Stochastic Processes and Applications

Assignment 1

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Report

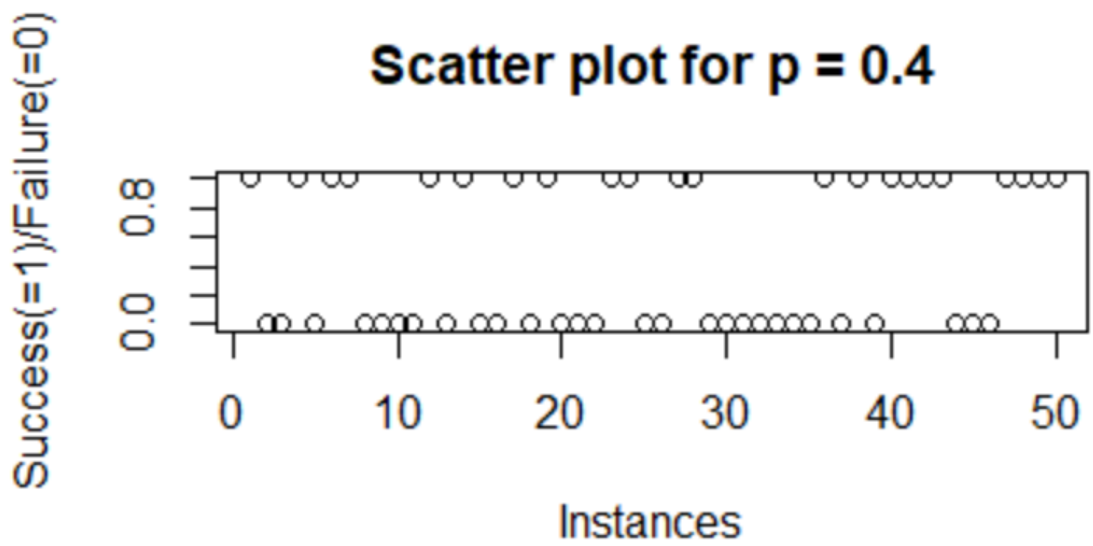
Q1) We are going to model a Bernoulli process. This is because the necessary conditions for a Bernoulli process have been met:

1. There are only 2 outcomes, either the earthquake will have a magnitude >= 6 or it won't (success or failure).
2. Each earthquake is independent of each other.
3. The probability of an earthquake having a magnitude >= 6 remains the same for all the trials/earthquakes.
4. We are studying the process over discrete and finite time (the number of instances is discrete).

We have given the option of taking input t, but here for the graphs attached in the report, we will take t = 50, i.e., the number of instances will be taken as 50.

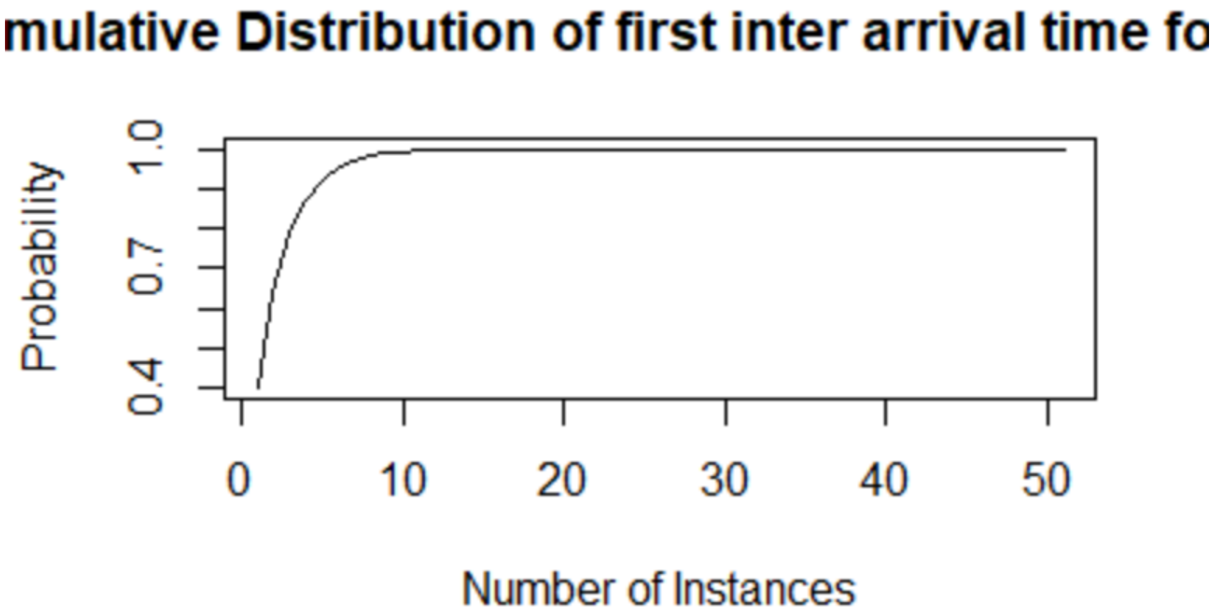
a)

