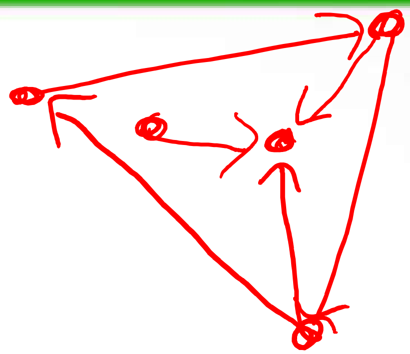
Introduction to Graphs

	Miles Jones	MTThF 8:30-9:50am	CSE 4140

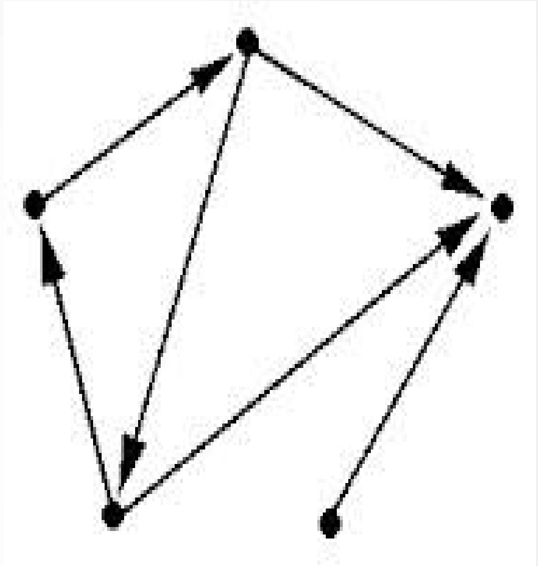
August 11, 2016

What is a graph?

(undirected)



A (directed) graph G is



- A nonempty **set of vertices** V, also called nodes

and

- A **set of edges** E, each pointing from one vertex to another (denoted with an arrow)

Variants of graphs

Rosen p. 644

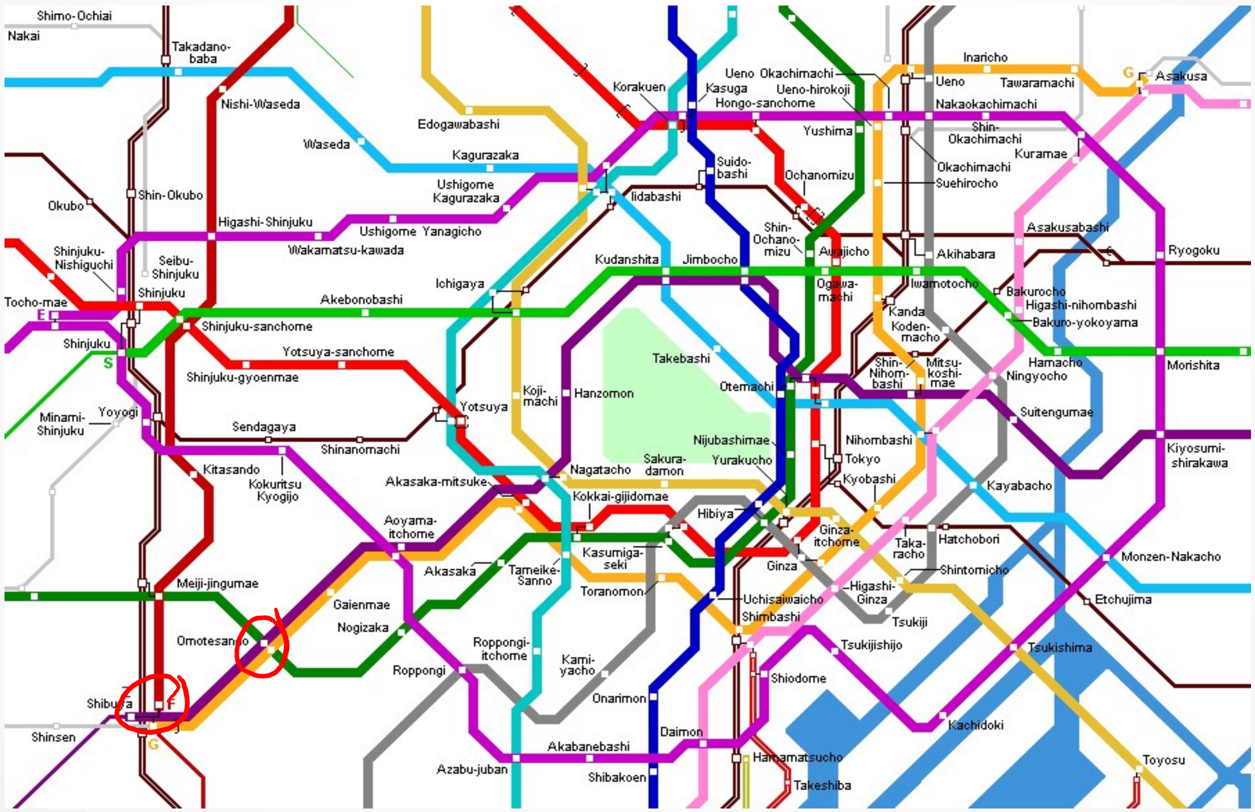
Undirected graph: don't need arrows on edges
if there's an edge from v to w then there's an edge from w to v.

