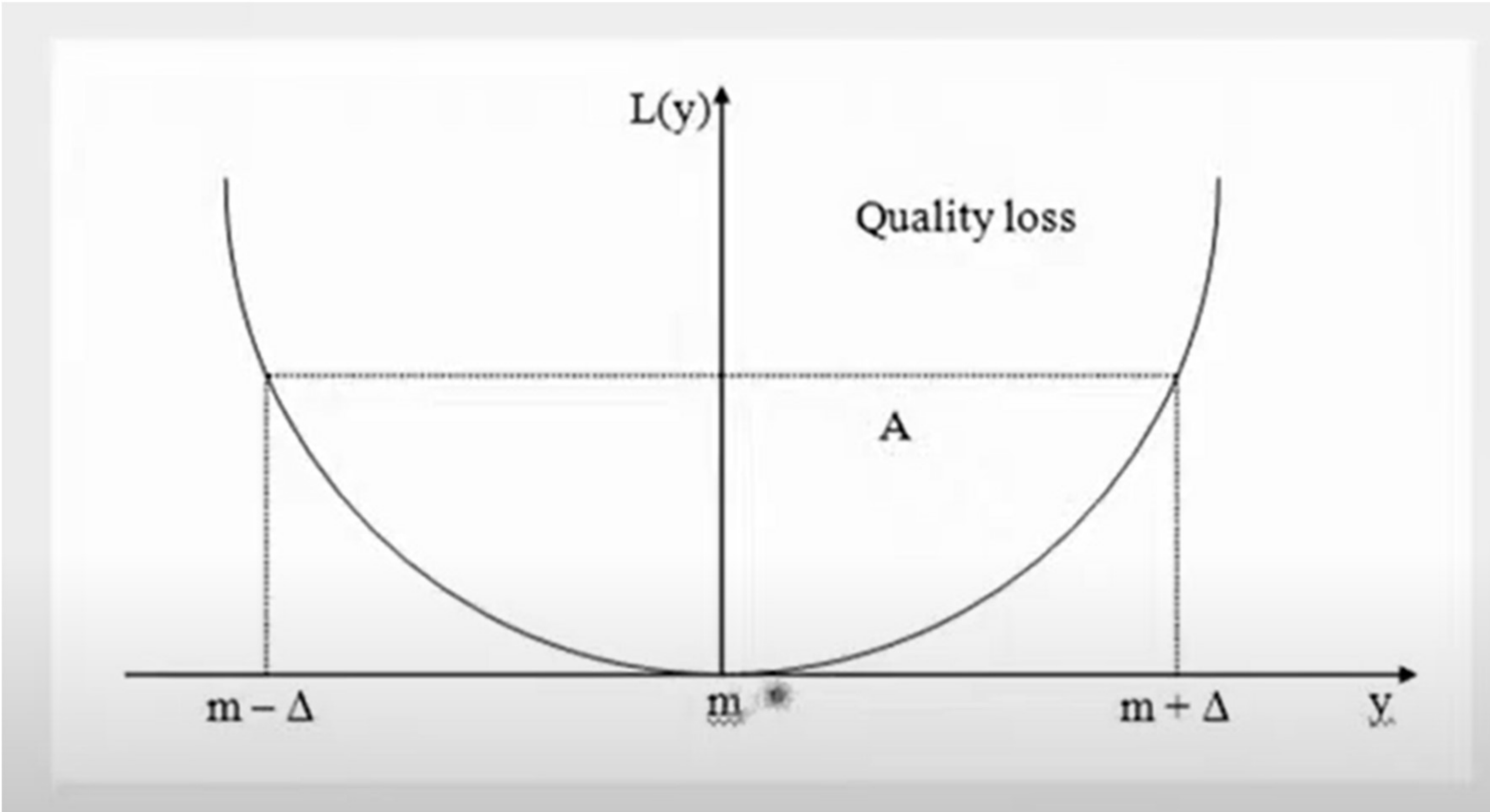
The Taguchi Quality/Quadratic Loss Function (QLF)



Where y is the critical performance parameter value,
L is the loss associated with a particular parameter y,
m is the nominal value of the parameter specification (ie. Target value of y),
k is a constant that depends on the cost at the specification limits. (ie. Quality loss coefficient)

The Taguchi Quality Loss Function (QLF)



The Taguchi Quality Loss Function (QLF)

- The **quality loss function** is given as

$$L(y) = k(y-m)^2 \qquad (1)$$

where

- **L(y)** = loss in dollars when the quality characteristics is equal to *y*. It is the average quality loss incurred by the customers.
- ***y*** = the value of the quality characteristic (i.e. length, width, concentration, surface finish, flatness, etc.)
- ***m*** = target value of y
- ***k*** = constant called *quality loss coefficient*

- The loss is proportional to square of the deviation of y from the target value.
- k is proportionality constant.

The Taguchi Quality Loss Function (QLF)

- A convenient way to determine k is to determine first the ***functional limits* for the value of *y***. Functional limit is the value of y at which the product would fail in half of the applications. **Let $m \pm \Delta$ be the functional limits**.
- Suppose, the **loss at $m \pm \Delta$ is A.** Then by substitution in Eq. (1), we have

$$k = \frac{A}{\Delta^2} \tag{2}$$

where A is the cost of repair or replacement of the product and includes the loss due to the unavailability of the product during the repair period.

- Substituting Eq. (2) in Eq. (1) we get

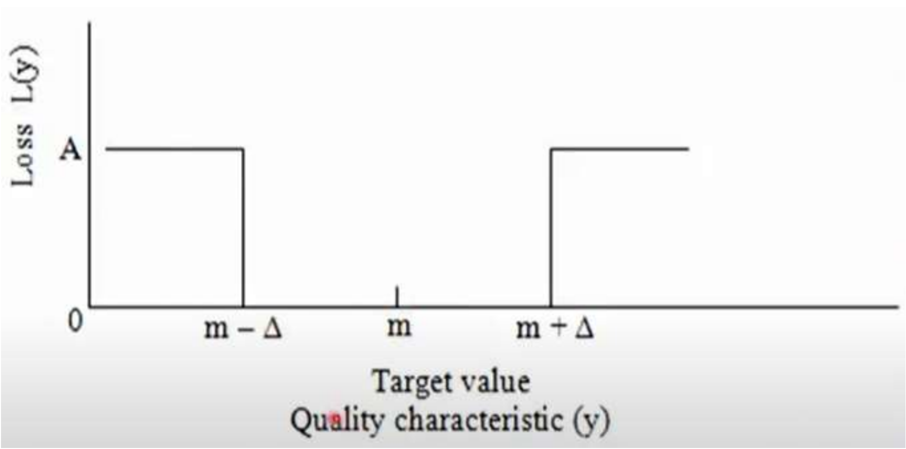
$$L(y) = \frac{A}{\Delta^2}(y - m)^2 \tag{3}$$

Quality Evaluations

- According to the **traditional concept of loss function that as long as the product's quality characteristics is within the specification limits or tolerances, there are no losses incurred.** The lose takes the form of a step function with a constant value **A** outside of the specification limits or tolerances.
- **Taguchi defines** quality as "**the total loss imparted to society from the time the product is designed to the time product is shipped to the customer**."

Quality Evaluations

- Loss function L(y) makes no distinction between products
  - Whose quality characteristic is exactly on target at **m**, or
  - Whose quality characteristic is just above lower specification limit **(LSL),** or
  - Whose quality characteristics is just below the upper specification limit **(USL)**



Quality Evaluations

- The conventional method of computing the cost of quality is based on the number of parts rejected and reworked in a production environment. This method of quality evaluation is incapable of distinguishing between two samples, both within the specification limits but with different distributions of targeted properties.

UAL and LAL in Figure 2-1 represent upper and lower acceptable limits of a design parameter, respectively. Normally the product is functionally acceptable if the value of the specified parameter is within the range between the UAL and LAL limits. No societal loss is assumed to occur; the product is shipped to the consumer. However, outside these limits, as shown by the crosshatched region, 100% functional deterioration occurs, and the product is either discarded, reworked, or subjected to salvage operations. Every attempt is made to control the manufacturing process to maintain the product within the acceptable limits.

However, according to Taguchi, there is no sharp cutoff in the real world on situations just before and beyond the LAL and UAL points. Typically, performance begins to gradually deteriorate as the design parameter deviates from its optimum value. Therefore, Taguchi proposed that the loss function be measured by the deviation from the ideal value. This function is continuous, as shown by the dotted line in Figure 2-1. Product performance begins to suffer when the design parameters deviate from the ideal or the target value. Taguchi's definition clearly puts more emphasis on concerned with the producer. Optimum customer satisfaction can be achieved by developing the products that meet the target value on a consistent basis. It may be worthwhile to mention that Taguchi allows for more than 100% loss imparted by a product. Such cases can occur when a subsystem results in a failure of the entire system or when a system fails catastrophically. Thus, the single most important aspect of Taguchi's quality control philosophy is the minimization of variation around the target value.

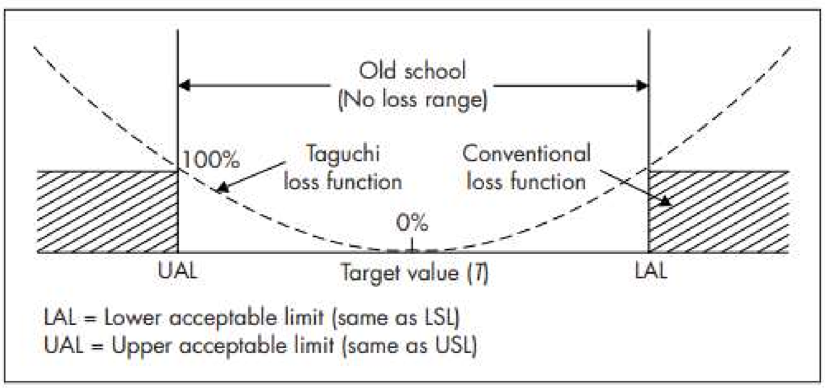


Figure 2-1. Taguchi and conventional loss functions

Taguchi's definition clearly puts more emphasis on customer satisfaction, whereas previously all definitions were concerned with the producer. Optimum customer satisfaction can be achieved by developing the products that meet the target value on a consistent basis.

https://brharnetc.edu.in/br/wp-content/uploads/2018/11/11.pdf

Quality Evaluations

- It is also doubtful that the loss function L(y) remains **constant** at a value of **A** beyond the specification limits **USL and LSL**.
- Since the Taguchi loss function increases quadratically with increasing deviation from the target value, the **Taguchi loss function can be applied to three different situations or cases, namely**
  - **Target is best (or) Nominal is best**
  - **Smaller is better**
  - **Larger is better**

Quality Evaluations

Larger is better

Suppose that we are investigating a pump to determine the best design parameters that produce the maximum flow rate. In this case, the quality of the design may be judged by the flow rate, measured in units of cubic feet per minute, which therefore will be of the characteristic "bigger is better."

Target is best (or) Nominal is best

When the object or process under study has a target value, as for a battery of 9.0 volts or a process to machine a cylinder with a 3.00 inch inside diameter, the measure of quality will possess the "nominal is best" characteristic.

Smaller is better

If, on the other hand, the purpose of the study is to determine the least noisy pump, the noise measured in units of, say, decibels, will be of the type described by "smaller is better." When the

Ranjit K . Roy, A Primer on the Taguchi Method, Second

Quality Evaluations

Target is best or The Nominal-the-Best Case

Taguchi's loss function is given by

$$L(y) = k(y-m)^2 \quad (1)$$

where **k** = a proportionality constant called *the quality loss coefficient*

m = the target value of y

y = the value of the quality characteristic

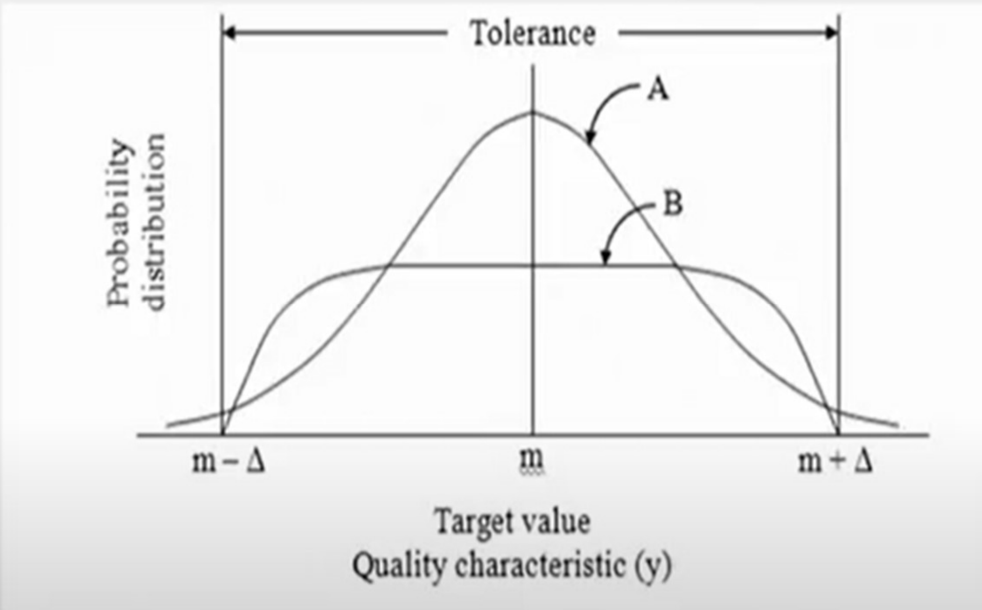
L(y) = loss in dollars when the quality characteristic is equal to y

Note that L(y) = 0 when y = m.

