DO 1: 04:00 PM – 04:50 PM
DO 2: 09:40 AM – 11:30 PM

Subject Code

18MEO113T - Design of Experiments

Handled by

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Disclaimer

The content prepared in the presentation are from various sources, only used for education purpose. Thanks to all the sources.

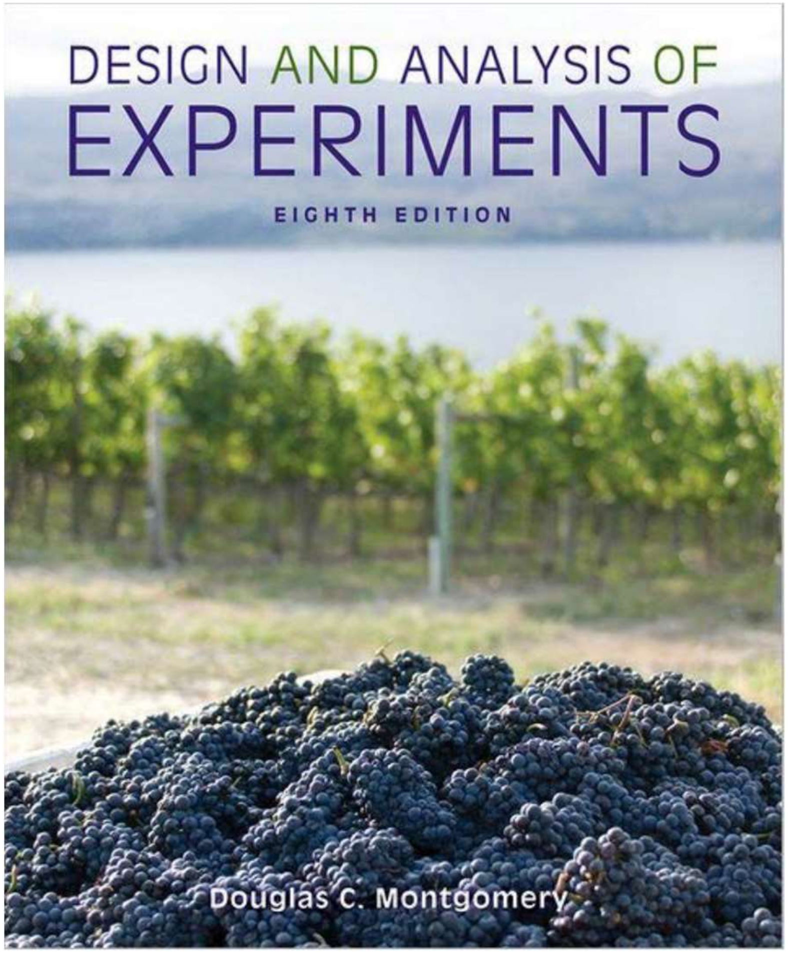
18MEO113T – Design of Experiments

- To learn the fundamentals of design of experiment techniques.
- To familiarize in how to setup experiments and accomplish all analyze tasks using software packages like Minitab, etc.
- After this course, You will be ready to apply the technique confidently to all of your projects .

Course Plan

Duration (hour)		9	9	9	9	9
S-1	SLO-1	Introduction in Design of experiments (DOE)	Need for DOE methodology	Introduction to Robust design, Loss functions	Background of response surface design	Introduction and uses of confounding
S-2	SLO-1	The fundamental and potential practical problems in experimentation	Barriers in the successful application of DOE	Eight steps in Taguchi methodology	Multiple Responses and Contour profile of response surface plot	2^3 factorial experiment with complete confounding
S-3	SLO-1	Statistical thinking and its role within DOE	Practical methodology of DOE and Analytical tools for DOE	Orthogonal array, Selecting the interaction, Linear graphs	Creation of response surface designs	2^3 factorial experiment with partial confounding
S-4	SLO-1	Basic principles of DOE and Degrees of freedom	The confidence interval for the mean response	S/N ratio: Larger-the-better, Smaller-the-better, Nominal-the-best	Central composite designs (Rotatable central composite design)	Confounding in the 2^n series and examples
S-5	SLO-1	Selection of quality characteristics for experiments	Introduction to Screening design	Analyze the data, factor effect diagram	Central composite designs (Rotatable central composite design)	Confounding of 3^n factorial and examples
S-6	SLO-1	Understanding key interaction in processes	Geometric and non-geometric P-B design	Levels of parameters	Box-Behnken design with case studies	ANOVA (One-way and two-way, higher-way ANOVA)
S-7	SLO-1	An alternative method for calculating two-order interaction effect	Introduction of full factorial design, Basic concepts of 2^2, 2^3 and 2^k designs	Confirmation test	Random factor models and its industrial application , Random Effects Models	MANOVA and ANCOVA overview
S-8	SLO-1	Synergistic interaction, Antagonistic interaction	Solving Case studies on Full factorial design with statistics software	Augmented design with simple case studies	Two Factor Factorial with Random Factors	Solving Case studies on ANOVA with statistics software
S-9	SLO-1	Synergistic interaction versus Antagonistic interaction	Solving Case studies on Full factorial design with statistics software	Solving case studies on robust design with statistics software	Two Factor Mixed Models with random factors	Regression Models and Regression Analysis

Course Plan


DESIGN AND ANALYSIS OF
EXPERIMENTS
EIGHTH EDITION
Douglas C. Montgomery

Unit 1

1: Introduction in Design of Experiments (DOE).

2: The fundamental and potential practical problems in experimentation

3: Statistical thinking and its role with DOE

4: Basic principles of DOE and Degrees of Freedom

5: Selection of quality characteristics for experiments

6: Understanding key interaction in processes

7: An alternative method for calculating two-order interaction effect

8: Synergistic interaction, Antagonistic interaction

9: Synergistic interaction verus Antagonistic interaction

What is Experiment?

- **Experiment** is an important activity performed by all in day to day activity.

- An **experiment is a test or a series of tests** conducted to analyse, understand or improve the system.

- The only way to **learn** anything about a system is to **disturb it and then observe it**.

What is Experiment?

- An **Experiment** as a test or series of runs in which **purposeful changes** are made to the **input variables** of a process or system so that we may **observe and identify** the reasons for changes that may be observed in the **output response**.
- Observations on a system or process can lead to theories or hypotheses about what makes the system work, but experiments are required to demonstrate that these theories are correct.

What is Experiment?

- **Efficiency** is doing the least amount of work and getting the most information.

- **Objective** is to improve the outcome and there are **two or more factors** we can **change** to make that improvement or influence the outcome.

- So, there's an outcome/response and there are **factors** that we want to improve.

What is Experiment?

Aluminium Pot

Constant Heat,
Same amount of water

Ceramic Pot

Outcome : Time taken to Boil

Other Variables : source of heat, without lid, Material, shape of pot etc…

What is Experiment?

Aluminium Pot | **Ceramic Pot**

Constant Heat,
Same amount of water

Control Variable: Aluminum Pot

Experiment Variable: Ceramic Pot

Independent Variable: Type of material used for pot

Dependent Variable: Time taken to Boil

Outcome : minimize the time taken to Boil

Other Variables : source of heat, without lid, Material, shape of pot etc…

What is Experiment?

Outcomes = Response
Variables / Attributes = Factors

Types of factors:
Numeric: Quantative
Categorical : Qualitative

Objective:
Combine outcome by adjusting choosing the variable
Some Examples: Maximize the profit, yield, Minimise the expense, loss, energy

What is Experiment?

Spike shoe

Sports shoe

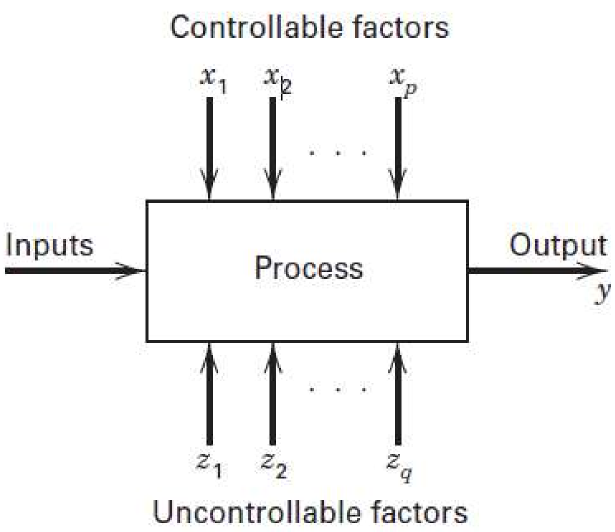
Same person, day, playground

Outcome : Time taken to reach Target

Other Factors : Weather, Cloth, location

Process Model

- Determining which Control variables (Input) X, are **most influential** on the response (output), y
- Determining where to **set the influential** 'x' so that 'y' is almost always near the desired nominal value.
- Determining where to set the influential 'x' so that **variability** in 'y' is small.
- Determining where to set the influential 'x' so that the **effect of uncontrollable variables** 'z' are minimized.



Objective with Mathematical expression

Output can be represented as function of input and controllable variables.

$$y = f(x, z); \qquad \nabla y = \frac{\partial f}{\partial x}\nabla X + \frac{\partial f}{\partial z}\nabla Z$$

The objective of the experiment may include the following:

a. To know about the input variable x and controllable variable z that will affect the value of output variable y; This phenomena is known as **process characterization**.

b. To capture the changes of x's [∇X] so that the value of y is always near the desired nominal value; This phenomena is known as **process control**.

Objective with Mathematical expression

$$y = f(x, z); \qquad \nabla y = \frac{\partial f}{\partial x}\nabla X + \frac{\partial f}{\partial z}\nabla Z$$

c. To capture the changes of x's $[\frac{\partial f}{\partial x}]$ so that the variability of y is minimized; This phenomena is known as **process optimization**.

d. To capture the changes of x's so that the effects of uncontrollable variables z's are minimized; here putting [∇y = 0] , Therefore, $\frac{\partial f}{\partial x}\nabla X = -\frac{\partial f}{\partial z}\nabla Z$. This phenomena is known as **robust design**.

DOE Terminology

- **Response**
  - A measureable outcome of interest. Eg: Time, etc.
- **Factors**
  - Controllable variables that are deliberately manipulated to determine the individual and joint effects on the response(s), OR Factors are those quantities that affect the outcome of an experiment, Example: heat source, lid, shape of pot etc.
- **Levels**
  - Levels refer to the values of factors i.e., the values for which the experiments will be conducted
  - Eg: Level – 1 heat input = low & Level – 2 heat input = high

DOE Terminology

- **Treatment**
  - A set of specified factor levels for an experiments or run
    - Treatment – 1: Heat input = low and with lid
    - Treatment – 2: Heat input = high and without lid
- **Noise**
  - Variables that affect product/ process performance, whose values cannot be controlled or are not controlled for economic reasons. Eg. Climate, environment, etc.…
- **Replication**
  - Replication is a **systematic duplication** of series of experimental runs. It provides the means of **measuring precision** by calculating the **experimental error**.

DOE Terminology

Factor

- A **variable or attribute** which influences or is suspected of influencing the characteristic being investigated.
- All input variables which affect the output of a system are factors
- Factors are varied in the experiment
- They can be controlled at fixed levels.
- They can be varied or set at levels of our interest.

DOE Terminology

Factor

- Two types of variables or factors : **quantitative and qualitative**
- **Quantitative** – how the range of settings to be measured and controlled

  e.g. temperature , pressure in the process yield experiment
- **Qualitative** – discrete in nature ; requires more levels

  e.g. types of raw materials, type of catalysts, type of supplier, etc.
- These are also called **independent variables**.

