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Integrated Production Inventory Routing Planning with Time Windows for Perishable Food*

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Abstract — **This paper investigates an integrated production inventory routing problem with time windows where a central depot is responsible for supplying single type of perishable food to multiple retailers within the planned time horizon. A mixed integer linear programming (MILP) model aiming at maximizing the total profit is formulated with explicitly tracing the food quality. To strengthen the formulation, a series of valid inequalities are introduced. Randomly generated instances with up to 40 retailers and 3 time periods are used to verify the effectiveness and the complexity of the proposed model, which is solved by the linear programming solver CPLEX. The computational results show that the proposed model is able to provide integrated plan for the decision makers, and instances with 20 retailers and 3 time periods are optimally solved with 102.97s on average. The results also indicate that the introduced valid inequalities are useful in helping CPLEX generate better upper bounds (maximization problem) for 20 out of 23 instances that are not optimally solved within the time limit.**

I. Introduction

Food supply chain (FSC), which has been more and more important to the food industry in today's business environment, refers to the process where farm produce moves from farms to customers. This process includes food production, processing, distribution, consumption and waste disposal. In a typical FSC, these activities are often planned and optimized sequentially based on upstream decisions. However, various studies and practices have shown that such decentralized FSC is less competitive and may lead to food quality distortion [1]. In addition, the availability of the advanced information and communication technologies (ICT) makes it possible and realistic to make more complex decisions in an integrated way. The need of close coordination along the FSC process and the development of the ICT have prompted the development of several new business models in FSC [2]. One representative is the development of vendor managed inventory (VMI), where the vendor has the responsibility to control the inventory of the supplied retailers or outlets, and to determine when and how much to deliver over a finite planning horizon. The advantage of the VMI policy with respect to traditional retailer managed inventory systems lies in a more overall efficient resource utilization due to the integration of the concurrently separately determined inventory and routing decisions [3][4]. This has motivated the development of inventory routing problem (IRP). Going one step further, one may expect to benefit more with the integration of production decision within the VMI policy, then comes the production inventory routing problem (PIRP). PIRP is an integrated operational planning problem that jointly optimizes production, inventory and routing decisions simultaneously. The total cost saving achieved by coordinating production and distribution planning ranged from 3% to 20% [2]. The integration and optimization of several activities within the FSC may not only significantly reduce the chain cost, but also provide better food quality management [4][5][6]. This is particularly true for FSC dealing with perishable food.

Perishable food has a relatively short shelf-life and starts deteriorating once produced. The perishability severely impacts the strategic supply chain planning process, while adding extra requirements to operational activities, e.g. cold storage, refrigerated transportation and special packaging. In addition, the perishability creates uncertainty for the buyer with respect to food quality, safety and reliability of supply, and imposes new constraints on the inventory quantity held in both the supplier and retailer. What's more, as the food quality decreases quickly, the food may neither meet the requirements of the customers nor be edible any more when reaching the customers. Therefore, to reduce food waste and chain cost induced by these uncertainties, stakeholders within the FSC should work in a more coordinated way to develop a leaner FSC. In terms of the customers, they are becoming more aware of the food they are buying and have more concerns on food quality, safety as well as traceability than ever before. Moreover, product freshness is highly related to customer satisfaction, so a good management of food perishability may entail strong competitive advantages [7]. This implies the need of taking the food perishability into account when making supply chain decisions. Especially, the shelf-life constraints should be carefully included when planning the production in cooperation with their supply chain partners [8]. The aforementioned situations inspire us to take an integrated

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view on perishable FSC and take full advantage of integrating different stages within FSC.

We can find a large body of literature on FSC dealing with perishability; however, most of it manages different sub-systems like production planning, inventory control or distribution separately [9][10]. Pahl *et al.* [11] conducted a review that focused on production planning and inventory management for perishable food. Ahumada and Villalobos [12] reviewed production and distribution planning models for perishable and nonperishable agri-foods based on agricultural crops. Akkerman *et al.* [13] reviewed quantitative operational management approaches on food distribution management, regarding food quality, safety and sustainability. A review on planning models handling perishability issues in production and distribution was conducted by Amorim *et al.* [7]. From these reviews, we find that literature handling different parts within the FSC in an integrated manner is relatively scarce. The following studies are among the few examples that examine the perishable FSC in an integrated or partially integrated way.

