1) $\begin{bmatrix}1\\2\end{bmatrix}$ Dimension = 2 $\in R^2$ $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}$ Dimension = 3 $\in R^3$

2) $\begin{bmatrix}8\\13\end{bmatrix} = 3\begin{bmatrix}2\\3\end{bmatrix} + 2\begin{bmatrix}1\\2\end{bmatrix}$

↳ linear combination of $\begin{bmatrix}2\\3\end{bmatrix}$ and $\begin{bmatrix}1\\2\end{bmatrix}$

3) Linear dependency is asked for a set of vector(s).

4) If atleast one vector of set can be represented as linear combination of others then set is linearly dependent.

5) If $w = 3u + \underline{0}v$ then v cannot be represented in terms of u and w using these coefficient. there may be other coef

6) A set containing zero vector is always LD.

7) If subset $\{u_1, u_2\}$ is LD then superset $\{u_1, u_2, u_3\}$ is also LD.

8) $c_1v_1 + c_2v_2 + c_3v_3 + \dots + c_nv_n = 0$

(Do you have atleast one $c_i \neq 0$)

- yes → LD (Non trivial sol also exist along with trivial sol)
- NO → LI (only one trivial sol, all $c_i = 0$) obvious

1) vector space: collection of vectors (must have a zero vector) Ex: R^2, R^3

2) Filling the vector space (span): Any 'n' LI vectors fills space of R^n. It means, any vector in R^n can be represented as linear combination of these 'n' LI vectors.

3) $\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$ is LI so it fills space of R^2. Its convenient vectors becos getting coefficient is so easy

$\begin{bmatrix}4\\-2\end{bmatrix} = 4\begin{bmatrix}1\\0\end{bmatrix} + (-2)\begin{bmatrix}0\\1\end{bmatrix}$

4) $\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\6\end{bmatrix}\right\}$ is LD so doesn't fills space of R^2. Some vectors in R^2 may be represented as linear combination of these two vector, but not all.

5) $a\begin{bmatrix}1\\2\end{bmatrix} + b\begin{bmatrix}9\\2\end{bmatrix}$ Its also a vector space filled by these 2 vectors.

6) Basis: Most efficient vector set that can span the space.

Ex: $\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}3\\5\end{bmatrix}\right\}$ These are 2 LI vectors ∴ It fills space of R^2.

Basis is $\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$

7) We can not have more than 'n' LI vectors in R^n.

use these intutions to test LD/LI:

- For 2 vectors check if 1 is multiple of other.
- If set has zero vector ⇒ LD
- If set has more than n vectors $\in R^n$ ⇒ LD

8) In a matrix 3×5, there are 5 vectors of R^3.

9) If there is just one vector in set $\{v\}$

- $v = 0$: In $cv = 0$, non zero c exists ∴ LD
- $v \neq 0$: In $cv = 0$, c has to be 0 LI

1) Matrix x Vector

$\begin{bmatrix}2 & 4\\3 & 5\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = a\begin{bmatrix}2\\3\end{bmatrix} + b\begin{bmatrix}4\\5\end{bmatrix}$

Linear combination of columns of matrix.

2) In $Ax = b$, b is linear comb. of columns of A and coefficient is from vector x.

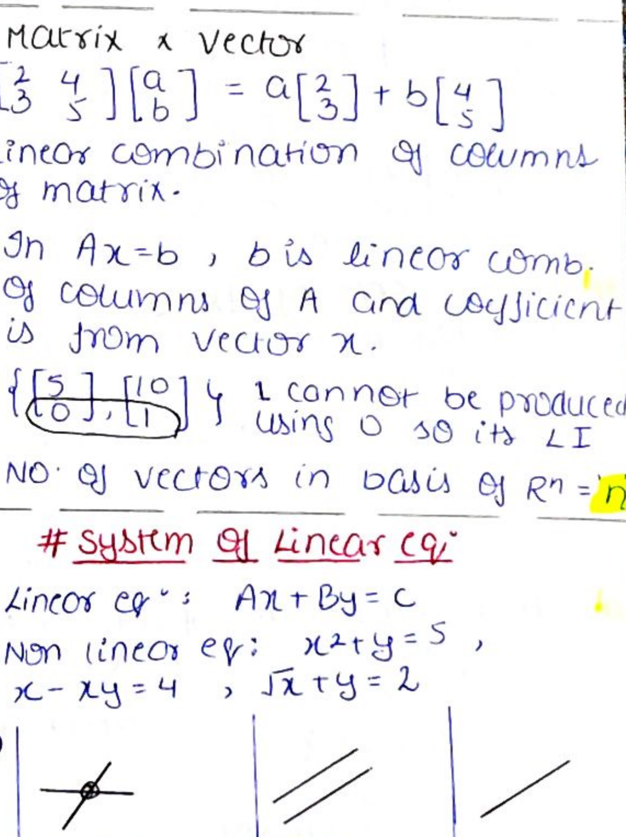
3) $\left\{\begin{bmatrix}5\\0\end{bmatrix}, \begin{bmatrix}10\\1\end{bmatrix}\right\}$ 1 cannot be produced using 0 so its LI