Quality Evaluations

Target is best or The Nominal-the-Best Case

- **It should be noted here that the form of the distribution of the quality characteristics also has an influence on the expected loss.** Two typical probability distributions of a quality characteristics are shown in Figure, where distribution A is a normal distribution and distribution B is a uniform distribution.



Quality Evaluations

- **Example Sony TV**

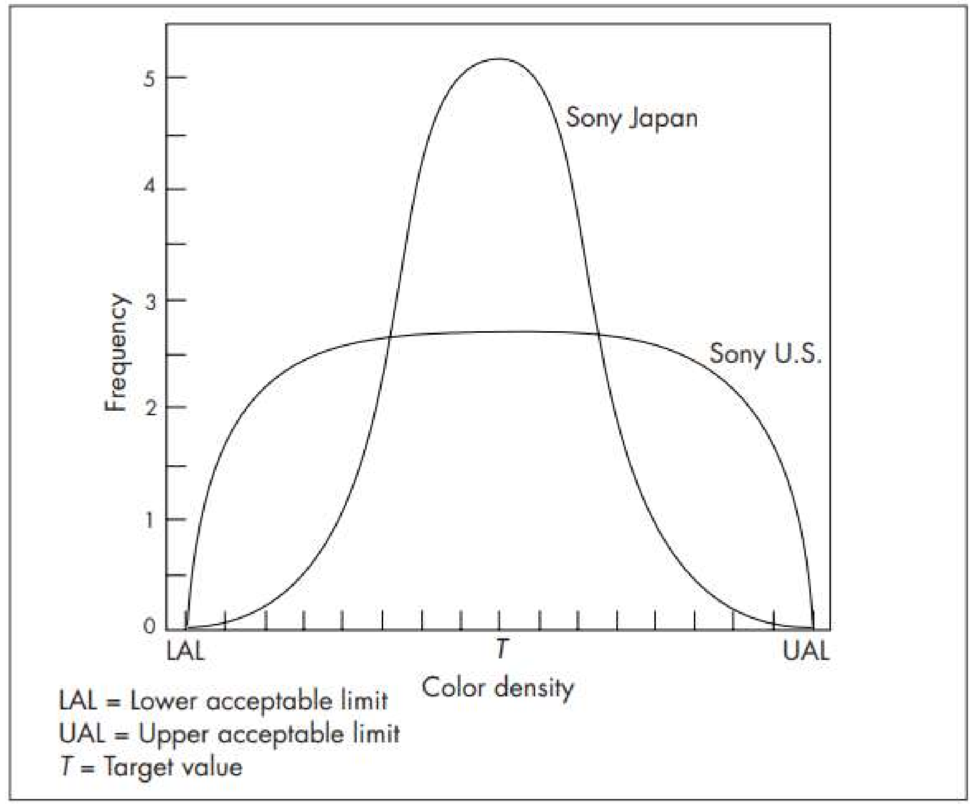


Figure 2-2. Color density distributions

https://brharnetc.edu.in/br/wp-content/uploads/2018/11/11.pdf

Quality Evaluations (S/N Ratio)

- Taguchi experiments **often** use a **2-step optimization process**.
- **In step 1,** use the signal-to-noise ratio to identify those control factors that reduce variability.
- **In step 2,** identify control factors that move the mean to target and have a small or no effect on the signal-to-noise ratio.

Quality Evaluations (S/N Ratio)

- Higher values of the **signal-to-noise ratio (S/N)** identify control factor settings that **minimize the effects of the noise factors**.
- The **signal-to-noise ratio** measures **how the response varies relative to the nominal or target value under different noise conditions**. You can choose from different signal-to-noise ratios, depending on the goal of your experiment.
- The important contribution of Taguchi is proposing the signal-to-noise (S/N) ratio. It was developed as a **proactive equivalent** to the **reactive loss function**.

Quality Evaluations (S/N Ratio)

$$S/N = 10\log\left(\frac{\mu^2}{\sigma^2}\right)$$

Mean

Standard Deviation or Variation

https://www.slideshare.net/rbalisnomo/Introduction-To-Taguchi-Method-05Sep08

Signal-to-Noise Ratio

- **The quality of a product or a process is greatly affected by two classes of input parameters:**
  - **design factors**
  - **disturbance factors**
- **The designer can specify the design factors or parameters rather freely while the disturbance factors are the parameters that are uncontrollable or not practical to control.**
- The **input and output parameter variability is classified into four categories**. As mentioned earlier, the term signal, or the average value of the quality characteristic, represents the desirable component, which will preferably close to a specified target value.
- **Noise represents the undesirable component and it is a measure of the variability of the output characteristic,** which should preferably be as small as possible.

Signal-to-Noise Ratio

- **Four categories of the variability of the input and output parameters are:**

Input Variability

1. Tolerances

   Tolerances are defined as the limit at which some economically measurable action is taken. Tolerances are the normal variability in design factors. The customer tolerance corresponds to the point at which a significant number of customers take economic action because of off-target performance.

2. Inner noise

   Inner noise is a result of **variation due to the deterioration of parts and material. Examples include wear out of parts due to friction, and increase in resistance of resistors with age**. Thus, inner noise is a long term change in product characteristics over time due to deterioration and wear.

Signal-to-Noise Ratio

Output Variability

3. Variational noise

Variational noise is the short term unit-to-unit variation due to the manufacturing process.

4. Outer noise

Outer noise corresponds to the variability of the disturbance factors that contribute to output variability. Environmentally related noise factors, which affect variation within a product (temperature, humidity, operators, etc.). The variables external to a product that affect the product performance are known as external noise factors. **Examples include variations in temperature, humidity, and dust.**

Signal-to-Noise Ratio

- Taguchi has combined the **two components signal and noise into one measure known as the signal-to-noise (s/N) ratio**.
- The **signal-to-noise metric is designed to be used to optimize the robustness of a product or process**.
- The **performance measure combines both the parameters, the mean and the variation around the mean**. Hence, when the performance measure is maximized, the expected loss will be minimized.
- **Taguchi uses the signal-to-noise ratio, s/N**, as the objective function to be optimized. Here, we consider three situations:
  - **Target is best or the nominal-the-best**
  - **Smaller is better**
  - **Large is better**

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Signal-to-Noise Ratio

Smaller-the better

$$S/N = -10\log\left[\frac{1}{n}\sum_{i=1}^{n} y_i^2\right] \quad (1)$$

where y_i denote the n observations of response variable

Larger-the-better

$$S/N = -10\log\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right] \quad (2)$$

Nominal-the-better

$$S/N = 10\log\frac{\mu^2}{\sigma^2} \quad (3)$$

where μ_i^2 denotes the square of mean and s^2 the variance of the observations of the replicated response values.

S/N Ratio

Smaller-is-better (variance of response):

This S/N ratio assumes that the target for the response is zero and is appropriate when specifications indicate an upper tolerance limit only.

$$S/N_S = -10\log(\frac{1}{n}\sum y_i^2)$$

The goal of an experiment for smaller-is-better situations is to minimize $\sum y_i^2$ and y. That is: we aim to maximize $-10\log(\frac{1}{n}\sum y_i^2)$.

Larger-is-better (mean of response):

This S/N ratio assumes that the goal is to maximize the response and is appropriate when specifications indicate a lower tolerance limit only.

$$S/N_L = -10\log\left(\frac{1}{n}\sum \frac{1}{y_i^2}\right)$$

Again, the goal of an experiment for larger-is-better situations is to maximize the response (e.g., yield of a process). But maximizing y is the same as minimizing 1/y. This means that we aim to maximize $-10\log\left(\frac{1}{n}\sum \frac{1}{y_i^2}\right)$.

Target-value-is-best (ratio of mean to variance of response):

This S/N ratio assumes that the given target is best and is appropriate when there is a target value with both upper and lower tolerance limits.

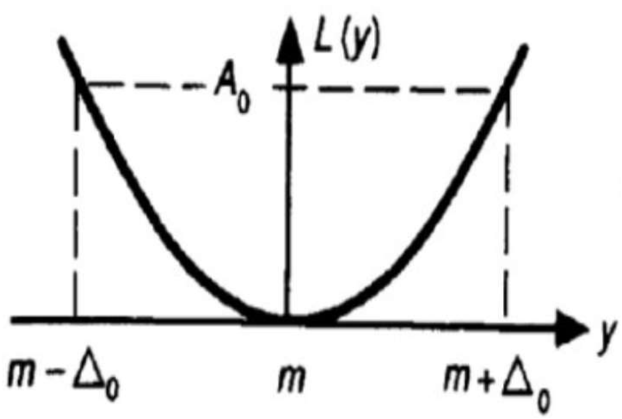
$$S/N_N = 10\log(\bar{y}^2 / S^2)$$

The goal of an experiment for target-value-is-best situations is to reduce variability around a specific target. When the variability of the response is reduced, relative to the average response, S/N_N will increase.

Taguchi Analysis (Example)

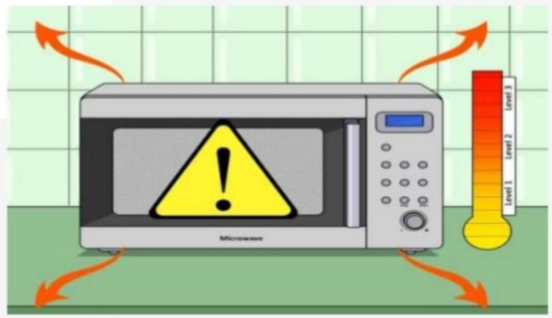
(a) Nominal-the-best

Ex. Colour Density and Brightness must be Optimum. Power output.

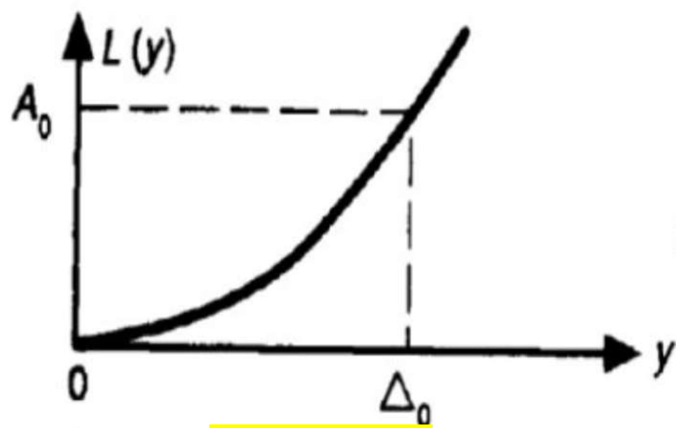


$L(y) = k\,(y - m)^2$

(b) Smaller-the-better



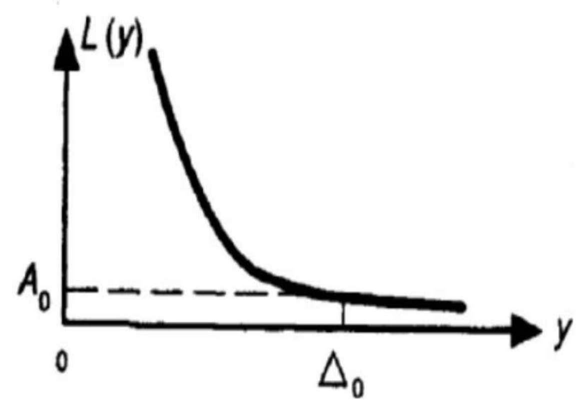
Ex. Radiation leakage in Microwave Oven; pollution; leakage current.



