In Earlier Example we had one Dependent variable ie y and one Independent variable ie x

So it was Case of Simple Linear Regression.

Ex sqftarea → price ⇒ $\hat{y} = \beta_0 + \beta_1 x$

But if

x_1 Size (feet)^2
x_2 No of Bedrooms
x_3 No of floor
x_4 location

→ Price y

← multiple linear Regression.

The linear Regression to express the above dependency is

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$$

General form of multiple linear Regression.

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n$$

Here $\beta_0, \beta_1, \beta_2 \ldots \beta_n$ < +ve OR -ve

$$h_\theta(x) = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n$$

$x_0 = 1$

Here $x_0, x_1, x_2 \ldots x_n$ = features

$\beta_0, \beta_1, \beta_2 \ldots \beta_n$ ⇒ Parameter / coefficients.

let X = Feature Vector = $\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$ $n+1 * 1$

n = no of features

θ = Parameter Vector = $\begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix}$ $n+1 * 1$

$$\therefore h_\theta(x) = \theta^T \cdot X = \underbrace{\begin{bmatrix} \beta_0 & \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix}}_{\theta^T} * \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} X$$

$$h_\theta(x) = \theta^T X = \beta_0 x_0 + \beta_1 x_1 + \cdots + \beta_n x_n.$$

let

x_0	X_1	X_2	y
1	7	2.6	78.5
1	1	2.9	74.3
1	11	5.6	104.3
1	11	3.1	87.6
1	7	5.2	95.9
1	11	5.5	109.2
1	3	7.1	102.7

3 2 2

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

To find β_0
β_1
β_2

Solve →

Feature Vector = $X = \begin{bmatrix} 1 & 7 & 2.6 \\ 1 & 1 & 2.9 \\ 1 & 11 & 5.6 \\ 1 & 11 & 3.1 \\ 1 & 7 & 5.2 \\ 1 & 11 & 5.5 \\ 1 & 3 & 7.1 \end{bmatrix}_{7\times3}$ & $y = \begin{bmatrix} 78.5 \\ 74.3 \\ 104.3 \\ 87.6 \\ 95.9 \\ 109.2 \\ 102.7 \end{bmatrix}_{7\times1}$

$$C = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}_{3\times1} = (X^T \cdot X)^{-1} \cdot X^T \cdot y$$

↑ coffiient Vector

Solution

$$X = \begin{bmatrix} 1 & 7 & 2.6 \\ 1 & 1 & 2.9 \\ 1 & 11 & 5.6 \\ 1 & 11 & 3.1 \\ 1 & 7 & 5.2 \\ 1 & 11 & 5.5 \\ 1 & 3 & 7.1 \end{bmatrix}$$

(1) find $X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 7 & 1 & 11 & 11 & 7 & 11 & 3 \\ 2.6 & 2.9 & 5.6 & 3.1 & 5.2 & 5.5 & 7.1 \end{bmatrix}$

(2) Find $X^T \cdot X \Rightarrow \begin{bmatrix} 7 & 51 & 32 \\ 51 & 471 & 235 \\ 32 & 235 & 163.84 \end{bmatrix}_{3\times3}$
$3\times7 \quad 7\times3$

(3) Find $(X^T \cdot X)^{-1}$

$$= \begin{bmatrix} 1.79 & -0.06 & -0.25 \\ -0.06 & 0.01 & -0.0011 \end{bmatrix}$$

$$= \begin{bmatrix} 1.79 & -0.06 & -0.25 \\ -0.06 & 0.01 & -0.0011 \\ -0.25 & -0.0011 & 0.0571 \end{bmatrix}$$

$$= (X^T \cdot X)^{-1} \cdot X^T \cdot y$$

$$\theta = \begin{bmatrix} 51.6 \\ 1.5 \\ 6.72 \end{bmatrix}_{3\times 1}$$

$\beta_0 = 51.6$

$\beta_1 = 1.5$

$\beta_2 = 6.72$

$$y = 51.6 + 1.5x_1 + 6.72x_2$$

what is y when $x_1 = 3$ $x_2 = 2$

$$y = 51.6 + (1.5 \times 3) + 6.72 * 2 = 69.54$$

Consider Formulae that will be used, when only 2 Independent Variables specified.

