MNIST Classification Report: Performance Evaluation of LDA, QDA, PCA, and FDA

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1. Introduction

This report presents the implementation and performance evaluation of a classification pipeline for handwritten digits **0, 1, and 2** from the **MNIST dataset**. The study incorporates **Maximum Likelihood Estimation (MLE), Principal Component Analysis (PCA), Fisher's Discriminant Analysis (FDA), and classification using Linear Discriminant Analysis (LDA) and Quadratic Discriminant Analysis (QDA)**.

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2. Data Preprocessing

- **Loaded the MNIST dataset** and filtered images for digits **0, 1, and 2**.
- **Converted images into feature vectors** (flattening each **28×28 image into 784-dimensional vectors**).
- **Normalized pixel values to the range [0,1]** to standardize input data.
- **Randomly selected 100 samples per class** for both training and testing, leading to:
    - **Train Set**: 300 samples (100 per class).
    - **Test Set**: 300 samples (100 per class).
- **OUTPUT :**

```
Train images shape: (60000, 28, 28)
Train labels shape: (60000,)
Test images shape:  (10000, 28, 28)
Test labels shape:  (10000,)
```

```
Filtered train images shape: (18623, 28, 28)
Filtered train labels shape: (18623,)
Filtered test images shape:  (3147, 28, 28)
Filtered test labels shape:  (3147,)
Final train images shape: (300, 28, 28)
Final train labels shape: (300,)
Final test images shape:  (300, 28, 28)
Final test labels shape:  (300,)
train_X shape: (300, 784) test_X shape: (300, 784)
train_y shape: (300,) test_y shape: (300,)
```

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3. Compute MLE Estimates

Using **MLE**, we estimated the class-wise mean vector (μc) and covariance matrix (Σc) under the assumption of a **Multivariate Gaussian Distribution**:

- Assume the data follows a multivariate Gaussian distribution:

$$P(\mathbf{x}|y=c) = \frac{1}{(2\pi)^{d/2}|\mathbf{\Sigma}_c|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_c)^T \mathbf{\Sigma}_c^{-1}(\mathbf{x}-\boldsymbol{\mu}_c)\right).$$

- The **mean image for each class** was computed and visualized.
- The **covariance matrices** for each class were displayed using heatmaps.
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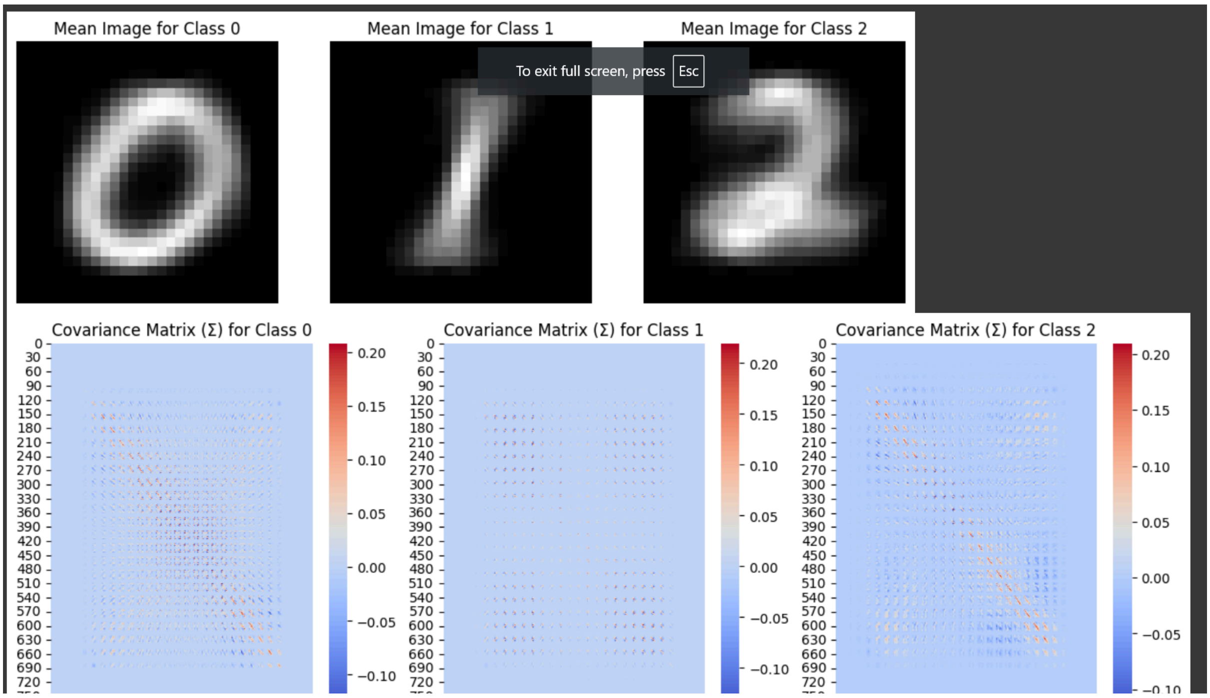
```
Done computing MLE estimates (mean vectors and covariance matrices) for each class.

print("Mean shape for class 0:", means[0].shape)  # Should be (784,)
print("Covariance matrix shape for class 0:", covariances[0].shape)  # Should be (784, 784)

Mean shape for class 0: (784,)
Covariance matrix shape for class 0: (784, 784)
```
- 

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4. Dimensionality Reduction using PCA

Steps followed:

- **Computed mean-centered data matrix** Xc.
- **Calculated covariance matrix S.**

- **Performed Eigen decomposition** and sorted eigenvalues.
- **Projected data onto principal components (PCs).**
- **Retained 95% variance**, reducing dimensions from **784 to 83 features**.
- **Alternative PCA settings:**
    - Retained **90% variance → 52 features**.
    - Retained **only first 2 principal components**.
- **OUTPUTS :**
- `Original dimension: 784`
- `Reduced dimension (95% var): 83`
- `Shape of Y_train (PCA-transformed data): (300, 83)`
- `Shape of Y_test (PCA-transformed test set): (300, 83)`