DOE Terminology

Level

– Specified value or setting of the factor being examined in the experiment.

– The values of a factor/independent variable being examined in an experiment.

– **If the factor is an attribute**, each of its **state is a level**.

For example, setting of a switch on or off are the two levels of the factor switch setting.

DOE Terminology

Level

- **If the factor is a variable**, the range is divided into required number of levels.

  For example, the factor temperature ranges from 1000 to 1200°C and it is to be studied at three values say 1000°C, 1100°C and 1200°C, these three values are the three levels of the factor temperature.

- **The levels can be fixed or random.**

DOE Terminology

Treatment

– One set of levels of all factors employed in a given experimental trial.

For example, an experiment conducted using **temperature T1 and pressure P1** would constitute one treatment.

– In the case of single factor experiment, each level of the factor is a treatment.

Run or trial :

– a trial or run is a certain combination of factor levels whose effect on the output (or performance characteristic) is of interest.

DOE Terminology

Experimental unit

- Facility with which an experimental trial is conducted such as samples of material, person, animal, plant, etc.

Response:

- The result/output obtained from a trial of an experiment. This is also called dependent variable. Examples are yield, tensile strength, surface finish, number of defectives, etc.

Effect :

- Effect of a factor is the change in response due to change in the level of the factor.

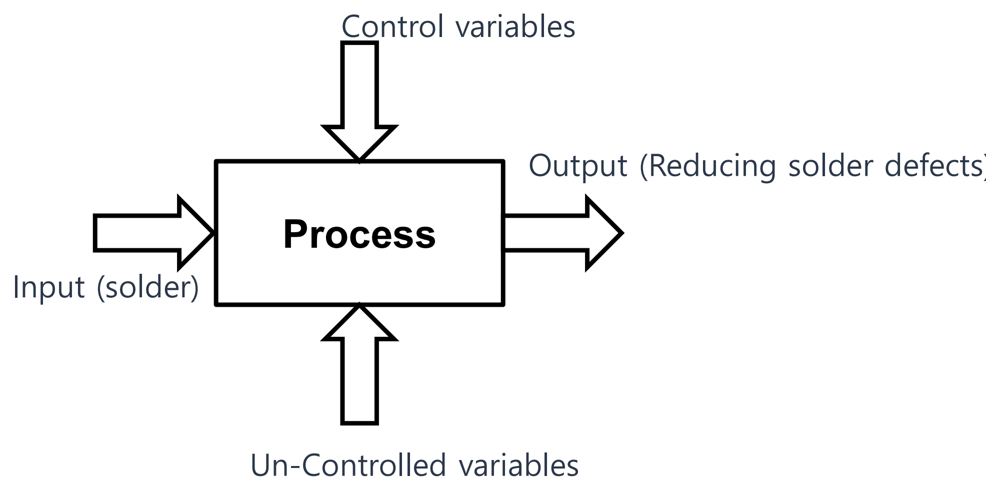
DOE Terminology

Experimental error:

- It is the **variation in response** when the same experiment is repeated, caused by **conditions not controlled** in the experiment.
- It is estimated as the **residual variation** after the effects have been removed.

Example

Example 1: A flow solder machine is used in the manufacturing process for printed circuit boards. The process engineer would like to conduct a designed experiments to determine which controllable variables is significantly affecting the output quality. And also, to find out the way to reduce the variability of output due to change in uncontrollable variable.

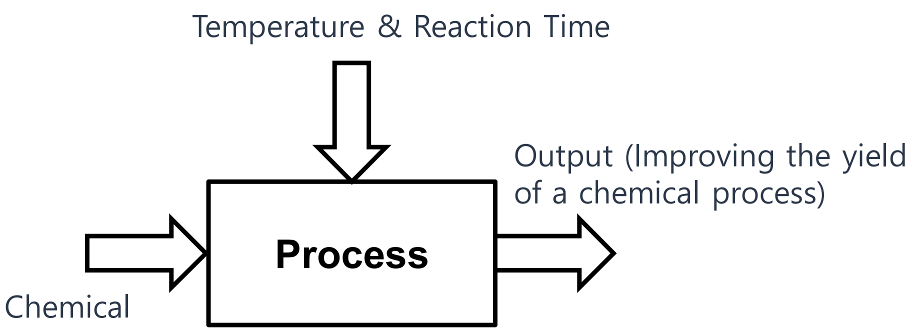


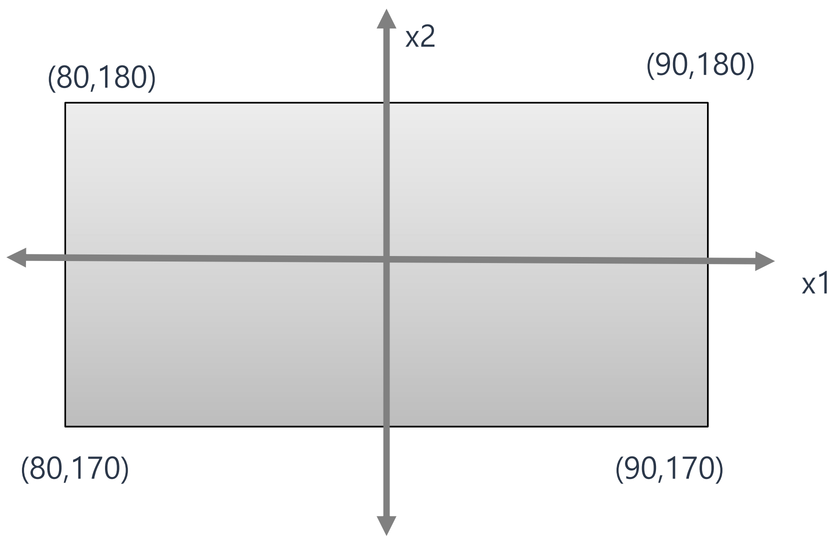
Example

Controllable factors	Un-Controllable factors
1. Solder temperature	1. Thickness of the printed circuit board
2. Preheat temperature	2. Types of components used on the board
3. Conveyor speed	3. Layout of the components on the board
4. Flux Type	4. Operator
5. Flux specific gravity	5. Production rate
6. Solder wave depth	
7. Conveyor angle	

Example

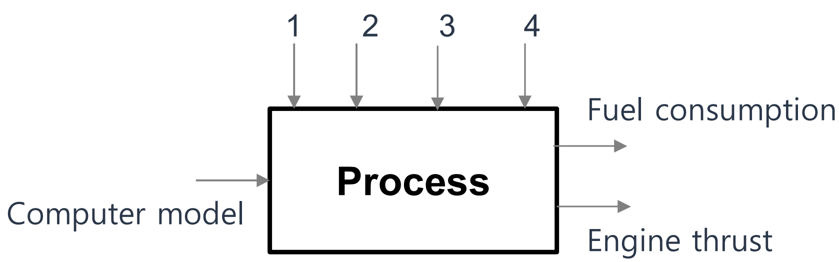
Example 2: In this example, we are interested to improve the yield of a chemical process through using the two most important process variables: reaction time (x1) and operating temperature (x2). As the given response surface is unknown to the process personnel, experimental set up will be required to investigate it.





Example

Example 3: The response variables of interest in designing an aircraft engine are fuel consumption and engine thrust. The engineer in-charge uses a computer model of the system where he can vary the following designed parameters (inlet flow, fan pressure ratio, overall pressure, stator outlet temperature) to optimize the performance of the engine.



Strategy of Experimentation

- Experimental Strategies
  - Best Guess
  - Conventional Strategy
    - One factor at a Time
      - Several Factors, one at a Time
        - Several factors, all at same time
  - Strategic Design

Best-Guess Approach

- **Best-Guess Approach**
  - Eg: **Play Golf** / Machining a part / Preparing a concrete / Preparing a sample for new application / etc.

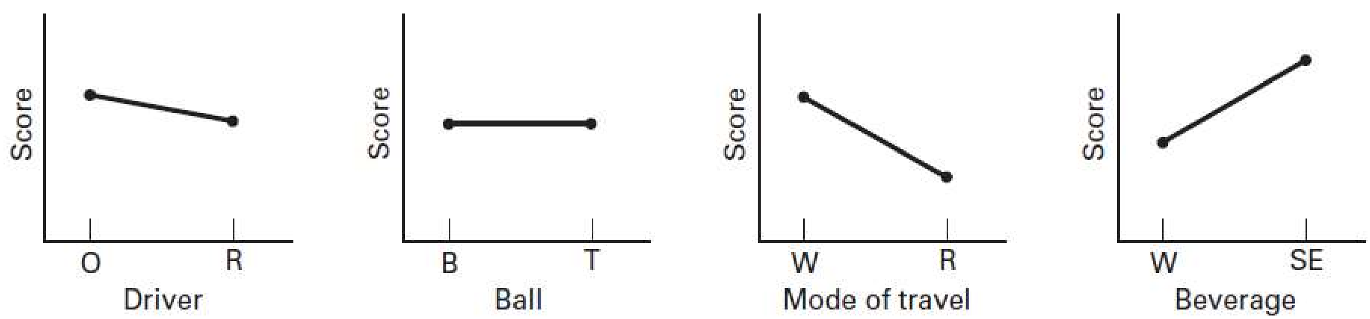
1. The type of driver used (oversized or regular sized)
2. The type of ball used (balata or three piece)
3. Walking and carrying the golf clubs or riding in a golf cart
4. Drinking water or drinking "something else" while playing
5. Playing in the morning or playing in the afternoon
6. Playing when it is cool or playing when it is hot
7. The type of golf shoe spike worn (metal or soft)
8. Playing on a windy day or playing on a calm day.

Best-Guess Approach

- **Best-Guess Approach**
  - Disadvantages.
  - **First**, suppose the initial best-guess does not produce the desired results. Now the experimenter has to take another guess at the correct combination of factor levels. This could continue for a long time, without any guarantee of success.
  - **Second**, suppose the initial best-guess produces an acceptable result. Now the experimenter is tempted to stop testing, although there is no guarantee that the best solution has been found.

Conventional Strategies

- **One Factor at a Time (OFAT)**
  - The OFAT method consists of **selecting a starting point, or baseline** set of levels, for each factor, and then successively varying each factor over its range with the other factors held constant at the baseline level. After all tests are performed, a series of graphs are usually constructed showing how the response variable is affected by varying each factor with all other factors held constant.

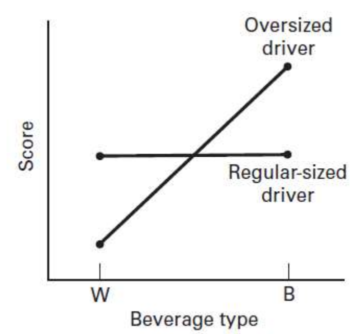


■ **FIGURE 1.2** Results of the one-factor-at-a-time strategy for the golf experiment

Conventional Strategies

One Factor at a Time (OFAT)

- **Disadvantage**. OFAT method **fails to consider** any possible interaction between the factors.
- Interactions between factors are very common, and if they occur, the OFAT strategy will usually produce **poor results**



■ **FIGURE 1.3** Interaction between type of driver and type of beverage for the golf experiment

Conventional Strategies

One Factor at a time

Trial	Factor	Test Result		Test Average
1	A1	-	-	Y1
2	A2	-	-	Y2

Several Factor One at a time

	Factor				Test Result		Test Average
Trial	A	B	C	D			
1	1	1	1	1	-	-	Y1
2	2	1	1	1	-	-	Y2
3	1	2	1	1	-	-	Y3
4	1	1	2	1	-	-	Y4
5	1	1	1	2	-	-	Y5

Conventional Strategies

Several Factors at same time

	Factor				Test Result		Test Average
Trial	A	B	C	D			
1	1	1	1	1	-	-	Y1
2	2	2	2	2	-	-	Y2

Disadvantage:

– Not possible to attribute the change in result to any of the factor/s.