[>2 features, use matrix Algebra].

Consider

X_1	X_2	y
3	8	-3.7
4	5	3.5
5	7	2.5
6	3	11.5
2	1	5.7
3	2	?

$$y = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

$\theta_0 = ?$

$\theta_1 = ?$

$\theta_2 = ?$

$$\theta_0 = \bar{y} - \theta_1 \bar{x_1} - \theta_2 \bar{x_2}$$

$\bar{y} =$
$\bar{x_1} =$ } mean
$\bar{y_1} =$

$$\theta_1 = \frac{(\Sigma x_2^2)(\Sigma x_1 y) - (\Sigma x_1 x_2)(\Sigma x_2 y)}{(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2}$$

$$\theta_2 = \frac{(\Sigma x_1^2)(\Sigma x_2 y) - (\Sigma x_1 x_2)(\Sigma x_1 y)}{(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2}$$

where

$$\Sigma x_1^2 = \Sigma x_1 x_2 - \frac{(\Sigma x_1)(\Sigma x_1)}{N}$$

$$\Sigma x_2^2 = \Sigma x_2 x_2 - \frac{(\Sigma x_2)(\Sigma x_2)}{N}$$

$$\Sigma x_2 = \Sigma x_2^2 - \frac{(\Sigma x_2)(\Sigma \quad)}{N}$$

$$\Sigma x_1 x_2 = \Sigma x_1 x_2 - \frac{(\Sigma x_1)(\Sigma x_2)}{N}$$

$$\Sigma x_1 y = \Sigma x_1 y - \frac{(\Sigma x_1)(\Sigma y)}{N}$$

$$\Sigma x_2 y = \Sigma x_2 y - \frac{(\Sigma x_2)(\Sigma y)}{N}$$

Sol^n = $\boxed{y = a_0 + a_1 x_1 + a_2 x_2}$

$a_0 = 2.796$, $a_1 = 2.28$, $a_2 = 1.67$

HW

* To Evaluate Performance of M.L models
* To find how good the model fits on given data

(1) Karl Pearsons Coefficient of Correlation (r)

→ To calculate Relationship bet two variable

$$r = \frac{N\Sigma xy - \Sigma x \Sigma y}{\sqrt{\left(N*\Sigma x^2 - (\Sigma x)^2\right)\left(N*\Sigma y^2 - (\Sigma y)^2\right)}}$$

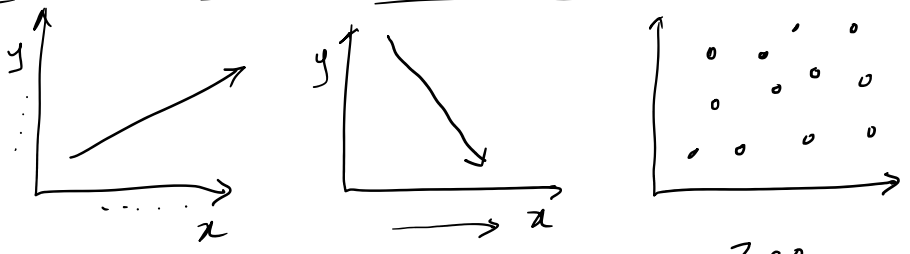
r = quantifies the strength of relationship bet^n two variables.

The value of r be between $+1$ and -1

If $r = 1$ ⇒ Total +ve correlation ⇒ If $x\uparrow$ then $y\uparrow$

If $r = -1$ ⇒ Total -ve correlation ⇒ If $x\downarrow$ then $y\uparrow$ or $x\uparrow$ then $y\downarrow$

→ It gives strength (degree) & direction of correlation.

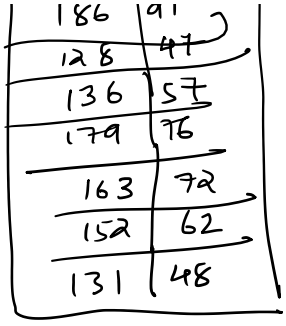


+ve Correlation — −ve Correlation — Zero Correlation.