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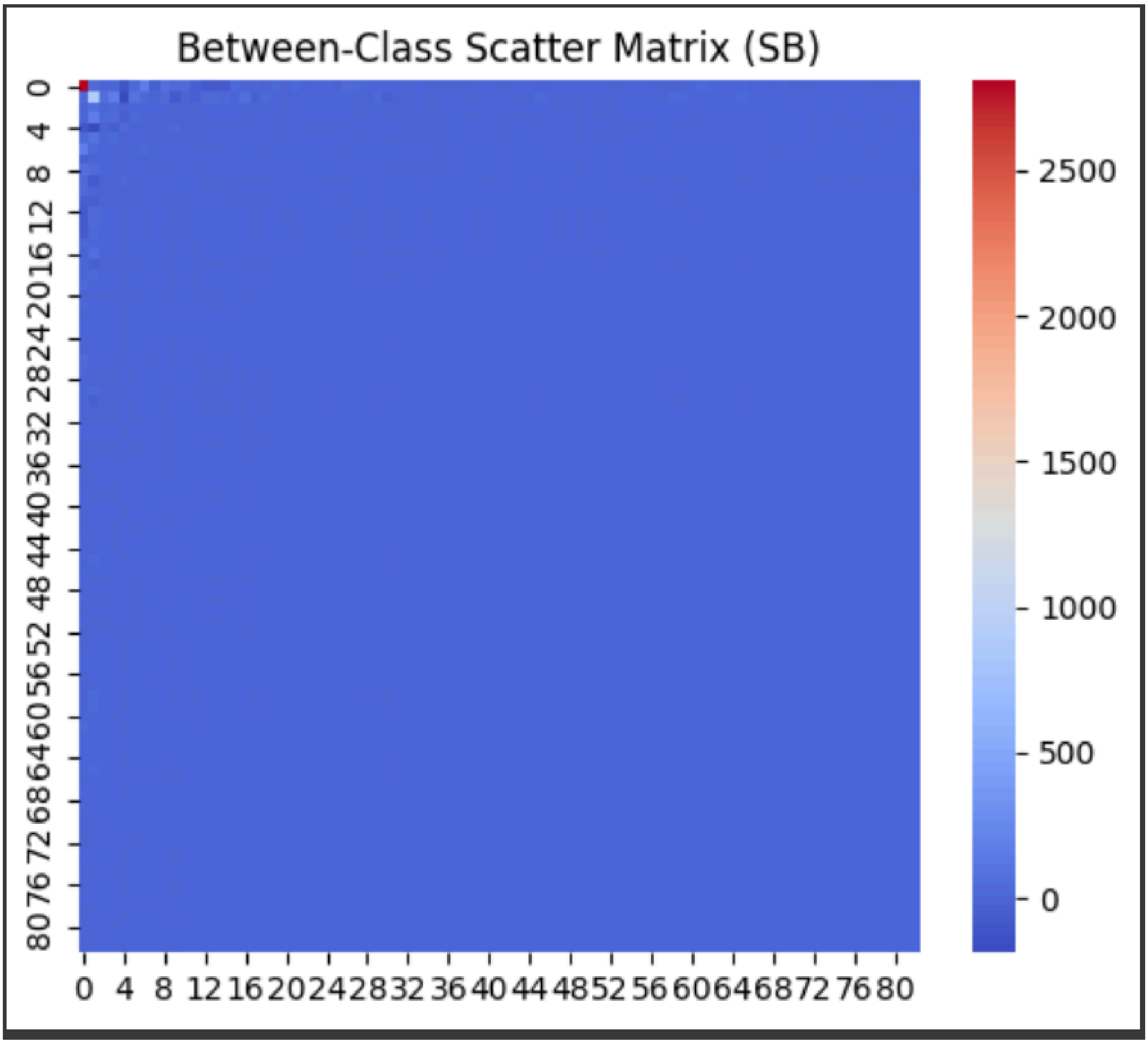
5. Class Projection using FDA

Steps followed:

- Computed **Between-Class Scatter Matrix SB**.

```
Between-Class Scatter Matrix (S_B):



[[ 2.80716959e+03  4.62159457e+01 -1.12374996e+01 ...
-2.28891972e+00

   2.90304789e+00 -2.24333331e+00]

 [ 4.62159457e+01  8.96497152e+02  6.84082105e+01 ...
-2.12429151e+00

  -9.41876201e-01  5.48995048e+00]

 [-1.12374996e+01  6.84082105e+01  5.29768066e+00 ...
-1.50624278e-01

  -8.74077900e-02  4.32215164e-01]

 ...

 [-2.28891972e+00 -2.12429151e+00 -1.50624278e-01 ...
6.72707722e-03

  -6.16704300e-05 -1.10456378e-02]

 [ 2.90304789e+00 -9.41876201e-01 -8.74077900e-02 ...
-6.16704300e-05

   4.09565665e-03 -8.42643265e-03]
```

