Random Geometric Graphs as Model of Wireless Sensor Networks

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Abstract— **Wireless Sensor Networks (WSN) are likely to be widely deployed in the near future because they greatly extend our ability to monitor and control the physical environment. Finding a good network model for WSN is of great importance and has been the subject of several publications. Many models have their origin in classic areas of theoretical computer science and applied mathematics: regardless of the radio technology used, from the topology point of view, at any instant in time a WSN can be represented as a graph with a set of vertices consisting of the nodes of the network and a set of edges consisting of the links between the nodes. However, future applications will involve a large number of sensors to deploy on inaccessible areas, leading to a random deployment which makes the classical graph models obsolete. In this paper, we show that the random strategy is the only way to deploy future WSN and therefore the modeling with Random Geometric Graphs is the most appropriate.**

Keywords: Wireless Sensor Networks, Future Applications of Wireless Sensor Networks, Random Deployment, Modeling, Random Geometric Graph.

I. INTRODUCTION

Advances in MEMS technology, wireless communications and digital electronics have enabled the development of multi-functional sensor nodes. Sensor nodes are small mini- aturized devices which consist of sensing, data processing and communicating components [1], [5], [15](see Figure 2). These inexpensive tiny sensors can be embedded or scattered onto target environments in order to monitor useful informa- tion in many situations. We categorize their applications into law enforcement, environment, health, home and other com- mercial areas. Moreover, it is possible to expand this classif- ication with more categories including space exploration [11] and undersea monitoring [16].

Many of the requisite features of a **WSN** such as connectivity and coverage (the definition of coverage depends on the use of the network, but usually is either the area covered by the sensors or the ability to detect moving objects) are significantly affected by the deployment strategy, capabilities of the sensors, the environment in which they are deployed, their locations within that envi- ronment, and their interactions with each other. Deployment strategies are designed to meet optimization criteria in coverage and cost of deployment under various constraints. Hence, sensor deployment strategies play a significant role in determining the performance of sensor networks. As oppo- sed to traditional networks, wireless networks are often composed of a number of nodes that can be several orders of magnitude higher than the size of conventional networks [1]. It is mainly due to the fact that, the tiny sensor nodes have limited physical capabilities, they are equipped with batteries with limited power. On most applications, it is infeasible to recharge or replace their batteries. Consequently, sensor deployments with reasonable densities are fundamental in order to prolong the networks lifetime : in an insufficiently dense network, it is necessary to have all nodes operating simultaneously in the active mode to increase the coverage of the monitored region. Sensor nodes can be deployed in several ways in a given deployment region and they are often deployed inside a medium. Therefore, the positions of these nodes need not be engineered or pre-determined. This allows rapid deployment in inaccessible terrains and suits well the specific needs to disaster-relief, law enforcement, collaborative computing and other special purpose appli- cations. As a result, practical deployment in large sensor networks is usually random.

From the computational perspective, it has been recognized that the successful design of algorithms performed on such networks, including routing, ranking and recommendation algorithms, depends on the modeling. Therefore, the expec- ted deployment requires efficient sensor network modeling techniques to facilitate initial programming of the sensor nodes and their eventual reprogramming once the network is deployed.

II. RELATED WORKS

There are two fundamentally different ways to deploy sensors: random deployment (such as Poisson point process or uniform distribution); or deterministic deployment (such as grid deployment). Intuitively, deterministic deployment seems to require a fewer number of sensor nodes to achieve a given degree of coverage and connectivity than random depl- oyment. In practice, however, it is usually infeasible to dev- ise a deployment strategy whereby each sensor is placed precisely at some location. Several researchers [3], [4], [6], [7], [9], [13], [17] have addressed various methods to study fundamental characteristics of the underlying graphs; since the structures of the graph affect almost all algorithmic aspects of the networks, such as connectivity, degrees, diameter and minimum spanning trees; when their nodes are randomly deployed.

The Erdős-Rényi random graph [12] is one of the best studied models of a network [19]. This model possesses the considerable advantage of being exactly solvable for many of its average properties. Erdős and Rényi gave a number of versions of their model. The most commonly studied is the one denoted $G_{n,p}$, in which each possible edge between two vertices is present with independent probability p and absent with probability $1-p$. It is obvious that the $G_{n,p}$ graph model is not a realistic representation of a **WSN**: in **WSN** two nodes at close range have a higher probability of being connected than nodes at farther distance. Other models are Regular lattice and Scale-free graphs (see [10] for a comparison of network models). However, like the $G_{n,p}$, these models are unrealistic when used to represent a **WSN**. Having considered the previous models, the random geometric graph denoted $G(n,r)$, where the n nodes (or sensors) locations are chosen uniformly and independently at random (according to some probability distribution) in a region and two points are connected by an edge if they are within a distance less then a certain value r, seems the most realistic. Random geometric graphs have been proposed to model **WSN** before, see for example (non-exhaustive list) [3], [7], [10], [13], without giving any actual arguments to justifie its use. We will give arguments to show that this model is the most relevant and of course, the most realistic.

