

MA374 – LAB02

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Roll Number: 200123081

Q1

Given Data:

$S(0)$ = Initial Stock Price = 100

K = Strike Price = 100

T = Time to maturity = 1 year

r = Risk Free Rate = 8%

σ = Volatility of the stock = 20%

At each time step, the stock's price can rise by a factor of u or fall by a factor of d .

The formula for u and d –

$$\text{Set1: } u = e^{\sigma\sqrt{\Delta t}}, \quad d = e^{-\sigma\sqrt{\Delta t}}$$

$$\text{Set2: } u = e^{\sigma\sqrt{\Delta t} + \left(r - \frac{\sigma^2}{2}\right)\Delta t}, \quad d = e^{-\sigma\sqrt{\Delta t} + \left(r - \frac{\sigma^2}{2}\right)\Delta t}$$

The probability (q) of an upward return in price is $R - d/u - d$, where $R = e^{r\Delta t}$ and $\Delta t = T/M$ with M being the number of subintervals in the timeinterval $[0, T]$.

At expiry, i.e, $t = T$, we calculate the price of the option using the respective payoff function for both the call and put option, i.e,

$$C_n^M = \max(S_n^M - K, 0) \text{ and } P_n^M = \max(-S_n^M + K, 0)$$

where $0 \leq n \leq M$ and C_n^M is the nth possible price of the call option for the Mth interval, and P_n^M is the nth possible price of the put option for the Mth interval

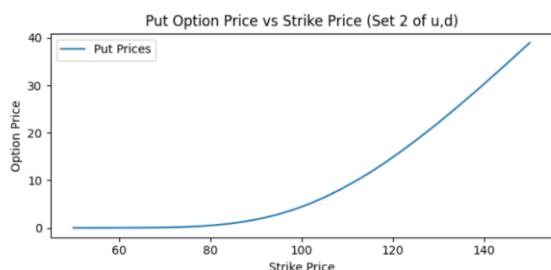
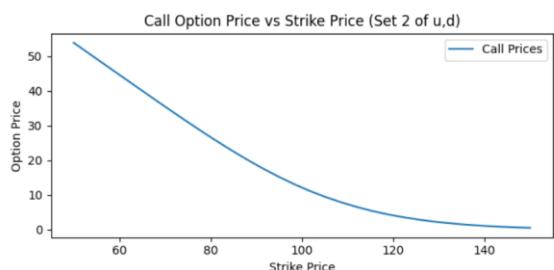
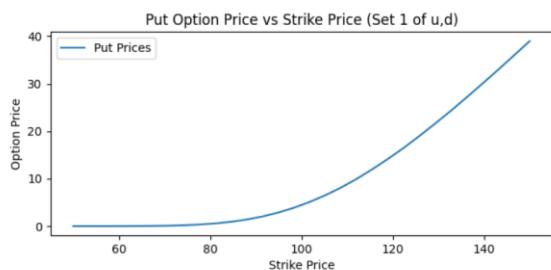
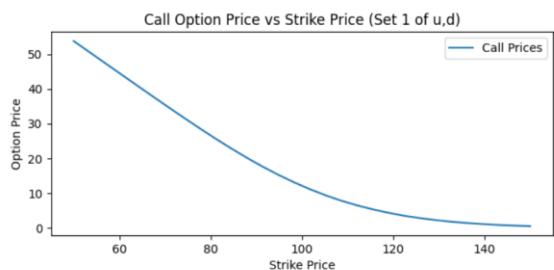
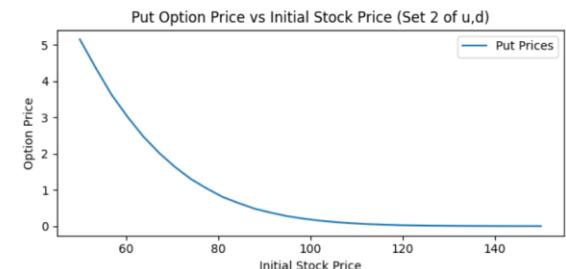
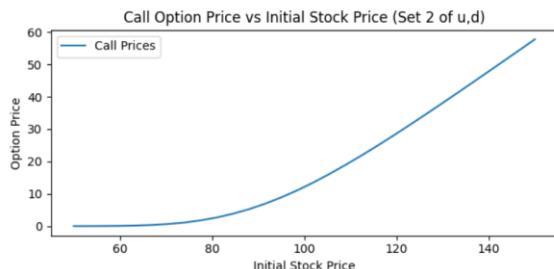
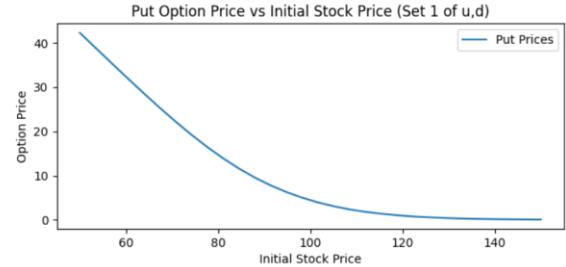
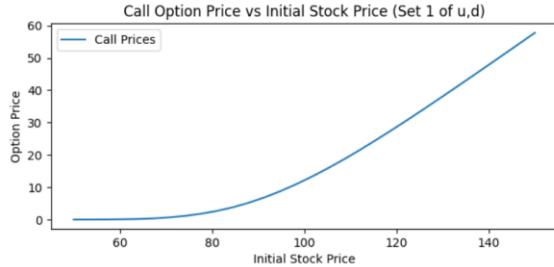
Now, we continuously apply **Backward Induction** to find out the option price at $t = 0$ by using following relation:

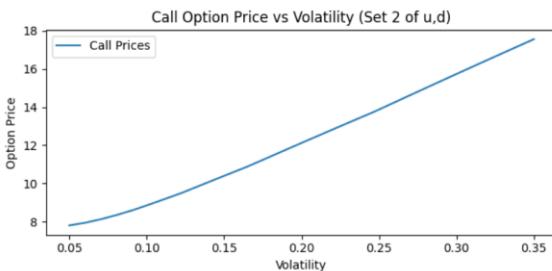
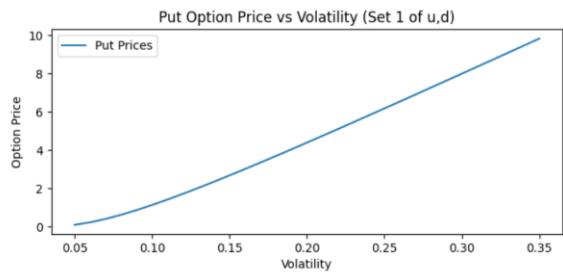
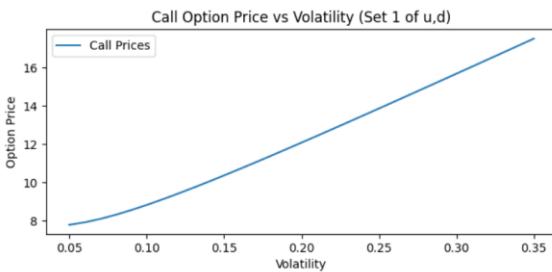
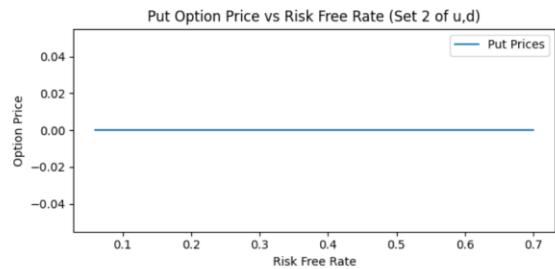
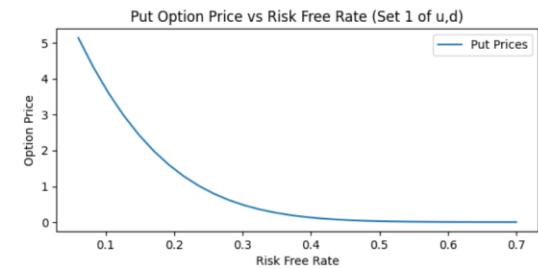
$$C_n^i = (1-p) C_{n+1}^{i+1} + (p) C_n^{i+1} \text{ and } P_n^i = (1-p) P_{n+1}^{i+1} + (p) P_n^{i+1}, \quad 0 \leq n \leq i \quad \& \quad 0 \leq i \leq M - 1$$

C_0^0 and P_0^0 are the required values, i.e, initial option prices.

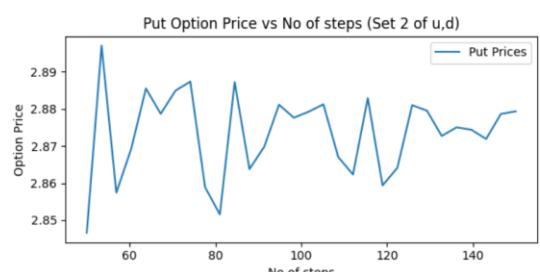
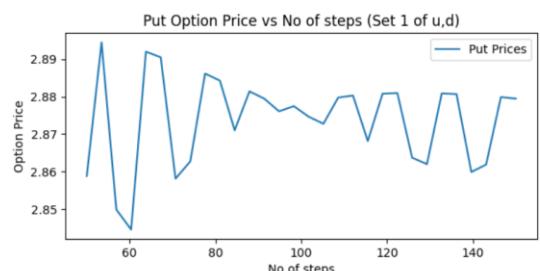
In the below graphs, we have done the sensitivity analysis first by changing only a single factor and then by changing 2 at a time.

Note: It took around 5 minutes for the code to run on my computer.

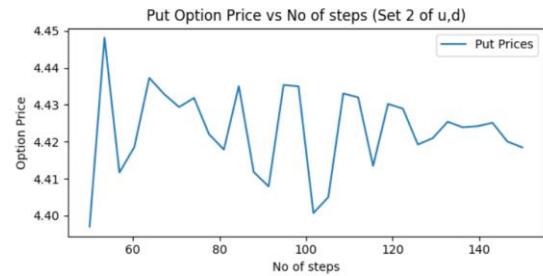
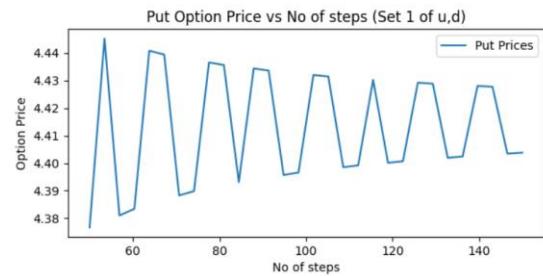
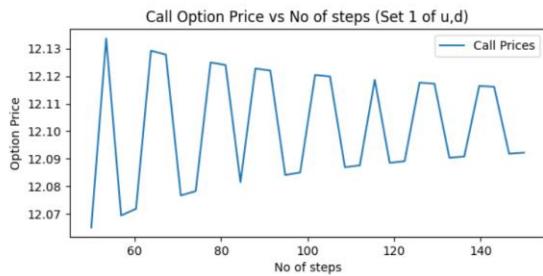




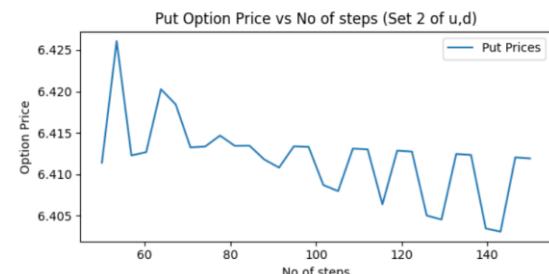
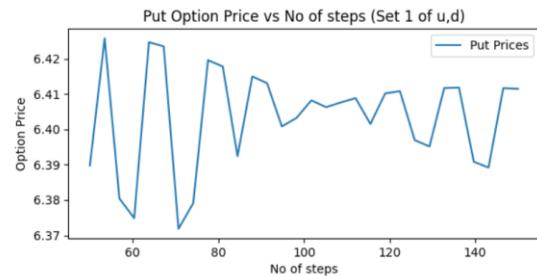
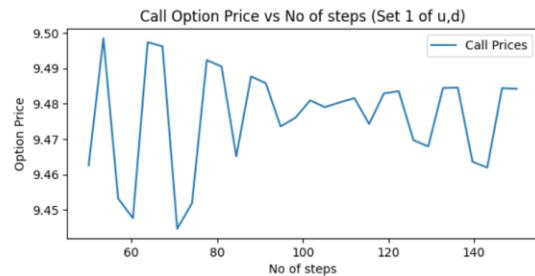
a) $K = 95$