Given p = 0.4, the following graph shows the scatter plot for a Bernoulli Process for a finite t. The x axis denotes the instances of earthquakes, and y axis denotes the success or failure of the earthquake having a magnitude greater than or equal to 6.



b)

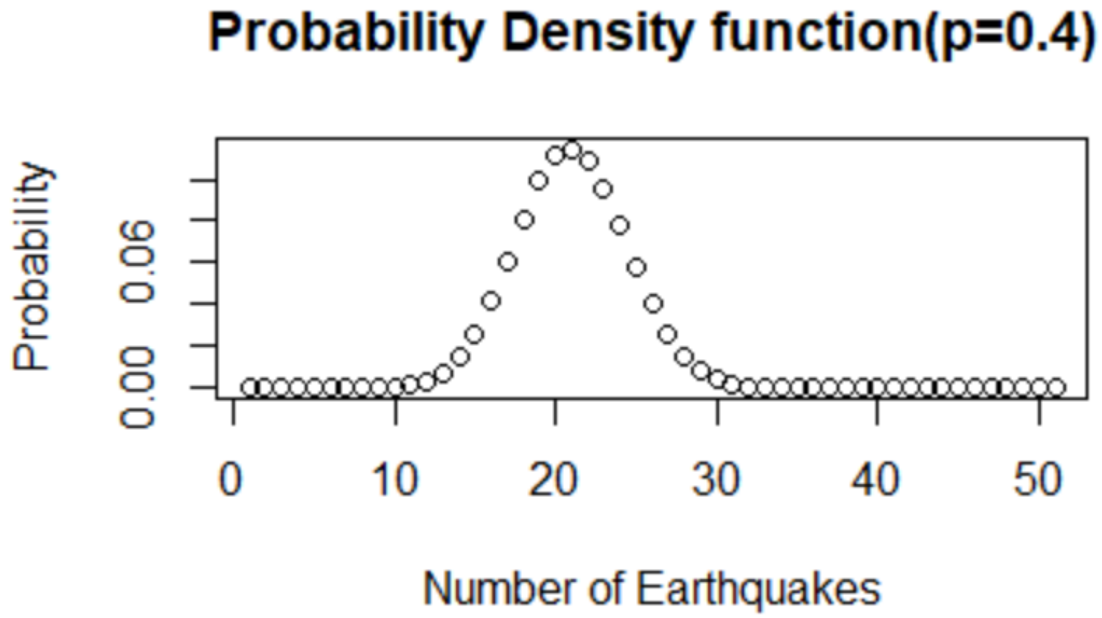
We need to plot the cumulative distribution for the first inter arrival time for time t. We know that the interarrival time follows a geometric distribution. Here, the x axis denotes the number of instances, while the y axis denotes the Probability of the first success being on that instance.