Chen *et al.* [14] proposed a nonlinear programming model to consider production scheduling and vehicle routing with time windows for perishable food products in the same framework. They considered a single period decision, thus no inventory was held neither in supplier nor in retailers. Rau *et al.* [15] studied an integrated inventory model for deteriorating items in a multi-echelon supply chain, where the total cost of the supplier, producer and buyer was minimized. However, their model only involved a single supplier, a single producer and a single buyer. Uncapacitated and capacitated production-distribution planning problems with direct transportation assumptions were studied by Ekşioğlu [16] and Ahuja *et al.* [17] respectively. Govindan *et al.* [18] formulated a multi-objective model on the location-routing problem with time windows for perishable food. Soysal *et al.* [19] presented a multi-period IRP model considering truck-load-dependent distribution costs for a comprehensive evaluation of CO_2 emission, fuel consumption, perishability, and service level constraints to meet uncertain demand. Amorim *et al.* [20] formulated a multi-objective model on the production-distribution problem for perishable products with fixed and loose shelf-life, and revealed that the economic benefits gained from using an integrated approach were much dependent on the freshness level of the products delivered. Le *et al.* [21] proposed a mathematical IRP model for perishable goods, and developed a column-generation-based heuristic. Note that the above papers all implicitly formulated the perishability with fixed or loose shelf-life constraints, which limited the possible storage time to avoid food waste.

Explicitly formulating the perishability into the FSC model with an age or quality index is beneficial [22]. Tekin *et al.* [23] and Duan & Liao [24] stated that the age-based inventory control policy was promising and had clear advantages. In addition, with the quick development of the ICT, we can obtain the food's instantaneous condition easily and economically through efficient information tracking systems using the radio frequency identification (RFID) technology [25], which is capable to reveal product information at an item-level in a way that is fully automatic, instantaneous, and touchless. We therefore can take full advantage of the ICT, and explicitly trace the food quality throughout the supply chain, which ensures good quality transparency and enhances the performance of the FSC. With these technologies, decision makers can intervene effectively with the instantaneous information, e.g. pricing adjustments, promotions, and waste disposals.

Mirzaei and Seifi [26] presented a mathematical IRP model considering the inventory age and lost sale, and developed two meta-heuristic methods to solve this complex problem. Rong *et al.* [27] modeled the generic two echelon supply chain with direct transportation where the product quality was explicitly traced throughout the supply chain with a quality index and made a combined optimization between supply chain and temperature control. Coelho and Laporte [28]developed a mixed integer linear programming (MILP) model for perishable products with explicitly tracing product age. The results indicated that the total gained revenue highly depended on the shape of the selling price. They solved the problem with a branch and cut method. The existing literature handling perishable food either assumes direct transportation or excludes the production decisions, making them only partially integrated. Yantong *et al*. [29] investigated the full integrated PIRP for perishable food, and the food quality are explicitly formulated using a quality index throughout the model.

In this paper, we proposed a PIRP model based on that of [29], especially the visiting time window in each retailer is included. Time window is one of the most common issues in transportation operations. Basically, there are two types of time window, namely the delivery period time window and the deliver time window within the delivered time period. The former imposes requirements on particular periods where a delivery can be made to retailers, while the latter specifies the visiting time inside each time period. It is pointed out in the latest reviews of Adulyasak *et al.* [3] and Díaz-Madroñero *et al.* [5] that the inclusion of time windows in production inventory routing problem is one of the most promising future research directions. In the PIRP, the vendor is responsible for the replenishment, i.e. the retailers can not determine the delivery period time window; however, they wish to be delivered during a specified time window in the visited time period, e.g. for the vegetable retailers, they are willing to receive the fresh vegetable in the early morning. Thus the problem we address here is an integrated production inventory routing problem with time windows (PIRPTW) for perishable food.