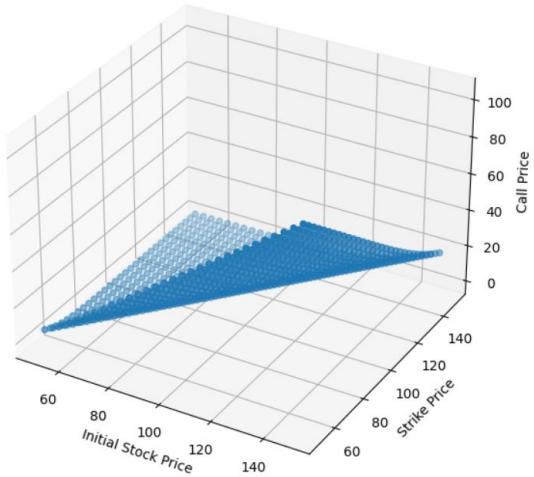
b) $K = 100$



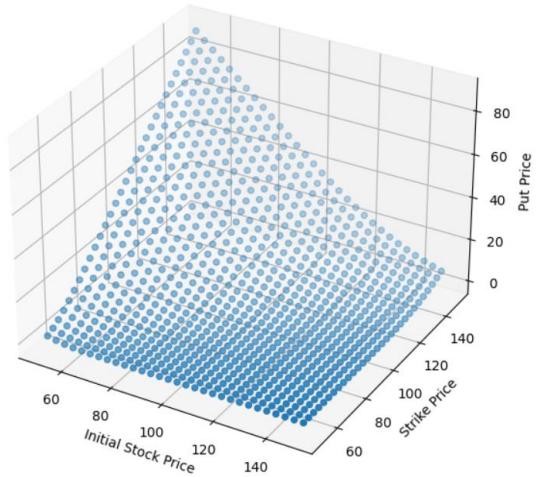
c) $K = 105$



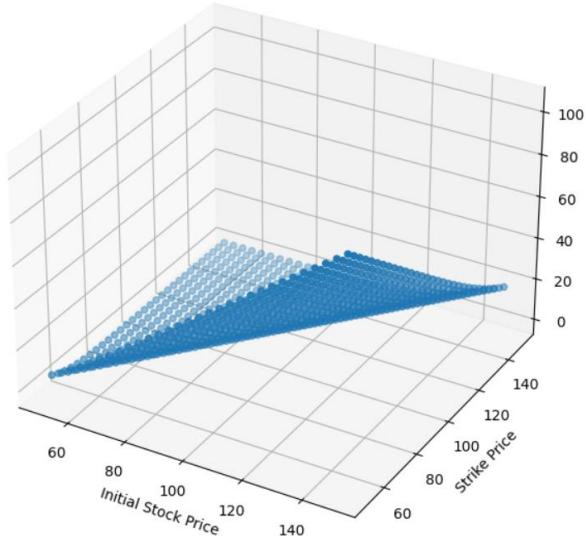
Call Price vs (Initial Stock Price and Strike Price) (Set 1 of u,d)



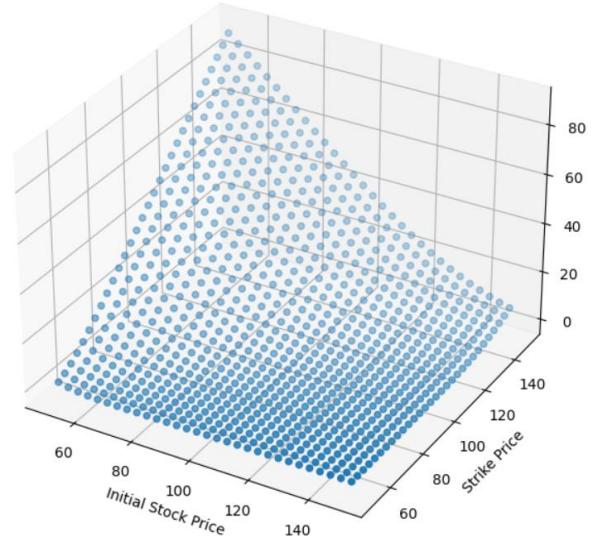
Put Price vs (Initial Stock Price and Strike Price) (Set 1 of u,d)



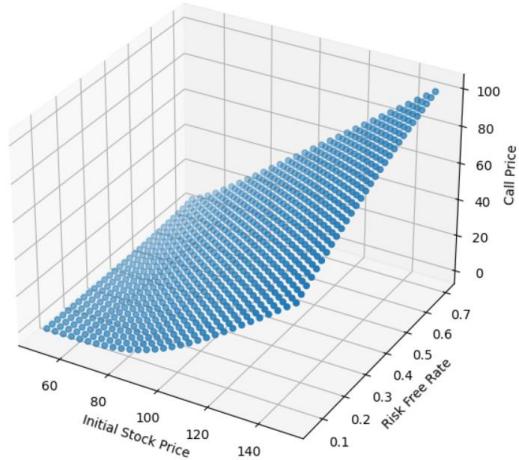
Call Price vs (Initial Stock Price and Strike Price) (Set 2 of u,d)



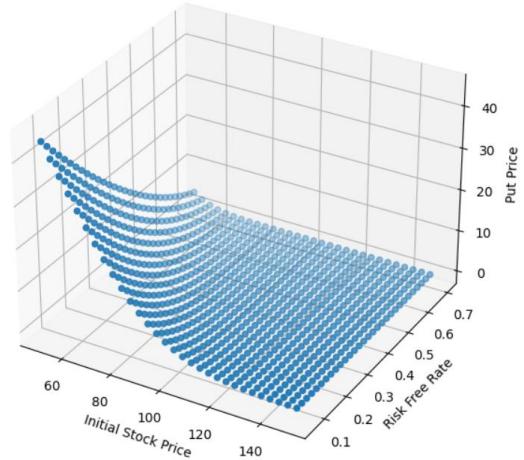
Put Price vs (Initial Stock Price and Strike Price) (Set 2 of u,d)



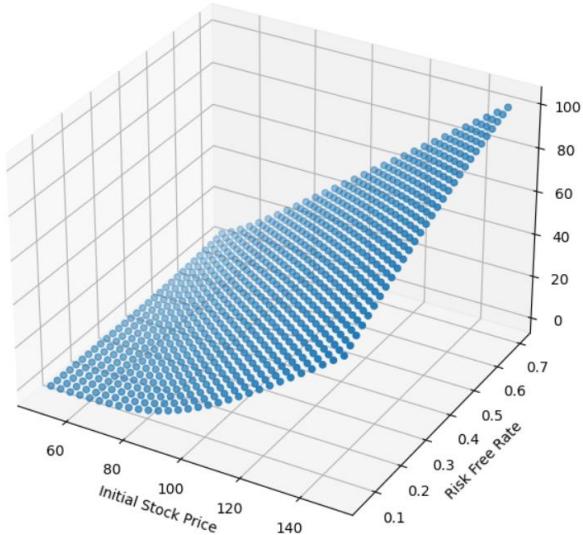
Call Price vs (Initial Stock Price and Risk Free Rate) (Set 1 of u,d)



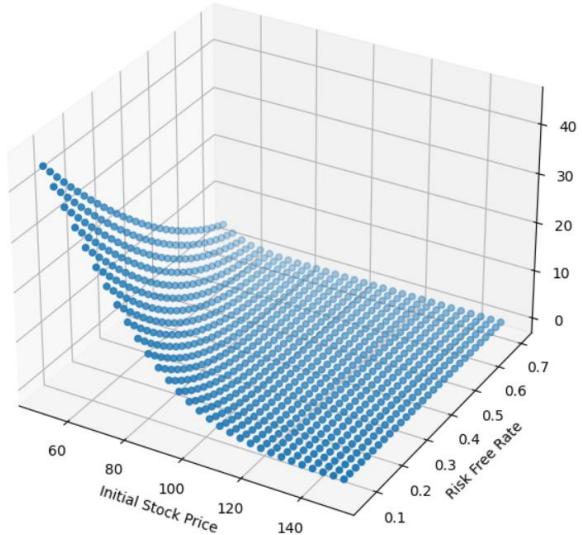
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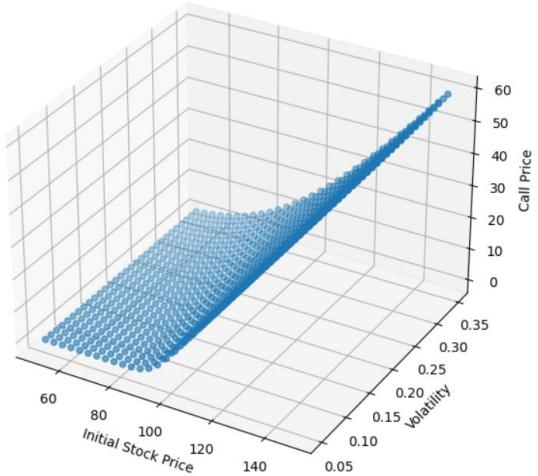
Call Price vs (Initial Stock Price and Risk Free Rate) (Set 2 of u,d)



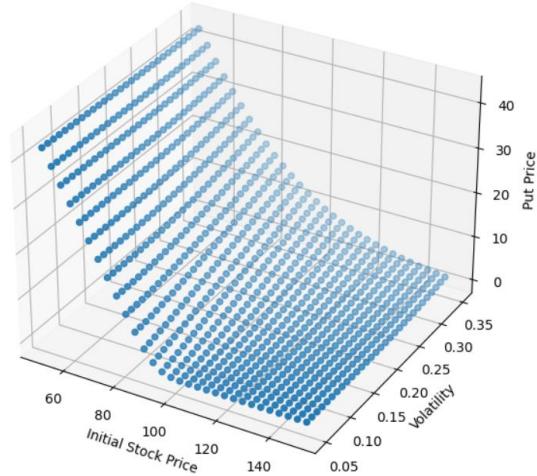
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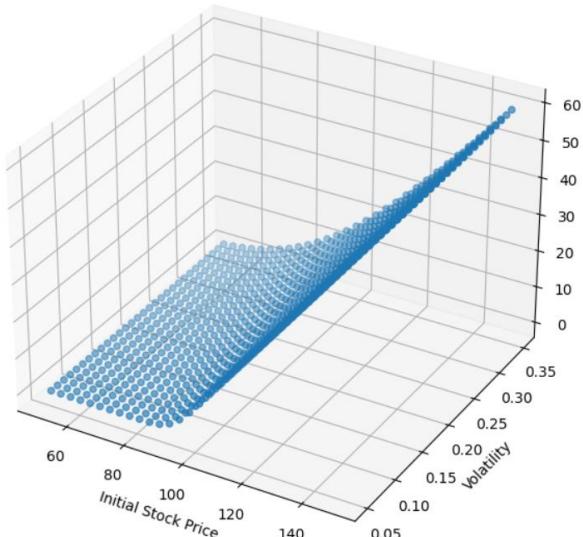
Call Price vs (Initial Stock Price and Volatility) (Set 1 of u,d)



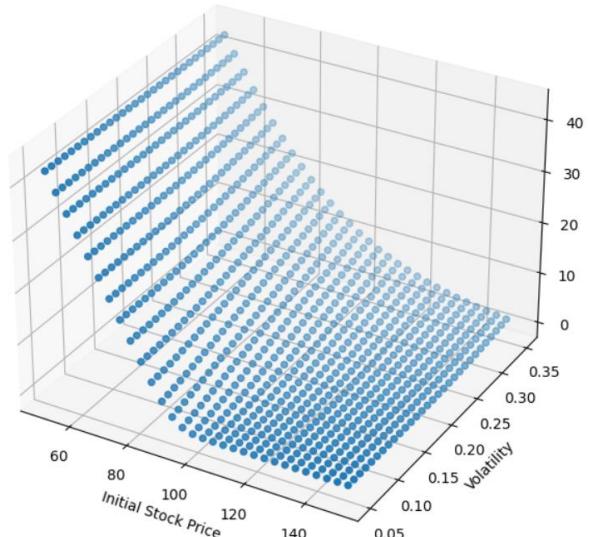
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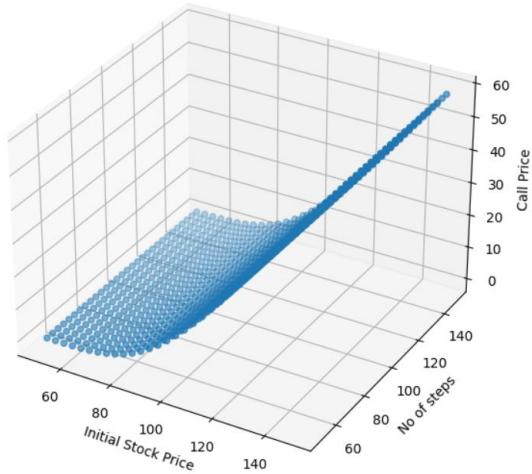
Call Price vs (Initial Stock Price and Volatility) (Set 2 of u,d)



