MA374: Financial Engineering Lab

Lab03

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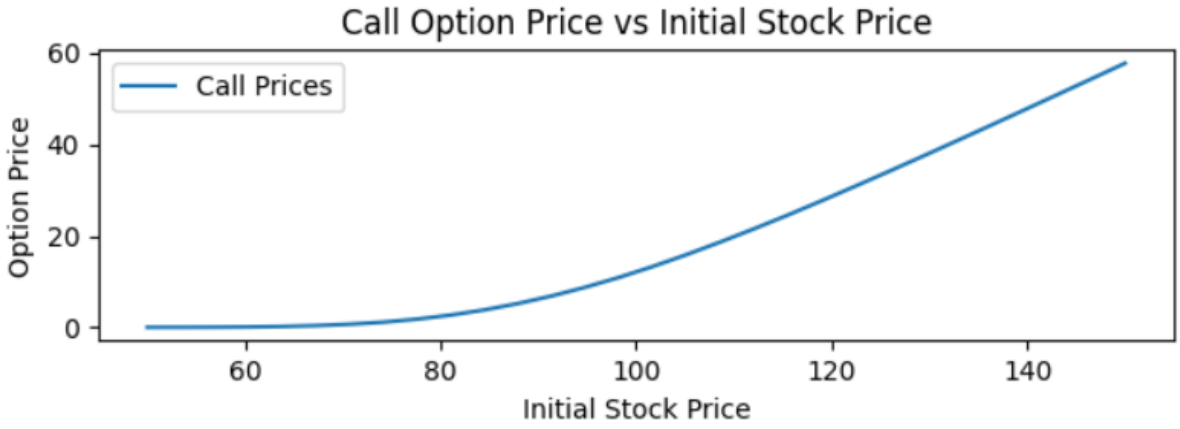
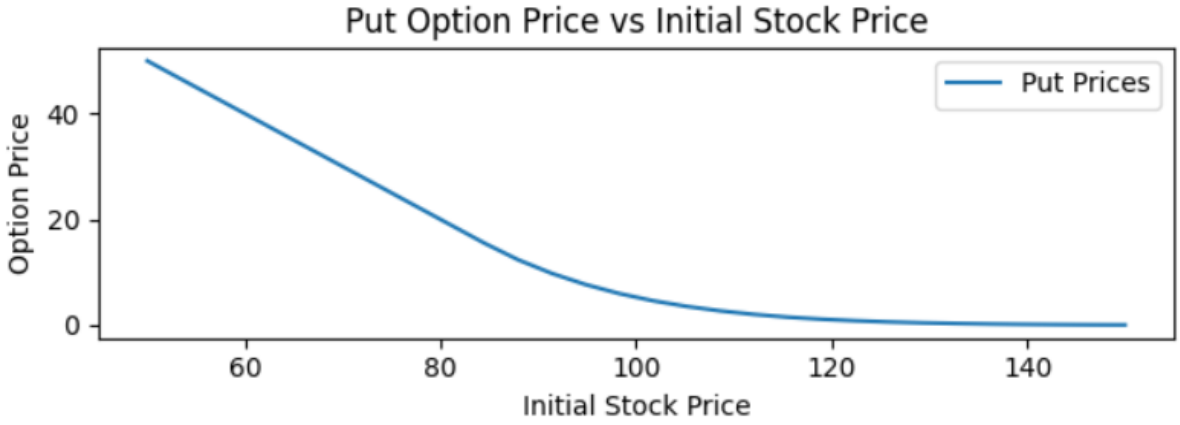
Question 1

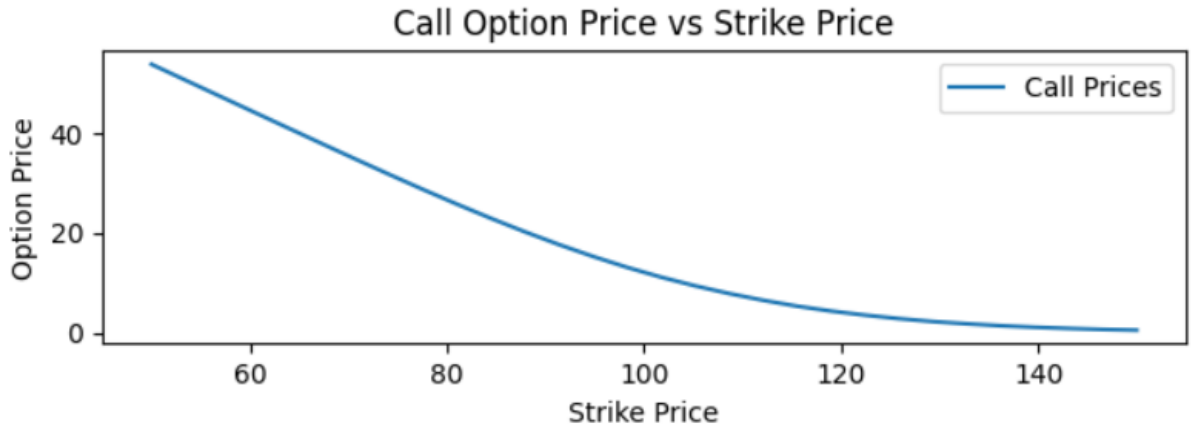
Given expression for u and d are:

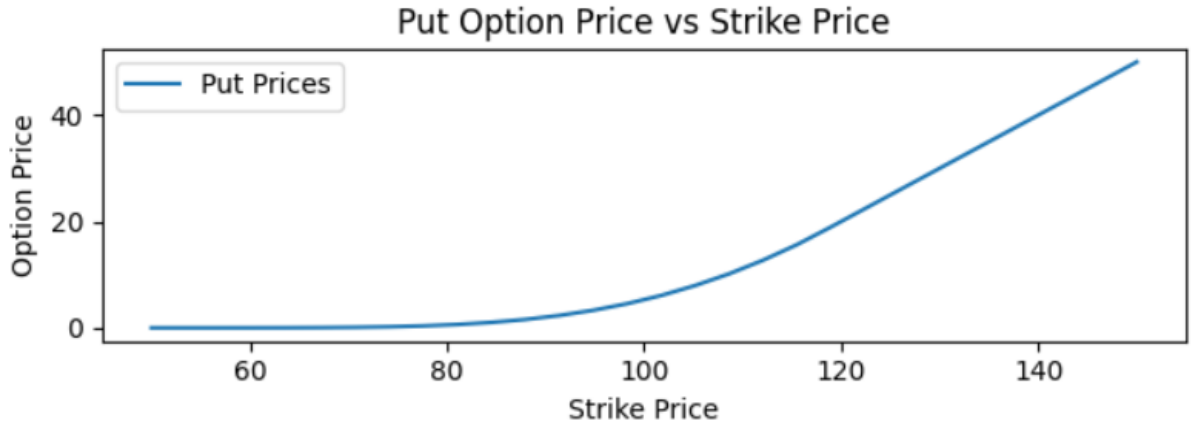
$$u = e^{\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t} \text{ and } d = e^{-\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}, \quad \text{where } \Delta t = {}^{T}/_{M},$$
$$M \text{ here is the number of subintervals in } [0, T].$$

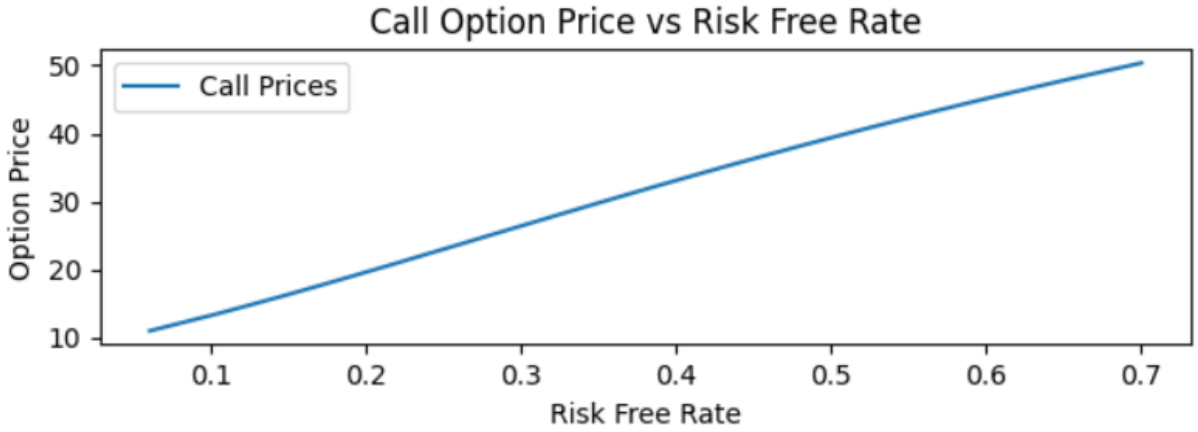
Initial price of the American Call option price = 12.123047074012304 and Put option price = 5.27983714598915.