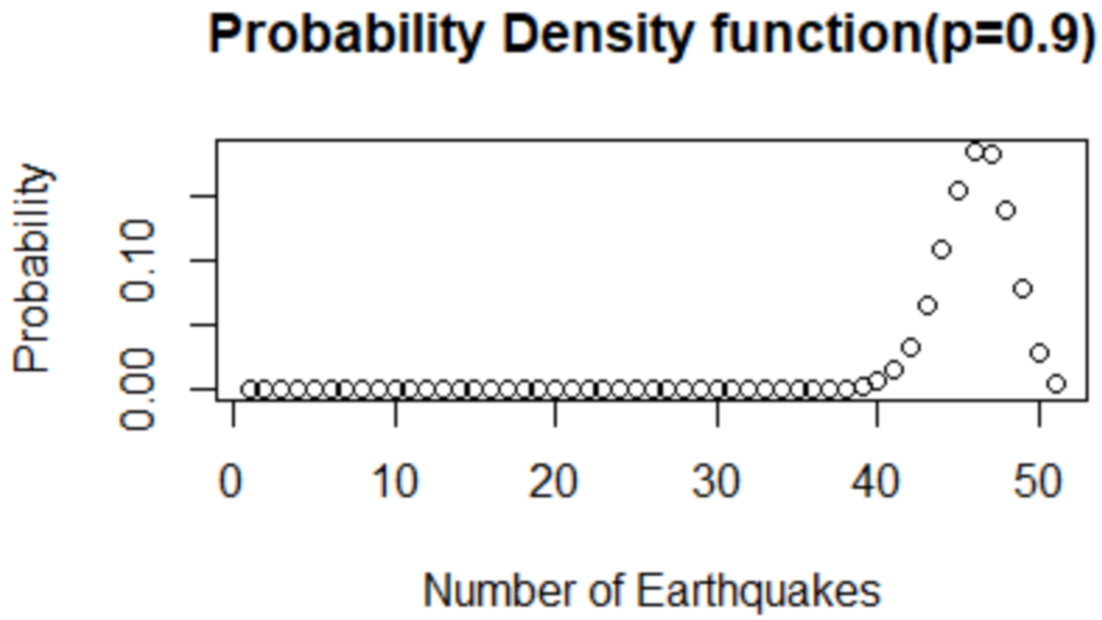
c)

To study and compare the changes in behaviour for p = 0.4, and p = 0.9, we will plot the probability distribution functions for them. Here, the x axis will denote the number of arrivals in time t, and y axis will denote the probability.

For p = 0.4:



For p = 0.9:



Here, we can see that for p = 0.4, the probability increases when the number of earthquakes is approximately 20. Whereas, for p = 0.9, the probability increases when the number of earthquakes is approximately 45. The probability increases for these values because the probability of a success is 0.4 and 0.9, and the number of trials being taken are 50, so the total number of expected successes can be taken as: probability of success X Number of trials = 50 * 0.4 = 20 (for p = 0.4) and 50*0.9 = 45 (for p = 0.9).

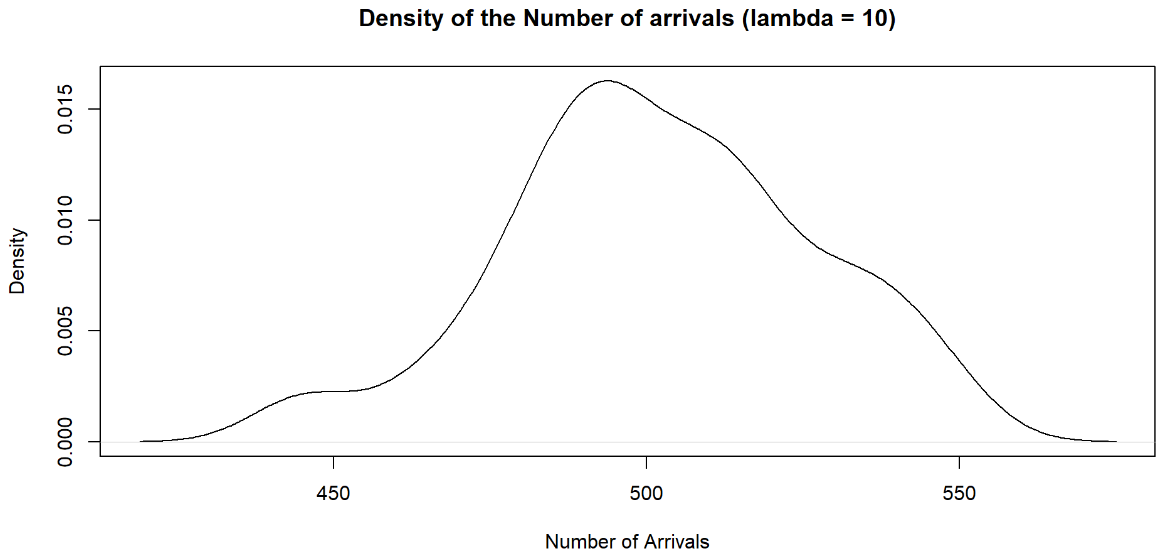
Q2)

We can model this process as a Poisson Process.

T can be considered as any value, but for this question, I have taken t as 50.