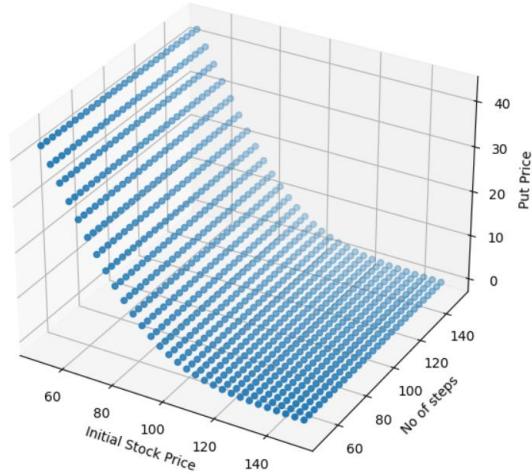
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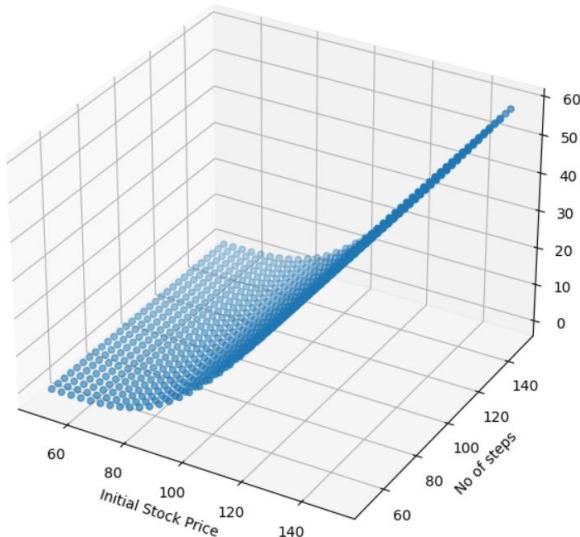
Call Price vs (Initial Stock Price and No of steps) (Set 1 of u,d)



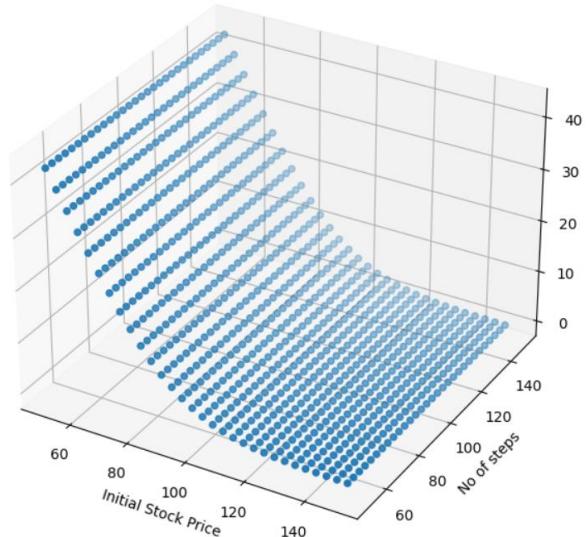
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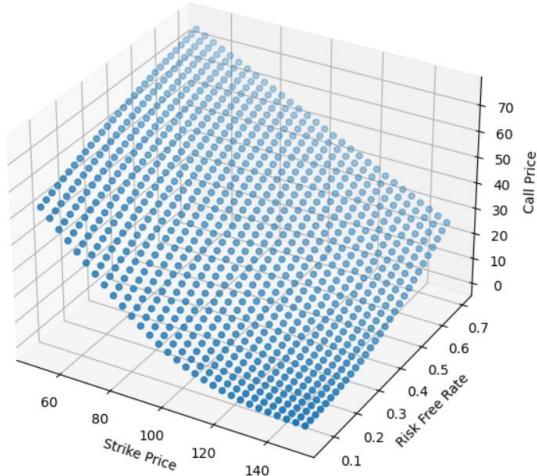
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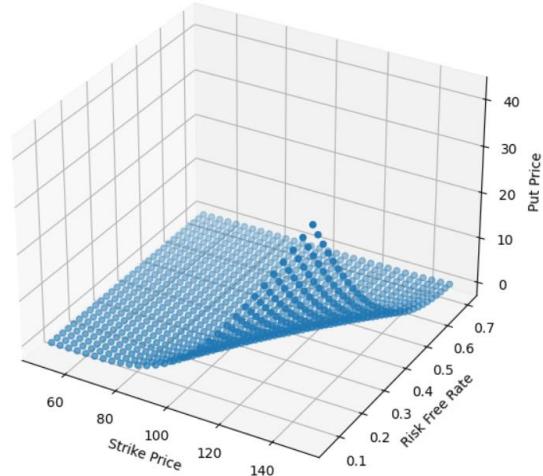
Put Price vs (Initial Stock Price and No of steps) (Set 2 of u,d)



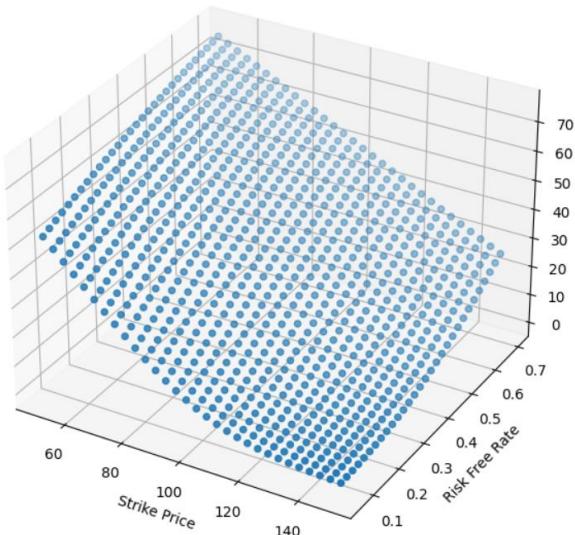
Call Price vs (Strike Price and Risk Free Rate) (Set 1 of u,d)



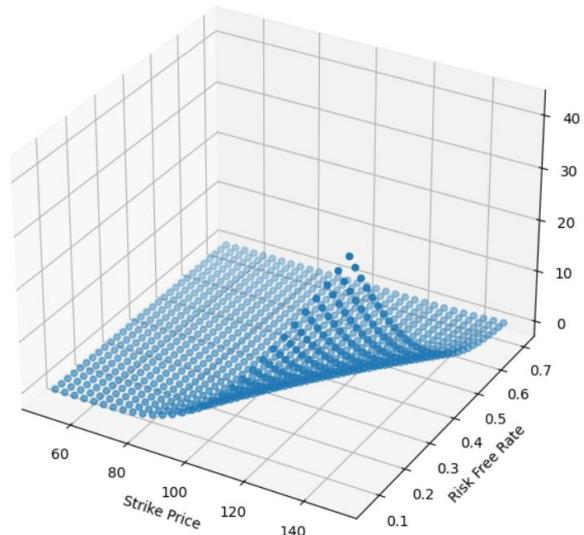
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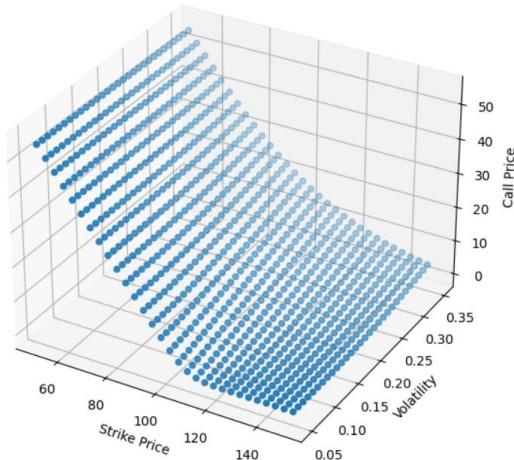
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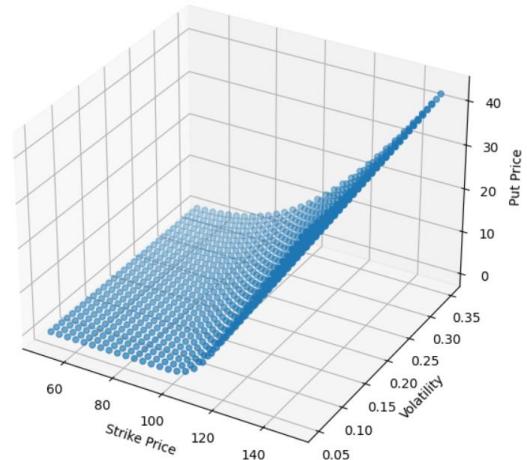
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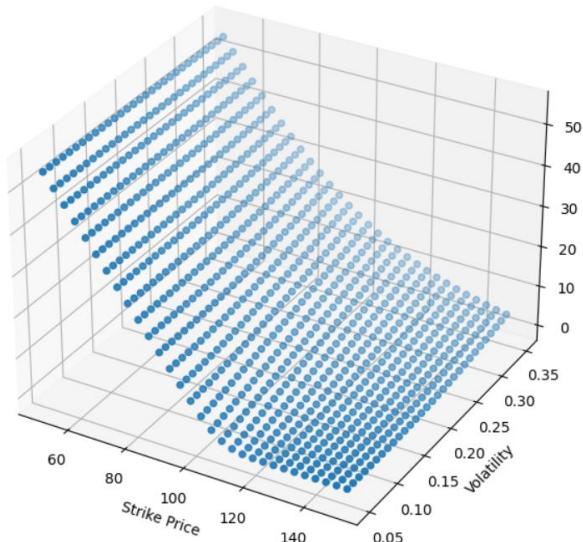
Call Price vs (Strike Price and Volatility) (Set 1 of u,d)



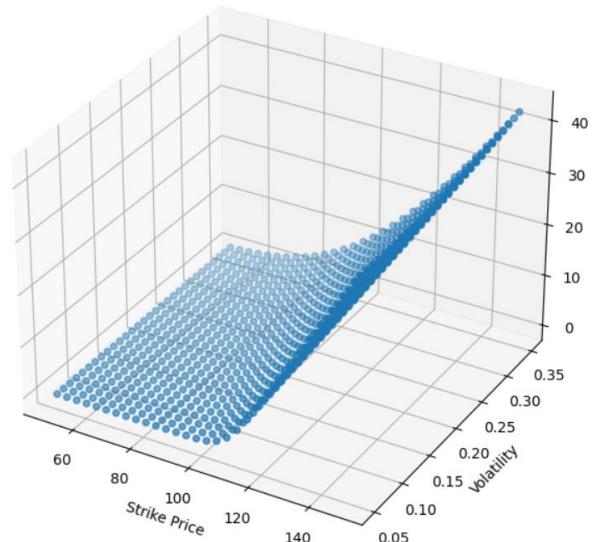
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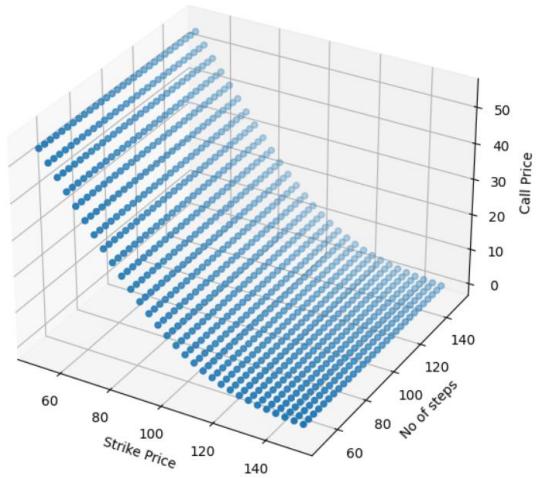
Call Price vs (Strike Price and Volatility) (Set 2 of u,d)