Multigraph: undirected graph that may have multiple edges between a pair of nodes. Such edges are called *parallel* edges.

Simple graph: undirected graph with no self-loops (edge from v to v) and no parallel edges.

Mixed graph: directed graph that may have multiple edges between a pair of nodes and self loops.

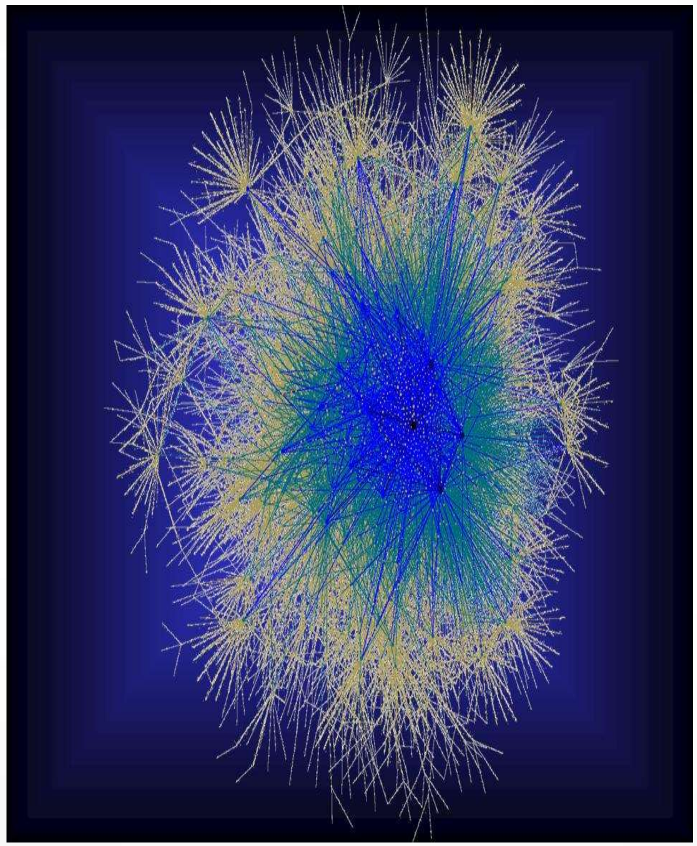
Graphs are everywhere



Graphs are everywhere

nodes
websites

edges
links.

