"श्री" Lecture:

Randomized 3-SAT:

→ Consider a formula ϕ

$$\phi = (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee \overline{x_3} \vee x_4) \wedge (\overline{x_1} \vee \overline{x_3} \vee \overline{x_4}) \wedge (x_3 \vee x_4 \vee \overline{x_5})$$

In each clause, we have 3 literals.
∴ It is 3-SAT type of formula.

→ Applying same strategy as given for Randomized 2-SAT, we will get following:

☆ Assume satisfying assignment A_i S

→ Let X_i be the number of matches in A_i with satisfying assignment S!

→ What is the $\Pr\left(\frac{X_{i+1} = k+1}{X_i = k}\right)$?

and

What is the $\Pr\left(\frac{X_{i+1} = k-1}{X_i = k}\right)$?

Ans: In A_i, let's say there is a clause

$$a \vee b \vee c$$

∵ A_i is not satisfying assignment.
∴ At least 1 of the truth values

i.e. either a or b or c will not be same with satisfying assignment

i.e. either a disagrees with s

or

a, $\bar{b}$ both disagrees with s

or

a, $\bar{b}$, c all disagrees with s

because if all a, $\bar{b}$, c agrees with s then this clause of A_i must have return true as s is satisfying assignment.

$\therefore$ Atleast 1 out of 3 will not match with s

$\therefore$ Pr. that you select a literal out of 3 literals in a clause which doesn't match with satisfying assignment is atleast $\frac{1}{3}$.

$$\therefore \Pr\left(\frac{X_{i+1} = k+1}{X_i = k}\right) \geq \frac{1}{3}$$

Reason:

as we will get increment only when we flip that truth value which is not matching with s at step i.