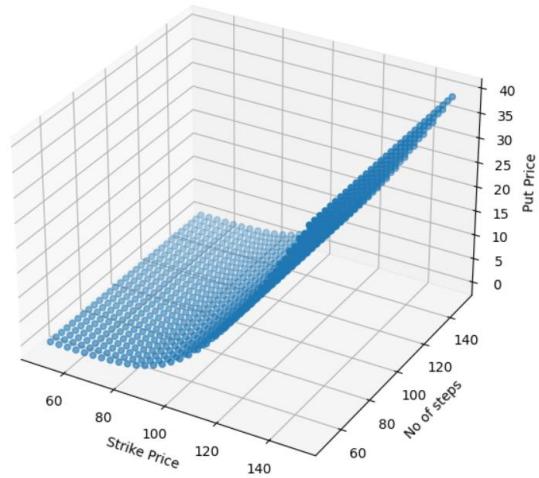
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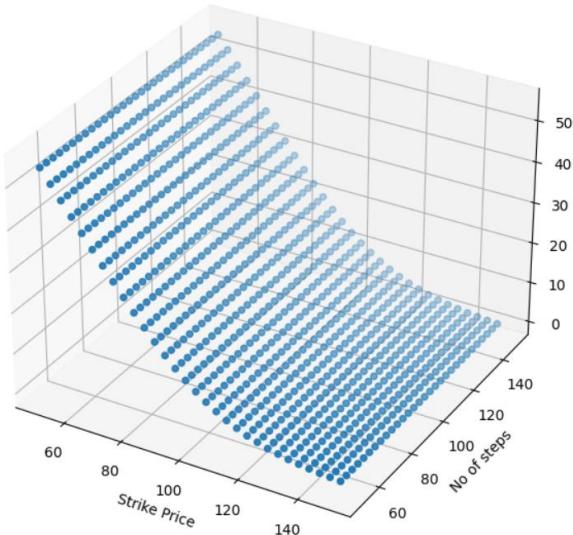
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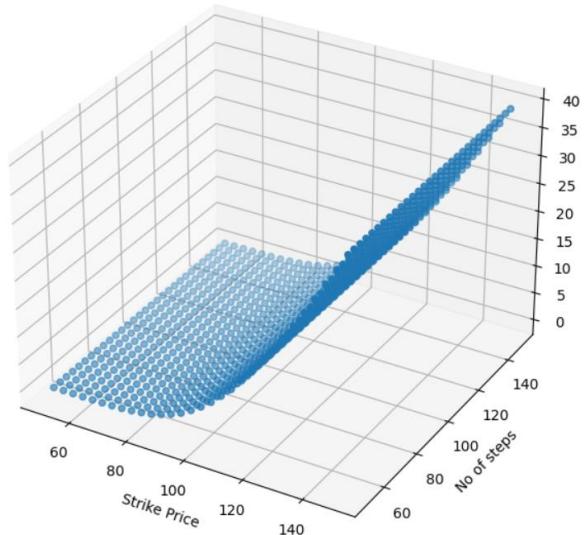
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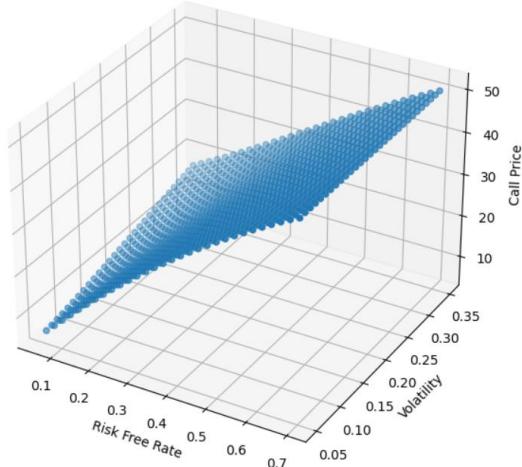
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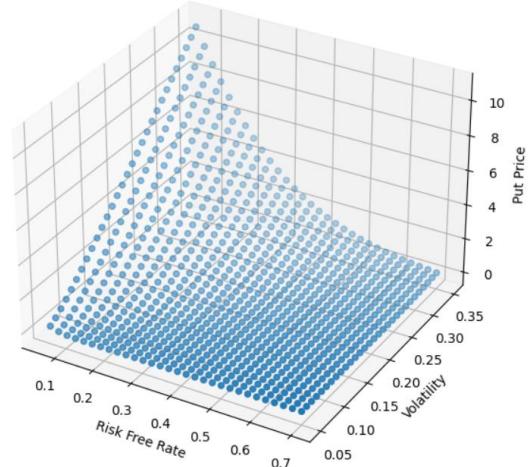
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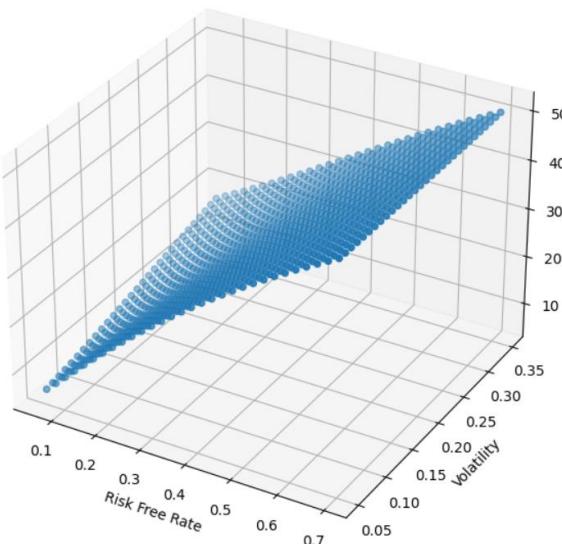
Call Price vs (Risk Free Rate and Volatility) (Set 1 of u,d)



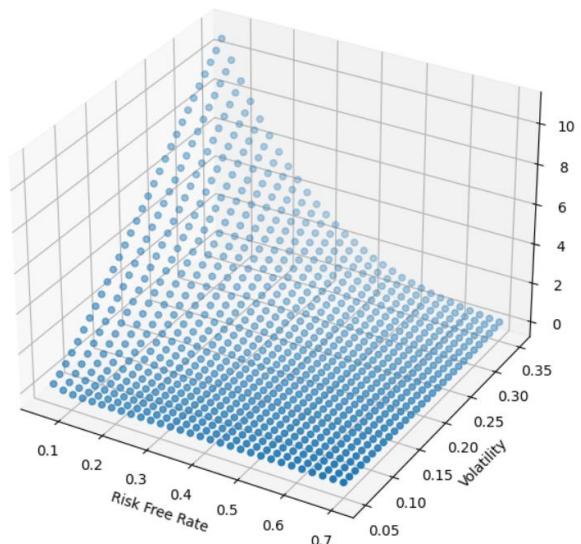
Put Price vs (Risk Free Rate and Volatility) (Set 1 of u,d)



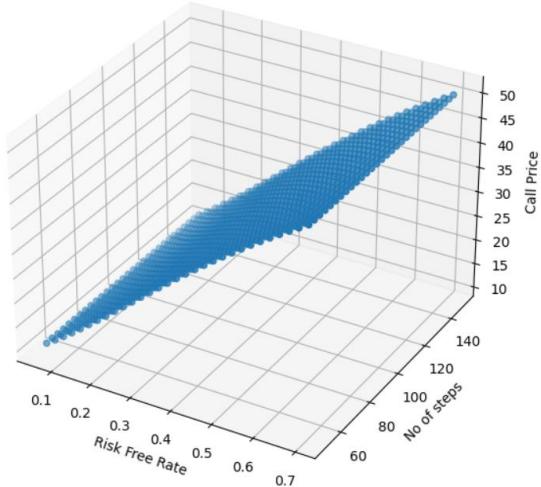
Call Price vs (Risk Free Rate and Volatility) (Set 2 of u,d)



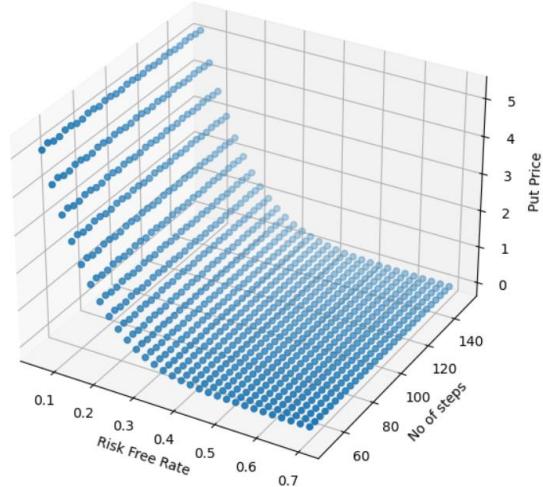
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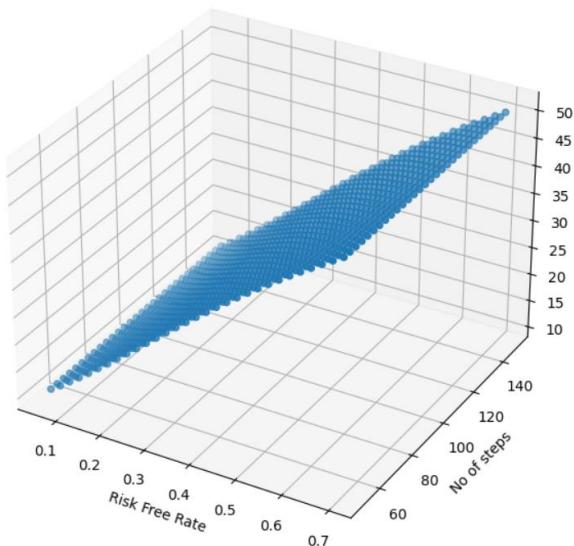
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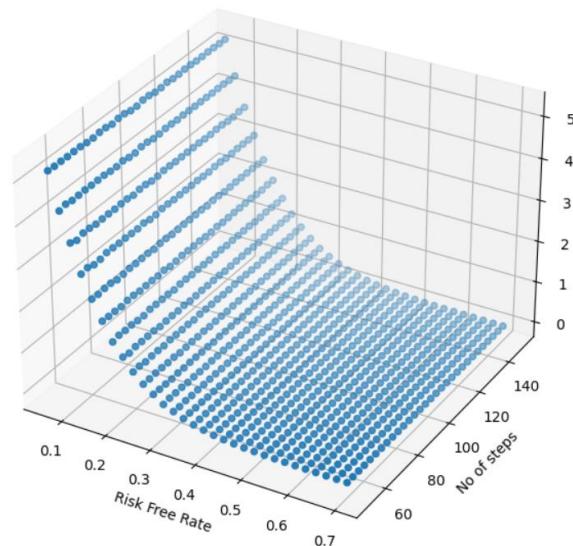
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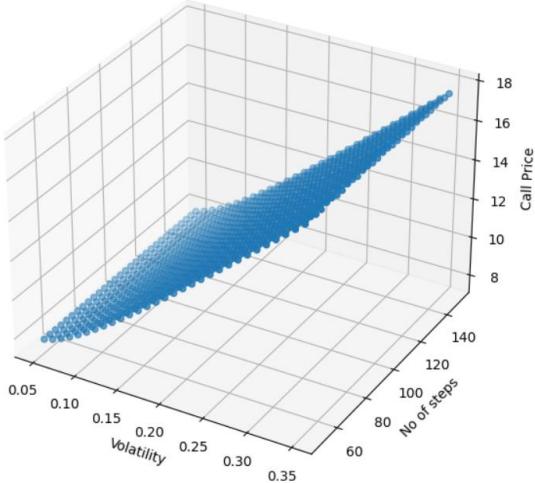
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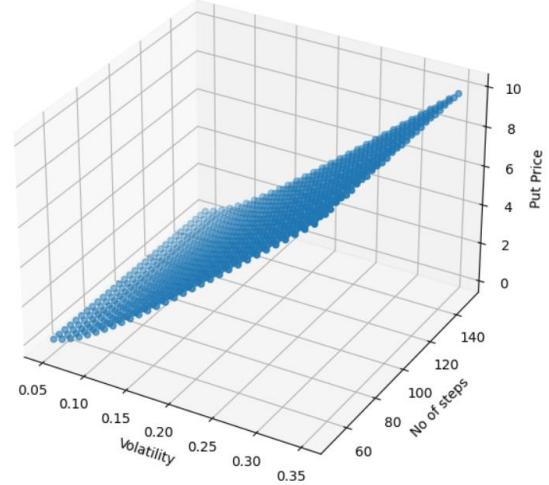
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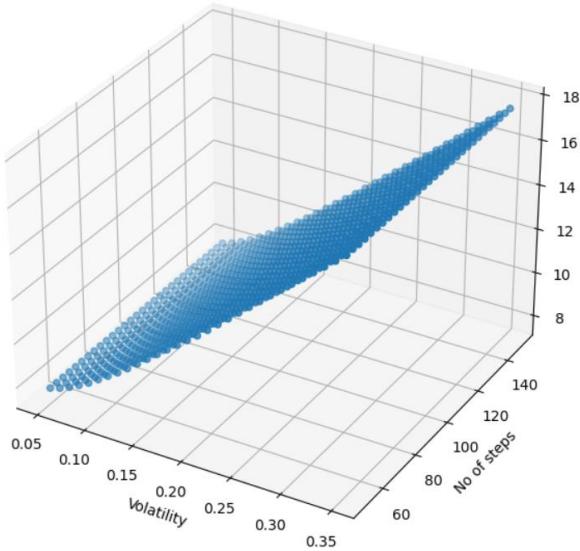
Call Price vs (Volatility and No of steps) (Set 1 of u,d)



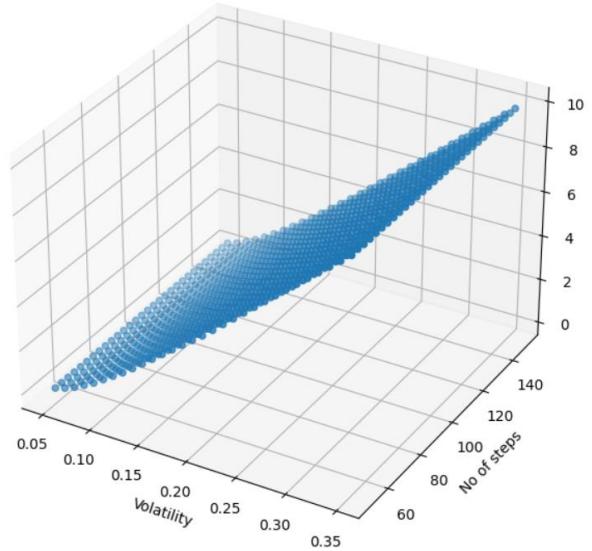
Put Price vs (Volatility and No of steps) (Set 1 of u,d)



Call Price vs (Volatility and No of steps) (Set 2 of u,d)



Put Price vs (Volatility and No of steps) (Set 2 of u,d)



Q2

This question is done by choosing the path dependent derivative to be Asian Options.

