CSE 21
Homework 1
Due: Friday April 1, 2016 at 11:59pm

INSTRUCTIONS

This homework assignment must be completed **individually** (and not in groups). Completing this assignment will set you up to use the tools you'll need for the rest of the course, and will give you and us an idea of your mastery of the prerequisite knowledge before you start the course.

For this assignment, you will receive full credit for completing all required steps. For the pre-test, this means an honest attempt at answering all the questions; correctness of the answers will not be used to calculate your score for this first homework.
REQUIRED READING Rosen Chapters 1 and 2, Sections 5.1-5.2 (covers prerequisite concepts).

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1. (10 points) Prove that for every $a > 1$ and $b > 1$, $a^{\log_2 b} = b^{\log_2 a}$.

   $a^{\log_2 b} = (2^{log_2 a})^{\log_2 b} = 2^{(\log_2 a \log_2 b} = (2^{\log_2 b})^{\log_2 a} = b^{\log_2 a}$

   The first equation uses the definition of logs, the second uses the algebraic rule of exponentiation $(x^y)^z = x^{yz}$, the third uses the same rule in reverse, and the fourth uses the definition of log. We need $a > 0$ and $b > 0$ since otherwise log is not well defined (at least as a real number).

2. Consider the following algorithm

   procedure Loops(n: a positive integer)

   ```
   for i := 1 to n
       for j := 1 to n
           print (i, j)
   ```

   (a) Write what the algorithm prints when $n = 4$.
   (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)

   (b) Describe what the algorithm prints in general terms.
   It prints all pairs of positive integers where both integers are at most n, ordered first by the first integer and then by the second

   (c) How many times does **print** routine get called?
   Since there are n possible i's and for each n j's, print is called n^2 times.

   (d) Describe (in words) a rule to decide, if (i_1, j_1) and (i_2, j_2) have both been printed for some n then which ordered pair was printed first?
   We print from smallest i to largest i. So if $i_1 < i_2$, the first pair is printed first, and if $i_1 > i_2$ the second pair. If $i_1 = i_2$, since within the same i, we print in increasing order of j, the first pair is first if $j_1 < j_2$ and second otherwise. Summing up, the first pair is printed first if and only if $i_1 < i_2$ or $i_1 = i_2$ and $j_1 < j_2$.

3. Prove by induction that

   $$1 + 3 + 5 + ... + (2n - 1) = n^2$$

   for all $n \geq 1$.

   Base case: $n = 1$: $2(1) - 1 = 1$ and $1^2 = 1$ so the base case is satisfied.

   Induction step: Suppose for some $k \geq 1$,

   $$1 + 3 + 5 + ... + (2k - 1) = k^2$$

   We want to show that

   $$1 + 3 + 5 + ... + (2(k + 1) - 1) = (k + 1)^2$$

   $$1 + 3 + 5 + ... + (2(k + 1) - 1) = 1 + 3 + 5 + ... + (2k - 1) + (2(k + 1) - 1)$$

   . By the inductive hypothesis, we have that

   $$1 + 3 + 5 + ... + (2k - 1) + (2(k + 1) - 1) = k^2 + (2(k + 1) - 1)$$

   And by algebra, we have that

   $$k^2 + (2(k + 1) - 1) = k^2 + 2k + 1 = (k + 1)^2.$$

   Therefore

   $$1 + 3 + 5 + ... + (2n - 1) = n^2$$

   for all $n \geq 1$.