MA 374 - Financial Engineering Lab07

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Q1

Formula used for calculating the Put and Call option price in the classical BSM framework using Black-Scholes-Merton PDE is:

$$C(s,t) = sN(d_1) - Ke^{-r(T-t)}N(d_2)$$
$$P(s,t) = Ke^{-r(T-t)}N(-d_2) - sN(-d_1)$$
where,

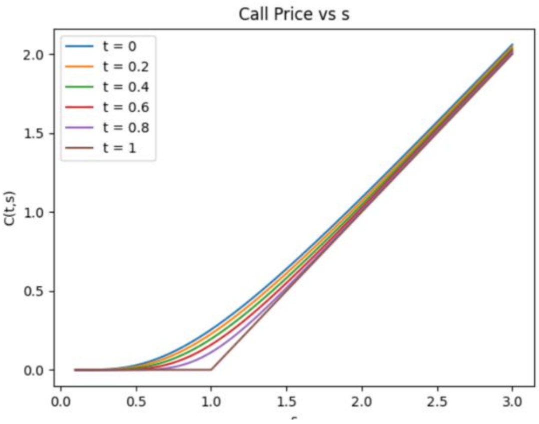
$$d_1 = \frac{\log\left(\frac{s}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = \frac{\log\left(\frac{s}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$
$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}y^2}\,dy$$

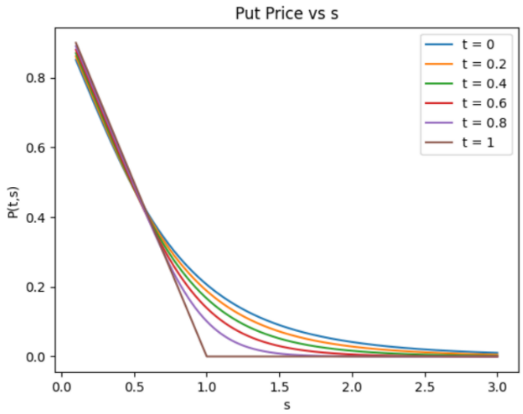
Function written to compute the required prices in this question is:

```
import math
import numpy as np
import random
import matplotlib.pyplot as plt
from scipy.stats import norm
from mpl_toolkits import mplot3d


def N(x):
    return norm.cdf(x)

def calc_price(s,T, K, r, sigma, t):
    if(t == T):
        return max(0,s - K), max(0, K-s)
    t = T - t
    d1 = (math.log(s/K) + (r + (sigma**2)/2)*t)/(sigma*math.sqrt(t))
    d2 = d1 - sigma*math.sqrt(t)
    C = N(d1)*s - N(d2)*K*math.exp(-r*t)
    P = -N(-d1)*s + N(-d2)*K*math.exp(-r*t)

    return C,P



T = 1
K = 1
r = 0.05
sigma = 0.6
t = [0,0.2,0.4,0.6,0.8,1]

C, P = calc_price(2, 1, 1, 0.05, 0.6,0.2)
print("Taking parameters: \ns=2\nT=1\nK=1\nr=0.05\nsigma=0.6\nt=0.2")
print("European Call Price =", C)
print("European Put Price =", P)
```

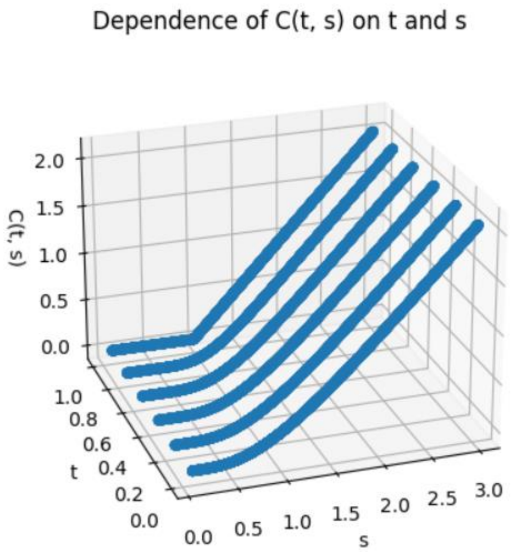
Q2

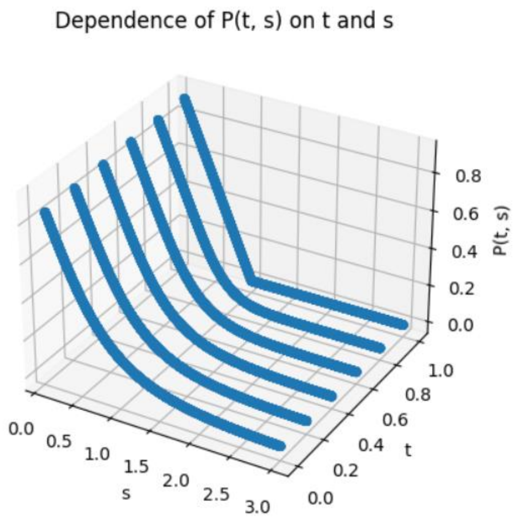
Using the given values of the parameters as given in the question and the function written in previous question, plot of C(t,s) and P(t,s) as a function of *s* for different values of *t* is obtained s:





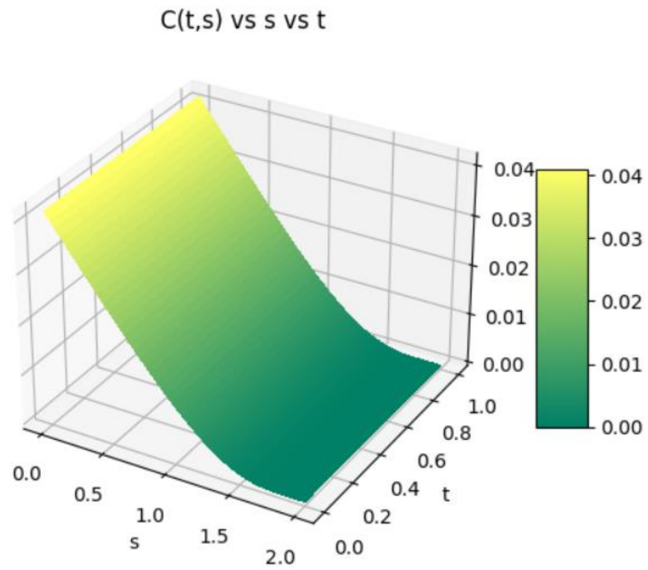
Showing the same information in 3-dimensional form:

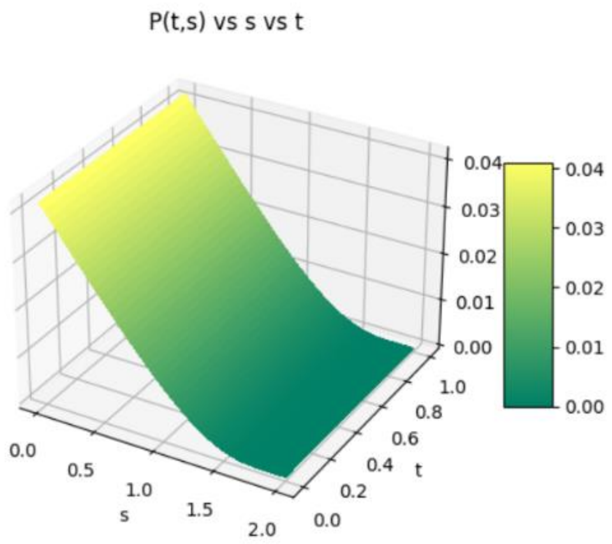




Q3

Required plots of C(t,s) and P(t,s) as smooth surface above the (t,s)-plane:





The parameter values are adjusted as needed, and when specific parameter values are needed, they are obtained as:

$$T = 1, K = 1, r = 0.05, \sigma = 0.6$$

Q4

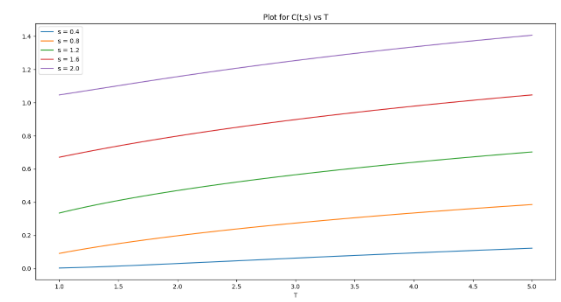
The parameter values are adjusted as needed, and when specific parameter values are needed, they are obtained as:

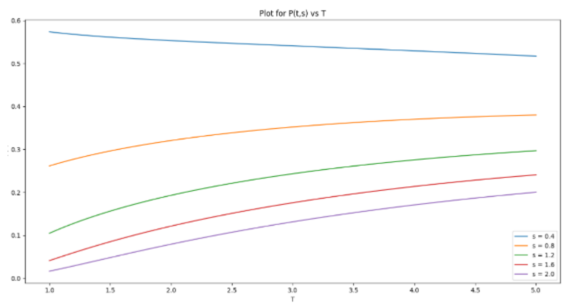
$$s = 2, t = 0.4, T = 1, K = 1, r = 0.05, \sigma = 0.6$$

