MAL7080 Mathematical Modeling and Simulations

Tutorial 1

1. Implement Gauss-Jacobi method and Gauss-Seidel method in Python. You need it to solve the next problem.

2. Modeling of steady state heat conduction phenomena leads to elliptic partial differential equations. One of the well known elliptic partial differential equations is so called Laplace's equation. Suppose we are solving the Laplace's equation

$$
\begin{aligned}
u_{xx} + u_{yy} &= 0 \text{ on } \Omega = \{(x, y)|0 \leq x \leq 4,\ 0 \leq y \leq 4\} \\
u(x, 0) &= 20 \\
u(x, 4) &= 180, \\
u(0, y) &= 80, \\
u(4, y) &= 0,
\end{aligned}
$$

using a 5-point finite difference numerical scheme with number of sub-intervals in x-direction $N_x = 4$, and number of sub-intervals in y-direction $N_y = 4$. It gives the following matrix equation:

$$
\begin{bmatrix}
-4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4
\end{bmatrix}
\begin{bmatrix}
p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9
\end{bmatrix}
=
\begin{bmatrix}
-100 \\ -20 \\ -20 \\ -80 \\ 0 \\ 0 \\ -260 \\ -180 \\ -180
\end{bmatrix}
\tag{1}
$$

Solve this matrix equation.

Ans: $\mathbf{P} = \begin{bmatrix} 55.7143 & ??? & 27.1429 & ??? & 70.0000 & ??? & 112.857 & ??? & 84.2857 \end{bmatrix}^T$.

3. Modeling in action: An oil company wants build 25 cuboid tanks with a combined capacity of 28,000 cubic meter. Three different types of tank designs are under consideration: a 10-meter wide tank with a capacity of 350 cubic meter, a 14-meter wide tank with a capacity of 700 cubic meter, and a 24-meter tank with a capacity of 1,400 cubic meter. How many of each type of tank the company can build?

4. Pick a city of your choice. Pick an year of your choice. Get the daily maximum and minimum temperature data for that year for that city. So you should have about 365 data points for each series. Analyse the temperature trend using the curve fitting technique. Mention your data source.

5. Data science in action: In a certain course taught by me, the performance of the students is as follows:
Sr.No: Student identifier, T1: Marks in Test 1, T2: Marks in Test 2, T3: Marks in Test 3, T4: Marks in Test 4, G: Grade point

Sr. No.	T1	T2	T3	T4	G
428	17	15	28	29	10
215	15	18	30	27	10
221	16	20	23	31	10
159	15	16	28	28	10
247	14	15	29	32	10
146	15	17	25	31	10
222	12	11	29	24	9
449	14	13	28	23	9
225	18	18	25	23	9
257	17	13	23	26	9
299	13	15	20	29	9
315	10	12	24	27	9
329	13	15	20	28	9
460	15	10	26	22	8
507	9	16	20	27	8
171	14	14	25	22	8
232	13	15	15	28	8
249	13	17	23	21	8
389	14	10	18	19	7
484	9	9	24	16	7
485	12	8	19	20	7
506	15	7	16	20	7
63	13	8	15	23	7
141	15	12	17	22	7
280	2	10	17	14	6
324	0	0	22	9	6
492	11	3	16	12	6
400	8	13	8	13	6
34	3	1	18	12	6

Analysing the above data, *recommend* the grade points for the following students:

Sr. No.	T1	T2	T3	T4	G
127	10	7	15	20	(?)
365	13	15	25	18	(?)
210	18	18	28	27	(?)
34	3	1	18	12	(?)

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