Q. M.10

x	y
151	63
174	81
138	56
186	91
128	47
136	57

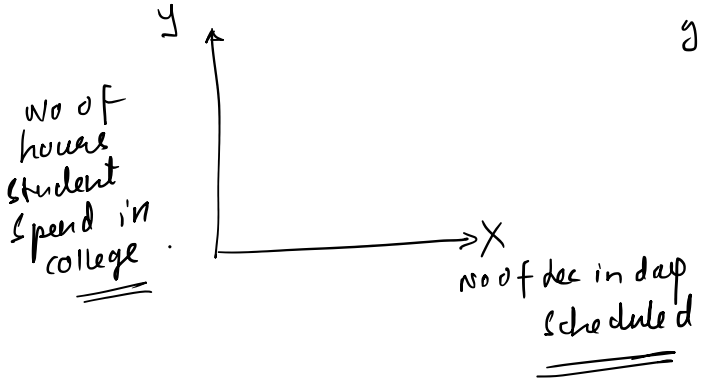
Find r = ? 0.9884


186 91
128 47
136 57
179 76
163 72
152 62
131 48

② R^2 method (R Square)

↳ It gives information about good of fit feature of the model.

↳



If $r^2 = 0.85$

Variation in no of hours that students spend in college is 85% dependent on no of dec scheduled.

↳ Indicate percentage of variance in dependent and Independent variable pair

↳ value varies from 0 to 1

If $r^2 = 1$ ⇒ no diff bet^n actual & predicted value.

$r^2 = 0$ means the model does not learn any relationship bet^n variables.

① SST (Sum of squares of Total) = $\Sigma (y - \bar{y})^2$

actual y ↑ mean of y.

② SSR (Sum of Squares due to regression) = $\Sigma (\hat{y} - \bar{y})^2$

predicted y ↑ mean of y.

③ SSE (Sum of Squares of Error) = $\Sigma (y - \hat{y})^2$

actual y ↑ predicted y

$$R^2 = SSR/SST = \frac{\Sigma(\hat{y}-\bar{y})^2}{\Sigma(y-\bar{y})^2}$$

Q. To find R^2

H.W

x	y	$\hat{y}$
151	63	
174	81	
138	56	
186	91	
128	47	
136	57	
179	76	
163	72	
152	62	
131	48	

To find Predicted i.e $\hat{y}$

$y = \beta_0 + \beta_1 x$

find $\beta_0 = -38.455$

$\beta_1 = 0.6746$

$y = -38.455 + 0.6746x$

Now $x = 151$ find $\hat{y} = 63.4$

$x = 174$ $\hat{y} = 78.92$

$x = 138$ $\hat{y} = 54.63$

Note $R^2 = \frac{\Sigma(\hat{y}-\bar{y})^2}{\Sigma(y-\bar{y})^2}$

if line fits properly

then $\hat{y} \approx y$

$\therefore R^2 \approx 1$

(3) Standard Error of Estimate →

* Measures the accuracy of Prediction.
* $\sigma_{est} = \sqrt{\frac{\Sigma(y-\hat{y})^2}{N}} = \sqrt{MSE} = \sqrt{\frac{SSE}{N}}$

($\uparrow$ mean square error)

y = actual value
$\hat{y}$ = predicted value.

* It reflects how well the Regression model fits the dataset.
* Smaller the value better it is
* Larger the value worst it is

Q. 11.10

x	y
151	63
174	81
138	56
186	91
128	47
136	57
179	76
163	72
152	62
131	48

$\sigma_{est} = 2.909$

Classification →

Ex Mail → Spam (+ve) / Not Spam (-ve)

Online Transaction → Fraudulent (+ve) / Non Fradulent (-ve)

Tumor → Malignent (+ve) / Benign (-ve)

where $y \in \{0, 1\}$ — 0 ⇒ -ve class; 1 ⇒ +ve class.

Here y has Discrete Value.