<u>Variation of C(t,s) and P(t,s) with Stock Price (s):</u>

This is case is already covered in Q2 and Q3

<u>Variation of C(t,s) and P(t,s) with Expiration Time (T):</u>

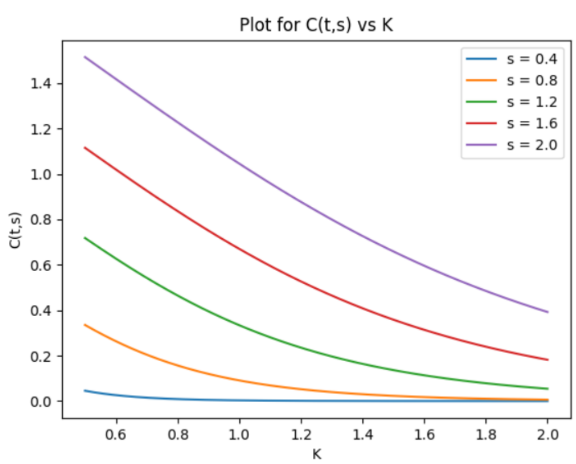


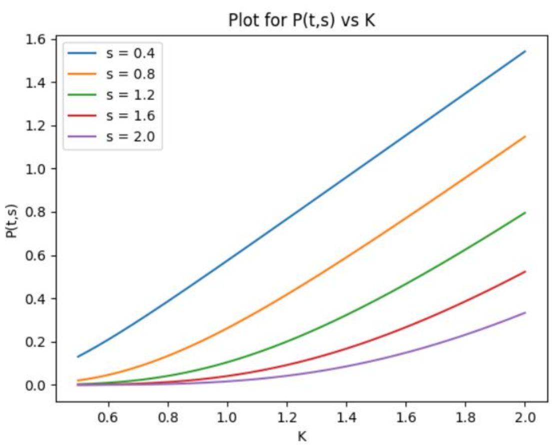


Some of the values are: (with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$)

T	C(t,s)	P(t,s)
1	0.33419	0.104635
1.4004	0.396218	0.147428
1.8008	0.447249	0.179605
2.2012	0.491255	0.205132
2.6016	0.530216	0.225978
3.002	0.565294	0.243301
3.4024	0.597245	0.25785
3.8028	0.626598	0.270145
4.2032	0.653742	0.280569
4.6036	0.678971	0.28941

<u>Variation of C(t,s) and P(t,s) with Strike Price (K):</u>

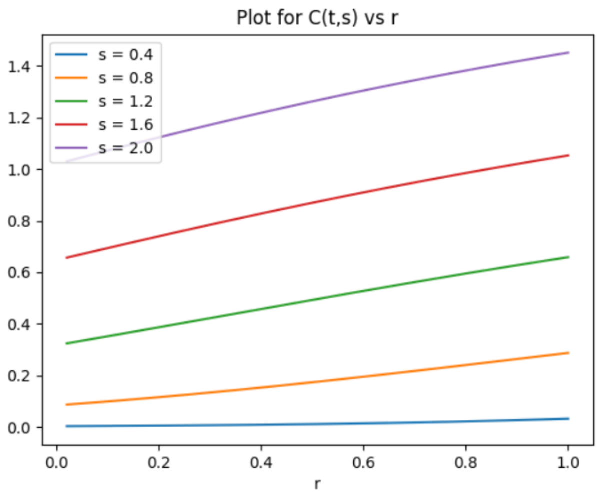


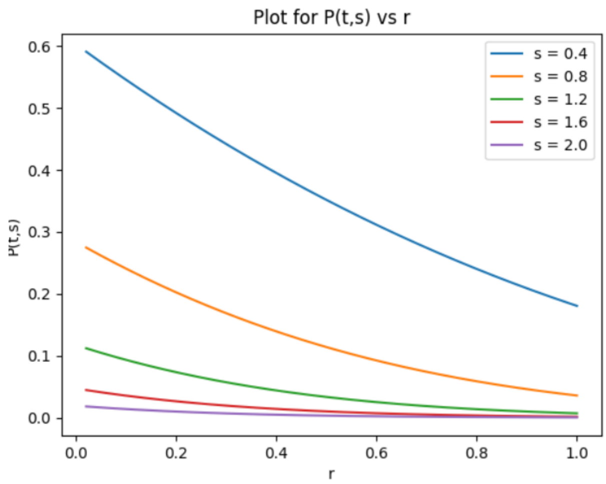


Some of the values are: (with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$)

K	C(t,s)	P(t,s)
0.5	0.71816	0.0033831
0.65015	0.584159	0.0150945
0.8003	0.464803	0.0414512
0.95045	0.363507	0.0858676
1.1006	0.280734	0.148807
1.25075	0.214947	0.228733
1.4009	0.163676	0.323174
1.55105	0.12425	0.42946
1.7012	0.0941987	0.545122
1.85135	0.0714199	0.668056

Variation of C(t,s) and P(t,s) with Rate of Interest (r):

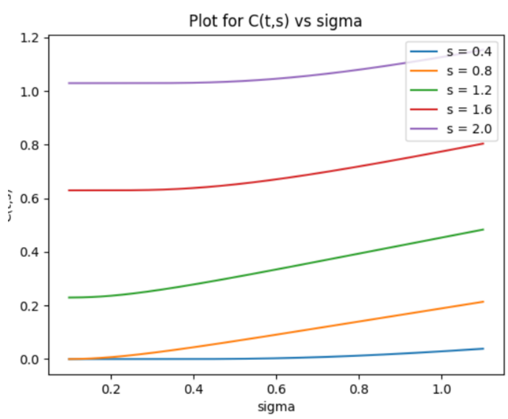


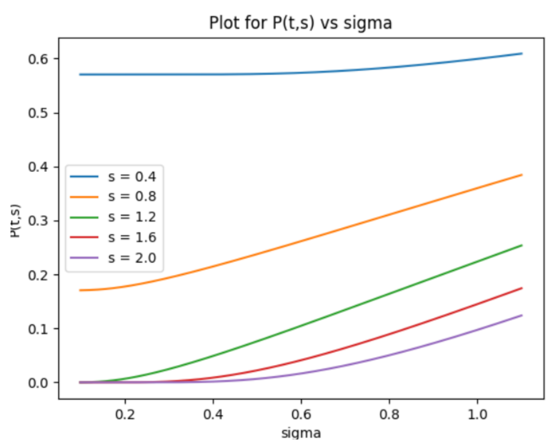


Some of the values are: (with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$)

r	C(t,s)	P(t,s)
0.02	0.323945	0.112017
0.118098	0.357721	0.0893144
0.216196	0.392126	0.0704696
0.314294	0.426867	0.0550036
0.412392	0.461658	0.0424585
0.51049	0.496235	0.0324045
0.608589	0.530355	0.0244455
0.706687	0.563808	0.0182239
0.804785	0.596413	0.0134225
0.902883	0.628024	0.00976515

Variation of C(t,s) and P(t,s) with Volatility (sigma):

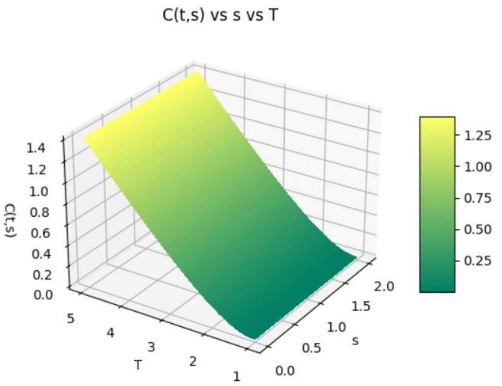


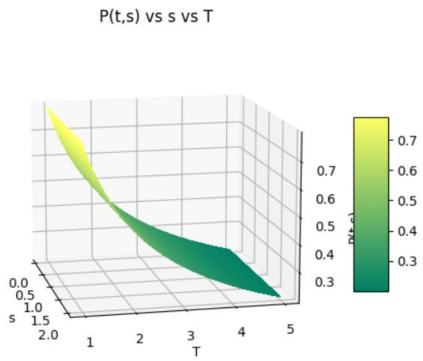


Some of the values are: (with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$)

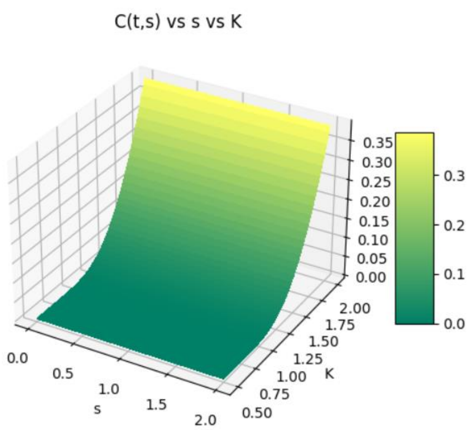
sigma	C(t,s)	P(t,s)
0.1	0.229632	7.7373e-05
0.2001	0.236093	0.00653828
0.3002	0.254056	0.0245013
0.4003	0.27828	0.048726
0.5004	0.30554	0.0759857
0.600501	0.334336	0.104782
0.700601	0.36391	0.134356
0.800701	0.393835	0.164281
0.900801	0.423846	0.194291
1.0009	0.453763	0.224209