The remainder of the paper is organized as follows. Section 2 depicts the problem and model formulation, followed by introducing some valid inequalities. Section 3 presents the computational experiments and analyses. The conclusion and future research are given in section 4.

II. NOTATION AND FORMULATION

A. *Description and Notation*

We consider a PIRP involving a depot, a set of retailers and a single type of food product with limited shelf-life. The depot has a limited production capacity and inventory capacity, and is responsible for serving the retailers with a fleet of homogeneous vehicles over a finite planning horizon. Once there is some production in each period, a setup cost is incurred in the depot. Each retailer has a limited inventory capacity and a deterministic period-dependent demand, and the out-of-stock situations are not allowed. The food quality degrades from period to period since it is produced, and we generalize it with a quality level index. Usually, the food quality degradation may impact the demand. To avoid this situation, we use a price markdown strategy based on the food quality level so that the demand remain unchanged even if the food quality decreases.

We assume that the newly produced food can be immediately used to replenish the retailer, and the delivered food can be directly used by the retailer to fulfill the demand of its customers. Moreover, in each time period, each retailer has a specified deliver time window, which remains the same for all time periods.

The objective of the model is to maximize the total profit. The decisions to be made are: (1) when and how much to produce; (2) when and how much to deliver; (3) which deliver routes to use; (4) how to fulfill the customer's demand with food of varying quality level.

To formulate the problem, we define a directed complete graph $E=(N,A)$, where $N=\{0,1,...n\}$ is the vertex set and $A=\{(i,j):i,j\in N, i\neq j\}$ is the edge set. Vertex 0 represents the depot and the remaining vertices $R=N\setminus\{0\}$ correspond to *n* retailers. The following notations are defined:

Indices and parameters:

t	time period index $t\in T=\{1,2,...,Nbp\}$
q	Food quality level index $q\in Q=\{0,...,Nbq\}$, where 0 represents the freshest food
k	vehicle index $k\in K=\{1,2,...,Nbv\}$
PC	production capacity
VC	vehicle capacity
IC_i	inventory capacity of vertex $i\in N$
CP_t	unit production cost in time period $t\in T$
CI_i^q	unit inventory cost of food with quality $q\in Q$ at vertex $i\in N$
CS_t	production set up cost in time period $t\in T$
CT_{ij}	transportation cost on arc $(i,j)\in A$, which is independent of vehicle load
d_i^t	demand at retailer $i\in R$ in time period $t\in T$
Inv_i^q	initial inventory quantity of vertex $i\in N$ with quality level $q\in Q$
s_i^q	food price with quality $q\in Q$ at retailer $i\in R$
e_i	starting time of time window of retailer $i\in R$
e_0	earliest departure time from depot 0
l_i	finishing time of time window of retailer $i\in R$
l_0	latest arrival time at depot 0
st_i	service time in retailer $i\in R$
tt_{ij}	travel time on arc $(i,j)\in A$

Decision variables:

p_t	production quantity in time period $t\in T$
w_t	1 if $p_t>0$ in time period $t\in T$; 0 otherwise
I_i^{qt}	inventory level at node $i\in N$ with quality $q\in Q$ at the end of time period $t\in T\cup\{0\}$
v_i^{kt}	1 if vehicle $k\in K$ visit retailer $i\in R$ in time period $t\in T$; 0 otherwise
x_{ij}^{kt}	1 if vehicle $k\in K$ traverses arc $(i,j)\in A$ in time period $t\in T$, 0 otherwise
y_i^{qkt}	delivery quantity to retailer $i\in R$ with quality $q\in Q$ by vehicle $k\in K$ in time period $t\in T$
δ_i^{qt}	food quantity with quality $q\in Q$ used to satisfy the demand at retailer $i\in R$ in time period $t\in T$
z_i^{kt}	time to serve retailer $i\in R$ by vehicle $k\in K$ in time period $t\in T$
ε_{0e}^{kt}	departure time of vehicle $k\in K$ in time period $t\in T$ from the depot
λ_{0l}^{kt}	arrival time of vehicle $k\in K$ in time period $t\in T$ at the depot

