Closures of a set of functional dependencies

A Closure is a set of FDs is a set of all possible FDs that can be derived from a given set of FDs. It is also referred as a complete set of FDs. If F is used to donate the set of FDs for relation R, then a closure of a set of FDs implied by F is denoted by F^+. Let's consider the set F of functional dependencies given below: F = {A -> B, B -> C, C -> D} from F, it is possible to derive following dependencies.

A -> A ...By using Rule-4, Self-Determination.

A -> B ...Already given in F.

A -> C ...By using rule-3, Transitivity.

A -> D ...By using rule-3, Transitivity.

Now, by applying Rule-6 Union, it is possible to derive A^+ -> ABCD and it can be denoted using A -> ABCD. All such type of FDs derived from each FD of F form a closure of F. Steps to determine F^+ example:

- Determine each set of attributes X that appears as a left hand side of some FD in F.
- Determine the set X^+ of all attributes that are dependent on X, as given in above example.
- In other words, X^+ represents a set of attributes that are functionally determined by X based on F. And, X^+ is called the Closure of X under F. All such sets of
- X^+, in combine, Form a closure of F.

Algorithm : Determining X^+, the closure of X under F.

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Input : Let F be a set of FDs for relation R.

Steps:
            1. X+ = X                             //initialize X+ to X
            2. For each FD : Y -> Z in F Do
               If Y ? X+  Then                    //If Y is contained in X+
               X+  =   X+  ?  Z                   //add Z to X+
               End If
               End For
            3. Return  X+                       //Return closure of X

Output : Closure  X+ of X under F
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