Similarly $\Pr\left(\frac{X_{i+1} = k-1}{X_i = k}\right)$

$$\leq 1 - \Pr\left(\frac{X_{i+1} = k+1}{X_i = k}\right)$$

$$= 1 - \frac{1}{3}$$

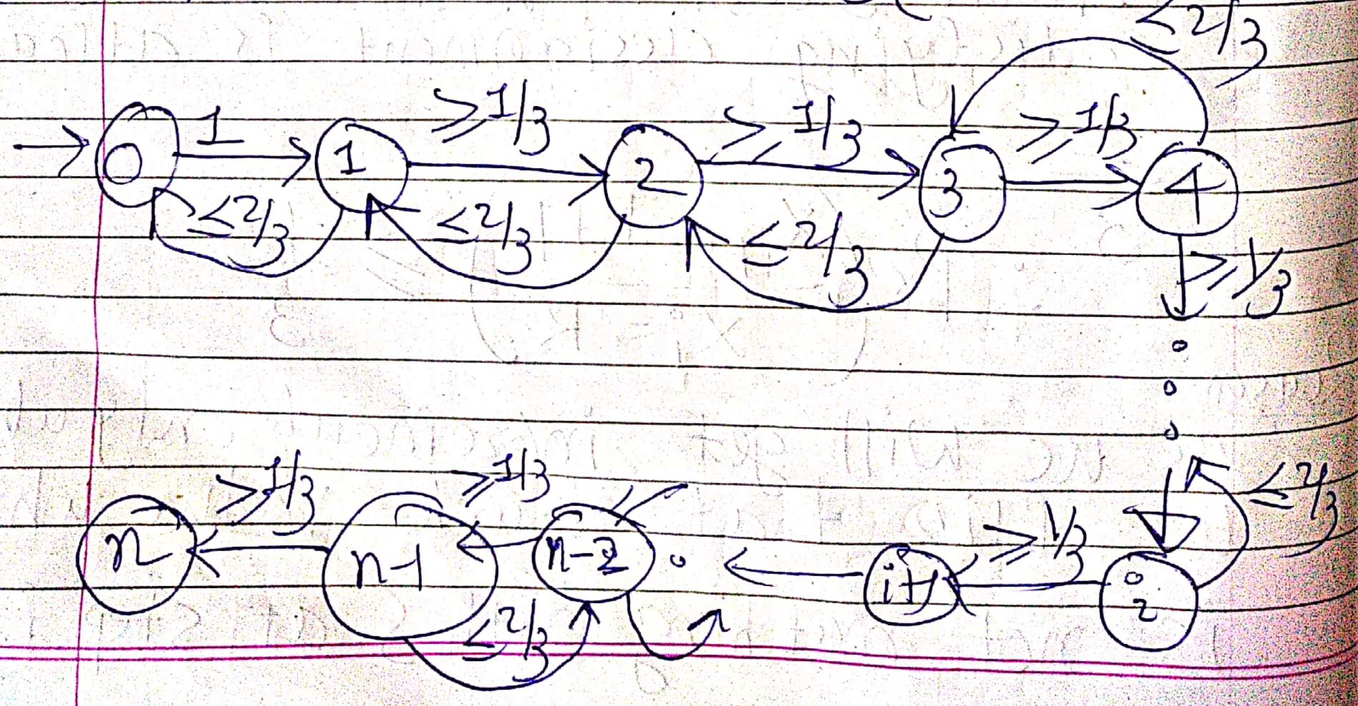
$$= \frac{2}{3}$$

$$\therefore \Pr\left(\frac{X_{i+1} = k-1}{X_i = k}\right) \leq \frac{2}{3}$$

and

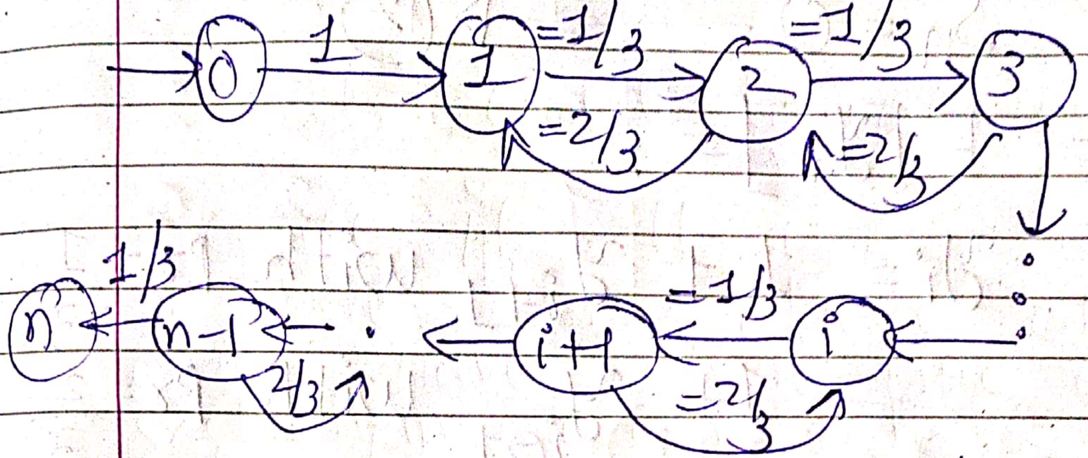
$$\Pr\left(\frac{X_{i+1} = k+1}{X_i = k}\right) \geq \frac{1}{3}$$

→ So, Random walk will be



→ Where (i) indicates that there are i matches with satisfying assignment.

☆ Simplified Walk:



As previous (Randomized 2-SAT)

Expected Number of steps for Walk 1

$\leq$

Expected Number of steps for Walk 2

i.e. Simplified Walk

→ So, Now we will ~~for~~ analyze Simplified Walk.

$$Pr\left(\frac{y_{i+1} = k+1}{y_i = k}\right) = \frac{1}{3}$$

$$Pr\left(\frac{y_{i+1} = k-1}{y_i = k}\right) = \frac{2}{3}$$

y_i's are just new Random variables for Walk-2

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→ Let $S_i = E[y_i]$ i.e. Expected number of steps to reach state n from state i.