B. Model Formulation

$$Max\begin{Bmatrix}\sum_{i\in R}\sum_{q\in Q}\sum_{t\in T}\delta_i^{qt}\cdot s_i^q-\sum_{t\in T}(p_t\cdot CP_t+w_t\cdot CS_t)\\ -\sum_{i\in N}\sum_{q\in Q}\sum_{t\in T}I_i^{qt}\cdot CI_i^q-\sum_{i\in N}\sum_{j\in N\setminus\{i\}}\sum_{k\in K}\sum_{t\in T}x_{ij}^{kt}\cdot CT_{ij}\end{Bmatrix}\tag{1}$$

subject to:

$$I_i^{q0}=Inv_i^q,\forall i\in N,q\in Q\tag{2}$$

$$I_0^{qt}=I_0^{q-1,t-1}-\sum_{i\in R}\sum_{k\in K}y_i^{qkt},\forall q\in Q\setminus\{0\},t\in T\tag{3}$$

$$I_0^{0t}=p_t-\sum_{i\in R}\sum_{k\in K}y_i^{0kt},\forall t\in T\tag{4}$$

$$I_i^{qt}=I_i^{q-1,t-1}+\sum_{k\in K}y_i^{qkt}-\delta_i^{qt},\forall i\in R,q\in Q\setminus\{0\},\forall t\in T\tag{5}$$

$$I_i^{0t}=\sum_{k\in K}y_i^{0kt}-\delta_i^{0t},\forall i\in R,\forall t\in T\tag{6}$$

$$p_t\le PC\cdot w_t,\forall t\in T\tag{7}$$

$$p_t\le\sum_{i\in R}\sum_{t'=t}^{Nbp}d_i^{t'},\forall t\in T\tag{8}$$

$$\sum_{q\in Q}I_i^{qt}\le IC_i,\forall i\in N,\forall t\in T\tag{9}$$

$$\sum_{q\in Q}\delta_i^{qt}=d_i^t,\forall i\in R,\forall t\in T\tag{10}$$

$$\sum_{i\in R}\sum_{q\in Q}y_i^{qkt}\le VC,\forall k\in K,t\in T\tag{11}$$

$$\sum_{q\in Q}y_i^{qkt}\le\sum_{t'=t}^{Nbp}d_i^{t'}\cdot v_i^{kt}\ ,\forall i\in R,k\in K,t\in T\tag{12}$$

$$\sum_{k\in K}v_i^{kt}\le 1,\forall i\in R,t\in T\tag{13}$$

$$\sum_{j\in N\setminus\{i\}}x_{ij}^{kt}=\sum_{j\in N\setminus\{i\}}x_{ji}^{kt}=v_i^{kt},\forall i\in R,k\in K,t\in T\tag{14}$$

$$\sum_{i\in R} x_{0i}^{kt} \le 1, k\in K, t\in T \tag{15}$$

$$z_i^{kt} + st_i + tt_{ij} - (l_i + st_i + tt_{ij})(1-x_{ij}^{kt}) \le z_j^{kt} \quad \forall i,j\in\{R \mid i\ne j\}, k\in K, t\in T \tag{16}$$

$$\varepsilon_{0e}^{kt} + tt_{0i} - M_{0i}(1-x_{0i}^{kt}) \le z_i^{kt}, \forall i\in R, k\in K, t\in T \tag{17}$$

$$z_i^{kt} + st_i + tt_{i0} - M_{i0}(1-x_{i0}^{kt}) \le \lambda_{0l}^{kt}, \forall i\in R, k\in K, t\in T \tag{18}$$

$$e_i\cdot v_i^{kt} \le z_i^{kt} \le l_i\cdot v_i^{kt}, \forall i\in R, \forall k\in K, \forall t\in T \tag{19}$$