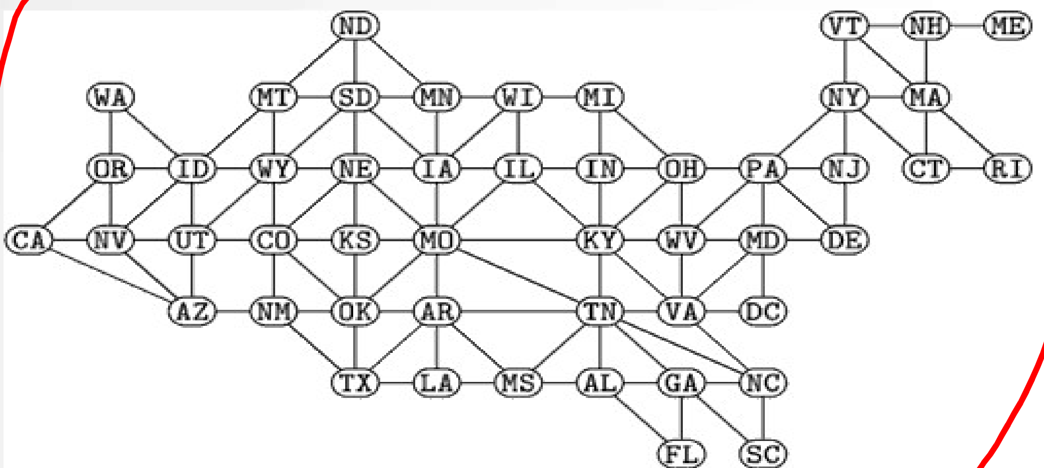


nodes
Computers

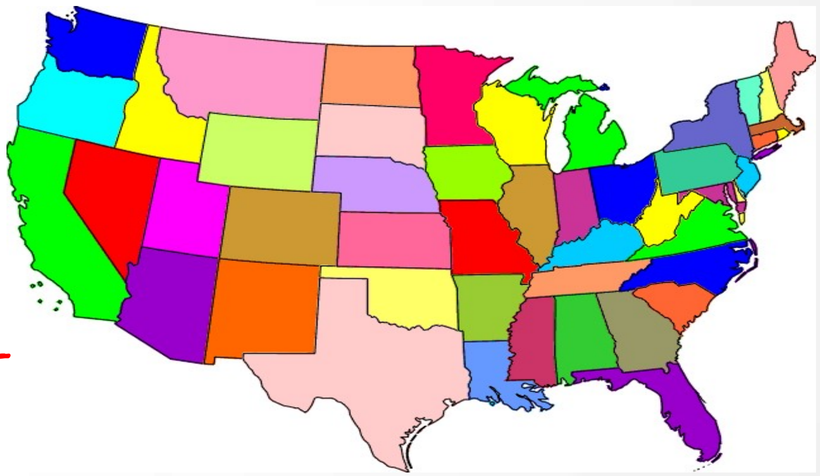
edges
connected by wire

The internet graph

Graphs are everywhere

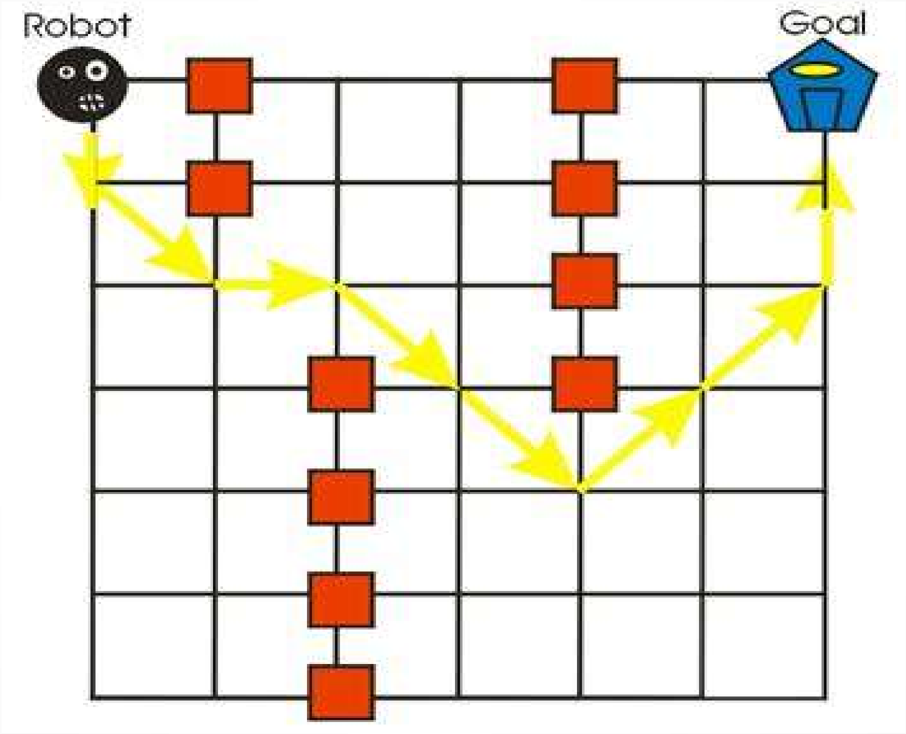


nodes: states
edges: borders.