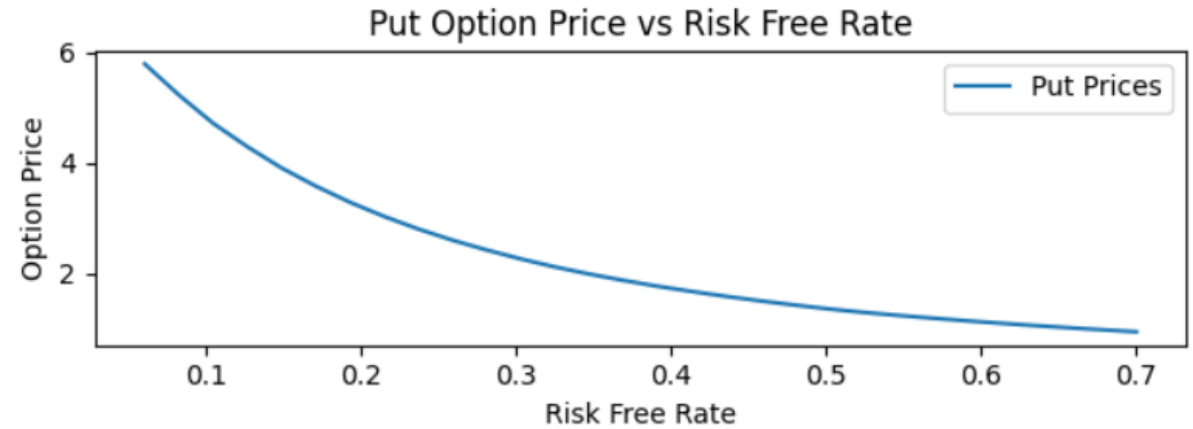


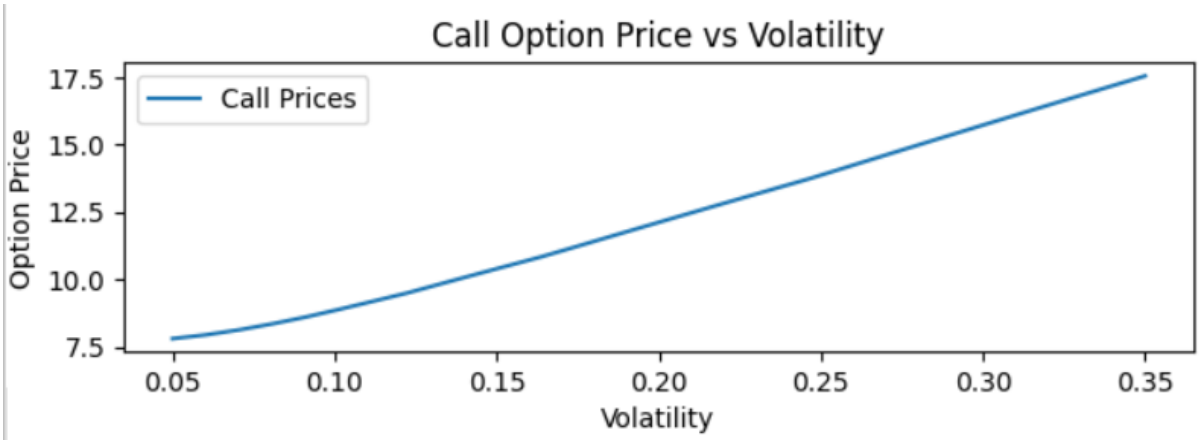


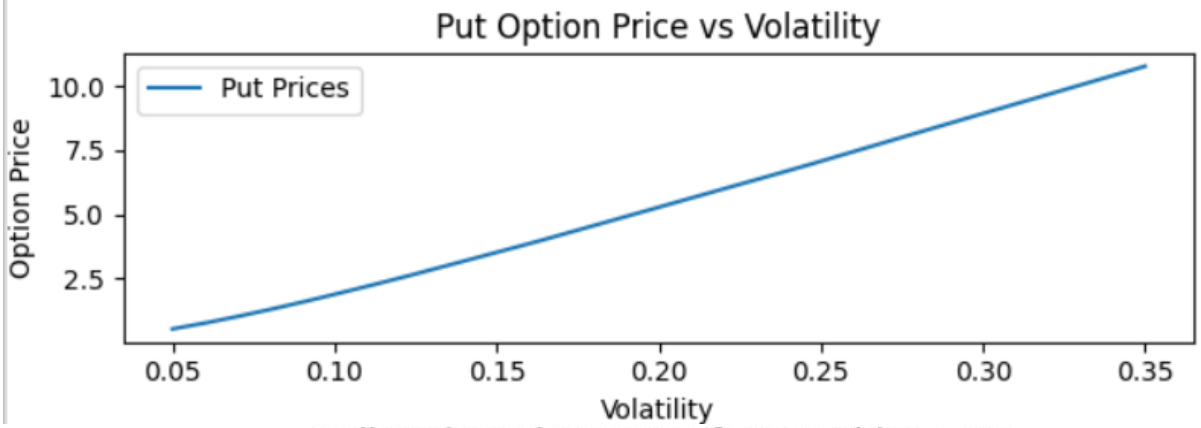

Call Option Price vs Strike Price
Call Prices
Option Price
0
20
40
60
80
100
120
140
Strike Price