For this question we have taken M as 10 because Algorithm for calculating price is $O(2^M)$.

The **payoff for Asian Option** at expiry T can be given as:

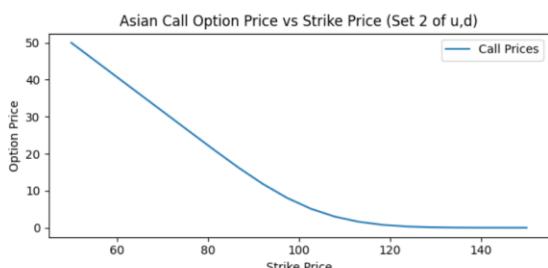
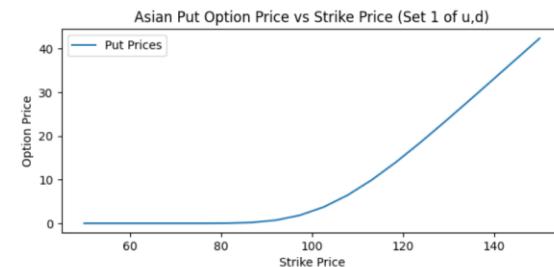
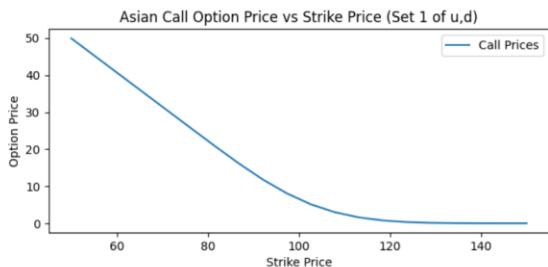
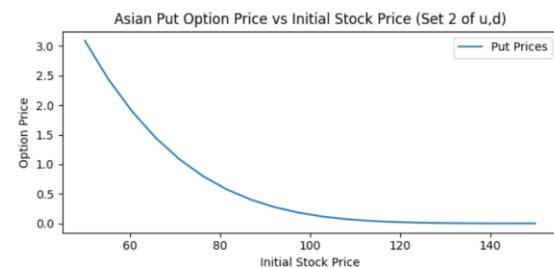
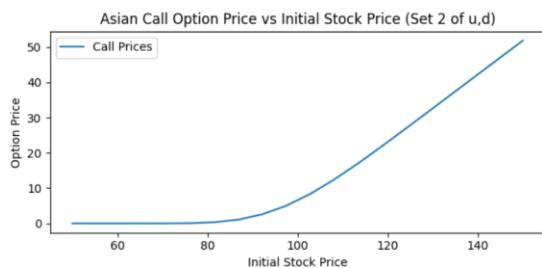
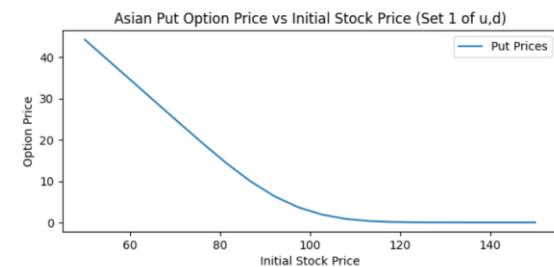
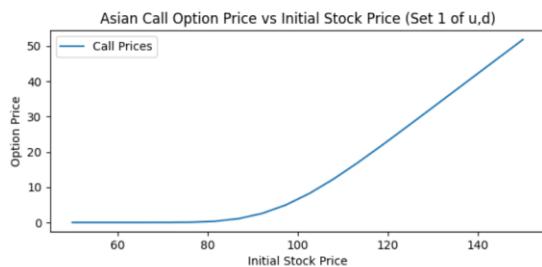
$$\frac{1}{e^{rT}} \sum q^{\text{number of ups}} (1 - q)^{M - \text{number of ups}} f(S_{avg})$$

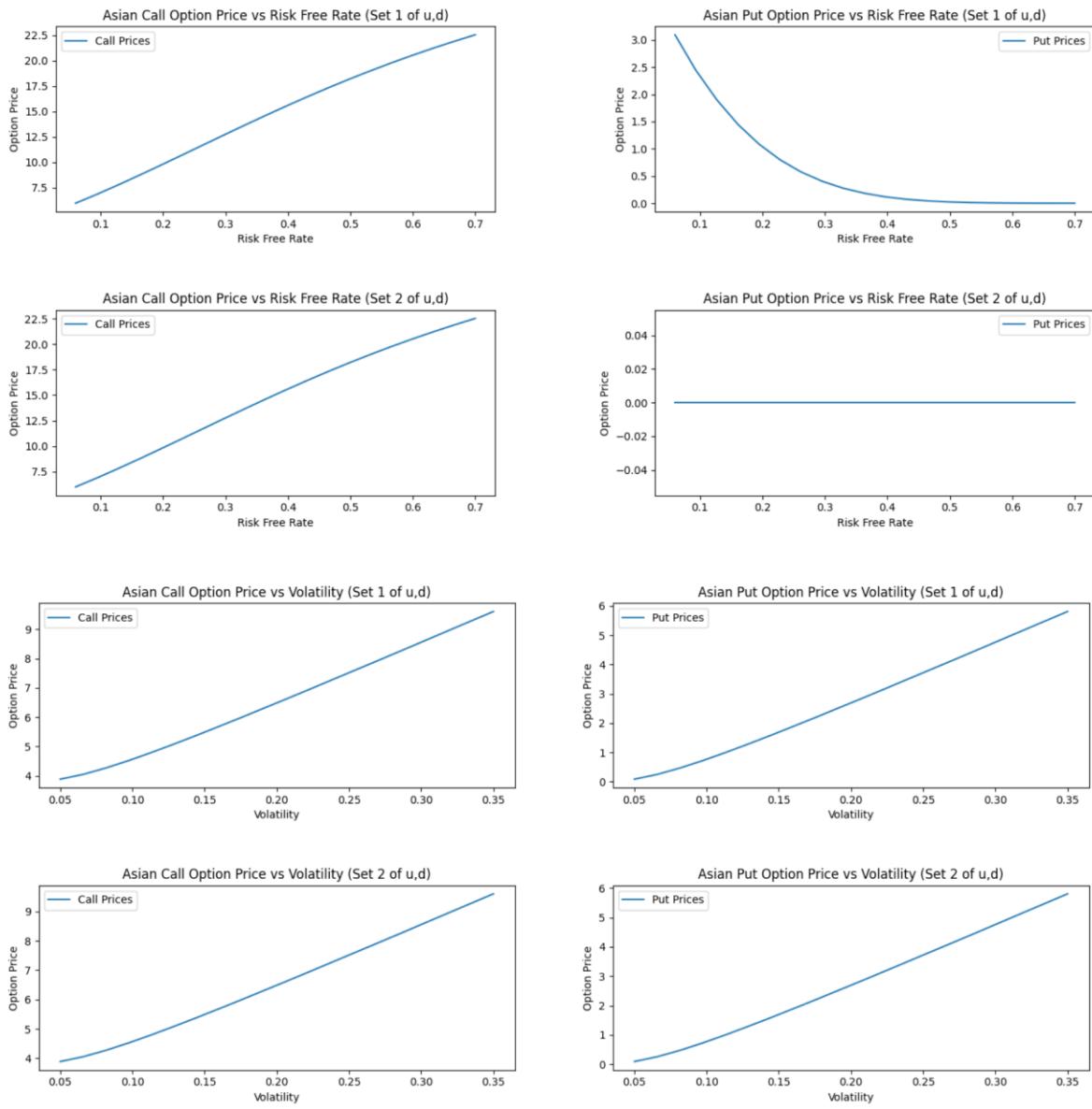
where $f(S_{avg})$ is the final payoff calculated using following formula –
 for call option – $\max(S_{avg}(T) - K, 0)$

for put option – $\max(K - S_{avg}(T), 0)$

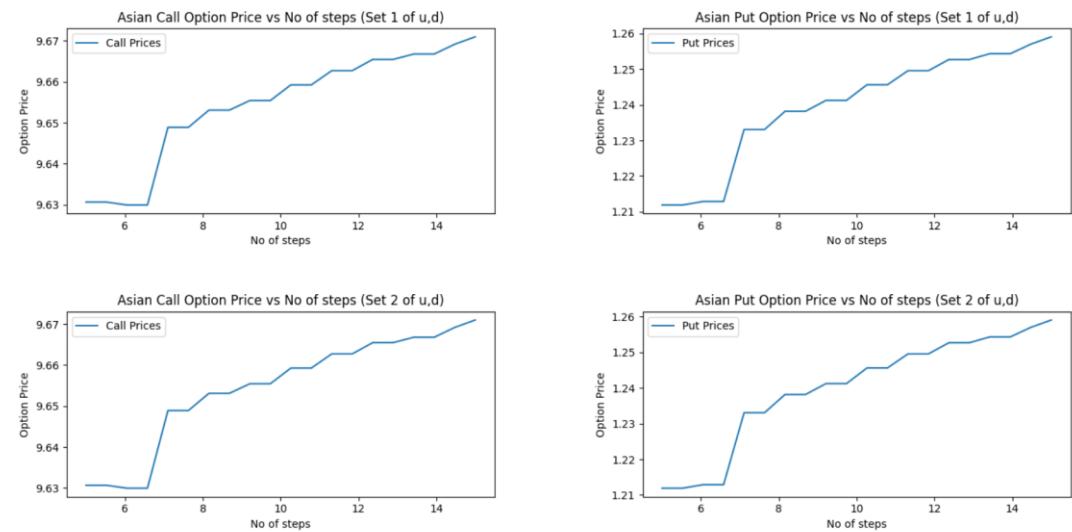
S_{avg} is the average stock price along any particular path.

In the below graphs, we have done the sensitivity analysis first by changing only a single factor and then by changing 2 at a time.