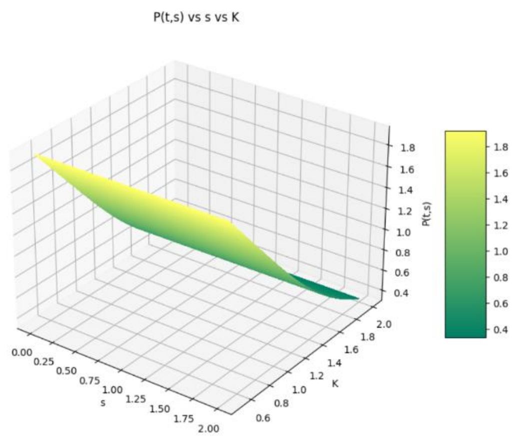
Variation of C(t,s) and P(t,s) with Stock Price (s) and Expiration Time (T):



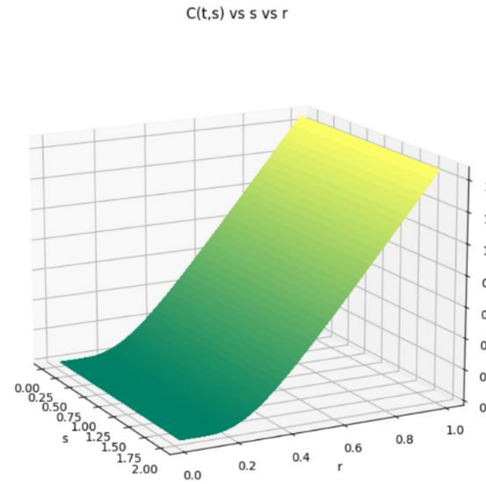


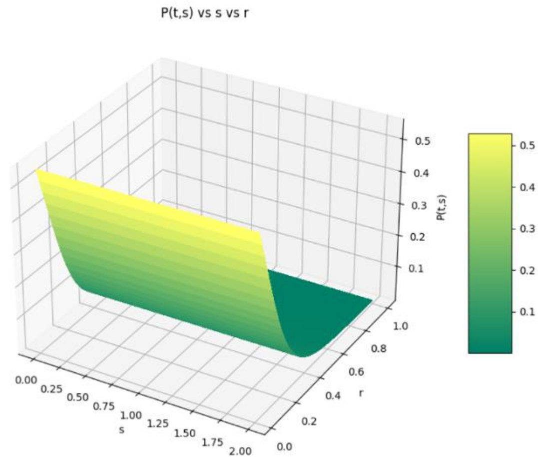
Variation of C(t,s) and P(t,s) with Stock Price (s) and Strike Price (K):



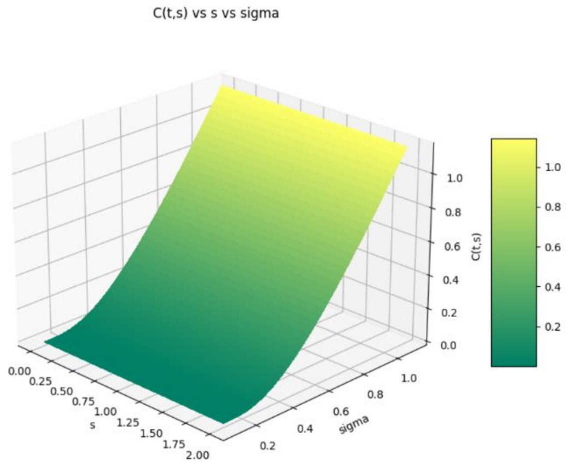


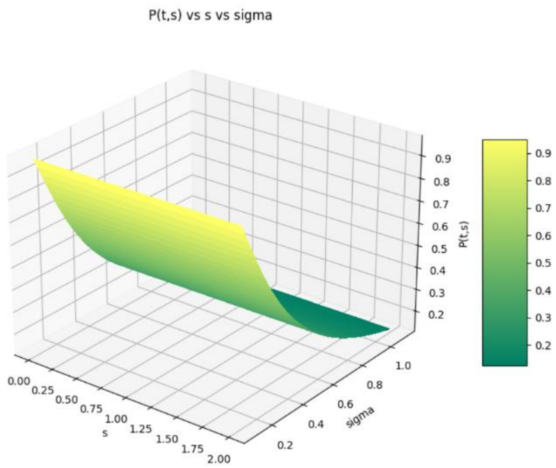
Variation of C(t,s) and P(t,s) with Stock Price (s) and Rate of Interest (r):



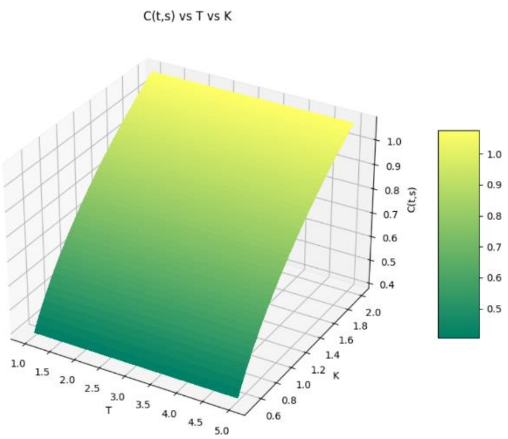


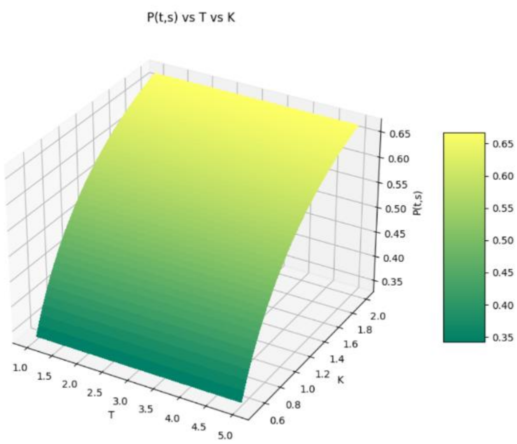
Variation of C(t,s) and P(t,s) with Stock Price (s) and Volatility (sigma):



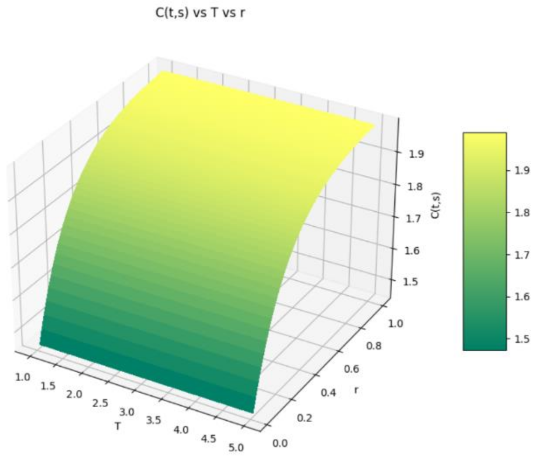


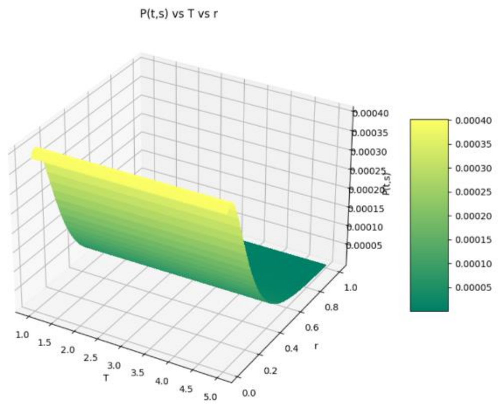
Variation of C(t,s) and P(t,s) with Expiry Time (T) and Strike Price (K):



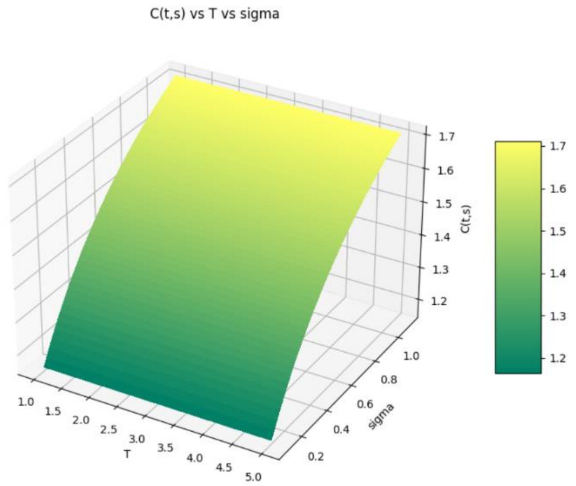


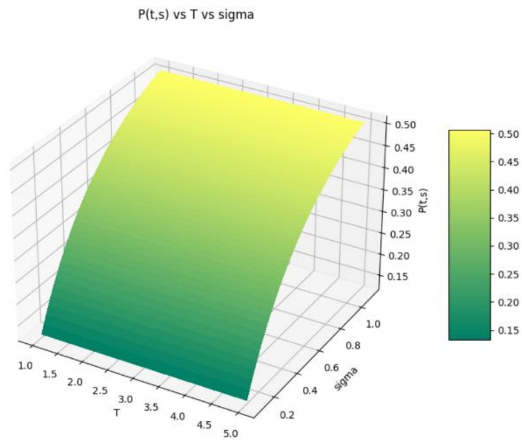
Variation of C(t,s) and P(t,s) with Expiry Time (T) and Rate of Interest (r):