$$e_0 \le \lambda_{0l}^{kt}, \forall k\in K, \forall t\in T \tag{20}$$

$$\varepsilon_{0e}^{kt} \le l_0, \forall k\in K, \forall t\in T \tag{21}$$

$$p_t \ge 0, \forall t\in T \tag{22}$$

$$w_t \in \{0,1\}, \forall t\in T \tag{23}$$

$$I_i^{qt} \ge 0, \forall i\in R, q\in Q, t\in T \tag{24}$$

$$\delta_i^{qt} \ge 0, \forall i\in R, q\in Q, t\in T \tag{25}$$

$$y_i^{qkt} \ge 0, \forall i\in R, q\in Q, k\in K, t\in T \tag{26}$$

$$v_i^{kt} \in \{0,1\}, \forall i\in R,, k\in K, t\in T \tag{27}$$

$$x_{ij}^{kt} \in \{0,1\}, \forall (i,j)\in A, k\in K, t\in T \tag{28}$$

$$z_i^{kt} \ge 0, \forall i\in R,, k\in K, t\in T \tag{29}$$

$$\varepsilon_{0e}^{kt}, \lambda_{0l}^{kt} \ge 0, \forall k\in K, t\in T \tag{30}$$

The objective function (1) maximizes the total profit which equals to the selling revenue minus the total supply chain cost. Constraints (2) link the inventory variables to the initial inventory. Constraints (3) - (6) correspond to the inventory balance with different product quality in the depot and retailers. Constraints (7) are the production capacity constraints. Constraints (8) limit the total production quantity in each period with the total remaining demand of all retailers. Constraints (9) are the inventory capacity constraints for the depot and retailers. Constraints (10) indicate that the customers' demand at each retailer in each period is fulfilled by food with different quality. Constraints (11) indicate that the vehicle capacity should be conformed with, i.e. the total quantity delivered by one vehicle in each time period must not exceed the vehicle capacity. Constraints (12) indicate that the deliver quantity to a retailer in a period must not exceed its remaining demand. If a retailer is not visited by a vehicle, then the deliver quantity to the retailer by that vehicle should be 0. Constraints (13) forbid the split delivery, i.e. each retailer can be visited at most once within each time period. Constraints (14) are the vehicle flow conservation constraints, which ensure that a vehicle goes into a node must leave that node. Constraints (15) indicate that one vehicle can only be used once within the same time period. Constraints (16)-(18) ensure the time consistency of the same vehicle, where $M_{ij} = l_i + st_i + tt_{ij}$. Specifically, these constraints indicate that if node *j* is the successive node of *i*, then the arriving time at *j* must be later than the arriving time at *i* plus the service time in *i* and the travel time between *i* and *j*. In addition, these constraints provide the benefit of eliminating the subtours. Constraints (19) are the time window constraints at each retailer. Constraints (20) and (21) specify the earliest departure time from the depot and the latest arrival time at the depot respectively. Constraints (22)-(30) are the non-negative and integer constraints.

We denote the objective (1) and constraints (2)-(30) as the original model, namely MODEL1. This PIRPTW is NP-hard, since it contains a classical vehicle routing problem with time windows (VRPTW) which has been well known as a NP-hard problem [30].

C. Valid Inequalities

In this section, we present valid inequalities that strengthen the formulation. The following valid inequalities are developed based on that introduced by Archetti *et al.* [31] and Coelho *et al.* [32].

$$\sum_{j\in R} x_{0j}^{kt} \ge \sum_{j\in R} x_{0j}^{k+1,t}, \forall j\in N, 1\le k< Nbv, t\in T \tag{31}$$

$$v_i^{kt} \le \sum_{j=1}^{i} v_j^{k-1,t}, \forall i\in R, k\in K/\{1\}, t\in T \tag{32}$$

$$\sum_{q\in Q} I_i^{qt} \ge \sum_{h=t+1}^{t+u} d_i^h\cdot(1-\sum_{k\in K}\sum_{h=t+1}^{t+u} v_i^{kh}), \forall i\in R, t\in T, 1\le u< Nbp-t \tag{33}$$

Constraints (31) and (32) break the symmetry in selecting the identical vehicles and to assign these vehicles to routes. Particularly, constraints (31) make sure that the vehicles with smaller number are always used first. Constraints (32) mean that if vehicle *k* is used to serve retailer *i*, then vehicle $k-1$ must have been used to serve retailers with smaller sequence number. Constraints (33) are used to strengthen the inventory routing part. These constraints refer that if a retailer is not visited during the time interval [*t*+1, *t*+*u*], then the inventory held in period *t* must be sufficient to meet the summation of the total demand in this interval.

