1. Choose True/False for the following statements. $1 \times 6 = 6$

   (a) When a strategic form game has multiple pure strategy Nash equilibria (PSNE), the payoffs for a player can be different in different PSNEs. True.

   (b) Every dominant strategy equilibrium is also a PSNE and an MSNE. True.

   (c) A game with a finite number of players and strategies has at least one Mixed Strategy Nash Equilibrium (MSNE). True.

   (d) Using mixed strategy, a game may have multiple strongly dominant strategy equilibria (SDSE). False.

   (e) In MSNE, the expected utility of a player i must be same for any strategy that has probability $(0, 1]$. True.

   (f) A player does not have any benefit of unilaterally moving to some other strategy from her MSNE strategy. True.

2. Give formal definitions of the following with respect to a strategy form game $\Gamma = \langle N, (S_i), (u_i) \rangle$. The notations bear conventional meanings. $1.5 \times 4 = 6$

   i) *Weakly dominated strategy* in Γ.

   ii) *Pure strategy Nash Equilibrium (PSNE)* in Γ.

   iii) *Best response correspondence* in Γ.

   iv) *Maxmin strategy* in Γ.

   Solution: Refer the mentioned sections of *Book-Narahari*

   i) Weakly dominated strategy: Section 5.2, Definition 5.4

   ii) Pure strategy Nash equilibrium (PSNE): Section 6.1, Definition 6.1

   iii) Best response correspondence: Section 6.1, Definition 6.2

   iv) Maxmin strategy: Section 6.6, Definition 6.3

3. Assume two bidders with valuations v_1 and v_2 for an object. Theird bids are in multiple of some unit (that is, discrete). The bidder with higher bid wins the auction and pays the amount that he has bid. If both bid the same amount, one of them gets the object with equal probability 0.5. Compute a pure strategy Nash equilibrium of this game, if exists. 4

4. Compute a two person game with $S_1 = [0, 1]$ and $S_2 = [3, 4]$ with the following utility functions:

$$u_1(x, y) = -u_2(x, y) \ = |\ x - y\ | \quad \forall (x, y) \in [0, 1] \times [3, 4]$$

   i) How many possible strategies each player has ? 1

   ii) Compute MSNE for the above game. 3

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5. Apply maximum number of iterations to eliminate strongly dominated strategies to the following problem and show the reduced matrix in each iteration. 4

		Player-2		
		X	Y	Z
Player-1	A	(5, 1)	(2, 2)	(1, 3)
	B	(2, 3)	(5, 4)	(6, 0)
	C	(3, 2)	(3, 0)	(0, 4)

Solution: The given payoff matrix is:

$$\begin{array}{c|ccc} & X & Y & Z \\ \hline A & (5,1) & (2,2) & (1,3) \\ B & (2,3) & (5,4) & (6,0) \\ C & (3,2) & (3,0) & (0,4) \end{array}$$

Iteration 1: Eliminate Strongly Dominated Strategies for Player 1 Strategy C is strongly dominated by B since:

$$3 < 5, \quad 3 < 5, \quad 0 < 6$$

Thus, we eliminate C.

$$\begin{array}{c|ccc} & X & Y & Z \\ \hline A & (5,1) & (2,2) & (1,3) \\ B & (2,3) & (5,4) & (6,0) \end{array}$$

Iteration 2: Eliminate Strongly Dominated Strategies for Player 2 Strategy X is strongly dominated by Y since:

$$1 < 2, \quad 3 < 4$$

Thus, we eliminate X.

$$\begin{array}{c|cc} & Y & Z \\ \hline A & (2,2) & (1,3) \\ B & (5,4) & (6,0) \end{array}$$

Iteration 3: Eliminate Strongly Dominated Strategies for Player 1 Strategy A is strongly dominated by B since:

$$2 < 5, \quad 1 < 6$$

Thus, we eliminate A.

	Y	Z
B	$(5,4)$	$(6,0)$

Iteration 4: Eliminate Strongly Dominated Strategies for Player 2 Strategy Z is strongly dominated by Y since:

$$4 > 0$$

Thus, we eliminate Z.

	Y
B	$(5,4)$

Thus, the final remaining strategy profile is:

$$(B,Y)$$

Any other solution with appropriate explanation is acceptable.

6. Consider a pickup-and-delivery application in which a set drones $D = \{1, 2, \ldots, n\}$ are engaged for the task. Also, there are a set of ground vehicles $G = \{1, 2, \ldots, m\}$. A drone can ride (piggyback) on a ground vehicle en route towards its destination and thus saves energy. Let, a T-length path of drone $i \in D$ is $p_i = (e_i^1, \ldots, e_i^T)$. A planning algorithm decides if an edge e_i^k, where $k = \{1, \ldots, T\}$, on p_i is assigned to some ground vehicle $j \in G$ or not. Mathematically, for drone i, an assignment $A_i := (a_i^1, \ldots, a_i^T)$ where $a_i^k = 1$ if some $j \in G$ is assigned to e_i^k, and $a_i^k = 0$ otherwise. *Find the cost* of p_i, denoted as $\texttt{cost}(p_i, A_i)$. Consider $\texttt{c}(e_i^k)$ denotes the cost of edge e_i^k. 3

7. Consider a industry warehouse in which some worker robots each of which has its fixed trajectory. A set of mobile rechargers meet the worker robots and recharge them as and when needed. Consider that there is an algorithm A which finds i) the time and location for meeting a worker robot with a recharger, and ii) the path of the rechargers. If we take a time horizon T, the goal is to *match* i) the initial locations of the worker robots at time $t = 1$ and $t = T$, ii) charge level of the worker robots at time $t = 1$ and $t = T$, and iii) locations of rechargers at time $t = 1$ and $t = T$. However, matching all these criteria simultaneously at $t = T$ is computationally intractable. Suggest an algorithm that reduces the computational burden. 3