$L(y) = ky^2$

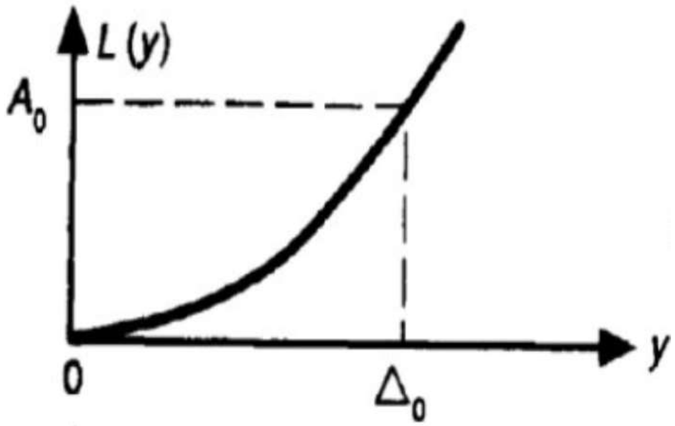
(c) Larger-the-better

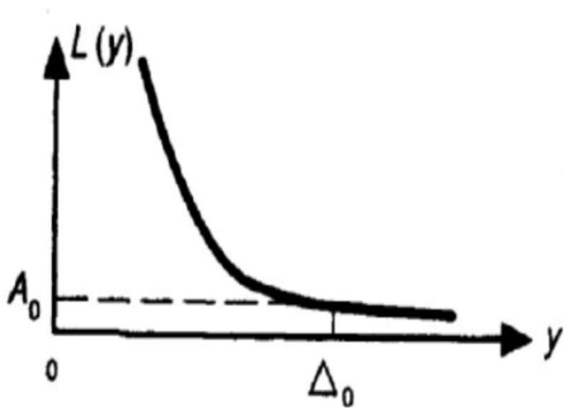
Ex. Bond strength of Adhesives.



$L(y) = k\,(1/y^2)$

Quality Evaluations





Smaller – the – better loss function

- No loss if the quality characteristics is below the upper limit Δ
- Loss is A_0 if the quality characteristics is above upper limit Δ
- $L(y) = ky^2$

Larger – the – better loss function

- No loss if the quality characteristics is above the lower limit Δ
- Loss is A_0 if the quality characteristics is below lower limit Δ
- $L(y) = k\,(1/y^2)$

https://brharnetc.edu.in/br/wp-content/uploads/2018/11/11.pdf

Tolerance Design and Tolerancing

- It is important for the designer **to establish the optimal values of a product's parameters and their tolerances besides a product's performance, appearance, and reliability**.
- **Examine the effect of tolerance design on the quality evaluation of a product and present methods for economic design of product tolerancing and tolerance.**
- The **nominal values and functional limits generally define the characteristics of a product**. The tolerance of a product characteristic have a significant affect on the total quality loss. Therefore, optimal tolerances are those that minimize the total quality loss.
- The **three sources of variation of product characteristics from their nominal values are:**
    - ➢ **Environmental factors**
    - ➢ **Deterioration factors**
    - ➢ **Imperfections in manufacturing processes.**

Tolerance Design and Tolerancing

- Product designer addresses the first two causes of variation by considering the **parameter design and the tolerance design**. The production and manufacturing people address on the third cause of variation, that is, variations due to manufacturing process imperfections.
- There are **three approaches for obtaining the optimal tolerances**. They are
  - **The nominal-the-best**
  - **The larger-the-better**
  - **The smaller-the-better.**

Tolerance Design and Tolerancing

- **The Nominal-the-Best**

In this type of tolerance, the nominal measurement of product characteristic such as dimensions and viscosity are considered and the variation is reduced to a minimum.

- **The Larger-the-Better**

In larger-the-better is applicable to product’s characteristic such as the hardness of materials, fuel efficiency, and thermal properties of materials. The product designer and the manufacturer aim for the larger value of the characteristic.

- **The Smaller-the-Better**

In this case, the product designer and the manufacturer aim for the smaller value of the characteristic of a product such as wear and tear, deterioration and noise level, etc.

Example (Quality Loss Function)

The critical dimension of a mechanical component made using a turning machine must be 0.45 ± 0.005 inches. The scrapping cost is $100 per component. Samples taken from the turning machine had the following dimensions: 0.451, 0.447, 0.448, 0.452, 0.450, 0.453, 0.449, 0.447, 0.454, 0.456, 0.450, and 0.452.

a) find the Taguchi loss equation for this operation

b) determine the average loss function for the parts made using this turning machine.

Example (Quality Loss Function)

a) $L(y) = k(y\text{-}m)^2 = \dfrac{A}{\Delta^2}(y-m)^2 = \dfrac{\$100}{(0.005)^2}(y-m)^2 = 4\text{x}10^{-6}(y-m)^2$

b)

$$Q = k[(\mu - m)^2 + \sigma^2]$$

$$\mu = \bar{x} = \frac{\begin{bmatrix} 0.451 + 0.447 + 0.448 + 0.452 + 0.450 + 0.453 + 0.449 \\ +0.447 + 0.454 + 0.456 + 0.450 + 0.452 \end{bmatrix}}{12}$$

$$= \frac{5.409}{12} = 0.451$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum(y_i - \mu)^2} = \sqrt{\frac{1}{12-1}}\sqrt{(0.000090)} = \sqrt{\frac{0.000090}{11}} = 0.00286$$

$$Q = 4 \times 10^6 [(0.451 - 0.450)^2 + (0.00286)^2] = 4 \times 10^6 [(0.001)^2 + (0.00286)^2]$$

$$= 4 \times 10^6 [(1 \times 10^{-6} + 8.180 \times 10^{-6}] = 36.72$$

Average quality loss = \$ 36.72.

8-Steps in Taguchi Methodology

1. Choose control factors and their levels
2. Identify uncontrollable (noise) factors and decide on how they will be simulated.
3. Select the response variable(s) and determine the performance measures (mean, standard deviation, SNR, etc.)
4. Setup the experimental layout (choose appropriate design array (s))
5. Conduct the experiments and collect data
6. Analyze the data (effects, ANOVA, regression)
7. Choose optimal control factor levels and predict the performance measure at these levels
8. Confirm the optimal levels by experimentation.

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- Imagine we are setting up an experiment in a grinding machine in our machine shop. We want to find out what settings in the grinding machine can give us maximum material removal efficiency, for a grinding wheel. We want to use **Grinding ratio (GR)** as the factor to be optimized.
- GR is given by the expression **material removed divided by wheel wear in grams**.

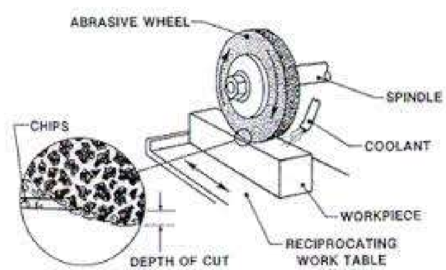


Fig 1 Elements of a grinding process

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 1: Problem definition:** We want to maximize the GR by selecting suitable machine parameters for the given wheel formulation.
- **Step 2:** Identify the Noise factor-i.e. Factors not in our control but are liable to affect the measured output (GR). Let us assume that all wheels of the same formulation may not work at the same efficiency level due to slight variations in processing. Let us also assume that different persons will experiment differently. This will also affect the results, but not in our control.

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 3:** Plan the evaluation method to take care of the noise factor:

✓ To overcome the first noise factor, namely wheel inconsistency, we can arrange to test two wheels of the same category.

✓ To overcome the second noise factor, namely operator difference, we can plan to have the wheels evaluated by two different operators.

✓ The correct identification of noise factors and correct planning to capture these effects is the key to the success of this method. Thus in our case, for each selected machine settings, we have to get GR for 4 wheels of the same formulation (2 wheels by 1 operator and another 2 by the second operator).

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 4:** Identify the objective function:

✓ Taguchi suggests different formula for different needs like Maximizing, minimizing, normalizing etc. Our aim is to maximize GR.

✓ So the formula for objective function ɳ is given as

- ɳ = -10log10 $(1/y^2)$
- Where y is the measured output function to be optimized. So, in our case, y=Grinding ratio.

$$S/N = -10\log\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right]$$

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 5:** Identify control factors and their levels:

✓ A control factor is one which has an effect on the output (Our case G.R) and which is controllable by us. A level indicates a setting.

Item	Control Factor	Level 1	Level 2
A	Spindle speed	33 mps	45 mps
B	Feed rate	2.5mm/rev	5.0 mm / rev
C	Depth of cut	0.5mm	1.0mm

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 6:** Design of experiment

✓ Taguchi method suggests the number of experiments to be used based on the number of factors. There are separate tables for two level and three level factors. The table giving the number of experiments is called Orthogonal array table.

For 2 Level factors	
No of control factors	Array to be Selected
2-3	L4
4-7	L8
8-11	L12
12-15	L16

For 3 Level factors	
No of control factors	Array to be Selected
2-4	L9
5-7	L18

From the above table, in our case, since there are three control factors of 2 level each, we choose orthogonal array L4, from the first table

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 6:** Design of experiment
- L4 looks as below. (Taguchi method lists such table for all Orthogonal arrays).

Expt.No	L4 orthogonal Array		
	Control Factor levels		
	A	B	C
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Expt. No	Spindle speed mps	Feed rate mm/rev	Depth of cut mm
1	33	2.5	0.5
2	33	5.0	1.0
3	45	2.5	1.0
4	45	5.0	0.5

The number of experiments at each setting is decided by us from steps 2 &3 to capture the noise factors. Please note that this is different from the number of machine settings as given in orthogonal array.

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 6:** Design of experiment
- In our case, since we have assumed 2 operators evaluating two wheels each , we will be doing experiments in 4 different settings with 4 repetitions for each setting.

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 7:** Conduct the experiments and tabulate the ‘y’ values
  - ✓ As decided in step 6, we conduct the experiments and measured the 'y' , which in our case is grinding ratio defined as.
  - ✓ G.R =Weight of material removed/Weight of wheel loss
  - ✓ Values in brackets denote the level whether 1 or 2.

Expt. No	Speed	Feed rate	Depth of cut	Measured values of 'y' G.R			
				1	2	3	4
1	33(1)	2.5(1)	0.5(1)	21.5	20.2	20.5	19.0
2	33(1)	5.0 (2)	1.0 (2)	18.5	17.5	18.0	18.0
3	45(2)	2.5 (1)	1.0 (2)	19.0	19.5	18.8	18.8
4	45(2)	5.0 (2)	0.5 (1)	16.0	16.2	16.6	16.6

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 8:** Calculate the objective function
- From step 4, for our type of problem of **larger the better** category

$$S/N = -10\log\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right]$$

η = - 10 log10 {1/4 ($1/21.5^2$+ $1/20.2^2$+ $1/20.5^2$+ $1/19^2$) } = 26.13

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 8:** Calculate the objective function

Expt. No	Speed	Feed rate	Depth of cut	Measured values of 'y' G.R				η
				1	2	3	4	
1	33(1)	2.5(1)	0.5(1)	21.5	20.2	20.5	19.0	26.13
2	33(1)	5.0 (2)	1.0 (2)	18.5	17.5	18.0	18.0	25.10
3	45(2)	2.5 (1)	1.0 (2)	19.0	19.5	18.8	18.8	25.58
4	45(2)	5.0 (2)	0.5 (1)	16.0	16.2	16.6	16.6	24.27
				Mean				25.27

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 9:** Determine the factor effects

This method helps in segregating the individual effect of each control factor at each level and is denoted as mA1 mB1 etc.

mA1 means the effect of factor A at level 1

mC2 means the effect of factor C at level 2

Let us see how this can be calculated From the table under step 7.

mA1 should be the effect of all those η values where factor A (speed) is used in level 1 (33mps).