Map coloring

Graphs are everywhere

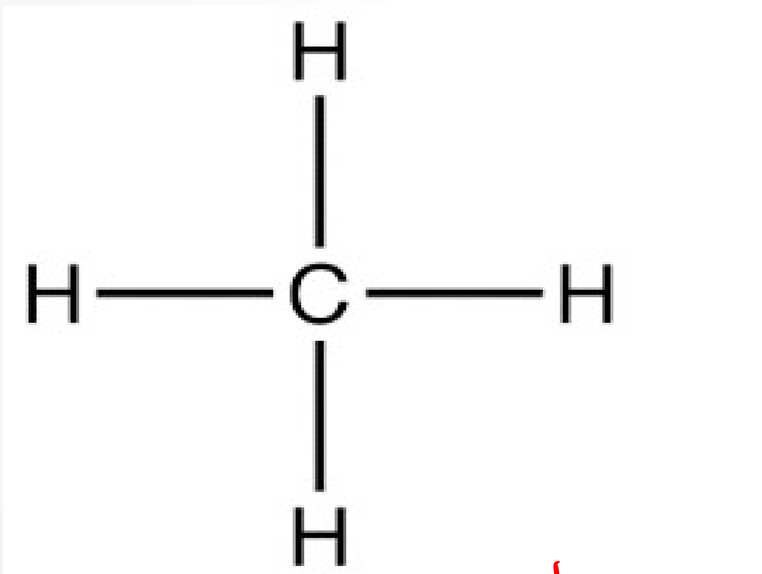


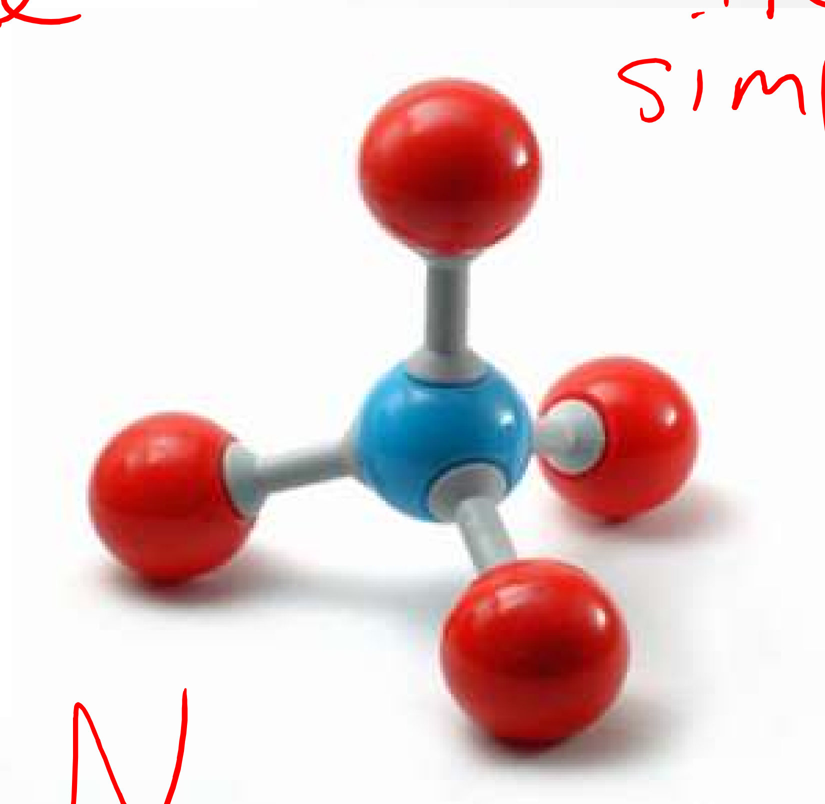
Path planning for robots

Graphs are everywhere

methane

undirected
simple

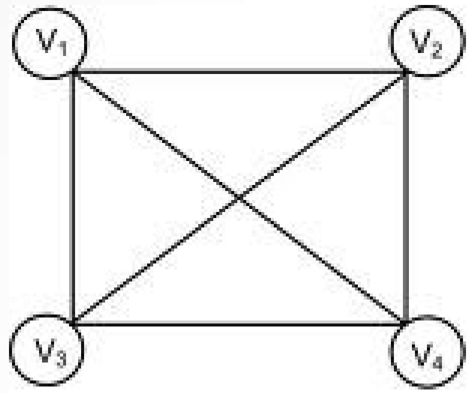


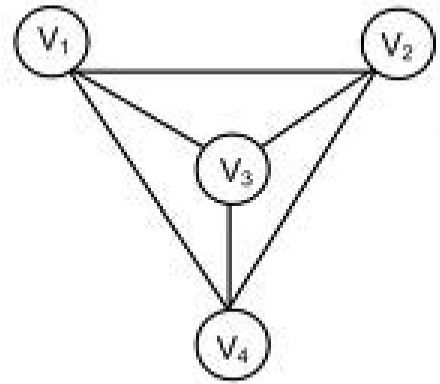


$O=O$

$N \equiv N$

Are these the same graph?





A. Yes: the set of vertices is the same.
B. Yes: we can rearrange the vertices so that the pictures look the same.
C. No: the pictures are different.
D. No: the left graph has a crossing and the right one doesn't.
E. None of the above.

Representing directed graphs

☺

Diagrams with vertices and edges

How many vertices?
For each ordered pair of vertices (v,w)
how many edges go from v to w?

in a simple graph
0 or 1

Representing directed graphs

directed graphs.

☺

Diagrams with vertices and edges

Simple graph

How many ordered pairs of vertices ~~are there~~? are possible

A. n
B. n(n-1)
C. n^2
D. n(n-1)/2 undirected
E. 2^n

How many vertices? **n**
