<u>**MA 374 - Financial Engineering Lab04**</u>

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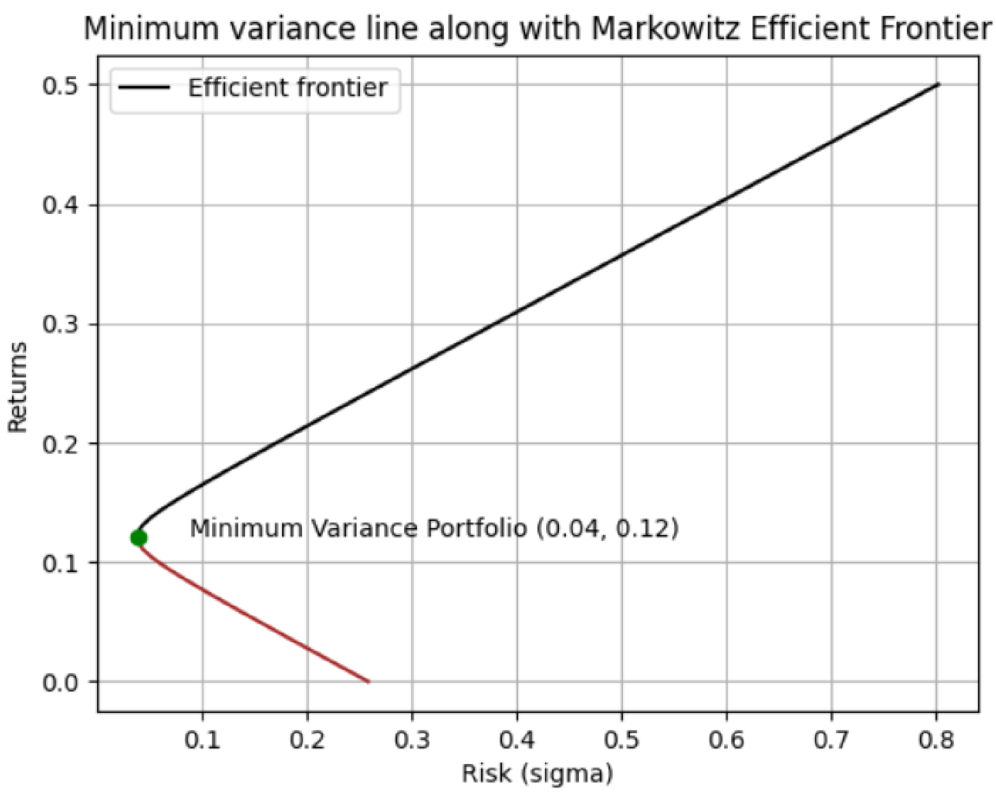
QUESTION - 1:

a) The Markowitz efficient frontier is as follows:



The minimum variance line is constructed using the following steps:

i) Obtain the required weights $\boldsymbol{w}$ using the following relation -

$$w = \frac{\begin{vmatrix} 1 & uC^{-1}M^T \\ \mu_v & MC^{-1}M^T \end{vmatrix} uC^{-1} + \begin{vmatrix} uC^{-1}u^T & 1 \\ MC^{-1}u^T & \mu_v \end{vmatrix} MC^{-1}}{\begin{vmatrix} uC^{-1}u^T & uC^{-1}M^T \\ MC^{-1}u^T & MC^{-1}M^T \end{vmatrix}}$$

where, μ_v = return,

$u = [1,1,1, \dots ,1]$ (with same dimension as that of number of assets)

ii) Obtain the risk using following relation -

$$\sigma_v^2 = wCw^T$$

and then take square root to obtain the risk in terms of std. deviation.

Minimum variance portfolio has weights:

$$w = \frac{uC^{-1}}{uC^{-1}u^T}$$

using which the corresponding point was calculated.