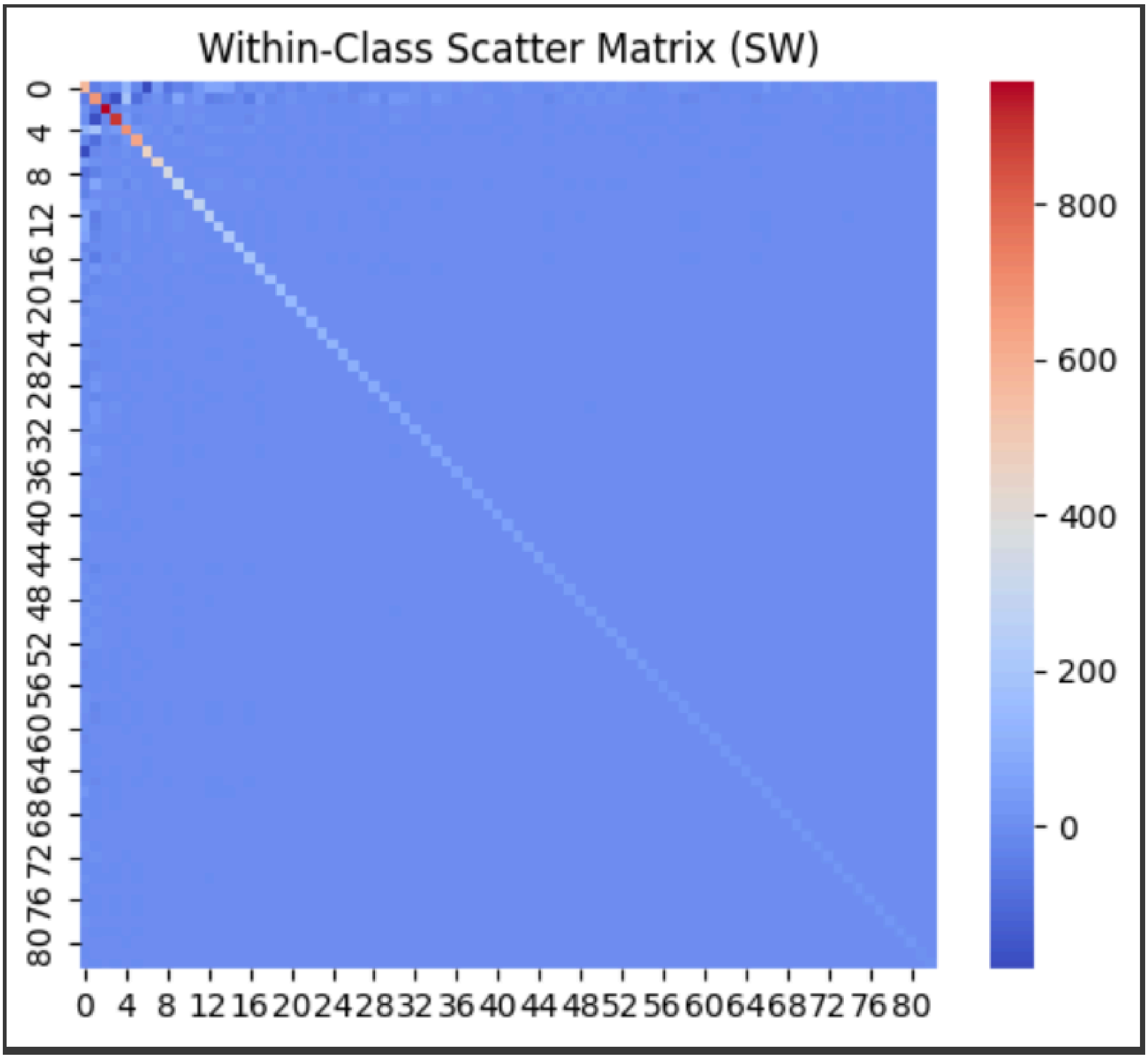
```
 [-2.24333331e+00  5.48995048e+00  4.32215164e-01 ...
-1.10456378e-02
  -8.42643265e-03  3.58947969e-02]]
```



- Computed **Within-Class Scatter Matrix SW**.

```
Within-Class Scatter Matrix (S_W):

[[ 5.73388110e+02 -4.62159457e+01  1.12374996e+01 ...  2.28891972e+00
  -2.90304789e+00  2.24333331e+00]
 [-4.62159457e+01  6.73628810e+02 -6.84082105e+01 ...  2.12429151e+00
   9.41876201e-01 -5.48995048e+00]
 [ 1.12374996e+01 -6.84082105e+01  9.57295020e+02 ...  1.50624278e-01
```

```
   8.74077900e-02 -4.32215164e-01]
 ...
 [ 2.28891972e+00  2.12429151e+00  1.50624278e-01 ...  1.74982462e+01
   6.16704300e-05  1.10456378e-02]
 [-2.90304789e+00  9.41876201e-01  8.74077900e-02 ...  6.16704300e-05
   1.69386073e+01  8.42643265e-03]
 [ 2.24333331e+00 -5.48995048e+00 -4.32215164e-01 ...  1.10456378e-02
   8.42643265e-03  1.65320455e+01]]
```



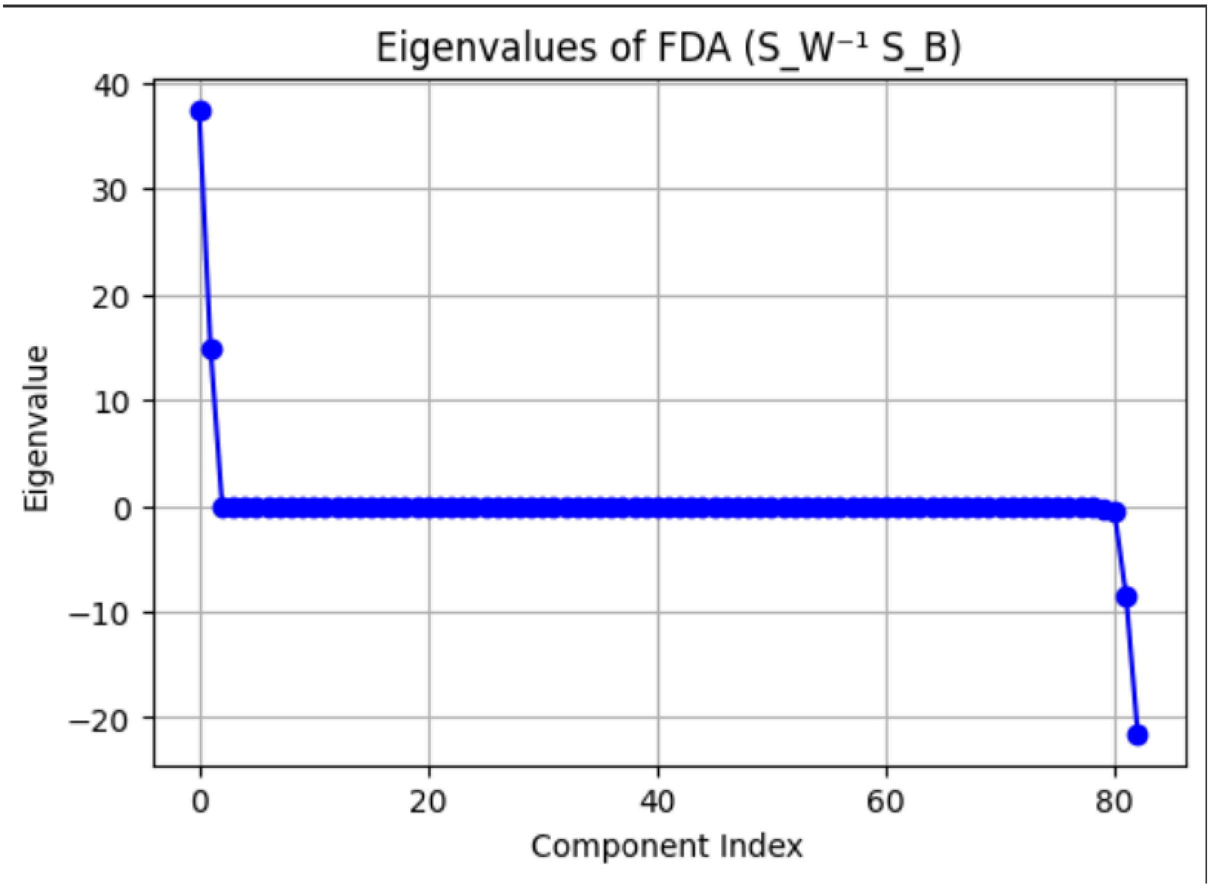
- **Solved the generalized eigenvalue problem**:
  - Compute the between-class scatter matrix $\mathbf{S}_B$ and within-class scatter matrix $\mathbf{S}_W$:

$$\mathbf{S}_B = \sum_c N_c(\boldsymbol{\mu}_c - \boldsymbol{\mu})(\boldsymbol{\mu}_c - \boldsymbol{\mu})^T$$

