Set A

SRM Institute of Science and Technology
College of Engineering and Technology
Department of Mechanical Engineering

Academic Year: 2022-23 Even **Semester: VI** **Mark: 50**

Subject Code: 18MEO113T **Title: Design of Experiments**

Duration: 90 min **Type of Test: CLA III**

ANSWER KEY

Q. No.	Part A Question
1	The objective of Response surface methodology is to _____________ a) Maximize the response b) Minimize the response c) Optimize the response d) Neglect the response
2	In the regression equation Y = a+bX, the X is called: a) Independent variable b) Dependent variable c) Continuous variable d) Intercept
3	The features of Central Composite design a) +Corner points b) +3 Levels c) +Less tests d) -No Corner points
4	___is the most popular class of designs used for fitting second order models. a) Central Composite Design b) Fractional Factorial Design c) Box-Behnken Design d) Plackett-Burman Design
5	Which of the CCD design in not rotable? a) Circumscribed b) Inscribed c) Face centered d) Body centered
6	The sum of squares measures the variability of the observed values around their respective treatment means a) Treatment b) Error c) Total d) Interaction
7	In One way anova problem if SS Total=120 and SS Treatment=80, then SS Error is a) 160 b) 120 c) 80 d) 40
8	For the ANOVA table <table><tr><td>Source of variations</td><td>Sum of Squares</td><td>Degree of freedom</td></tr><tr><td>Between treatment</td><td>75</td><td>3</td></tr><tr><td>Error</td><td>48</td><td>16</td></tr><tr><td>Total</td><td>123</td><td>19</td></tr></table>The F-Statistics is a) 8.99 b) 8.33 c) 8.6 d) 7.33
9	How is the significance of an ANOVA test determined? a) By calculating the chi-squared statistic b) By calculating the t-statistic c) By calculating the F-statistic d) By calculating the p-value
10	What is the most common type of ANOVA?

	a) One-way ANOVA b) Two-way ANOVA c) Three-way ANOVA d) Four-way ANOVA
	Part B Question
	Answer any TWO (Unit 4)
11	**Write the four steps to check whether the first-order model fits or not.** In order to check, **whether the first-order model fitted or not**, we have to check the following: 1. Estimate the Error (based on central point observation) 2. Test for Interaction ($\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2$) whether β_{12} significant or not? 3. Test for Quadratic effects ($\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2$) whether $\beta_{11} + \beta_{22}$ significant or not? 4. If step 2 and step 3 is equal to zero, then we can say the first-order model is fit. If not, we have to analysis further.
12	**Briefly discuss about two main types of response surface designs.** **There are two main types of response surface designs:** • **Central Composite designs (CCD)** – Central Composite designs can fit a full quadratic model. They are often used when the design plan calls for sequential experimentation because these designs can include information from a correctly planned factorial experiment. • **Box-Behnken designs (BBD)** – Box-Behnken designs usually have fewer design points than central composite designs, thus, they are less expensive to run with the same number of factors. They can efficiently estimate the first- and second-order coefficients; however, they **can't include runs** from a factorial experiment. Box-Behnken designs **always have 3 levels per factor**, unlike central composite designs which can have up to 5. Also unlike central composite designs, Box-Behnken designs never include runs where all factors are at their extreme setting, such as all of the low settings.
13	**Write the various phases for optimisation.**