4) No. of vectors in basis of R^n = n

System of Linear eq

1) Linear eq: $Ax + By = C$

2) Non linear eq: $x^2 + y = 5$, $x - xy = 4$, $\sqrt{x} + y = 2$

3) 

Unique sol · NO sol · Infinite sol

4) $Ax = 0$ is always consistent as it has trivial sol $x = 0$.

$a_1x_1 + a_2x_2 + \dots = 0$

Is there any non zero x_i (Non trivial sol)

- yes → columns of A are LD
- NO → LI

5] Sol exist or not for $Ax=b$?
- If column vectors of A fills space then sol always exist.
- Otherwise it depends on b
  - if b is linear comb. of columns of A then sol exist
  - else no.

Multiplying 2 matrices :-

1) $[AB]_{12} = $ (1st row of A).(2nd column of A)

2) $\underset{A}{\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}} \underset{B}{\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 3 \\ 0 \end{bmatrix} & 2\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 0 \end{bmatrix} \end{bmatrix}$

3) $[A][B] = [C]$
If there are 2 LI vectors in A; C can have atmost 2 LI vectors as its the linear comb. of columns of A.

4) If system has m eq with n variables then matrix formed is $m \times n$.

5) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix}$ This is a way to generate rank 1 matrix (ie no. of LI vectors = 1)

6) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ col2 & col3 are redundant as it can be generated using col1.

Sol of $Ax=b$

$A_{m \times n}$ has m LI vectors (columns) ?
- yes → Sol always exist
- No → b is linear comb. of columns of A ?
  - yes → sol exist
  - No → No sol

b is linear comb. of col of A ?

- yes → Always a sol → col of A are LI ?
  - yes → unique sol
  - No → infinite sol
- No → Never a sol

1) Column space: linear combination of columns.

2) Echelon form / Row echelon form
- All non zero rows are above zero rows.
- All ele below leading entry in a column are 0.
- The leading entry of any row occurs to the right of leading entry of row above it.

3) Leading entry: 1st non zero ele of a row.

Trick: Make stair with leading entries, if its stair with a step in each row then its in echelon form.

4) Pivot column → LI col
free column → LD col on pivot

5) In echelon form, column having leading entry is called pivot (or Basic) variable / column.
Remaining are free var/column.

Ex: $x + 3y + z = 2$
$0x - 2y + 2z = 5$
$0x + 0y + 0z = 0$

$\begin{array}{ccc} x & y & z \end{array}$
$\begin{bmatrix} 1 & 3 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ in echelon form

x & y are pivot var/column
z is free var/column.

6) Augmented matrix $[A|b]$ line is optional
A is coefficient matrix.

7) Don't label b as pivot or free as its not associated with any var.

8) which are pivot & free in augm. matrix
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ don't consider b
pivot

9) Elementary row operation :-
- $R_i \leftrightarrow R_j$
- $R_i \to cR_i$, $c \neq 0$
- $R_i \to R_i + cR_j$

10) Just make sure that no info is lost by our row operation.

11) Gaussian Elimination
Matrix ⟶ Echelon form

Rank

1) Rank = # linearly independent columns = # linearly independent rows = # Non zero rows in echelon form = # pivot variables in echelon form.

2) Rank ≥ 0

3) Rank is zero only for NULL (zero) matrix.

Is not true for $Ax=b$ becoz b also need to be changed accordingly.

4) To find rank, convert matrix to echelon form & find rank.