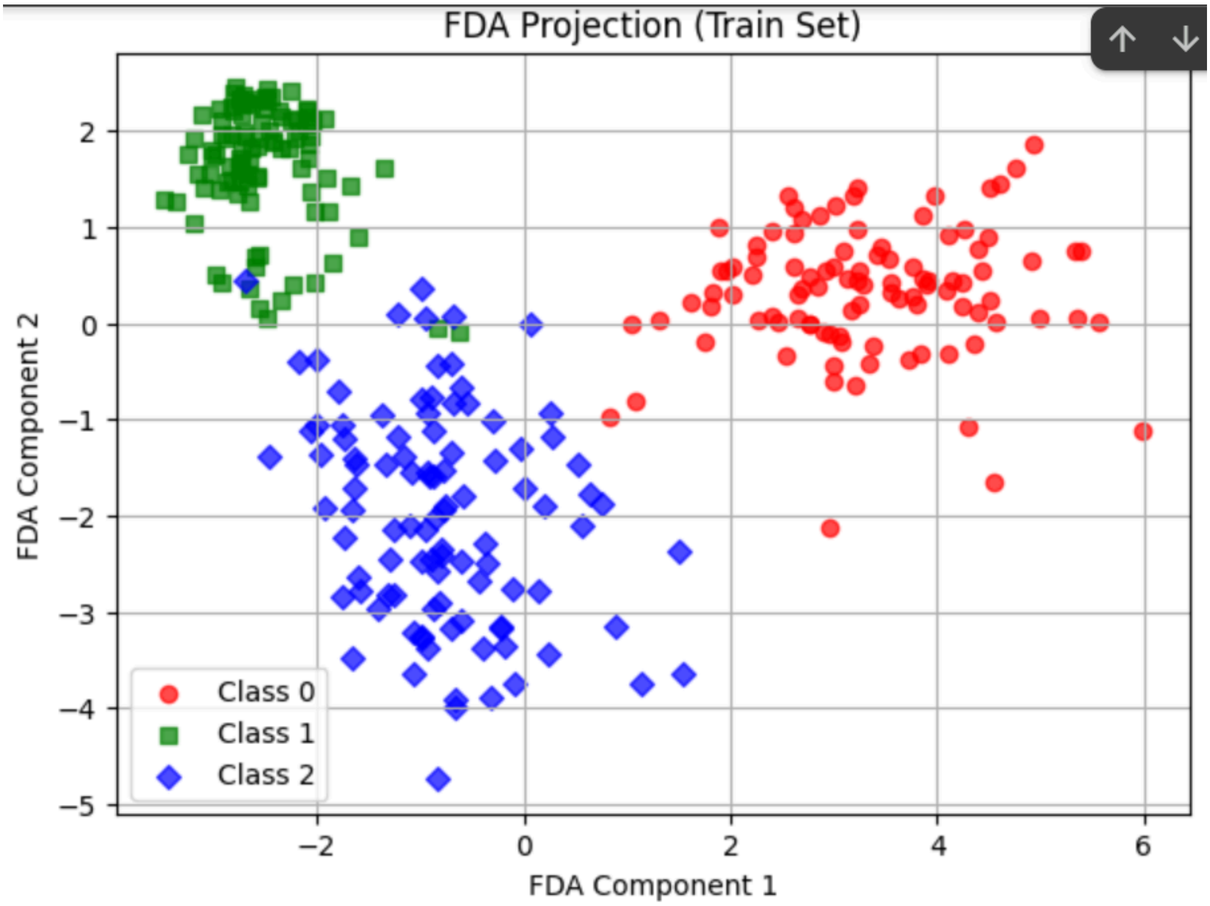
$$\mathbf{S}_W = \sum_c \sum_{\mathbf{x}_i \in C_c} (\mathbf{x}_i - \boldsymbol{\mu}_c)(\mathbf{x}_i - \boldsymbol{\mu}_c)^T.$$

  - Compute the optimal projection matrix $\mathbf{W}$ that maximizes class separability:

$$\mathbf{W} = \arg\max_{\mathbf{W}} \det(\mathbf{W}^T\mathbf{S}_B\mathbf{W})/\det(\mathbf{W}^T\mathbf{S}_W\mathbf{W}).$$



- **Projected data into 2D FDA space** and visualized the transformed feature space.



6. Evaluate and Compare Performance

Classification Accuracy of LDA & QDA on FDA

Model	Train Accuracy	Test Accuracy
FDA + LDA	97.67%	95.33%
FDA + QDA	98.67%	96.67%