	• *screening* (determine which factors really influence the outcome; tool: screening designs like fractional factorial) • *improvement* (approach optimum by repeated change of factor settings; tools: Box/simplex or steepest ascent approach) • *determination of optimum* (find optimal settings of factor settings; tool: response surface designs like CCD or Box-Behnken + analysis of response surface using eigenvalues)
	Answer any TWO (Unit 5)
14	**Write principle of ANOVA and write its types.** • Principle of ANOVA – We have to make two estimates of population variance viz., one based on between samples variance and the other based on within samples variance. Then the said two estimates of population variance are compared with F-test, wherein we work out. $F = \frac{\text{Estimate of population variance based on between samples variance}}{\text{Estimate of population variance based on within samples variance}}$ – This value of F is to be compared to the F-limit for given degrees of freedom. If the F-value we work out is equal or exceed the F-limit value, we may say that there are significant difference between the sample means. • ANOVA is two types: – One Way ANOVA: Only one factor is investigated • One independent variable (With 2 levels) • Analysis of Variance could have one independent variable – Two Way ANOVA: Investigate two factors at the same time • Two independent variables (can have multiple levels) • Analysis of Variance could have two independent variables – Two way ANOVA without replication – Two way ANOVA with replication
15	**Discuss one way ANOVA with an example.**

	• A one way ANOVA is used to compare two means from two independent (unrelated) groups using the F-distribution. • The **null hypothesis** for the test is that the **two means are equal**. • Therefore, a significant result means that the **two means are unequal**. • **Alternate hypothesis …? Two means are not equal**.. Examples of when to use a one-way ANOVA • Situation 1: You have a group of individuals randomly split into smaller groups and completing different tasks. For example, you might be studying the effects of tea on weight loss and form three groups: green tea, black tea, and no tea.
16	**What is the F-test in ANOVA? Write about null hypothesis.** The F-test in one-way analysis of variance (ANOVA) is used to assess whether the expected values of a quantitative variable within several pre-defined groups differ from each other. A null hypothesis is a type of statistical hypothesis that proposes that no statistical significance exists in a set of given observations.
	Part C (Unit 4)
17	Write the various features for selecting response surface design.

	When selecting a response surface design, some of the features of a desirable design are as follows: 1. Provides a reasonable distribution of data points (and hence information) throughout the region of interest 2. Allows model adequacy, including lack of fit, to be investigated 3. Allows experiments to be performed in blocks 4. Allows designs of higher order to be built up sequentially 5. Provides an internal estimate of error 6. Provides precise estimates of the model coefficients 7. Provides a good profile of the prediction variance throughout the experimental region 8. Provides reasonable robustness against outliers or missing values 9. Does not require a large number of runs 10. Does not require too many levels of the independent variables 11. Ensures simplicity of calculation of the model parameters
	OR
18	**Response surface for certain manufacturing process was defined by the given equation ($Z = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2$). Determine the approximate optimum operating point using the method of steepest ascent. The starting point of research should be X_1=2 and X_2=3 and step size C=4.**

$Z = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2$
Step size = 4.0
$x_1 = 2; x_2 = 3$
$Z(2,3) = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2 = 17(2) + 27(3) - 2^2 - 0.9(3^2) = 102.9$

$Gradient\ of\ x_1 = G1.P. = \frac{\partial z}{\partial x_1} = 17 - 2x_1$

$Gradient\ of\ x_2 = G2.P. = \frac{\partial z}{\partial x_2} = 27 - 1.8x_2$

$Magniture\ m_1 = \mathrm{Sqrt}\left((\mathrm{G_1P})^2 + (\mathrm{G_2P})^2\right)$
$G_1P = 17 - 2x_1, where\ x_1 = 2$
$G_1P = 13$
$G_2P = 27 - 1.8x_2, where\ x_2 = 3$
$G_2P = 21.6$

Hence, $Magniture\ m_1 = 25.2$

New, x1 and x2..i.e., $x_1' and x_2'$

$x_1' = x_1 + C\left(\frac{G_1P}{m}\right) = 4.063$

$x_2' = x_2 + C\left(\frac{G_2P}{m}\right) = 6.43$

$Then, Z(x_1', x_2') = Z(4.063, 6.43) = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2 = 188.981$
$G_1P = 17 - 2x_1, where\ x_1 = 4.063$
$G_1P = 8.874$
$G_2P = 27 - 1.8x_1, where\ x_2 = 6.43$
$G_2P = 15.426$
$Magniture\ m_1 = \mathrm{Sqrt}\left((\mathrm{G_1P})^2 + (\mathrm{G_2P})^2\right) = 17.79$
New, x1 and x2..i.e., $x_1' and x_2'$

