MAL7080 Mathematical Modeling and Simulations

10 marks | Tutorial 3 | Due date: 1 May, 2024

- You need to submit this tutorial in Jupyter notebook form.
- Late submissions will not be considered.

1. Implement the following numerical differentiation and numerical integration methods: **[6]**

   (a) First order forward finite difference approximation for y'
   (b) Second order central finite difference approximation for y'
   (c) First order forward finite difference approximation for y''
   (d) Second order central finite difference approximation for y''
   (e) Composite Trapezoidal rule for integration
   (f) Composite Simpson's 1/3 rule

2. Consider the following data of a distance travelled by a vehicle: **[1]**

t [s]	0	2	4	6	8	10	12	14	16
x [m]	0	0.7	1.8	3.4	5.1	6.3	7.3	8.0	8.4

   (a) Find the velocity and the acceleration at $t = 10$ s.
   (b) Find the velocity and the acceleration at $t = 5$ s.

3. Write a code to obtain first derivative estimates for the following unequally paced data: **[1]**

x [s]	0.6	1.5	1.6	2.5	3.5
$f(x)$ [m]	0.9036	0.3734	0.3261	0.08422	0.01596

   where $f(x) = 5xe^{-2x}$. Compare your results with the true derivatives.

4. The amount of mass transported via a pipe over a period of time can be computed as **[1]**

$$M = \int_{t_1}^{t_2} Q(t)c(t)\, dt$$

   where M= mass (mg), t_1= the initial time (min), t_2= the final time (min), $Q(t)$= flow rate (m^3/min), and $c(t)$= concentration (mg/m^3). Suppose that the temporal variation in flow and concentration are defined as:

$$\begin{aligned} Q(t) &= 9 + 5\cos^2(0.4t), \\ c(t) &= 5e^{-0.5t} + 2e^{0.15t} \end{aligned}$$

Determine the mass transported between $t_1 = 2$ and $t_2 = 8$ min with (a) Composite Trapezoidal rule for integration, (b) composite Simpson's 1/3 rule, (c) composite Simpson's 3/8 rule. Compare your results with the in-built function *quad*.

5. Find the value of $\displaystyle\int_0^1 xe^{-x}\,dx$ using the above discussed numerical integration methods. Play with the number of sub-intervals. Compare your answer with the exact answer $1 - \dfrac{2}{e}$. [**1**]

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