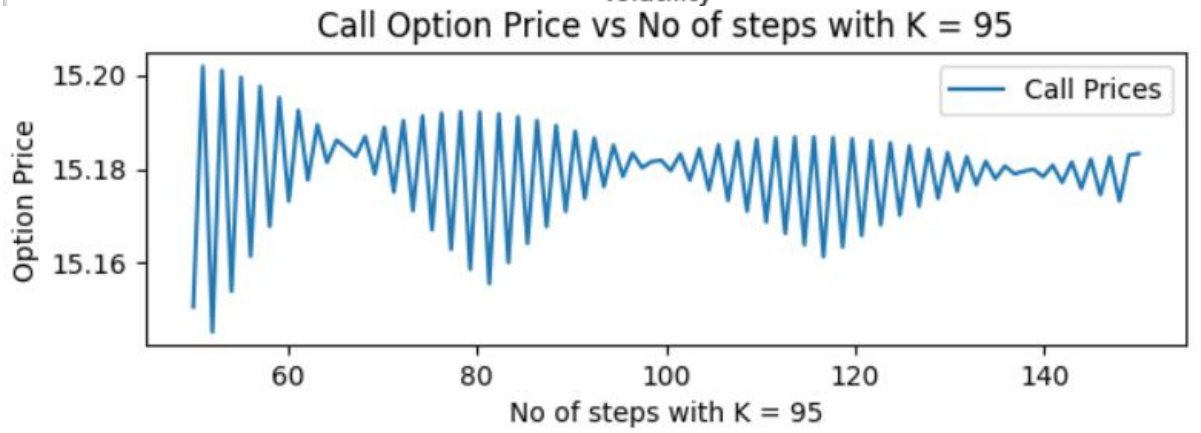

Put Option Price vs Strike Price
Put Prices
Option Price
0
20
40
60
80
100
120
140
Strike Price

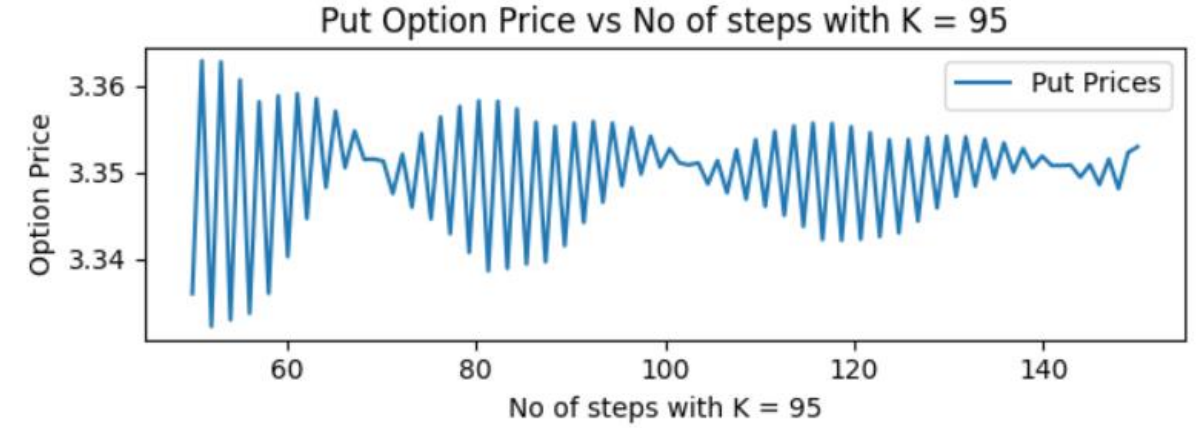

Call Option Price vs Risk Free Rate
Call Prices
Option Price
10
20
30
40
50
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Risk Free Rate

