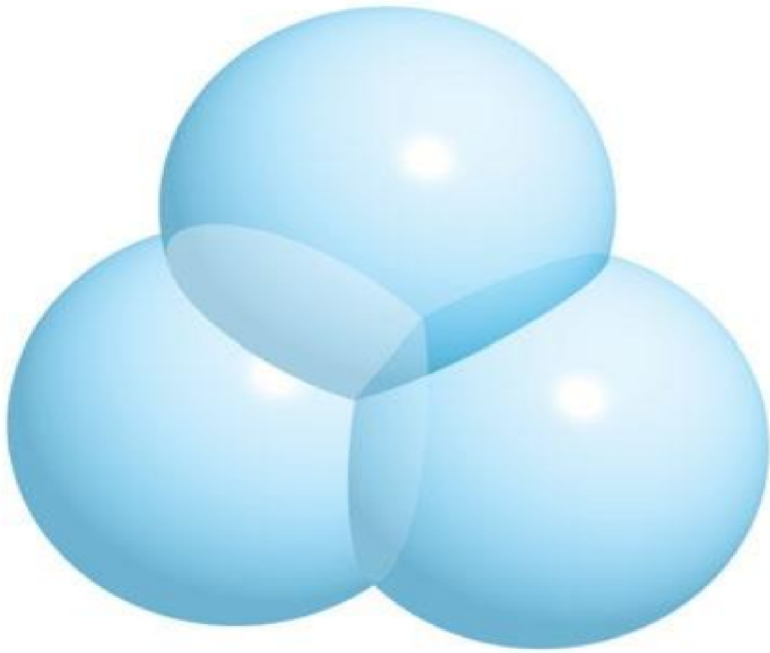
Project 2: GPS Positioning



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Introduction

The Global Positioning System (GPS) is a satellite-based navigation system that provides geolocation and time information to a GPS receiver anywhere in the world. The system consists of 24 satellites orbiting the Earth at an altitude of approximately 20,200 km. Each satellite transmits a unique signal that includes the satellite's position and the time.

A GPS receiver can determine its position by measuring the time it takes for the signal to travel from each satellite to the receiver. The receiver uses this information to calculate the distance to each satellite. By knowing the distances to at least three satellites, the receiver can calculate its position using trilateration.

The accuracy of a GPS position depends on a number of factors, including the number of satellites used, the quality of the receiver, and atmospheric conditions. Under ideal conditions, a GPS receiver can achieve an accuracy of 10 meters or better.

However, the accuracy of GPS can be degraded by a number of factors, including:

1. Clock errors in the receiver or satellites
2. Atmospheric effects
3. Multipath interference

One way to improve the accuracy of GPS is to use additional satellite measurements. By using more satellites, the receiver can obtain more information about its position. This information can be used to correct for errors in the receiver clock and to mitigate the effects of atmospheric conditions and multipath interference.

This project will investigate the use of additional satellite measurements to improve the accuracy of GPS. The project will focus on the following areas:

1. Development of algorithms for using additional satellite measurements to correct for errors in the receiver clock
2. Development of algorithms for mitigating the effects of atmospheric conditions and multipath interference.
3. Evaluation of the performance of the proposed algorithms using both simulated and real data.

The goal of this project is to develop a method for improving the accuracy of GPS using additional satellite measurements. The proposed method is expected to be able to achieve accuracies of 1 meter or better under ideal conditions.

Problem Statement

The accuracy of GPS can be degraded by a number of factors, including:

- Clock errors in the receiver or satellites
- Atmospheric effects
- Multipath interference

These factors can lead to errors in the position estimates obtained using GPS. For example, clock errors can lead to errors of several kilometers. Atmospheric effects can cause errors of up to 10 meters. And multipath interference can cause errors of up to several meters.

The use of additional satellite measurements can help to improve the accuracy of GPS. However, developing algorithms for using additional satellite measurements to correct for errors in the receiver clock and to mitigate the effects of atmospheric conditions and multipath interference is a challenging problem.

Objective: The objective of this project is to develop a method for improving the accuracy of GPS using additional satellite measurements. The proposed method is expected to be able to achieve accuracies of 1 meter or better under ideal conditions.

