MA374: Financial Engineering Lab10

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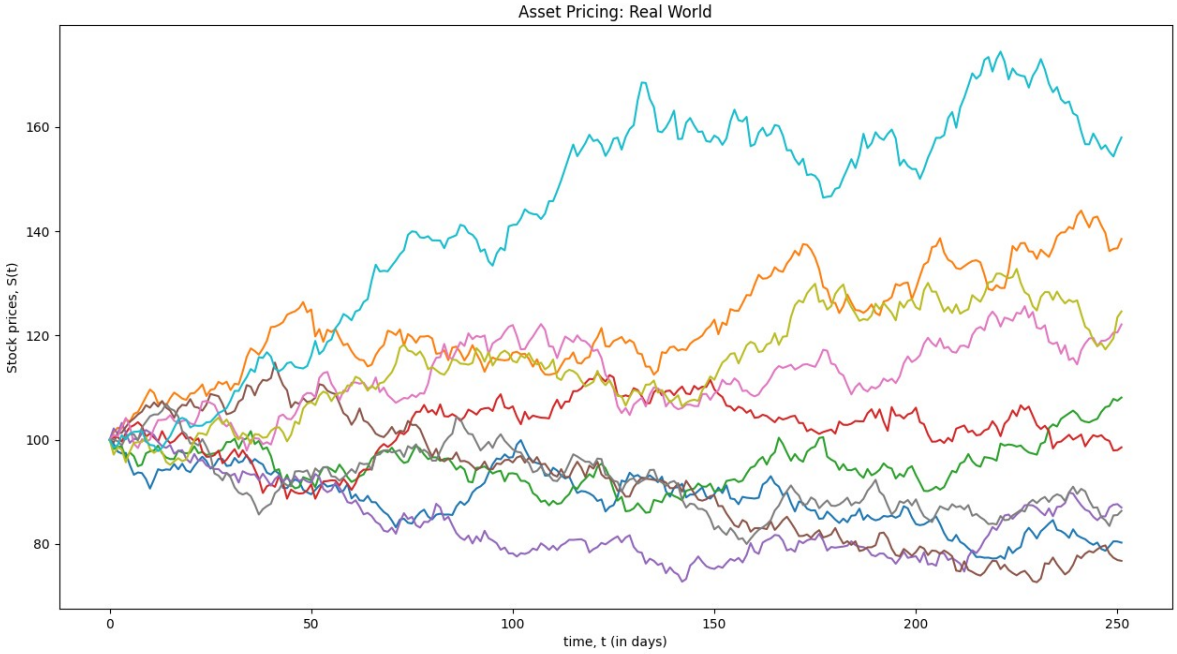
Q1

10 differnet paths of the asset prices generated using GBM model in real world using the differential equation:

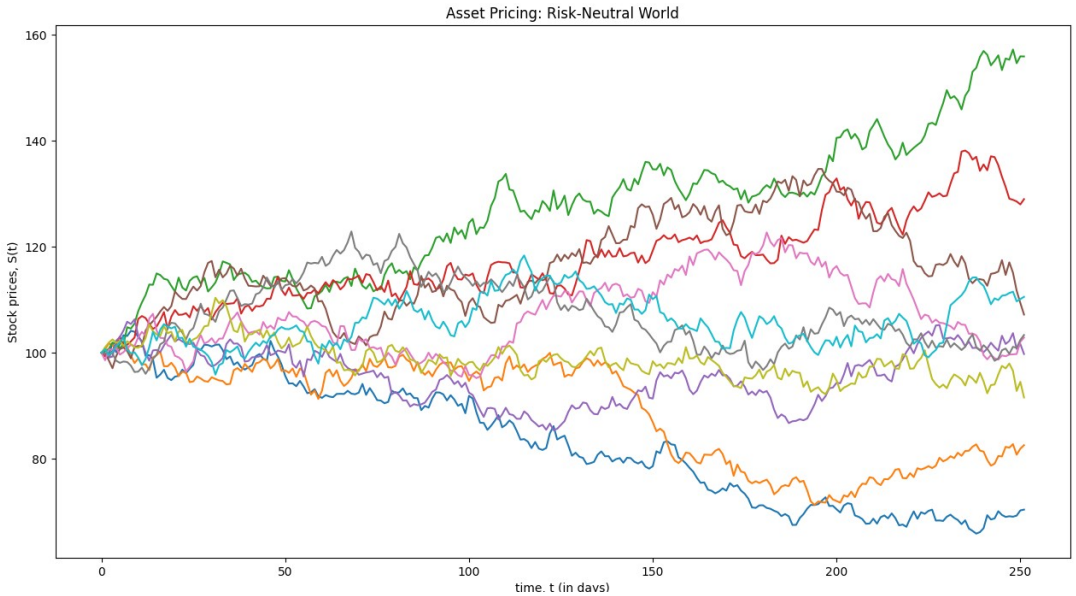
$$dS = \mu S dt + \sigma s dw(t)$$



10 differnet paths of the asset prices generated using GBM model in risk-neutral world using the differential equation:

$$dS = r S dt + \sigma S d\widetilde{W}(t)$$

where $\widetilde{W}$ denotes Brownian Motion in Risk-Neutral Probability measure:

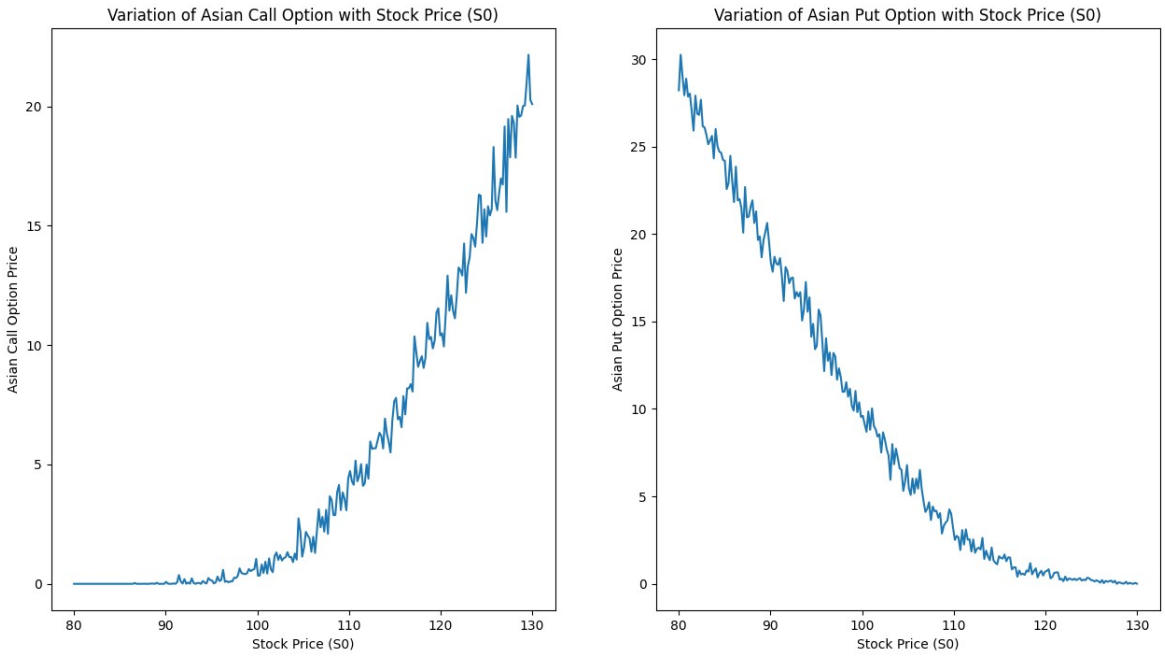


Prices of six month fixed-strike Asian option with different strike prices are as:

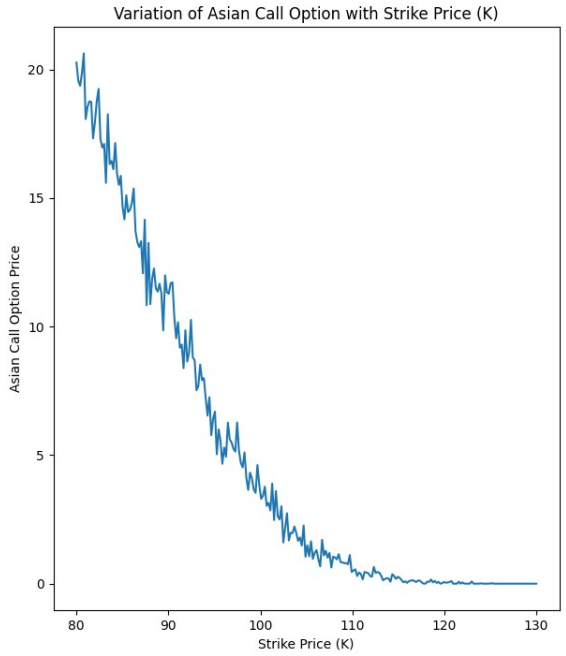
```
For K = 105
Asian Call Option Price:                2.999710463160364
Asian Call Option Variance:             35.954400852023845
Asian Put Option Price:                 6.191119941139521
Asian Put Option Variance:              48.11778451701077



For K = 110
Asian Call Option Price:                1.4705677598324929
Asian Call Option Variance:             17.182114038165302
Asian Put Option Price:                 10.123651695471242
Asian Put Option Variance:              69.98363260865652



For K = 90
Asian Call Option Price:                11.328224448323072
Asian Call Option Variance:             97.30502479385908
Asian Put Option Price:                 0.8323144076359099
Asian Put Option Variance:              6.056003360980276
```