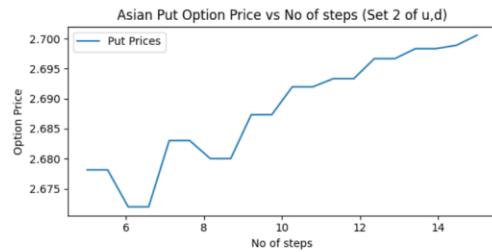
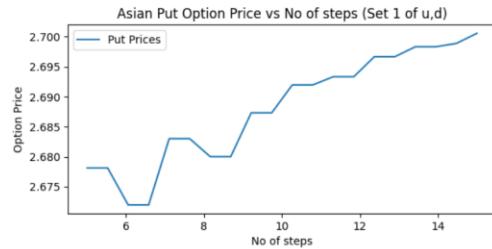




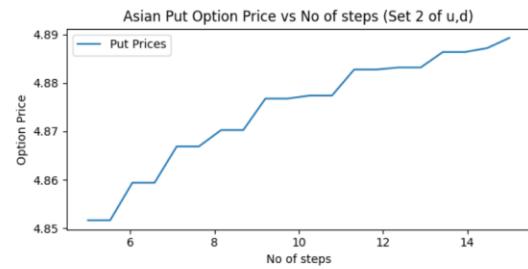
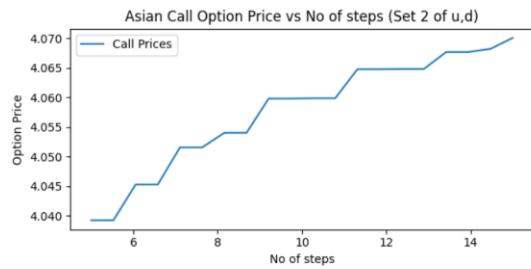
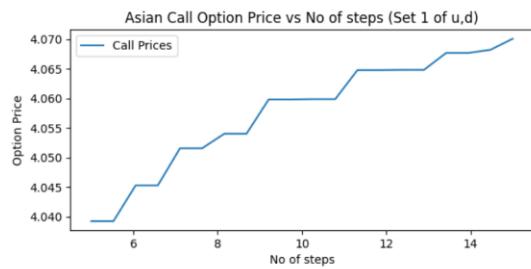
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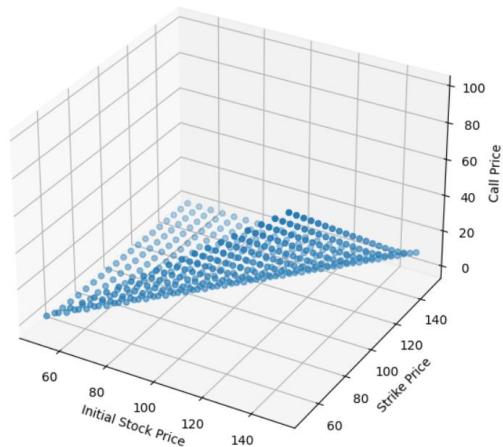
b) $K = 100$



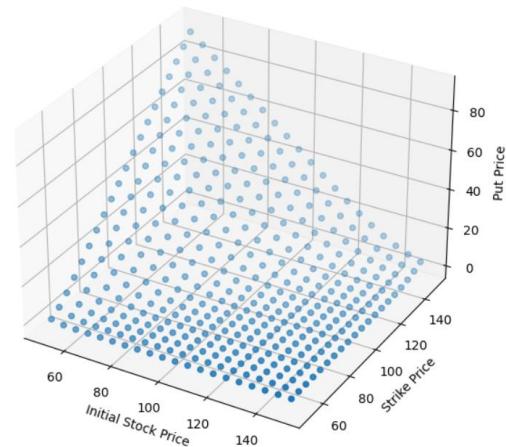
c) $K = 105$



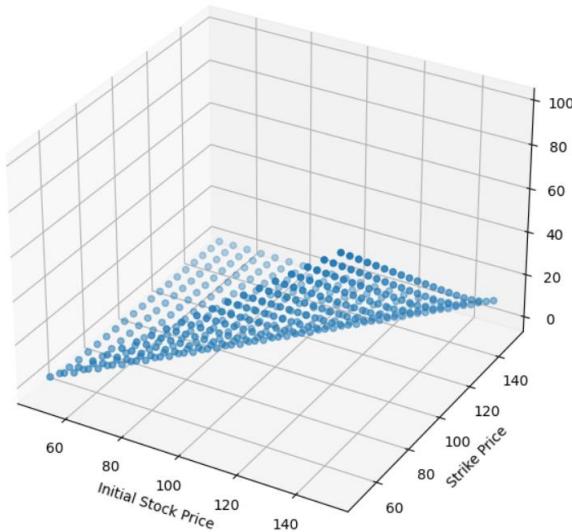
Asian Call Price vs (Initial Stock Price and Strike Price) (Set 1 of u,d)



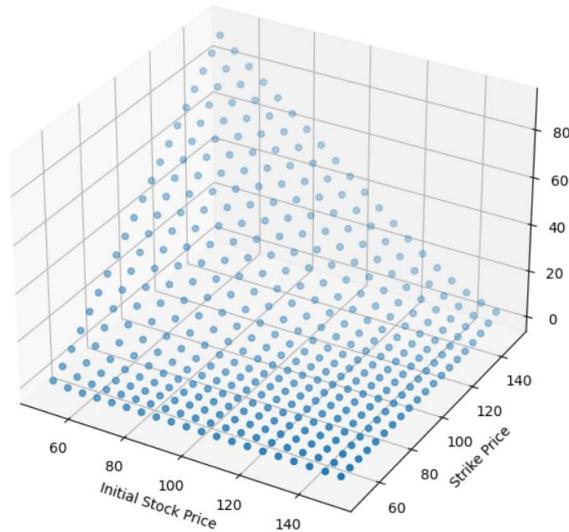
Asian Put Price vs (Initial Stock Price and Strike Price) (Set 1 of u,d)



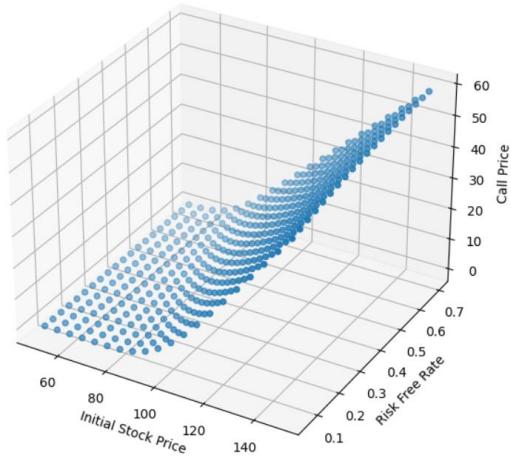
Asian Call Price vs (Initial Stock Price and Strike Price) (Set 2 of u,d)



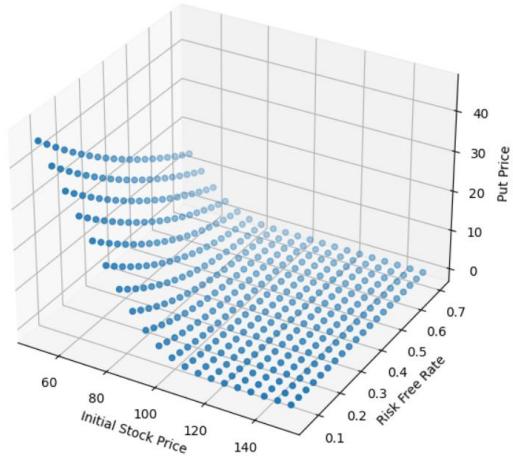
Asian Put Price vs (Initial Stock Price and Strike Price) (Set 2 of u,d)



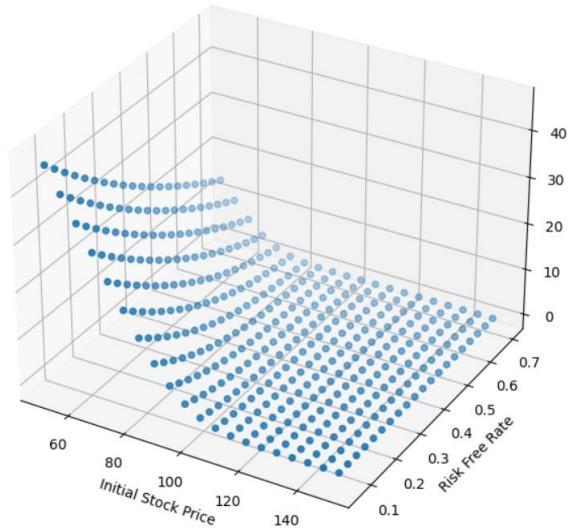
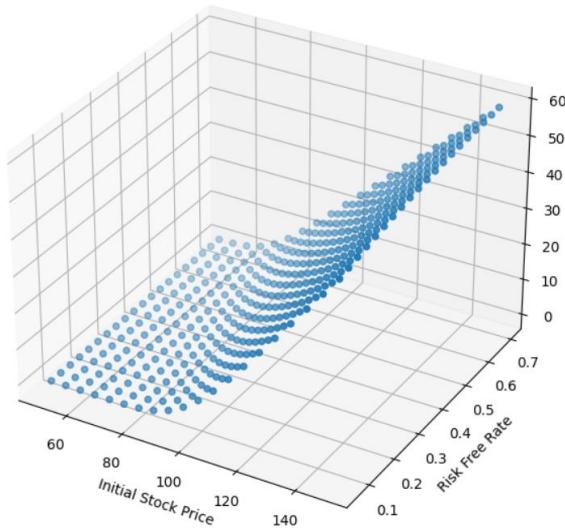
Asian Call Price vs (Initial Stock Price and Risk Free Rate) (Set 1 of u,d)



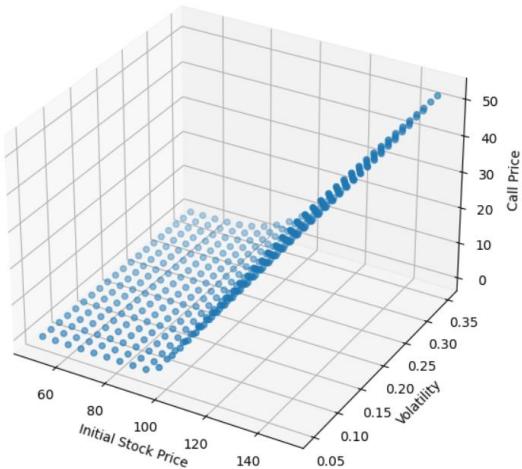
Asian Put Price vs (Initial Stock Price and Risk Free Rate) (Set 1 of u,d)



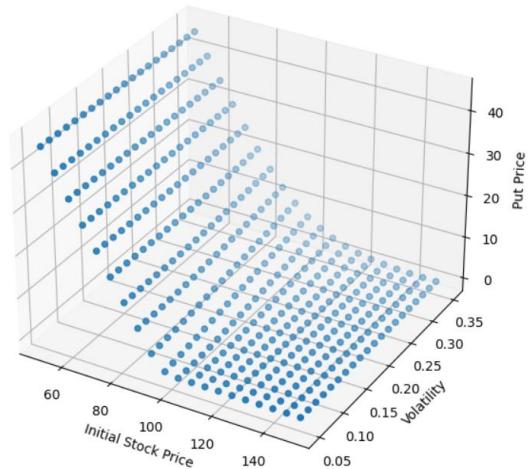
Asian Call Price vs (Initial Stock Price and Risk Free Rate) (Set 2 of u,d) Asian Put Price vs (Initial Stock Price and Risk Free Rate) (Set 2 of u,d)



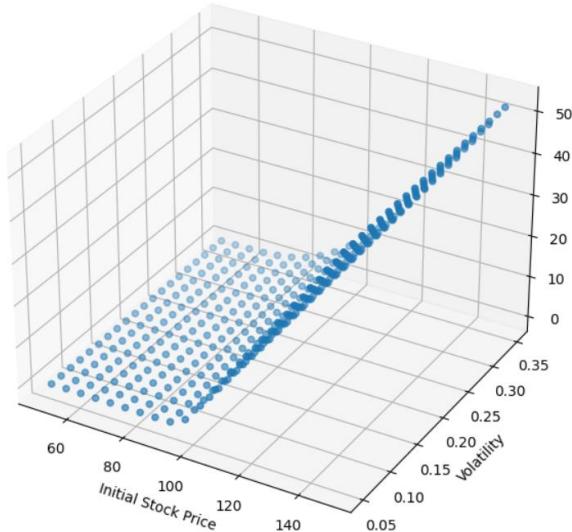
Asian Call Price vs (Initial Stock Price and Volatility) (Set 1 of u,d)



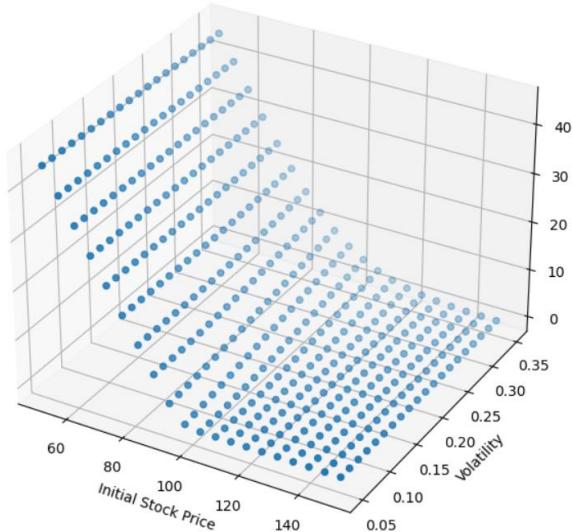
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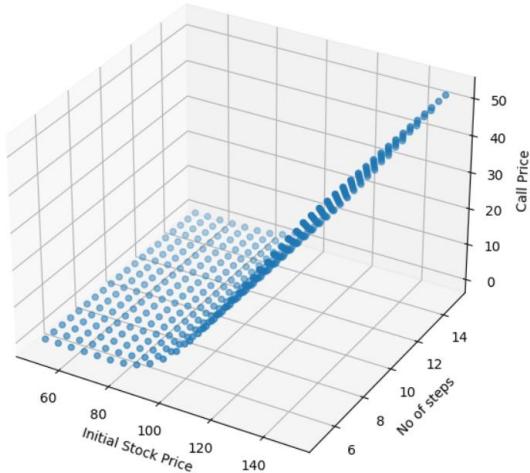
Asian Call Price vs (Initial Stock Price and Volatility) (Set 2 of u,d)