– Effect of interaction between factors **cannot be studied**

– These are **poor experimental strategies** and there is no scientific basis. The results cannot be validated

Strategy Design

Full Factorial Design

	Factor			
Trial	A	B	Response	
1	1	1	-	-
2	1	2	-	-
3	2	1	-	-
4	2	2	-	-

- It is balanced and also orthogonal
- Factor A does not influence the estimate of the effect of factor B and vice versa.
- Both factor and interaction effects can be estimated.

Limitation

- Only few factors can be investigated
- When several factors are to be investigated, the number of experiments to be run under full factorial design is very large.

Need for Strategy Design

- To obtain **unambiguous** results at minimum cost.
- Permits **consideration of all possible** variables/factors simultaneously
- A valid evaluation of the **main factors effects** and the **interaction effects** can be obtained.
- With limited number of experiments the **crucial factors** can be obtained.
- The statistical concepts used in the design form the basis for **validating** the results

Statistical Thinking and its Role within DOE

- One of the success factors for the effective deployment of DOE in any organization is the **uncompromising commitment** of the senior management team and visionary leadership.

- However, it is not essential that the senior managers have a good technical knowledge of the working mechanisms of DOE.

Statistical Thinking and its Role within DOE

Statistical thinking is a philosophy of learning and action based on the following three fundamental principles (Snee, 1990)

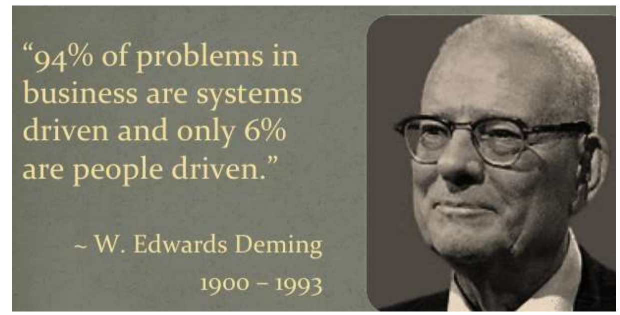
(1) All work occurs in a system of interconnected processes.

(2) Variation exists in all processes.

(3) Understanding and reducing variation are the key to success.

Statistical Thinking and its Role within DOE

- The importance of statistical thinking derives from the fundamental principle of quality put forth by *Deming*: '**Reduce variation and you improve quality**'.
- Customers of today and tomorrow value products and services that consistently perform, which can be achieved **by systematically eliminating variation** in business processes (*ASQ, 1996*).



Statistical Thinking and its Role within DOE

However, our managers lack statistical thinking and some of the possible reasons for this are as follows:

1. A shift in the organization's priorities - Global competition has forced managers to rethink how organisations are run and to search for better ways to manage. Problem solving in manufacturing and R&D, while important, **is not seen** as particularly relevant to the needs of management.

Statistical Thinking and its Role within DOE

2. Managers view statistics as a tool for ‘fire fighting’ actions - One of the most difficult challenges for every manager is to figure out how to use statistical thinking effectively to help them make effective decisions.

When a problem arises in the business, managers want to fix it as soon as possible so that they can deal with their day-to-day activities. However, what they do not realise is that the majority of problems are in systems or processes that can only be tackled with the support of senior management team.

The result is that management spends too much time ‘fire fighting’, solving the same problem again and again because the system was not changed.

Statistical Thinking and its Role within DOE

3. A change in the mindset of people in the enterprise - 'If you cannot change your mind, you cannot change anything' *by Philosopher George Bernard Shaw.*

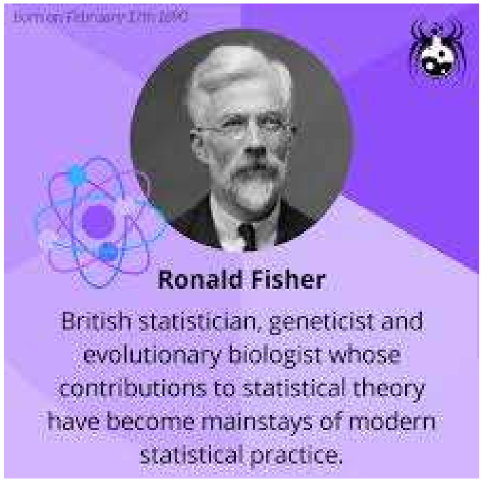
It is clear that managers, quality professionals and statisticians all have new roles that require new skills. Change implies discontinuity and the destruction of familiar structures and relationships. Change can be resisted because it involves confrontation of the unknown and loss of the familiar (Huczynski and Buchanan, 2001)

Statistical Thinking and its Role within DOE

4. Fear of statistics by managers - 'Even if managers were taught statistics at university, it was usually focused on complex maths and formulas **rather than** the application of statistical tools for problem solving and an effective decision-making process. Usually managers have their **first experience** with statistical thinking in a workshop inside the company, applying some tools with the guidance of an expert. **Although** this is the best learning method for understanding and experiencing statistical thinking, managers may **still struggle to apply** the principles to a different problem. This fundamental problem can be tackled by teaching usable and practical statistical techniques through real case studies at the university level.

Brief history of DOE

- **The agricultural origins, 1918-1940s**
  - R. A. Fisher & his co-workers
  - Profound impact on agricultural science
  - Factorial designs, ANOVA
- **The First industrial era, 1951- late 1970s**
  - Box & Wilson, response surfaces
  - Applications in the chemical & process industries
- **The Second industrial era, late 1970s – 1990s**
  - Quality improvement initiatives in many companies
  - Taguchi and robust parameter design, process robustness
- **The modern era, beginning circa 1990**



Skills required for an industrial designed experiment

Planning skills: Understanding the significance of experimentation for a particular problem, time and experimental budget required for the experiment, how many people are involved with the experimentation, establishing who is doing what, etc.

Statistical skills: The statistical analysis of data obtained from the experiment, assignment of factors and interactions to various columns of the design matrix (or experimental layout), interpretation of results from the experiment for making sound and valid decisions for improvement, etc.

Skills required for an industrial designed experiment

Teamwork skills: Understanding the objectives of the experiment and having a shared understanding of the experimental goals to be achieved, better communication among people with different skills and learning from one another, brainstorming of factors for the experiment by team members, etc.

Engineering skills: Determination of the number of levels of each factor and the range at which each factor can be varied, determination of what to measure within the experiment, determination of the capability of the measurement system in place, determination of what factors can be controlled and what cannot be controlled for the experiment, etc.

Characteristics of a well-planned experiment

As per, Cox 1958

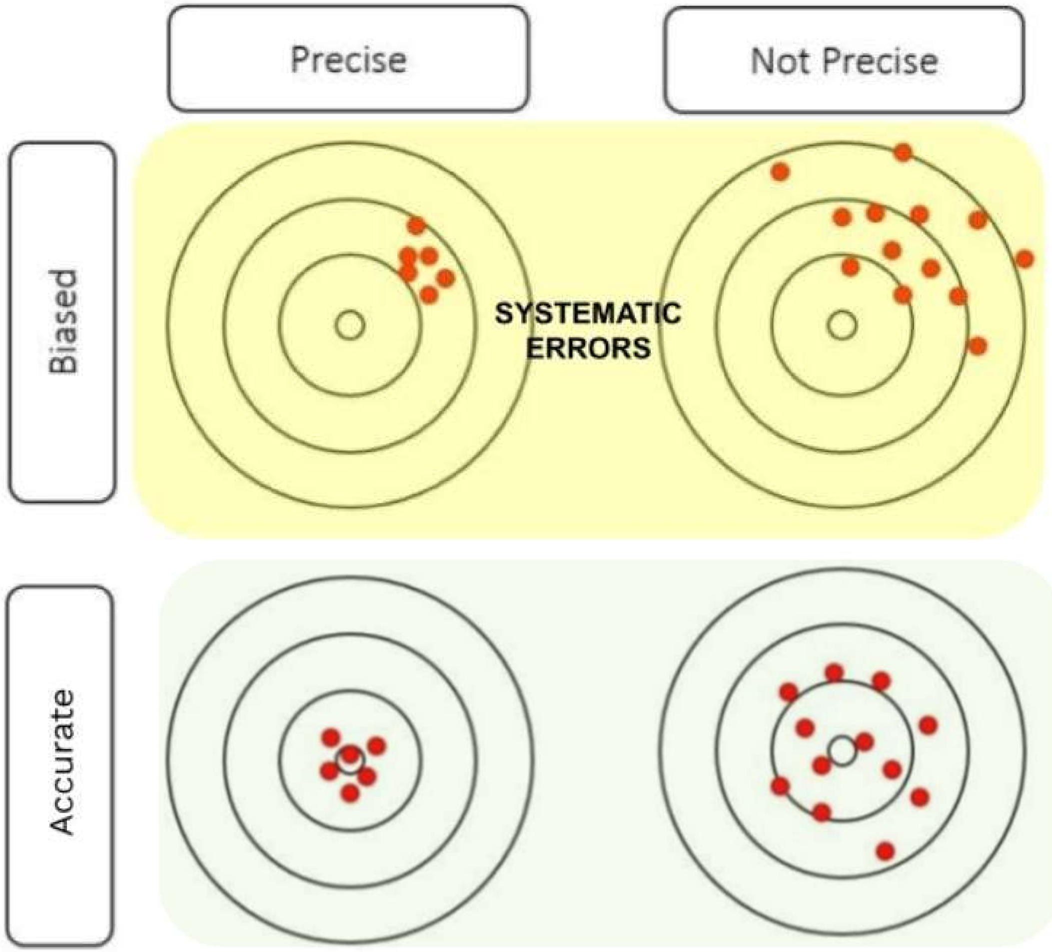
1. **Degree of precision** → the probability should be high that the experiment will be able to measure differences with the degree of precision the experimenter desires. This implies an appropriate design and sufficient replication. (i.e., the standard deviation should be **minimal** from the experiment)
2. **Simplicity** → the design should be as **simple** as possible, **consistent** with the **objective** of the experiment.
3. **Absence of systematic error** → Experimental units receiving one treatment **should not differ** in any systematic way from those receiving another treatment so that an **unbiased estimate** of each treatment effect can be obtained.

Characteristics of a well-planned experiment

4. **Range of validity of conclusions** → Conclusions should have as wide a **range of validity** as possible.

5. **Calculation of degree of uncertainty** → the experiment should be designed so that it is possible to calculate the possibility of obtaining the observed result by chance alone.

Characteristics of a well-planned experiment



Unbiased

Basic principles of experimentation

The basic principles of experimental design are **Randomization**, **Replication** and **Blocking** (Montgomery 2014). These principles make a **valid test** of significance possible.

