1. Choose True/False for the following statements. $1 \times 3 = 3$

   (a) Conditional Entropy for two random variables is symmetric.

   (b) Simulated Annealing (SA) algorithm may get stuck in a local optima.

   (c) In Ant Colony Optimization (ACO) algorithm, pheromone intensity monotonically increases for a given edge.

2. (a) Give a mathematical expression for *Conditional Entropy*. 1

   (b) Give a mathematical expression for *Mutual Information*. 1

   (c) Draw two circles to represent entropy $H(X)$ and $H(Y)$ respectively. Indicate conditional entropy and mutual information in that figure. 1

3. Consider X and Y are the beliefs of two different robots. $H(.)$ denotes entropy.

   (a) Prove that $H(X \mid Y) = 0$ if and only if $X = g(Y)$ for some function g. 1.5

   (b) Prove that the information revealed by Y about X is same as the information revealed by X about Y. 2.5

4. Consider a mobile robot in a workspace $S = \{s_1, s_2, \ldots, s_n\}$. Belief of the robot at current time t is given by $P(X \mid \xi_t, a_t)$ where X is the random variable representing the environment and ξ_t is the history of states and actions until time t. Mathematically, $\xi_t = \langle s_1, a_1, \ldots, s_t \rangle$. An effective exploration strategy is to select an action that gives maximum information about the environment, i.e. maximizes the reduction in uncertainty about the environment. From the current state s_t the next state distribution is denoted as S_{t+1}.

   (a) Write down the mathematical expression for selecting the action that maximally reduces uncertainty in X between time $t + 1$ and t. 2

5. Consider solving the Traveling Salesman Problem using Ant Colony Optimization (ACO) algorithm. Also consider that in a given iteration the *pheromone* intensity is $\tau(i, j)$ of edge (i, j) in graph G. Write a mathematical expression to update $\tau(i, j)$ by the agents (ants) in the system. 2

6. We want to design an optimization algorithm which runs over iterations to achieve a global optimum solution. For an arbitrary $i \in \mathbb{Z}^+$, consider that in iteration i and

$i+1$ the solution states are x_i and x_{i+1} respectively. It may happen that x_{i+1} is a worse solution than x_i in terms of proximity to the optimum solution. That is, it may happen that $\texttt{dist}(x_i, x^*) < \texttt{dist}(x_{i+1}, x^*)$. In order to ensure that the algorithm probabilistically moves to a worse solution, we need to define a probability function $P(x_i, x_{i+1}, T)$ where $T = f(i)$ (function of iteration number).

(a) Given the above criteria, define $P(.)$ so that the probability of moving to a worse solution keeps on decreasing as the iterations proceed. 2

(b) Consider that instead of a single agent if we consider multiple agents. These agents work cooperatively to calculate the probabilities of transition. Suggest a cooperative approach to calculate $P(.)$ cooperatively. 2

7. Both ACO and SA are probabilistic metaheuristic algorithms. Mention two basic differences between these two algorithms. 2