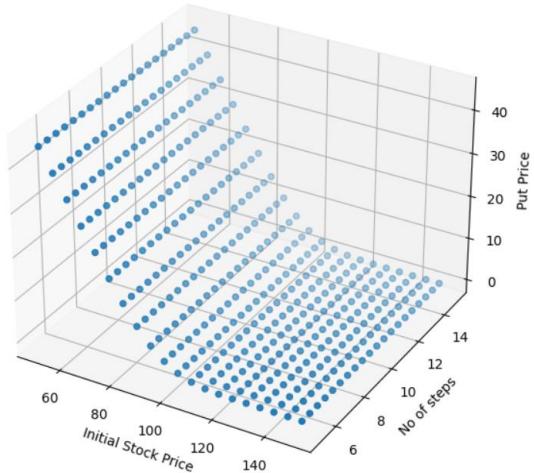
Asian Put Price vs (Initial Stock Price and Volatility) (Set 2 of u,d)



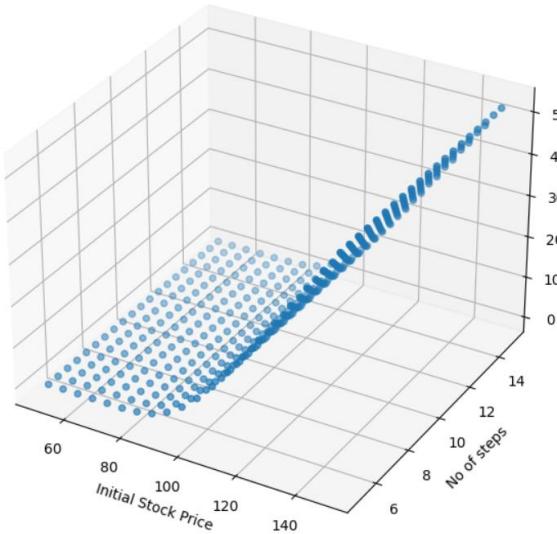
Asian Call Price vs (Initial Stock Price and No of steps) (Set 1 of u,d)



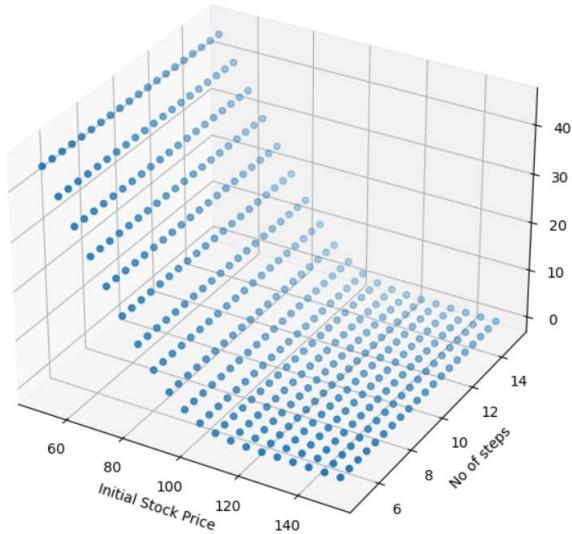
Asian Put Price vs (Initial Stock Price and No of steps) (Set 1 of u,d)



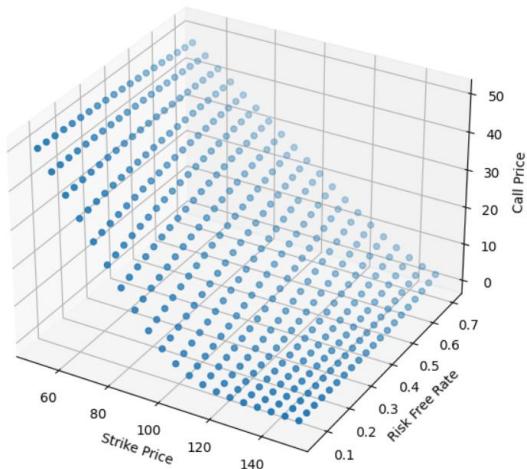
Asian Call Price vs (Initial Stock Price and No of steps) (Set 2 of u,d)



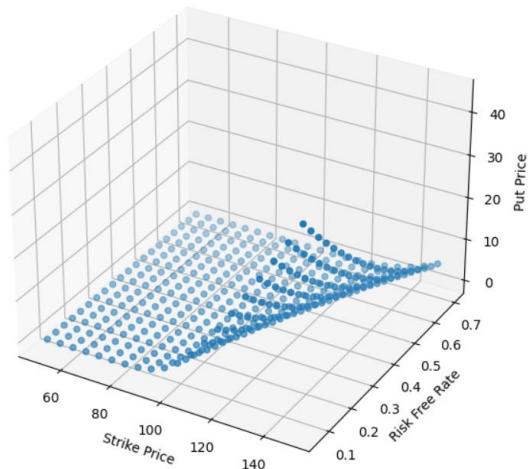
Asian Put Price vs (Initial Stock Price and No of steps) (Set 2 of u,d)



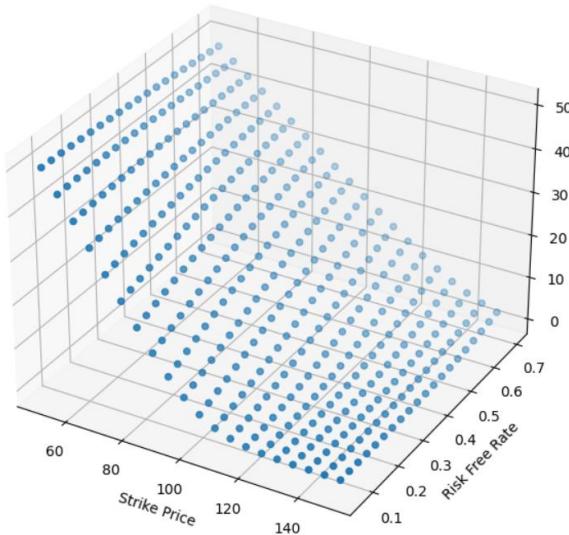
Asian Call Price vs (Strike Price and Risk Free Rate) (Set 1 of u,d)



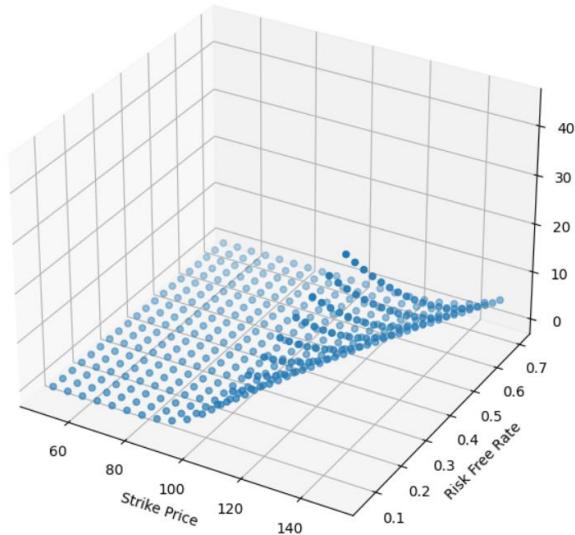
Asian Put Price vs (Strike Price and Risk Free Rate) (Set 1 of u,d)



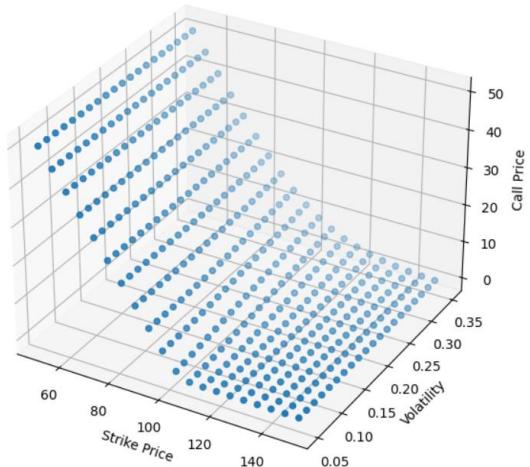
Asian Call Price vs (Strike Price and Risk Free Rate) (Set 2 of u,d)



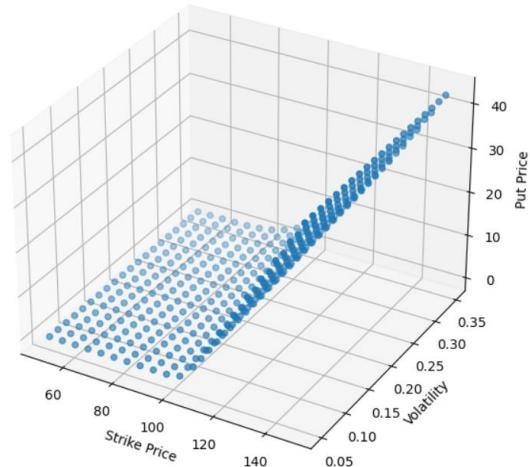
Asian Put Price vs (Strike Price and Risk Free Rate) (Set 2 of u,d)



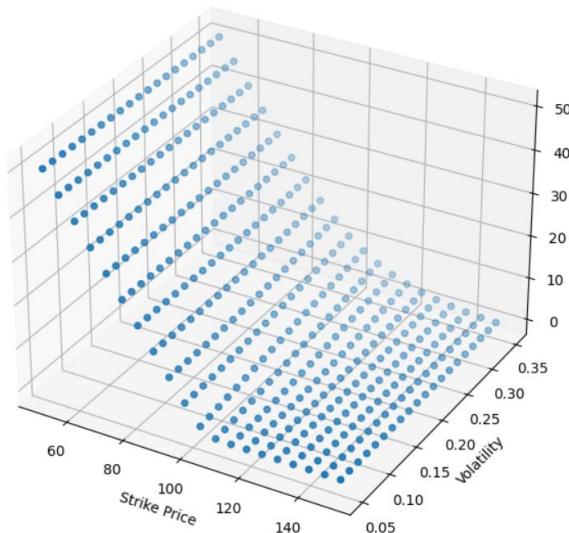
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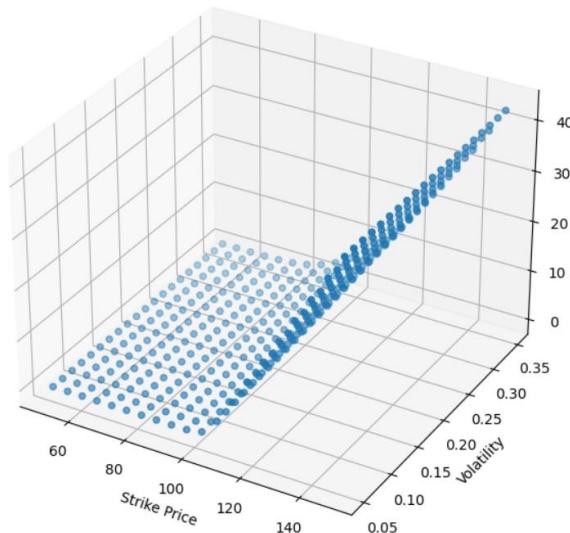
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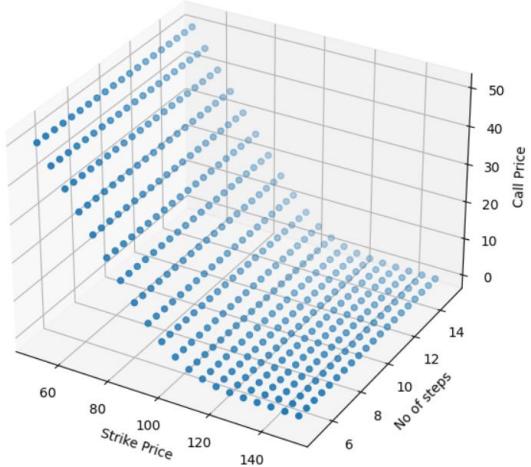
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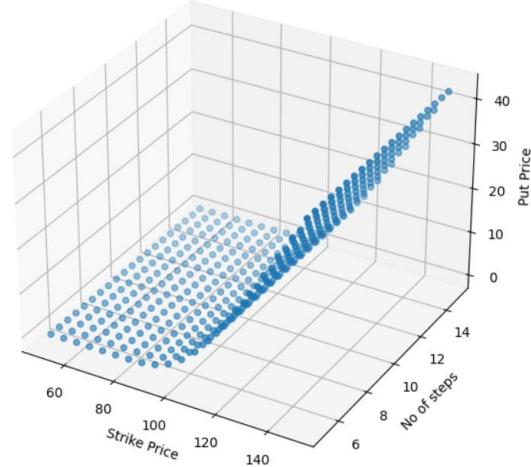
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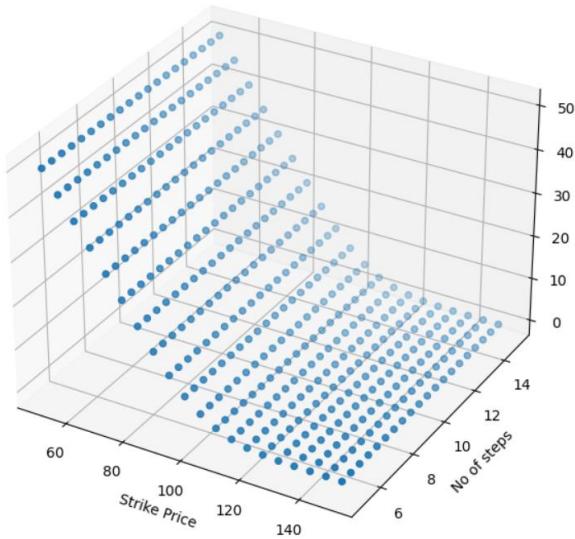
Asian Call Price vs (Strike Price and No of steps) (Set 1 of u,d)



