Physics 112, Fall 2017: Holzapfel
Problem Set 10 (5 Problems). **Due Monday, November 20**, 5 PM

Problem 1: **Cooling of a solid to $T = 0$K**
a) Kittel 8.9

b) Repeat part (a) for a metal with a heat capacity given by $C = aT^3 + bT$.

Problem 2: **Room air conditioner**
Kittel 8.6

Problem 3: **Photon Carnot Engine**
Kittel 8.3

Problem 4: **Total available work**
Two identical bodies, each characterized by a heat capacity at constant pressure C_P, which is independent of temperature, are used as heat reservoirs for a heat engine. The bodies remain at constant pressure and undergo no changes of phase. Initially, their temperatures are T_1 and T_2, respectively; finally, as a result of the operation of the heat engine, the bodies will attain a common final temperature T_f.

a) What is the total work W done by the engine? Express the answer in terms of C_P, T_1, T_2, and T_f.

b) Use an argument based on entropy considerations to derive an inequality relating T_f to the initial temperatures T_1 and T_2.

c) For the given initial temperatures T_1 and T_2, what is the maximum amount of work obtainable from the engine?

Problem 5: **Maximum Power**
To get more than an infinitesimal amount of work out of a Carnot engine, we would have to keep the temperature of its working substance below that of the hot reservoir and above that of the cold reservoir. Consider, then, a Carnot cycle in which the working substance is at temperature T_{hw} as it absorbs heat for the reservoir, and at temperature T_{cw} as it expels heat to the cold reservoir. Under most circumstances, the rates of heat transfer are directly proportional to the temperature differences:

$$\frac{Q_h}{\Delta t} = K(T_h - T_{hw}) \text{ and } \frac{Q_c}{\Delta t} = K(T_{cw} - T_c).$$

For simplicity, assume that the thermal conductivity K is the same for both processes. Also, assume that both processes are given the same time, Δt, to proceed.

a) Assuming that no new entropy is created during the cycle except during the two heat transfer processes, derive an equation that relates the four temperatures T_h, T_c, T_{hw}, and T_{cw}.

b) Assuming that the time for the adiabatic steps is negligible, find an expression for the power output of this engine. Express your answer in terms of the four temperatures and K, then use the results of part (a) to eliminate T_{cw}.

c) When the cost of building an engine is much greater than the cost of fuel (as is sometimes the case), it is desirable to optimize the engine for maximum power output, rather than maximum efficiency. Show that, for fixed T_h and T_c, the answer from part (b) has a maximum value at

$$T_{hw} = \frac{1}{2}(T_h + \sqrt{T_h T_c}) \, .$$

Find the corresponding expression for T_{cw}.

d) Show that the efficiency of this engine is $1 - \sqrt{T_h/T_c}$. Evaluate this expression for a typical coal-fired steam turbine with $T_h = 600$C and $T_c = 25$C, and compare with the Carnot efficiency. The achieved efficiency of a typical coal-fired power plant is about 40%.