Randomization

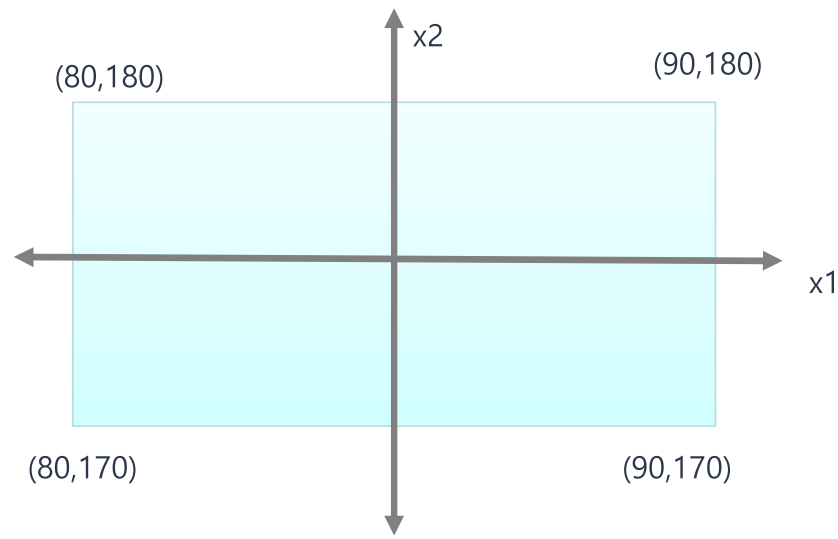
A statistical tool used to **minimize potential uncontrollable biases** in the experiment by **randomly** assigning material, people, order in the experimental trails to be conducted.

Purpose: to remove bias and variation

Example: chemical processing

X1: Reaction Time (80~90 min)

X2: Temperature (170 ~180 degree)



Randomization

Dorian Shainin - '**failure to randomize** the trial conditions **mitigates/reduces** the **statistical validity of an experiment**'.

- The allocation of experimental units (samples) for conducting the experiment as well as the order of experimentation should be random.
- When complete randomization is not possible, appropriate statistical design methods shall be used to tackle restriction on randomization.

Randomization

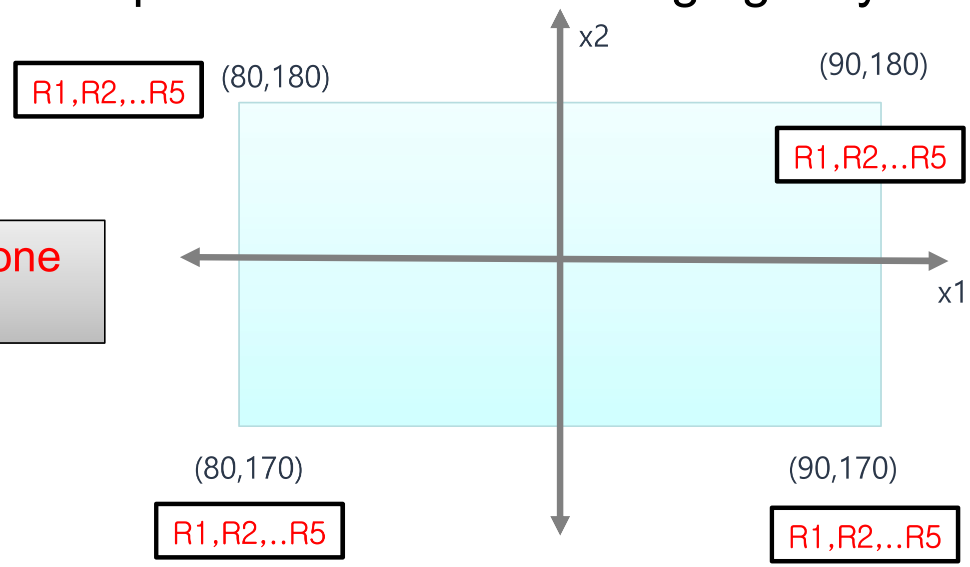
The following questions are useful if you decide to apply randomization strategy to your experiment.

- What is the cost associated with change of factor levels?
- Have we incorporated any noise factors in the experimental layout?
- What is the set-up time between trials?
- How many factors in the experiment are expensive or difficult to control?
- Where do we assign factors whose levels are difficult to change from one to another level?

Replication

The replication is a repetition of the basic experiment without changing any factor settings.

Each run needs to be conduct more than one time in random order.



Purpose:

- To ensure a more accurate estimate of the experimental error
- To increase the precision of estimate of error, which is a measure of the variability of the experimental error
- Obtain a more precise estimate of the mean effect of a treatment.

Replication

Replication involves the repetition of the experiment and obtaining the response from the same experimental set up once again on **different experimental unit (samples)**.

The purpose of replication is to obtain the **magnitude of experimental error**. This error estimate (error variance) is used for testing statistically the observed difference in the experimental data.

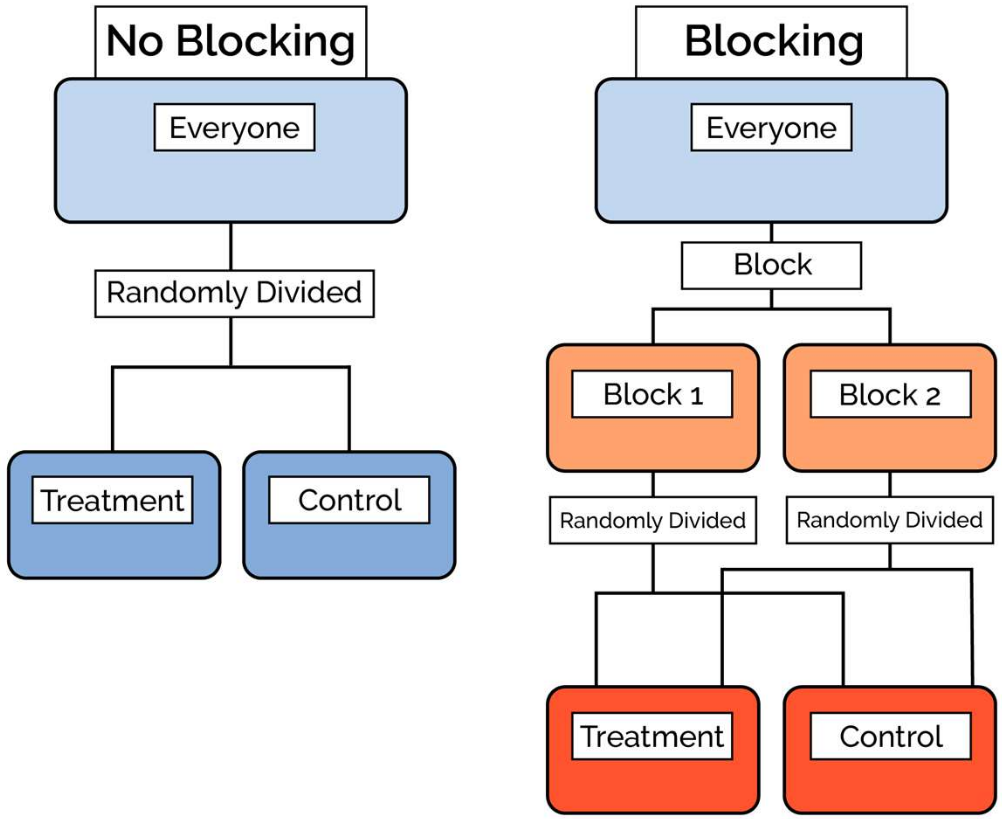
Blocking

A technique used to **increase the precision** of an experiment by **breaking the experiment** into **homogeneous** segments (blocks) in order to control block variability. It basically deals with noise factors.

Purpose: the main purpose of the principle of blocking is to **increase the efficiency** of an experimental design by **decreasing** the experimental error.

A metallurgist wants to improve the strength of a steel product. Four factors are being considered for the experiment, which might have some impact on the strength. It is decided to study each factor at 2-levels (i.e. a low setting and a high setting). An eight-trial experiment is chosen by the experimenter but it is possible to run only four trials per day. Here each day can be treated as a separate block.

Blocking



No Blocking
Everyone
Randomly Divided
Treatment
Control
Blocking
Everyone
Block
Block 1
Block 2
Randomly Divided
Randomly Divided
Treatment
Control

Steps in experimentation

Little and Hills 1978 suggested that:

- Define the problem
- Determine the objectives
- Select the treatments
- Select the experimental material
- Select the experimental design
- Select the experimental unit and number of replications
- Ensure proper randomization and layout

Steps in experimentation

- Ensure proper means of data collection
- Outline the statistical analysis before doing the experiment
- Conduct the experiment
- Analyze the data and interpret the results
- Prepare complete and readable reports.

Steps in experimentation

1. Problem statement
2. Selection of factors, levels and ranges
3. Selection of response variable
4. Choice of experimental design
5. Conducting the experiment
6. Analysis of data
7. Conclusions and recommendations

1. Problem statement

- A clear definition of the problem contributes to better understanding of the process being studied and the final solution of the problem.
- An **insight** into the problem leads to a better design of the experiment.

Example:

1. If there is a rejection in bore diameter, specify whether the rejection is due to oversize or undersize or both.
2. A variation in the compression force of a shock absorber., specify clearly if there is less force or more force or both.
3. A defect like crack, blister, etc., specify whether its observed as random phenomenon on the product or concentrated to one specific area.

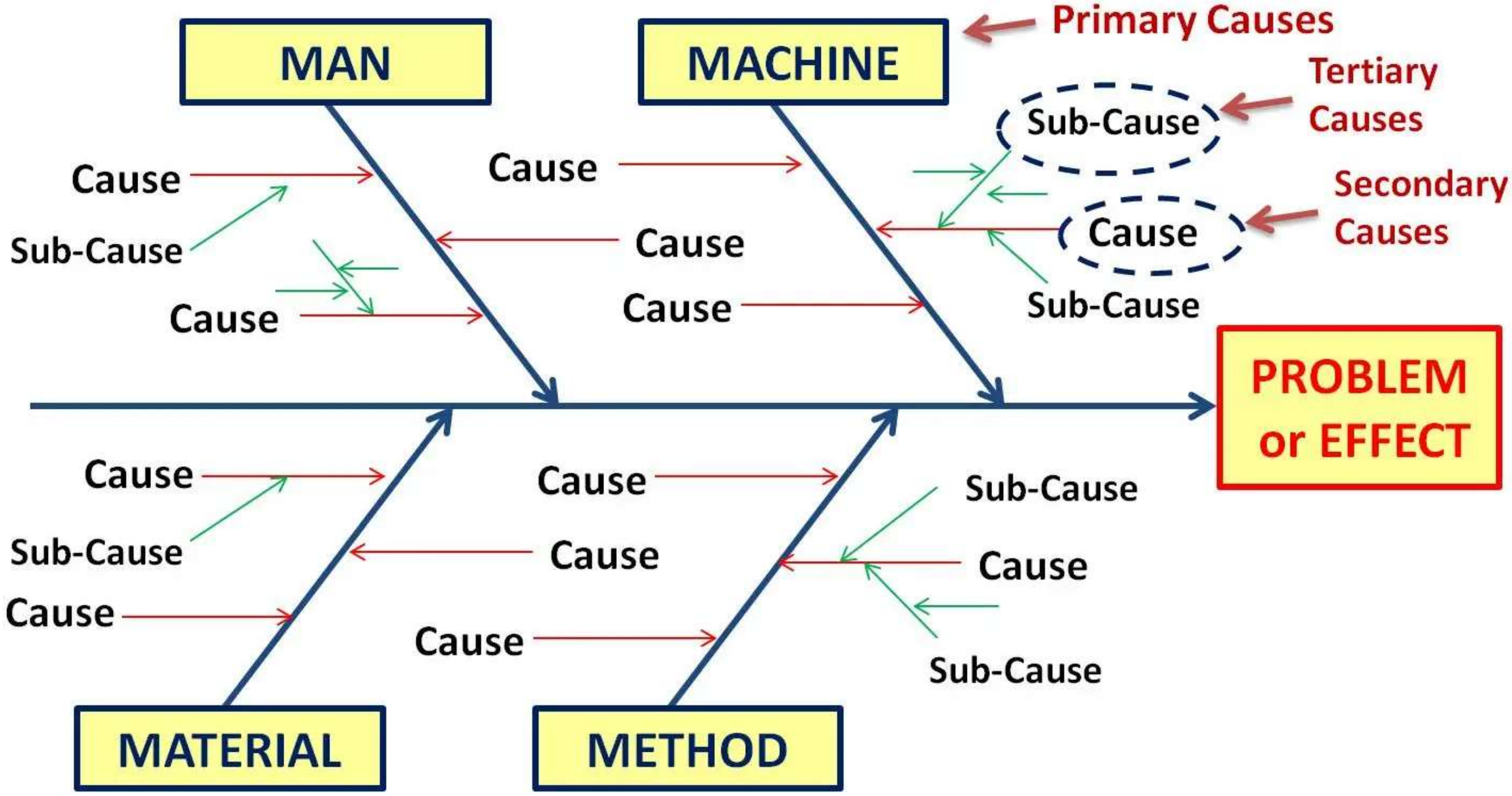
2. Selection of factors, levels and ranges