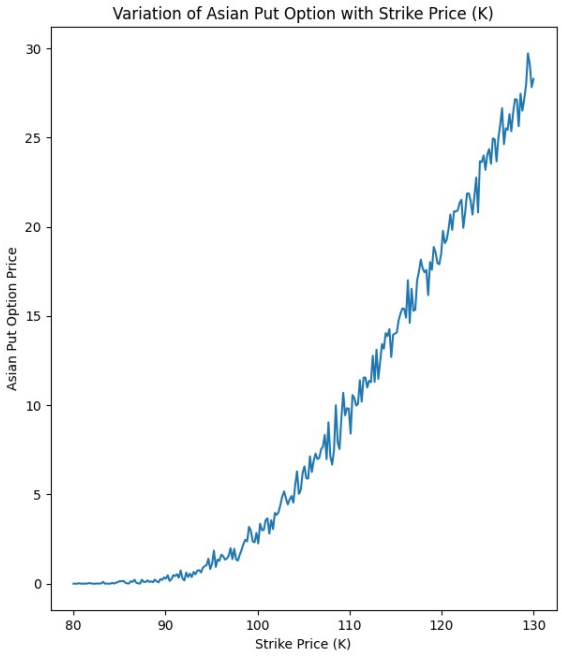
Sensitivity Analysis

- **Variation of Asian Option Prices with Stock Price:**

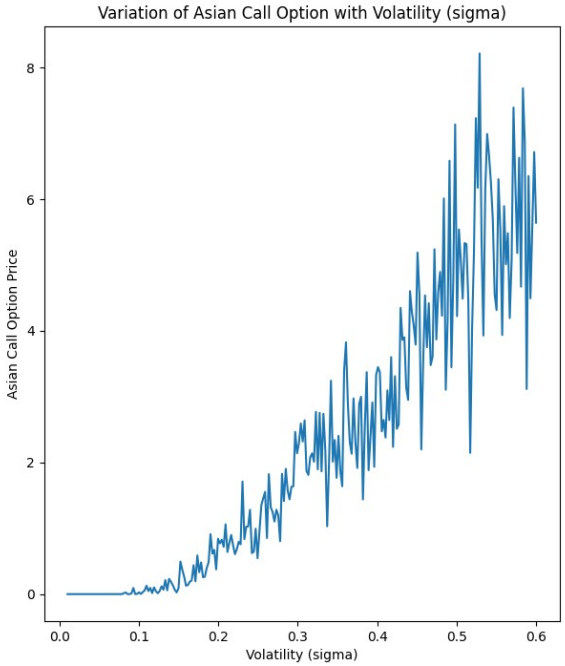


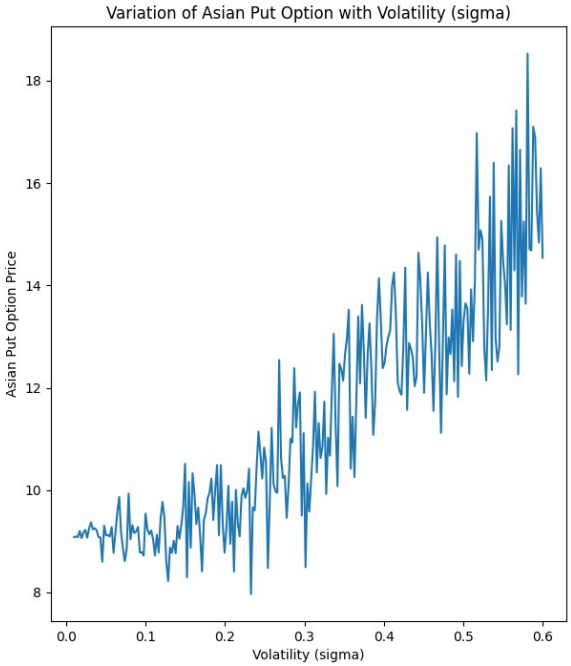
- **Variation of Asian Option Prices with Strke Price**



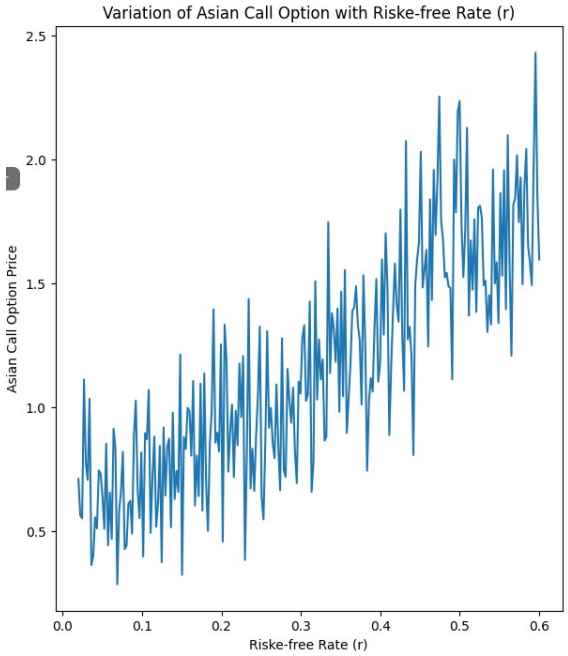


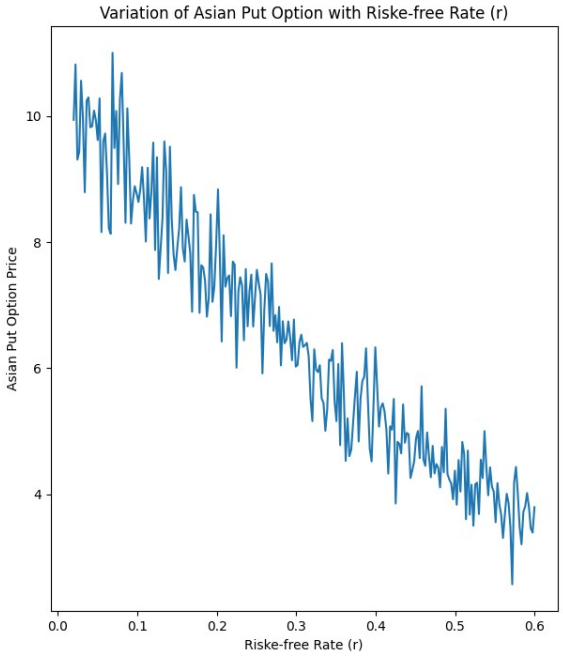
- **Variation of Asian Option Prices with Volatility**





- **Variation of Asian Option Prices with r**





Observations:

- As the original asset price, S0, rises, the price of the call option rises while the price of the put option falls.
- As Strike Price, the price of the call option falls while the price of the put option rises.
- With an increase in the risk-free interest, the price of the call option rises while the price of the put option falls. r
- As volatility rises, the price of both call and put options rises.
- There appear to be some variations in the graphs, which we attempt to reduce in the next question using variance reduction methods

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Q2

Variance Reduction Technique:

The method of Control Variate is used as a variance reduction tool. This technique uses knowledge about estimation errors in known quantities to minimise estimation errors in unknown quantities. The prices of European Option Prices are taken as the control variaate here, i.e., for Call Option Prices the payoff **max(S(t) – K, 0)** and for Put Option Prices the payoff **max(K – S(t), 0)** is taken as the control variate.