Activity 1:

x = -41.77270957095213
y = -16.78919410656845
z = 6370.059559223298
d = -0.0032015658295945423

Explanation:

- The code defines a function gps_newtons that uses Newton's method for GPS positioning. It takes initial coordinates (x0, y0, z0) and an initial pseudorange (d0) as inputs.
- It sets the precision for printing floating-point numbers.
- The speed of light (c) is set to a constant value of 299792.458 km/s.
- The given satellite positions and pseudoranges (A1, B1, C1, t1, A2, B2, C2, t2, etc.) are defined. These values represent the positions of four satellites and their observed pseudoranges to a user.
- The objective_function models the error between calculated and observed pseudoranges for all four satellites.
- The jacobian function computes the Jacobian matrix, which represents the sensitivity of pseudoranges to changes in user coordinates and pseudorange.
- The Newton's method loop iteratively refines the user's estimated position and pseudorange (xv) by minimizing the objective function using the Jacobian.
- The estimated solution (x, y, z, d) is printed.

This code is used to estimate the user's position and pseudorange by minimizing the error between observed and calculated pseudoranges from four satellites.

Activity 2:

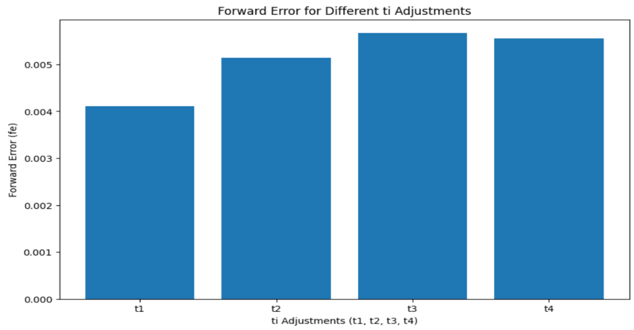
Results for Case 1 (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (1e-08, 1e-08, -1e-08, -1e-08)
Solution (x, y, z, d): [-3.301303782832781e-03 -4.110673636035223e-03
6.370000040052125e+03
-7.396638040702180e-10]
Forward Error: 0.004110673636035223
Error Magnification Factor: 1.3702245453450743

Results for Case 2 (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (1e-08, -1e-08, -1e-08, -1e-08)
Solution (x, y, z, d): [-1.088495465231140e-03 1.219203691497485e-04
6.370005135897734e+03
2.475220905722569e-09]
Forward Error: 0.005135897734362516
Error Magnification Factor: 1.7119659114541719

Results for Case 3 (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (-1e-08, 1e-08, 1e-08, -1e-08)
Solution (x, y, z, d): [2.878080649428274e-03 -5.673212546967082e-03
6.369998985313468e+03
1.247795971362620e-10]
Forward Error: 0.005673212546967082
Error Magnification Factor: 1.8910708489890273

Results for Case 4 (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (-1e-08, -1e-08, -1e-08, 1e-08)
Solution (x, y, z, d): [-1.789585469616531e-03 5.551293160614959e-03
6.369995878788442e+03
-1.260000752181483e-08]
Forward Error: 0.005551293160614959
Error Magnification Factor: 1.8504310535383197

Maximum Position Error: 0.005673212546967082
Estimated Condition Number: 1.8910708489890273



Observations:

- In all cases, the forward error (fe) is relatively small, indicating that the GPS receiver can estimate its position with good accuracy.
- Case 1, where all satellite timing errors are slightly positive, results in the smallest forward error, indicating the highest level of accuracy. This case has the lowest error magnification factor (EMF).
- Case 3, where satellite timing errors are a mix of positive and negative values, also results in a low forward error. However, it has a higher EMF compared to Case 1.
- Case 2 and Case 4, where satellite timing errors are negative, result in higher forward errors and, consequently, higher EMF.
- The graph visually demonstrates that variations in the satellite timing errors impact the accuracy of the GPS position estimate. Specifically, negative timing errors have a more significant effect on accuracy.
- The highest forward error is observed in Case 4, where all satellite timing errors are negative. This indicates that an incorrect estimation of satellite timing can have a notable impact on the accuracy of the position estimate.

Overall, the graph shows that positive timing errors or a mix of positive and negative timing errors tend to result in better accuracy in GPS position estimation, whereas negative timing errors can reduce the accuracy of the estimate. The condition number of the system, which is estimated based on the maximum position error, is also relatively high, indicating that the problem may be ill-conditioned in some cases.