5) Nullity: No. of free variables.

n columns in coeff matrix

Pivot var (r) — Rank

Free var ($n-r$) — Nullity

6) Total no. of variables (columns) = Rank + Nullity.

7) System of linear eqn:-
- Homogeneous $Ax=0$
- Non Homogeneous (Heterogeneous) $Ax=b$

8) To solve system of linear eqn:- Find Augmented matrix → convert A into echelon form → form equations and solve.

Number of soln:

Find echelon form of A in [A|b]

[0 0 0 0 | Nonzero] exist?

- yes → No soln (Inconsistent)
- No → consistent
  - Free var exist?
    - yes → Infinite soln
    - No → Unique soln

9) For infinite sol:

No. of LI soln = No. of free var in A
= Nullity of A
= No. of parameters in soln

*Take free var as some parameter.

10) $\text{rank}(A_{m\times n}) \leq \min(m,n)$

11) when we add vector b to A

- b is linear comb. of columns of A
  ↓
  b is LD on coln of A
  ↓
  $\text{Rank}(A) = \text{Rank}(A|b)$
  - $\text{Rank}(A) = \text{Rank}(A|b) < n$ (Free var exist) Infinite soln.
  - $\text{Rank}(A) = \text{Rank}(A|b) = n$ (No free var) Unique soln.
- b is not linear comb. of coln of A
  ↓
  b is LI
  ↓
  $\text{Rank}(A|b) = \text{Rank}(A)+1$
  ↓
  $\text{Rank}(A|b) \neq \text{Rank}(A)$
  No Solution

* col dependency in matrix, echelon, reduced echelon form is same as all r obtained via elementary row op.

12) Elementary row operations does not change linear dependency between columns.

$A \xrightarrow{\text{unknow ele. row operation}} B$

$C_3 = C_1 + 2C_2$ $\qquad$ $C_3 = C_1 + 2C_2$

13) Ele. row operations preserve row space but not column space

14) $a\begin{bmatrix}1\\2\\0\end{bmatrix} + b\begin{bmatrix}3\\4\\1\end{bmatrix}$ → Dimension of vector space = No of LI vector = 2

→ dimension of vector = 3

1) Row reduced echelon form or Reduced echelon form:
It has these additional properties:
- Each leading entry is 1
- All ele above & below leading entry is 0.

2) converting into reduced ech. form:
- convert to echelon form
- Make leading entry 1
- Make all ele above leading entry 0.

3) Adv: Pivot columns are convenient vectors so finding coeffs is easy.

4) Echelon form of a matrix is not unique whereas reduced echelon form is unique.
dependency b/w column

5) $Ax=0$ has "only trivial soln" means system has unique soln.

Determinant

- Here all matrices are square mat.

1) det(A) or $|A|$ is scalar.

2) $|I| = 1$

3) det. changes sign when two rows (or two columns) are interchanged

4) Linearity is applied on a row or column at a time.

$$\begin{vmatrix} a+b & c+d \\ e & f \end{vmatrix} = \begin{vmatrix} a & c \\ e & f \end{vmatrix} + \begin{vmatrix} b & d \\ e & f \end{vmatrix}$$

$$\begin{vmatrix} ka & kb \\ c & d \end{vmatrix} = k\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

5) $|A+B| \neq |A| + |B|$

6) If any 2 rows (or columns) are equal/proportional then det is 0.

7) $R_i \to R_i \pm cR_j$
det remains unchanged.

8) whatever you do with rows is also applicable for columns in det.

9) Matrix with row or column of zeroes has det 0.

10) Det of diagonal/triangular matrix is product of diagonal elements.

11) Finding det using permutation
Take all perm (n! terms); such that each row & each column has 1 nonzero ele. then make it into diagonal matrix & take product of diagonal ele.

12) using it we can find sign of terms
Ex: $a_{13}\, a_{22}\, a_{34}\, a_{41}$
$3241 \to 1243 \to 1234$
2 swaps $\Rightarrow (-1)^2 \Rightarrow +ve$

13) Calculate determinant :-
- using permutation
- using co-factor
- convert into echelon form (triangular matrix), take product of diagonal elements.

14) Minor : M_{ij} : leave ith row and jth column, take determinant.

15) Cofactor : $C_{ij} = (-1)^{i+j} M_{ij}$

16) Adjoint : transpose of cofactor element matrix.

