

CSE 21
Practice Exam for Midterm 2
Summer Academy 2016

This practice exam should help prepare you for the second midterm on Friday, August 27.

- 1. Representing Problems as Graphs** I have \$10, and I plan to spend some or all of my money on three types of candy, which I will buy one piece at a time:

- chocolate bars cost \$3,
- almond rocca cost \$2, and
- caramel chunks cost \$5.

I want to know what combinations of candy I can afford; I might buy more than one of the same type.

- (a) Describe how you would model this situation using a directed graph, where paths in your graph should represent possible sequences of candy purchases. What are the vertices, and when are two vertices connected with an edge?
- (b) How can this graph be used to determine which amounts of change I might have left over when I have had my fill of candy?
- (c) Give the adjacency list representation for this graph.
- (d) Is your graph a DAG? Explain why or why not.

- 2. Graphs, Trees, Tours, and Counting** A complete bipartite graph is an undirected graph where the vertex set V can be partitioned into two sets V_1 and V_2 so that

- (i) there is an edge between every vertex in V_1 and every vertex in V_2 ,
- (ii) there are no edges between the vertices of V_1 , and
- (iii) there are no edges between the vertices of V_2 .

Answer the following questions about complete bipartite graphs, and show how you came to your conclusions.

- (a) How many vertices are there in a complete bipartite graph with $|V_1| = m$ and $|V_2| = n$?
- (b) How many edges are there in a complete bipartite graph with $|V_1| = m$ and $|V_2| = n$?
- (c) For which values of m and n is a complete bipartite graph an unrooted tree?
- (d) For which values of m and n does a complete bipartite graph have an Eulerian tour that starts and ends at different vertices?
- (e) For which values of m and n does a complete bipartite graph have a Hamiltonian tour?
- (f) If $m = n$ how many Hamiltonian tours does a complete bipartite graph have?

- 3. Counting** In each of the following problems, a hand of five cards will be dealt from a standard deck of cards, with thirteen cards in each of four suits, and no jokers or wild cards. A hand is a *set* of five cards, so the order in which the cards are dealt does not matter. Say how many different hands of the following types are possible. Here, as well as on the exam, you can leave your answer as an unsimplified algebraic expression involving binomial coefficients, factorials, or exponents. You do not need to simplify.

- (a) Straight: A hand where the numbers of the cards are five consecutive integers (with Jack = 11, Queen = 12, King = 13, and Ace counting as 1 or 14).
- (b) Three of a kind: A hand with three cards of one number, one card of a second number, and one card of a third number.
- (c) Two pair: A hand with two cards of one number, two cards of a second number, and one card of a third number.

4. Encoding and Decoding Consider strings containing the symbols 0, 1, 2, 3 and 4. Say that we look at the set of such strings of length n that never have the same symbol appearing twice in a row. For example, 01242343 is such a string of length $n = 8$.

- (a) How many such strings of length n are there that never have the same symbol appearing twice in a row?
- (b) How many bits (0's and 1's) are required to represent such a string of length n that never has the same symbol appearing twice in a row?
- (c) Describe how to encode such strings of length n that never has the same symbol appearing twice in a row, using the number of bits you gave in part (b). Illustrate your description with an example.
- (d) Describe how to decode an encoding to recover such a string of length n that never has the same symbol appearing twice in a row, if it has been encoded using the method you described in part (c). Illustrate your description with an example.