The efficient frontier is a concept in finance that refers to the portfolio with the highest expected return for a given level of risk, as measured by standard deviation. The efficient frontier is represented on a graph as a curve with points that have a higher return than the minimum variance portfolio, indicating a trade-off between risk and reward.

b) The weights, return and risk of the portfolios for 10 different values on the efficient frontier:

Sl No.	Weights	Return	Risk
1.	$[1.83550649, -0.1653936, -0.67011288]$	0.02405612017613421	0.04995499549954996
2.	$[1.11983859, 0.11903851, -0.2388771]$	0.0034570647912315977	0.09995999599959997
3.	$[0.40417069, 0.40347062, 0.19235869]$	0.005229455948986914	0.14996499649964998
4.	$[-0.3114972, 0.68790274, 0.62359447]$	0.029373293649400157	0.19996999699969997
5.	$[-1.0271651, 0.97233485, 1.05483025]$	0.0758885778924713	0.24997499749975
6.	$[-1.742833, 1.25676696, 1.48606604]$	0.14477530867820082	0.29997999799979996
7.	$[-2.4585009, 1.54119907, 1.91730182]$	0.23603348600658744	0.34998499849985
8.	$[-3.17416879, 1.82563119, 2.34853761]$	0.34966310987763205	0.3999899989999
9.	$[-3.88983669, 2.1100633, 2.77977339]$	0.4856641802913356	0.44999499949995003
10.	$[-4.60550459, 2.39449541, 3.21100917]$	0.6440366972476959	0.5

c) For 15% risk,

Maximum return $= 0.1895689568956896$

Weights of the portfolio $= [-0.16263828, 0.62874086, 0.53389743]$

Minimum return $= 0.052455245524552455$

Weights of the portfolio $= [1.79972309, -0.151172, -0.64855109]$

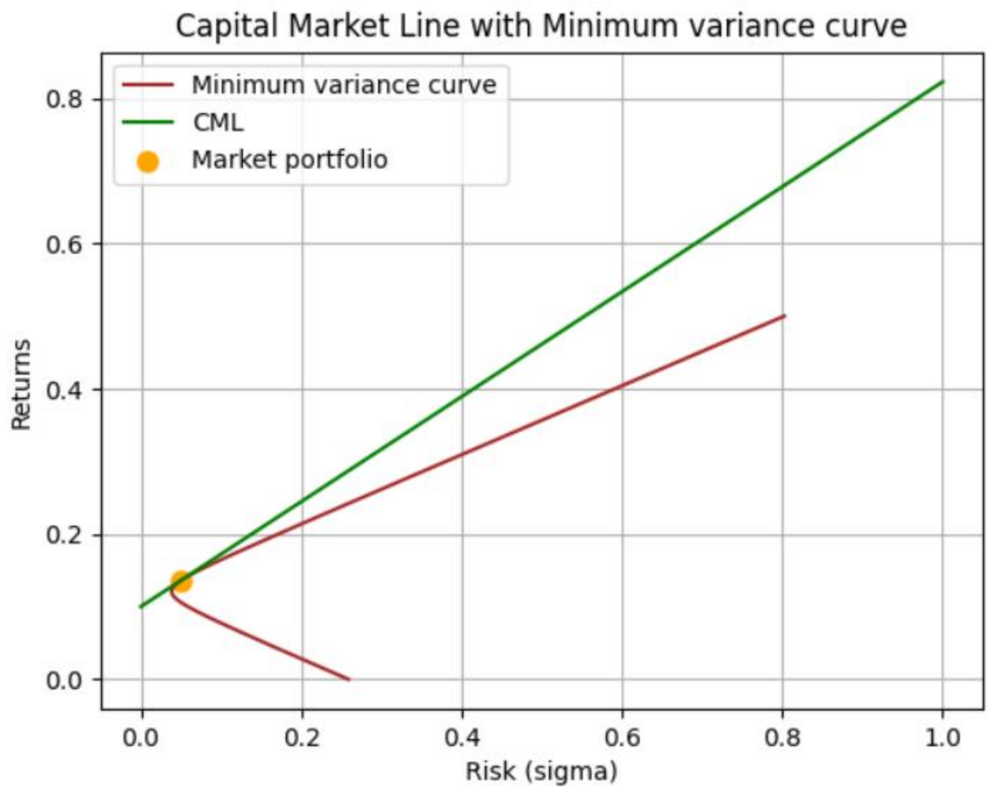
d) For a 18% return, the minimum risk portfolio is:

Minimum risk for 18% return $= 13.056827100982519\%$

Weights of the portfolio $= [-0.02568807, 0.57431193, 0.45137615]$

e) The market portfolio is:

Market Portfolio Weights	=	$[0.59375, 0.328125, 0.078125]$
Return	=	0.13671875
Risk	=	5.081128919221593%



The equation of the CML is:

$$\mu = 0.72\sigma + 0.1$$

The equation is obtained using the following formula:

$\mu = \frac{\mu_M - \mu_{rf}}{\sigma_M}\sigma + \mu_{rf}$ where,

$\mu_M =$ return corresponding to market portfolio
$\mu_{rf} =$ risk free return
$\sigma_M =$ risk corresponding to market portfolio

f) The required portfolio with risk at 10% is:

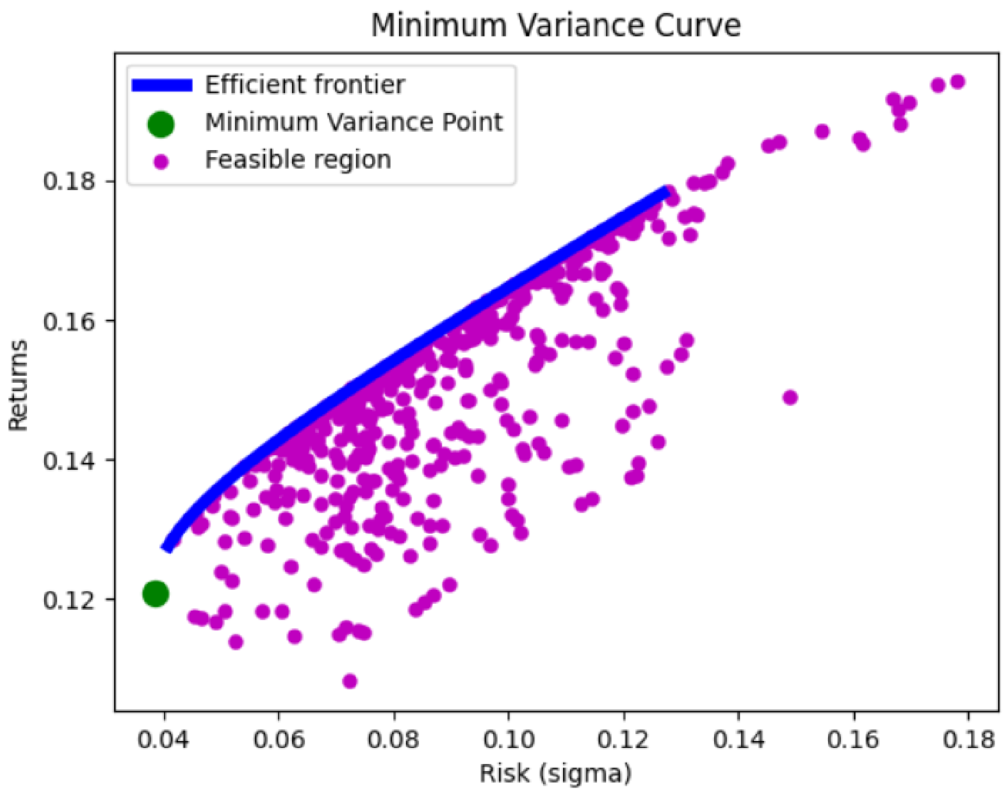
Risk-free weights	$=$	-0.9680665771282883
Risky Weights	$=$	$[1.16853953, 0.64577185, 0.1537552]$
Returns	$=$	0.17226494462892933

