

Numericals based on
N-gram model

Q1. Find the total count of unique bigrams for which likelihood will be estimated.

Alice went to the café

Bob was waiting for Alice

Alice and Bob went to the museum

Total Bigrams = 20

Unique Bigrams = 17

Solⁿ - $\langle s \rangle$ Alice went to the cafe $\langle /s \rangle$

$\langle s \rangle$ Bob was waiting for Alice $\langle /s \rangle$

$\langle s \rangle$ Alice and Bob went to the museum $\langle /s \rangle$

$\langle s \rangle$ Alice
Alice went
went to
to the
the cafe
cafe $\langle /s \rangle$

⑥

$\langle s \rangle$ Bob
Bob was
was waiting
waiting for
for Alice
Alice $\langle /s \rangle$

⑥

$\langle s \rangle$ Alice
Alice and
and Bob
Bob went
went to
to the
the museum
museum $\langle /s \rangle$

⑤

Q2. Find the probability of the statement: <S> Michael and Zack played at the playground </s> from the following corpus. Assume a trigram Language model.

<S>the school was open</S>

<S>Michael and Zack went to the school</S>

<S>the playground at the school was huge</S>

<S>Bob and Zack played at the playground</S>

<S>Bob, Michael and Zack were friends</S>

Solⁿ - Calculate the trigram's probability / $p(\text{at} | \text{Zack played}) = \frac{1}{1}$

$$p(\text{Michael} | \text{<s>}) = \frac{1}{5}$$

$$p(\text{and} | \text{<s> Michael}) = \frac{1}{1}$$

$$p(\text{Zack} | \text{Michael and}) = \frac{2}{2}$$

$$p(\text{played} | \text{and Zack}) = \frac{1}{3}$$

$$p(\text{the} | \text{played at}) = \frac{1}{1}$$

$$p(\text{playground} | \text{at the}) = \frac{1}{2}$$

$$p(\text{</s>} | \text{the playground}) = \frac{1}{2}$$

$$\begin{aligned} p(\text{<s> Michael and Zack played at the playground </s>}) \\ = \frac{1}{5} \times \frac{1}{1} \times \frac{2}{2} \times \frac{1}{3} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{2} \times \frac{1}{2} \\ = \frac{1}{60} \end{aligned}$$

Q3. What is the perplexity of the statement in Q2

Solⁿ

$p(s)$ = Probability

n = Total token (including $\langle s \rangle$ and $\langle /s \rangle$)

$$\text{Perplexity} = p(s)^{-1/n}$$

$$= \left(\frac{1}{60} \right)^{-1/9}$$

$$= 1.576$$

Q4. For the given corpus, find the bigram probability estimate by laplace model for
 $P(\text{do} | \langle s \rangle)$, $P(\text{do} | \text{Sam})$, $P(\text{Sam} | \langle s \rangle)$, $P(\text{Sam} | \text{do})$, $P(\text{I} | \text{Sam})$, $P(\text{I} | \text{do})$, $P(\text{like} | \text{I})$

$\langle S \rangle \text{I am Sam} \langle /S \rangle$

$\langle S \rangle \text{Sam I am} \langle /S \rangle$

$\langle S \rangle \text{Sam I like} \langle /S \rangle$

$\langle S \rangle \text{Sam I do like} \langle /S \rangle$

$\langle S \rangle \text{do I like Sam} \langle /S \rangle$

Solⁿ

Tokens	Frequency
$\langle s \rangle$	5
$\langle I \rangle$	5
I	5
am	2
Sam	5
like	3
do	2

Unique tokens = 7

$$P(\text{do} | \langle s \rangle) = \frac{1+1}{5+7} = \frac{2}{12} = \frac{1}{6}$$

$$P(\text{do} | \text{Sam}) = \frac{0+1}{5+7} = \frac{1}{12}$$

$$P(\text{Sam} | \langle s \rangle) = \frac{3+1}{5+7} = \frac{4}{12} = \frac{1}{3}$$

$$P(\text{Sam} | \text{do}) = \frac{0+1}{2+7} = \frac{1}{9}$$

$$P(\text{I} | \text{Sam}) = \frac{3+1}{5+7} = \frac{4}{12} = \frac{1}{3}$$

$$P(\text{I} | \text{do}) = \frac{1+1}{2+7} = \frac{2}{9}$$

$$P(\text{like} | \text{I}) = \frac{2+1}{5+7} = \frac{3}{12} = \frac{1}{4}$$

Q5. For a corpus, MLE for bigram "battery life" is 0.27, frequency of "battery" is 800. After applying Laplace smoothing the MLE for "battery life" becomes 0.025. What is the vocabulary size of the corpus?

Soln

$$P(w_i | w_{i-1}) = \frac{\text{Count}(w_{i-1}, w_i)}{\text{Count}(w_{i-1})}$$

$$P_{MLE}(\text{Battery life}) = \frac{f(\text{battery life})}{f(\text{battery})}$$

$$0.27 = \frac{f(\text{battery life})}{800}$$

$$f(\text{battery life}) = 800 \times 0.27 = 216$$

With Laplace Smoothing

$$P_{MLE}(\text{life} | \text{Battery}) = \frac{f(\text{battery life}) + 1}{f(\text{battery}) + V}$$

$$0.025 = \frac{216 + 1}{800 + V}$$

$$\therefore \boxed{V = 7880}$$

insert - 1
delete - 1
substitute - 1 or 2

$$D(i, j) = \min \begin{cases} D(i-1, j) + 1 \\ D(i, j-1) + 1 \\ D(i-1, j-1) + \begin{cases} 2 & \text{if } s_1(i) \neq s_2(j) \\ 0 & \text{if } s_1(i) = s_2(j) \end{cases} \end{cases}$$

Edit distance

N	9	8	9	10	11	12	11	10	9	8
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	0	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

$$\begin{array}{r} 1+1 \\ 1+1 \\ 0+2 \end{array}$$
$$\begin{array}{r} 2+1 \\ 2+1 \\ 1+2 \end{array}$$
$$\begin{array}{r} 7+1 \\ 7+1 \\ 6+0 \end{array}$$
$$D(i, j-1)$$
$$D'(i-1, j-1)$$
$$D(i-1, j)$$

Q8 In the sentence-"The only thing we have to fear is fear itself" , the ratio between total number of word token and word types is:

Solⁿ Token = 10 Ratio of $\frac{\text{Token}}{\text{Type}} = \frac{10}{9}$
 Type = 9

Q9 Let the rank of two words W1 and W2 in a corpus be 1600 and 400 respectively. Let m1 and m2 represent the number of meanings of w1 and w2 respectively. The ratio m1:m2 would be?

Solⁿ Zipf's Law $m \propto \sqrt{F}$, $F \propto \frac{1}{\text{rank}}$

$$m_1 = \frac{1}{\sqrt{r_1}} = \frac{1}{\sqrt{1600}} = \frac{1}{40} \quad \therefore m_1 : m_2 = 20 : 40$$
$$m_2 = \frac{1}{\sqrt{r_2}} = \frac{1}{\sqrt{400}} = \frac{1}{20}$$

$m_1 : m_2 = 1 : 2$

Q10 What is the size of unique words in a document where total number of words=1200 and $k=3.71$, $\beta=0.69$

Solⁿ

Heap's Law

$$\begin{aligned} |V| &= \text{size of vocabulary} = k \cdot N^\beta \\ &= 3.71(1200)^{0.69} \\ &= 494.32 \end{aligned}$$