For each ordered pair of vertices (v,w)
how many edges go from v to w?

Representing directed graphs

☺ Diagrams with vertices and edges

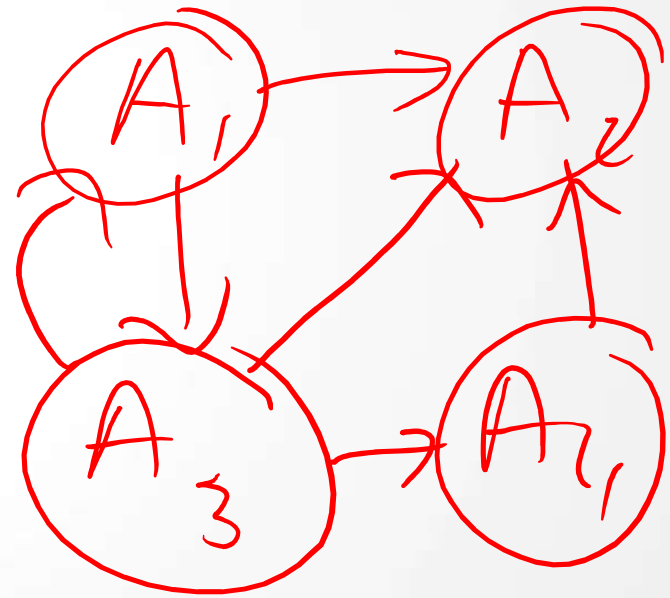
How many vertices? **n**
For each ordered pair of vertices (v,w)
how many edges go from v to w?
Need to store n(n-1) ints

Representing directed graphs

Adjacency matrix n x n matrix:

entry in row i and column j is the number of edges from vertex i to vertex j

$$
\begin{array}{c|cccc}
 & A_1 & A_2 & A_3 & A_4 \\
\hline
A_1 & 0 & 1 & 1 & 0 \\
A_2 & 0 & 0 & 0 & 0 \\
A_3 & 1 & 1 & 0 & 1 \\
A_4 & 0 & 1 & 0 & 0
\end{array}
$$



Rosen p. 669

Representing directed graphs

Adjacency matrix n x n matrix:

entry in row i and column j is the number of edges from vertex i to vertex j

What can you say about the adjacency matrix of a **loopless** graph?

A. It has all zeros.
B. All the elements below the diagonal are 1.
C. All the elements are even.
D. All the elements on the diagonal are 0.
E. None of the above.