III. WSN CHARACTERISTICS

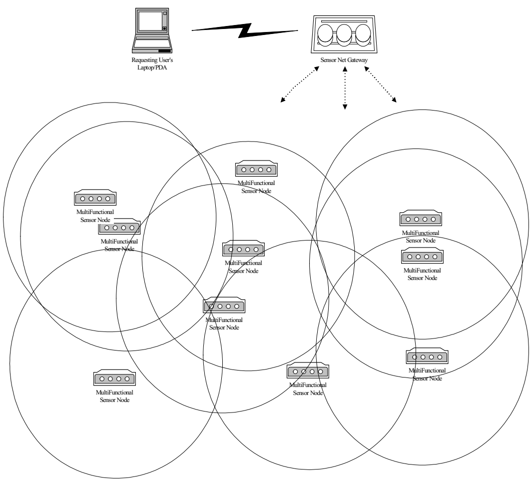

Figure 1. Typical multihop WSN architecture

First, we give some definitions:

- *Sensor:* The device that measures a physical quantity and converts it into a signal which can be read by an observer or by an instrument (another sensor). Typically, it consists of these components: sensing hardware, memory, battery, embedded processor, and trans-receiver (and much more, see Figure 2).
- *Observer*: The end user interested in obtaining information disseminated by the sensor network about the phenomenon. The observer may indicate interests (or queries) to the network and receive responses to these queries. Multiple observers may exist in a sensor network.
- *Phenomenon*: The entity of interest to the observer that is being sensed and potentially analyzed/ filtered by the sensor network. Multiple phenomena may be under observation concurrently in the same network.

In a sensing application, the observer is interested in monitoring the behavior of the phenomenon under some specified performance requirements. In a typical sensor network, the individual sensors sample local values (measurements) and disseminate information as needed to other sensors and eventually to the observer.

We define a sensor wireless network as a group of nodes that can communicate with each other over a wireless channel. The nodes (or processors) come without ready-made links and without any centralized controller. Since large number of sensor nodes are densely deployed, neighbor nodes may very close to each other: each node may be connected to other nodes in its vicinity. Hence, multihop communication in sensor networks is expected to consume less power than the traditional single hop communication (see Figure 1.).

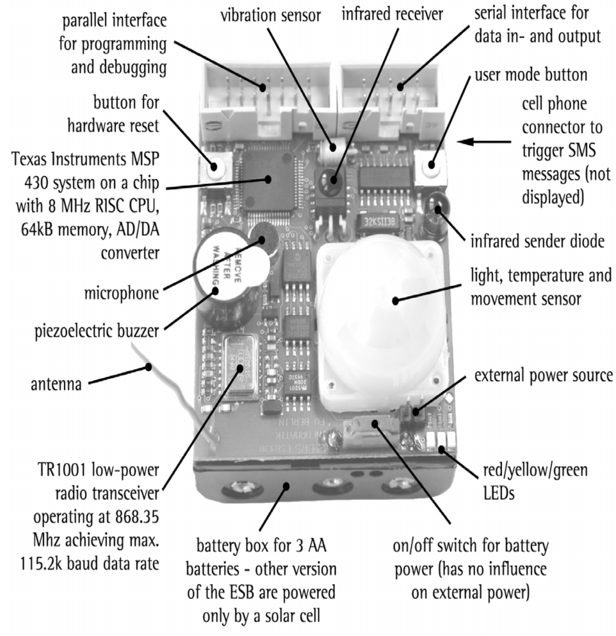

Figure 2. Overview of the components of the Embedded Sensor Board (ESB) from the FU-Berlin

IV. DEPLOYMENT

In this section, the problem of large **WSN** deployment is formulated and solutions are identified.

Sensors can be placed in an indoor environment at convenient locations. Alternatively, hundreds or thousands (or much more as argued below) of them can be placed in a field. For example, sensor nodes can be air-dropped from a helicopter to cover an open field and report on vehicular activity or troop movement. In some regions, such as desert or the jungle, there is no terrestrial communications infrastructure. In other regions, access is unavailable because of destruction of or the damage to the local communications infrastructure. Space sensor scenarios are also envisioned, consisting of a large sensor field; hundreds of sensor nodes air-dropped from a spacecraft over a geographical area in another solar system planet; and multiple rovers also placed randomly in this field and in charge of collecting periodic measurements and relaying data and commands with Earth users.

