CSE-619: End-Semester Examination

Monsoon 2024

December 12, 2024

Full Marks: 70 **Time: 2:30 hours**

General Instructions: This is a cheat based examination. You can carry one A4-paper where on both sides, basic definitions of graphs, and any specific graph class(es) etc. can be written. You have to submit it along with your answer scripts at the end of the examination. You can use any result that was taught in the class unless that is what explicitly you have to design an algorithm for. For instance, if you have to use that "VERTEX COVER in bipartite graph can be computed in polynomial-time", you can use that as a subroutine to design an algorithm for some problem you are asked to solve here.

1. Let $G = (V, E)$ be a simple undirected graph of n vertices. A *cut* of G is a partition (A, B) of $V(G)$ into nonempty parts (i.e. A and B are nonempty) and the *size* of (A, B) is the number of edges with one endpoint in A and the other endpoint in B. Formally, the cardinality of this set $\{(u, v) \in E(G) \mid u \in A, v \in B\}$ is the *size* of the cut (A, B). A cut (A, B) is a *maximum cut* if the size of (A, B) is the maximum among all the cuts.

   Let $\mathbb{T} = (T, \mathcal{X}_{t \in V(T)})$ be a tree decomposition of G of width k. Given this tree decomposition, design a dynamic programming based algorithm that runs in $O(2^k \text{poly}(n))$-time and computes the size of a maximum cut of G. **(15 Marks)**
   (**N.B.** You can assume that $\mathbb{T}$ is a nice or nicer tree decomposition whichever you feel comfortable with. But mention which one you use.)

2. Given a simple undirected graph $G = (V, E)$, ONE-OVERLAP K_4-PACKING asks to find a collection $C_1, C_2, \ldots, C_k$ of at least k cliques each having four vertices and for every $i \neq j$, $|C_i \cap C_j| \leq 1$. Design a color-coding based dynamic programming algorithm for this problem. **(10 Marks)**

   N.B. (You can ignore the derandomization part)

3. DOMINATING SET SPLIT GRAPH is defined as follows.
   Input: (G, k) such that G is a split graph, that is $V(G)$ can be partitioned into C and I such that C is a clique and I is an independent set.
   Parameter: k.
   Question: Does there exist a set $S \subseteq V(G)$ of at most k vertices such that for every $u \in V(G)$, $u \in S$ or u has a neighbor v such that $v \in S$?
   Prove that DOMINATING SET SPLIT GRAPH is W-hard.
   Hint: Reduction from DOMINATING SET in general graph. **(10 Marks)**

4. DISJOINT-COMPRESSION-VC is defined as follows.
   Input: An undirected graph $G = (V, E)$, and a vertex cover $S \subseteq V(G)$.
   Goal: Find the minimum possible size of a vertex cover X of G such that $X \cap S = \emptyset$.

   (a) Design a polynomial-time algorithm for DISJOINT-COMPRESSION-VC.

   (b) Explain how the above algorithm can be used to design a $2^k \cdot \text{poly}(n)$-time algorithm for VERTEX COVER when parameterized by solution size k.

   (c) Design a kernel for DISJOINT-COMPRESSION-VC with $O(1)$ vertices and edges.

   (4 Marks + 3 Marks + 3 Marks)

5. CONNECTED VERTEX COVER problem is defined as follows.
Input: An undirected graph $G = (V, E)$ and an integer k.
Question: Does there exist $S \subseteq V(G)$ with at most k vertices such that $G[S]$ is connected and S is a vertex cover of G?

(a) If the parameter is k (solution size), design an algorithm that runs in $6^k \cdot \text{poly}(n)$-time.

(b) If the parameter is tw (treewidth of G), then assuming a nice tree decomposition T given with the input graph, design a dynamic programming based algorithm that for CONNECTED VERTEX COVER that runs in $\text{tw}^{O(\text{tw})}\text{poly}(n)$-time.

(5 Marks + 10 Marks)

6. PARTIAL VERTEX COVER is defined as follows.
Input: An undirected graph $G = (V, E)$ and two integers k and ℓ.
Parameter: ℓ.
Question: Does there exist a set $S \subseteq V(G)$ of at most k vertices such that the number of edges incident to the vertices of S is at least ℓ?
Design an algorithm for PARTIAL VERTEX COVER that runs in $2^{O(\ell)}\text{poly}(n)$-time.

(10 Marks)