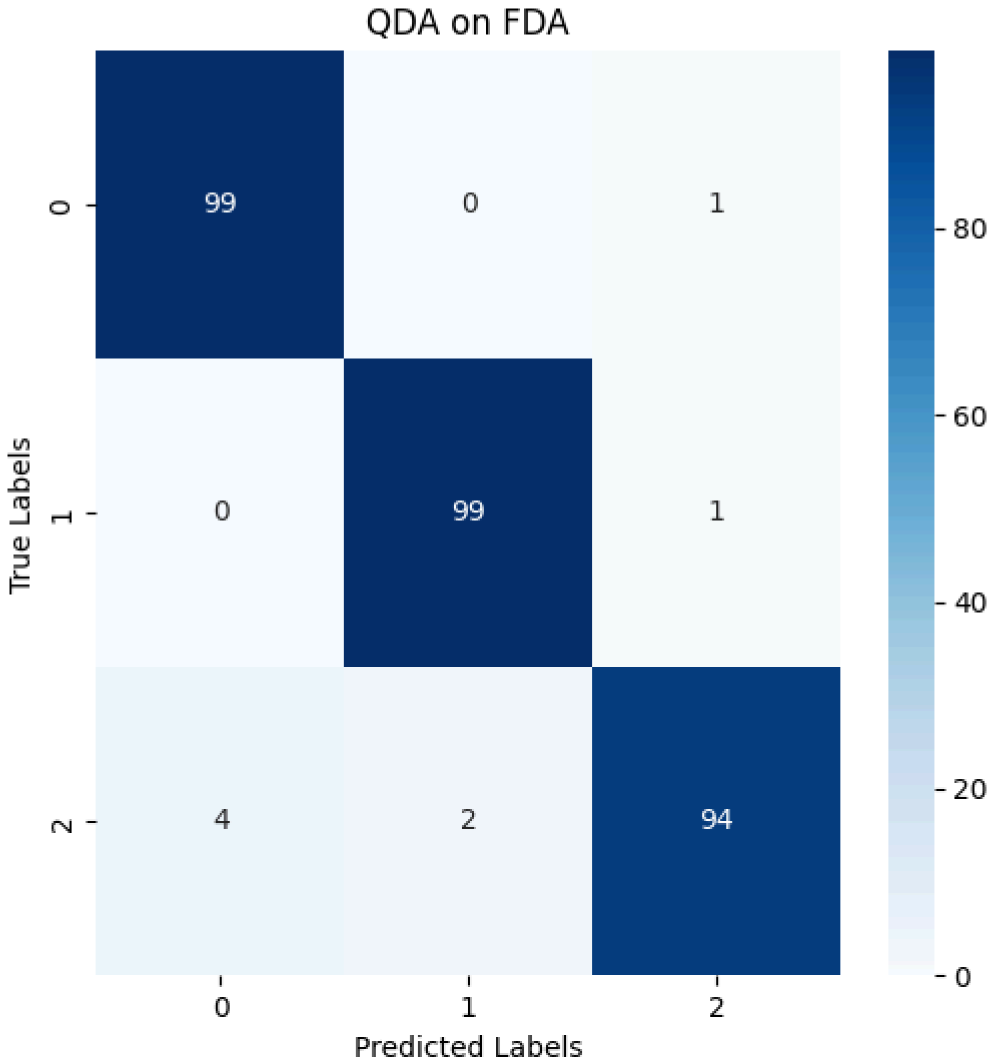
Classification Accuracy of LDA on PCA

Method	Train Accuracy	Test Accuracy
PCA (95%) + LDA	99.00%	96.00%
PCA (90%) + LDA	98.67%	96.67%
PCA (2D) + LDA	92.33%	90.00%

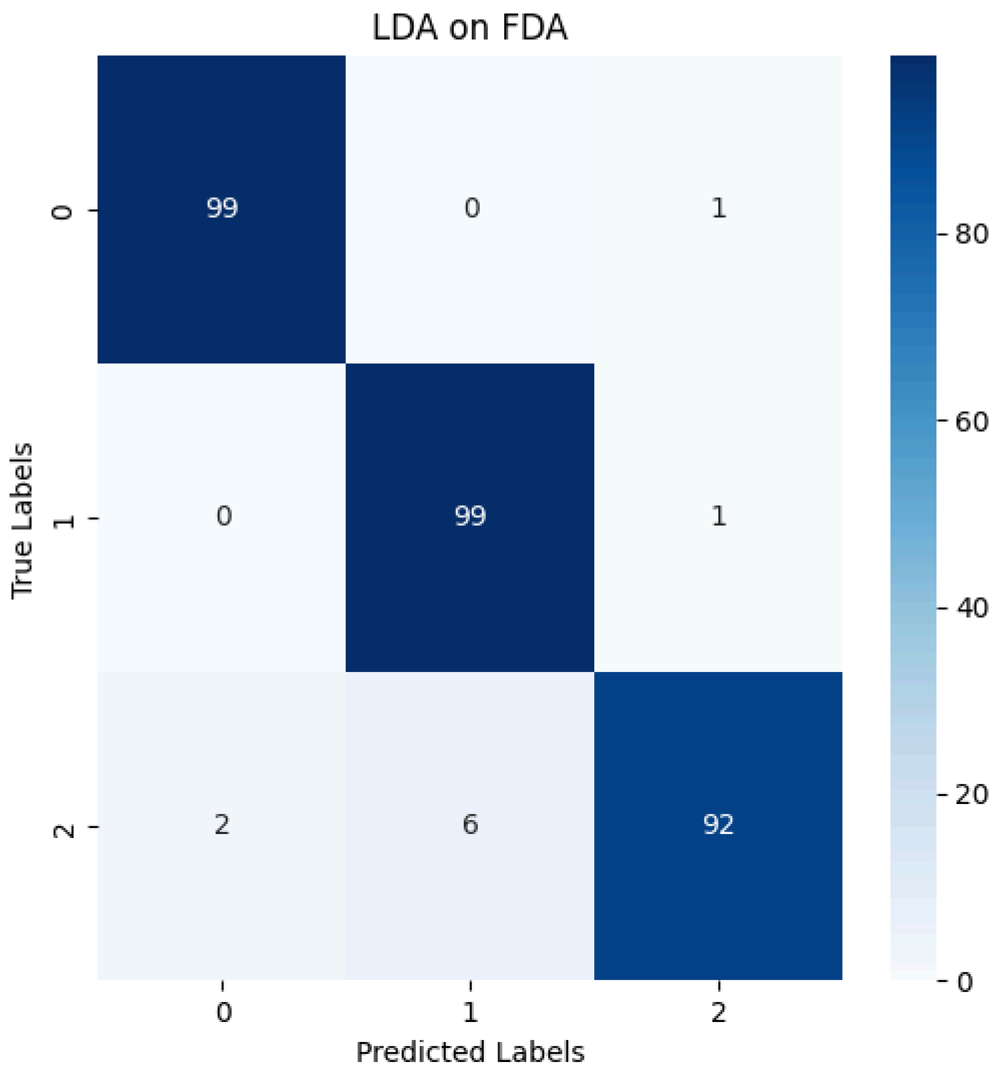
Analysis of PCA Effect on Accuracy

- **Dropping variance retention from 95% to 90% (reducing dimensions from 83 to 52) had minimal impact on accuracy.**
- **PCA (2D) performed significantly worse (~90%)**, confirming that retaining only two principal components leads to **high class overlap and information loss**.