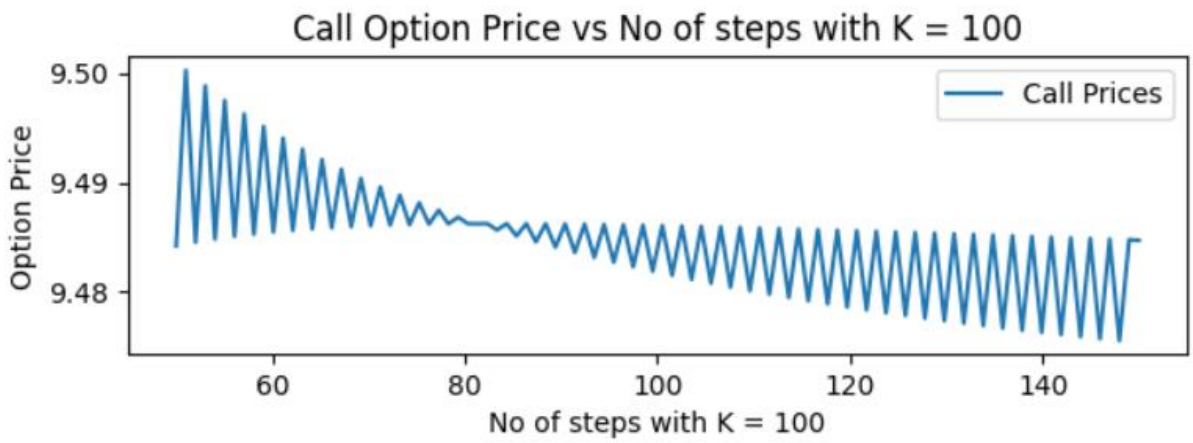

Put Option Price vs Risk Free Rate
Put Prices
Option Price
2
4
6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Risk Free Rate

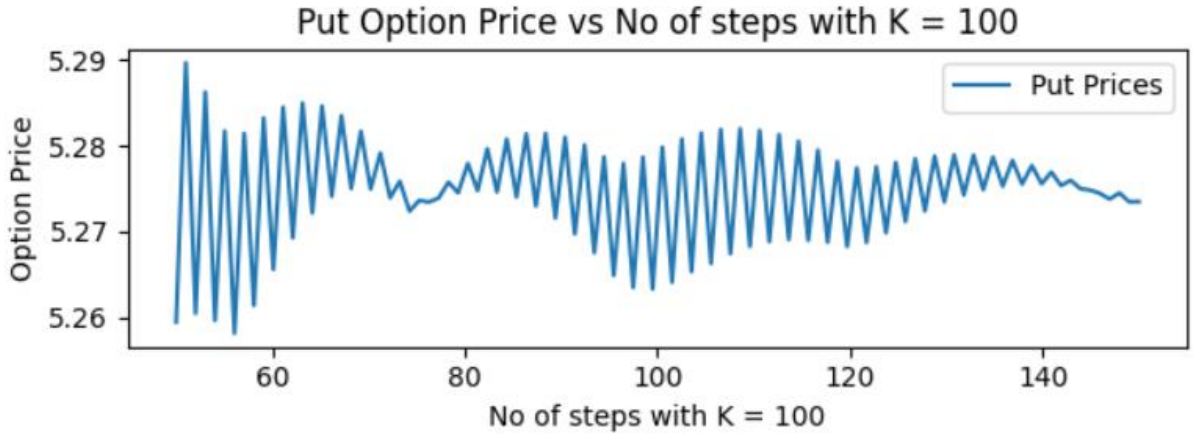

Call Option Price vs Volatility
Call Prices
Option Price
17.5
15.0
12.5
10.0
7.5
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Volatility

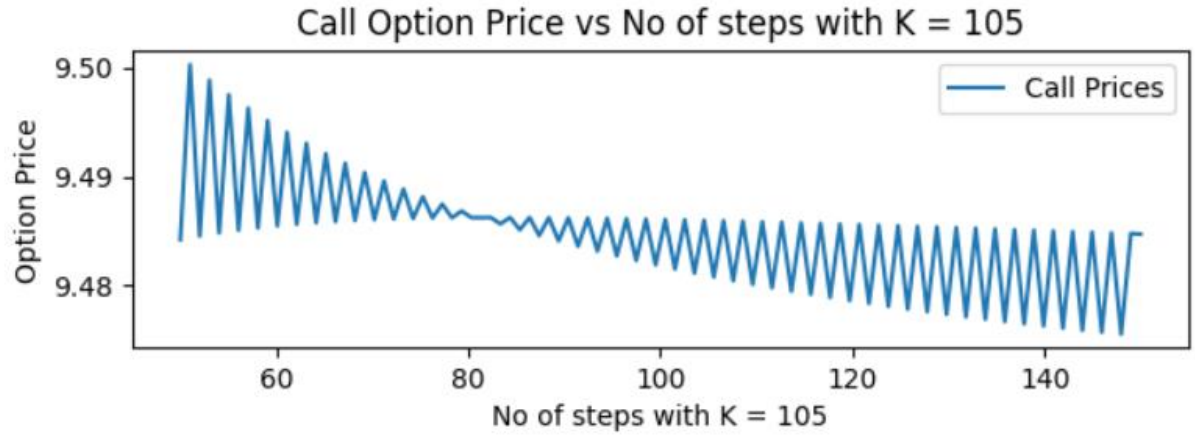

Put Option Price vs Volatility
Put Prices
Option Price
10.0
7.5
5.0
2.5
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Volatility

