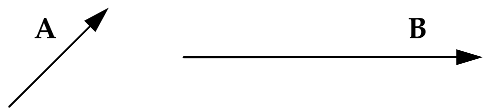
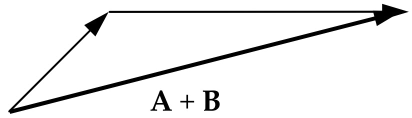
1. The Basics

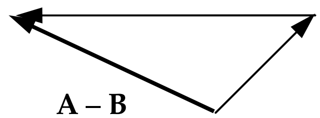
a) Given vectors **A** and **B** as shown, draw the following:



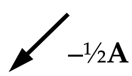
- **A** + **B**



- **A** – **B**



- –½ **A**

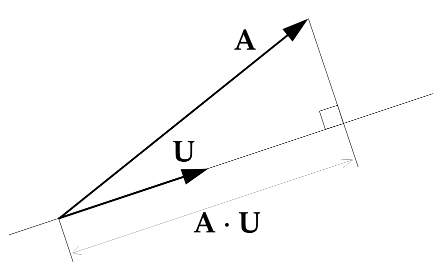


b) Write two equations for calculating the dot product **A** · **B**, where **A** = [A_x A_y A_z] and **B** = [B_x B_y B_z].

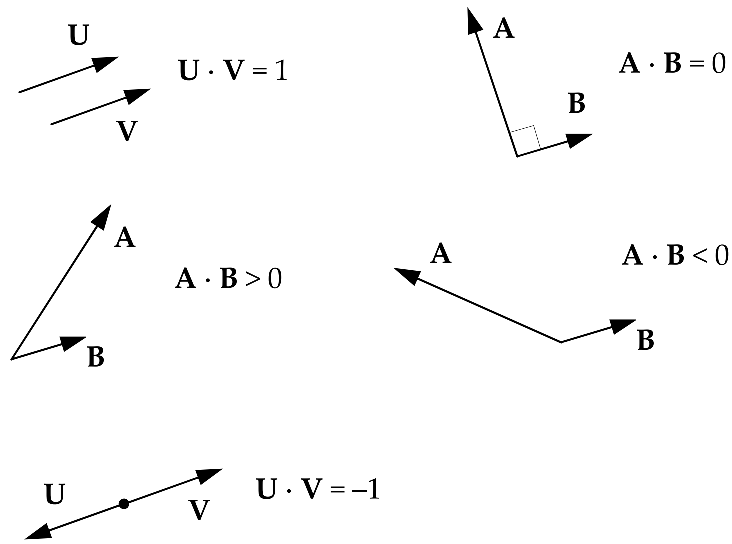
- **A** · **B** = $A_xB_x + A_yB_y + A_zB_z$

- **A** · **B** = |**A**| |**B**| cos θ

c) Draw **A** · **U** on the diagram, given that |**U**| = 1.

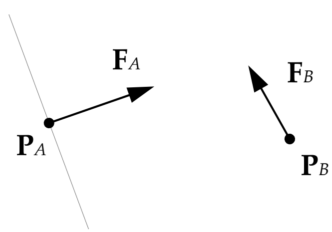


d) For each pair of vectors **A** and **B**, or **U** and **V**, write an inequality indicating the sign of the dot product… or if possible, write the exact value of the dot product. Note that $|\mathbf{U}| = |\mathbf{V}| = 1$, while $|\mathbf{A}| \neq 1$ and $|\mathbf{B}| \neq 1$.



2. Can you see me?

Two characters are standing on a roughly horizontal planar surface. The position of character A is $\mathbf{P}_A$ and its forward-facing unit vector is $\mathbf{F}_A$. Likewise the position and forward vector of character B are $\mathbf{P}_B$ and $\mathbf{F}_B$ respectively.



Top View

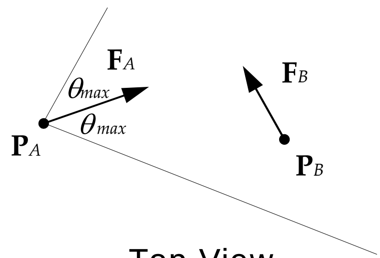
a) Use the sign of a dot product to determine whether character B is in front of or behind character A.

let $\mathbf{C} = \mathbf{P}_B - \mathbf{P}_A$.

if $(\mathbf{C} \cdot \mathbf{F}_A) \geq 0$ then character B is in front of A,

otherwise B is behind A.

b) Assume both characters have a vision cone extending θ_{max} radians to either side of their **F** vectors. Write an expression (using a dot product) indicating whether or not character A can "see" character B.
BONUS: How can we avoid finding the inverse cosine, $\cos^{-1}(\theta_{max})$?