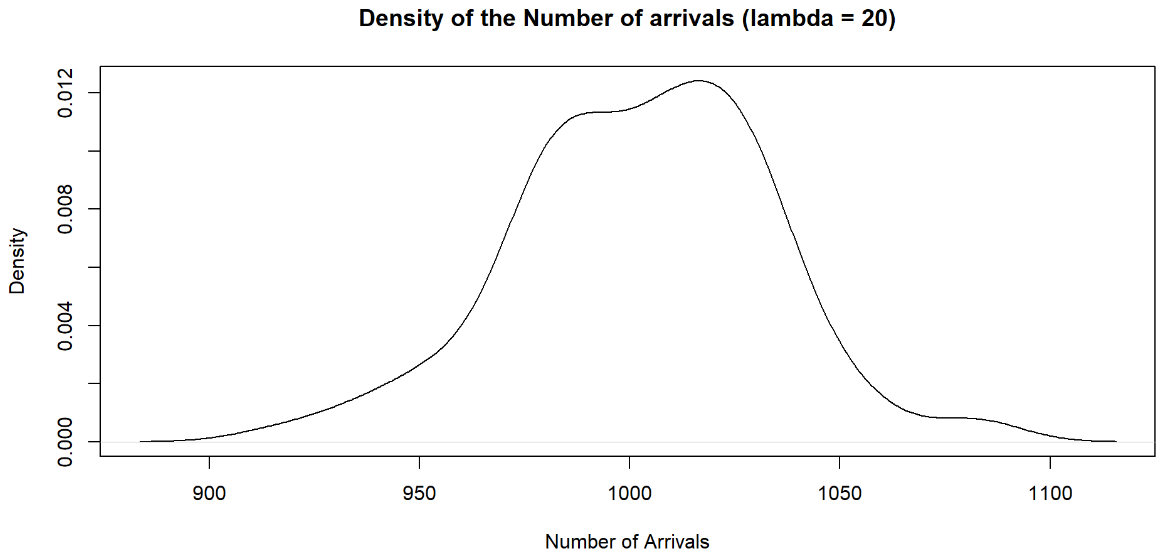
a)

The plot below shows the density of the number of arrivals until time t for a website that receives an average of 10 visitors per hour. The x axis shows the number of arrivals in time t, and the y axis shows the density.



b)

The plot below shows the density of the number of arrivals until time t for a website that receives an average of 20 visitors per hour. The x axis shows the number of arrivals in time t, and the y axis shows the density.

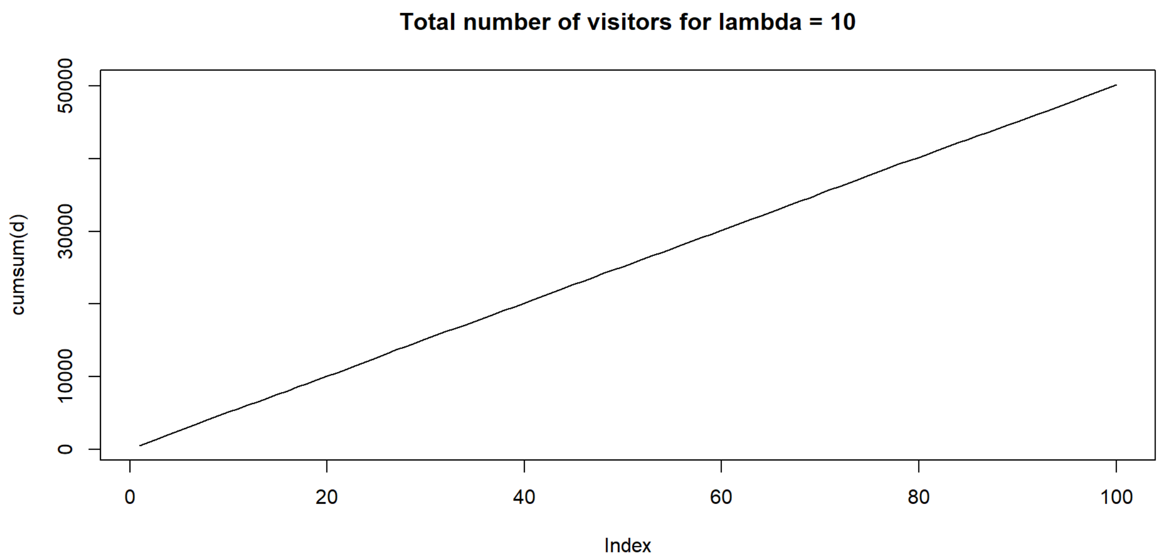


We can see that in (a) part, the density is the highest when the number of arrivals is around 500, whereas in (b) part, the density is the highest when the number of arrivals is around 1000. The number of arrivals is double compared to (a) part, because the average rate of receiving visitors is also twice that of the first site. We can see that the number of arrivals increase as the rate is increased.

