CSE 21 Spring 2016
Homework 5
Due: Tuesday, August 16 at 11:59pm

Instructions

Homework should be done in groups of **one to two** people. You are free to change group members at any time throughout the quarter. Problems should be solved together, not divided up between partners. A **single representative** of your group should submit your work through Gradescope. Submissions must be received by 11:59pm on the due date, and there are no exceptions to this rule.

Homework solutions should be neatly written or typed and turned in through **Gradescope** by 11:59pm on the due date. No late homeworks will be accepted for any reason. You will be able to look at your scanned work before submitting it. Please ensure that your submission is legible (neatly written and not too faint) or your homework may not be graded.

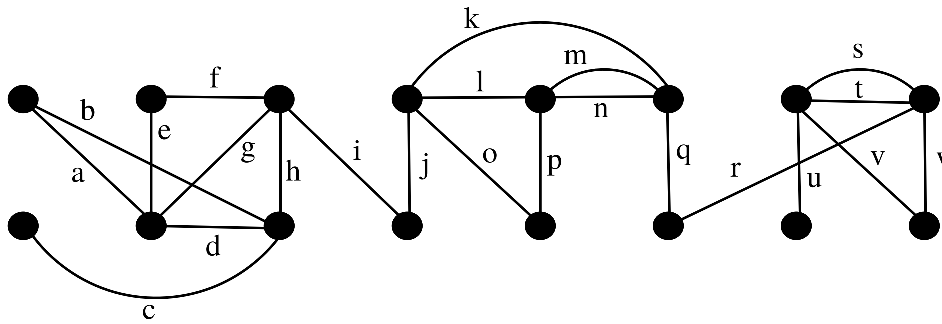
Students should consult their textbook, class notes, lecture slides, instructors, TAs, and tutors when they need help with homework. Students should not look for answers to homework problems in other texts or sources, including the internet. Only post about graded homework questions on Piazza if you suspect a typo in the assignment, or if you don't understand what the question is asking you to do. Other questions are best addressed in office hours.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

For questions that require pseudocode, you can follow the same format as the textbook, or you can write pseudocode in your own style, as long as you specify what your notation means. For example, are you using "=" to mean assignment or to check equality? You are welcome to use any algorithm from class as a subroutine in your pseudocode. For example, if you want to sort list A using InsertionSort, you can call InsertionSort(A) instead of writing out the pseudocode for InsertionSort.

Required Reading Rosen 10.1, 10.2, 10.3, 10.4 through Theorem 1, 10.5 through Example 7.

Key Concepts Graphs (definitions, modeling problems using graphs), Hamiltonian tours, Eulerian tours, Fleury's algorithm.

1. 

   (a) (3 points) Which of the edges in the graph above are bridges?

   (b) (3 points) Use Fleury's algorithm to find an Eulerian tour of the graph above. Suppose that whenever the algorithm allows you a choice for which edge to take, you always take the edge whose label comes first alphabetically. For example, if you were in a position where you could take edge b, edge c, or edge h, you would take edge b. Write down the Eulerian tour you find by listing the edges of your tour in order.

   (c) (3 points) Draw a connected graph with 5 vertices that has no Eulerian tour.

2. We say a matrix has dimensions $m \times n$ if it has m rows and n columns. If matrix A has dimensions $x \times y$ and matrix B has dimensions $z \times w$, then the product AB exists if and only if $y = z$. In the case where the product exists, AB will have dimensions $x \times w$. In this problem, we are given a list of matrices and their dimensions, and we want to determine if there is an order in which we can multiply all the matrices together, using each matrix exactly once. For example, here is a possible list of matrices and their dimensions:

   A is 3×5
   B is 4×3
   C is 4×4
   D is 2×5
   E is 5×2
   F is 5×3

   (a) (3 points) Given any list of matrices and dimensions, describe how to draw a graph so that each order in which we can multiply the matrices corresponds to a Hamiltonian tour of your graph. Carefully describe what the vertices of your graph represent, and when two vertices are connected with an edge.

   (b) (1 point) Draw the graph described in part (a) for the example list of matrices given above.

   (c) (3 points) Given any list of matrices and dimensions, describe how to draw a graph so that each order in which we can multiply the matrices corresponds to an Eulerian tour of your graph. Carefully describe what the vertices of your graph represent, and when two vertices are connected with an edge.

   (d) (1 point) Draw the graph described in part (c) for the example list of matrices given above.

   (e) (2 points) For the given example list of matrices, give one order in which we can multiply those matrices, or say that no such order exists.

3. (10 points) Say that two actors are co-stars if they have been in the same movie. Show that in any group of six actors, we can either find a group of three such that all pairs in the group are co-stars, or a group of three so that no two in the group are co-stars.

4. A *tournament* is a directed graph where between every two distinct vertices u and v, there is either an edge from u to v or an edge from v to u, but not both. Prove that every tournament has a Hamiltonian tour. (Hint: use induction on $|V|$, and show that the last element can be inserted somewhere on a tour that spans the first $n-1$.) (10 points).