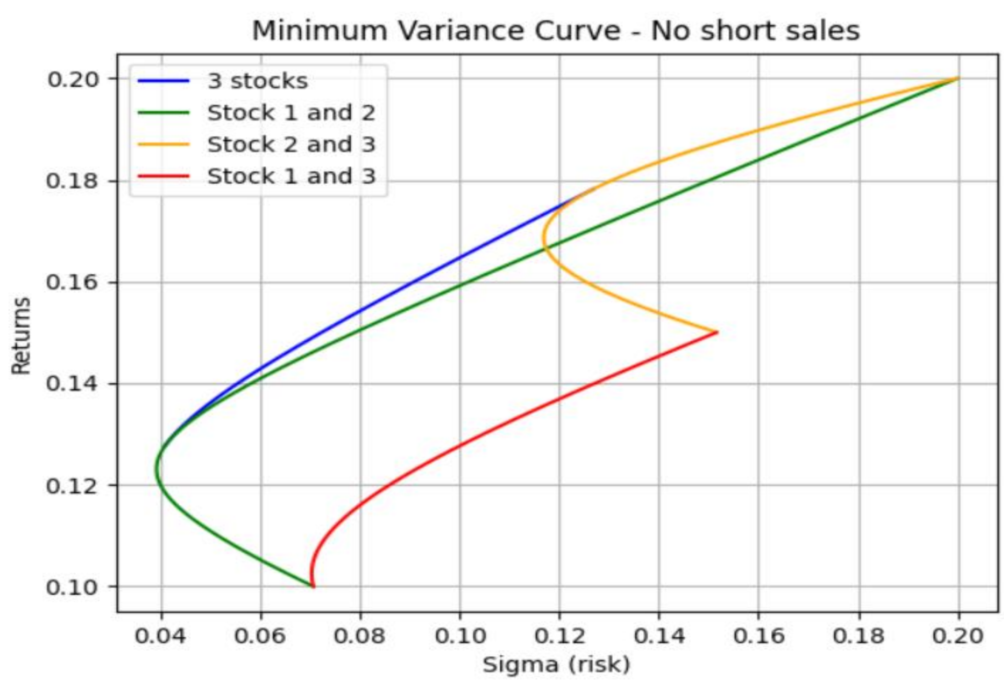
The required portfolio with risk at 25% is:

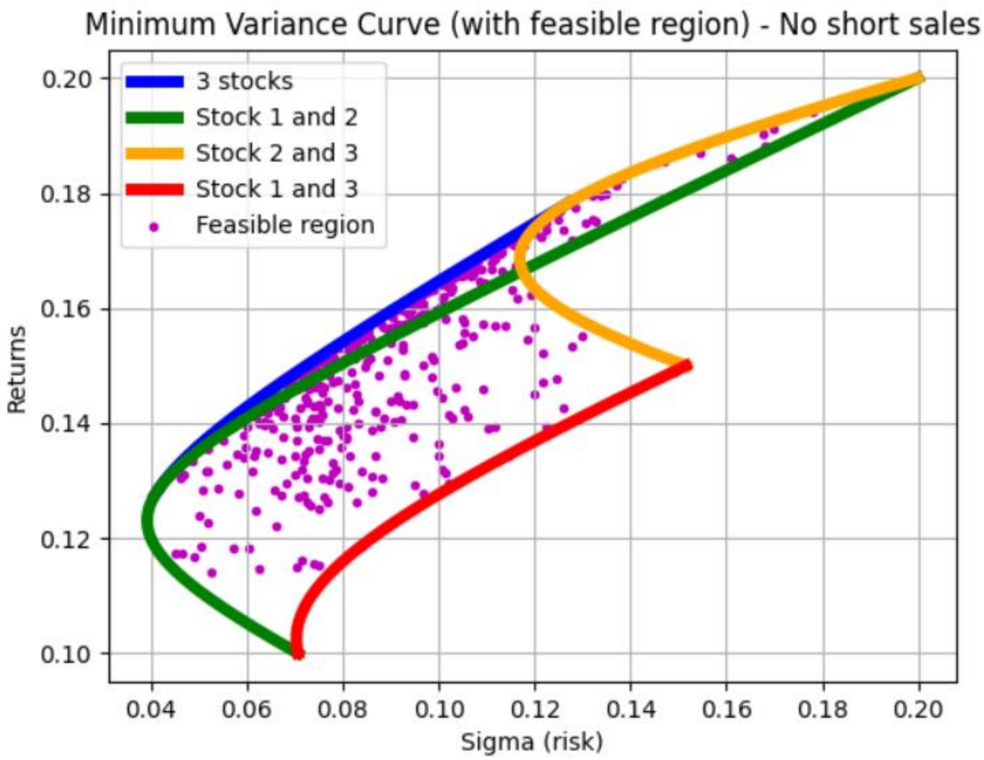
Risk-free weights	$=$	-3.9201664428207224
Risky Weights	$=$	$[2.92134883, 1.61442961, 0.384388]$
Returns	$=$	0.2806623615723234

QUESTION - 2:

The various plots (assuming short sales are not allowed, i.e., weights are non-negative) are:

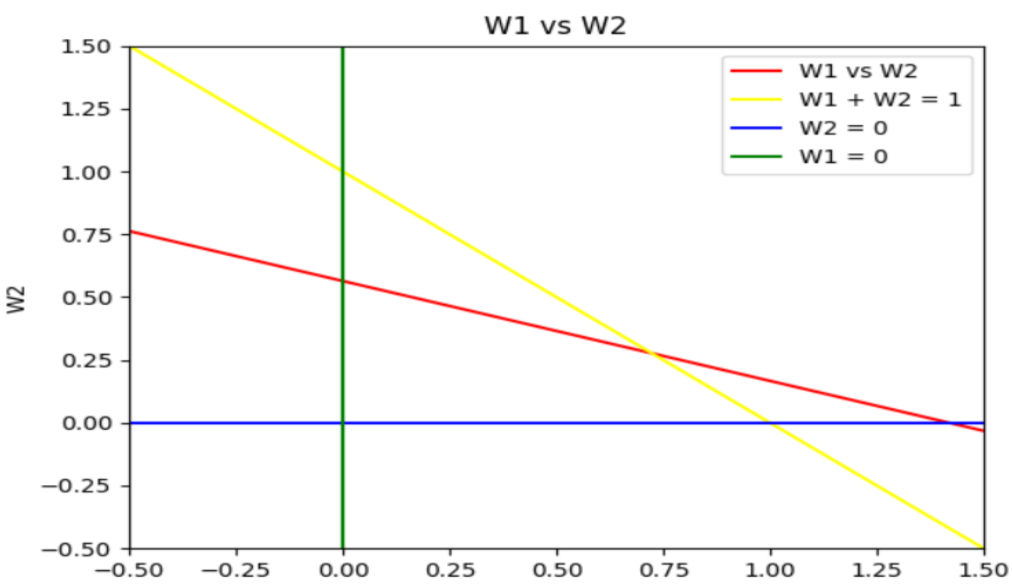




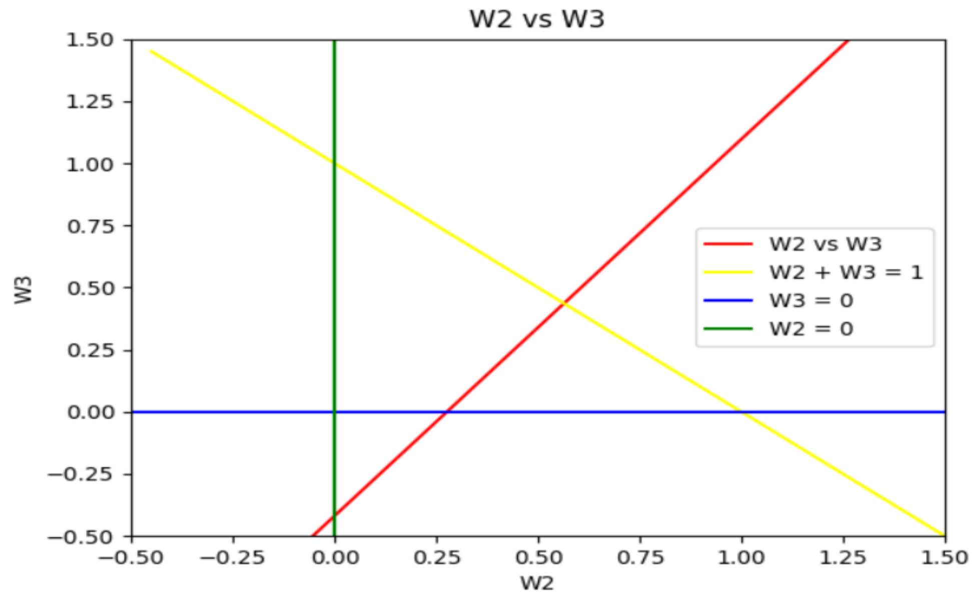


Plots corresponding to the corresponding weights vs the minimum variance curve are:

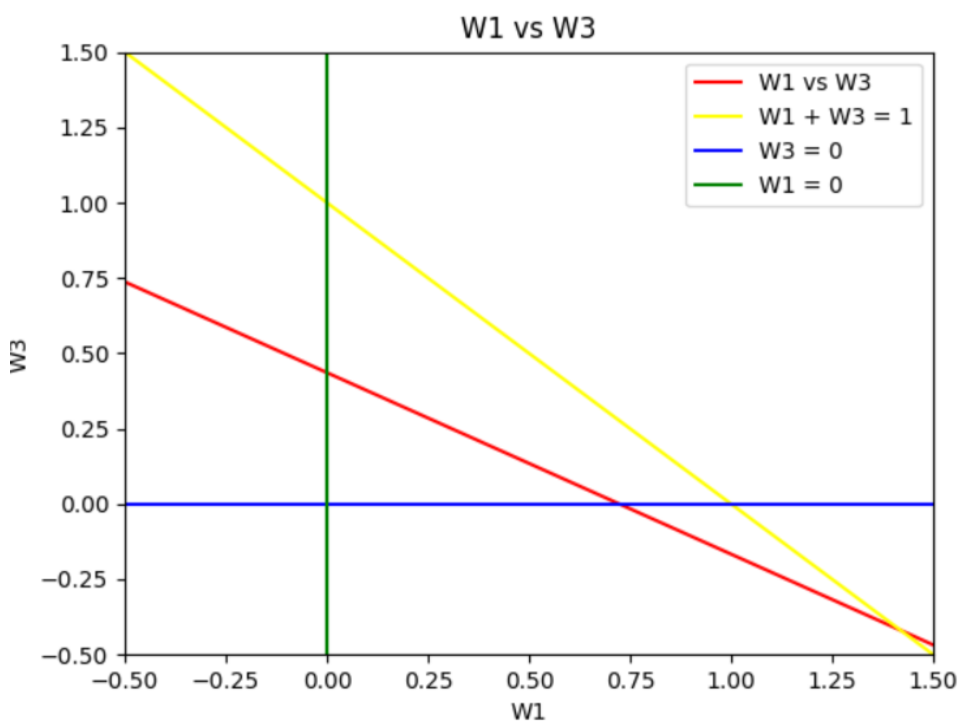
Equation of $W1$ vs W2: $W2 = -0.4W1 + 0.56$