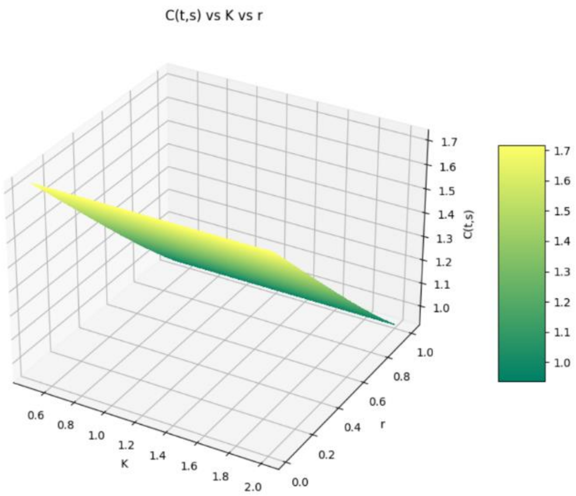


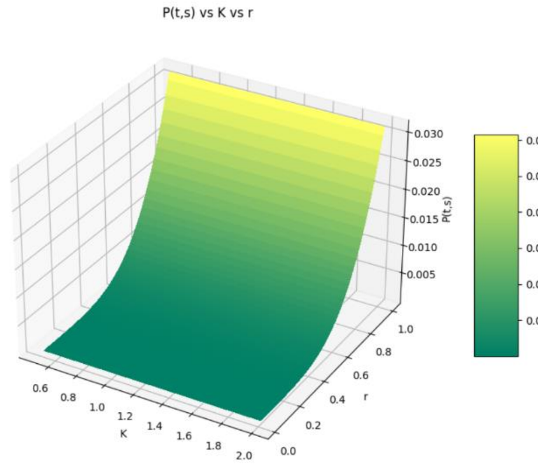
Variation of C(t,s) and P(t,s) with Expiry Time (T) and Volatility (sigma):



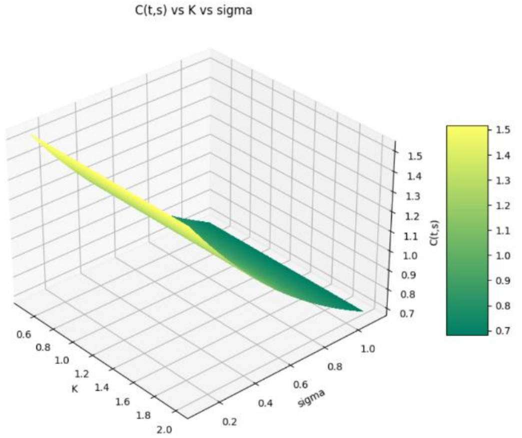


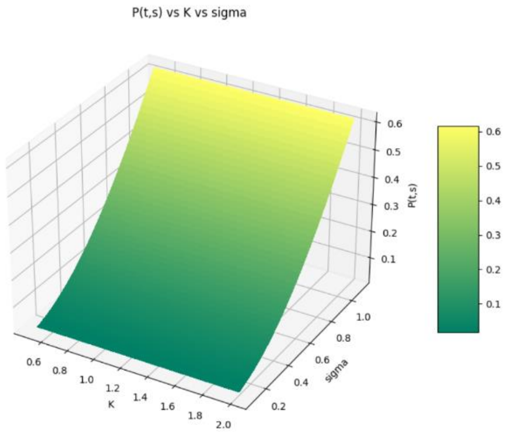
Variation of C(t,s) and P(t,s) with Strike Price (K) and Rate of Interest (r):



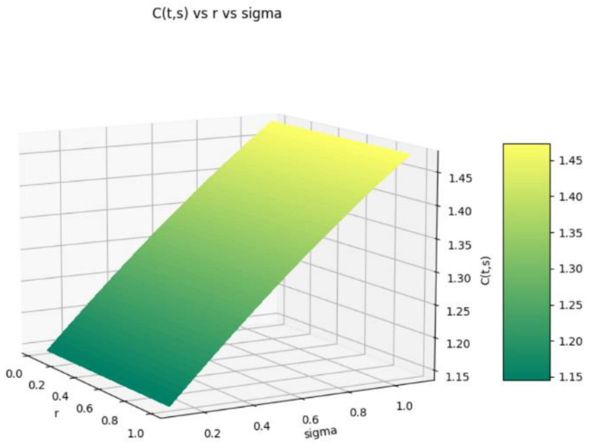


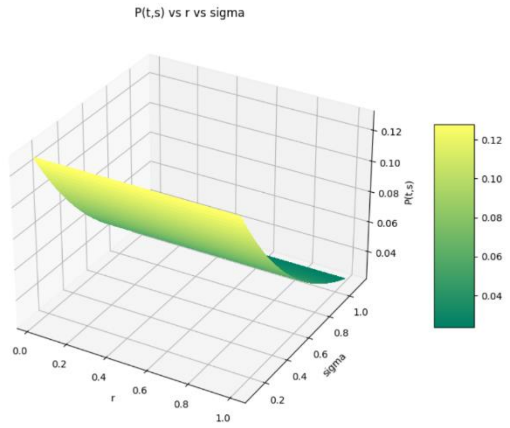
Variation of C(t,s) and P(t,s) with Strike Price (K) and Volatility (sigma):





Variation of C(t,s) and P(t,s) with Rate of interest (r) and Volatility (sigma):





Q5

Formula used for calculating the Put and Call option price in the classical BSM framework with dividend paying stocks is:

$$C(s,t) = se^{-a(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2)$$
$$P(s,t) = Ke^{-r(T-t)}N(-d_2) - se^{-a(T-t)}N(-d_1)$$
where,

$$d_1 = \frac{\log\left(\frac{s}{K}\right) + \left(r - a + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

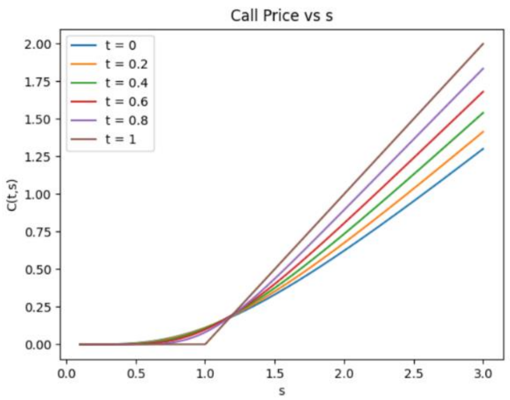
$$d_2 = \frac{\log\left(\frac{s}{K}\right) + \left(r - a - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

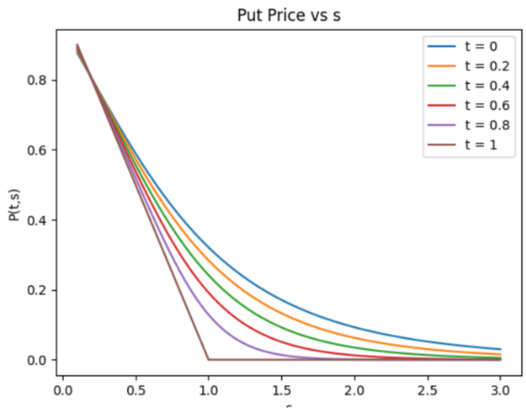
$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}y^2}\,dy$$

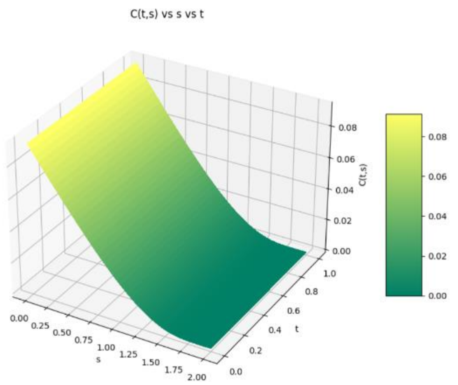
The parameter values are adjusted as needed, and when specific parameter values are needed, they are obtained as:

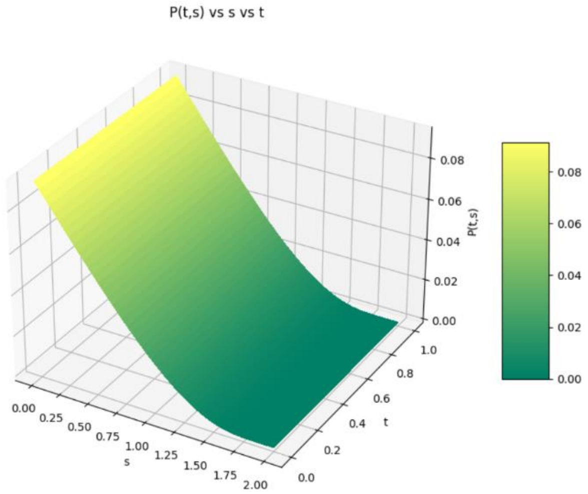
$s = 2, t = 0.4, T = 1, K = 1, r = 0.05, \sigma = 0.6$, a = 0.3

Variation of C(t,s) and P(t,s) with Stock Price (s):

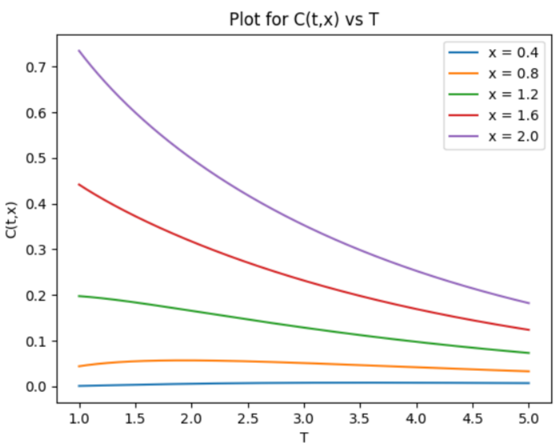


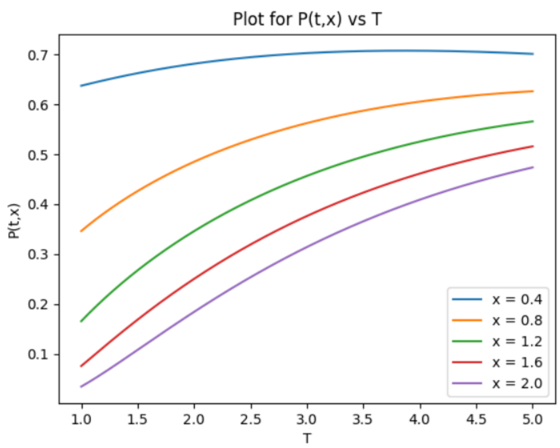






Variation of C(t,s) and P(t,s) with Expiration Time (T):

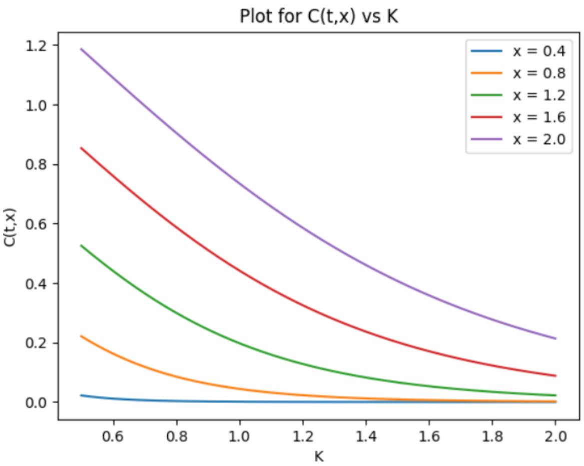


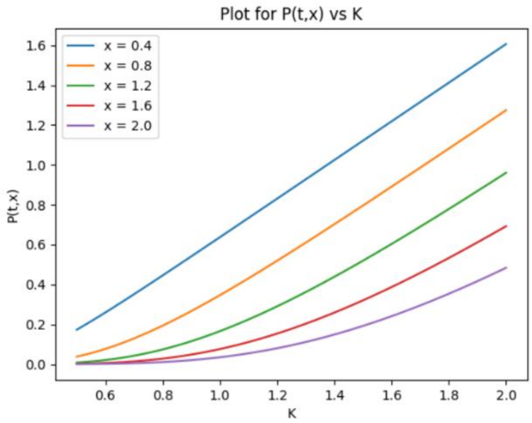


Some of the values are: (with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$, a $=$ 0.3)

T	C(t,x)	P(t,x)
1	0.197624	0.165745
1.4004	0.187183	0.249518
1.8008	0.173004	0.317093
2.2012	0.157838	0.372669
2.6016	0.142867	0.418706
3.002	0.128636	0.456887
3.4024	0.115392	0.488465
3.8028	0.103227	0.514424
4.2032	0.0921517	0.535564
4.6036	0.0821295	0.552551

Variation of C(t,s) and P(t,s) with Strike Price (K):

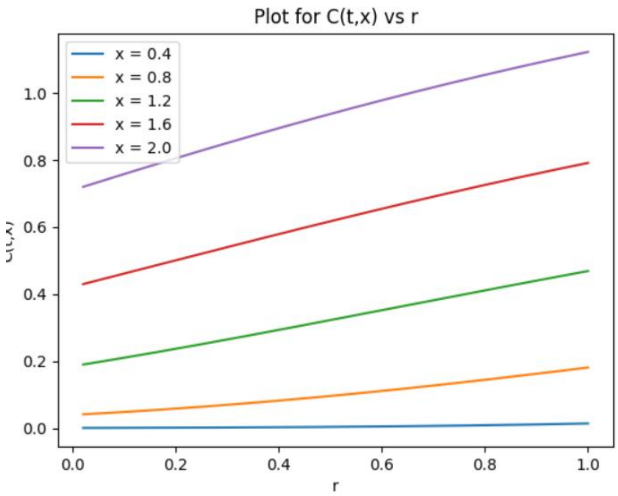


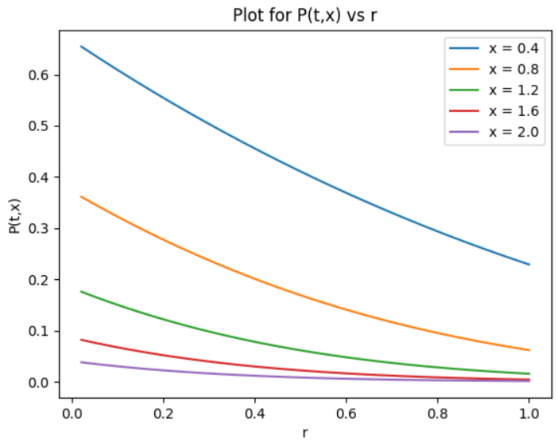


Some of the values are:(with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$, a $=$ 0.3)

K	C(t,x)	P(t,x)
0.5	0.525195	0.00809305
0.65015	0.401894	0.0305053
0.8003	0.299719	0.0740427
0.95045	0.219588	0.139624
1.1006	0.159092	0.224841
1.25075	0.114544	0.326005
1.4009	0.0822467	0.439421
1.55105	0.0590437	0.56193
1.7012	0.042452	0.691051
1.85135	0.0306068	0.824918

Variation of C(t,s) and P(t,s) with Rate of Interest (r):

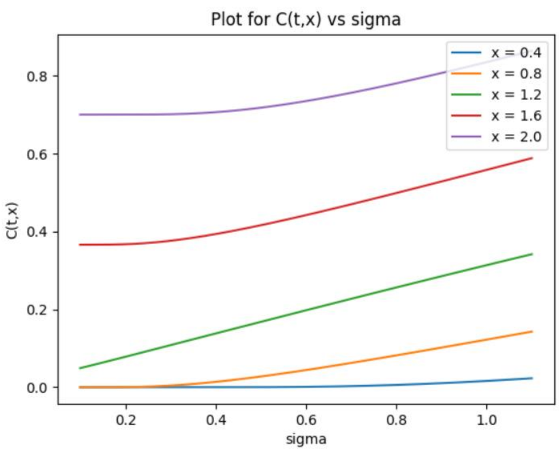


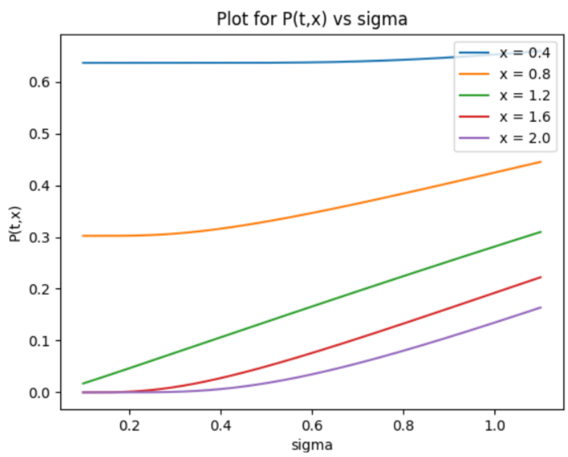


Some of the values are: (with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$, a $=$ 0.3)

r	C(t,x)	P(t,x)
0.02	0.190085	0.175833
0.118098	0.215208	0.144477
0.216196	0.241574	0.117593
0.314294	0.268962	0.0947749
0.412392	0.297136	0.0756124
0.51049	0.32585	0.0596956
0.608589	0.354858	0.0466238
0.706687	0.383922	0.0360135
0.804785	0.412819	0.027504
0.902883	0.441346	0.0207628