Factors

- Several **factors** may influence the performance of a process or product. The following methods may be used for factor selection keeping customers in mind
  1. Brainstorming
  2. Flowcharting (especially for processes)
  3. Cause-effect diagrams

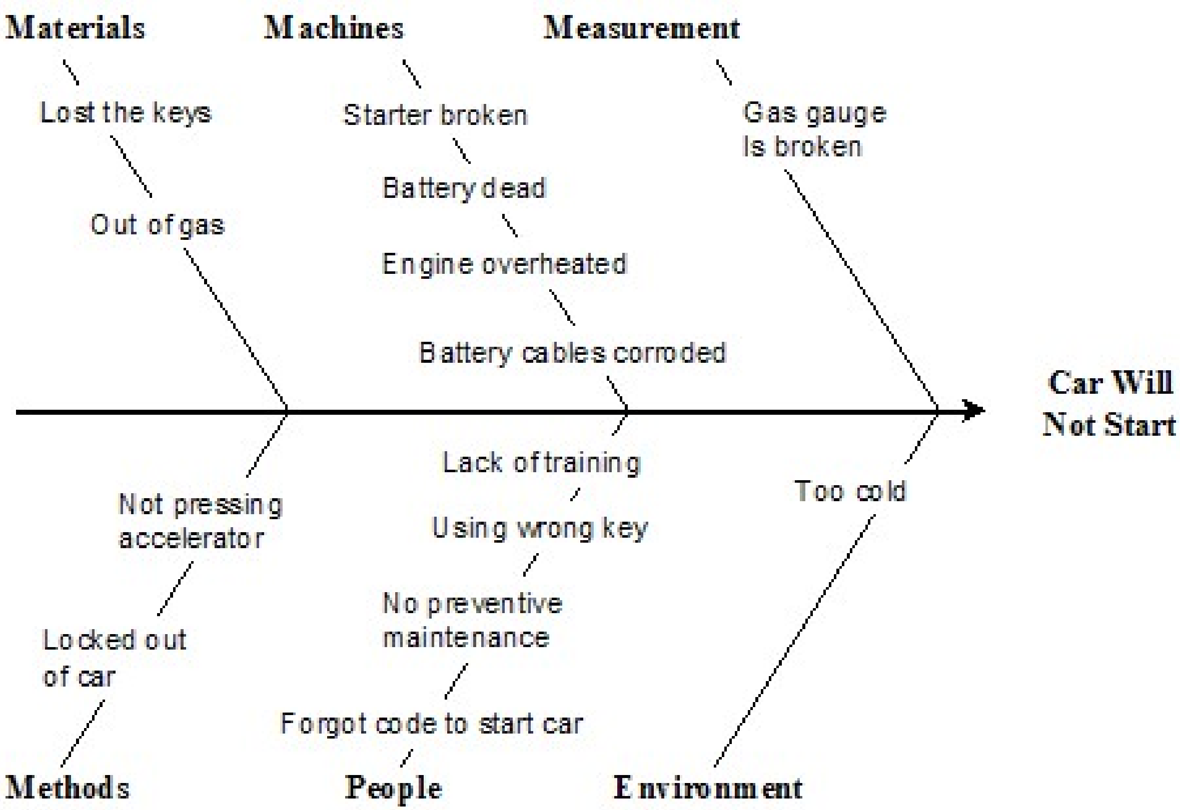
2. Selection of factors, levels and ranges

- **Cause-Effect diagram / Fishbone / Ishikawa diagram**



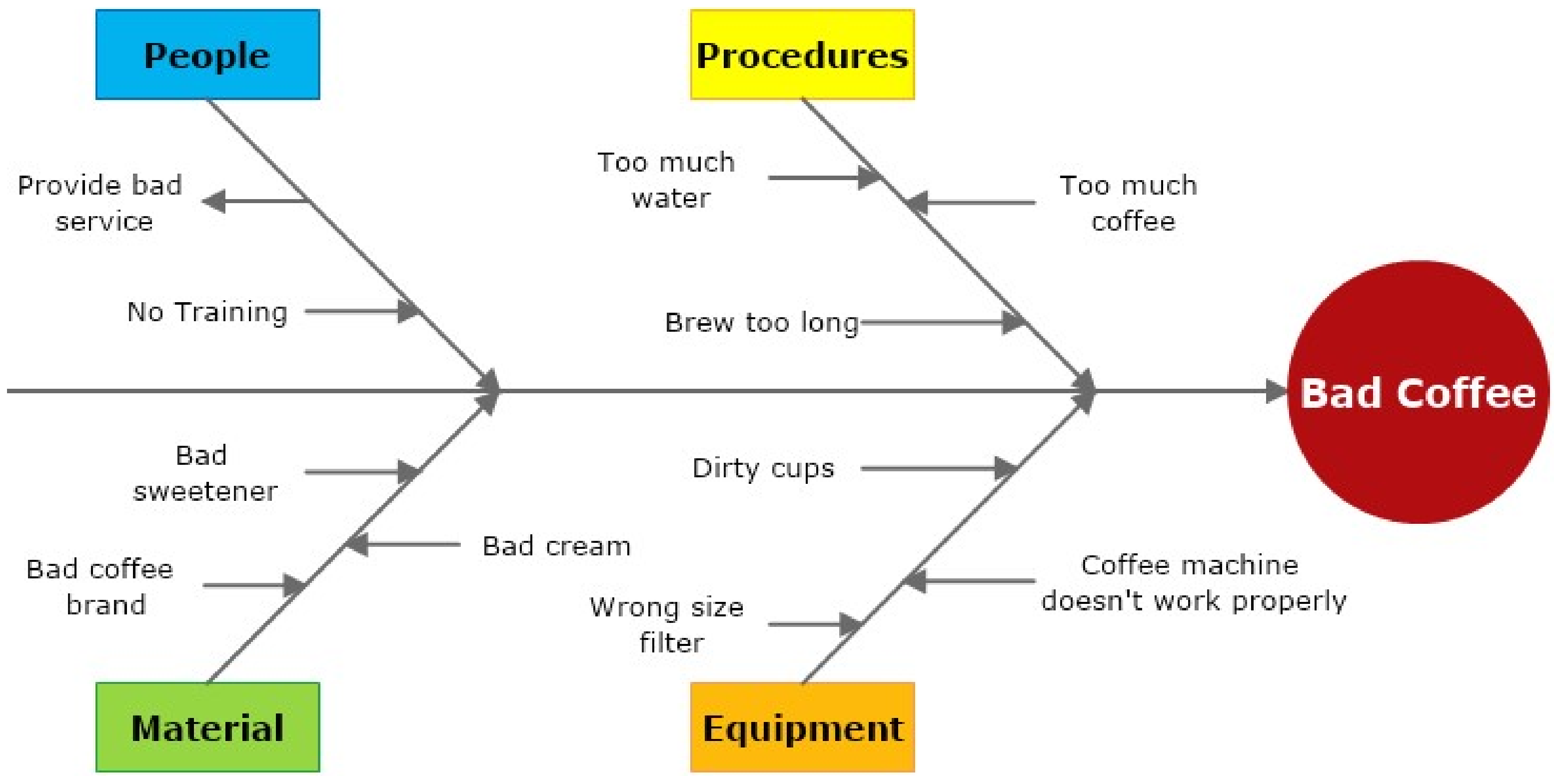
2. Selection of factors, levels and ranges

- **Cause-Effect diagram / Fishbone / Ishikawa diagram**



2. Selection of factors, levels and ranges

- **Cause-Effect diagram / Fishbone / Ishikawa diagram**



2. Selection of factors, levels and ranges

Levels

- to estimate factor effect, a **minimum of two levels** should be used in **preliminary** experimentation.
- when **large number of factors** are included in the study, the number of levels is **limited to two**.
- Detailed experimentation with a **few factors**, more **number of levels** can be considered

2. Selection of factors, levels and ranges

Range

- Selection of levels for **qualitative variables** is easy as the level would be Sometimes ‘on’ and ‘off’ or ‘yes’ and ‘no’ or ‘high’ and ‘low’.
- Selection of the **extreme** (within the operating range) is always safe because the system would function safely.
- If levels are chosen **too close together**, one **may not see any difference** and **may miss something** important outside the range of experimentation.
- Better to include experts and operators who are knowledgeable.

3. Selection of response variable

- The response variable selected must be **appropriate** to the problem under study and provide useful information.

- A **clear statement** is required of the the response to be studied, i.e., the dependent variables to be evaluated.

Example: tensile strength, surface finish, % elongation, leakages, % defectives, etc.

4. Choice of experimental design

- Several experimental designs like factorial designs - single factor and multifactor, fractional factorial designs, confounding designs etc.
- Selecting the **right type of design** for the problem under study is very important, **else** the results will be **misleading**.
- Choice of design involves the consideration of the number of **factors** and their **levels**, **resources** required such as **sample size** (number of replications), availability of samples, time to complete one replication, randomization restrictions, applicability of blocking etc…

5. Conducting the experiment

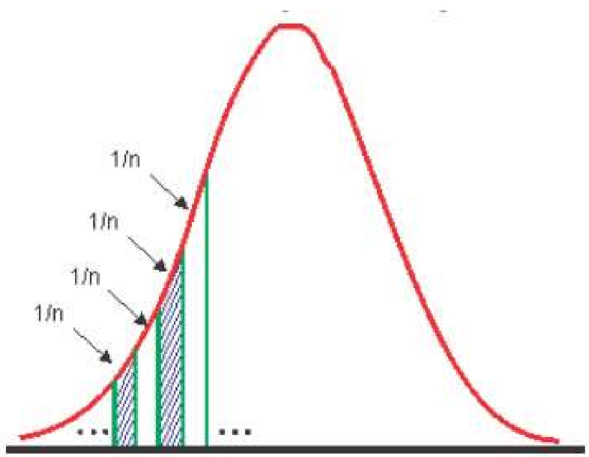
- Experiments should be **monitored** & carried out **as per the plan**.
- The allocation of sample as well as the order of experimentation should be **random** to even out the effect of unknown & controlled factors.
- **Complete Randomization**
  - any trial has an equal chance of being selected for the first test.
- **Simple Repetition**
  - any trial has an equal chance of being selected for the first test, but once that trial is selected all the **repetitions** are tested for that trial.
- **Complete Randomization within Blocks**
  - where one factor may be very difficult or expensive to change the test set up for, but others are very easy. If factor A were difficult to change, the experiment could be completed in two blocks. All A1 trials could be selected randomly and then all A2 trials could be randomly selected.

