

SML 2025, Winter, Quiz 2, Dur. 1 hr 10 mins. Marks 9.5

[CO3] Q1 Consider the following 8 data points in a two-dimensional feature space, labeled into three classes: X, Y, and Z.

Index	Feature 1	Feature 2	Class
A	3	7	X
B	2	8	X
C	7	2	Y
D	8	1	Y
E	6	3	Y
F	7	7	Z
G	5	8	Z
H	6	6	Z

Perform a single split using the condition:

$$\text{Feature 1} \leq 5.5$$

Evaluate Gini index for left node, right node and total Gini index. [2]

[CO3] Q2. Consider a two-dimensional dataset with 6 data points as shown below:

Index	x_1	x_2	y
A	1	.5	+1
B	1	.3	+1
C	2	2.8	-1
D	3	1	-1
E	3	3	-1
F	3	2	-1

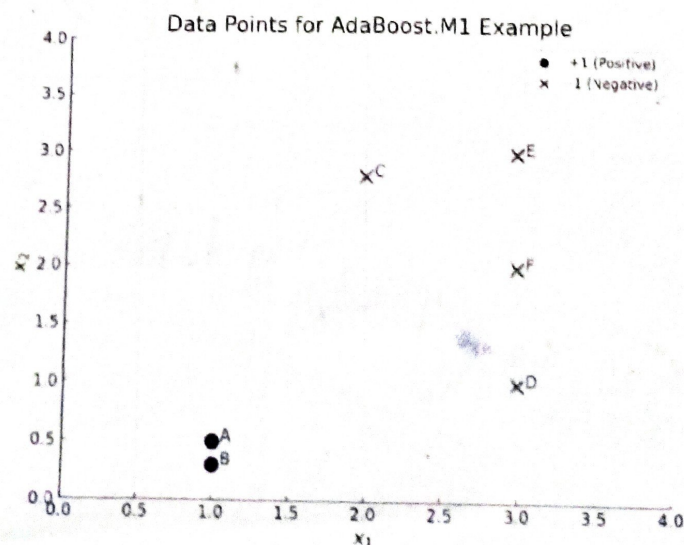


Figure 1: Illustration of the data

In the first iteration of AdaBoost.M1, the selected weak classifier $h_1(x)$ is a vertical split defined as:

$$h_1(x) = \begin{cases} +1, & \text{if } x_1 \leq 2.5 \\ -1, & \text{if } x_1 > 2.5 \end{cases}$$

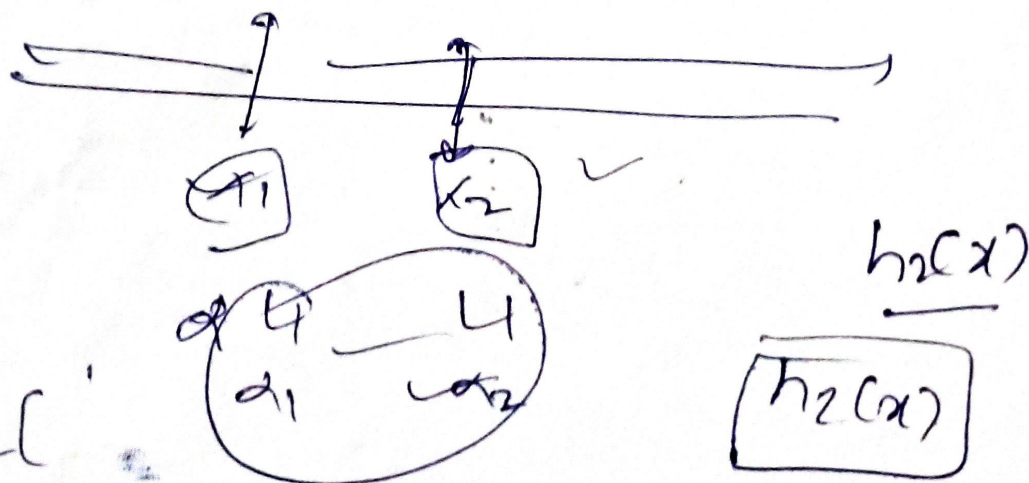
a. Find $h_2(x)$ using the boosting algorithm. Consider 2 splits only at $x_2 = 1.5$ and at $x_2 = 2.5$. You need to evaluate which cut is better. In case there is an equal proportion of samples from the classes in a region, then the decision would be the opposite sign of the other region. [3]

b. Find the boosted classifier $f(x)$ using the $h_1(x)$ and $h_2(x)$. Note $f(x)$ should give value of +1 or -1. Find the value of $f(x)$ when $\bar{x} = [1.5, 4]^T$. [1.5]

[CO1] Q3. Consider the regression model without noise $y = f(x)$. Consider a train dataset D used to obtain predictor $\hat{f}(x)$. The error decomposition can be written in the form of bias and variance. Now suppose the regression model is $y = f(x) + \eta$, where $\eta \sim \mathcal{N}(0, \sigma^2)$. Find the error decomposition for this model in terms of bias, variance and σ^2 . [3]

$$E \left(\frac{\hat{f}(x) - f(x)}{2} \right) = \hat{f}(x) + \eta$$

$$f(x) = \alpha_1 h_1(x) + \alpha_2$$



$$f = \sum w_i I(y_i \neq h_i(x))$$