Top View

let $\mathbf{C} = \mathbf{P}_B - \mathbf{P}_A$.

$(\mathbf{C} \cdot \mathbf{F}_A) = |\mathbf{C}| \cos \theta$, {recalling that $|\mathbf{F}_A| = 1$}

$\therefore\ \theta = \cos^{-1}((\mathbf{C} \cdot \mathbf{F}_A) / |\mathbf{C}|)$.

if $\theta \leq \theta_{max}$, then B can be seen by A.

or much more simply and less expensively…

if $((\mathbf{C} \cdot \mathbf{F}_A) / |\mathbf{C}|) \geq \cos \theta_{max}$, then B can be seen by A.

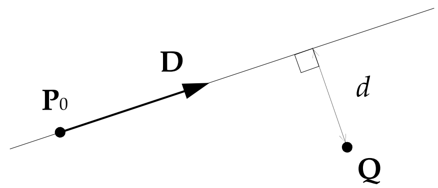
3. Wind Tunnel

The designers want to implement a shaft of wind that will affect any character or object that enters its cylindrical boundary.

a) You are given an arbitrary point **Q** in 3D space, and an infinite line represented by the locus of points $\mathbf{P}(t)$ defined as follows:

$$\mathbf{P}(t) = \mathbf{P}_0 + t\mathbf{D},$$

where $\mathbf{P}_0$ is a fixed point on the line, and **D** is a unit vector defining the line's direction. Find the perpendicular distance d from **Q** to the line.



let $\mathbf{C} = \mathbf{Q} - \mathbf{P}_0$.

break into components parallel and perpendicular to **D**, respectively:

$\mathbf{C} = \mathbf{C}_\parallel + \mathbf{C}_\perp$.

using the dot product, we have…

$\mathbf{C}_\parallel = (\mathbf{C} \cdot \mathbf{D})\mathbf{D}$, and then

$\mathbf{C}_\perp = \mathbf{C} - \mathbf{C}_\parallel$.

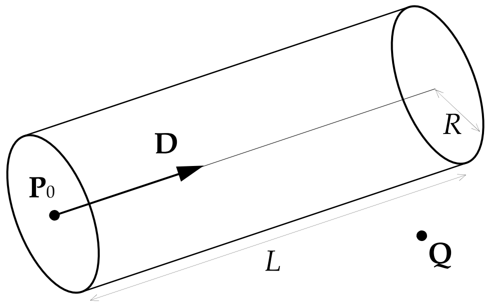
$\therefore\ d = |\mathbf{C}_\perp| = |\mathbf{C} - (\mathbf{C} \cdot \mathbf{D})\mathbf{D}|$.

or more simply…

$|\ \mathbf{C} \times \mathbf{D}\ | = |\mathbf{C}|\,|\mathbf{D}| \sin\theta = |\mathbf{C}| \sin\theta$ {remembering that $|\mathbf{D}| = 1$}

$\therefore\ d = |\ \mathbf{C} \times \mathbf{D}\ |$

b) The cylindrical wind tunnel can be defined by adding a radius r and length L to the infinite line from part (a). Assuming the position of our object or character is **Q**, write an expression that can be used to determine whether it will be affected by the wind or not.



let $\mathbf{C} = \mathbf{Q} - \mathbf{P}_0$,

let $s = |\mathbf{C}_{\parallel}| = (\mathbf{C} \cdot \mathbf{D})$, and

let $d = |\mathbf{C}_{\perp}| = |\mathbf{C} - \mathbf{C}_{\parallel}| = |\mathbf{C} - (\mathbf{C} \cdot \mathbf{D})\mathbf{D}| = |\ \mathbf{C} \times \mathbf{D}\ |$.

if $0 \leq s \leq L$ and $d \leq R$, then point Q is affected by the wind,

otherwise it isn't.