Full marks: 20

1. Choose True/False for the following statements.

 $1 \times 5 = 5$

- (a) Consider three strategies s_i , s'_i , and s''_i of a player i. Also, i) s_i weakly dominates s'_i and ii) s'_i weakly dominates s''_i then it is possible that s''_i weakly dominates s_i .
- (b) A player with m number of strategies has either 0 or 1 number of strongly dominant strategy.
- (c) A PSNE is always a dominant strategy equilibrium.
- (d) At Pure Strategy Nash Equilibrium (PSNE), a player may not find optimal payoff.
- (e) A player does not have any benefit of unilaterally moving to some other strategy from her PSNE strategy.
- 2. Consider the following instance of the Prisoner's Dilemma problem.

| | | Player-2 | | |
|----------|----|----------|----------|--|
| | | С | NC | |
| Player-1 | С | (-x,-x) | (-x,-3) | |
| | NC | (-3,-x) | (-6, -6) | |

Find the value of x for which

i) The profile (C, C) is a strongly dominant strategy equilibrium.

1

- ii) The profile (C, C) is a weakly dominant strategy equilibrium, but not a strongly dominant strategy equilibrium 2
- 3. Find the PSNE, maxmin values and maxmin strategies for the following game.

| | | Player-2 | | | |
|----------|---|----------|---------|--------|--|
| | | X | Y | Z | |
| | A | (6, 6) | (8, 20) | (0, 8) | |
| Player-1 | В | (10, 0) | (5, 5) | (2, 8) | |
| | С | (8, 0) | (20, 0) | (4, 4) | |

Answer the following for the above game:

i) Find the PSNE.

2

ii) Find the maxmin value (or values).

1

iii) Find the maxmin strategy (or strategies).

1

See next page

- 4. In First Bid Auction, there is no strongly dominant strategy for any player. Give reason.
- 5. Compute PSNE for the two person game with $S_1 = \{0,1\}$ and $S_2 = \{3,4\}$ with the following utility functions:

$$u_1(x,y) = -u_2(x,y) = |x-y| \quad \forall (x,y) \in \{0,1\} \times \{3,4\}$$

- 6. Consider a game with n players. The mapping $b_i: S_{-i} \mapsto 2^{S_i}$ is the best response conrespondence for player i. If $b_i(s_{-i}) \neq \emptyset \ \forall s_{-i} \in S_{-i}$, there may not exist a PSNE of the game. Give reason.
- 7. In auction-based multi-robot routing discussed in the class, suppose the team objective of n robots is

$$\min_{\mathcal{A}} f(g(r_1, \mathcal{A}_1), \dots, g(r_n, A_n))$$

where f and g are the performance meansures of the team and individual robot respectively, $\mathcal{A} = \{A_1, \ldots, A_n\}$ is a partition of the set of targets. Let (S_1, \ldots, S_n) be the current allocation of targets to the robots. Now, if an unallocated target t appears, what bid value should robot r bid on target t? Give the general mathematical expression only.