17) Determinant is sum of product of any row (or column) ele with its cofactor.

18) C_{ij} do not depend on ele at position (i,j).

19) If ele of a row or column is multiplied by cofactor of any other row or column, its Σ is 0.
Ex: $a_{21}C_{11} + a_{22}C_{12} + a_{23}C_{13} = 0 \Rightarrow \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

20) $A^{-1} = \frac{1}{|A|} adj(A) \Rightarrow A \cdot adjA = |A| \cdot I$

21) If A is singular (ie $|A| = 0$) then A^{-1} does not exist.

22) $AA^{-1} = A^{-1}A = I$

23) For square matrix (Both r inverse of each other.)
$AB = I \Leftrightarrow BA = I$

Trick to find adjoint :-

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \Rightarrow \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \quad \begin{matrix} 5 & 8 & 2 & 5 \\ 6 & 9 & 3 & 6 \\ 4 & 7 & 1 & 4 \\ 5 & 8 & 2 & 5 \end{matrix} \quad \begin{bmatrix} -3 & - & - \\ 6 & - & - \\ - & - & - \end{bmatrix}$$

24) Det evaluated across any row or column is same.

25) $|A^T| = |A|$

26) $|AB| = |B||A| = |A||B|$

27) $|A^n| = (|A|)^n$

28) $|A^{-1}| = \frac{1}{|A|}$

29) $A \cdot (adjA) = (adjA) \cdot A = |A| \cdot I$

30) $adj(AB) = adj(B)\, adj(A)$

31) $|adj A| = |A|^{n-1}$

32) $|adj(adj A)| = |A|^{(n-1)^2}$

33) singular matrix : $|A| = 0$
Non " " : $|A| \neq 0$

34) A is invertible if $|A| \neq 0$ i.e A^{-1} exist

35) cramers rule : used to solve system of linear eq^n. $TC = O(n^3)$
- condition: $|A| \neq 0$ ie A^{-1} exist ie unique sol^n exist

$$x = \frac{1}{|A|}\begin{bmatrix} |A_1| \\ |A_2| \\ |A_3| \end{bmatrix} \qquad \begin{matrix} Ax = b \\ x = A^{-1}b \end{matrix}$$

$$A_1 = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix}, A_2 = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}, A_3 = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

36) System of linear eq^n can also be solved as $x = A^{-1}b$, using echelon form

* To find 3rd column of A^{-1}
$A^{-1}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = x \Rightarrow Ax = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ solve for x

* Methods to solve system of linear eq^n
i) using echelon form
ii) $x = A^{-1}B$
iii) using cramers rule

Eigen value & Eigen vector

- We will talk about square matrix.

1) When we multiply a vector with matrix and we get vector in same direction then its eigen vector.

2) $Ax = \lambda x$ — x: Eigen vector, λ: Eigen value (scalar)

3) $\lambda = 1$ No change
   $\lambda < 1$ Shrink
   $\lambda > 1$ Stretched

4) λ can be 0 but eigen vector can't be zero vector.

5) If x is eigen vector of A then kx ($k \neq 0$) is also eigen vector of A. Eigen value is same for both. (∞ Evectors corresponds to each Evalue.)

6) Every matrix has infinitely many eigen vector, and atmost n LI Evector. (n×n matrix ie R^n space)

7) Calculating Evalue & Evector:
   - Find Evalues from $|A - \lambda I| = 0$ ← char eq
   - Find Evectors from $(A - \lambda I)x = 0$ — col r LD bcoz $x \neq 0$ ∴ $|A - \lambda I| = 0$

8) Every square matrix of degree n×n has n Evalues. (may be repeated)

9) If matrix is of order n×n then char eq is of degree n.

10) $\lambda_1 \neq \lambda_2 \Rightarrow e_1$ & e_2 are LI
    $\lambda_1 = \lambda_2 \Rightarrow$ If λ is repeating twice ($\lambda = 4, 4$) then we may get atmost two LI Evectors.

11) $(\lambda-1)^2(\lambda-5)(\lambda-7)^3$ — Matrix must be of order 6×6
    # Evectors LI: atleast 3, atmost 6

12) $(\lambda - \lambda_1)^m$
    Algebraic mul.: (m) no of times λ_1 is repeated.
    Geometric multiplicity: No of LI Evectors corresponding to λ_1

13) AM ≥ GM

14) For $A_{n \times n}$, n Evalues exist and atmost n LI Evectors.

