1. Choose True/False for the following statements. $1 \times 4 = 4$

   (a) *Belief* of a robot is the posterior distribution over the states of the environment conditioned on past actions.

   (b) Bayes filter makes a *Markov assumption* according to which the state is a complete summary of the past.

   (c) A Uniform probability distribution has higher entropy value compared to a non-uniform probability distribution.

   (d) Consider a non-negative random variable $Q \geq 0$ and a given value $b > 0$. To calculate the maximum probability of Q exceeding b, which bound (inequality) we should use ?

2. (a) In a robot workspace, the state of the environment (and the robot) at time k is expressed as a random variable X_k. Give a mathematical expression of "belief" $b[X_k]$ of the robot at time k. 1.5

   (b) Generally, in the expression of $b[X_k]$ the state of the environment can be represented as a random variable, however random variables cannot be used for the other parameters in the above expression. Why ? 1.5

   (c) Write the Bayes filter algorithm that outputs $b[X_k]$. Clearly mention the inputs and the steps. 3

3. A random variable X, where $X \sim P$, has the following probability distribution:

$$P(X) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{256}, \ldots, \frac{1}{256}\right)$$

   (a) Calculate entropy of X. 2

   (b) Now, consider another random variable Y with the following probability distribution

$$\left(\frac{3}{4}, \frac{1}{256}, \ldots, \frac{1}{256}\right)$$

   Without calculating entropy of Y, answer which one of X and Y has higher entropy. Give reason. 1.5

   (c) Shannon Entropy of a random variable Z is a non-negative value. Why ? 1.5

4. See Figure-1. In an environment monitoring application a sensor is deployed. Suppose Day-1 is sunny (this is known for a fact), and in the subsequent two days our sensor observes *cloudy*, *cloudy*. What is the probability that Day-2 is indeed cloudy as predicted by our sensor ? 3

		our sensor tells us...		
		sunny	cloudy	rainy
	sunny	.6	.4	0
the actual weather is...	cloudy	.3	.7	0
	rainy	0	0	1

Figure 1:

5. In the paper Kundu2024iros discussed in class, the planning algorithm decides the joint action for the next time step i.e. horizon length $L = 1$. Instead, we want to modify our algorithm in a way that plans for an arbitrary length horizon i.e. $L \geq 1$. What challenges the modified algorithm may face ? 2