Lab 5 Line Segment Intersection

Point $P = (x_1, y_1)$

$Q = (x_2, y_2)$

How to check point P is at which side of Q with reference to origin O(0,0)?

We need to find following

$$\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} = P \times Q \text{ (Notation)}$$

y_1 → Point P, y_2 → Point Q

$$\det = x_1 y_2 - x_2 y_1$$

if $\det > 0 \Rightarrow$ P is clockwise to Q

if $\det < 0 \Rightarrow$ P is Anticlockwise to Q

E.g. $P = (1, 3)$

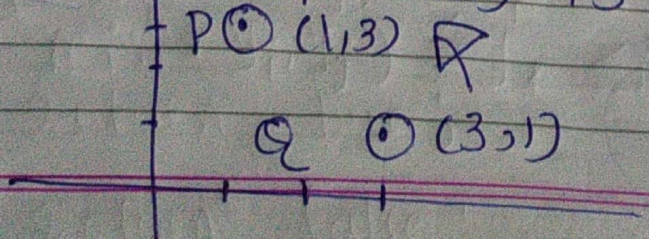
$Q = (3, 1)$

$$\det = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 1 - 9$$

$$= -8$$

$$< 0$$

$\det < 0 \Rightarrow P = (1,3)$ is Anticlkuise to Q



☆ Consider $P(x_1, y_1)$
$Q(x_2, y_2)$
$R(x_3, y_3)$

Now we need to see that $P \to Q \to R$ makes what type of turn at Q i.e. $P \to Q \to R$ is Left turn or Right turn.

We need to translate P to Origin

$\therefore P(x_1, y_1)$ will be $P(0,0)$
$Q(x_2, y_2)$ will be $Q(x_2 - x_1, y_2 - y_1)$
$R(x_3, y_3)$ will be $R(x_3 - x_1, y_3 - y_1)$

Then we need to fin

$$\det = \begin{vmatrix} x_3 - x_1 & x_2 - x_1 \\ y_3 - y_1 & y_2 - y_1 \end{vmatrix} \Leftarrow \overline{(P_k - P_i) \times (P_j - P_i)} = \overline{(P_3 - P_1) \times (P_2 - P_1)}$$

$$= (x_3 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_3 - y_1)$$

If $\det < 0 \Rightarrow$ R is Anticlkwise to Q

If $\det > 0 \Rightarrow$ R is clkwise to Q

Direction (P_i, P_j, P_k)
{
return $(P_k - P_i) \times (P_j - P_i)$
}

Segments Intersect (P_1, P_2, P_3, P_4)

{

d_1 = Direction (P_3, P_4, P_1)

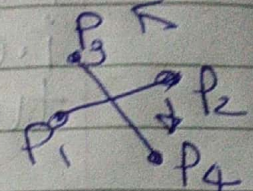
d_2 = Direction (P_3, P_4, P_2)

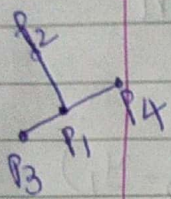
d_3 = Direction (P_1, P_2, P_3)

d_4 = Direction (P_1, P_2, P_4)

if ($d_1 * d_2 < 0$ and $d_3 * d_4 < 0$)

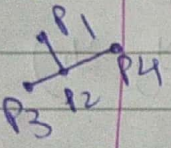
return true;





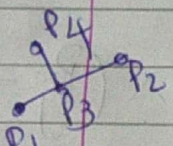
else if $d_1 == 0$ and onsegment (P_3, P_4, P_1)

return true;



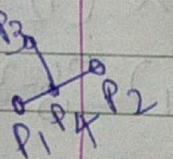
else if $d_2 == 0$ and onsegment (P_3, P_4, P_2)

return true;



else if $d_3 == 0$ and onsegment (P_1, P_2, P_3)

return true;



else if $d_4 == 0$ and onsegment (P_1, P_2, P_4)

return true;

else

return false;

}

Onsegment (P_i, P_j, P_k)

{

if ($\min(x_i, x_j) \le x_k \le \max(x_i, x_j)$ and

$\min(y_i, y_j) \le y_k \le \max(y_i, y_j)$)

return true;

else

return false;

}