Consequently, the deployment strategies are defined by two criteria: the network density (number of nodes) to deploy and the type of area to cover. More precisely, the aim of this section is to give arguments for random deployment.

A. Density : The network size

Define network size usually refers to the number of network nodes, but it can also refers to the geographical area covered by the network; both are critical parameters for coordinating network actions with distributed control mechanisms. Taken together, the number of nodes over a given geographic area defines the network *density*. Numerous strategies exist, aided by fact that sensor technology provides an expansive array of available hardware.

A fundamental dichotomy exists in the approach to sensing an environment based on the number and the complexity of sensors. For a fixed cost, one can deploy a small number of sophisticated “global" sensors with high signal complexity and precise readings. In contrast, one can deploy a large number of small, coarse, "local" devices that may have large uncertainties in their readings.

With technological advances, future applications will involve a very large number of inexpensive and sophisticated sensors to cover large areas in order to perform specified tasks. Minimizing the number of nodes deployed may not be critical when the cost of node is negligible and we have excessive number of nodes. It is expected that the cost of sensors will decrease as the technologies advance. As a result, if we consider a small density, it is easy to place the nodes one by one, but soon the number grew, assume only a hundred nodes; it becomes more difficult and too expansive to deploy those nodes in a deterministic way.

B. Areas type : Inaccessible lands

1) Environment

To improve environmental monitoring sensors can be scattered from airplanes and balloons to report any environ-mental problems (fire, pollution, epidemics, etc..) in order to improve our knowledge of the environment and indicate effective ways to quickly solve those problems.

On industrial sites, nuclear for example, the sensors can be deployed in a network to detect leaks of toxic products (gas, chemicals, radioactive elements, oil, etc...) to alert users, to allow effective intervention. A large quantity of micro-sensors could be deployed in the dense forest like Amazon rainforest or in some protected areas to collect information on the status of natural habitats and behavior of wildlife. Sensors eaten by animals or under their skin are already used, so it becomes and will be more possible to observe "Biodiversity" without disturbing the animal species vulnerable to disturbance and difficult to study in their natural environment, and propose more effective solutions for wildlife conservation. Until now, the lack of environmental appropriate data in certain areas like the north and south Poles, which is difficult to study because of dangerous terrain and a highly varied climate, prevented the development of tracking devices and simulation sufficiently reliable. To remedy to this and in order to measure and exploit the environmental data, the deployment of smart sensors capable of recovering some very accurate data and make measurements of parameters such as rainfall, temperature and wind in areas considered inhospitable will be the solution. These heterogeneous sensor networks will be interconnected to dedicated software that can transmit information to different research groups. The aim is to refine methods for monitoring and forecasting of the environment in arctic regions. These infrastructures, which will predict more reliably the effects of climate changes on arctic areas, will improve our ability to predict, and ultimately to improve our models for predicting natural disasters such ice melting. The possible consequences of the mass dispersion of micro-sensors in the environment have raised several concerns. Indeed, they are usually equipped with a micro-battery containing harmful metals. However, the deployment of one million sensors of 1 cubic millimeter each represents a total volume of one liter. Even if all the volume was made up of batteries, it would not have a disastrous impact on the environment.

2) Space scenarios : The red planet

The projects focus on space has other great features. Most scientific missions are unique, with very few opportunities for continuous learning as permit applications on earth. The NASA Mars rovers are continuing their mission of exploration in the Martian soil. The mission was originally scheduled for 3 months only, but since, they cling to the Red planet. Avoiding the shadows and putting their robots on the slopes exposed to sun, engineers at the Jet Propulsion Laboratory were able to give them the extra vitality that allowed them to survive in the extreme cold of the Martian winter, -65 degree. The Martian atmosphere is also coming to the rescue robots. The 'dust devil', gusts of wind swirling, came opportunely clean their solar panel. But the weight of age begins to weigh, limiting scientific experiments. The use of self-organizing sensor networks is envisioned to play a key role in space exploration, such as for future in situ exploration of Mars. In this mission concept, numerous sensor nodes are deployed to the surface of Mars via spacecraft. Each node is equipped with one or more measurement instruments, a radio, and some associated processing capability. The nodes communicate with each other in order to cooperate autonomously and to collect scientific measurements (seismic, chemicals, temperature...). A sensor field can thus be viewed as a distributed instrument providing time and position dependent measurements. One or more landers or rovers function as base stations by periodically (or on demand) collecting measurements and relaying the aggregated sensor field results to Earth (possible via an intermediate relay orbiter). Reliable even in extreme conditions, these sensors supplied with energy from solar panels, will remain permanently on Mars, with a life expectancy of years.