The optimal value of *b* calculated which minimises the variance of our estimator is:

$$b_n = (\sum(X_i - \overline{X})(Y - \overline{Y}))/((Y_i - \overline{Y})^2)$$

Here Yi are outputs from n replications of the simulations and Xi are corresponding output using the control variate.
Using SLLN, we coculde that b* is the required optimal value with bn -> b* with probability 1 and hence using this value we calculate the Control Value Estimator to achieve variance reduction.

The variance reduction value depends on the Correlation of X and Y.

Prices of six month fixed-strike Asian option with different strike prices after performing variance reduction:

```
For K = 105
Asian Call Option Price:                2.809533274783352
Asian Call Option Variance:             11.633375455201104
Asian Put Option Price:                 6.6511417836861835
Asian Put Option Variance:              16.52687981411049



For K = 110
Asian Call Option Price:                1.8958834585514288
Asian Call Option Variance:             8.238528278772293
Asian Put Option Price:                 9.953095106764247
Asian Put Option Variance:              20.460044374225145



For K = 90
Asian Call Option Price:                12.072407158734636
Asian Call Option Variance:             28.144091584951784
Asian Put Option Price:                 0.7708418075631741
Asian Put Option Variance:              2.5841627541627012
```

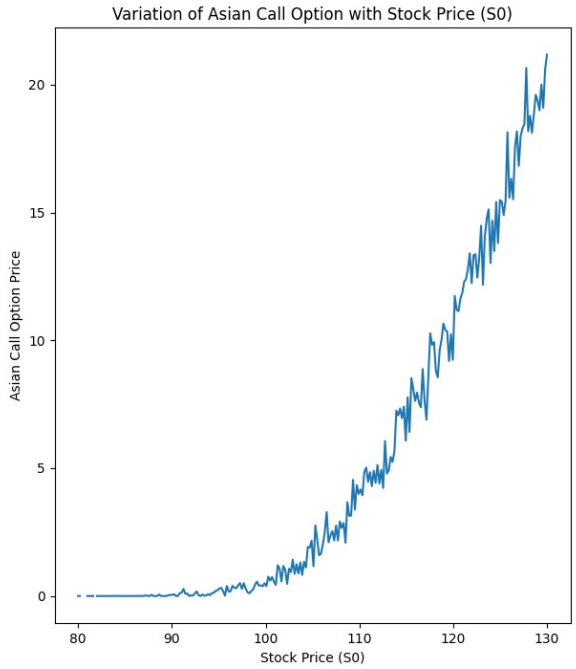
Table to compare the variance:

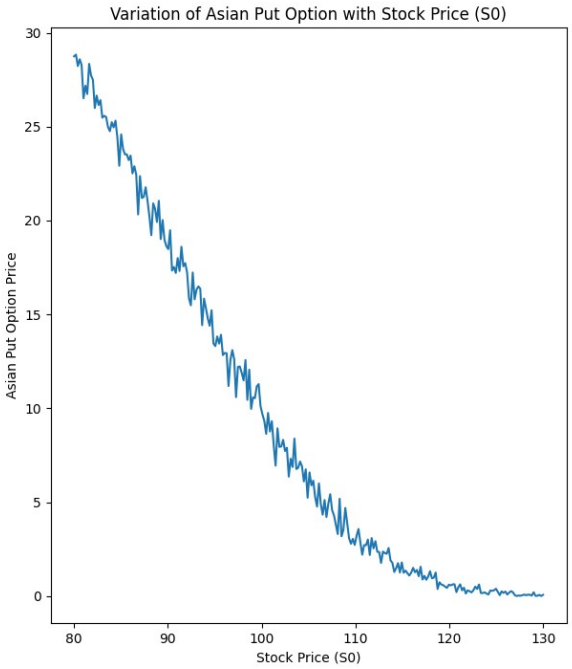
Strike Price	Asian Call Variance	Asian Call Variance after Variance Reduction	Asian Put Variance	Asian Put Variance after Variance Reduction
105	11.633375455201104	11.633375455201104	16.52687981411049	16.52687981411049
110	17.182114038165302	8.238528278772293	69.98363260865652	20.460044374225145
90	97.30502479385908	28.144091584951784	6.056003360980276	2.5841627541627012

Clearly, we can see that the variance has reduced after applying the variance reduction technique.