Variation of C(t,s) and P(t,s) with Volatility (sigma):

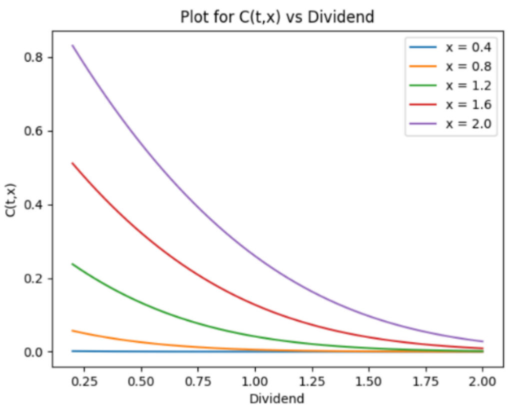


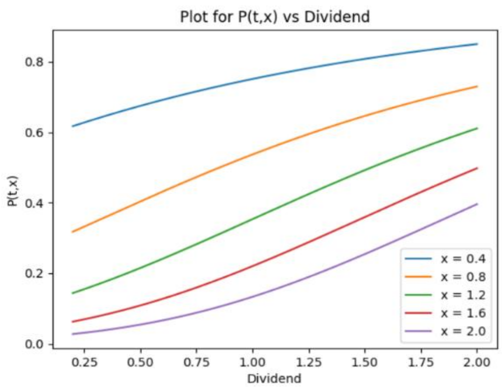


Some of the values are: (with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$, a = 0.3)

sigma	C(t,x)	P(t,x)
0.1	0.0490265	0.0171477
0.2001	0.0781885	0.0463098
0.3002	0.108115	0.076236
0.4003	0.138123	0.106244
0.5004	0.168038	0.136159
0.600501	0.197772	0.165893
0.700601	0.227264	0.195385
0.800701	0.256462	0.224584
0.900801	0.285322	0.253443
1.0009	0.3138	0.281921

Variation of C(t,s) and P(t,s) with Dividend (a):

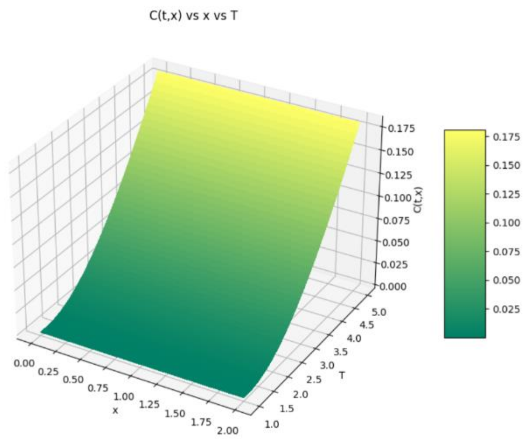


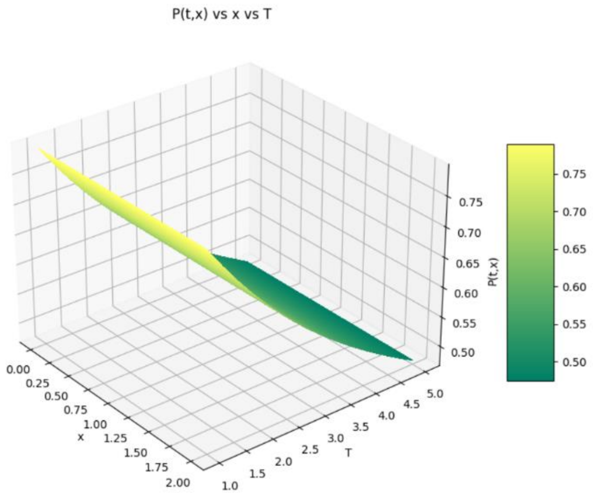


Some of the values are: (with parameters as $x = 2, t = 0.4, K = 1, r = 0.05$ and $\sigma = 0.6$, a = 0.3)

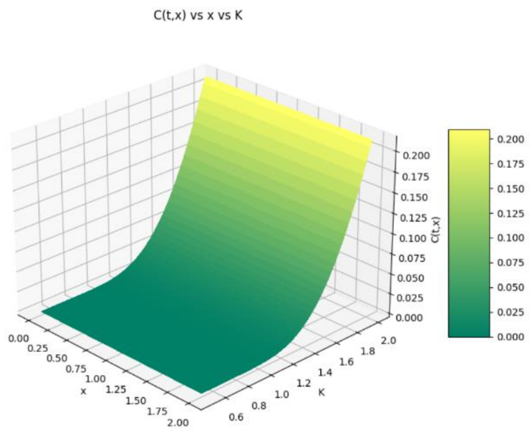
Dividend	C(t,x)	P(t,x)
0.2	0.237486	0.143627
0.38018	0.169429	0.184629
0.56036	0.117211	0.230294
0.740541	0.0784549	0.279392
0.920721	0.0507063	0.330494
1.1009	0.0315849	0.382144
1.28108	0.0189294	0.433008
1.46126	0.0108985	0.481988
1.64144	0.00601962	0.528278
1.82162	0.00318577	0.571369

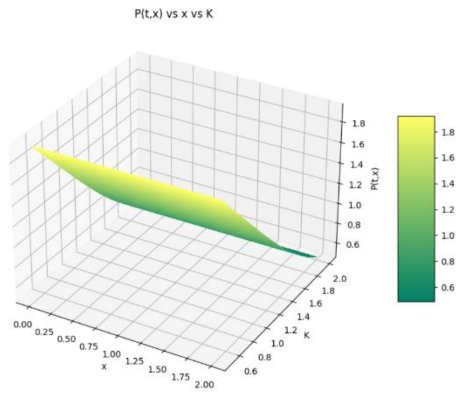
Variation of C(t,s) and P(t,s) with Stock Price (s) and Expiration Time (T):



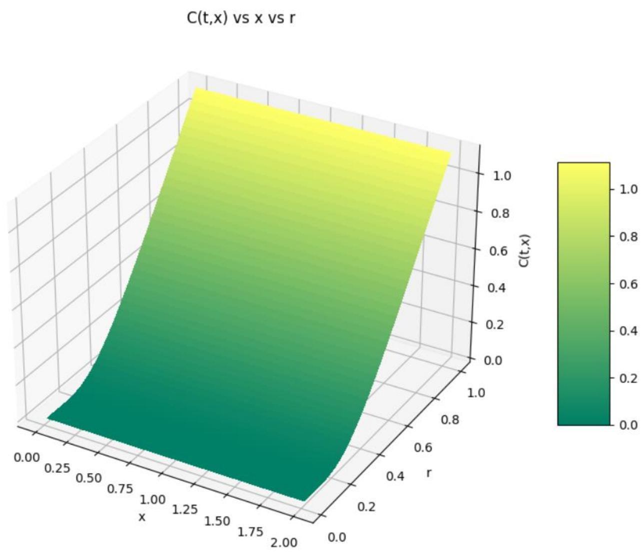


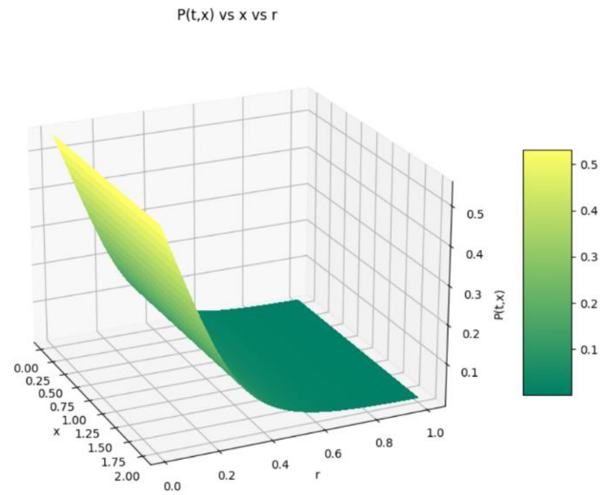
Variation of C(t,s) and P(t,s) with Stock Price (s) and Strike Price (K):