Similarly, we derive the following constraints to strengthen the formulation by considering the food perishability. These constraints denote that there must be at least one visit to each retailer within the *Nbq*+1 time periods. This must happen since the food can not be stored longer than *Nbq*+1 periods, e.g. if one retailer is visited in time period t, then the next visit is no later than time period *t*+*Nbq*+1. Constraints (33) can be very strong when the food has a very short shelf-life.

$$\sum_{k\in K}\sum_{h=t}^{t+Nbq} v_i^{k,h} \ge 1, \forall i\in R, 1\le t\le Nbp-Nbq \tag{34}$$

The objective function (1) and the constraints (2)-(34) form the strengthened model, namely MODEL2.

III. Numerical Experiments

In this section, in order to verify the effectiveness of the proposed model and the proposed valid inequalities, we have tested the two models with randomly generated instances. We first tested the original model 1, and then tested the model 2 which contains all the valid inequalities. The formulation was coded in C++ language using Microsoft Visual Studio 2010 and CPLEX version 12.6.0 with default settings was used to solve the model. All test runs were performed on a CORE CPU 2.5 GHz with 8GB RAM, with the windows 10 operating system.

A. Instance Generation

As there are no benchmark instances available for the proposed model, we generate 9 different scenarios with 5 instances for each, yielding a total of 45 instances. The size of the generated instances are based on that in Coelho and Laporte [28], where an inventory routing problem is solved. All the data concerning the inventory routing part including the network is taken from their instances, and the parameters concerning the production part are generated as follows:

PC $\sum_{i\in R}\sum_{t\in T} d_i^t / Nbp\cdot\beta$, where β is randomly generated form range $[2,4]$

IC_0 $PC\cdot\gamma$, where γ is randomly generated from $[1.5,2]$

CP_t Randomly generated from $[4,7]$

CS_t $PC\cdot\mu$, where μ is randomly generated from $[0.3,0.5]$

Concerning the parameters for the time windows, we use the principle proposed by Solomon [30]. The earliest departure time from the depot e_0 and the latest arrival time to the depot l_0 are set to be 0 and 1200 respectively. The service time in retailer st_i is randomly generated from interval [60, 120]. Travel time between node i and j is set to be $t_{ij}=CT_{ij}/10$. To fully generate the time window parameters, we need to define two additional parameters, namely the time window center and width. The time window in each retailer is generated as follows:

$center_i$ Randomly generated $[e_0+tt_{0i}, l_0-tt_{i0}-st_i]$

$width_i$ Randomly generated from $(l_0-e_0)\cdot\alpha$, where α is randomly generated from $[1/8, 1/4]$

e_i $\max\ [center_i - width_i/2,\ e_0+tt_{0i}]$

l_i $\min\ [center_i + width_i/2,\ l_0-tt_{i0}-st_i]$

B. Computational results

In this section, we report the average solution values and the computational time for the generated instances, which are denoted by the number of retailers n, the minimum quality level Nbq, the number of available vehicle Nbv, and the length of the planning horizon Nbp. The code 10-2-1-3 represents the instance set with 10 retailers, minimum quality level 2, 1 available vehicle and 3 time periods. Average computational results are given in Table 1. The notation "Ins" denotes the instance set with different scenario. The best solution value obtained is denoted by "LB". The “UB$_1$” and “UB$_2$” denote the best upper bound found with the original MODEL1 and strengthened MODEL2 respectively within the time limit 7200 seconds. “Gap” denotes the average gap between the obtained lower bound and the upper bound by MODEL1, i.e. (UB$_1$-LB)/LB×100%. The notation "Tim" indicates the total computational CPU time. The column “Opt” reports the number of instances optimally solved by CPLEX. We have tested the generated 9 sets of total 45 intances, which cover 10 retailers with up to 10 periods and 40 retailers with up to 3 periods.