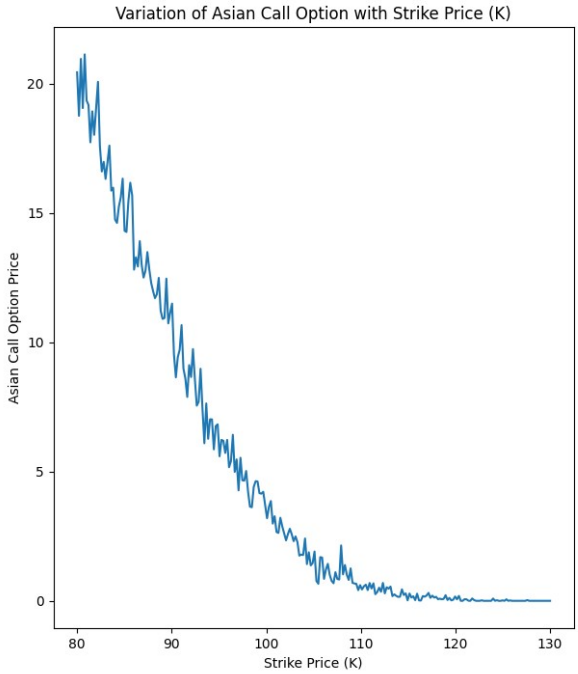
Sensitivity Analysis After Variance Reduction:

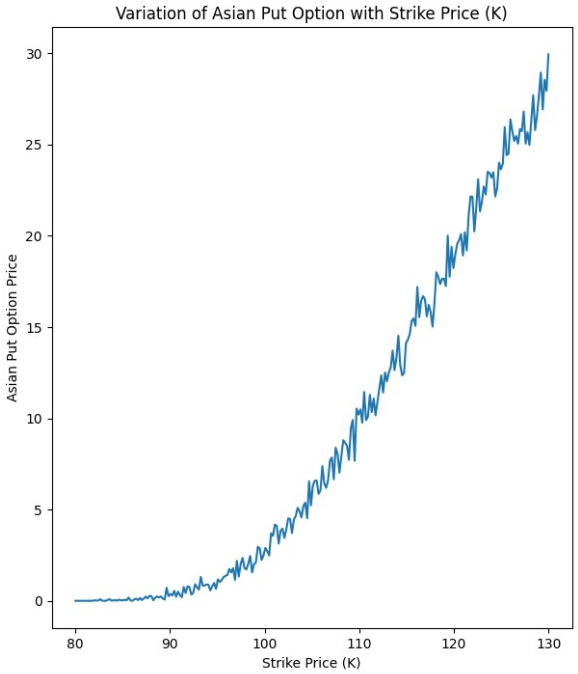
- **Variation of Asian Option Prices with Stock Price:**