This occurs in expt.1 and expt. 2. So, using the values of these experiments,

$$\mathbf{mA1 = 1/2\ (\eta 1 + \eta 2) = 0.5\ (26.13 + 25.10) = 25.62}$$

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 9:** Determine the factor effects

Expt.No	L4 orthogonal Array		
	Control Factor levels		
	A	B	C
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Similarly, we can calculate the effect of other factors also as below:

$mA_1 = 1/2\ (\eta_1 + \eta_2) = 0.5\ (26.13 + 25.10) = 25.62$

$mA_2 = 1/2\ (\eta_3 + \eta_4) = 0.5\ (25.58 + 24.27) = 24.92$

$mB_1 = 1/2\ (\eta_1 + \eta_3) = 0.5\ (26.13 + 25.58) = 25.86$

$mB_2 = 1/2\ (\eta_2 + \eta_4) = 0.5\ (25.10 + 24.27) = 24.68$

$mC_1 = 1/2\ (\eta_1 + \eta_4) = 0.5\ (26.13 + 24.27) = 25.2$

$mC_2 = 1/2\ (\eta_2 + \eta_3) = 0.5\ (25.10 + 25.58) = 25.34$

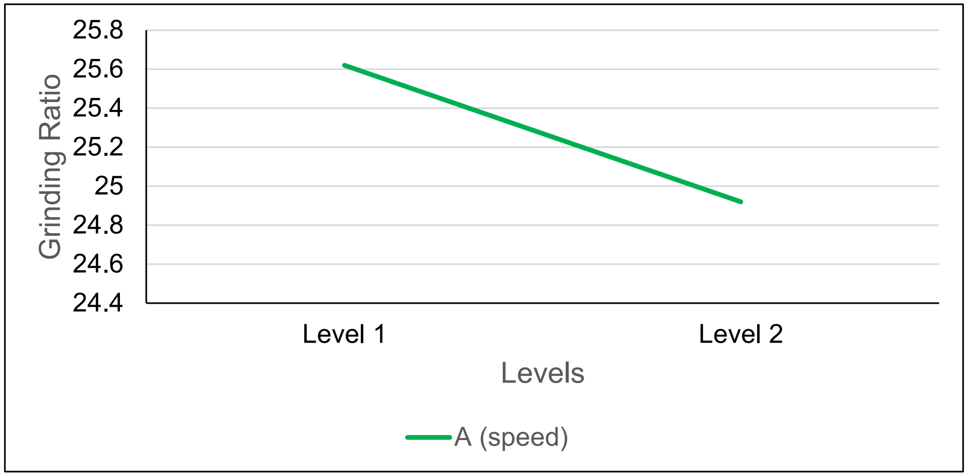
Factor	Level	
	1	2
A (speed)	25.62	24.92
B (Feed rate)	25.86	24.68
C (depth of cut)	25.2	25.34

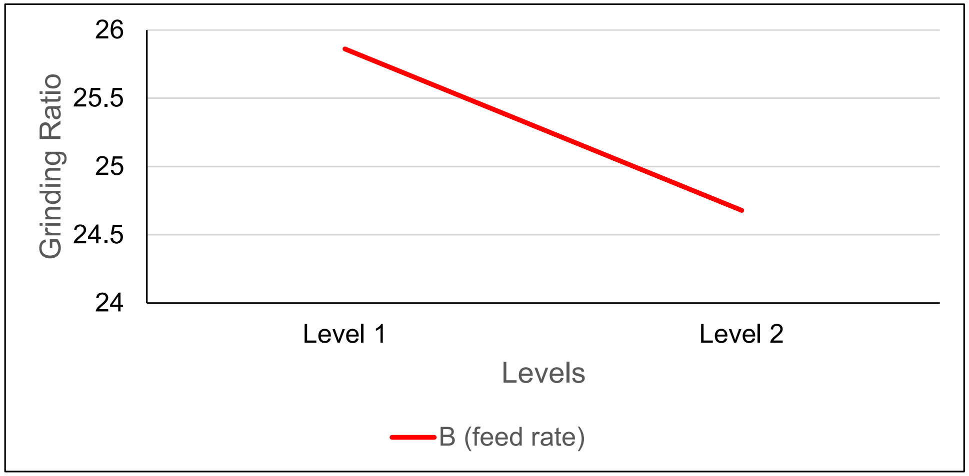
These are tabulated and plotted with control factors at three individual points along x axis.

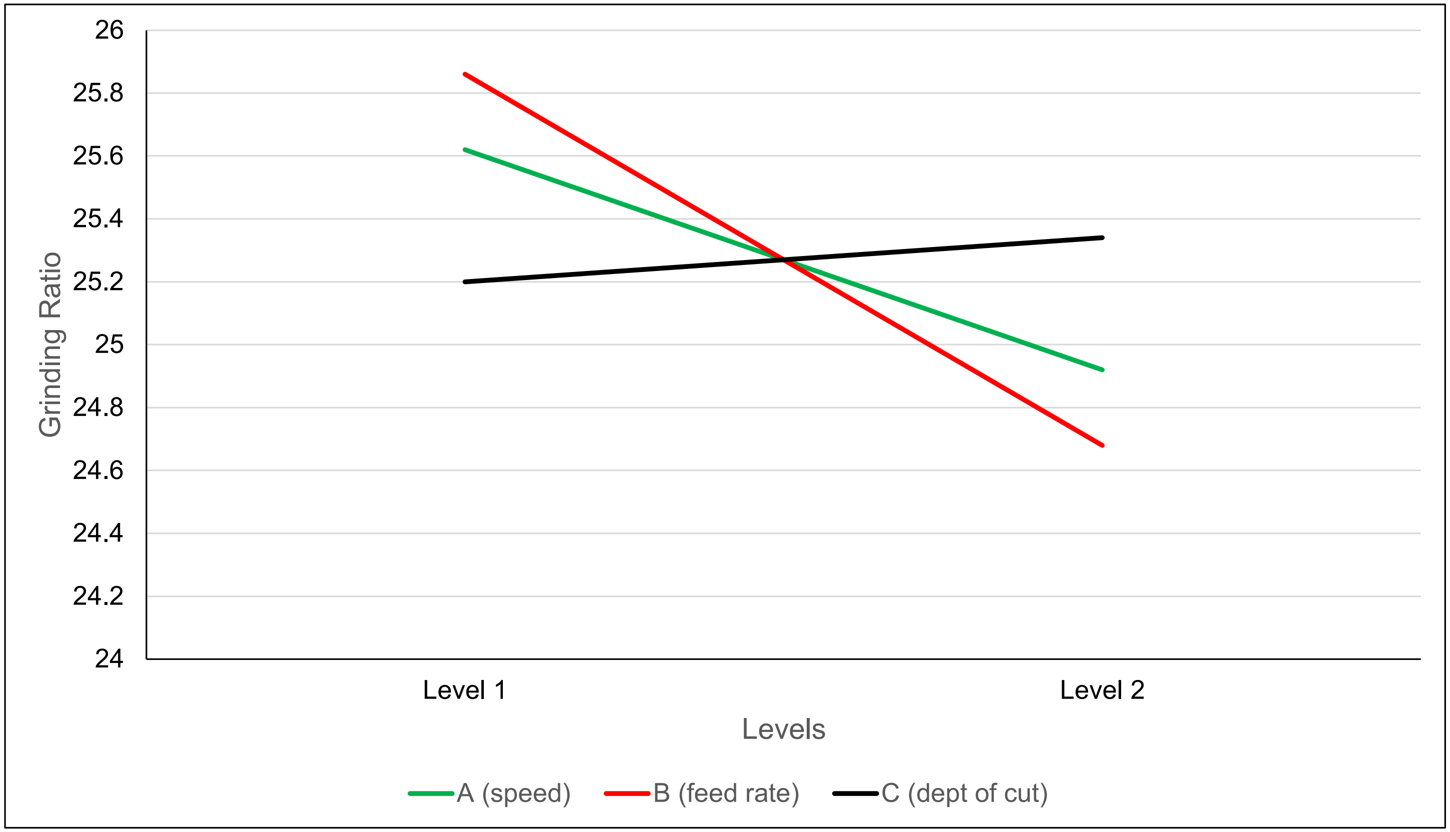
Taguchi Analysis (Example)

- **Improving the Grinding Ratio.**

From the graph, level 1 of A, level 1 of B and level 2 of C give the maximum effect in improving G.R.






26
25.8
25.6
25.4
25.2
25
24.8
24.6
24.4
24.2
24
Grinding Ratio
Level 1
Level 2
Levels
A (speed)
B (feed rate)
C (dept of cut)

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 9:** Determine the factor effects
- Naturally, Al, B1, C2 is the best combination. i.e. a speed of 33mps , a feed rate of 2.5m/min and a depth of cut of 1mm will give the maximum G.R.
- It is of interest to see that we have not tried this combination. The combinations tried by us are given below:

Expt 1 was Al, B1, C1

Expt 2 was A1, B2, C2

Expt 3 was A2, B1, C2

Expt 4 was A2, B2, C1

Other inferences from the graph **Factor B has the largest effect on GR; Factor C has the least effect on GR.** The lowest GR is from A2, B2, C1.

Taguchi Analysis (Example)

Improving the Grinding Ratio.

- **Step 10:** Predicting GR for Untried combinations

This is the most exciting part of Taguchi method. Any combination of control factor settings, even for experiments not conducted, can be estimated, using the formula.

Factor	Level	
	1	2
A (speed)	25.62	24.92
B (Feed rate)	25.86	24.68
C (depth of cut)	25.2	25.34

$$\eta = \mu + (mA - \mu) + (mB - \mu) + (mC - \mu)$$

where μ = mean

for example, for the combination of A2 B2 C1.

$$\eta = \mu + (mA_2 - \mu) + (mB_2 - \mu) + (mC_1 - \mu)$$

$$= 25.27 + (24.92 - 25.27) + (24.68 - 25.27) + (25.20 - 25.27)$$

$$= 24.26$$

Which is very close to 24.27 in the table under step 7 for expt 4.

Taguchi Analysis (Example)

Already did on the following combination

Expt 1 was Al, B1, C1

Expt 2 was A1, B2, C2

Expt 3 was A2, B1, C2

Expt 4 was A2, B2, C1

Improving the Grinding Ratio.

Step 10: Predicting GR for Untried combinations

So, for the expected combination of A1 B1 C2 ,

η expectable = μ + (mA1 - μ) + (mB1 - μ) + (mC2 - μ)

= 25.27 + (25.62 - 25.27) + (25.86 – 25.27) + (25.34 - 25.27)

= 26.28

From our definition, η = -10 log10 (1 / y^2) = 26.28

log10 (1 / y^2) = 26.28/(-10)

log10 (1 / y^2) = -2.628

(1 / y^2) = antilog (-2.628)

(1 / y^2) = 10^(-2.628)

(1 / y^2) = 0.002355

(y^2) = 1/(0.002355) = 424.6196

So, y = SQRT (424.6196) = 20.61 GR max.

Factor	Level	
	1	2
A (speed)	25.62	24.92
B (Feed rate)	25.86	24.68
C (depth of cut)	25.2	25.34

Taguchi Analysis (Example)

- Instead of 8 combinations, we tried only four combinations of setting and from these four we were able to predict the GR achievable for those combinations of controls which we have not tried.
- For 3 — two level factors, the combinations possible are only 2^3= 8. Here we tried 4 experiments.
- But suppose there are **seven** factors like speed, infeed, dressing traverse etc. each with **three** setting levels, then the possibility is 3^7 = 2187 combinations.
- The effect of all these 2187 experiments can be predicted with just 18 experiments. (for 3 level factors of 7 numbers, we use L18 array).

Taguchi Analysis (Orthogonal Array)

For 2 Level factors	
No of control factors	Array to be Selected
2-3	L4
4-7	L8
8-11	L12
12-15	L16

For 3 Level factors	
No of control factors	Array to be Selected
2-4	L9
5-7	L18

	L4 orthogonal Array		
	Control Factor levels		
Expt.No	A	B	C
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Taguchi Analysis (Orthogonal Array)

L8 (2^7) ORTHOGONAL ARRAY

Expt. No	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

L9 (3^4) ORTHOGONAL ARRAY

Expt.No	Columns			
	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Taguchi Analysis (Orthogonal Array)

Orthogonal Array	No. Runs	Max. Factors	Max. of columns at these levels			
			2-level	3-level	4-level	5-level
L4	4	3	3			
L8	8	7	7			
L9	9	4		4		
L12	12	11	11			
L16	16	15	15			
L'16	16	5			5	
L18	18	8	1	7		
L25	25	6				6
L27	27	13		13		
L32	32	31	31			
L'32	32	10	1		9	
L36	36	23	11	12		
L'36	36	16	3	13		
L50	50	12	1			11
L54	54	26	1	25		
L64	64	63	63			
L'64	64	21			21	
L81	81	40		40		

Taguchi Analysis (Example Problem)

- The objective of the experiment is to obtain minimum surface roughness of the parts machined by wire cut EDM machine using Taguchi's DOE technique of process parameters.

Column	Factors	Units	Level-1	Level-2
1	Pulse on time	μ - sec	125	130
2	Pulse off time	μ - sec	35	40
3	Peak current	Amps	11	12
4	Wire tension	Grams	1000	1200
5	Wire feed	M / min	7	8
6	Fluid rate	Lpm	8	9
7	Fluid pressure	Kg /cm^2	13	15

- Surface roughness(Ra) values were obtained using Taylor-Hobson Surtronic instruments and the sequence are 2.5275, 2.3520, 2.2540, 2.4650, 2.7000, 2.8125, 2.350, 2.2875.
- Find out the Main effect and draw the factor effect diagram and identify the optimum WEDM process parameters.

