BOOLEAN ALGEBRA

- A lattice which is complimented as well as distributive is called a boolean lattice or boolean algebra
- It is generally denoted by $(B, +, \cdot, ', 0, 1)$ where

$$\begin{array}{cc} + & \cdot \\ \vee & \wedge \\ \cup & \cap \\ + & * \end{array}$$

- B is called a boolean algebra if all properties are satisfied

1. Identity law $a+0=a$ $a\cdot 1=a$
2. commutative $a+b=b+a$ $a\cdot b=b\cdot a$
3. Associative $a+(b+c)=(a+b)+c$ $(a\cdot b)\cdot c=a\cdot(b\cdot c)$
4. Distributive $a+(b\cdot c)=(a+b)\cdot(a+c)$
   $a\cdot(b+c)=a\cdot b+a\cdot c$
5. compliment:
   $a+a'=1$
   $a\cdot a'=0$

- Boolean Algebra is an Algebric structure which is based on the principle of logics

1. Closure law:
   $\forall a,b \in B \Rightarrow a+b \in B$ and $a\cdot b \in B$
2. commutative:
   $\forall a,b \in B$ $a+b=b+a$ and $a\cdot b=b\cdot a$
3. Distributives
   $\forall a,b,c \in B$ $\cancel{a+(b+c)=(a+b)}+c$ $a+(b\cdot c)=(a+b)\cdot(a+c)$
   $\cancel{(a\cdot b)\cdot c=a\cdot(b\cdot c)}$ $a\cdot(b+c)=a\cdot b+a\cdot c$

4. Identity:

$\forall a, b \in B$

$a+0=a$

$a \cdot 1 = a$

5. compliment:

$\forall a \in B$

$a+a'=1$

$a \cdot a' = 0$

6. Associative:

$\forall a, b, c \in B$

Q show that the algebric structure $(B, +, \cdot, ', 0, 1)$ is a booleanalgebra where, $B=\{0,1\}$ & $+, \cdot$ are two binary operation & compliment is unary operation on B

+	0	1
0	0	1
1	1	1

*	0	1
0	0	0
1	0	1

a	a'
0	1
1	0

A closurelaw:

$\forall 0, 1 \in B$

$0+1 \in B$

$0 \cdot 0 \in B$

commutative:

compliment:

distributive

Hence, allproperties are satisfied

identity

$\therefore (B, +, *, ', 0, 1)$ is an boolean algebra.

Q check D_{30} is boolean algebra or not

* Theorems of Boolean Algebra:-

1. Idempotent law:

$\forall a \in B$

$a + a = a \quad a \cdot a = a$

2. Dominance law:

$\forall a \in B,$

$a + 1 = 1$

$a \cdot 0 = 0$

3. Absorption law:

$\forall a, b \in B$

$a \cdot (a + b) = a$

$a + (a \cdot b) = a$

4. Involution law:

$\forall a \in B$

$(a')' = a$

5. Demorgan's law.

$\forall a, b \in B$

$(a + b)' = a' \cdot b'$

$(a \cdot b)' = a' + b'$

* Prove that $0' = 1$ and $1' = 0$

SIA

in complimented distributive lattice =)

compliment is unique

* Simplification of Boolean Algebra:-

1. Algebric method
2. K-map.

1. Algebric method:
Bring the expression in SOP form by boolean laws and demorgan theorm
(ii) Simplify SOP expression by checking the product terms for common factors.

→ Minterms → each individual term in the SOP form is called minterm.

$Y = \underbrace{ABC} + \underbrace{A\bar{B}C} + \underbrace{\bar{A}BC}$, denoted by $m_0, m_1, ...$

→ Max terms → each individual term in POS form

$Y = \underbrace{(A+B)} \cdot \underbrace{(A+\bar{B})}$ denoted by $M_0, M_1, M_2 ...$

literals

Q1. How to represent logical exp. using min terms and max terms. $Y = ABC + \bar{A}BC + A\bar{B}\bar{C}$, represent this logical exp with min terms

~~$Y = m_0 + m_4 + m_3$~~

$Y = m_7 + m_3 + m_4$ $\quad Y = \sum m(3,4,7)$

A	B	C	min	max
0	0	1	$\bar{A}\bar{B}\bar{C} (m_0)$	$A+B+C$ (M_0)
0	0	1		
0	1	0		
0	1	1		