Rosen p. 669

Representing directed graphs

Adjacency matrix n x n matrix:

entry in row i and column j is the number of edges from vertex i to vertex j

What can you say about the adjacency matrix of a graph with no **parallel** edges?

A. It has no zeros.
B. It is symmetric.
C. All the entries above the diagonal are 0.
D. All entries are either 0 or 1.
E. None of the above.

Rosen p. 669

Representing directed graphs

Adjacency matrix n x n matrix:

entry in row i and column j is the number of edges from vertex i to vertex j

What can you say about the adjacency matrix of an **undirected** graph?

A. It has no zeros.
B. It is symmetric.
C. All the entries above the diagonal are 0.
D. All entries are either 0 or 1.
E. None of the above.

Rosen p. 669

Representing undirected graphs

Simple undirected graph:

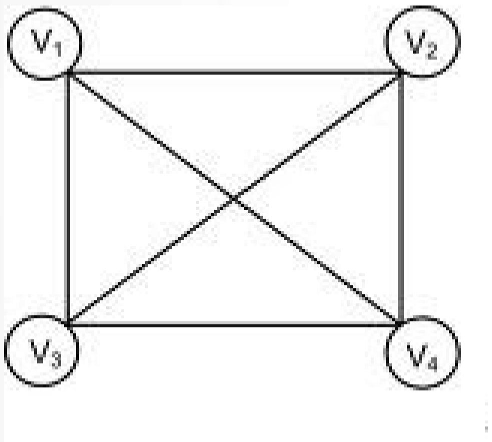
* Only need to store the adjacency matrix above diagonal.

What's the maximum number of **edges** a simple undirected graph with n vertices can have?

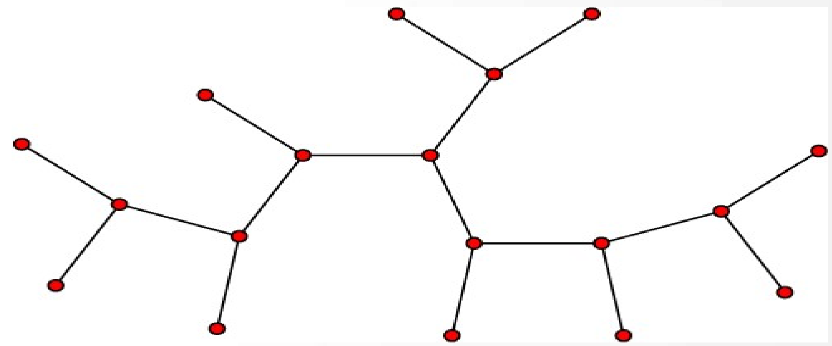
A. n^2
B. $n^2/2$
C. $n(n-1)/2$
D. $n(n+1)/2$
E. n

Efficiency?

When is an adjacency matrix an inefficient way to store a graph?



High density of
edges compared to
number of vertices



Low density of
edges compared to
number of vertices

Representing directed graphs

Adjacency list (list of lists):

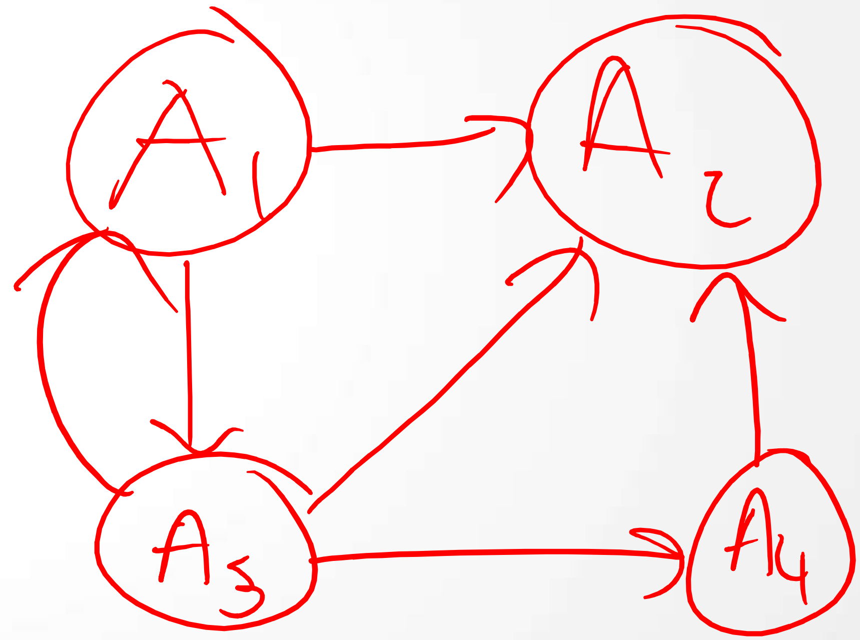
for each vertex v, associate list of all **neighbors** of v.