$New\ x_1 = x_1 + C\left(\frac{G_1P}{m}\right) = 6.06$

$New\ x_2 = x_2 + C\left(\frac{G_2P}{m}\right) = 9.89$

$Then, Z(x_1', x_2') = Z(6.06, 9.89) = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2 = 245.3$

$Then, Z(x_1', x_2') = Z(6.06, 9.89) = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2$ = 245.3
$G_1P = 17 - 2x_1, where\ x_1 = 6.06$
$G_1P = 4.88$
$G_2P = 27 - 1.8x_1, where\ x_2 = 9.89$
$G_2P = 9.198$
$Magniture\ m_1 = \mathrm{Sqrt}\ ((\mathrm{G_1P})^2 + (\mathrm{G_2P})^2) = 10.41$
New, x1 and x2..i.e., $x_1' and x_2'$

$$New\ x_1 = x_1 + C\left(\frac{G_1P}{m}\right) = 7.935$$

$$New\ x_2 = x_2 + C\left(\frac{G_2P}{m}\right) = 13.42$$

$Then, Z(x_1', x_2') = Z(7.935, 13.42) = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2$ = 272.19

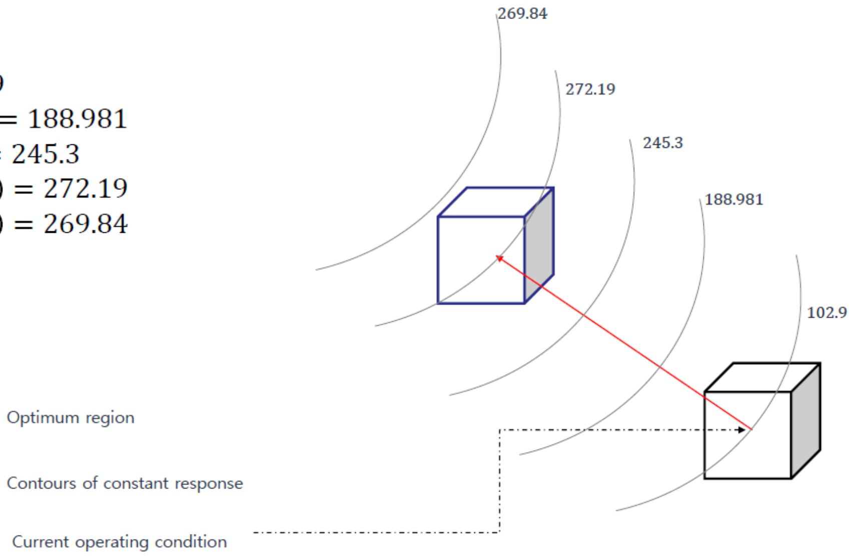
$Then, Z(x_1', x_2') = Z(7.935, 13.42) = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2$ = 272.19
$G_1P = 17 - 2x_1, where\ x_1 = 7.935$
$G_1P = 1.13$
$G_2P = 27 - 1.8x_1, where\ x_2 = 13.42$
$G_2P = 2.844$
$Magniture\ m_1 = \mathrm{Sqrt}\ ((\mathrm{G_1P})^2 + (\mathrm{G_2P})^2) = 3.06$
New, x1 and x2..i.e., $x_1' and x_2'$

$$New\ x_1 = x_1 + C\left(\frac{G_1P}{m}\right) = 9.412$$

$$New\ x_2 = x_2 + C\left(\frac{G_2P}{m}\right) = 17.13$$

$Then, Z(x_1', x_2') = Z(7.935, 13.42) = 17x_1 + 27x_2 - x_1^2 - 0.9x_2^2$ = 269.84