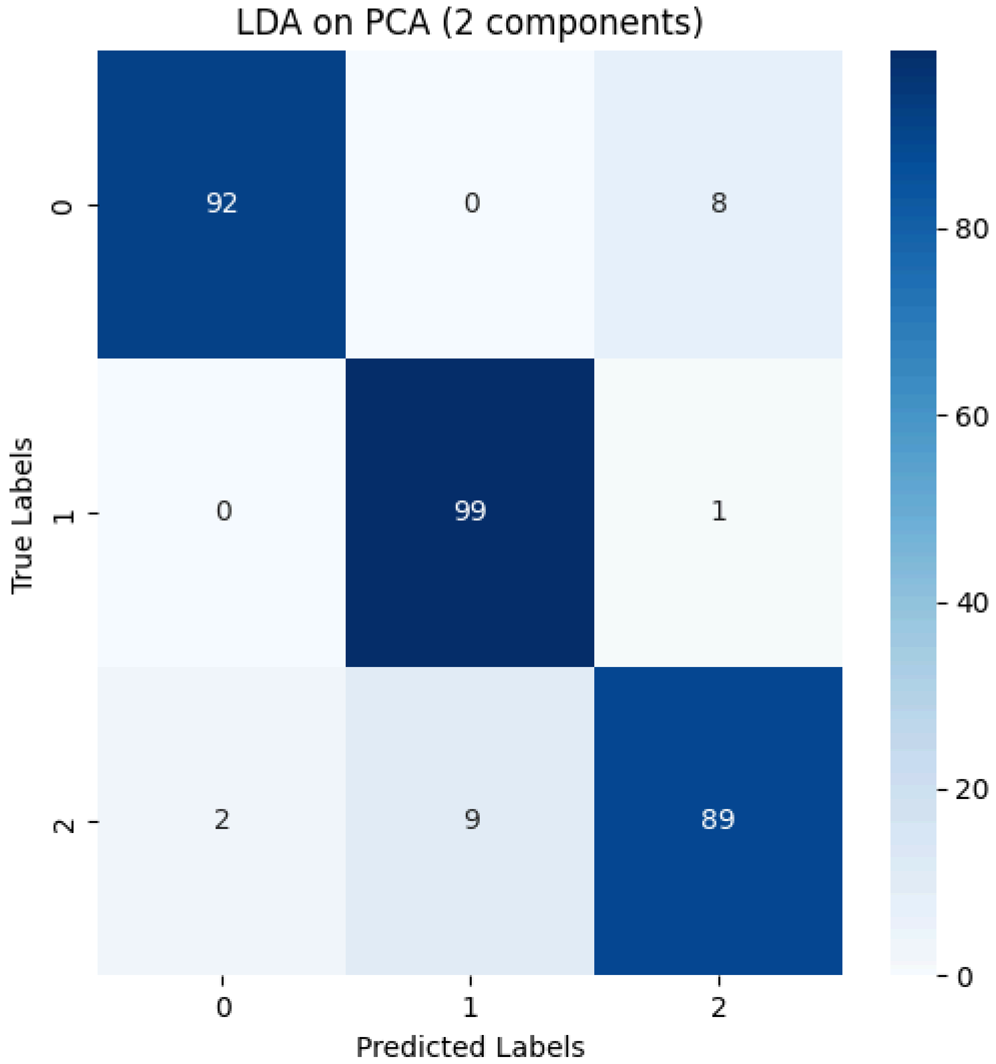
CONFUSION MATRIX



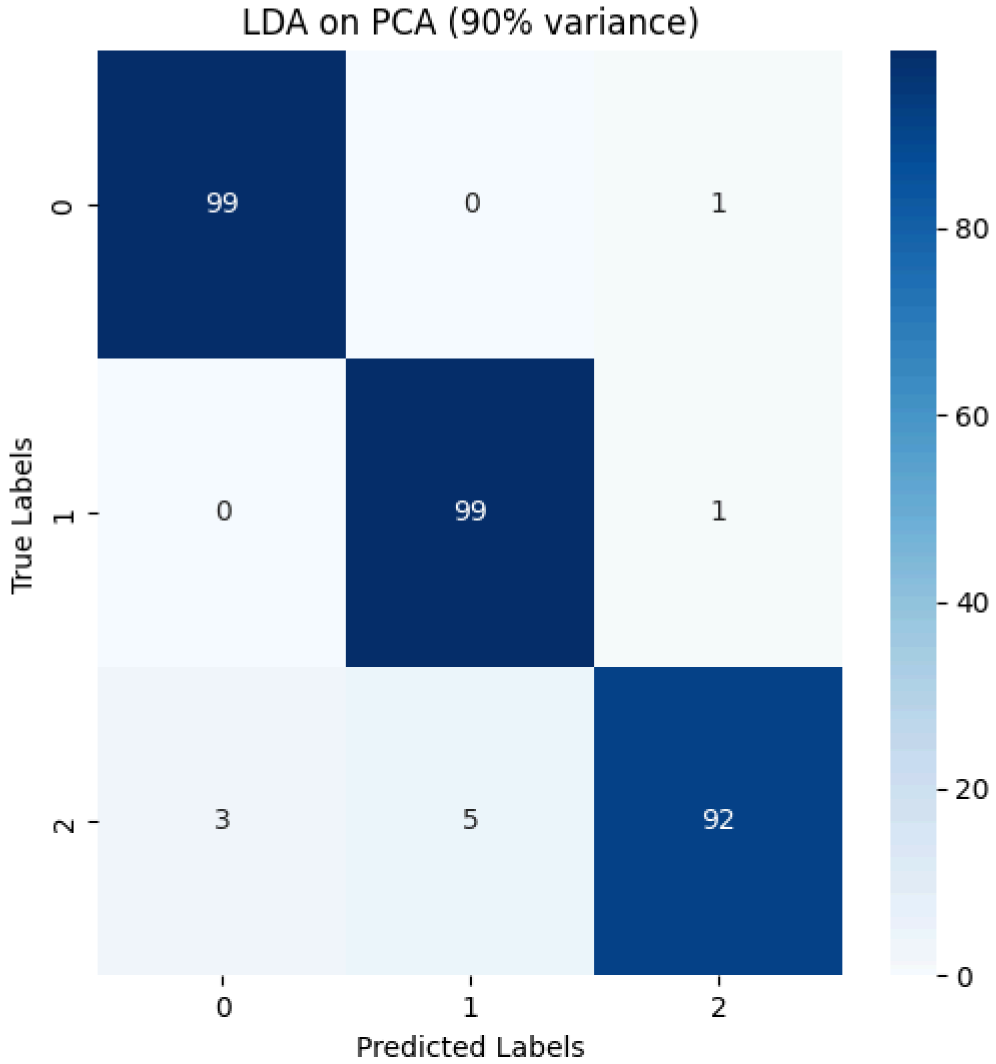
Accuracy: 0.9733
Precision: 0.9735
F1-Score: 0.9732