The results of set 1, 3, 5 and 8 show that CPLEX is capable to optimally solve small instances with 10 retailers up to 10 periods, and 20 retailers up to 3 time periods. The average computational time is less than 102.97s. In addition, 2 out of 5 set of instances with 20 retailers and 6 periods are solved to optimality, but with longer computational time. The computational time increases dramatically with the increasing of retailer number and the length of planning horizon, which has been shown by instance sets 1-3 and 2, 6, 9 respectively. Looking at the instance set 3 with 30 retailers and 3 periods, the average gap reaches 22.27%, which shows the complexity of the proposed problem. Note that the average computational time is less than the time limit, because most of the tests, if not solved optimally, run out of memory before reaching the time limit. We also note that the gap of set 4 with 40 retailers is smaller than that of set 3 with 30 retailers, which is not normal, this may happen because of the different network structure. Concerning the computational results with all valid inequalities, results in column UB$_2$ obviously indicate that the all upper bounds have decreased on average comparing with that in column UB$_1$.

TABLE I. COMPUTATIONAL RESULTS

NO.	Ins	LB	UB$_1$	UB$_2$	Gap(%)	Tim(s)	Opt
1	10-2-1-3	13449	13449	13449	0.00	0.28	5
2	20-2-2-3	33656	33656	33656	0.00	102.97	5
3	30-2-2-3	39606	48079	47793	22.27	1714.58	0
4	40-2-2-3	63964	73016	72927	14.87	4931.71	0
5	10-3-1-6	34279	34279	34279	0.00	3.44	5
6	20-3-2-6	76397	77770	77699	1.85	1642.84	2
7	30-3-2-6	111336	122412	122211	10.03	4667.89	0
8	10-5-1-10	71921	71921	71921	0.00	32.38	5
9	20-5-2-10	151998	154367	154040	1.60	4742.49	0

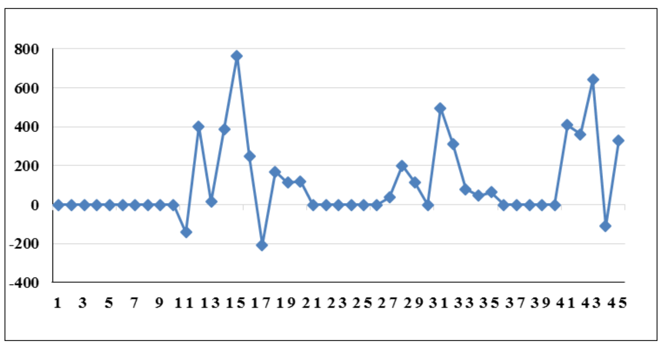

Figure 1. Upper bound value decrease for all instances

Figure 1 gives the detailed results of the upper bound value decrease by including the valid inequalities, where axis x denotes the instance number and the axis y is the upper bound decrease value, i.e. UB$_1$-UB$_2$. We can observe that the upper bounds for 20 out of 23 instances which have not been solved optimally have decreased, with the largest value being 764 for instance 15. The results show that these valid inequalities are useful in helping CPLEX generate better upper bounds.

IV. Conclusion and Future Research

In this paper, we formulate a MILP model for the PIRP with time windows, where food quality is explicitly traced throughout the supply chain with a quality level index. Valid inequalities are introduced to strengthen the formulation. Experimental results show that our proposed model is capable of providing integrated plans for the decision makers. As the problem is quite complex, only small instances can be solved optimally by CPLEX. The computational time increases significantly with the increasing of the retailer number and the length of planning horizon. However, in reality, there may be hundreds of retailers, and we are expected to make long term plan. Therefore, fast and effective solution methods are strongly required to solve the problem in realistic sizes.

In future study, we may focus on developing efficient algorithms for solving this complex problem. Decomposition based heuristic can be considered to obtain good feasible solutions within an acceptable time. Moreover, we expect to get better upper bounds with lagrangian relaxation or column generation methods. Furthermore, the proposed model can be extended to form many other variants with some additional considerations. Particularly, demand uncertainty should be included in future research.

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