Taguchi Analysis (Example Problem)

- Since, the number of factors given are 7 with 2 levels. Hence, we can choose L8 orthogonal array.

Expt. No	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Taguchi Analysis (Example Problem)

- L8 orthogonal array.

- **Smaller-the-Better: η = -10 log10(y^2)**
- **Hence solving for η**

Expt. No	Columns							Surface Roughness (y)	η	
	Pull on Timet (A)	Pulse off time (B)	Peak Current(C)	Wire Tension (D)	Wire Feed (E)	Fluid Rate (F)	Fluid Pressure (G)			
1	125 (1)	35 (1)	11 (1)	1000 (1)	7 (1)	8 (1)	13 (1)	2.5275		**η1**
2	125 (1)	35 (1)	11 (1)	1200 (2)	8 (2)	9 (2)	15 (2)	2.3520		**η2**
3	125 (1)	40 (2)	12 (2)	1000 (1)	7 (1)	9 (2)	15 (2)	2.2540		**η3**
4	125 (1)	40 (2)	12 (2)	1200 (2)	8 (2)	8 (1)	13 (1)	2.4650		**η4**
5	130 (2)	35 (1)	12 (2)	1000 (1)	8 (2)	8 (1)	15 (2)	2.7000		**η5**
6	130 (2)	35 (1)	12 (2)	1200 (2)	7 (1)	9 (2)	13 (1)	2.8125		**η6**
7	130 (2)	40 (2)	11 (1)	1000 (1)	8 (2)	9 (2)	13 (1)	2.350		**η7**
8	130 (2)	40 (2)	11 (1)	1200 (2)	7 (1)	8 (1)	15 (2)	2.2875		**η8**
								µ		

Taguchi Analysis (Example Problem)

- L8 orthogonal array.

- **Smaller-the-Better: η = -10 log10(y^2)**
- **Hence solving for η**

Expt. No	Columns							Surface Roughness (y)	η	
	Peak current (A)	Pulse on time (B)	Peak Current(C)	Wire Tension (D)	Wire Feed (E)	Fluid Rate (F)	Fluid Pressure (G)			
1	125 (1)	35 (1)	11 (1)	1000 (1)	7 (1)	8 (1)	13 (1)	2.5275	-8.053823285	**η1**
2	125 (1)	35 (1)	11 (1)	1200 (2)	8 (2)	9 (2)	15 (2)	2.3520	-7.428746348	**η2**
3	125 (1)	40 (2)	12 (2)	1000 (1)	7 (1)	9 (2)	15 (2)	2.2540	-7.059078234	**η3**
4	125 (1)	40 (2)	12 (2)	1200 (2)	8 (2)	8 (1)	13 (1)	2.4650	-7.836338472	**η4**
5	130 (2)	35 (1)	12 (2)	1000 (1)	8 (2)	8 (1)	15 (2)	2.7000	-8.627275283	**η5**
6	130 (2)	35 (1)	12 (2)	1200 (2)	7 (1)	9 (2)	13 (1)	2.8125	-8.981850622	**η6**
7	130 (2)	40 (2)	11 (1)	1000 (1)	8 (2)	9 (2)	13 (1)	2.350	-7.421357245	**η7**
8	130 (2)	40 (2)	11 (1)	1200 (2)	7 (1)	8 (1)	15 (2)	2.2875	-7.187222055	**η8**
								μ	-7.82446144	

Taguchi Analysis (Example Problem)

- **Determine the factor effects**

mA_1 =1/4 ($\eta_1 + \eta_2 + \eta_3 + \eta_4$) = 0.25 * (-8.053-7.428-7.059-7.836) = -7.5945

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Taguchi Analysis (Example Problem)

- **Determine the factor effects**

mA_1 =1/4 ($\eta_1 + \eta_2 + \eta_3 + \eta_4$) = 0.25 * (-8.053-7.428-7.059-7.836) = -7.5945
mA_2 =1/4 ($\eta_5 + \eta_6 + \eta_7 + \eta_8$) = 0.25 * (-8.627-8.981-7.42-7.187) = -8.053
mB_1 =1/4 ($\eta_1 + \eta_2 + \eta_5 + \eta_6$) =
mB_2 =1/4 ($\eta_3 + \eta_4 + \eta_7 + \eta_8$) =
mC_1 =
mC_2 =
mD_1 =
mD_2 =
mE_1 =
mE_2 =
mF_1 =
mF_2 =
mG_1 =
mG_2 =

Taguchi Analysis (Example Problem)

- **Determine the factor effects**

$mA_1 = 1/4\ (\eta_1 + \eta_2 + \eta_3 + \eta_4) = 0.25 * (-8.053-7.428-7.059-7.836) = -7.5945$

$mA_2 = 1/4\ (\eta_5 + \eta_6 + \eta_7 + \eta_8) = 0.25 * (-8.627-8.981-7.42-7.187) = -8.053$

$mB_1 = 1/4\ (\eta_1 + \eta_2 + \eta_5 + \eta_6) = -8.272$

$mB_2 = 1/4\ (\eta_3 + \eta_4 + \eta_7 + \eta_8) = -7.37$

$mC_1 = 1/4\ (\eta_1 + \eta_2 + \eta_7 + \eta_8) = -7.522$

$mC_2 = 1/4\ (\eta_3 + \eta_4 + \eta_5 + \eta_6) = -8.125$

$mD_1 = 1/4\ (\eta_1 + \eta_3 + \eta_5 + \eta_7) = -7.789$

$mD_2 = 1/4\ (\eta_2 + \eta_4 + \eta_6 + \eta_8) = -7.858$

$mE_1 = 1/4\ (\eta_1 + \eta_3 + \eta_6 + \eta_8) = -7.82$

$mE_2 = 1/4\ (\eta_2 + \eta_4 + \eta_5 + \eta_7) = -7.827$

$mF_1 = 1/4\ (\eta_1 + \eta_4 + \eta_5 + \eta_8) = -7.925$

$mF_2 = 1/4\ (\eta_2 + \eta_3 + \eta_6 + \eta_7) = -7.722$

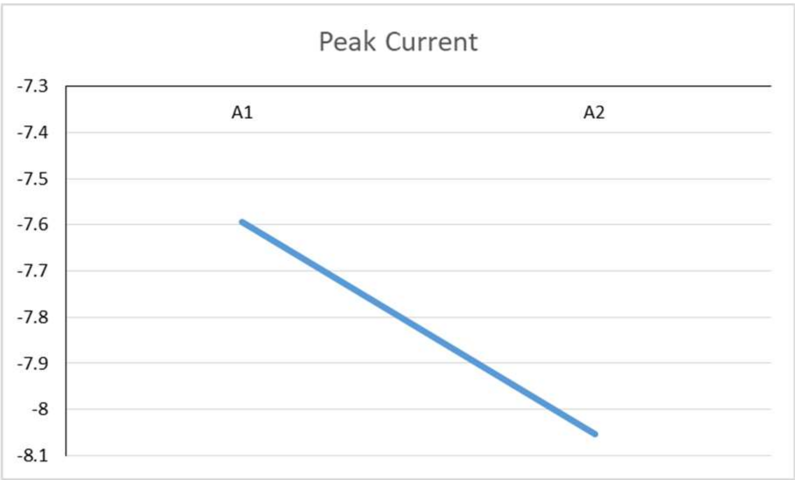
$mG_1 = 1/4\ (\eta_1 + \eta_4 + \eta_6 + \eta_7) = -8.072$

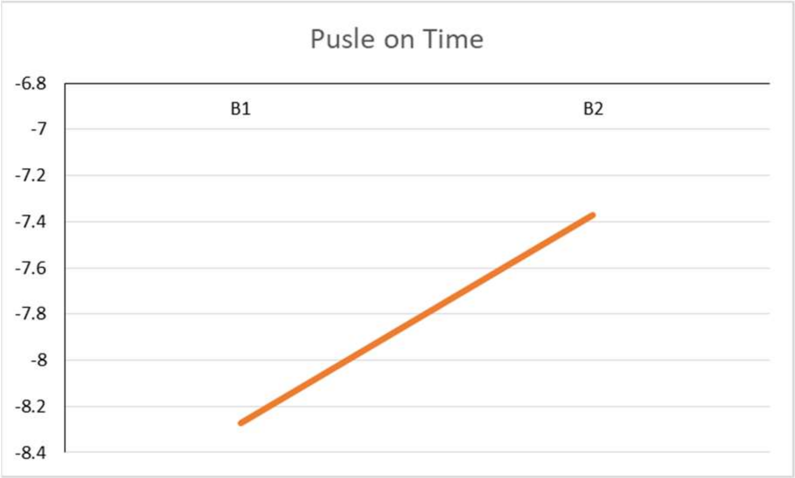
$mG_2 = 1/4\ (\eta_2 + \eta_3 + \eta_5 + \eta_8) = -7.575$

Expt. No	Columns							Surface Roughness (y)	η	
	Peak current (A)	Pulse on time (B)	Peak Current(C)	Wire Tension (D)	Wire Feed (E)	Fluid Rate (F)	Fluid Pressure (G)			
1	125 (1)	35 (1)	11 (1)	1000 (1)	7 (1)	8 (1)	13 (1)	2.5275	-8.053823285	η1
2	125 (1)	35 (1)	11 (1)	1200 (2)	8 (2)	9 (2)	15 (2)	2.3520	-7.428746348	η2
3	125 (1)	40 (2)	12 (2)	1000 (1)	7 (1)	9 (2)	15 (2)	2.2540	-7.059078234	η3
4	125 (1)	40 (2)	12 (2)	1200 (2)	8 (2)	8 (1)	13 (1)	2.4650	-7.836338472	η4
5	130 (2)	35 (1)	12 (2)	1000 (1)	8 (2)	8 (1)	15 (2)	2.7000	-8.627275283	η5
6	130 (2)	35 (1)	12 (2)	1200 (2)	7 (1)	9 (2)	13 (1)	2.8125	-8.981850622	η6
7	130 (2)	40 (2)	11 (1)	1000 (1)	8 (2)	9 (2)	13 (1)	2.350	-7.421357245	η7
8	130 (2)	40 (2)	11 (1)	1200 (2)	7 (1)	8 (1)	15 (2)	2.2875	-7.187222055	η8
								μ	-7.82446144	

Taguchi Analysis (Example Problem)

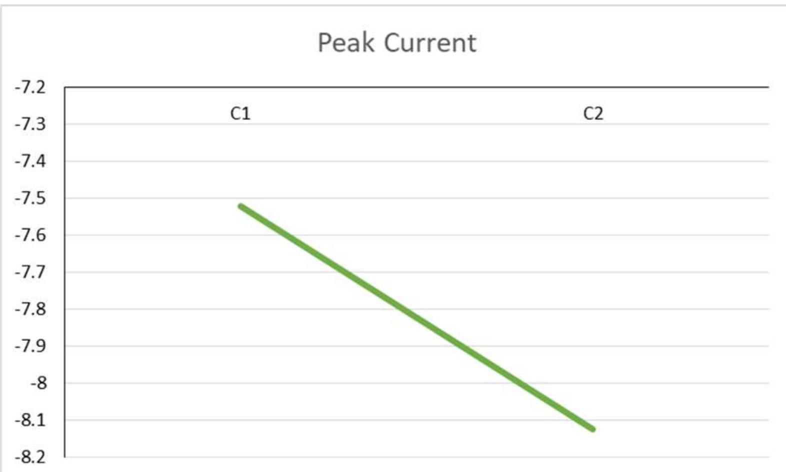
- **Determine the factor effects (graph)**

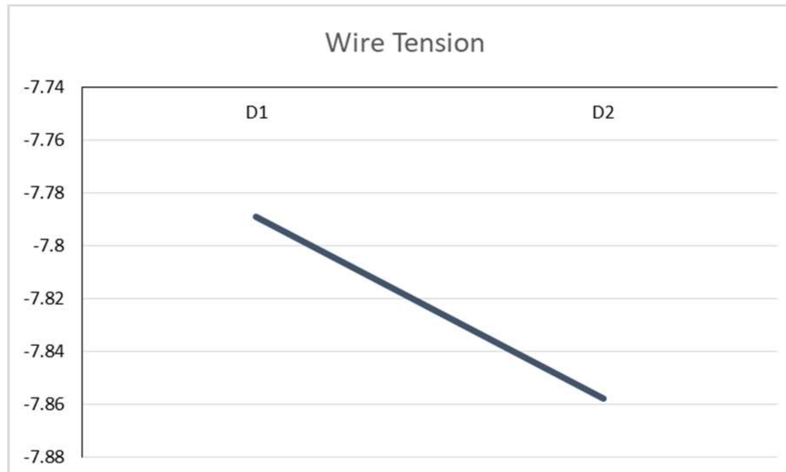




Taguchi Analysis (Example Problem)