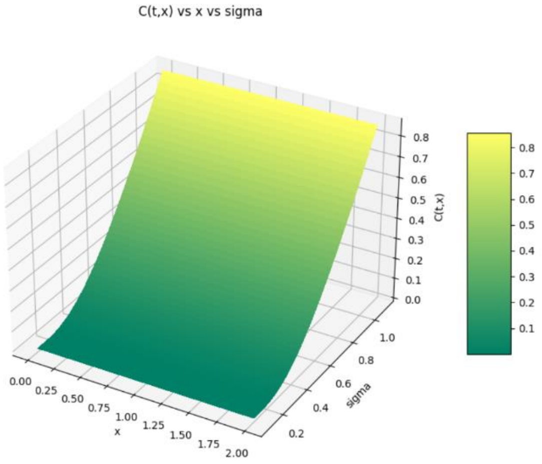


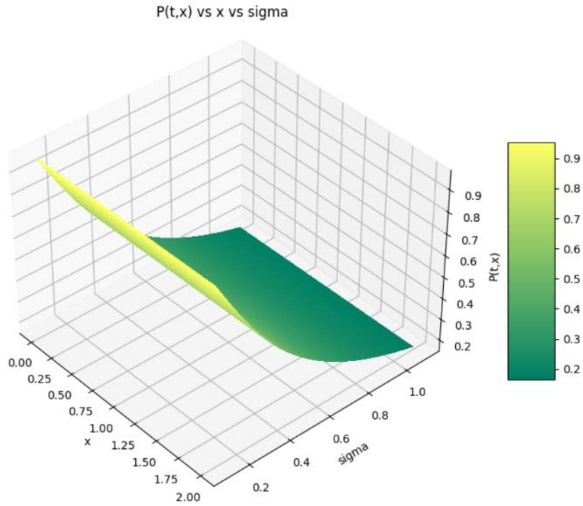
Variation of C(t,s) and P(t,s) with Stock Price (s) and Rate of Interest (r):



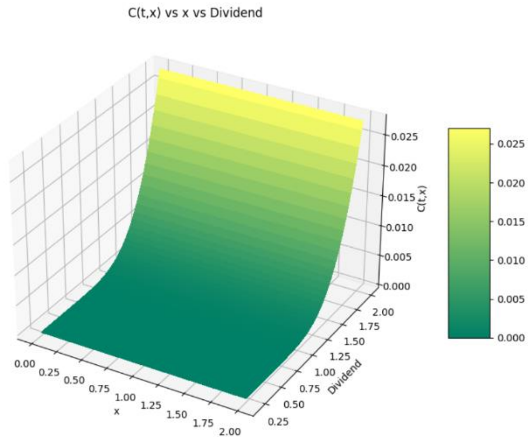


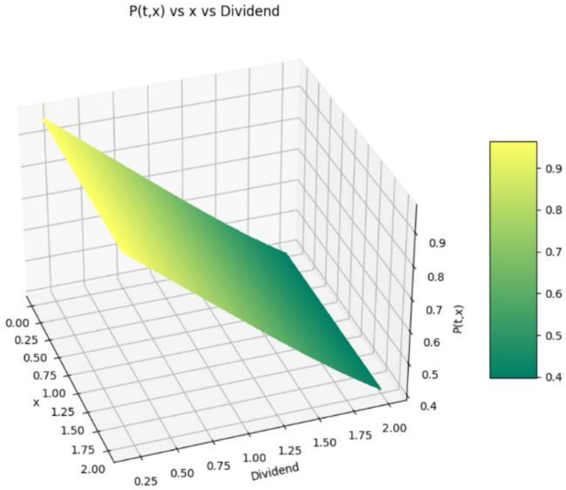
Variation of C(t,s) and P(t,s) with Stock Price (s) and Volatility (sigma):



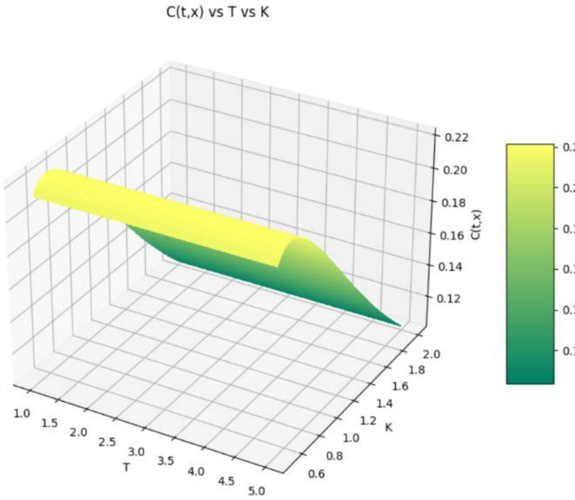


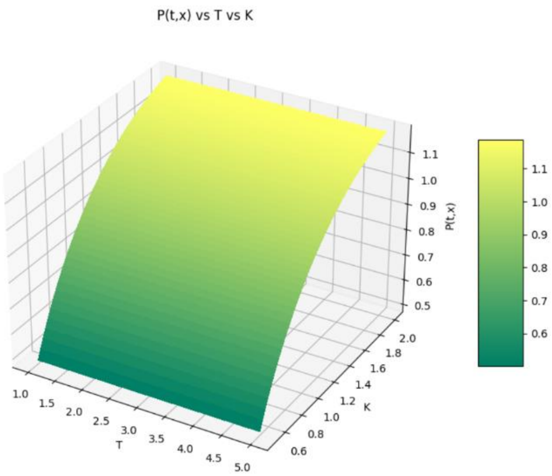
Variation of C(t,s) and P(t,s) with Stock Price (s) and Dividend (a):



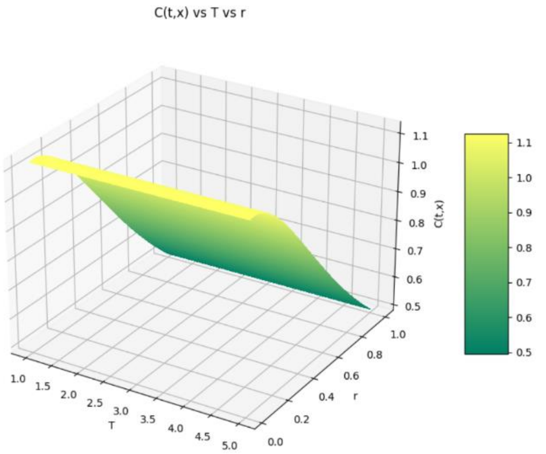


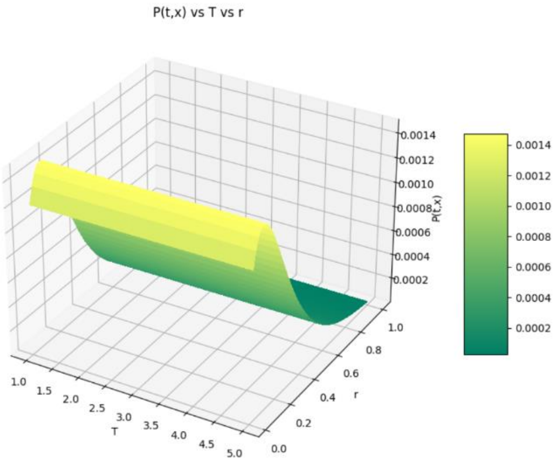
Variation of C(t,s) and P(t,s) with Expiry Time (T) and Strike Price (K):



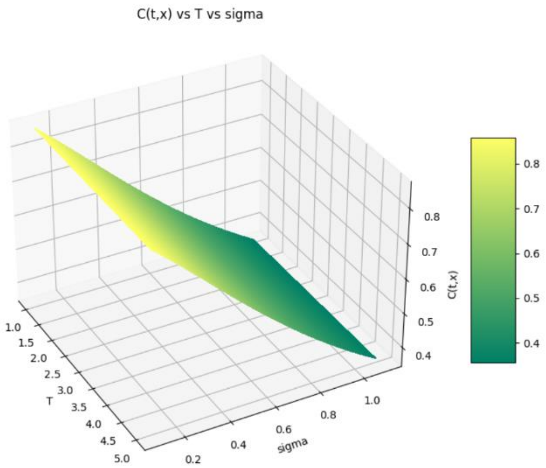


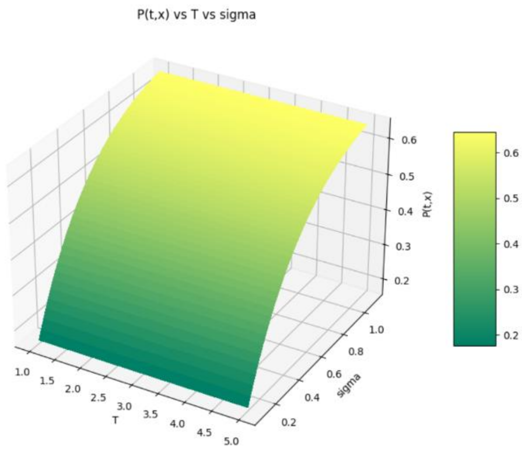
Variation of C(t,s) and P(t,s) with Expiry Time (T) and Rate of Interest (r):



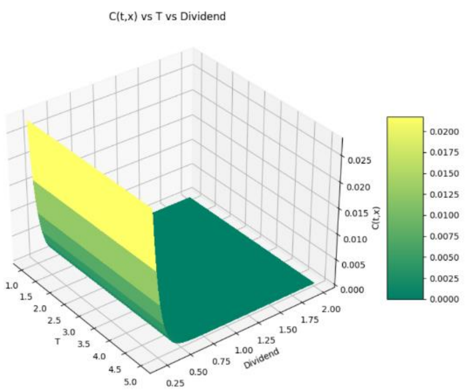


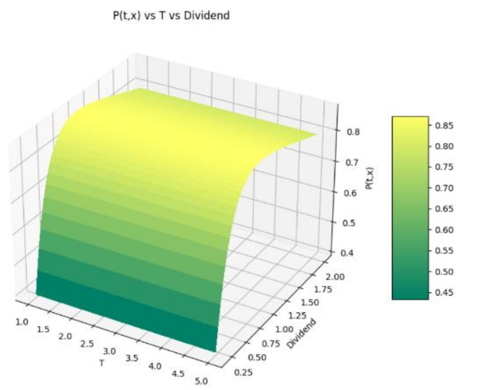
Variation of C(t,s) and P(t,s) with Expiry Time (T) and Volatility (sigma):