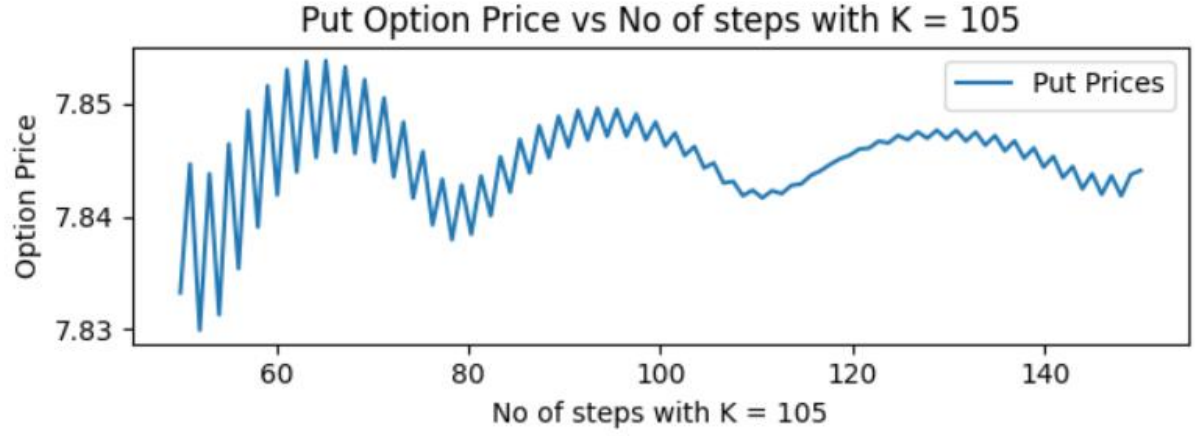

Call Option Price vs No of steps with K = 95
Call Prices
Option Price
15.20
15.18
15.16
60
80
100
120
140
No of steps with K = 95


Put Option Price vs No of steps with K = 95
Put Prices
Option Price
3.36
3.35
3.34
60
80
100
120
140
No of steps with K = 95


Call Option Price vs No of steps with K = 100
Call Prices
Option Price
9.50
9.49
9.48
60
80
100
120
140
No of steps with K = 100


Put Option Price vs No of steps with K = 100
Put Prices
Option Price
5.29
5.28
5.27
5.26
60
80
100
120
140
No of steps with K = 100


Call Option Price vs No of steps with K = 105
Call Prices
Option Price
9.50
9.49
9.48
60
80
100
120
140
No of steps with K = 105


Put Option Price vs No of steps with K = 105
Put Prices
Option Price
7.85
7.84
7.83
60
80
100
120
140
No of steps with K = 105

Question 2

For the European Option, we use the following data,

$$S(0) = 100, T = 1, r = 8\%, \sigma = 20\%$$

The payoff of the lookback option is given as,

$$V = \max_{0 \le i \le M} S(i) - S(M)$$

$S(i) = S(i\Delta t)$ and $u = e^{\sigma\sqrt{\Delta t} + \left(r - \frac{1}{2}\sigma^2\right)\Delta t}$ and $d = e^{-\sigma\sqrt{\Delta t} + \left(r - \frac{1}{2}\sigma^2\right)\Delta t}$, $where\ \Delta t = {}^{T}/_{M}$, $M\ here\ is\ the\ number\ of\ subintervals\ in\ [0, T]$.

a) The initial value of Option Prices for different M are as:

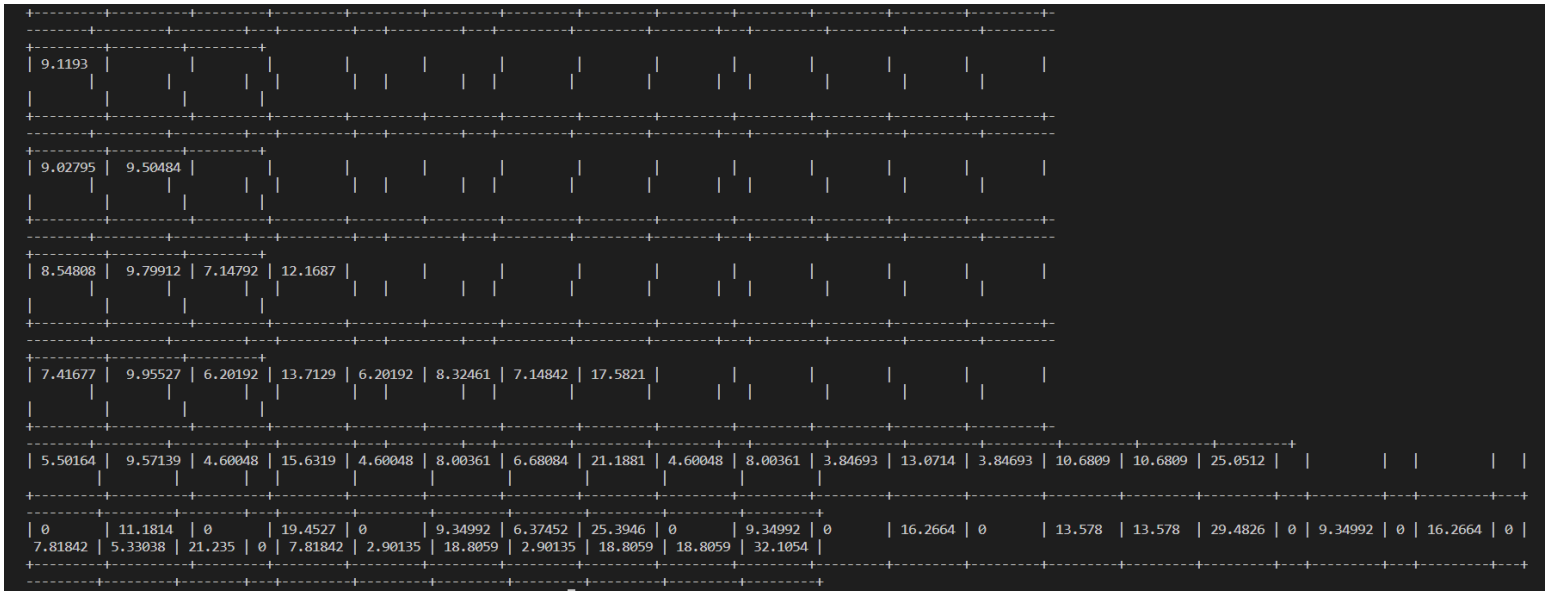
M	Option Price
5	9.11929898586469
10	10.08058290683101
25	Computationally Infeasible
50	Computationally Infeasible

b)

M	Option Value
5	9.1193
6	9.41543
7	9.60909
8	9.80637
9	9.93676
10	10.0806
11	10.1759
12	10.2869
13	10.3672
14	10.453
15	10.5192

From the table it is evident that the Option Value increase with M.

c) Below image shows the table for all intermediate values for M = 5. Here each row denotes the option price at each intermediate level. Last level has 32 values, coming in 2 lines in image.



Question 3

Initial payoff the Lookback Option using Markov based algorithm are as:

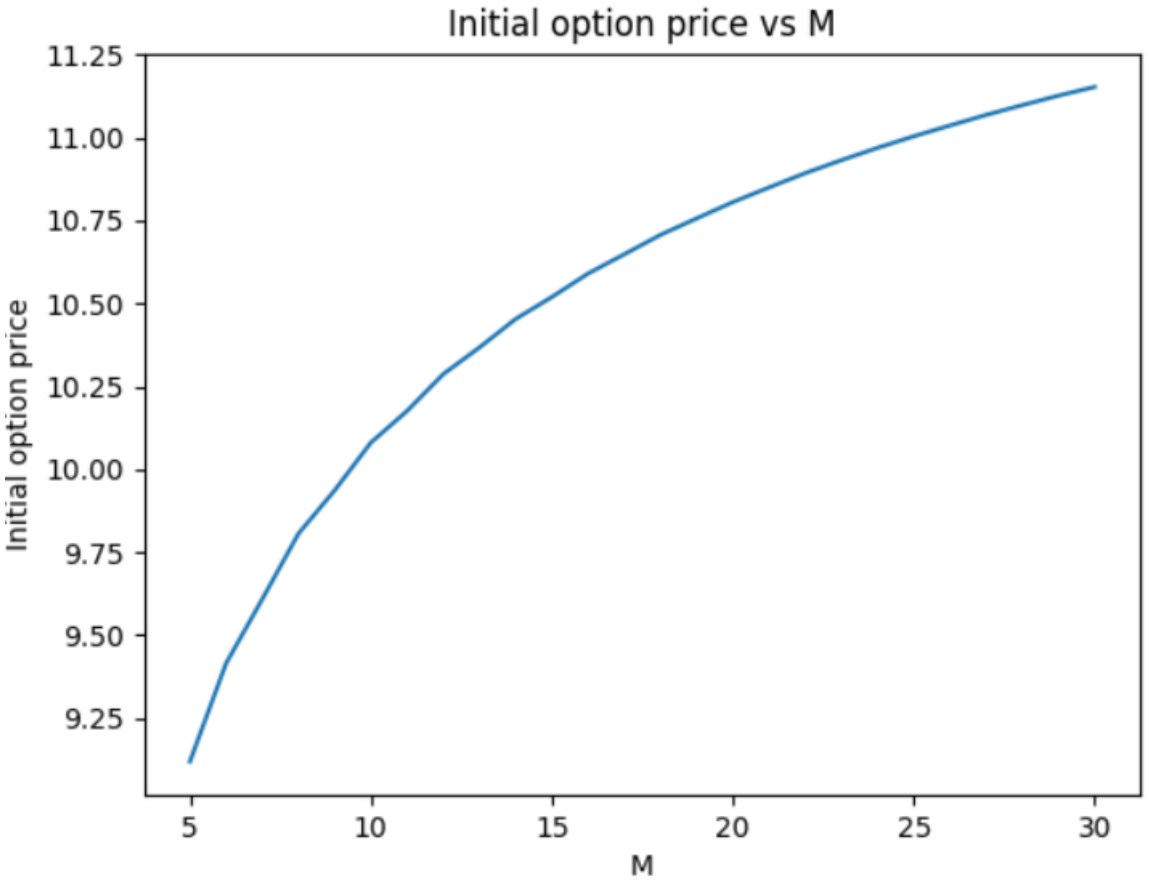
M	Option Value	Time taken
5	9.1193	0
10	10.0806	0
15	10.5192	0.0156255
20	10.8051	0.0156255
25	11.0035	0.0312679
50	11.5109	3.06085

Note: 0-time value does not mean that the time taken is 0 but it is coming because of the precision of the system.

Comparison

We can see that for binomial model time taken is high and also it is not feasible for high values of M which is not the case with Markov Algorithm.

Also, the initial Option Value increases with M as evident from the graph shown below.



Maximum value of M for the algorithm to run in reasonable in time:

- For binomial: 15
- For Markov: 50

Time Complexity

- Time complexity for binomial algorithm is O(2^M) because we are exploring every path of the binomial tree.
- Markov algorithm depends on 2 states, the current stock price and maximum stock price encountered along the path till now. Time complexity of this algorithm is O(M^4), because number of unique paths is bounded by O(M^2) and hence, maximum stock prices is also bounded by O(M^2).

Question 4

Output on running the code, following is the output:

```
PS D:\OneDrive - Indian Institute of Technology Guwahati\Sem 6\MA374> python -u "d:\OneDrive - Indian Institute of Technology Guwahati\Sem 6\MA374\Lab03\200123081_q4.py"
Using Markov based Computation for European Call
Option Value for M = 5 is 12.163185946764594 and time taken is 0.0
Option Value for M = 10 is 12.277327819222982 and time taken is 0.0
Option Value for M = 15 is 12.052004991882892 and time taken is 0.0
Option Value for M = 20 is 12.17470849895534 and time taken is 0.0
Option Value for M = 25 is 12.136745963232972 and time taken is 0.0009982585906982422
Option Value for M = 50 is 12.085361510072186 and time taken is 0.0019998550415039062
Option Value for M = 100 is 12.123047074012481 and time taken is 0.00899958610534668
PS D:\OneDrive - Indian Institute of Technology Guwahati\Sem 6\MA374>
```

In a reasonable amount of time, we found that

- $M_{max} = 20$ for Binomial
- $M_{max} = 1000\ approx.$ for Markov

Below is the table showing the comparison time and initial Option Value.

M	Time for Binomial	Time for Markov	Option Value
5	0.0	0.0	12.163185946764594
10	0.0010020732879638672	0.0	12.277327819222982
20	0.0010013580322265625	0.0	12.052004991882892
25	$Infeasible$	0.0009982585906982422	12.136745963232972
50	$Infeasible$	0.0019998550415039062	12.085361510072186
100	$Infeasible$	0.00899958610534668	12.123047074012481

Time Complexity

- Time complexity for binomial algorithm is O(2^M) because we are exploring every path of the binomial tree.
- Markov algorithm depends on 2 states, the step number and count of up steps encountered along the path till now. Time complexity of this algorithm is O(M^3), because number of unique states is bounded by O(M^2) and hence, number of up states is also bounded by O(M).