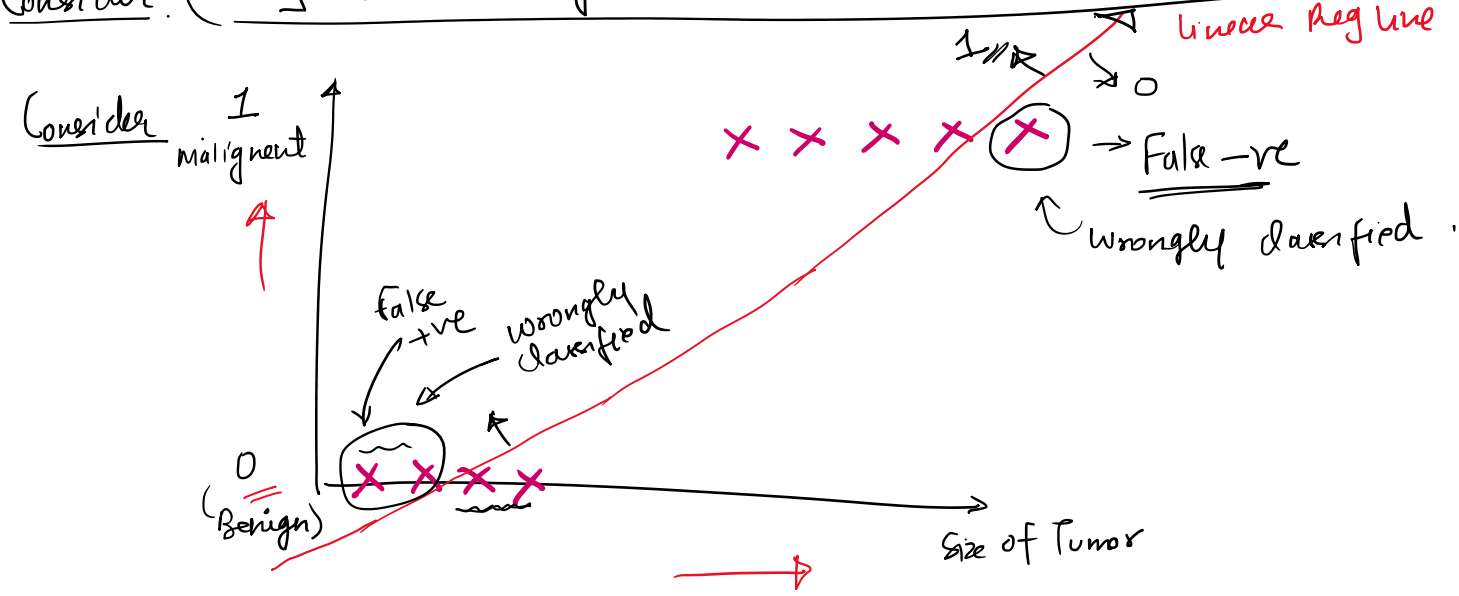
In above case o/p has only 2 class (0, 1) } Binary classification.

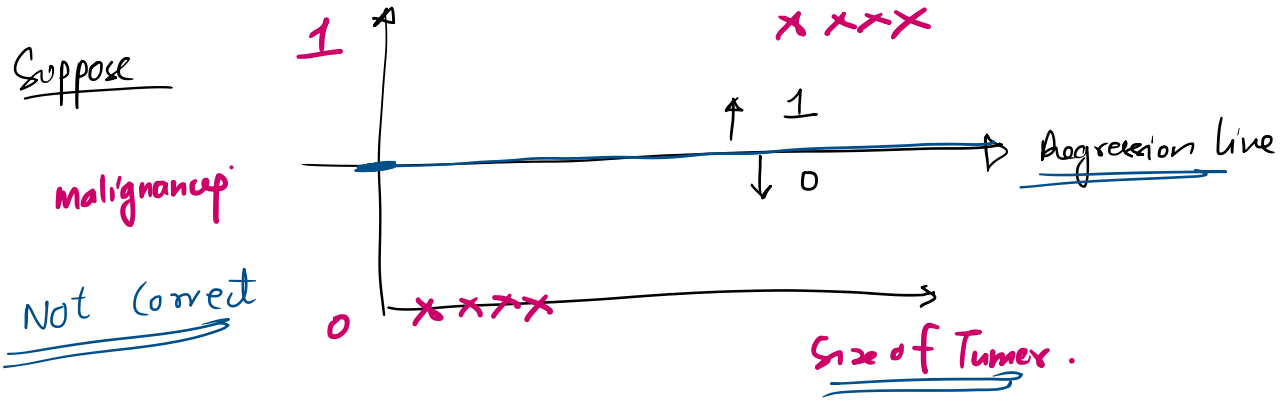
To check Weather.