* orthogonal ⇏ linearly independent

15) Real symmetric matrix always has n linearly inde. Evectors even if eigenvalues are repeating. ie. AM = GM

16) Real sym matrix has:
   - n real Evalue.
   - n orthogonal (LI) Evectors.

17) LI Evectors of sym. matrix are orthogonal, when we put its unit vector in matrix, its orthogonal matrix

18) Diagonalizable matrix ↔ has n LI Evectors.

19) Symmetric matrix is diagonalizable

20) Determinant of any matrix is product of eigen values.

21) Trace (sum of principle diagonal ele) = sum of eigen values.

22) If rows/columns of matrix are dependent ie. $|A| = 0$ then atleast one eigen value is 0.

Evalue of A = 0?
- yes → Rank(A) < n. No. of LI vectors corresponding to $\lambda = 0$ = Nullity of A.
- NO → Rank = n

* If no λ is 0 means $|A| \neq 0$ ⇒ rank(A) = n

23) $(A - \lambda I)x = 0$
    It always has free var as $|A - \lambda I| = 0$ ∴ Infinite Evectors ∀ λ.

24) No. of LI Evectors corresponding ($\lambda = 0$) = Nullity of A.

25) Cayley Hamilton theorem: Every square matrix satisfies its own characteristics eq.
   - Used to find A^{-1} and A^k

NOTE: Elementary row opr do not preserve Evalues, so never apply row opr in order to find Evalues

26)

	AB	BA
Evalue:	λ	λ ($\lambda \neq 0$)
Evector:	x	Bx

27) AB and BA shares non-zero Evalues. (copy nonzero Evalues & fill remaining with 0's)

7

→ Non zero Evalues of A^TB & BA^T same

28) If $A_{4\times3}$, $B_{3\times4}$ then there can be atmost 3 nonzero Evalues in AB & BA as AB & BA shares nonzero Evalues.

29) If Evalue of A is λ then Evalue of A^K is λ^K. (Evector same 'x' in point 29,30,31)

30) If Evalue of A is λ then Evalue of $A^K+3A^{K-1}+I$ is $(\lambda^K+3\lambda^{K-1}+1)$.

31) If Evalue of A is λ then Evalue of A^{-1} is $1/\lambda$.

32) If $(a+bi)$ is Evalue of A then $(a-bi)$ is also can Evalue of A.

33) If A is a column vector then
$A^TA = |A|^2$
If $A=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $|A|=\sqrt{1^2+2^2+3^2}$ Magnitude/length or Euclidean norm

NOTE: If Evector is of dimension 2x1 then matrix A is of " 2x2

34) If $\lambda=0 \Rightarrow |A|=0 \Rightarrow A^{-1}$ does not exist
If $\lambda\neq0 \Rightarrow |A|\neq0 \Rightarrow A^{-1}$ exist.

⇒ LU Decomposition:

- Matrix 'A' can be decomposed only if A can be reduced to echelon form without row interchange.

$$A = \begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix}\begin{bmatrix}U_{11}&U_{12}&U_{13}\\0&U_{22}&U_{23}\\0&0&U_{33}\end{bmatrix} \Rightarrow A=LU$$
L U

we may have 1's in diagonals of L or U.

Method1: Make eq^n from above & solve.

Method2: $A=\begin{bmatrix}1&4&-3\\-2&8&5\\3&4&7\end{bmatrix}$ $L=\begin{bmatrix}1&0&0\\-2&1&0\\-&-&1\end{bmatrix}$

convert A to echelon form

$R_2 \to R_2 + 2R_1$

it become $\begin{bmatrix}1&4&-3\\0&16&-1\\3&4&7\end{bmatrix}$ 0 so add -2 into L

similarly, keep filling L and A will be reduced to U.

* Solving system of linear eq^n $Ax=b$
- Decompose $A=LU$ then $LUx=b$
- Let $Ux=y$ then $Ly=b$
  Solve it to get y.
- Solve $Ux=y$ to get x.

NOTE: Eigen values of diagonal/triangular matrix are diagonal elements.