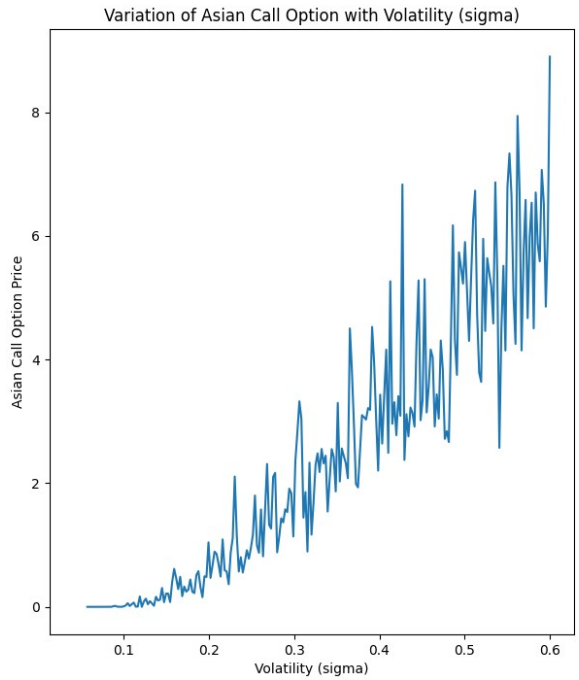


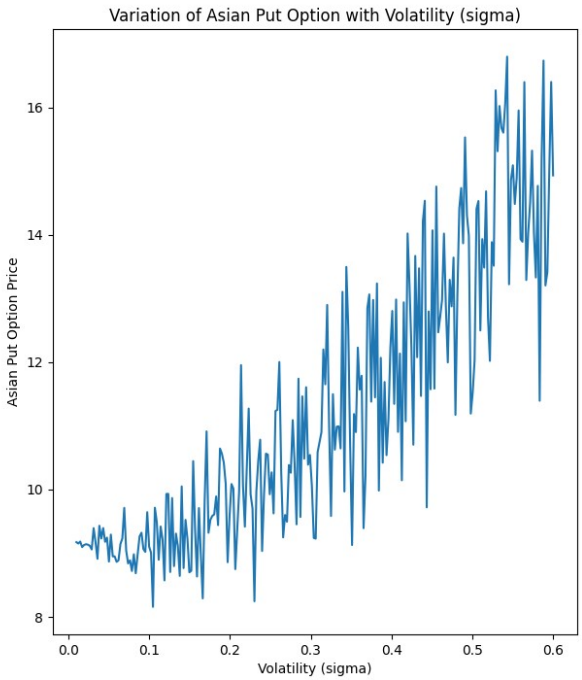
- **<u>Variation of Asian Option Prices with Strike Price:</u>**



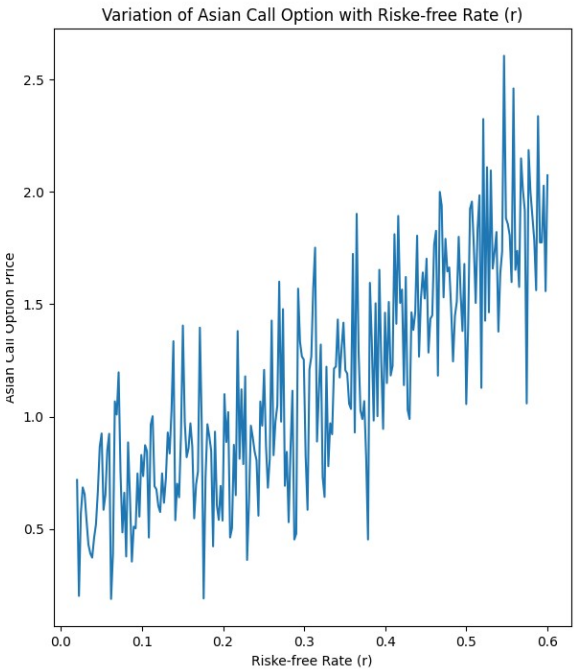


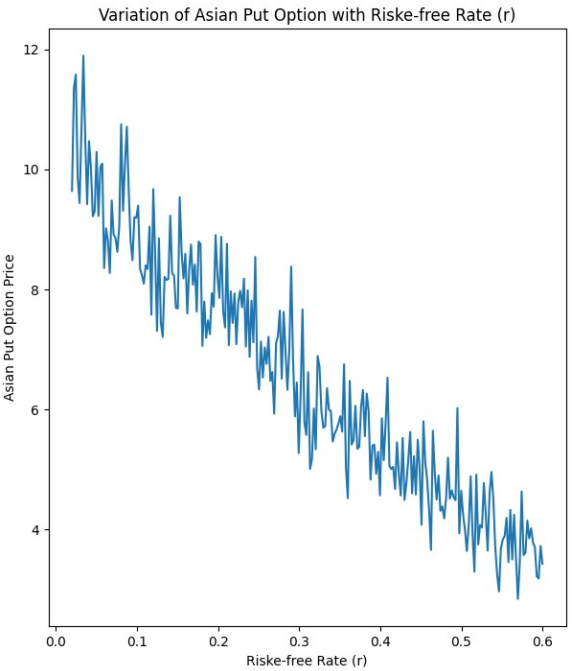
- **<u>Variation of Asian Option Prices with Volatility:</u>**





- **<u>Variation of Asian Option Prices with Riske-free Rate:</u>**





<u>Observations:</u>

Behavior of all the graphs is same as that of the previous question. Other observations made due to the variance reduction technique are as:

- We have previously proven numerically that the variance decrease is accomplished. The plots created further corroborate this assertion.
- After thorough examination, the fluctuations in the graphs appear to be less than when variance reduction was not used. As a result, the plan succeeds.