Q2. $Y = (A+\bar{B}+C)(A+B+C)(\bar{A}+\bar{B}+C)$

$$Y = M_2 . M_0 . M_6 = \prod M(0,2,6) = \prod_M (0,2,6)$$

Q3> $Y = AB + (A+B).(\bar{A}+B)$

$$Y = AB + A\bar{A} + AB + B\bar{A} + B.B$$

$$Y = AB + 0 + AB + \bar{A}B + B$$

$$Y = AB + \bar{A}B + B = \cancel{(A+\bar{A})B + B} = (A+\bar{A}+1)B$$

$$Y = B$$

Q4> Prove that $A + \bar{A}B + AB = A + B$

Q5> $A\bar{B} + \bar{A}B + \bar{A}\bar{B} + AB$

Q6> $A\bar{B}C + \bar{A}BC + ABC$

Q7> Prove that: $AB + ABC + A\bar{B} = A$

K-Map

K-Map is a graphical representation that provide systematic method for simplifying boolean expression.

* prime implicants:

if $E + P = E$ then P is called prime implicant

expression

The condition is that $x + E \neq E$ and $z + E \neq E$ $\quad xz = E$

* Two Variable K Map:

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	00	01
A 1	10	11

A \ B	$\bar{B}$	B
$\bar{A}$	$\bar{A}\bar{B}$	$\bar{A}B$
A	$A\bar{B}$	AB

* Three variable K-map:
* can be drawn in 2 ways

A \ BC	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
0 $\bar{A}$	0	1	3	2
1 A	4	5	7	6

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

* four variable K Map:

AB \ CD	00	01	11	10
$\bar{A}\bar{B}$ 00	0	1	3	2
$\bar{A}B$ 01	4	5	7	6
BC 11	12	13	15	14
$B\bar{C}$ 10	8	9	11	10

Q using K-map simplify:

$F(A,B,C,D) = \sum m(4,5,6,7,8,9,10,11,12,13,14,15)$

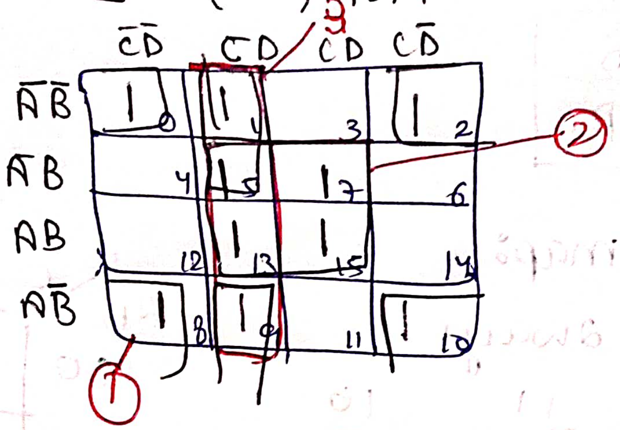
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	(0)	(1)	(3)	(2)
$\bar{A}B$	1 (4)	1 (5)	1 (7)	1 (6)
AB	1 (12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$	1 (8)	1 (9)	1 (11)	1 (10)

B

A

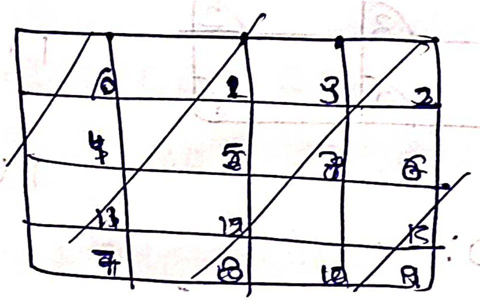
$= A+B$

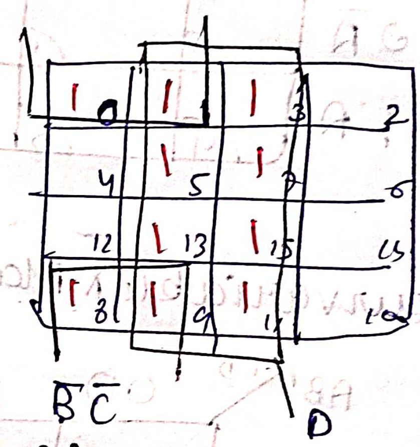
2. $f(A,B,C,D) = \sum m(0,1,2,5,7,8,9,10,13,15)$



$B D + \bar{B}\bar{D} + \bar{B}\bar{C}D$

3. $\sum m(0,1,3,5,7,8,9,11,13,15)$





4. $f(A,B,C) = \sum m(2,3,4,5) + \sum d(6,7)$