Equation of W2 vs W3: $W3 = 1.52\ \mathrm{W2} - 0.42$



Equation of $W1$ vs W3: $W3 = -0.60W1 + 0.44$

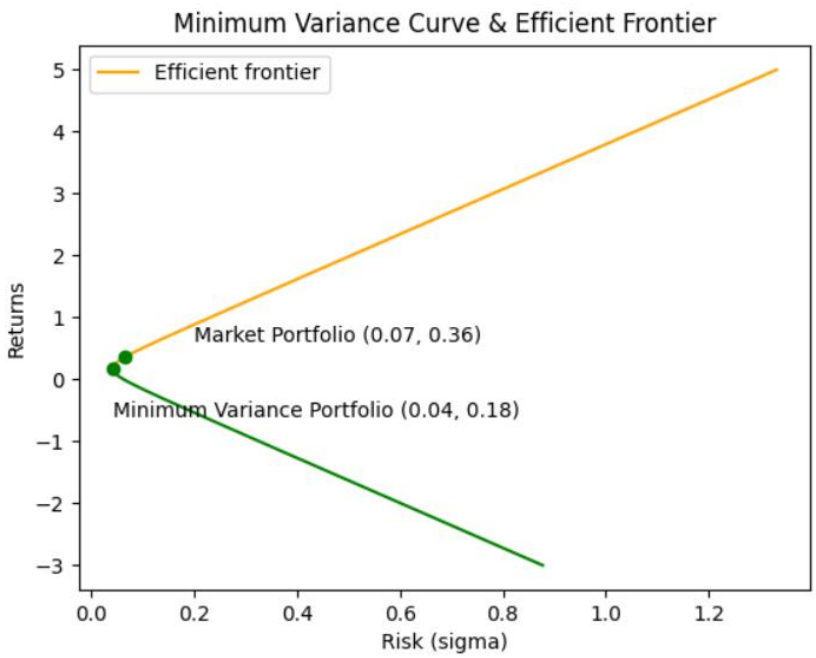


QUESTION - 3:

The data of monthly prices were obtained for 10 stocks each with 60 data points all taken at the same duration over 5 years (Feb 01, 2016 - Feb 01, 2021) from https://in.finance.yahoo.com/. The following companies were considered:

SBI, Asian Paints, BharatiAirtel, CIPLA, IOC, JSW Steel, Maruti, Wipro, Axis Bank, ONGC

Following a similar approach in question 1 after having obtained the Mean Return Vector and Covariance Matrix, we get the following graphs and results. a) The Markowitz efficient frontier:



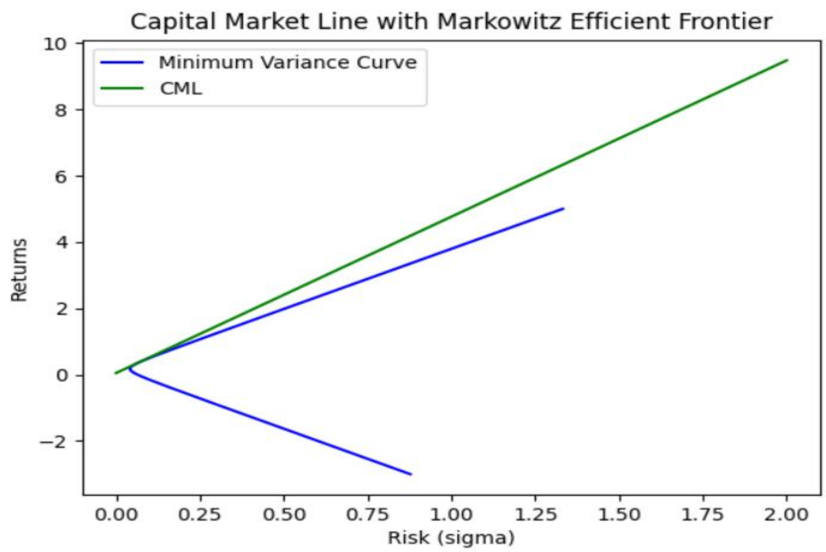
b) Market Portfolio:

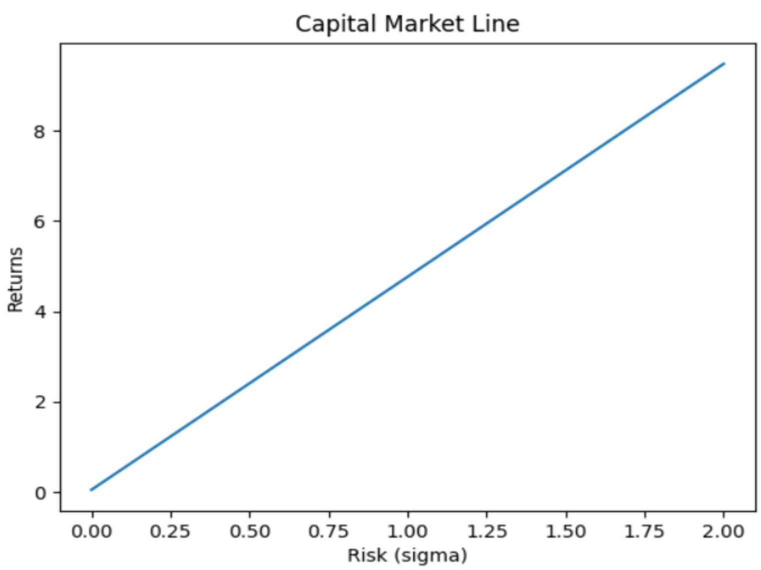
Market Portfolio Weights = [0.20029321, 0.44470774, 0.13445172, 0.18251229, 0.0116113, 0.37736222, 0.13165099, 0.21499233, -0.12999798, -0.56758383]

Return = 0.35714947818352405

Risk = 6.515911088194555 %

c) Equation of Capital Market Line comes out as: *y = 4.71x + 0.05*

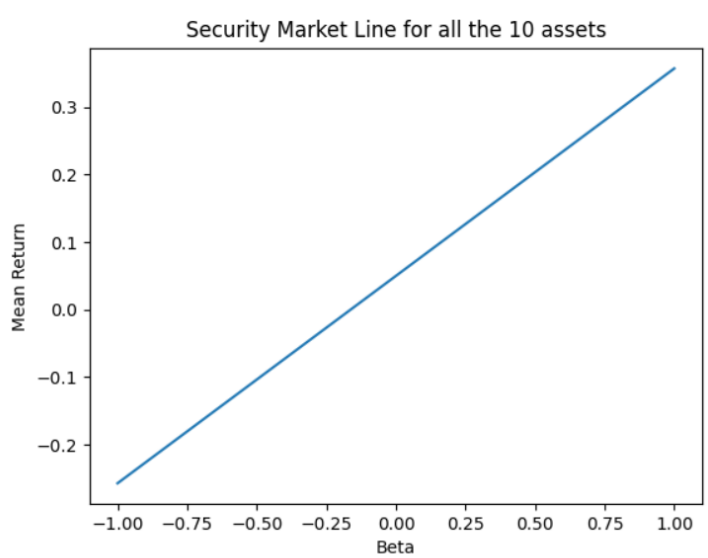




d) Equation of Security Market Line comes out as: μ (mu) = 0.31β + 0.05 The Security market line is obtained using the following formula:

$$\mu = (\mu_M - \mu_{rf})\beta + \mu_{rf}$$

where,



$\mu_M \quad =$ return corresponding to market portfolio
$\mu_{rf} \quad =$ risk free return