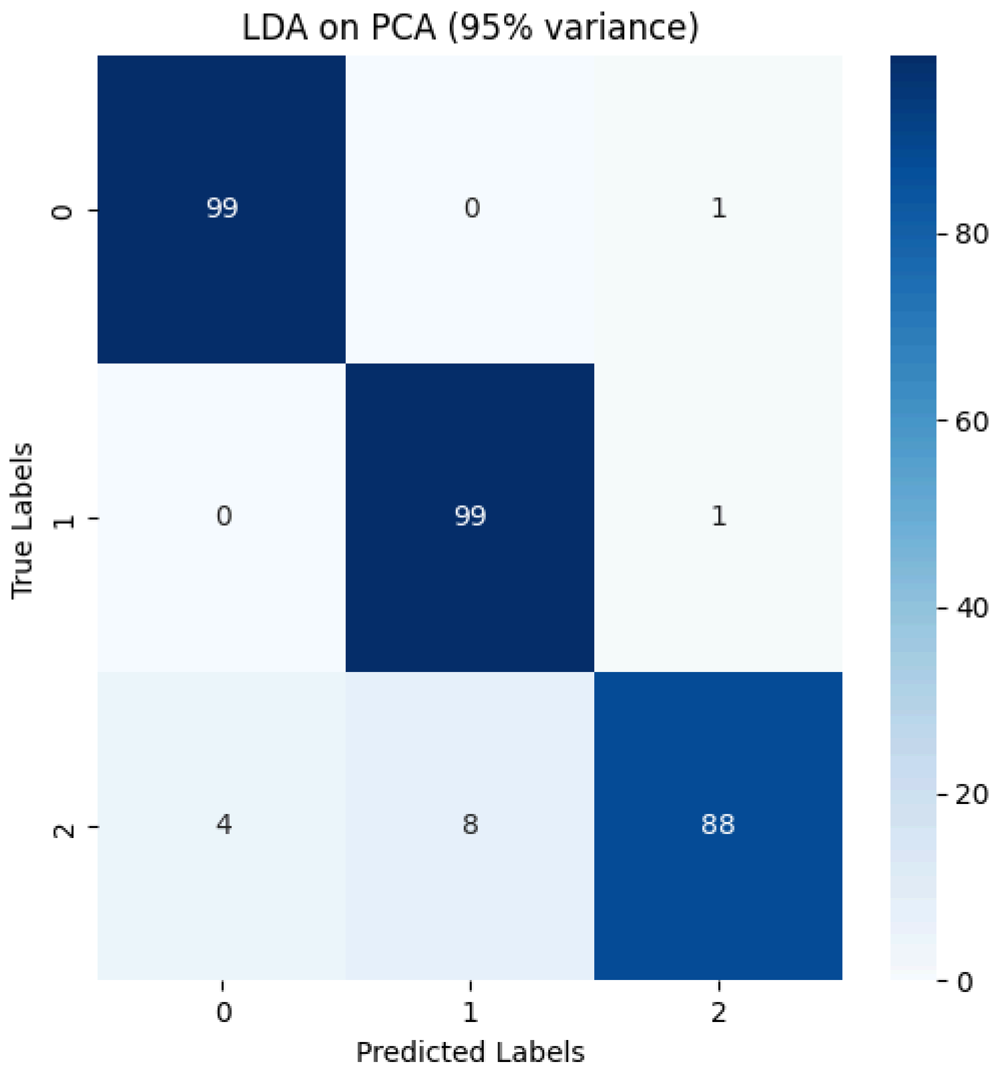
Accuracy: 0.9667
Precision: 0.9673
F1-Score: 0.9665



Accuracy: 0.9333
Precision: 0.9345
F1-Score: 0.9331



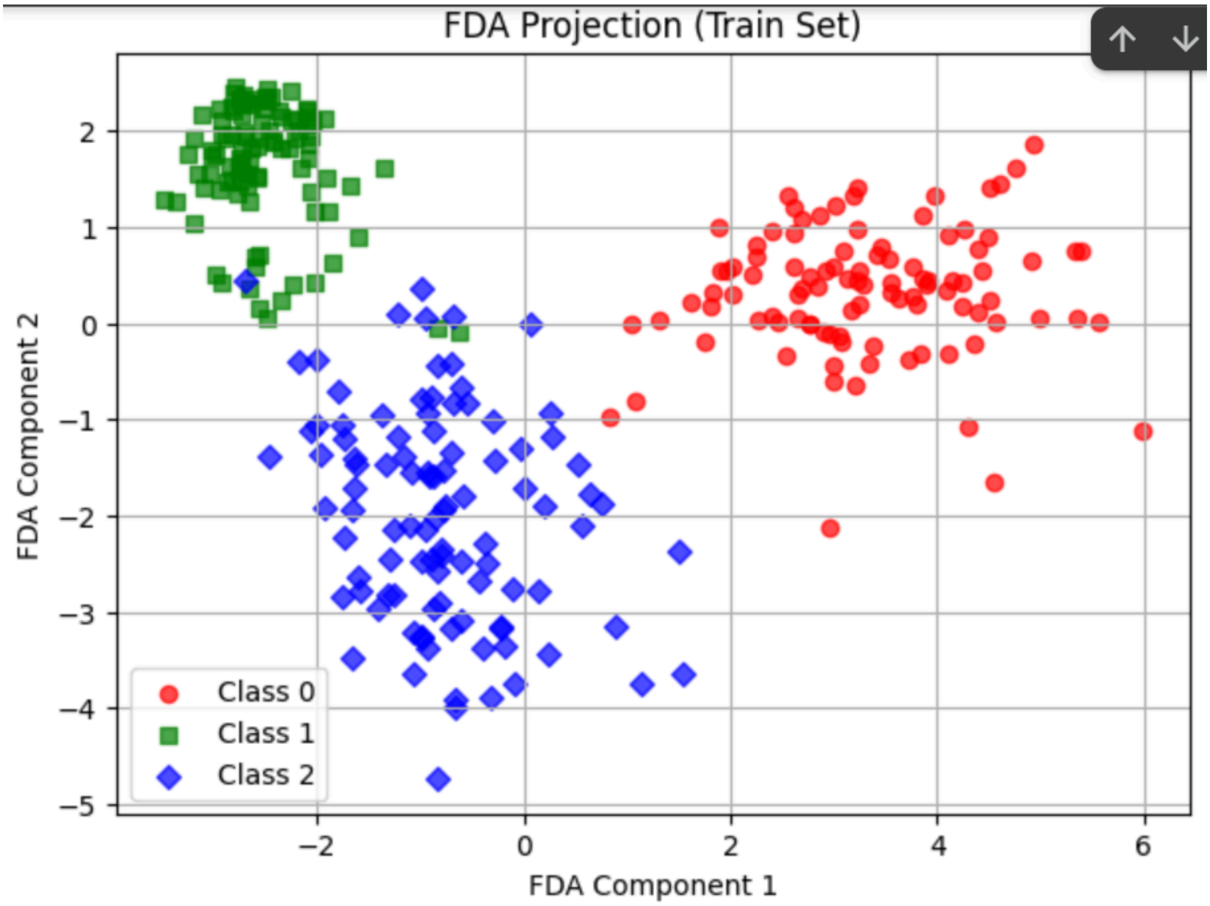
Accuracy: 0.9667
Precision: 0.9671
F1-Score: 0.9664



