# Algorithms (CSL 351)

## Assignment-7

( SAHIL — 2016UCS0008)

Ans-1 There are 'n' pieces of the jigsaw puzzle. As we connect two pieces of the puzzle, the no. of remaining sections decrease by 1.

Thus, after $x$ moves, the no. of remaining sections $= n - x$

This does not depend on the order in which we connect the pieces.

Thus, to complete the puzzle,

$$n - x = 1$$

$$\Rightarrow \boxed{x = n - 1}$$

Thus, we require $(n-1)$ moves atleast. ~~at the~~

~~Ans-2~~ Algorithm –

```
numMoves = 0
while (puzzle not completed)
    - find two compatible pieces and
      connect them
    - numMoves += 1
print(numMoves)
```

Ans-2 (a) The seating plan can be modelled by the Hamiltonian circuit problem.
The graph can be modelled as follows, $G = (V, E)$ where
$V$ = students in the class, $|V| = 33$.
$E$ = set of edges where student $i$ & student $j$ are connected if they are not ~~frien~~ enemies.

(b) There does not always exist a seating plan.
According to Dirac's Theorem,
Let $G$ be a graph of $n$ vertices, $n \geq 3$
If the degree of every vertex of $G$ is greater than or equal to $n/2$, then $G$ is Hamiltonian.
So, the seating plan exists if each student does not hate atleast $33/2 = 16$ other students.

# Proof of Dirac's Theorem (reference taken from Internet)

Let $v_1, v_2, ---, v_t$ be a maximal path in $G$. Every vertex adjacent to $v_1$ must have been in the set $\{v_2, --, v_t\}$ by maximality. This means that $t-1 \geq deg(v_1) \geq n/2$, i.e. $t \geq \frac{n}{2}+1$.

Since $n \geq 3$, $t \geq 3$. Similarly, every vertex adjacent to $v_t$ must also be in $\{v_1, v_2, ---, v_{t-1}\}$.

→ Claim – $\exists\, k$, $1 \leq k \leq t-1$, such that $v_1$ is adjacent to $v_{k+1}$ and $v_t$ is adjacent to $v_k$.

• If not, since $v_1$ is adjacent to atleast $n/2$ other vertices, in $\{v_2, -- , v_t\}$, $v_t$ is also adjacent to atleast $n/2$ vertices in $\{v_1, ---, v_{t-2}\}$. Since the degree of $v_t$ is atleast $n/2$ and as stated before every vertex adjacent to $v_t$ must also be in $\{v_1, --, v_{t-1}\}$, we find that $t-1 \geq n/2 + n/2 = n$, i.e. $t \geq n+1$. This is impossible.

So, claim is true.

Claim – The cycle $C$: $v_1 v_{k+1} v_{k+2} --- v_t v_k v_{k-1} -- v_1$ is a Hamiltonian cycle.

Suppose $w$ is a vertex not in $C$. Since $C$ has $t \geq n/2+1$ vertices & $deg(w) \geq n/2$, $w$ is adjacent to atleast one vertex $v_s$ in $C$. Then $w, v_s$ and remaining vertices of $C$ in sequence define a longer path, contradicting maximality.

(C) Algorithm to solve the problem

1. Create the graph as describe in part (a).
2. ~~Try~~ Generate all possible seating arrangements : $\frac{(33-1)!}{2} = \frac{32!}{2}$

For each arrangement, ~~ch~~ check if there is an edge between two consecutive vertices of this configuration and there is an edge from the last vertex to the first.

If true, print this configuration.