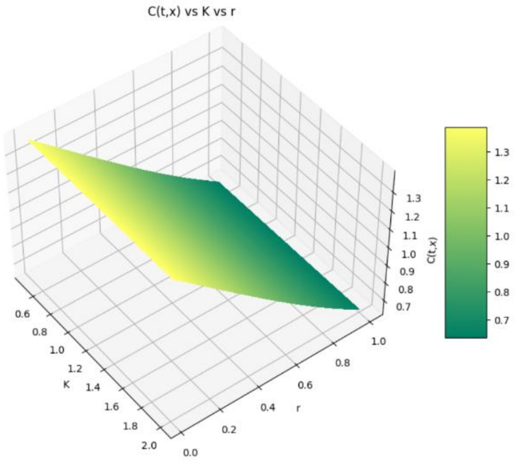


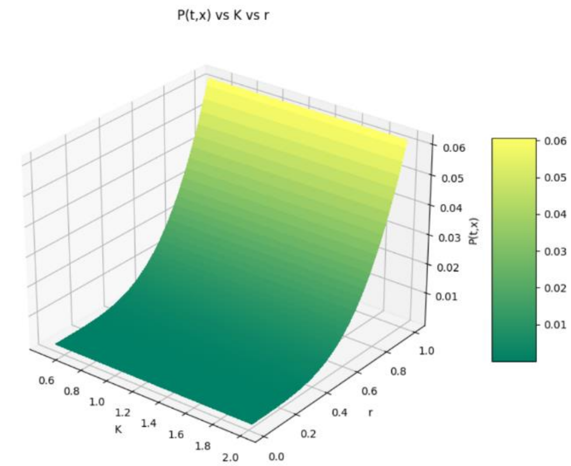
Variation of C(t,s) and P(t,s) with Expiry Time (T) and Dividend (a):



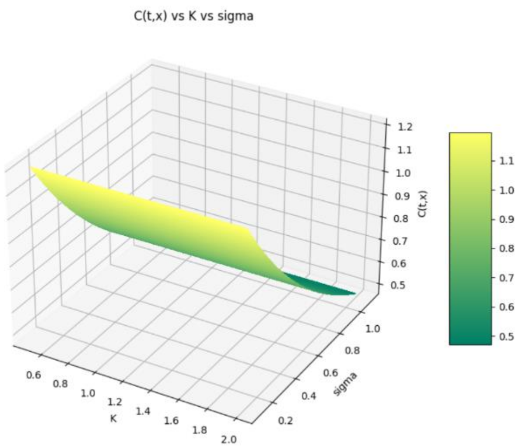


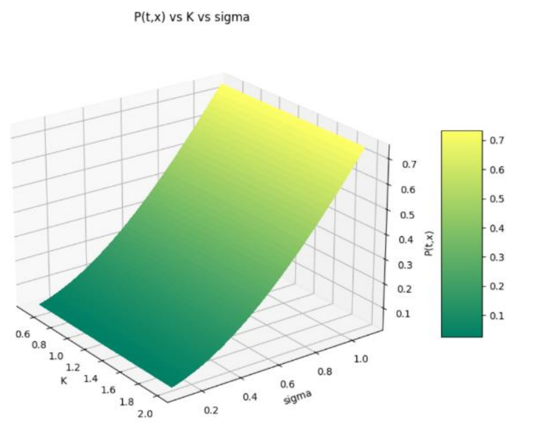
Variation of C(t,s) and P(t,s) with Strike Price (K) and Rate of Interest (r):



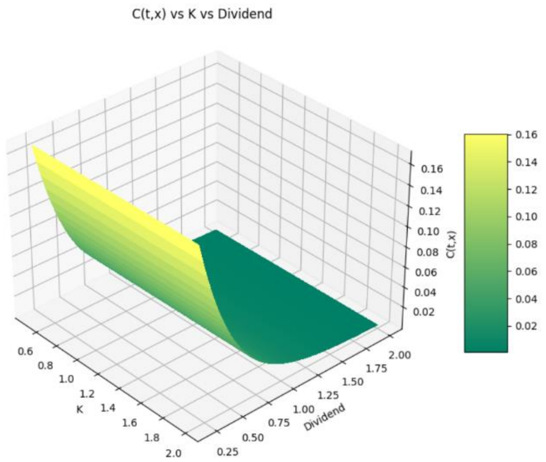


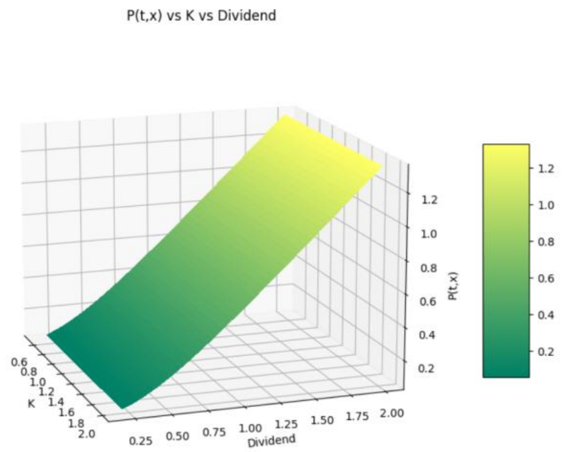
Variation of C(t,s) and P(t,s) with Strike Price (K) and Volatility (sigma):



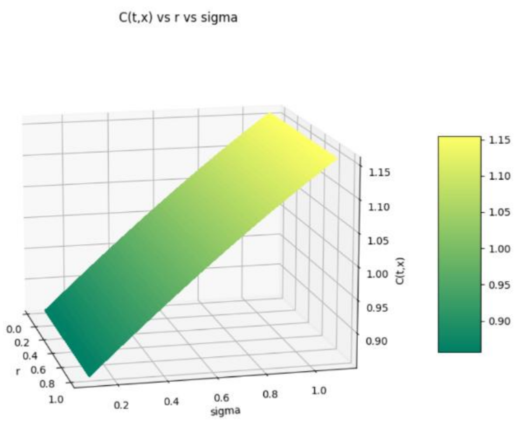


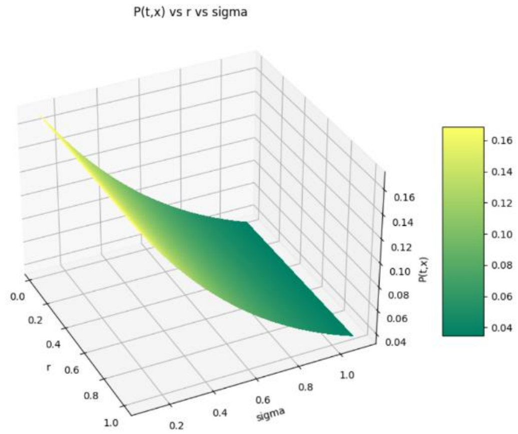
Variation of C(t,s) and P(t,s) with Strike Price (K) and Dividend (a):



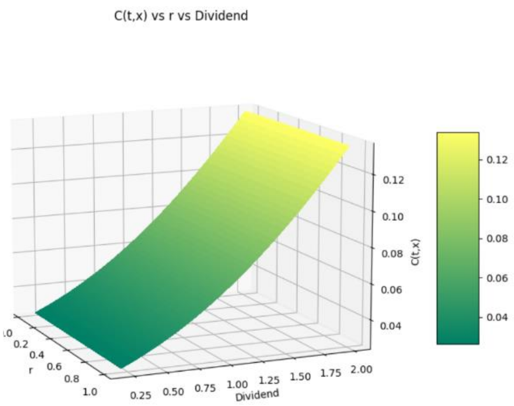


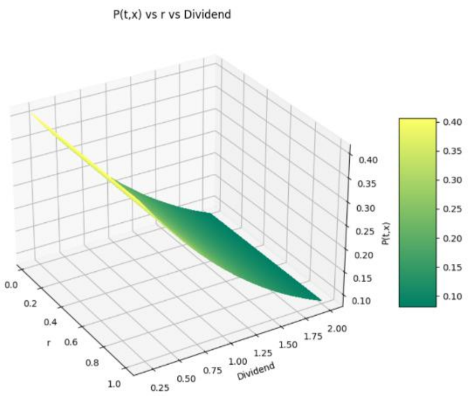
Variation of C(t,s) and P(t,s) with Rate of Interest (r) and Volatility (sigma):





Variation of C(t,s) and P(t,s) with Rate of Interest (r) and Dividend (a):





Variation of C(t,s) and P(t,s) with Volatility (sigma) and Dividend (a):