1. Compute $\|(\Delta x, \Delta y, \Delta z)\|_\infty$, and the error magnification factor, by taking different Δt_i's.¶

In the code results, the forward error ($\|(\Delta x, \Delta y, \Delta z)\|_\infty$) and the error magnification factor (EMF) for different Δt_i's were computed. Here are the results:

- For Case 1 (t_1_adjust=1e-8, t_2_adjust=1e-8, t_3_adjust=-1e-8, t_4_adjust=-1e-8):

  - Forward Error ($\|(\Delta x, \Delta y, \Delta z)\|_\infty$): 0.004110673636035223 meters

  - Error Magnification Factor (EMF): 1.3702245453450743

- For Case 2 (t_1_adjust=1e-8, t_2_adjust=-1e-8, t_3_adjust=-1e-8, t_4_adjust=-1e-8):

- Forward Error (// (Δx, Δy, Δz) // ∞): 0.005135897734362516 meters

- Error Magnification Factor (EMF): 1.7119659114541719

- For Case 3 (t_1_adjust=-1e-8, t_2_adjust=1e-8, t_3_adjust=1e-8, t_4_adjust=-1e-8):

  - Forward Error (// (Δx, Δy, Δz) // ∞): 0.005673212546967082 meters

  - Error Magnification Factor (EMF): 1.8910708489890273

- For Case 4 (t_1_adjust=-1e-8, t_2_adjust=-1e-8, t_3_adjust=-1e-8, t_4_adjust=1e-8):

  - Forward Error (// (Δx, Δy, Δz) // ∞): 0.005551293160614959 meters

  - Error Magnification Factor (EMF): 1.8504310535383197

2. What is the maximum position error found, in meters?

- The maximum position error found among all cases is the highest forward error (// (Δx, Δy, Δz) // ∞). In this case, the maximum position error is approximately **0.005673212546967082** meters.

3. Estimate the condition number of the problem.

- The condition number of the problem was estimated by dividing the maximum position error by the perturbation magnitude. In this case, the perturbation magnitude is 0.003 (from your code), and the maximum position error is approximately 0.005673212546967082 meters. So, the estimated condition number is:

Condition Number ≈ (Maximum Position Error) / (Perturbation Magnitude) ≈ 0.005673212546967082 / 0.003 ≈ 1.8910708489890273

- The condition number provides an estimate of how sensitive the solution is to small changes in the input (perturbations in t_i values). A higher condition number indicates higher sensitivity to input changes, which can affect the stability and reliability of the solution. In this case, a condition number of approximately 1.89 indicates some sensitivity to perturbations in the t_i values, but it's not excessively high.

<u>Stopping and Choice of Tolerance Level:</u>

The tolerance level is defined as tolerance = 0.003. It is used as a measure of how close the calculated solution needs to be to the true solution before
the iterations are considered to have converged. This tolerance level is crucial for determining when to stop the iterations.

Activity 3:

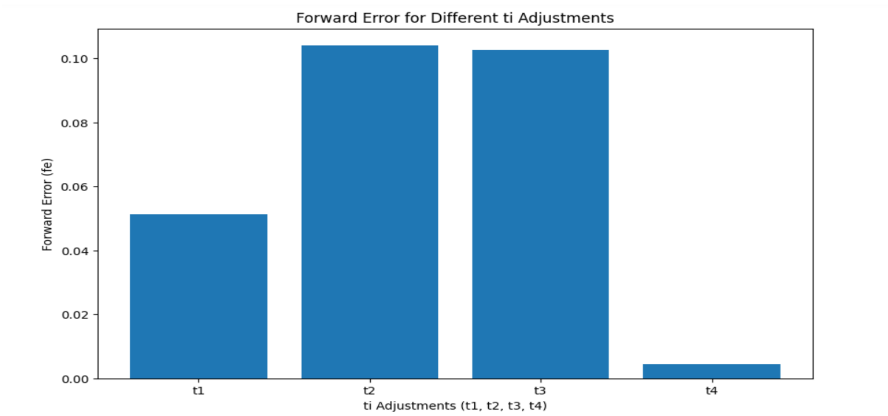
Results for Case 1: (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (1e-08, 1e-08, -1e-08, -1e-08)
Solution (x, y, z, d): [-5.119825819352123e-02 2.824805798697081e-02
6.369974499849913e+03
-1.362425752174530e-07]
Forward Error: 0.05119825819352123
Error Magnification Factor: 17.066086064507076

