SML 2025, Monsoon, Quiz 1, Dur. 1 hr 10 mins. Total marks 8

Q1. Consider a two-category problem. Let $\mathbf{x}_1 \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$, and $y_1 \sim Bernoulli(p)$. Let $\mathbf{x}_2 \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{I})$, and $y_2 \sim Bernoulli(q)$. Bernoulli(p) means that the probability that Bernoulli random variable takes 1 is with probability p. Likelihood of class 1 is given as $p(\mathbf{x}_1, y_1|\omega_1)$ and that of class 2 is $p(\mathbf{x}_2, y_2|\omega_2)$. Assume equiprobable priors. Also assume that $\mathbf{x}_i$ and y_i are statistically independent. Derive an expression for the discriminant function of class 1. [1]

Q2. Using likelihood ratio test, find the class of $\mathbf{x} = [0.5, 0.25]^\top$. Assume the classes have identity covariance and equal priors. The values of $\lambda = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$. The means are $\mu_1 = [0, 0]^\top$ and $\mu_2 = [1, 1]^\top$. μ_1 and $\mu_2 \in R^d$, where $d = 2$. [2]

Q3. Consider two exponential distributions in one dimension

$$p(x|\omega_1) = \lambda \exp\{-\lambda x\}, \quad x \geq 0 \tag{1}$$

$$p(x|\omega_2) = \lambda \exp\{\lambda x\}, \quad x \leq 0 \tag{2}$$

Assume $P(\omega_1) = P(\omega_2)$.
(a) Determine the decision boundary. [.5]
(b) For this boundary, what is the error rate for classifying ω_2 as ω_1? [.5]

Q4. Determine β^* corresponding to Chernoff bound for two category case where both the categories follow a Gaussian distribution. Both categories have same mean. Variance of category 1 is 1. Second category has variance of 2. Assume equal priors. Hint: [2]
$k(\beta) = \frac{\beta(1-\beta)}{2}(\mu_2 - \mu_1)^\top [\beta\Sigma_1 + (1-\beta)\Sigma_2]^{-1}(\mu_2 - \mu_1) + .5 \ln \frac{|\beta\Sigma_1 + (1-\beta)\Sigma_2|}{|\Sigma_1|^\beta |\Sigma_2|^{1-\beta}}$.

Q5. Suppose we have two equi-probable categories with the following underlying distributions:

$$p(x|\omega_1) \sim N(0, 1)$$
$$p(x|\omega_2) \sim N(1, 1)$$

Show that the minimum probability of error is given by [2]

$$P_e = \frac{1}{\sqrt(2\pi)} \int_{0.5}^{\infty} e^{-\frac{-y^2}{2}} dy$$