3. If no such arrangement exists, print that not possible.

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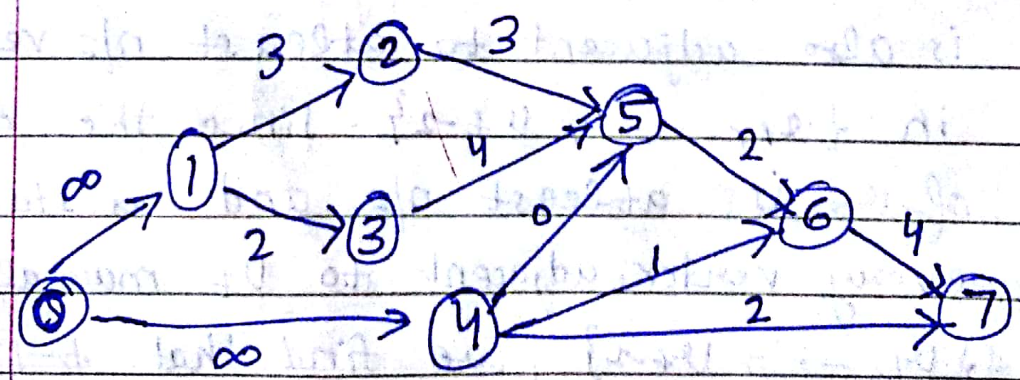
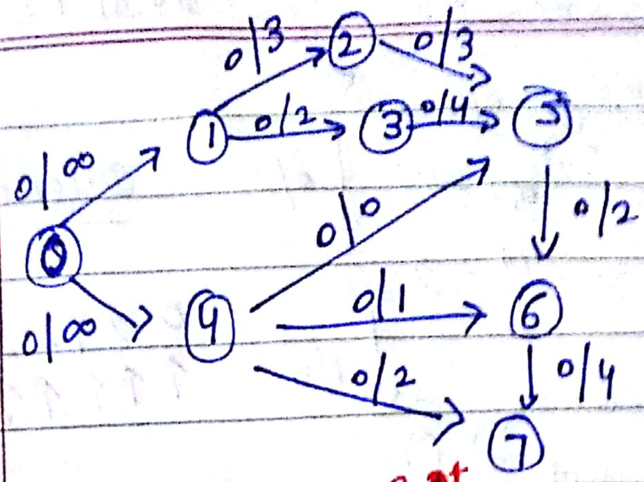
Ans-3




Since we have two sources, 1 & 4, we can assume a dummy source '0' with infinite capacity connected to 1 & 4.

Now, source is '0' & sink is 7.

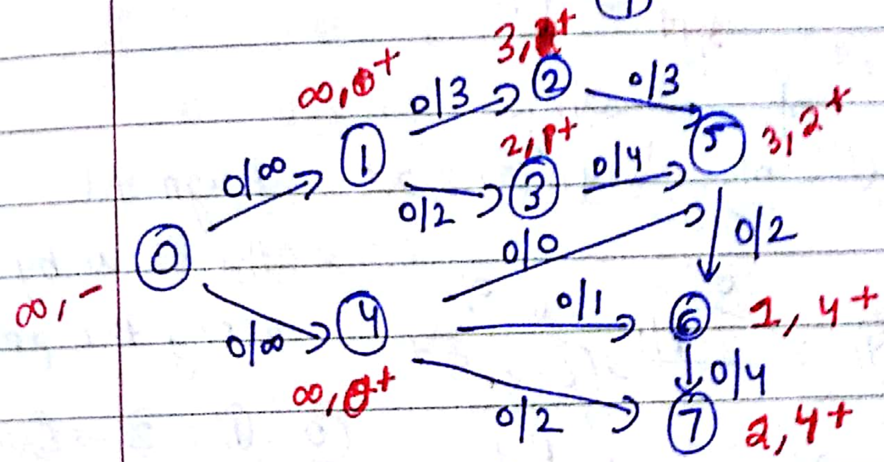


Queue:

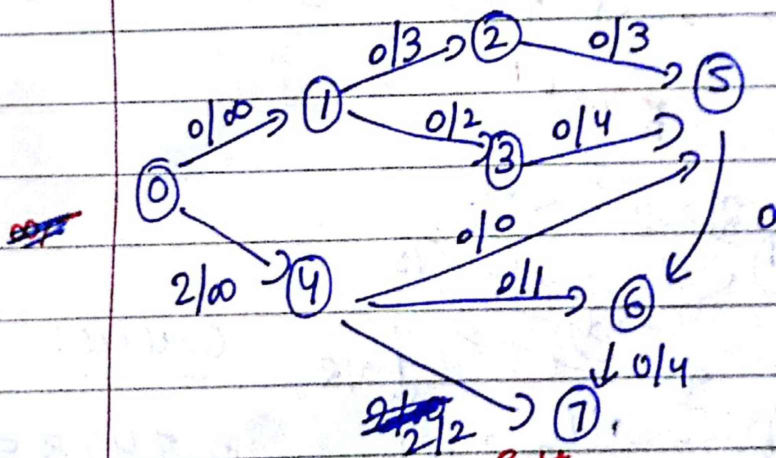
0 1 4 2 3 6 7 5

↑ ↑ ↑

not deleted from queue



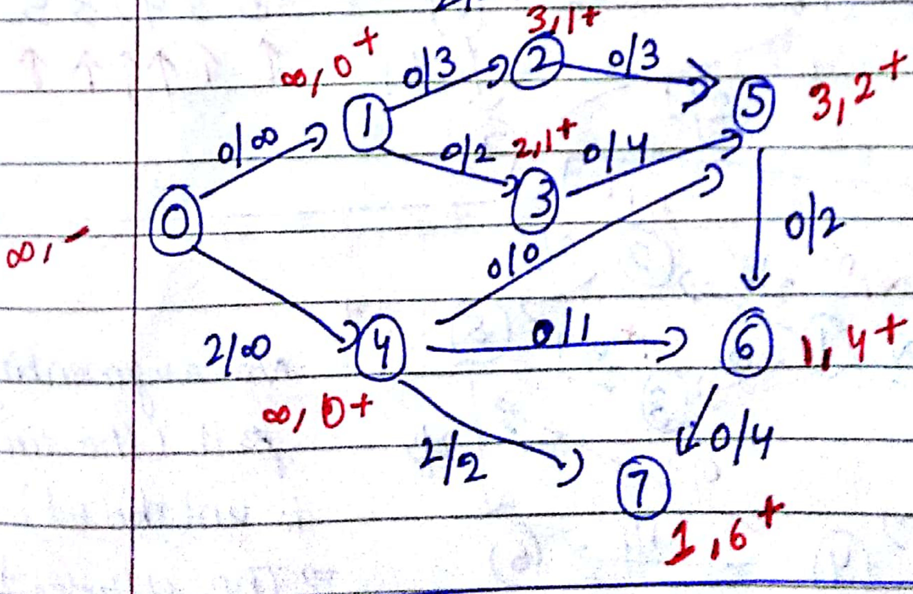
Augment the flow by 2, along 0→4→7



Queue:

0 1 4 2 3 6 7 5

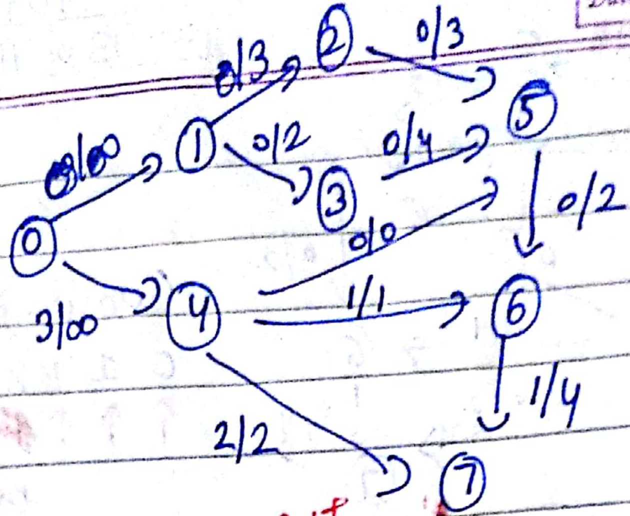
↑ ↑ ↑ ↑ ↑ ↑



~~No augmenting path (the sink is unlabeled)~~

Augment the flow by 1 along the path 0→4→6→7

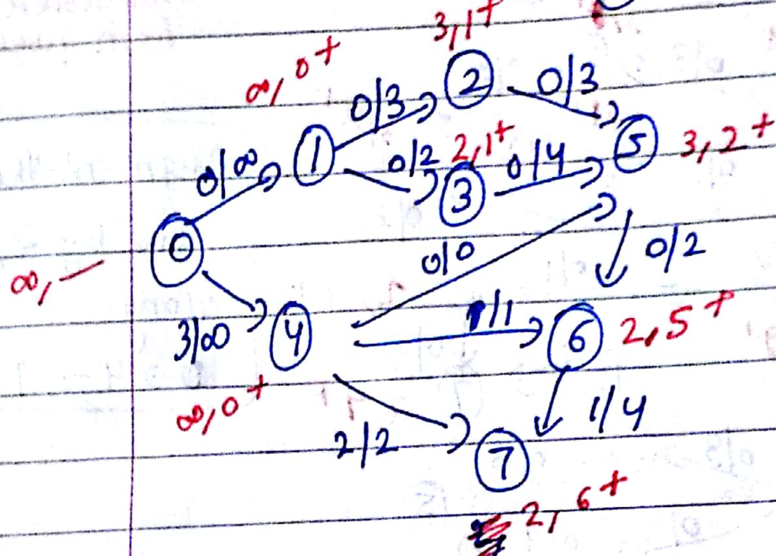
Date 30/11/18



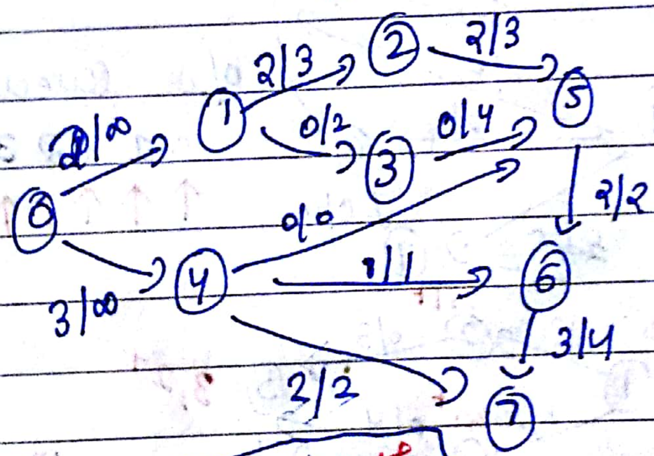

Queue:

0 1 4 2 3 5 6 7

↑ ↑ ↑ ↑ ↑ ↑ ↑




Augment the flow by 2 along the path $0 \to 1 \to 2 \to 5 \to 6 \to 7$

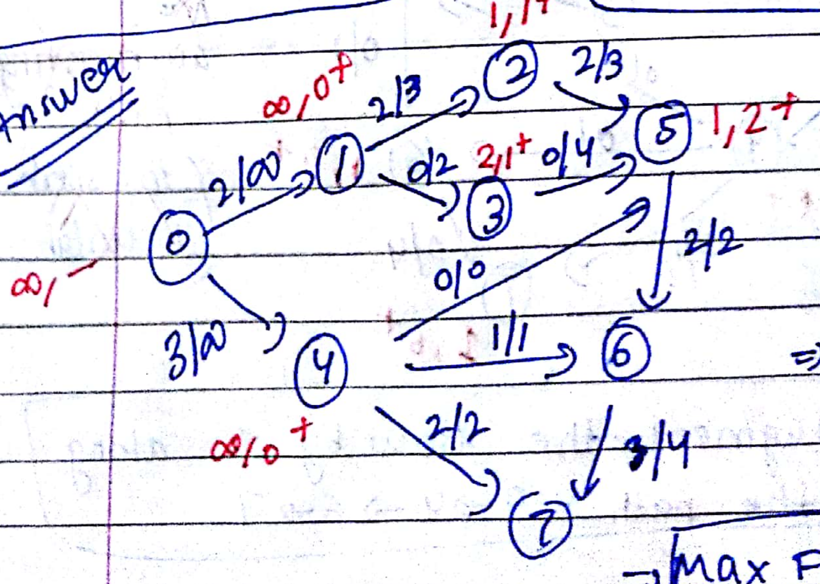



Queue:

0 1 4 2 3 5

↑ ↑ ↑ ↑ ↑ ↑

Answer

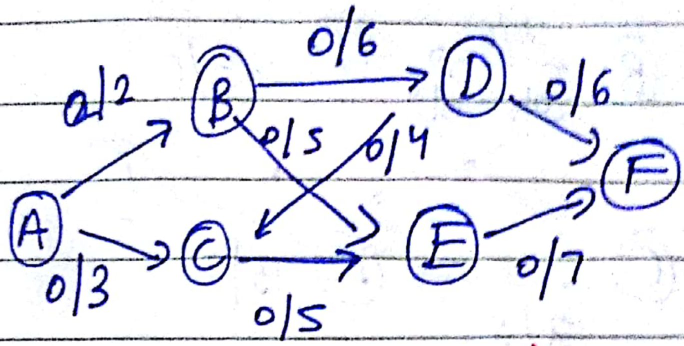



No augmenting path (the sink is unlabeled)

⇒ The current flow is maximal.

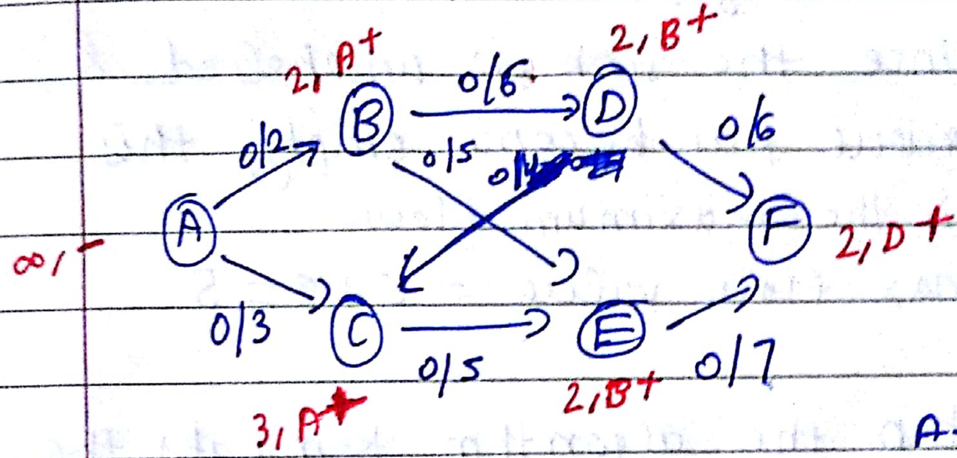
⇒ $\boxed{\text{Max Flow} = 5}$

Ans-4

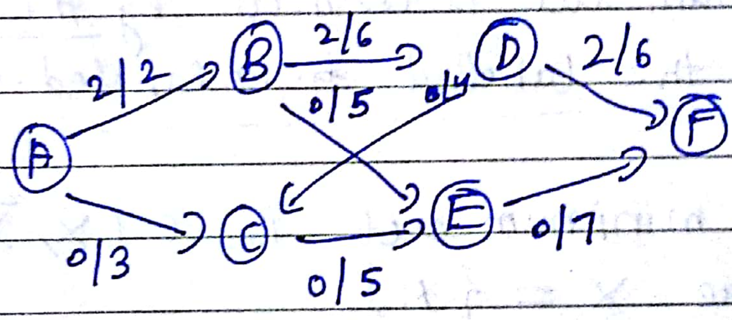



Queue:

A B C D E F

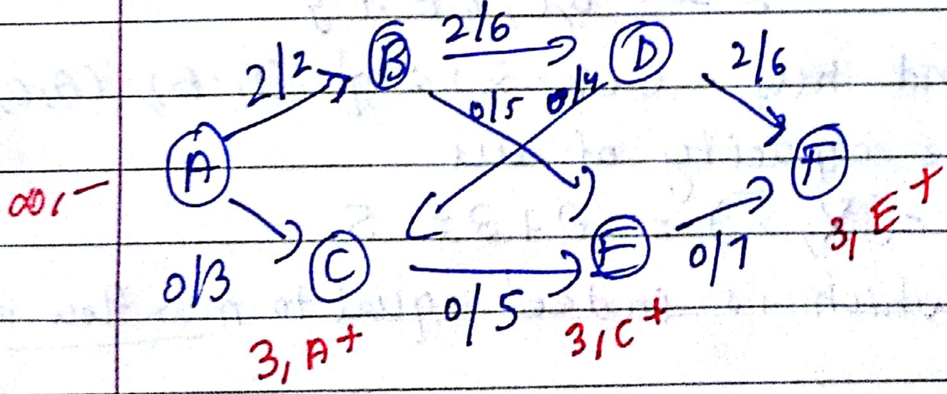



Augment the flow by 2 along the path A→B→D→F

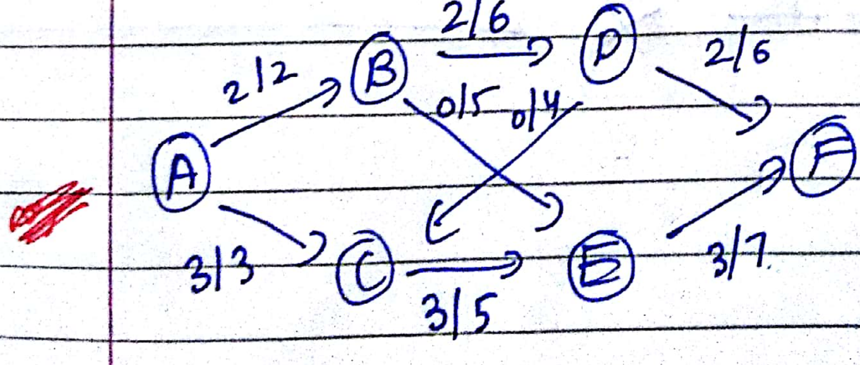



Queue:

A C E F

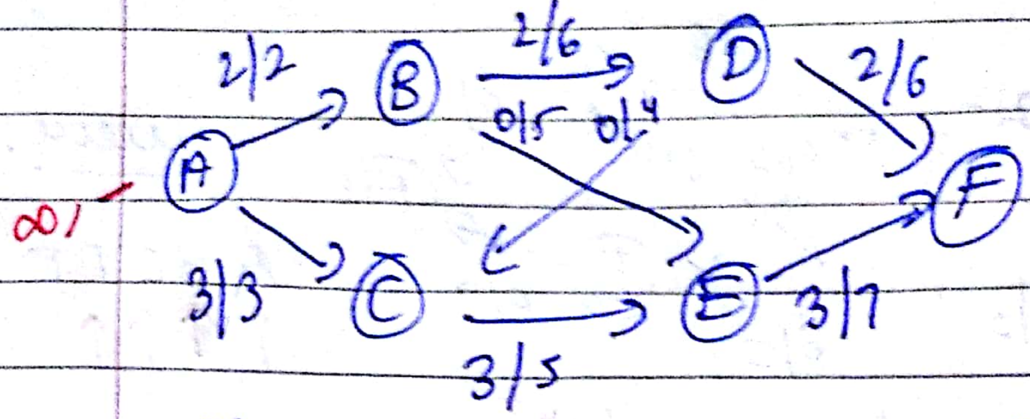



Augment the flow by 3 along the path A→C→E→F




Queue

A




Since the sink is unlabelled & queue has become empty, this is the maximum flow.

Max flow value $= 2+3 = 5$.

When the algorithm terminates, the minimum cut is formed by the edges from the labeled to unlabeled vertices.

Thus, minimum cut is $C(X, \bar{X})$

where $X = \{A\}$,

$\bar{X} = \{B, C, D, E, F\}$

and thus, $C(X, \bar{X}) = \{(A,B), (A,C)\}$

The capacity of cut

$c(X, \bar{X}) = 2+3 = 5$

which is indeed equal to max flow value.