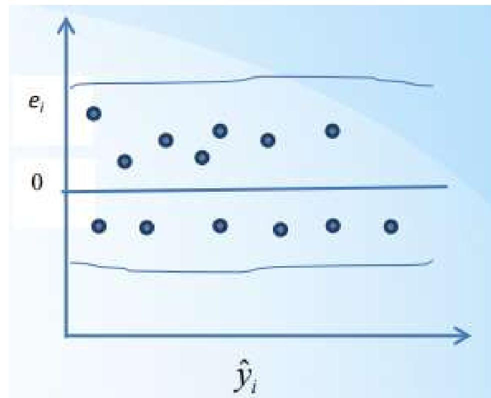
6. Analysis of data

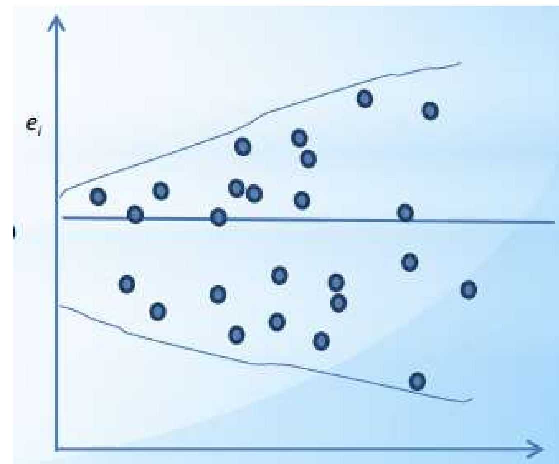
- **Statistical methods** should be used to analyse the data.
- **Analysis of variance (ANOVA)** is widely used to test the statistical significance of the effects through F-Test.
- **Confidence interval** estimation is also part of the data analysis. empirical models are developed relating the dependent (response) and independent variables (factors).
- **Residual analysis and model adequacy** checking are also part of the data analysis procedure.

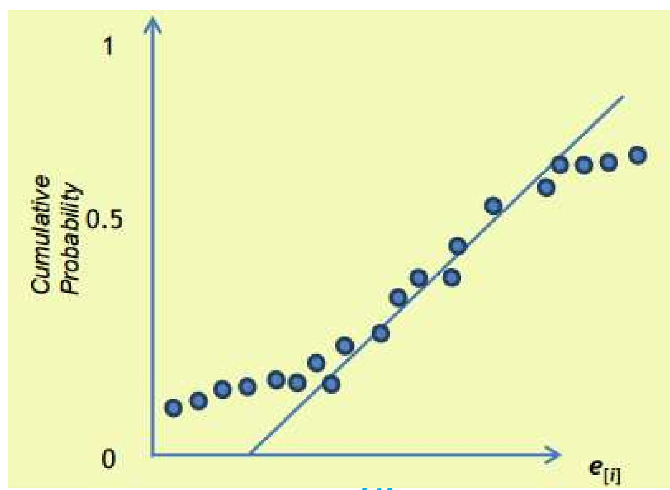
Statistical analysis of data is a must for academic and scientific purpose. graphical analysis and normal probability plot of the effects may be preferred by Industry

6. Analysis of data









7. Conclusions and recommendations

- The experimenter must **interpret the results** and recommend the same for possible implementation.
- The recommendations include the **settings of levels** for all the **factors** (input variables studied) that **optimizes the output** (response).
- It is always better to conduct **conformation tests** using the recommend levels for the factors to validate the conclusions.

Selection of Quality Characteristics for Industrial Experiments

- The selection of an appropriate quality characteristic is vital for the success of an industrial experiment.
- **To identify** a good quality characteristic, it is suggested to start with the engineering or economic goal.
- Having determined this goal, identify the fundamental mechanisms and the physical laws affecting this goal.

Selection of Quality Characteristics for Industrial Experiments

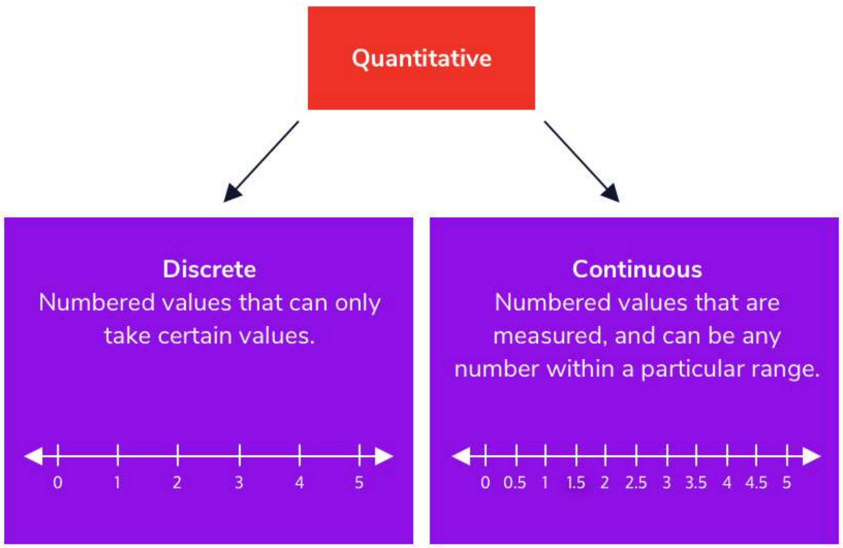
Quality Factors | Quality Characteristics

1. Reliability
2. Suitability
3. Durability
4. Affordability
5. Maintainability
6. Aesthetic look
7. Satisfaction to customer
8. Safe and foolproof workability
9. Versatility
10. Economical

Selection of Quality Characteristics for Industrial Experiments

1. Try to use quality characteristics that are easy to measure.
2. Quality characteristics should, as far as possible, be continuous variables.
3. Use quality characteristics which can be measured precisely, accurately and with stability.
4. For complex processes, it is best to select quality characteristics at the sub-system level and perform experiments at this level before attempting overall process optimisation.
5. Quality characteristics should cover all dimensions of the ideal function or the input–output relationship.

Selection of Quality Characteristics for Industrial Experiments



Discrete and Continuous Data

Discrete data can only take on certain individual values.

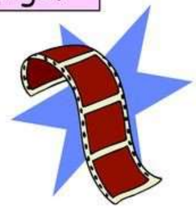
Continuous data can take on any value in a certain range.

Example 1

Number of pages in a book is a **discrete variable**.

Example 2

Length of a film is a **continuous variable**.



Example 3

Shoe size is a **Discrete variable**. E.g. 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$ etc. Not in between.

Example 4

Temperature is a **continuous variable**.

Example 5

Number of people in a race is a **discrete variable**.

Example 6

Time taken to run a race is a **continuous variable**.

Choice of sample size

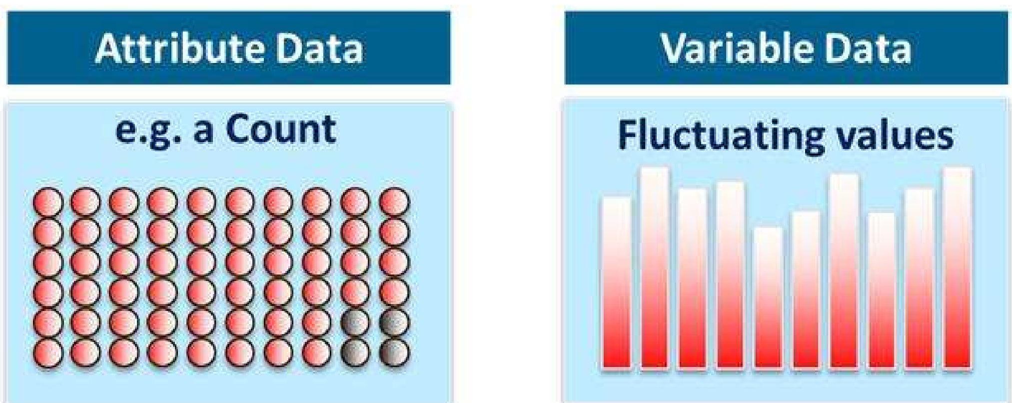
Variable Data:

- Selection of sample size is important.
- A minimum of **one test** for each trial is required.
- **More than one test per trial increases the sensitivity** of experiment to **detect small changes** in averages of populations.
- The sample size (the number of replications)depends on the resources available (the experimental units) and the time required for conduct of the experiment.
- The sample size should be as **large as possible** so that it meets the statistical principles and also facilitate a better estimate of the effect.

Choice of sample size

Attribute Data:

- Provides **less discrimination** than variable data.
- when an item is classified as defective, the **measure of how bad** is not indicated. Because of this reduced discrimination, many pieces of attribute data are required.



Choice of sample size

Attribute Data:

- Example: In a study on defective parts, if **20 defectives** are required and Suppose the past defective percent in a process is **5%**. Then, the total sample size required (all trials/ runs) shall be **400**.

Degrees of freedom (DOF)

- In the context of statistics, DOF refers to the number of independent and fair comparisons that can be made in a set of data.
- In the context of DOE,

  Degrees of freedom for a main effect = Number of levels - 1

- The **number of degrees of freedom** for the entire experiment is equal to one less than the total number of data points or observations.

Degrees of freedom (DOF)

0mg	50mg	100mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2

Factors= Dosage

Levels = 0mg,50 mg,100 mg

$$df_{Between} = a - 1 = 3 - 1 = \mathbf{2}$$

$$df_{Within} = N - a = 21 - 3 = \mathbf{18}$$

$$df_{Total} = N - 1 = 21 - 1 = \mathbf{20}$$

Degrees of freedom (DOF)

- In an **8 trial** experiment , where each trial was replicated **twice**
- So the total observations are **2 x 8 = 16** , therefore the DOF for each experiment is equal to **15** (i.e. **16 -1**)
- The degrees of freedom for **an interaction** is equal to the **product of the degrees of freedom associated with each factor** involved in that particular interaction effect.

Degrees of freedom (DOF)

Dosage ‘a’			Dosage ‘b’		
0mg	50mg	100mg	0mg	50mg	100mg
9	7	4	9	7	4
8	6	3	8	6	3
7	6	2	7	6	2
8	7	3	8	7	3
8	8	4	8	8	4
9	7	3	9	7	3
8	6	2	8	6	2

$df_{Between} = a - 1 = 3 - 1 = \mathbf{2}$

$df_{Between} = b - 1 = 3 - 1 = \mathbf{2}$

Degrees of freedom for Interaction = (a-1) x (b-1)

Robust Design

- **Reduce time** to design/develop new products & processes
- **Improve** performance of existing processes
- Improve **reliability** and performance of products
- Achieve product & process **robustness**
- **Evaluation** of materials, design alternatives, setting component & system tolerances, etc.

Example 1

- Consider a certain painting process which results in various problems such as
  - Orange peel
  - poor appearance
  - Voids, etc.
- Often, experimenters measure these characteristics as data and try to optimize the quality characteristic.
- It is not the function of the coating process to produce an orange peel.
- The problem could be due to excess variability of the coating process due to noise factors such as variability in viscosity, ambient temperature, etc.