Types of Matrices:

1) Identity matrix (I): Square matrix with
$a_{ij} = \begin{cases}1 & \text{if } i=j\\0 & \text{else}\end{cases}$

* Inverse:
- $A^{-1}A = AA^{-1} = I$
- $(A^{-1})^{-1} = A$
- $(KA)^{-1} = \frac{1}{K}A^{-1}$, $K\neq0$
- $(AB)^{-1} = B^{-1}A^{-1}$
- For square matrix $AB=I \Leftrightarrow BA=I$

NOTE: AB can be zero even though none of A, B is zero. (then $|A| = |B| = 0$)

* Transpose: $a_{ij} \leftrightarrow a_{ji}$
- $(A^T)^T = A$
- $(A+B)^T = A^T+B^T$
- $(AB)^T = B^TA^T$
- $(CA)^T = CA^T$
- $(A^T)^{-1} = (A^{-1})^T$

2) Lower Δlar matrix: all ele above diagonal are 0.
Upper Δlar matrix: all ele below diagonal are 0.
(sq matrix)

3) Diagonal matrix: need not be sq
$a_{ij} = \begin{cases}0 & \text{if } i\neq j\\ \text{otherwise anything}\end{cases}$

4) Symmetric matrix: (sq matrix)
$A^T = A$

5) Skew symmetric matrix: (sq)
$A^T = -A$
- Diagonal ele are 0.

6) For any sq matrix A^TA and AA^T are symmetric.

7) Any sq real matrix can be represented as sum of symmetric & skew sym matrix
$$A = \underbrace{\frac{1}{2}(A+A^T)}_{\text{symmetric}} + \underbrace{\frac{1}{2}(A-A^T)}_{\text{skew symmetric}}$$

8) Orthogonal vectors: (⊥ vector)
$a\cdot b = |a||b|\cos\theta = |a||b|\cos 90 = 0$

9) Orthonormal vectors:
Orthogonal vectors having unit length.

10) Orthogonal matrix: sq matrix whose rows & columns are orthonormal vectors.

- $A^TA = AA^T = I$
- $A^{-1} = A^T$

11) Idempotent matrix: $A^2 = A$ (sq)

- $A^n = A \quad \forall n > 0$

NOTE: $(a^b)^c = (a^c)^b = a^{bc}$

$a^{b^c} = a^{(b^c)}$

12) Involutary matrix: $A^2 = I$ (sq)

- $A^{-1} = A$
- If A is involutary then A^k is involutary.

13) Nilpotent matrix: sq matrix such that $\exists k$ such that $A^k = 0$

- min value of k is called index.

14) Hermitian matrix: $A^\theta = A$ (sq)

* Tranjugate = Transpose & conjugate (A^θ or A^* or A^H)

- $A^\theta = (\bar{A})^T = (\overline{A^T})$ $\quad |A^\theta| = |A|$
- $(A^\theta)^\theta = A$
- $A^\theta = A^T$ for real matrix
- $(A+B)^\theta = A^\theta + B^\theta$
- $(AB)^\theta = B^\theta A^\theta$
- $(A^\theta)^{-1} = (A^{-1})^\theta$
- $(CA)^\theta = \bar{C} A^\theta$, c is complex no.
- Diagonal ele are purely real nos.

15) Skew Hermitian matrix: $A^\theta = -A$

- Diagonal ele are purely imagenary or 0.

16) unitary matrix: $AA^\theta = I$

NOTE: If A is symm matrix then all its eigen vectors are orthogonal. False

- All LI Evectors are orthogonal.
- If x is EV then kx is also EV but both are not orthogonal.

Q: No. of matrix idempotent & invertible

$A^2 = A \Rightarrow A = AA^{-1} \quad AA^{-1} = I$

$\Rightarrow A = I$ only 1 matrix.

* $A_{3\times2}$ & $B_{2\times3}$ then $|AB|$ must be 0

↳ atmost 2 LI vectors so $AB_{3\times3}$ can have atmost 2 LI " $\Rightarrow |AB| = 0$

Q: can a symmetric matrix have $A^2 = -I$

$A^2 + I = 0 \Rightarrow \lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i$

But sym matrix has real eigen values

$\therefore$ Not possible.

NOTE: $A^{2024} B^{2023} v$

In such ques just check v is eigen vector of matrix A & B with eigen value λ_1 & λ_2 respectively.

so $\Rightarrow \lambda_1^{2024} \lambda_2^{2023} v$