CSE-619: Mid-Semester Examination

Monsoon 2024

October 6, 2024

Full Marks: 50 **Time: 2:30 hours**

General Instructions: This is a cheat based examination. You can carry one A4-paper where on both sides, basic definitions of graphs, and any specific graph class(es) etc. can be written. You have to submit it along with your answer scripts at the end of the examination.

1. Let $L \subseteq \Sigma^* \times \mathbb{N}$ be a parameterized problem. Prove the following statement for L.

   Statement: L admits a polynomial-time algorithm if and only if L admits a kernel of $O(1)$ size. (**5 Marks**)

2. Given a graph $G = (V, E)$, a subset of $F \subseteq E(G)$ of edges form an *odd subgraph* of G the endpoints of the edges of F, i.e. V_F and the edge set F forms a subgraph G_F where every vertex of V_F is odd degree in that graph G_F. Formally, speaking, every vertex of the subgraph $G_F = (V_F, F)$ has odd degree in G_F.

   Consider the problem Odd Subgraph that is defined as follows.

   - **Input:** An undirected graph $G = (V, E)$ and an integer k.
   - **Parameter:** k.
   - **Question:** Are there at least k edges in G that forms an odd subgraph?

   Design a kernel with $O(k^2)$ vertices for Odd Subgraph. (**10 Marks**)

3. In the Vertex Cover/OCT problem, we are given an undirected graph $G = (V, E)$, a subset $X \subseteq V(G)$ such that $|X| \leq k$ and $G - X$ is bipartite, and an integer ℓ. The objective is to test whether G has a vertex cover of size at most ℓ. Show that Vertex Cover/OCT admits an algorithm that runs in $2^k poly(n)$-time. Your answer must explain why your algorithm runs in your claimed running time. (**10 Marks**)

4. An undirected graph $G = (V, E)$ is a split graph if $V(G)$ can be partitioned into a clique C and an independent set I. In the Split Vertex Deletion problem, given (G, k), the objective to test whether G has a set of at most k vertices whose deletion results in a split graph. Design an algorithm for Split Vertex Deletion that runs in $4^k poly(n)$-time. Please give a clear explanation why your algorithm runs in your claimed running time.

   (**10 Marks**)

5. In the Independent Feedback Vertex Set (IFVS) problem, the input instance is an undirected graph $G = (V, E)$ and an integer k. The objective is to check if G has a set S of at most k vertices such that $G - S$ is acyclic and S is an independent set. Design an algorithm for Independent Feedback Vertex Set that runs in time $5^k poly(n)$.

   (**15 Marks**)