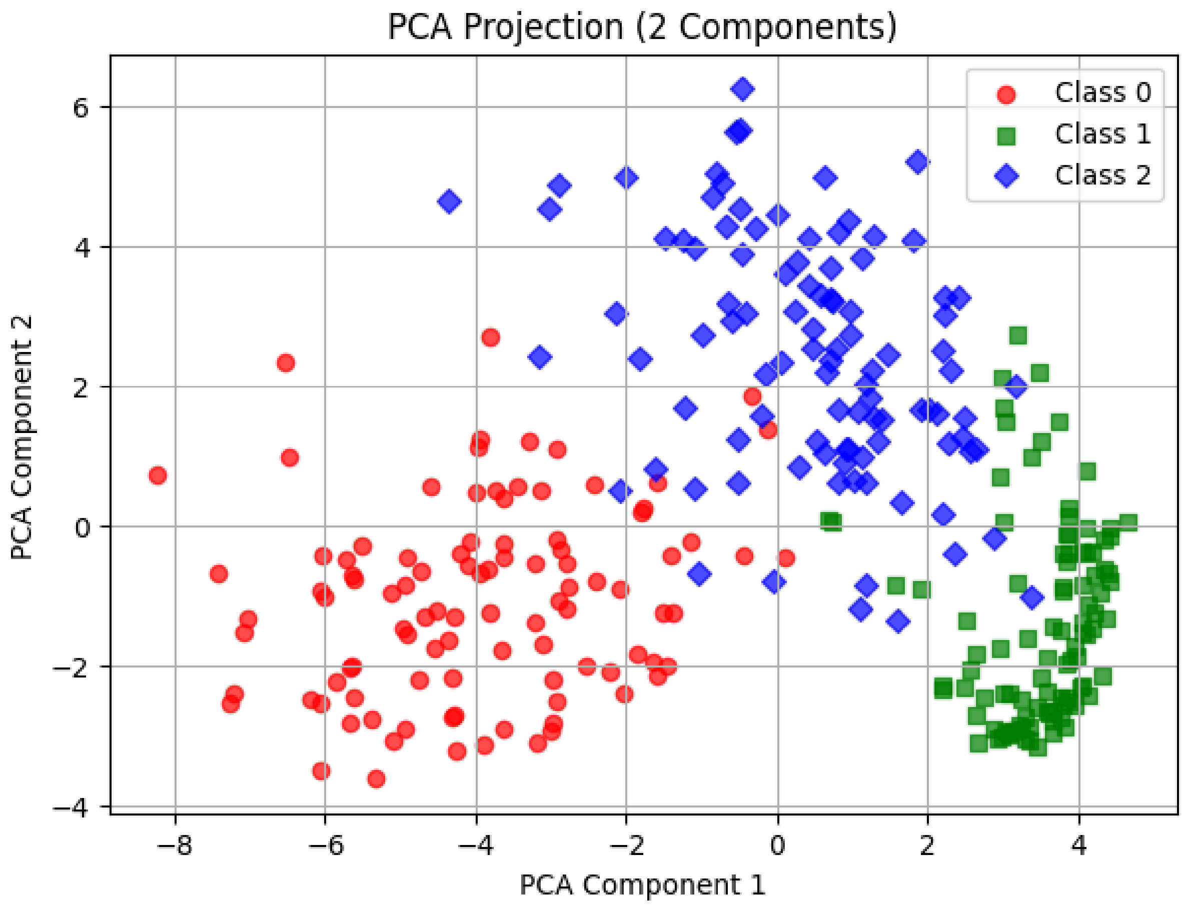
Accuracy: 0.9533
Precision: 0.9547
F1-Score: 0.9527

Visualizing FDA vs. PCA (2D) Projections

- **FDA Projection:** Clearly separated clusters → **higher classification accuracy.**



- **PCA (2D) Projection:** More class overlap → **lower classification accuracy.**



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7. Conclusion

- **FDA-based classifiers performed the best** due to its **direct optimization for class separability**.
- **PCA (95% and 90%) retained enough variance for high accuracy**, but performed slightly worse than FDA.
- **PCA (2D) significantly reduced accuracy due to poor class separation.**
- **QDA slightly outperformed LDA**, suggesting some **non-linearity in the data**.

Final Takeaway:

- **Best Method: FDA + QDA (96.67% Test Accuracy).**
- **Strong Alternative: PCA (90%) + LDA (96.67% Test Accuracy).**
- **Worst Method: PCA (2D) + LDA (90.00% Test Accuracy) due to information loss.**

FDA should be the preferred method for MNIST classification when reducing dimensions.