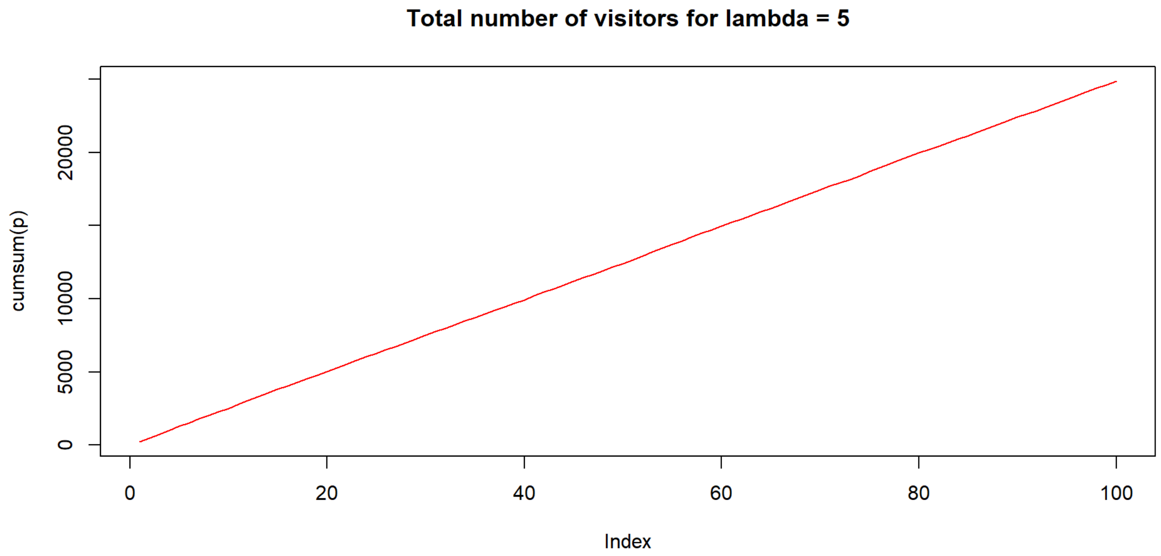
c)

We want to find the total number of visitors in time t. To do this, we will plot the cumulative distribution of the number of visitors.

For rate = 10 visitors per hour:

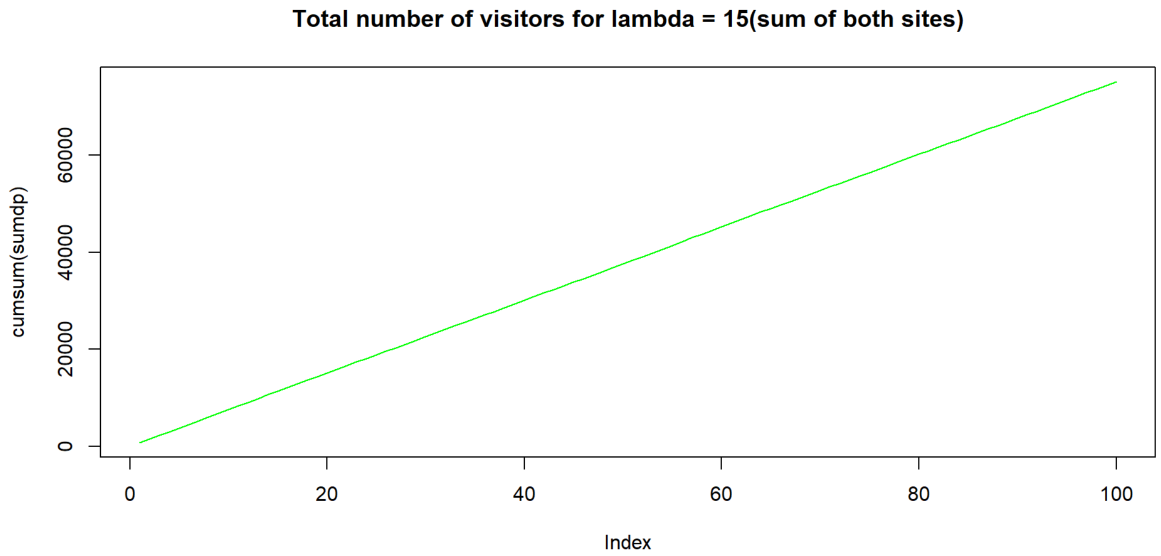


For rate = 5 visitors per hour:



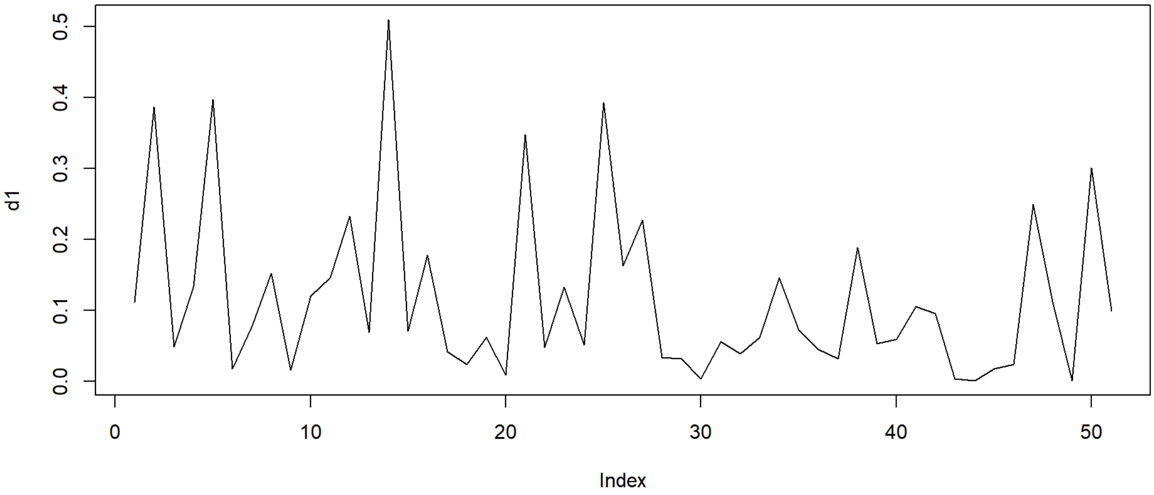
We will also plot the sum of both the websites. For sum, we can simply add the rate of both the Poisson processes to create a new Poisson process, which will have a rate that is the sum of both the rates of the processes.

For the sum of both the websites, i.e., rate = 15 visitors per hour:

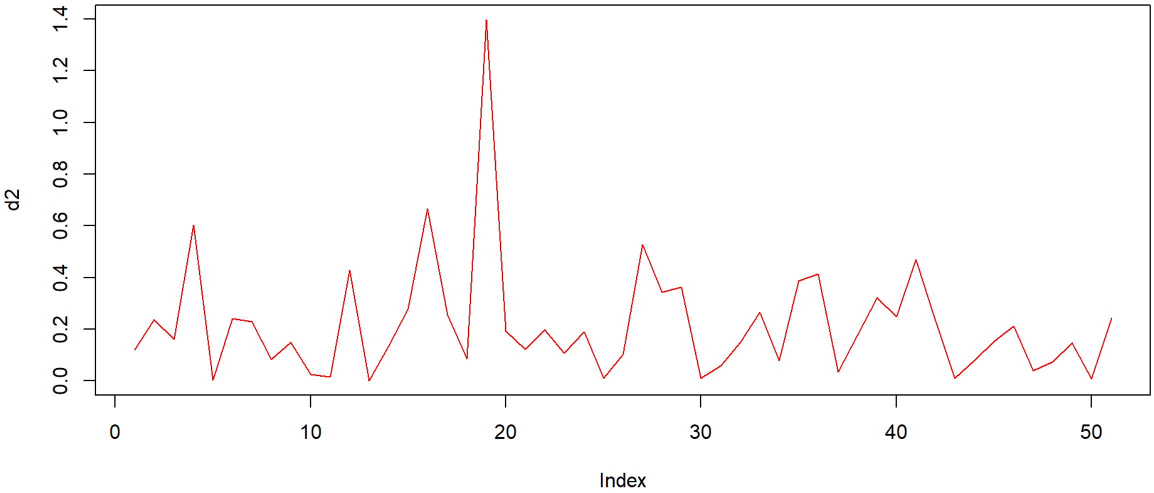


The inter arrival time for a Poisson process has an exponential distribution. The value of lambda for the exponential distribution will be the value of the rate of visitors received.