V. MODELING

In this section, we define the random geometric graph and give the main assumptions of the studied system model.

Before an actual deployment, we need a theoretical study of such networks. How the topology of the nodes can be modeled in a realistic way? To develop algorithms for sensor networks and in order to give mathematical proofs of their correctness and performance, appropriate models are needed. On the one hand, a model should be as simple as possible such that the analysis of a given algorithm remains tractable. On the other hand, however, a model must not be too simplistic in the sense that it neglects important properties of the network. Many models for sensor network have their origin in classic areas of theoretical computer science and applied mathematics. Models for reachability (communication) graph are essential to the theoretical analysis and the design of efficient algorithms and their simulation.

Regardless of the radio technology used, from the topology point of view, at any instant in time a **WSN** can be represented as a graph with a set of vertices consisting of the nodes of the network and a set of edges consisting of the links between the nodes. Two nodes can communicate or are connected with each other if they are close, which can be formalized: if and only if the Euclidean distance between them is smaller or equal to a transmission range. It is reasonable to assume that wireless links are bidirectional in the sense that an edge *(n,m)* exists if and only if there exists *(m,n)*.

A random **WSN** cannot be modeled as a pure random graph, as discussed in previous sections, in a random **WSN** the actual set of connections, in contrast, to random or scale-free graphs, depends on the geometric distance between nodes. Based on these observations, the most appropriate topology model is the random geometric graph; such random graphs are created by choosing n independent and identically distributed points from some probability distribution on d-dimensional space. Then, two vertices are joined by an edge exactly when they lie within a certain distance, of each other. In comparison to the well-known and well-studied Erdős-Rényi random graph [12], properties of random geometric graphs are less developed (see for instance the problems left open in[13]).

For correctness, we consider the case where nodes lie in planar Euclidean domain. Each node can perform some sensing task within a certain radially symmetric neighborhood. Within this *coverage disk*, the sensor performs its specified task. We assume the region R to be monitored is of a square region with side length l and of size $S=l^2$. We further assume torus convention (a.k.a. the toroidal model, see [9, p. 23] and [14]) i.e., each disk that protrudes one side of the region R enters R again from the opposite side. This eliminates consideration of boundary effects. Each sensor node can detect an event of interest within a distance of r, and this distance is termed as the sensing range. The disk centered at a sensor node and with a radius of r is termed as the coverage disk of this node. Without loss of generality, we assume that each sensor node has a sensing range of $r=1/\pi$ and thus each sensor node can cover a disk of unit area and we assume that $l >>r$, see Figure 3. for an example with 1000 nodes.

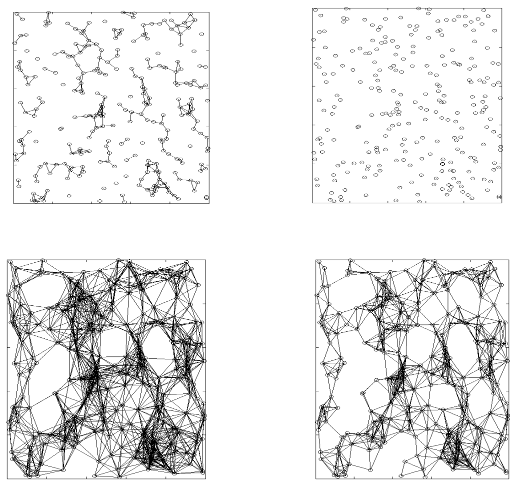

Figure 3. A typical radio network is generated according to the uniform distribution of coordinates of the devices. The transmission ranges of stations are gradually increasing from left to right.

We now have a well defined mathematical tool, with interesting properties and very close to reality. It is therefore possible to study accurately the behavior of **WSN**, such as connectivity (see[7]and [13]), k-coverage and lifetime that we have discussed in a previous article [18].

VI. FINAL REMARKS

The main purpose of this paper has been that of investigating the deployment strategy of a large WSN and their modeling. We briefly gave an overview of future applications, whether the high density or inaccessible area, to show that their deployment will necessarily be random and therefore the modeling with random geometric graphs is the most appropriate. We have argued that the modeling of WSN is needed to study such networks, however it some-times oversimplifies the reality. The next step will be to imp-rove it by taking into account more technical aspects.

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