$Final\ points:$
$\mathrm{Z}(2,3) = 102.9$
$\mathrm{Z}(4.063,6.43) = 188.981$
$\mathrm{Z}(6.06,9.89) = 245.3$
$\mathrm{Z}(7.935,13.42) = 272.19$
$\mathrm{Z}(9.412,17.13) = 269.84$



	Part C Question (Unit 5)
19	A clinical psychologist has run a between-subjects experiment comparing two treatments for depression (cognitive-behavioral therapy (CBT) and client-centered therapy (CCT) against a control condition. Subjects were randomly assigned to the experimental condition. After 12 weeks, the subject's depression scores were measured using the CESD depression scale. The data are summarized as follows:

$$\begin{array}{c} \\ \text{control} \\ \text{CBT} \\ \text{CCT} \end{array}\left[\begin{array}{ccc} \mathbf{n} & \mathbf{mean} & \mathbf{sd} \\ 40 & 21.4 & 4.5 \\ 40 & 16.9 & 5.5 \\ 40 & 19.1 & 5.8 \end{array}\right]$$

Use one-way ANOVA with α=.01 for the test.
$F_{critical}$ (2,117) =4.79

Solution

$Var_1 = 4.5^2 = 20.25$

$Var_2 = 5.5^2 = 30.25$

$Var_3 = 5.8^2 = 33.64$

$MS_{error} = \frac{20.25+30.25+33.64}{3} = 28.05$ *Note: this is just the average within-group variance; it is not sensitive to group mean differences!*

Calculating the remaining *error* (or *within*) terms for the ANOVA table:

$df_{error} = 120 - 3 = 117$

$SS_{error} = (28.05)(120 - 3) = 3281.46$

Intermediate steps in calculating the variance of the sample means:

Grand mean ($\bar{x}_{grand}$) = $\frac{21.4+16.9+19.1}{3} = 19.13$

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group mean grand mean deviations sq deviations
      21.4      19.13       2.27          5.15
      16.9      19.13      -2.23          4.97
      19.1      19.13      -0.03          0.00
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Sum of squares $(SS_{means}) = 10.12$

$Var_{means} = \frac{10.12}{3-1} = 5.06$

$MS_{between} = (5.06)(40) = 202.4$ *Note: This method of estimating the variance IS sensitive to group mean differences!*

Calculating the remaining *between* (or *group*) terms of the ANOVA table:

$df_{groups} = 3 - 1 = 2$

$SS_{group} = (202.4)(3 - 1) = 404.8$

Test statistic and critical value

$F = \frac{202.4}{28.05} = 7.22$

$F_{critical}(2, 117) = 4.79$

$\boxed{\text{Decision: reject H0}}$

ANOVA table

source	SS	df	MS	F
group	404.8	2	202.4	7.22
error	3281.46	117	28.05	
total	3686.26			

OR

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An engineer is studying methods for improving the ability to detect targets on a radar scope. Two factors she considers to be important are the amount of background noise, or "ground clutter," on the scope and the "type of filter" placed over the screen. The response variable is intensity level. It is experienced that the ground clutter can be categorized into three levels, ie., low, medium, and high and two filter types are available in the market.

Factor	Filter Types							
Ground Clutter	Type -1				Type -2			
Low (1)	90	96	100	92	86	84	92	81
Medium(2)	102	106	105	96	97	90	97	80
High (3)	114	112	108	98	93	91	95	83

F (2,23) = 3.4224
F (1,23) = 4.2793
F (2,23) = 3.4224

Two factors: Ground Clutter type & Filter Type
Response variable: Intensity level
DF for clutter = a – 1 = 3-1 = 2
DF for Filter = b – 1 = 2 – 1 = 1
DF for interaction = (a -1) (b-1) = 2
DF for errors = ab(n-1) = 2 * 3 *(4-1) =18
DF for total = N – 1 = 24 – 1 = 23

a = number of levels in factor 1 (clutter)
b = number of levels in factor 2 (filter)
n = number of replicates in each condition
N = abn