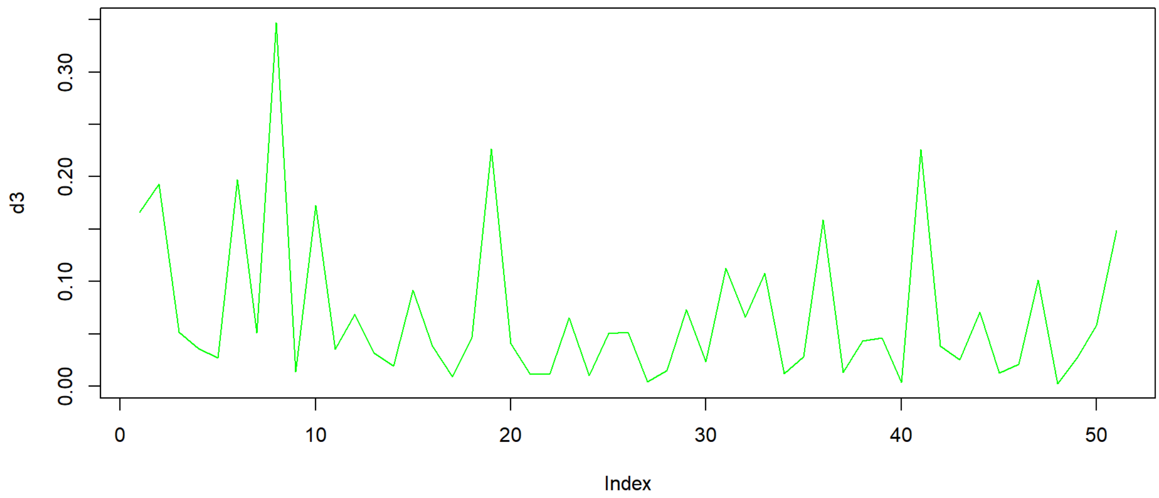
For lambda = 10:



For lambda = 5:

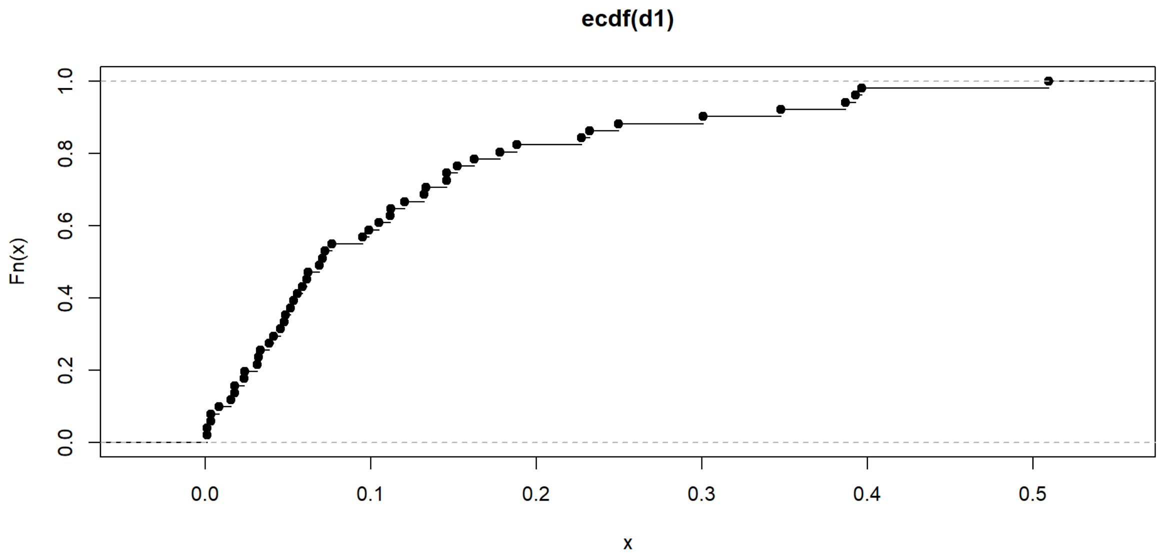


For lambda = 15:

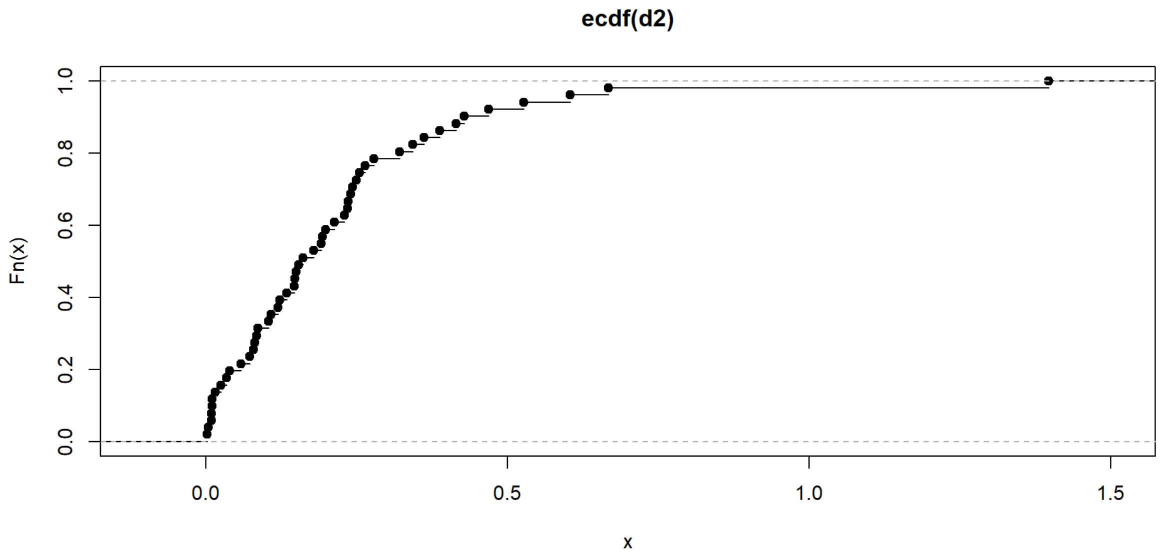


Now, we will plot the cumulative distribution function for the simulation of the first inter arrival times for different values of lambda.

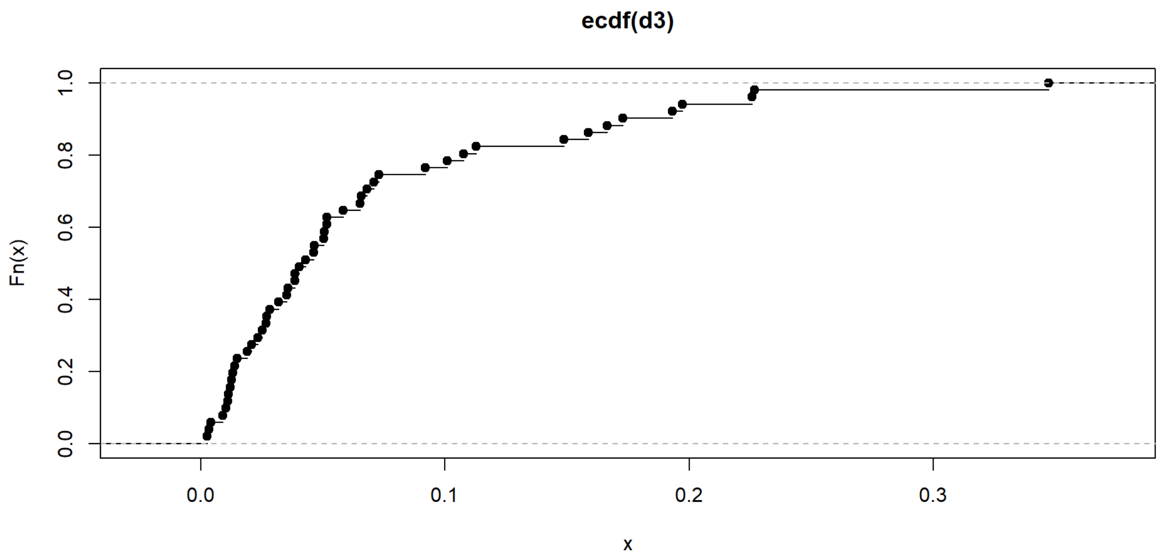
For lambda = 10:



For lambda = 5:



For lambda = 15:



From the graphs, we can see that the cumulative distribution function reaches 1, by 0.5 (when lambda = 10), by 1.5 (when lambda = 5), and by 0.3 (when lambda = 15). As rate gets higher, the first inter arrival time also becomes smaller.