A_1 : A_2 , A_3

A_2 :

A_3 : A_1 , A_2 , A_4

A_4 : A_2

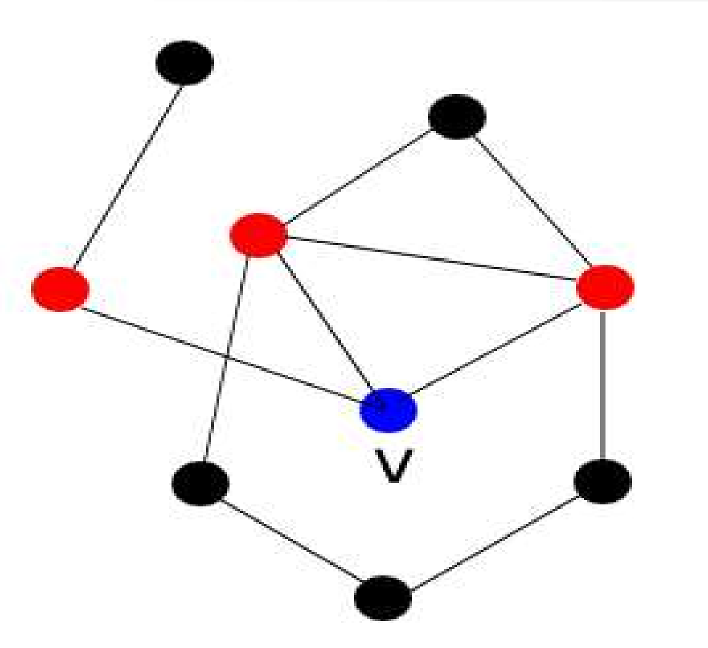


Rosen p. 668

Neighbors

The **neighbors** of a vertex v are all the vertices w for which there is an edge whose endpoints are v,w.

If two vertices are neighbors then they are called **adjacent** to one another.



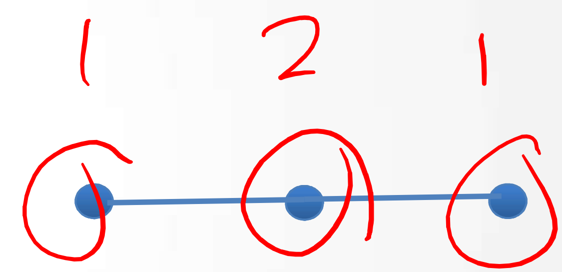
Rosen p. 651

Degree (undirected graph

The **degree** of a vertex in an undirected graph is the total number of edges **incident** with it, except that a loop contributes twice.

What's the maximum degree of a vertex in this graph?

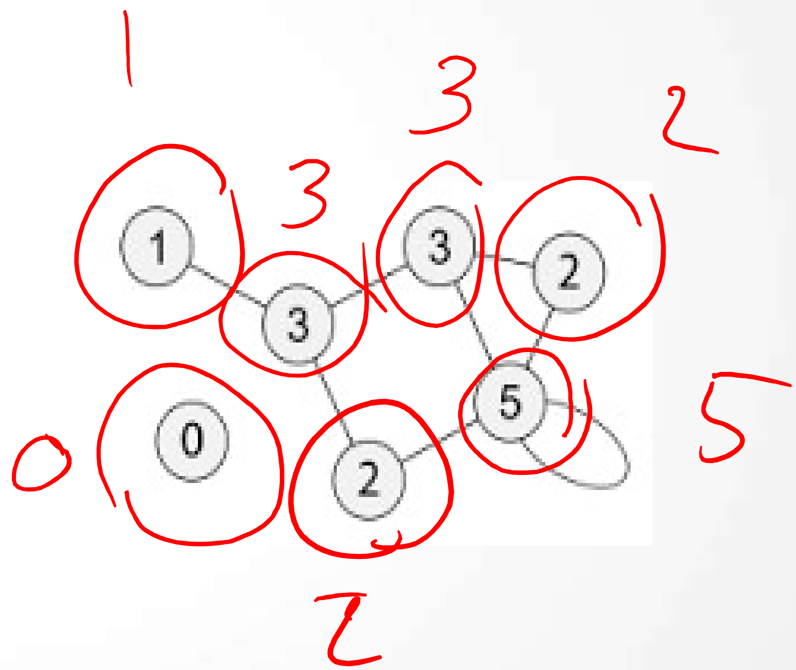
A. 0.
B. 1
C. 2
D. 3
E. None of the above.