Factor	Filter Types							
Ground Clutter	Type - 1				Type 2			
Low (1)	90	96	100	92	86	84	92	81
Medium (2)	102	106	105	96	97	90	97	80
High (3)	114	112	108	98	93	91	95	83

Step 1: Calculate Row, Column, and Grand Total

Factor	Filter Types								
Ground Clutter	Type - 1				Type 2				
Low (1)	90	96	100	92	86	84	92	81	721
Medium (2)	102	106	105	96	97	90	97	80	773
High (3)	114	112	108	98	93	91	95	83	794
	306	314	313	286	276	265	284	244	**2288**

Grand Total

Step 2: Count the total number of observations = N = 24

Step 3: Calculate Correction Factor (C) = (Grand Total^2) / N = 2288^2/24 = 218122.7

Step 4: SS Total = Sum of squares of all factors responses – Correction Factor (C)

$=(90^2+96^2+100^2+92^2+86^2+84^2+92^2+81^2+102^2+106^2+105^2+96^2+97^2+90^2+97^2+80^2+114^2+112^2+108^2+98^2+93^2+91^2+95^2+83^2) - 218122.7$

SS Total = 1985.3333

Step 5: SS Between Clutter = $\frac{1}{bn} * Sum\ of\ square\ of\ factor\ 1\ - Correction\ factor$

$= (\frac{721^2}{2*4} + \frac{773^2}{2*4} + \frac{794^2}{2*4}) - 218122.7 =$ 353.0833

Factor	Filter Types									
Ground Clutter	Type - 1				Type 2					
Low (1)	90	96	100	92	86	84	92	81		721
Medium (2)	102	106	105	96	97	90	97	80		773
High (3)	114	112	108	98	93	91	95	83		794

Step 6: SS Between Filter = $\frac{1}{an} * Sum\ of\ square\ of\ factor\ 2\ - Correction\ factor$

$= \frac{1}{3*4} * [(306+314+313+286)^2+(276+265+284+244)^2] - 218122.7 =$ 937.5

Factor	Filter Types							
Ground Clutter	Type - 1				Type 2			
Low (1)	90	96	100	92	86	84	92	81
Medium (2)	102	106	105	96	97	90	97	80
High (3)	114	112	108	98	93	91	95	83
	306	314	313	286	276	265	284	244

Step 7: SS Interaction = $\frac{1}{n} * Sum\ of\ square\ of\ both\ factor\ 2\ - Correction\ factor$ – SS Clutter – SS Filter

$= \frac{1}{4}(378^2 + 409^2 + 432^2 + 343^2 + 364^2 + 362^2) - 218122.7 - 353.08333 - 937.5$

SS Interaction = 81.25

Factor	Filter Types									
Ground Clutter	Type - 1					Type 2				
Low (1)	90	96	100	92	378	86	84	92	81	343
Medium (2)	102	106	105	96	409	97	90	97	80	364
High (3)	114	112	108	98	432	93	91	95	83	362

Step 8: SS Error = SS Total – SS Clutter – SS Filter – SS Interaction

= 1985.333 – 353.0833 – 937.5 – 81.25

SS Error = 613.5

Step 9: Make ANOVA Table

Sources of variation	SS	DOF	MS	F-Value		F- Critical Value from Table	
Ground Clutter Type	353.08333	2	176.5417	5.179707	>	F(2,23) = 3.4224	Significant / Reject H0
Filter Type	937.5	1	937.5	27.50611	>	F(1,23) = 4.2793	Significant / Reject H0
Interaction	81.25	2	40.625	1.191932	<	F(2,23) = 3.4224	Non-Significant / Accept H0
Error	613.5	18	34.08333				
Total	1985.333	23	86.31883				