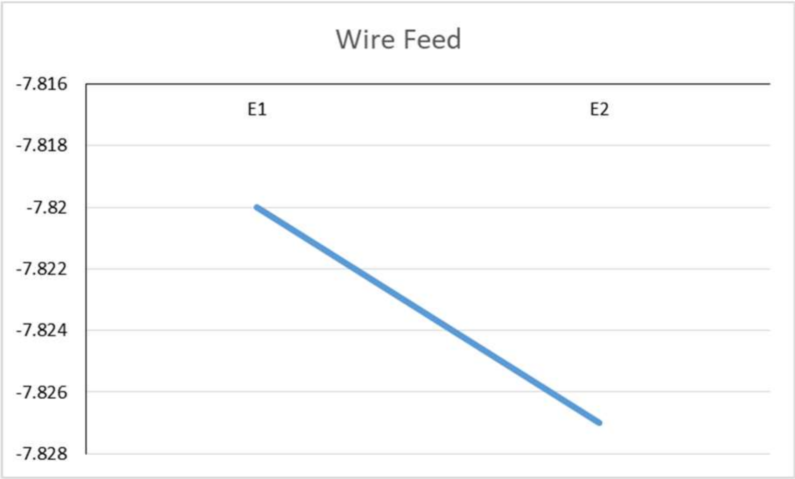
- **Determine the factor effects (graph)**

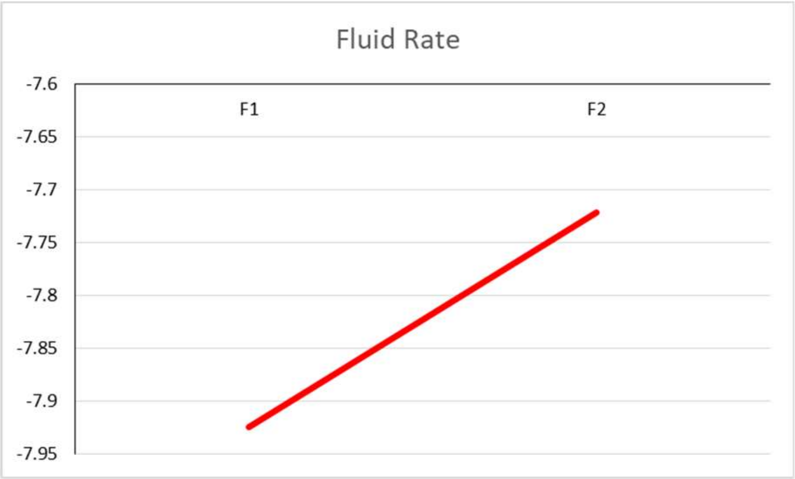




Taguchi Analysis (Example Problem)

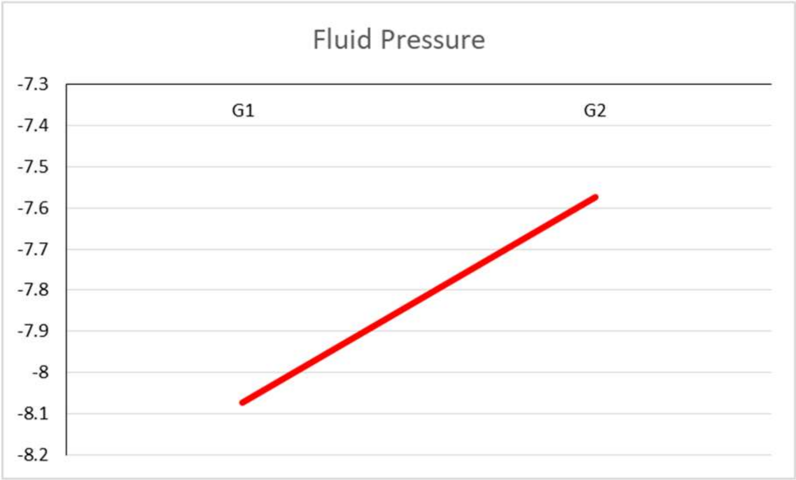
- **Determine the factor effects (graph)**





Taguchi Analysis (Example Problem)

- **Determine the factor effects (graph)**



Taguchi Analysis (Example Problem)

- A1, B2, C1, D1, E2, F2, G2 giving the minimum effect in reducing the surface roughness.

η = μ + (mA1 - μ) + (mB2 - μ) + (mC1 - μ) + (mD1 - μ) + (mE2 - μ) + (mF2 - μ) + (mG2 - μ)

= -7.823 + (-7.594+7.823) + (-7.37+7.823) + (-7.597+7.823) + (-7.789+7.823) + (-7.827+7.823) + (-7.722+7.823) + (-7.573+7.823)

= -6.533

η = -6.533

i.e., η = **-10 log10(y^2) =** -6.533

log10(y^2) = 0.6533

$\boldsymbol{y^2}$ = antilog (0.6533)

$\boldsymbol{y^2}$ = 4.5009

$\boldsymbol{y}$ = 2.12

Hence, **Surface Roughness = 2.12 with the above combination.**

Taguchi Analysis (Example Problem)

Problem: The objective of the research is an experimental investigation to find optimization of machining parameters of EDM machine for machining steel material EN-8 using Taguchi's DOE.

Column	Factors	Units	Level -1	Level -2	Level -3
1	Peak current	Amps	3	5	7
2	Pulse on time	μ-sec	25	52	100
3	Gap voltage	Volts	45	50	55
4	Fluid pressure	Kg /cm^2	0.5	0.75	1.0

- Ra values obtained according to experiment sequence are 2.625, 1.905,2.5275,2.99,4.4325,4.10,3.38,3.015, 3.312.
- Find out the Main effect and draw the factor effect diagram and identify the optimum process parameters.

Taguchi Analysis (Example Problem)

Solution:

How many factors ?

How many levels ?

Which Orthogonal Array?

Column	Factors	Units	Level -1	Level -2	Level -3
1	Peak current	Amps	3	5	7
2	Pulse on time	μ-sec	25	52	100
3	Gap voltage	Volts	45	50	55
4	Fluid pressure	Kg /cm^2	0.5	0.75	1.0

Orthogonal Array	No. Runs	Max. Factors	Max. of columns at these levels			
			2-level	3-level	4-level	5-level
L4	4	3	3			
L8	8	7	7			
L9	9	4		4		
L12	12	11	11			
L16	16	15	15			
L'16	16	5			5	
L18	18	8	1	7		
L25	25	6				6
L27	27	13		13		
L32	32	31	31			
L'32	32	10	1		9	
L36	36	23	11	12		
L'36	36	16	3	13		
L50	50	12	1			11
L54	54	26	1	25		
L64	64	63	63			
L'64	64	21			21	
L81	81	40		40		

Taguchi Analysis (Example Problem)

Solution: Hence, 4 factors and 3 levels, we have to choose L9 orthogonal array.

L9 (3^4) ORTHOGONAL ARRAY

Expt.No	Columns			
	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Expt.No	Columns			
	1	2	3	4
1	3	25	45	0.5
2	3	52	50	.75
3	3	100	55	1
4	5	25	50	1
5	5	52	55	0.5
6	5	100	45	0.75
7	7	25	55	0.75
8	7	52	45	1
9	7	100	50	0.5

Taguchi Analysis (Example Problem)

Solution: solving for signal-to-noise ratio (Efficiency)

Expt.No	Columns					
	1	2	3	4	Ra	S.N Ratio
1	3(1)	25(1)	45(1)	0.5(1)	2.625	-8.3826
2	3(1)	52(2)	50(2)	0.75(2)	1.905	-5.5979
3	3(1)	100(3)	55(3)	1(3)	2.5275	-8.0538
4	5(2)	25(1)	50(2)	1(3)	2.99	-9.5134
5	5(2)	52(2)	55(3)	0.5(1)	4.4325	-12.9330
6	5(2)	100(3)	45(1)	0.75(2)	4.10	-12.2557
7	7(3)	25(1)	55(3)	0.75(2)	3.38	-10.5783
8	7(3)	52(2)	45(1)	1(3)	3.015	-9.5857
9	7(3)	100(3)	50(2)	0.5(1)	3.312	-10.4018

Smaller-the-Better: η = -10 log10(y^2)
Hence solving for η

Taguchi Analysis (Example Problem)

Solution: Determination of factor effects

Expt.No	Columns					
	1	2	3	4	Ra	S.N Ratio
1	3(1)	25(1)	45(1)	0.5(1)	2.625	-8.3826
2	3(1)	52(2)	50(2)	0.75(2)	1.905	-5.5979
3	3(1)	100(3)	55(3)	1(3)	2.5275	-8.0538
4	5(2)	25(1)	50(2)	1(3)	2.99	-9.5134
5	5(2)	52(2)	55(3)	0.5(1)	4.4325	-12.9330
6	5(2)	100(3)	45(1)	0.75(2)	4.10	-12.2557
7	7(3)	25(1)	55(3)	0.75(2)	3.38	-10.5783
8	7(3)	52(2)	45(1)	1(3)	3.015	-9.5857
9	7(3)	100(3)	50(2)	0.5(1)	3.312	-10.4018

$mA_1 = 1/3\ (\eta_1 + \eta_2 + \eta_3) = -7.345$

$mA_2 = 1/3\ (\eta_4 + \eta_5 + \eta_6) = -11.567$

$mA_3 = 1/3\ (\eta_7 + \eta_8 + \eta_9) = -10.189$

$mB_1 = 1/3\ (\eta_1 + \eta_4 + \eta_7) = -9.491$

$mB_2 = 1/3\ (\eta_2 + \eta_5 + \eta_8) = -9.372$

$mB_3 = 1/3\ (\eta_3 + \eta_6 + \eta_9) = -10.237$

$mC_1 = 1/3\ (\eta_1 + \eta_6 + \eta_8) = -10.075$

$mC_2 = 1/3\ (\eta_2 + \eta_4 + \eta_9) = -8.504$

$mC_3 = 1/3\ (\eta_3 + \eta_5 + \eta_7) = -10.522$

$mD_1 = 1/3\ (\eta_1 + \eta_5 + \eta_9) = -10.572$

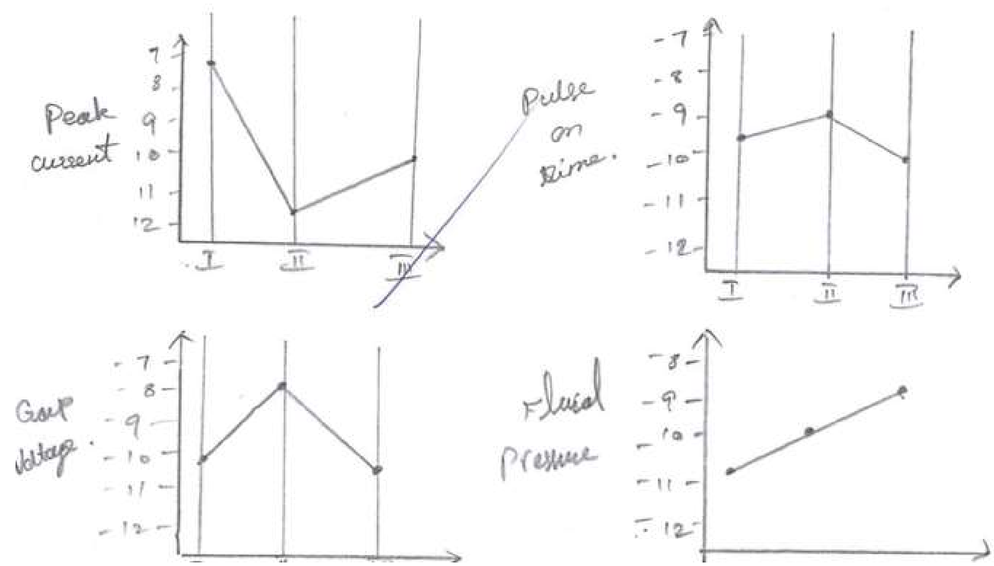
$mD_2 = 1/3\ (\eta_2 + \eta_6 + \eta_7) = -9.477$

$mD_3 = 1/3\ (\eta_3 + \eta_4 + \eta_8) = -9.051$

Taguchi Analysis (Example Problem)

Solution: Determination of factor effects

	A	B	C	D
1	-7.345	-9.491	-10.075	-10.572
2	-11.567	-9.372	-8.504	-9.477
3	-10.489	-10.237	-10.522	-9.054



Factor effect graphs.