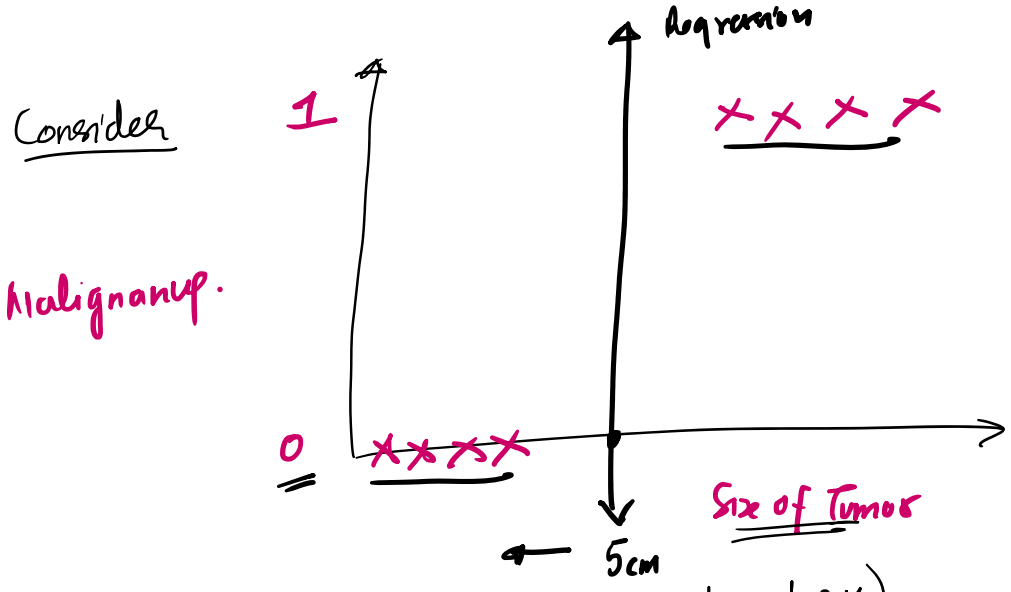
Possible o/p — Windy, Sunny, Cloudy, Rainy } o/p has more than one class [multi class classification]

* Classification bothers about label and not the Exact value.

Consider (Why linear Regression is not used for Classification)







Here we can observe for above Regression line

If tumor size $\leq$ 5cm $\rightarrow$ Yes (1) Malignant

If tumor size $\leq 5cm$ → Yes (1) Malignant

tumor size $> 5m$ → No (0) Benign

We can say let 'P' denote Probability that $y = 1$ when $X = x$.

$$y = P_r(y = 1 \mid X = x) = [\beta_0 + \beta_1 x]$$

In linear Reg

P = probability lies bet^n 0 to 1

But linear function are unbounded.

and Expected o/p here is 0 or 1

So we cannot use Regression to build Classifier

∴ linear Regression is not suitable for Classification.

For Classification we will use Logistic Regression.

* In Logistic Regression we get probability score.
* It predicts the probability of occurance of event

Odd = $\frac{\text{No of time the Event happens}}{\text{No of time the Event will not happen}}$

[Represents chances that the event will occur]

Ex ⇒ If odd of India winning against W.I is 4:1 = $\frac{\text{No of India win}}{\text{No of India not win}}$

$= \frac{4}{1}$

Best Case ⇒ The odd of India winning against WI = ∞

If odd of W.I winning against India is 1:4

$= \frac{\text{No of W.I win}}{\text{No of W.I not win}} = \frac{1}{4}$

Worst Case ⇒ W.I is winning 0 match = Odd = 0

Range of value that odd can take = 0 to ∞

Relationship between Odd & Probability = Odd = $\frac{P}{1-P}$