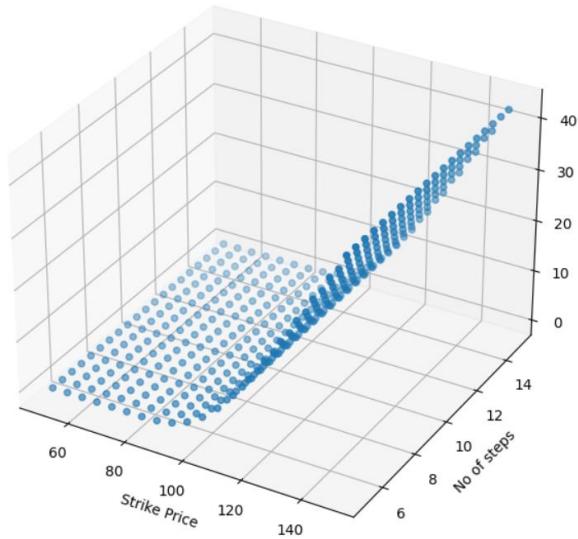
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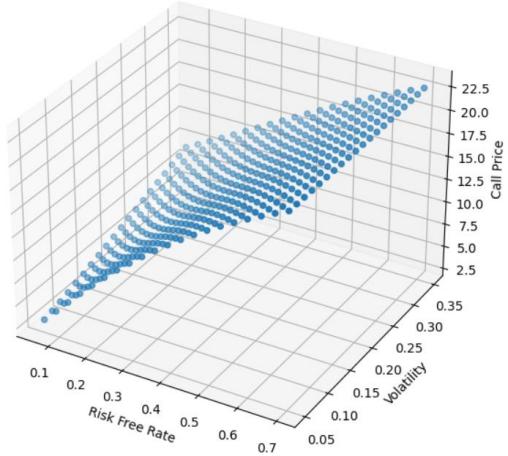
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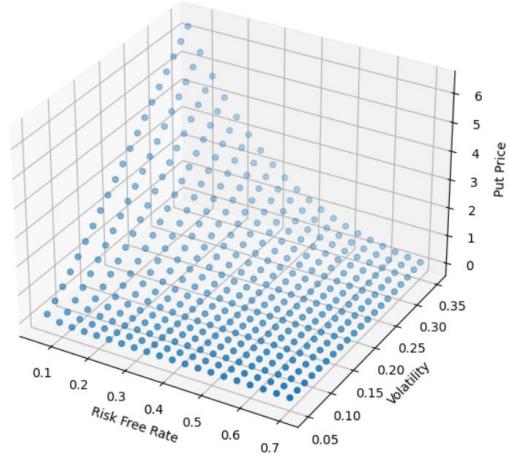
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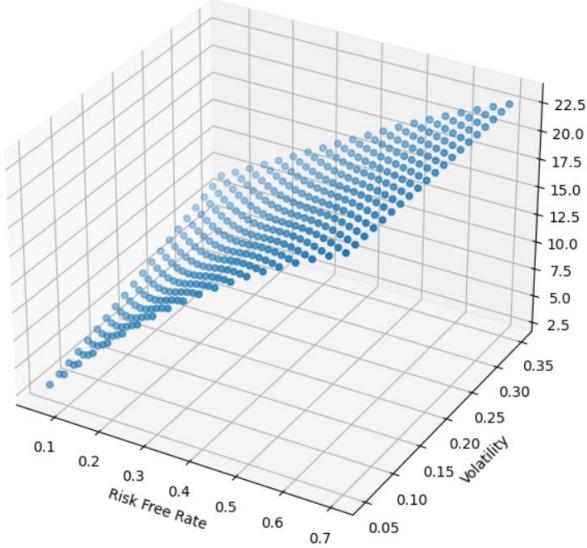
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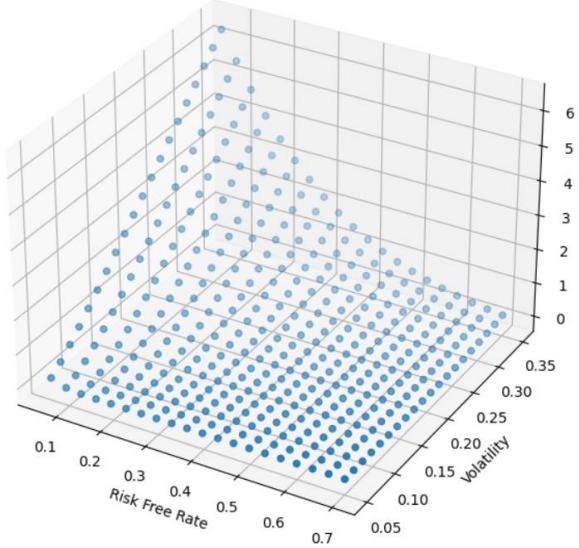
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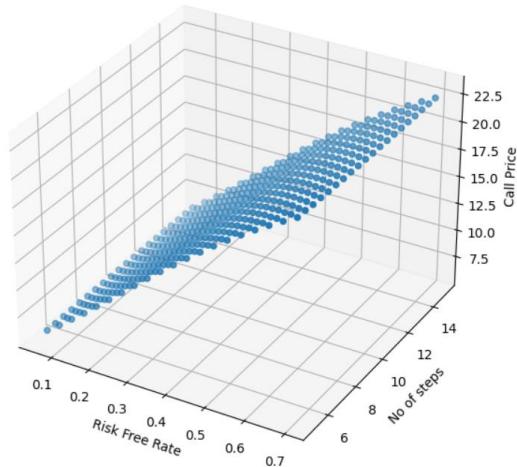
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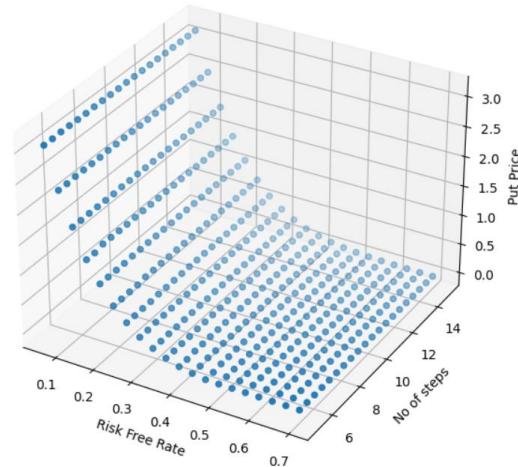
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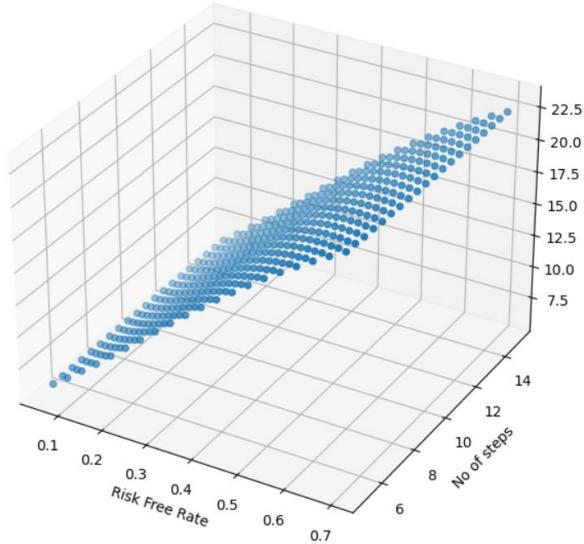
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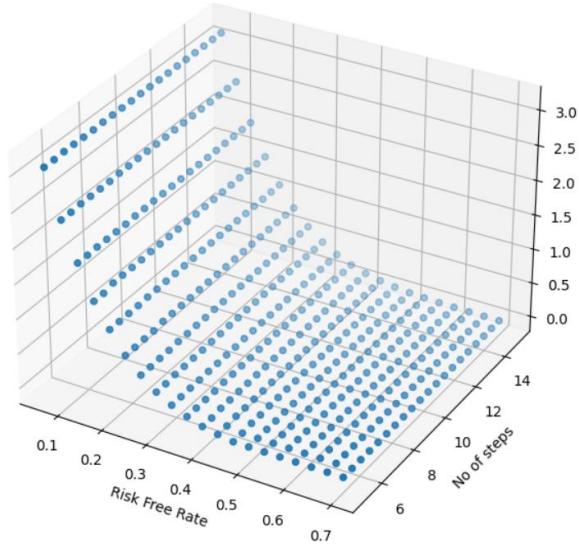
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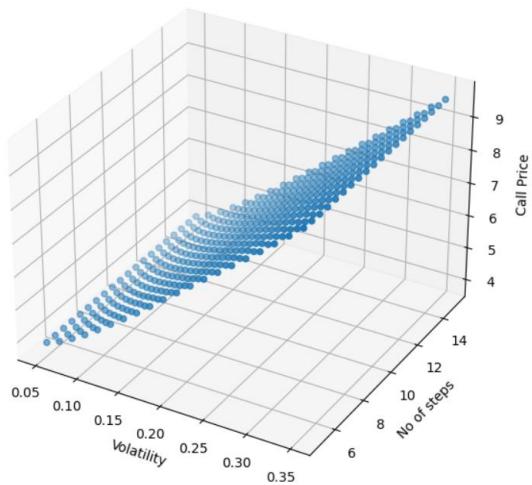
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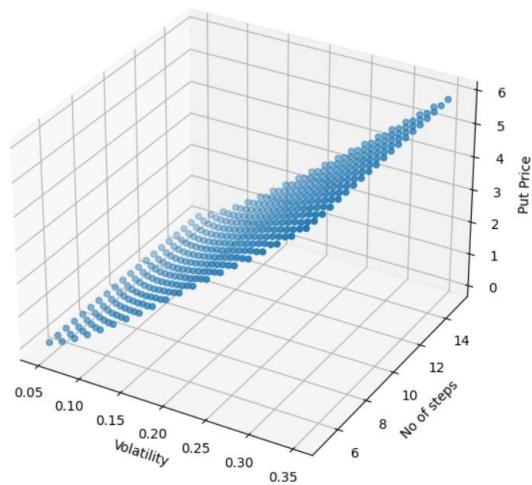
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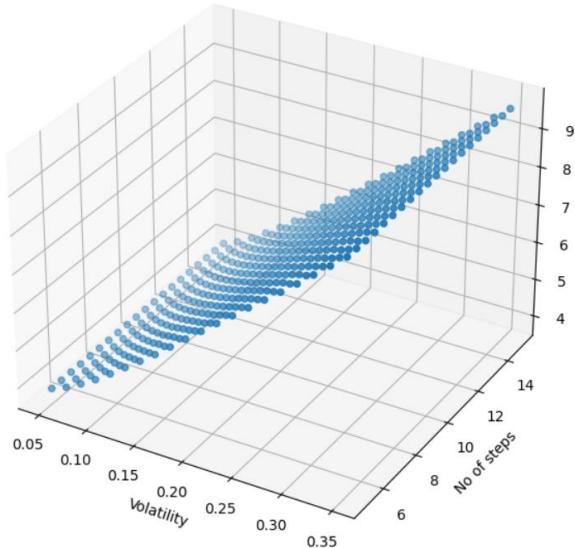
Asian Call Price vs (Volatility and No of steps) (Set 1 of u,d)



Asian Put Price vs (Volatility and No of steps) (Set 1 of u,d)



Asian Call Price vs (Volatility and No of steps) (Set 2 of u,d)



Asian Put Price vs (Volatility and No of steps) (Set 2 of u,d)