Example 1

- **We should make every effort to gather data that relate to the engineering function itself and not to the symptom of variability.**
- One fairly good characteristic to measure for the coating process is the **coating thickness**.
- It is important to understand that excess variability of coating thickness from its target value. This could **lead to** problems such as orange peel or voids.
- The sound engineering strategy is to design and analyse an experiment so that best process parameter settings can be determined in order to yield a **minimum variability** of coating thickness around the specified target thickness.

Example 2

- **In the service organisations**, the selection of quality characteristics is not very straight forward due to the human behavioural characteristics present in the delivery of the service.
- In the banking sector
  - The number of processing errors
  - The processing time for certain transactions
  - transactions, the waiting time to open a bank account, etc.
- It is important to measure those quality characteristics which have an impact on **customer satisfaction**.

Example 3

- Health care services
  - The proportion or fraction of medication errors
  - The proportion of cases with inaccurate diagnosis
  - The waiting time to get a treatment
  - The waiting time to be admitted to an A&E department
  - The number of malpractice claims in a hospital every week or month, etc.

Understanding key interactions in a process

- For modern industrial processes, the **interactions between the factors or process parameters are a major concern to many engineers and managers**, and therefore should be studied, analyzed and understood properly for problem solving and process optimisation problems.
- For many process **optimisation problems** in industries, the **root cause of the problem** is sometimes due to **the interaction between the factors rather than the individual effect of each factor** on the output performance characteristic (or response).
- Here performance characteristic is the characteristic of a product/ service which is most **critical to customers** (Logothetis, 1994)

Understanding key interaction in processes

- **Example :** Wave-soldering process of a PCB assembly line in a certain electronic industry.
- **Aim : To reduce the number of defective solder joints obtained from the soldering process.**
- The **average defect rate** based on the existing conditions is 410 ppm (parts per million)
- To perform a simple experiment to understand the influence of wave-soldering process parameters on the number of defective solder joints.

Understanding key interaction in processes

- **First Attempt through OVAT approach** – 2 levels, low level (represented by −1) and high level (represented by +1) for each parameter.

Table 1 List of Process Parameters and Their Levels

Table 3.1 List of Process Parameters and Their Levels

Labels	Process Parameters	Units	Low Level (–1)	High Level (+1)
A	Flux density	g/c/c	0.85	0.90
B	Conveyor speed	ft/min	4.5	5.5
C	Solder temperature	°C	230	260

Understanding key interaction in processes

What is OFAT ?

- **One Factor at a Time (OFAT)**
  - The OFAT method consists of **selecting a starting point, or baseline** set of levels, for each factor, and then successively varying each factor over its range with the other factors held constant at the baseline level. After all tests are performed, a series of graphs are usually constructed showing how the response variable is affected by varying each factor with all other factors held constant.

Understanding key interaction in processes

- **The experimental layout (or design matrix)**

Labels	Process Parameters	Units	Low Level (−1)	High Level (+1)
A	Flux density	g/c/c	0.85	0.90
B	Conveyor speed	ft/min	4.5	5.5
C	Solder temperature	°C	230	260

Table 3.2 OVAT Approach to Wave-Soldering Process

Run	A	B	C	Response (ppm)
1	−1	−1	−1	420 First Trial with all process parameters kept at low level
2	+1	−1	−1	370
3	+1	+1	−1	410
4	+1	+1	+1	350

Understanding key interaction in processes

Table 3.2 OVAT Approach to Wave-Soldering Process

Run	A	B	C	Response (ppm)
1	−1	−1	−1	420
2	+1	−1	−1	370
3	+1	+1	−1	410
4	+1	+1	+1	350

- The difference in the responses between the trials 1 and 2 provides an estimate of the effect of process parameter ‘A’. From Table 3.2, the effect of ‘A’ (370 − 420 = −50) was estimated when the levels of ‘B’ and ‘C’ were at low levels. There is **no guarantee** whatsoever that ‘A’ will have the same effect for different conditions of ‘B’ and ‘C’.
- Similarly, the effects of ‘B’ and ‘C’ can be estimated.
- In the above experiment, the response values corresponding to the combinations A (−1) B (+1), A (−1) C (+1) and B (−1) C (+1) are **missing**.
- Therefore OVAT to experimentation **can lead to unsatisfactory conclusions** and in many cases it would even lead to false optimum conditions. In this case, the team failed to study the effect of each factor at different conditions of other factors. In other words, the team failed to study the interaction between the process parameters.

Understanding key interaction in processes

- Interactions occur when the effect of one process parameter depends on the level of the other process parameter. In other words, the effect of one process parameter on the response is different at different levels of the other process parameter.

- **In order to study interaction effects among the process parameters, we need to vary all the factors simultaneously (Anderson and Whitcomb, 2000).**

Understanding key interaction in processes

- For the above wave-soldering process, the engineering team employed a **Full Factorial Experiment (FFE)** and each trial or run condition was replicated twice to observe variation in results within the experimental trials.

- The results of the FFE are given in Table 3.3.

- Each trial condition was **randomised** to minimise the effect of undesirable disturbances or external factors which were uncontrollable or expensive to control during the experiment.

Understanding key interaction in processes

- The experimental layout (or design matrix) **– using Full Factorial**

Labels	Process Parameters	Units	Low Level (−1)	High Level (+1)
A	Flux density	g/c/c	0.85	0.90
B	Conveyor speed	ft/min	4.5	5.5
C	Solder temperature	°C	230	260

2^3 Full Factorial Experiment

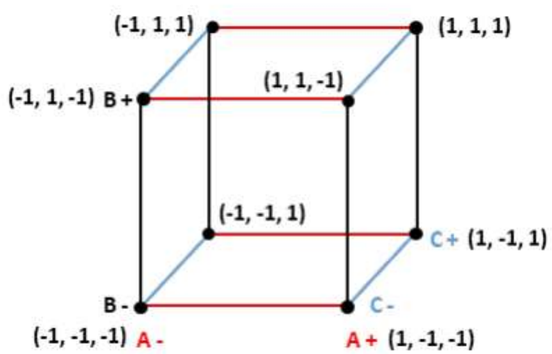


Figure 2: 2^3 Full Factorial Design

Table 2: 2^3 Factorial Design with Interactions

	Effects						
Run	A	B	C	AB	AC	BC	ABC
1	-1	-1	-1	1	1	1	-1
2	1	-1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	-1	1
4	1	1	-1	1	-1	-1	-1
5	-1	-1	1	1	-1	-1	1
6	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1

Understanding key interaction in processes

- The experimental layout (or design matrix) **– using Full Factorial**

Labels	Process Parameters	Units	Low Level (–1)	High Level (+1)
A	Flux density	g/c/c	0.85	0.90
B	Conveyor speed	ft/min	4.5	5.5
C	Solder temperature	°C	230	260

2^3 Full Factorial Experiment

Run (Standard Order)	Run (Randomised Order)	A	B	C	Response (ppm)
1	5	–1	–1	–1	420, 412
2	7	+1	–1	–1	370, 375
3	4	–1	+1	–1	310, 289
4	1	+1	+1	–1	410, 415
5	8	–1	–1	+1	375, 388
6	3	+1	–1	+1	450, 442
7	2	–1	+1	+1	325, 322
8	6	+1	+1	+1	350, 340

Understanding key interaction in processes

- **As it is an FFE**, it is possible to study **all the interactions** among the factors A, B and C. The interaction between two process parameters (say, A and B) can be computed using the following equation:

$$I_{A,B} = \frac{1}{2}(E_{A,B(+1)} - E_{A,B(-1)}) \tag{3.1}$$

where $E_{A,B\,(+1)}$ is the effect of factor 'A' at high level of factor 'B' and where $E_{A,B(-1)}$ is the effect of factor 'A' at low level of factor 'B'.

Understanding key interaction in processes

- For the above example, **three two-order interactions** and a third-order interaction can be studied. Third-order and higher order interactions are not often important for process optimisation problems and therefore not necessary to be studied.
- In order to study the interaction between A (flux density) and B (conveyor speed), it is important to form a table for average ppm values at the four possible combinations of A and B (i.e. A(−1) B(−1), A(−1) B(+1), A(+1) B(−1) and A(+1) B(+1)).

Run (Standard Order)	A	B	Average ppm
1, 5	−1	−1	398.75
3, 7	−1	+1	311.50
2, 6	+1	−1	409.25
4, 8	+1	+1	378.75

Understanding key interaction in processes

- From the previous Table, the effect of ‘A’ (i.e., going from low level (–1) to high level

$$
\begin{aligned}
(+1)\,\text{at high level of B (i.e. } +1)) &= 378.75 - 311.50 \\
&= 67.25\ \text{ppm}
\end{aligned}
$$

$$
\begin{aligned}
\text{Similarly, the effect of A at low level of B} &= 409.25 - 398.75 \\
&= 10.5\ \text{ppm}
\end{aligned}
$$

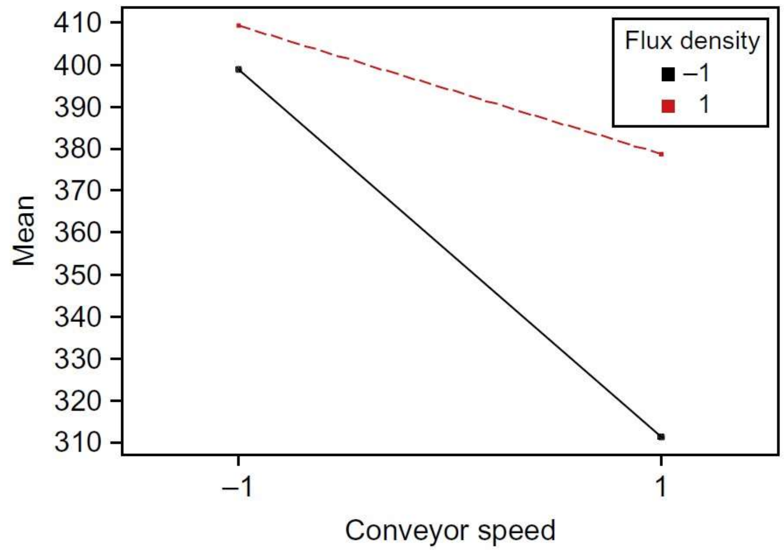
$$
\begin{aligned}
\text{Interaction between A and B} &= \frac{1}{2}[67.25 - 10.5] \\
&= 28.375
\end{aligned}
$$

Understanding key interaction in processes

- In order to determine whether two process parameters are interacting or not, one can use a simple but powerful graphical tool called interaction graphs.
- If the **lines in the interaction plot are parallel**, there is **no interaction** between the process parameters (Barton, 1990).
- This implies that the change in the mean response from low to high level of a factor does not depend on the level of the other factor.
- On the other hand, if the lines are **non-parallel**, an **interaction exists** between the factors. The **greater the degree of departure from being parallel, the stronger the interaction effect.**

Understanding key interaction in processes

Figure 1 Interaction plot between flux density and conveyor speed



The interaction graph between flux density and conveyor speed shows that the effect of conveyor speed on ppm at two different levels of flux density is not the same. This implies that there is an interaction between these two process parameters. The defect rate (in ppm) is minimum when the conveyor speed is at high level and flux density at low level.