Rosen p. 652

Degree

What's the degree of vertex 0?
A. 5
B. 3
C. 2
D. 1
E. None of the above.



Handshakes

If there are **n people** in a room, and each shakes hands with **d people**, how many handshakes take place?

A. n
B. d
C. nd
D. (nd)/2
E. None of the above.

Handshakes

If there are **n people** in a room, and each shakes hands with **d people**, how many handshakes take place?

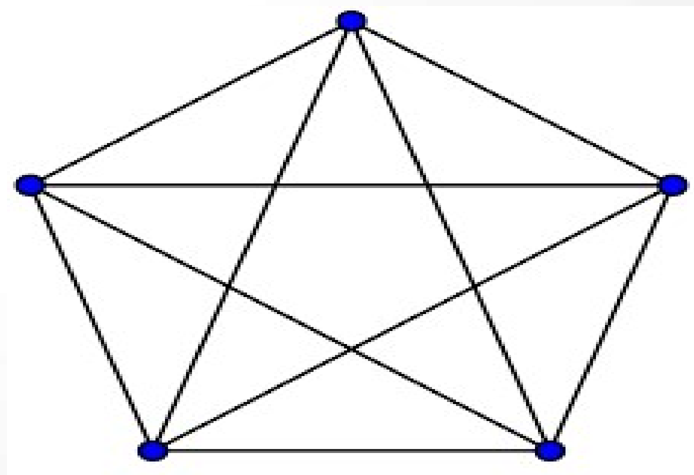
A. n
B. d
C. nd
D. (nd)/2
E. None of the above.

Don't double-count each handshake!

Handshakes "in" graphs

If a simple graph has **n vertices** and each vertex has **degree d**, how many edges are there?

$$2\,|E| = n*d$$

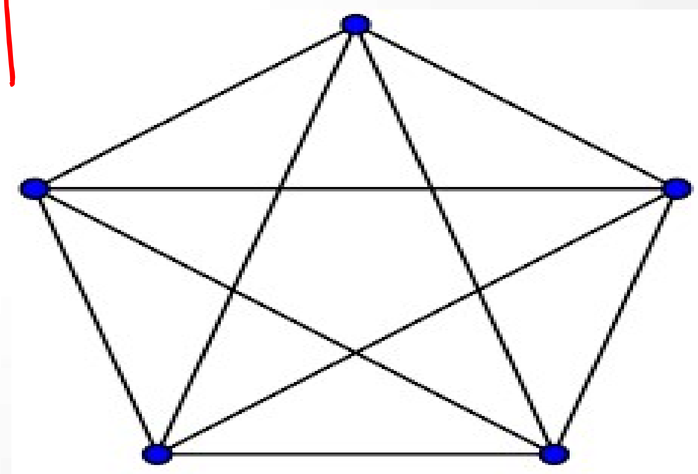


Handshakes "in" graphs undirected graphs

If any graph has **n vertices**, then

2 |E| = sum of degrees of all vertices

$$\sum_{v \in V} \deg(v) = 2|E|$$



Handshakes "in" graphs

If any graph has **n vertices**, then

$$\sum_{v \in V} deg(v) = 2|E|$$

2 |E| = sum of degrees of all vertices

What can we conclude?

A. Every degree in the graph is even.
B. The number of edges is even.
C. The number of vertices with odd degree is even.
D. The number self loops is even.
E. None of the above.