CSE 21 Winter 2016
Homework 8
Due: Tuesday, August 30, 2016 at 11:59pm

Instructions

Homework should be done in groups of **one to two** people. You are free to change group members at any time throughout the quarter. Problems should be solved together, not divided up between partners. A **single representative** of your group should submit your work through Gradescope. Submissions must be received by 11:59pm on the due date, and there are no exceptions to this rule.

Homework solutions should be neatly written or typed and turned in through **Gradescope** by 11:59pm on the due date. No late homeworks will be accepted for any reason. You will be able to look at your scanned work before submitting it. Please ensure that your submission is legible (neatly written and not too faint) or your homework may not be graded.

Students should consult their textbook, class notes, lecture slides, instructors, TAs, and tutors when they need help with homework. Students should not look for answers to homework problems in other texts or sources, including the internet. Only post about graded homework questions on Piazza if you suspect a typo in the assignment, or if you don't understand what the question is asking you to do. Other questions are best addressed in office hours.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

For questions that require pseudocode, you can follow the same format as the textbook, or you can write pseudocode in your own style, as long as you specify what your notation means. For example, are you using "=" to mean assignment or to check equality? You are welcome to use any algorithm from class as a subroutine in your pseudocode. For example, if you want to sort list A using InsertionSort, you can call InsertionSort(A) instead of writing out the pseudocode for InsertionSort.

Required Reading Rosen 7.1 through 7.4

Key Concepts basic probability principles, independence, conditional probability, expected value, conditional expectation, linearity of expectation, variance and concentration, Bayes' Theorem

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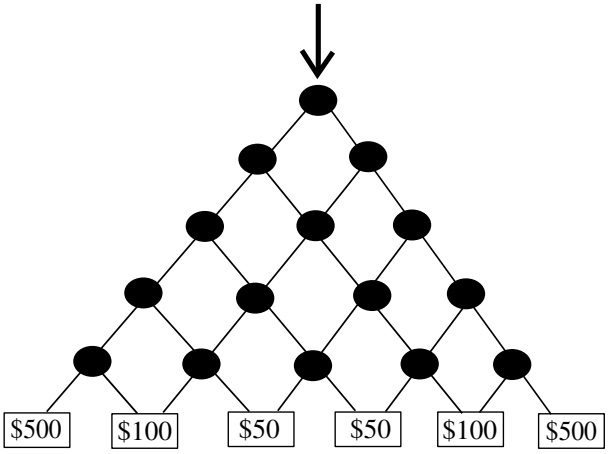
1. (a) (4 points) Assume that every time you go on a job interview, your chance of getting a job offer is 25%. How many job interviews must you go on so that the probability of your getting a job offer is greater than 95%?

   (b) (4 points) Suppose that you roll two dice and don't get to look at the outcome. Your friend looks at the outcome and tells you honestly that at least one of the dice came up 6. What is the probability that the sum of your two dice is 8?

2. In a variant of the game Plinko from the gameshow The Price is Right, the player has one disk which they insert into the top of the board shown below. Each time the disk hits a peg (shown as a black circle), it has a 50% chance of falling to the left, and a 50% chance of falling to the right. Eventually, the disk lands in one of the bins at the bottom of the board. Each bin is marked with a dollar amount, and the player wins the amount of money shown on the bin in which the disk lands.



   (a) (3 points) Suppose that bin 1 is the leftmost bin, and bins are numbered from 1 to 6 reading from left to right. In terms of i, find a formula for the probability that the disk falls into bin i.

   (b) (3 points) How much money does the player expect to win at this game?

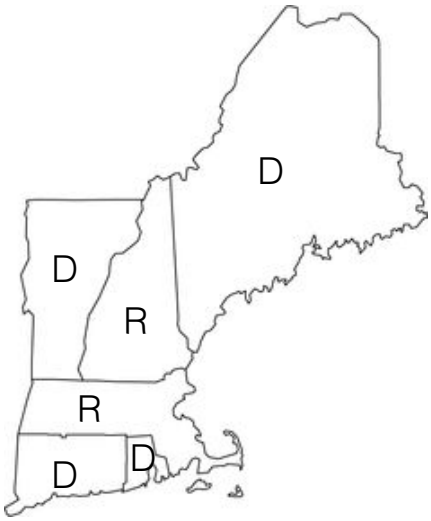
   (c) (3 points) How much money does the player expect to win if you know that the disk falls to the right the first time it hits a peg?

   (d) (3 points) How much money does the player expect to win if you know that the disk falls to the right the first and second time it hits a peg?

3. (a) (5 points) Suppose there are n people assigned to m different tasks. Assume that each person is randomly assigned a task and that for each person, all tasks are equally likely. Use linearity of expectation to find the expected number of people working on a task alone.

   (b) (5 points) Suppose each of the 50 states of the USA votes either Democrat or Republican in the next election. Assume that each state votes Democrat with a 50% probability, and Republican with a 50% probability. If there are 109 borders between states, use linearity of expectation to find the expected number of borders that separate a Democratic voting state from a Republican voting state.

   For example, the following map shows 6 states with 7 borders between states. There are 5 borders that separate a Democratic voting state from a Republican voting state.



4. Suppose we flip a fair coin n times where $n > 2$ and consider the sample space of all possible coin toss sequences.

   - Let A be the event that the first flip is Tails.
   - Let B be the event that the first and second flips are the same.
   - Let X be the random variable that counts the number of Tails that appear.
   - Let Y be the random variable that counts the number of Heads that appear minus the number of Tails that appear.

   For each pair of quantities, decide which of them is greater, or if they are both the same. Justify your answer in words.

   (a) (3 points) $P(A|B)$ and $P(A)$

   (b) (3 points) $E(Y|A)$ and $E(Y)$

   (c) (3 points) $V(X+Y)$ and $V(X)$

   (d) (3 points) $V(2X+Y)$ and $V(Y)$

5. At a party everyone brings a guest so there are $2n$ people. Mr. X and Mrs. X arrive at the party together

   (a) If all the people are randomly paired with a dance partner, what is the probability of Mr. X dancing with Mrs. X?

   (b) If all the people are randomly paired with their dance partner, use linearity of expectation to find the expected number of people who get to dance with the person they came with. Show your work.