From the graph, A1 B2 C2 D3 giving the minimum surface roughness. So, we can finanlize

Taguchi Analysis (Example Problem)

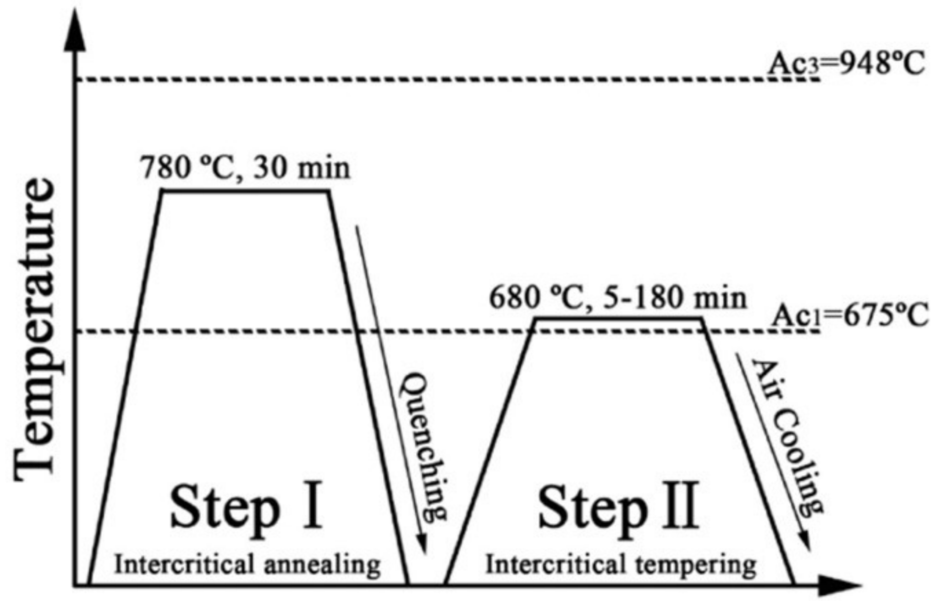
Problem: Heat treatment process used to harden steel components, using Taguchi analysis. Determine which process parameters have the greatest impact on the hardness of the steel components. Hardness values are 57, 59, 65, 45, 67, 73, 51, 48. Hardness value (HV)

Parameter number	Parameters	Level 1	Level 2
1	Temperature(^{O}C)	760	900
2	Quenching rate(^{O}C/s)	35	140
3	Cooling time(sec)	1	300
4	Carbon contents (Wt% c)	1	6
5	Co $_2$concentration (%)	5	20

Taguchi Analysis (Example Problem)

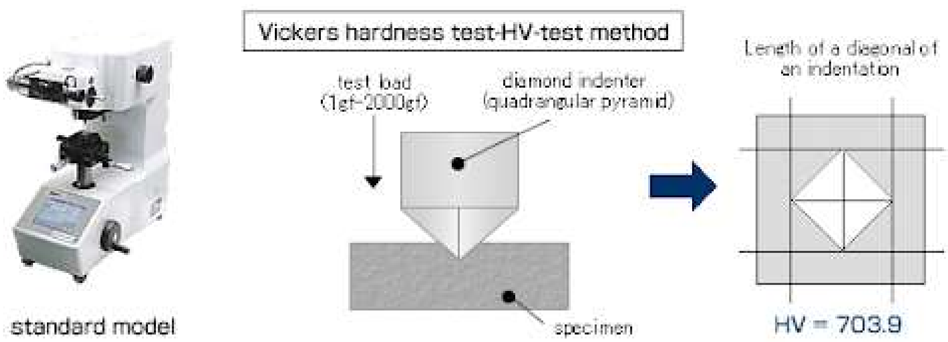
Problem: Heat treatment process

- *Heat Treatment process is a series of operations involving the* ***Heating and Cooling*** *of metals in the solid state.*
- *Its purpose is to change a mechanical property or combination of mechanical properties so that the metal will be more useful, serviceable, and safe for definite purpose.*
- *By heat treating, a metal can be made harder, stronger, and more resistant to impact, heat treatment can also make a metal softer and more ductile.*



Taguchi Analysis (Example Problem)

Problem: Determine which process parameters have the greatest impact on the hardness of the steel components. Hardness values are 57, 59, 65, 45, 67, 73, 51, 48. Hardness value (HV)



Parameter number	Parameters	Level 1	Level 2
1	Temperature(^{O}C)	760	900
2	Quenching rate(^{O}C/s)	35	140
3	Cooling time(sec)	1	300
4	Carbon contents (Wt% c)	1	6
5	Co$_2$concentration (%)	5	20

Taguchi Analysis (Example Problem)

Solution: How many factors? How any levels?

Orthogonal Array	No. Runs	Max. Factors	Max. of columns at these levels			
			2-level	3-level	4-level	5-level
L4	4	3	3			
L8	8	7	7			
L9	9	4		4		
L12	12	11	11			
L16	16	15	15			
L'16	16	5			5	
L18	18	8	1	7		
L25	25	6				6
L27	27	13		13		
L32	32	31	31			
L'32	32	10	1		9	
L36	36	23	11	12		
L'36	36	16	3	13		
L50	50	12	1			11
L54	54	26	1	25		
L64	64	63	63			
L'64	64	21			21	
L81	81	40		40		

Factors: 5

Level: 2

Orthogonal array: L8

Parameter number	Parameters	Level 1	Level 2
1	Temperature(^{O}C)	760	900
2	Quenching rate(^{O}C/s)	35	140
3	Cooling time(sec)	1	300
4	Carbon contents (Wt% c)	1	6
5	Co_2 concentration (%)	5	20

Expt. No	Columns						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Taguchi Analysis (Example Problem)

Solution: L8 orthogonal array

Expt. No	Columns				
	1	2	3	4	5
1	760	35	1	1	5
2	760	35	1	6	20
3	760	140	300	1	5
4	760	140	300	6	20
5	900	35	300	1	20
6	900	35	300	6	5
7	900	140	1	1	20
8	900	140	1	6	5

Parameter number	Parameters	Level 1	Level 2
1	Temperature(^{O}C)	760	900
2	Quenching rate(^{O}C/s)	35	140
3	Cooling time(sec)	1	300
4	Carbon contents (Wt% c)	1	6
5	Co_2 concentration (%)	5	20

Taguchi Analysis (Example Problem)

Solution: L8 orthogonal array.

Hardness value (HV) of steel: **larger is better**

Hence, S/N ratio: -10 * Log10(sum(1/y^2)/n)

S/N ratio = -10 * Log10(sum(1/y^2)/n) = 35.1175 = 35.12

..

..

..

..

S/N ratio = -10 * Log10(sum(1/y^2)/n) = 33.62

Expt. No	Columns						
	1	2	3	4	5	HV	S/N ratio
1	760	35	1	1	5	57	35.12
2	760	35	1	6	20	59	35.42
3	760	140	300	1	5	65	36.26
4	760	140	300	6	20	45	33.06
5	900	35	300	1	20	67	36.52
6	900	35	300	6	5	73	37.27
7	900	140	1	1	20	51	34.15
8	900	140	1	6	5	48	33.62

Taguchi Analysis (Example Problem)

Solution: main effects of factors

$mA_1 = 1/4\ (\eta_1 + \eta_2 + \eta_3 + \eta_4) = 0.25 * (35.12+35.42+36.26+33.06) = 34.96$
$mA_2 = 1/4\ (\eta_5 + \eta_6 + \eta_7 + \eta_8) = 35.39$
$mB_1 = 1/4\ (\eta_1 + \eta_2 + \eta_5 + \eta_6) = 36.08$
$mB_2 = 1/4\ (\eta_3 + \eta_4 + \eta_7 + \eta_8) = 34.27$
$mC_1 = 1/4\ (\eta_1 + \eta_2 + \eta_7 + \eta_8) = 34.58$
$mC_2 = 1/4\ (\eta_3 + \eta_4 + \eta_5 + \eta_6) = 35.78$
$mD_1 = 1/4\ (\eta_1 + \eta_3 + \eta_5 + \eta_7) = 35.51$
$mD_2 = 1/4\ (\eta_2 + \eta_4 + \eta_6 + \eta_8) = 34.84$
$mE_1 = 1/4\ (\eta_1 + \eta_3 + \eta_6 + \eta_8) = 35.57$
$mE_2 = 1/4\ (\eta_2 + \eta_4 + \eta_5 + \eta_7) = 34.79$

Expt. No	Columns							
	1	2	3	4	5	HV	S/N ratio	η
1	760 (1)	35(1)	1 (1)	1 (1)	5 (1)	57	35.12	1
2	760 (1)	35(1)	1 (1)	6 (2)	20 (2)	59	35.42	2
3	760 (1)	140(2)	300 (2)	1 (1)	5 (1)	65	36.26	3
4	760 (1)	140(2)	300 (2)	6 (2)	20 (2)	45	33.06	4
5	900 (2)	35(1)	300 (2)	1 (1)	20 (2)	67	36.52	5
6	900 (2)	35(1)	300 (2)	6 (2)	5 (1)	73	37.27	6
7	900 (2)	140(2)	1 (1)	1 (1)	20 (2)	51	34.15	7
8	900 (2)	140(2)	1 (1)	6 (2)	5 (1)	48	33.62	8

Taguchi Analysis (Example Problem)

Solution: main effects of factors

$mA_1 = 1/4\ (\eta_1 + \eta_2 + \eta_3 + \eta_4) = 0.25 * (35.12+35.42+36.26+33.06) = 34.96$

$mA_2 = 1/4\ (\eta_5 + \eta_6 + \eta_7 + \eta_8) = 35.39$

$mB_1 = 1/4\ (\eta_1 + \eta_2 + \eta_5 + \eta_6) = 36.08$

$mB_2 = 1/4\ (\eta_3 + \eta_4 + \eta_7 + \eta_8) = 34.27$

$mC_1 = 1/4\ (\eta_1 + \eta_2 + \eta_7 + \eta_8) = 34.58$

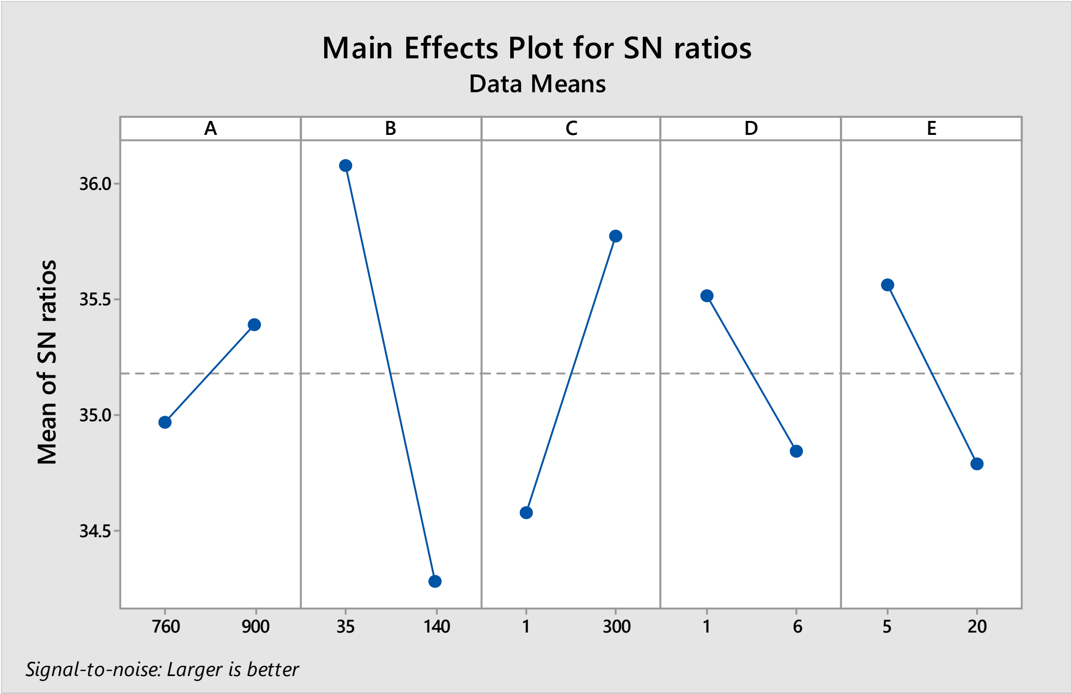
$mC_2 = 1/4\ (\eta_3 + \eta_4 + \eta_5 + \eta_6) = 35.78$