Alternative Method for Calculating the Two-Order Interaction Effect

In order to compute the interaction effect between flux density and conveyor speed, we need to first multiply both factors effect

Table 3.5 Alternative Method to Compute the Interaction Effect

A	B	$A \times B$	Average ppm
−1	−1	+1	398.75
−1	+1	−1	311.50
+1	−1	−1	409.25
+1	+1	+1	378.75

Having obtained column 3, we then need to calculate the average ppm at high level of $(A \times B)$ and low level of $(A \times B)$. The difference between these will provide an estimate of the interaction effect.

$$
\begin{aligned}
A \times B &= \text{Average ppm at high level of } (A \times B) - \text{Average ppm at low level of } (A \times B) \\
&= \frac{1}{2}(398.75 + 378.75) - \frac{1}{2}(311.50 + 409.25) \\
&= 388.75 - 360.375 \\
&= 28.375
\end{aligned}
$$

Alternative Method for Calculating the Two-Order Interaction Effect

Labels	Process Parameters	Units	Low Level (−1)	High Level (+1)
A	Flux density	g/c/c	0.85	0.90
B	Conveyor speed	ft/min	4.5	5.5
C	Solder temperature	°C	230	260

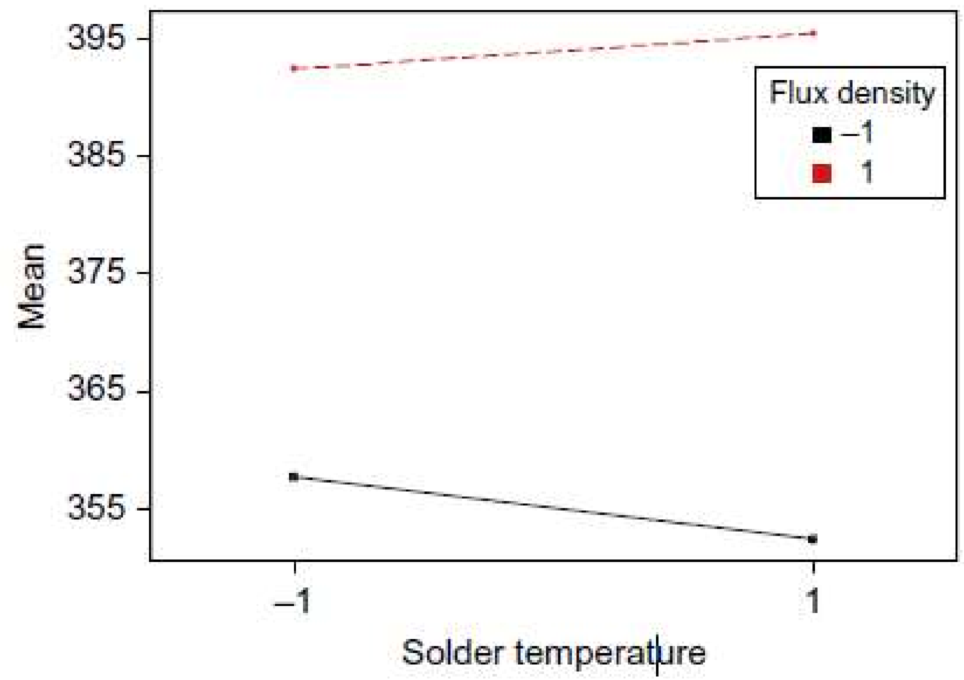


Figure 3.2 Interaction plot between solder temperature and flux density.

Alternative Method for Calculating the Two-Order Interaction Effect

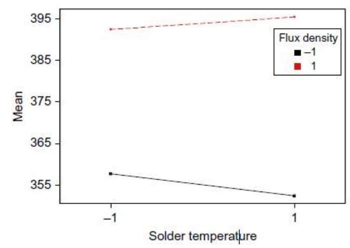


Figure 3.2 Interaction plot between solder temperature and flux density.

Now consider the interaction between flux density (A) and solder temperature.

- The graph shows that the effect of solder temperature at different levels of flux density is almost the same. Moreover, the lines are **almost parallel**, which indicates that there is **little interaction** between these two factors.
- The interaction plot suggests that the mean solder defect rate is minimum when solder temperature is at high level and flux density at low level.
- ***Note*: Non-parallel lines are an indicator of the existence of interactions between two factors and parallel lines indicate no interactions between the factors.**

Synergistic Interaction Vs Antagonistic Interaction

The effects of process parameters can be either fixed or random.

- ***Fixed Process Parameters***
  - ✓ **Fixed process parameter** effects occur when the process parameter levels included in the experiment are **controllable and specifically** chosen because they are the only ones for which **inferences are desired**.
  - ✓ **For example,** if you want to determine the effect of temperature at 2-levels (180°C and 210°C) on the viscosity of a fluid, then both180°C and 210°C are considered to be fixed parameter levels.

Synergistic Interaction Vs Antagonistic Interaction

The effects of process parameters can be either fixed or random.

- ***Random Process Parameters***
  - ✓ **Random process parameter** effects are associated with those parameters whose levels are randomly chosen from a large population of possible levels.
  - ✓ Inferences are not usually desired on the specific parameter levels included in an experiment, but rather on the population of levels represented by those in the experiment.
  - ✓ **For example,** Factor levels represented by batches of raw materials drawn from a large population

Synergistic Interaction Vs Antagonistic Interaction

- **Synergistic interaction** means that the effect of two chemicals taken together is greater than the sum of their separate effect at the same doses. An example is pesticide and fertilizer. (*i.e., Combined effect of two factors is **greater than** the individual factors on the response. In other words, when higher level of one independent variable enhances the effect of another independent variable*)

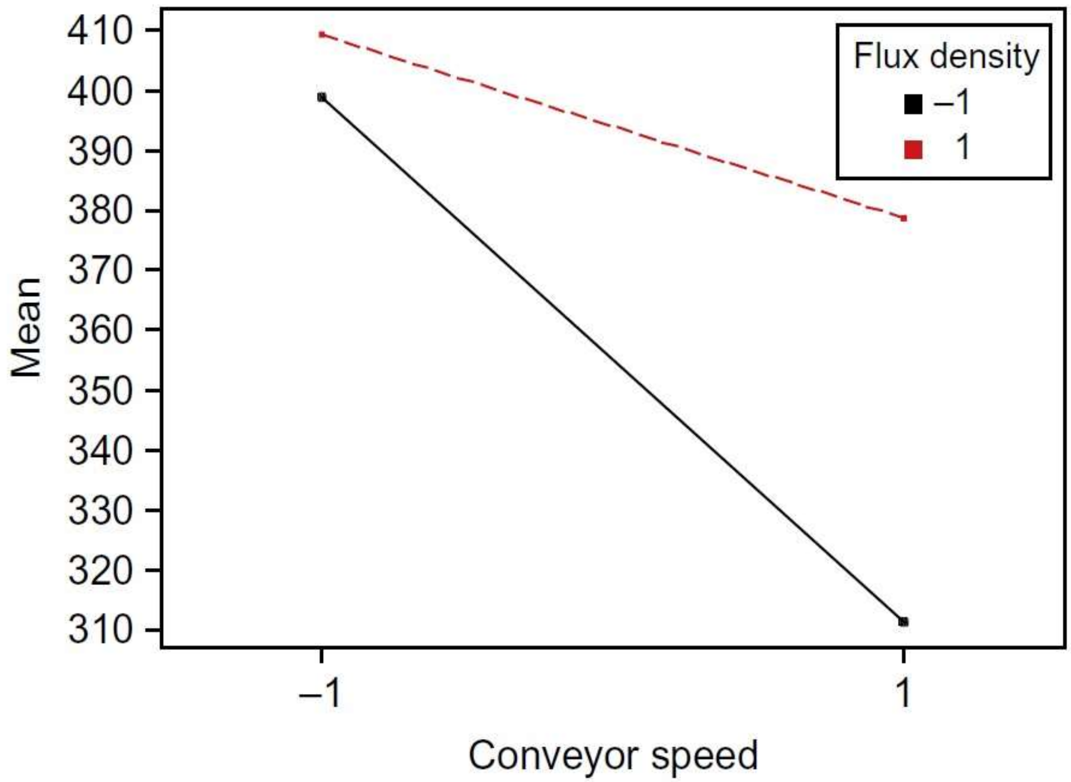
Synergistic Interaction Vs Antagonistic Interaction

- **Antagonistic interaction** means that the effect of two chemicals is actually less than the sum of the effect of the two drugs taken independently of each other. This is because the second chemical increases the excretion of the first, or even directly blocks its toxic actions. (*i.e., Combined effect of two factors is* ***lesser than*** *the individual factors on the response. In other words, When main effect is non-significant and interaction is significant. In this situation the two independent variables tend to reverse each others effect.*

Synergistic Interaction

The lines on the plot do not cross each other

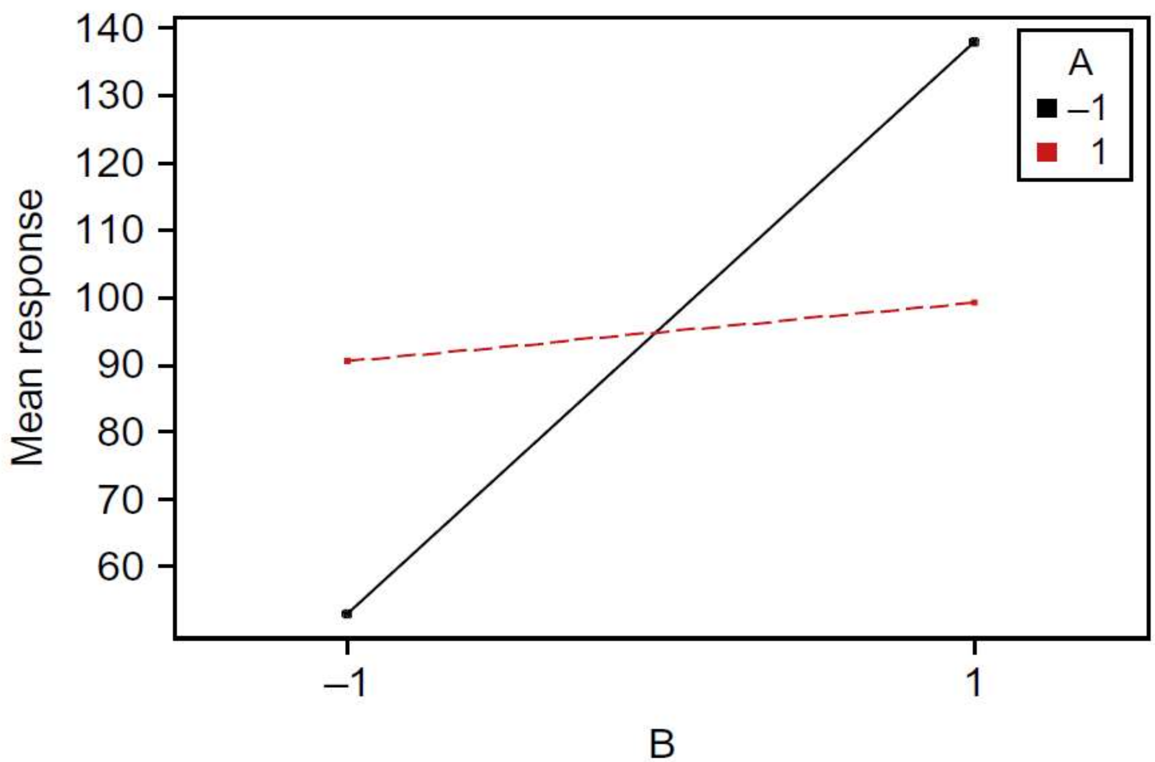
Figure 3 Synergistic interaction between two factors A and B.



Antagonistic Interaction

The lines on the plot cross each other

Figure 4 Antagonistic interaction between two factors A and B.



Thank you