$$S_0 = 1 + S_1 \longrightarrow (1)$$

$$S_n = 0 \longrightarrow (2)$$

☆ $S_i = E[y_i] = ?$

$$y_i = 1 + y_{i+1} \text{ with } pr = \frac{1}{3}$$
$$= 1 + y_{i-1} \text{ with } pr = \frac{2}{3}$$

$$\therefore E[y_i] = \frac{1}{3}(1 + E[y_{i+1}]) + \frac{2}{3}(1 + E[y_{i-1}])$$

$$= \frac{1}{3} + \frac{1}{3}E[y_{i+1}] + \frac{2}{3} + \frac{2}{3}E[y_{i-1}]$$

$$= \frac{1}{3} + \frac{2}{3} + \frac{E[y_{i+1}] + 2E[y_{i-1}]}{3}$$

$$\therefore S_i = 1 + \frac{S_{i+1} + 2S_{i-1}}{3}$$

$$\therefore 3S_i = 3 + S_{i+1} + 2S_{i-1}$$

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☆ $S_{i+1} = 3S_i - 2S_{i-1} - 3$

→ Put $i=1$

$S_2 = 3S_1 - 2S_0 - 3$

$= 3S_1 - 2(S_1+1) - 3 \quad (\because S_0 = S_1+1)$

$= S_1 - 5$

$\therefore S_1 - S_2 = 5 = 2^3 - 3 = 2^{2+1} - 3 \rightarrow (i)$

→ Put $i=2$

$S_3 = 3S_2 - 2S_1 - 3$

$= 3S_2 - 2(S_2+5) - 3$

$= S_2 - 13$

$\therefore S_2 - S_3 = 13 = 2^4 - 3 = 2^{3+1} - 3 \rightarrow (ii)$

→ Put $i=3$

$S_4 = 3S_3 - 2S_2 - 3$

$= 3S_3 - 2(S_3+13) - 3$

$= S_3 - 29$

$\therefore S_3 - S_4 = 29$

$= 2^5 - 3 = 2^{4+1} - 3 \rightarrow (iii)$

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$$\therefore S_{n-1} - S_n = 2^{n+1} - 3$$

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$$S_0 - \cancel{S_1} = 1$$

$$\cancel{S_1} - \cancel{S_2} = 2^3 - 3$$

$$\cancel{S_2} - \cancel{S_3} = 2^4 - 3$$

$$\vdots$$

$$S_i - S_{i+1} = 2^{i+2} - 3$$

$$\vdots$$

$$\cancel{S_{n-1}} - S_n = 2^{n+1} - 3$$

$$\therefore S_0 - S_n = 1 + (2^3 - 3) + (2^4 - 3) + \ldots + (2^{n+1} - 3)$$

$$\therefore S_0 - 0 = 1 + \sum_{j=3}^{n+1} (2^j - 3)$$

$$= 1 + \sum_{j=3}^{n+1} 2^j - \sum_{j=3}^{n+1} 3$$

$$= 1 + \left(\sum_{j=0}^{n+1} 2^j - (2^0 + 2^1 + 2^2) \right)$$

$$- 3 \sum_{j=3}^{n+1} 1$$

$$= 1 + \frac{1(2^{n+2} - 1)}{2-1} - 7 - 3(n+1-3+1)$$

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$$\therefore S_0 = \cancel{1} + 2^{n+2} - \cancel{1} - 7 - 3(n-1)$$
$$= 2^{n+2} - 7 - 3n + 3$$
$$= 2^{n+2} - 3n - 4$$

$$\therefore S_0 = 2^{n+2} - 3n - 4$$
$$= O(2^n)$$

i.e. Expected # of steps to reach to State n from state 0 is exponential to the # of Variables.

⟹ Shows No benifit because it is $O(2^n)$ steps

Reasons:

1. Prob. to move ahead is $\frac{1}{3}$ which is less than $\frac{2}{3}$ which is Prob. to go backward. i.e. It has tendancy to go back.

2. According to Well-Known Cook's Theorem 3-SAT is Np-Complete i.e. No Polytime algorithm is known yet.

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