$mD_1 = 1/4\ (\eta_1 + \eta_3 + \eta_5 + \eta_7) = 35.51$

$mD_2 = 1/4\ (\eta_2 + \eta_4 + \eta_6 + \eta_8) = 34.84$

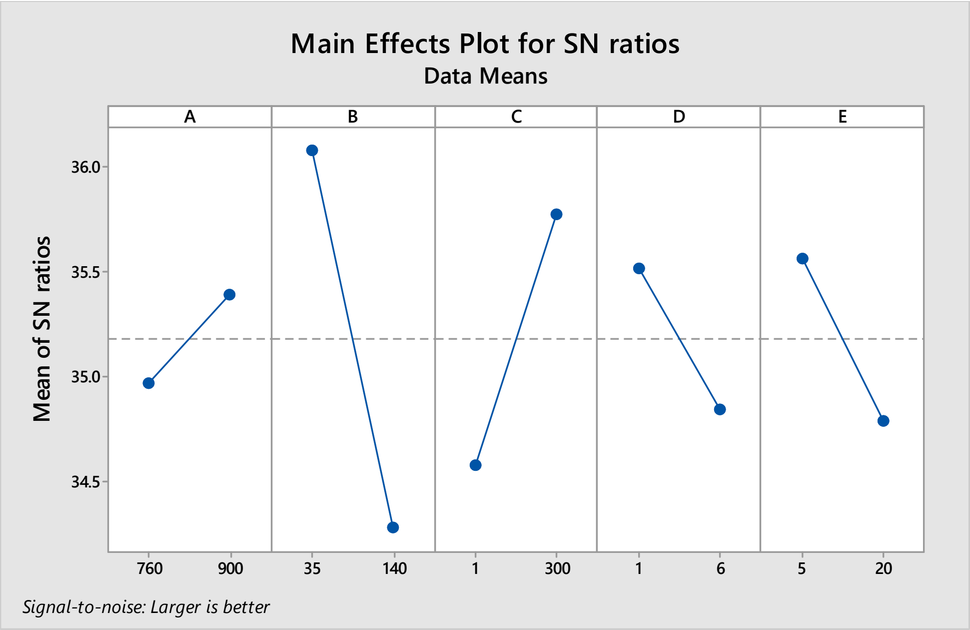
$mE_1 = 1/4\ (\eta_1 + \eta_3 + \eta_6 + \eta_8) = 35.57$

$mE_2 = 1/4\ (\eta_2 + \eta_4 + \eta_5 + \eta_7) = 34.79$



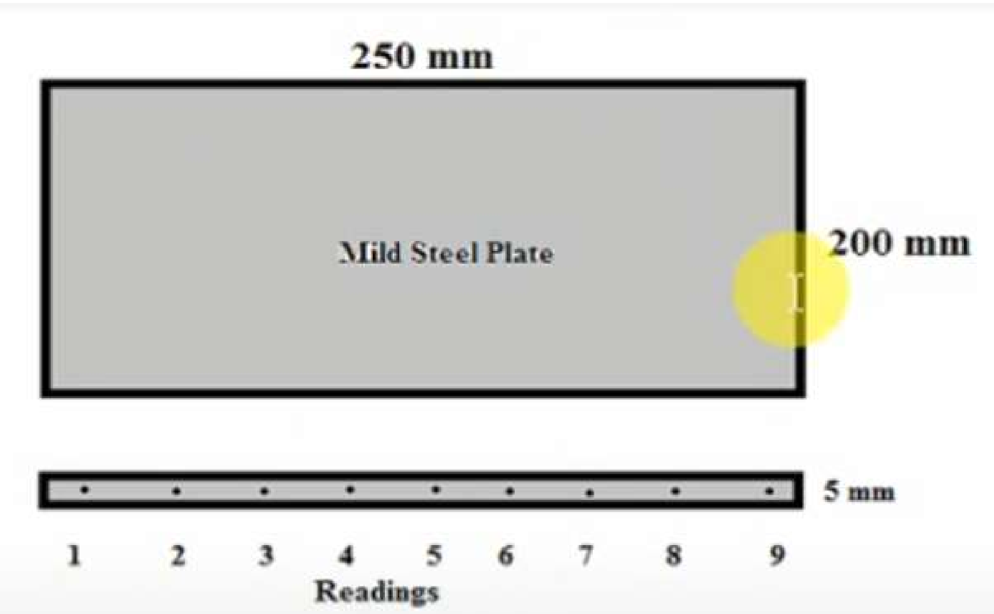
Taguchi Analysis (Example Problem)

Solution: A2, B1, C2, D1, E1 has largest value in S/N ratio. Hence, we can consider this as best combination of factors and its level to get higher hardness value (HV).



Taguchi Analysis (Example Problem)

Problem: Find out the optimum value for MS plate & Value of these shown in table.



Expt No.	Surface Roughness (μm)	Kerf (mm)
1	198.01	1.99
2	198.9	0.877
3	199.05	0.95
4	199.01	1.99
5	199.14	0.86
6	199.237	0.3
7	199.31	0.69
8	199.842	1.158
9	196.79	3.21

Given parameter

Cutting speed (mm/min)	Gas Provides (Bar)	Air Gap (mm)
500	1	1.5
600	2	1.8
700	3	2

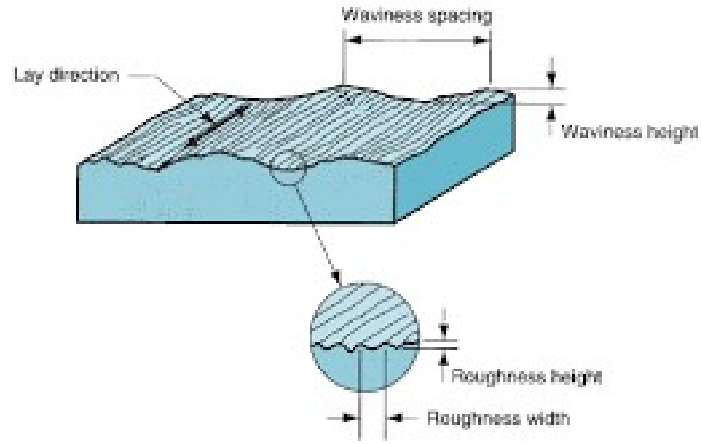
Control factors is:- Cutting speed, Gas Provider and Air Gap (stand Off distance)
Uncontrollable factors:- Surface Rpoughness and Kerf

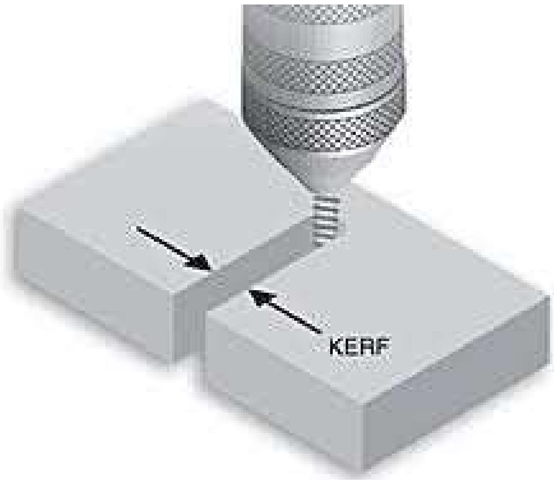
Taguchi Analysis (Example Problem)

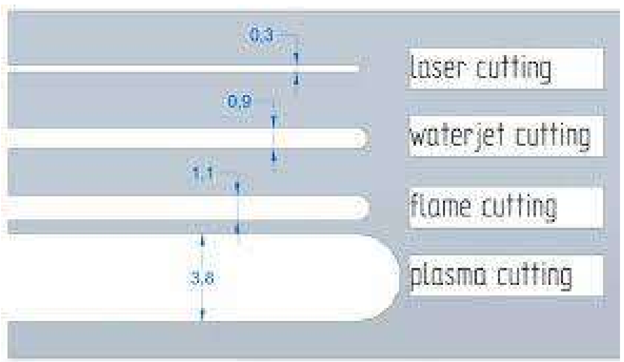
Problem:

Surface roughness:

Kerf:







Taguchi Analysis (Example Problem)

Solution:

How many factors? How many level? Which Orthogonal Array? Which S/N Ratio?

Expt No.	Surface Roughness (μm)	Kerf (mm)
1	198.01	1.99
2	198.9	0.877
3	199.05	0.95
4	199.01	1.99
5	199.14	0.86
6	199.237	0.3
7	199.31	0.69
8	199.842	1.158
9	196.79	3.21

Given parameter

Cutting speed (mm/min)	Gas Provides (Bar)	Air Gap (mm)
500	1	1.5
600	2	1.8
700	3	2

Control factors is:- Cutting speed, Gas Provider and Air Gap (stand Off distance)
Uncontrollable factors:- Surface Rpoughness and Kerf

Taguchi Analysis (Example Problem)

Orthogonal Array	No. Runs	Max. Factors	Max. of columns at these levels			
			2-level	3-level	4-level	5-level
L4	4	3	3			
L8	8	7	7			
L9	9	4		4		
L12	12	11	11			
L16	16	15	15			
L'16	16	5			5	
L18	18	8	1	7		
L25	25	6				6
L27	27	13		13		
L32	32	31	31			
L'32	32	10	1		9	
L36	36	23	11	12		
L'36	36	16	3	13		
L50	50	12	1			11
L54	54	26	1	25		
L64	64	63	63			
L'64	64	21			21	
L81	81	40		40		

Solution:

How many factors? 3 factors

How many level? 3 levels

Which Orthogonal Array? L9 orthogonal array

Which S/N Ratio? Both surface roughness and Kerf needs to be smaller.

Hence, Smaller is better to be used to calculate S/N ratio.

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Which Orthogonal Array? L9 orthogonal array

Which S/N Ratio? Surface roughness needs to be smaller.

Hence, Smaller is better to be used to calculate S/N ratio.

Smaller-the-Better: η = -10 log10(y^2)

Hence solving for η

Taguchi Analysis (Example Problem)

Solution:

Smaller-the-Better: η = -10 log10(y^2)

Hence solving for η

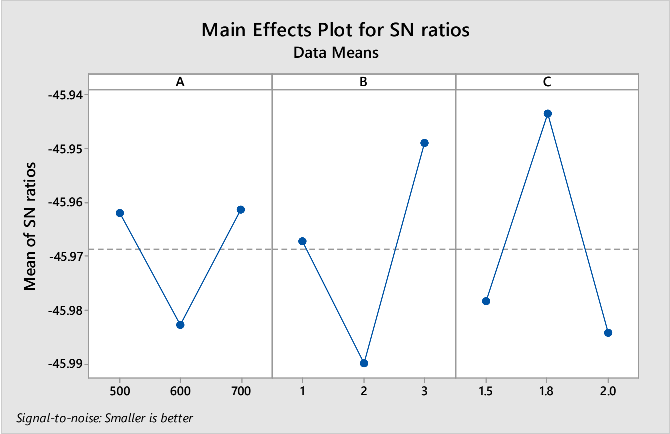
A	B	C
1	1	1
1	2	2
1	3	3
2	1	2
2	2	3
2	3	1
3	1	3
3	2	1
3	3	2

A	B	C	Surface Roughness	S/N Ratio
500	1	1.5	198.01	-45.9337
500	2	1.8	198.9	-45.9727
500	3	2	199.05	-45.9792
600	1	1.8	199.01	-45.9775
600	2	2	199.14	-45.9832
600	3	1.5	199.237	-45.9874
700	1	2	199.31	-45.9906
700	2	1.5	199.842	-46.0137
700	3	1.8	196.79	-45.8801

η = -10 log10(y^2)

```
Smaller is better

Level        A       B       C
1       -45.96  -45.97  -45.98
2       -45.98  -45.99  -45.94
3       -45.96  -45.95  -45.98
Delta     0.02    0.04    0.04
Rank         3       1       2
```



Application of Optimization Techniques

- Design of aircraft structures at minimum weight.
- Finding the optimum trajectory of space vehicle.
- Design of civil engineering structures like frames, foundation, bridges, towers, chimney at minimum weight.
- Selection of machining condition in metal cutting process for minimum production cost.
- Optimum design of electrical networks.
- Allocation of resources / services to several activities to maximize the benefits.
- Controlling the idle time, waiting time and queuing in production line to reduce the cost.
- Design of pumps, turbines and heat transfer equipments for maximize the efficiency.
- Shortest route to be taken by sales person to visit different cities.
- Design of material handling equipments for minimize the cost.

Thank you

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