Cost

	a	b	c	d	e	f	g	h
a	0	4	10	4	10	8	20	17
b	4	0	2	8	10	4	16	5
c	10	2	0	18	20	10	26	5
d	4	8	18	0	2	4	8	17
e	10	10	20	2	0	2	2	13
f	8	4	10	4	2	0	4	5
g	20	16	26	8	2	4	0	13
h	17	5	5	17	13	5	13	0

Cost Matrix (Distance is Squared for Comfort)

Algo: MST_Prim(G, w, r)

```
{
  G: Graph
  w: Cost
  r: root

  for each u ∈ G.V
  {
      u.key = ∞;
      u.parent = NIL;
  }                    (T)

  r.key = 0;
  Q = G.V
  while Q ≠ ϕ
  {
      u = Extract_Min(Q);
      for each v ∈ G.adj[u]
      { if v ∈ Q and w(u,v) < v.key
          { v.parent = u; v.key = w(u,v); }
      }
  }
```

Here Root r is Vertex a

Initially

	Key	Parent
a	∞	NIL
b	∞	NIL
c	∞	NIL
d	∞	NIL
e	∞	NIL
f	∞	NIL
g	∞	NIL
h	∞	NIL

→ Root r = a

Now **a.key = 0**

Q = G.V
= {a, b, c, d, e, f, g, h}

→ Q ≠ empty
∴ u = Extract_Min(Q)

i.e. Find a Vertex with Min key value Here it is Vertex 'a' and delete from Q.
∴ u = a and Q = {b, c, d, e, f, g, h}

→ for each adjacent v of u, if v ∈ Q and w(u,v) < v.key then v.Parent = u and v.key = w(u,v)

→ Initially all key's other than 'a' have value ∞
∴ ~~if stmt will be true for all~~ Vertices adjacent to 'a'.

☆ Vertex	old key	New key	old π	New π
b	∞	4	—	a
c	∞	10	—	a
d	∞	4	—	a
e	∞	10	—	a
f	∞	8	—	a
g	∞	20	—	a
h	∞	17	—	a

Here — ∴ NIL : Edge selected (a, b)

☆ $Q = \{b, c, d, e, f, g, h\} \neq$ empty

∴ Extract from Q vertex with Min. key value

There are two b, d.

Let's select $u = b$

Delete it from Q

∴ $Q = \{c, d, e, f, g, h\}$

☆ Vertex	old key	New key	old π	New π
c	10	2	a	b
d	4	4	a	a
e	10	10	a	a
f	8	4	a	b
g	20	16	a	b
h	17	5	a	b

Edge selected: (b, d)

Date
Page

☆ $Q = \{c, d, e, f, g, h\} \neq$ empty

∴ Find a vertex with Min. key value. Here it's 'c'

∴ $u = 'c'$

Delete it from Q

∴ $Q = \{d, e, f, g, h\}$

☆

Vertex	old key	New key	old π	New π
d	4	4	a	a
e	10	10	a	a
f	4	4	b	b
g	16	16	b	b
h	5	5	b	b

Now $Q = \{d, e, f, g, h\} \neq$ empty

∴ Find a vertex with Min. key

There are two 'd', 'e' with key 4

Here Let's select 'd'

∴ Next edge selected will be (a, d) because Parent of 'd' is 'a'

$\therefore u = \{e\}$

delete it from Q

$\therefore Q = \{e, f, g, h\}$

★

Vertex	Old key	New key	Old π	New π
e	10	2	a	d
f	4	4	b	b
g	16	8	b	d
h	5	5	b	b

★ $Q = \{e, f, g, h\} \neq$ empty

$\therefore$ Delete vertex with Min. key.

Here it is e, $\therefore u = \{e\}$

$\therefore$ Delete it from Q

$\therefore Q = \{f, g, h\}$

★ Next edge selected is (d, e).

Data Path

Vertex	Old key	New key	Old π	New π
f	4	2	b	e
g	8	2	d	e
h	5	5	b	b

★ Edge: (e, f)

★ There are Two Vertices f and g with same Minimum key value. Q = {f, g, h} ≠ empty

We will select u = 'f' ⇒ Q = {g, h}

Vertex	Old key	New key	Old π	New π
g	2	2	e	e
h	5	5	b	b

Selected edge: (e, g)

★ Q = {g, h} ≠ empty

∴ Delete u = 'g' (Min key = 2)

∴ Q = {h}

Vertex	Old key	New key	Old π	New π
h	5	5	b	b

★ Q = {h} ≠ empty

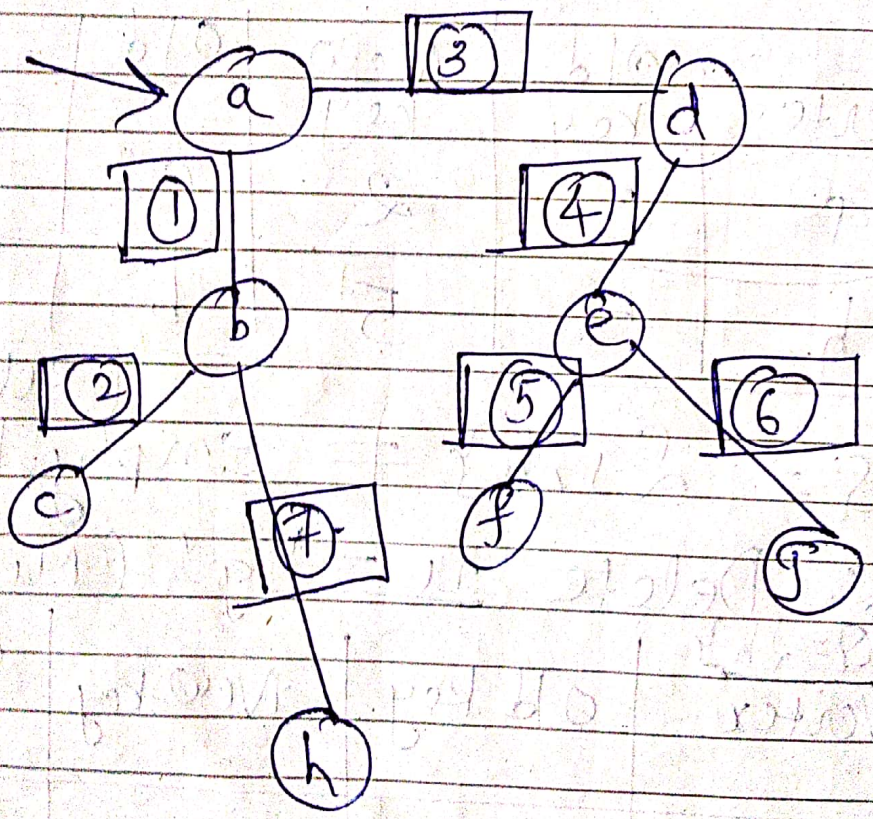
Delete h ⇒ Q = ϕ

Selected Edge: (b, h).

∴ Selected Edges

(a, b)
(b, c)
(a, d)
(d, e) → (e, f)
(e, g) ←
(b, h)

∴ Min. Spanning Tree :



Cost of MST = $\sqrt{4} + \sqrt{2} + \sqrt{4} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{5}$

Square root because we did Square

$= 2 + 1.41 + 2 + 1.41 + 1.41 + 1.41 + 2.24 \simeq 11.88$