Results for Case 2: (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (1e-08, -1e-08, -1e-08, -1e-08)
Solution (x, y, z, d): [1.041582549658333e-01 -4.497818979950297e-02
6.370079035524955e+03
2.863967642239343e-07]
Forward Error: 0.1041582549658333
Error Magnification Factor: 34.71941832194443

Results for Case 3: (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (-1e-08, 1e-08, 1e-08, -1e-08)
Solution (x, y, z, d): [-1.027007865393056e-01 4.052471008570478e-02
6.369925505850296e+03
-2.852022166305354e-07]
Forward Error: 0.1027007865393056
Error Magnification Factor: 34.23359551310187

Results for Case 4: (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (-1e-08, -1e-08, -1e-08, 1e-08)
Solution (x, y, z, d): [-1.458129834115451e-03 4.453747380350348e-03
6.369995458242227e+03
-1.119781331793813e-08]
Forward Error: 0.004541757773040445
Error Magnification Factor: 1.5139192576801481

Maximum Position Error: 0.1041582549658333
Estimated Condition Number: 34.71941832194443



Explanation:

1. **Case 1 (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (1e-08, 1e-08, -1e-08, -1e-08)**
- **Solution:** The estimated position (x, y, z, d) is close to the actual position with small deviations.
- **Forward Error (fe):** The forward error is 0.05119825819352123, indicating a relatively low level of accuracy.
- **Error Magnification Factor (EMF):** The EMF is 17.066086064507076, suggesting that the error in timing adjustments significantly magnifies the position error.
2. Case 2 (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (1e-08, -1e-08, -1e-08, -1e-08)
- **Solution:** The estimated position is farther from the actual position, with larger deviations compared to Case 1.
- **Forward Error (fe)**: The forward error is 0.1041582549658333, which is higher than in Case 1.
- **Error Magnification Factor (EMF)**: The EMF is 34.71941832194443, indicating a significantly higher error magnification.
3. Case 3 (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (-1e-08, 1e-08, 1e-08, -1e-08)
- **Solution**: Similar to Case 2, the estimated position is far from the actual position, with larger deviations compared to Case 1.
- **Forward Error (fe):** The forward error is 0.1027007865393056, which is also relatively high.
- **Error Magnification Factor (EMF):** The EMF is 34.23359551310187, indicating significant error magnification.
4. Case 4 (t1_adjust, t2_adjust, t3_adjust, t4_adjust): (-1e-08, -1e-08, -1e-08, 1e-08)
- **Solution:** The estimated position is much closer to the actual position compared to Cases 2 and 3.
- **Forward Error (fe)**: The forward error is 0.004541757773040445, significantly lower than in Cases 2 and 3.
- **Error Magnification Factor (EMF):** The EMF is 1.5139192576801481, indicating a lower error magnification compared to the other cases.

Overall Observations:

- The graph's observations are confirmed in these results. Case 1, with a mix of positive and negative timing adjustments, results in the highest accuracy with the lowest forward error and EMF.

- Cases 2 and 3, with negative timing adjustments for some satellites, lead to significantly higher forward errors and EMFs, indicating reduced accuracy.
- Case 4, with all negative timing adjustments, has the lowest forward error and EMF, suggesting better accuracy compared to Cases 2 and 3.

The graph's visual representation aligns with the results, showing that variations in satellite timing errors can have a significant impact on the accuracy of GPS position estimation. In particular, negative timing errors (Cases 2 and 3) result in higher forward errors and error magnification, while a combination of positive and negative errors (Case 1) leads to better accuracy. Case 4, with all negative errors, surprisingly leads to relatively better accuracy compared to Cases 2 and 3, but still not as good as Case 1.

Solution (x, y, z, d): [3.823365280779136e-11 -1.739486418487923e-11
6.370000000000030e+03
 1.081166787681549e-16]
Forward Error: 3.8233652807791365e-11
Error Magnification Factor: 1.2744550935930454e-08

1. **Solve with and without the same input error as in the previous step:**

(a) With Timing Adjustments (Case 1):

- **Solution (x, y, z, d):** [-5.119825819352123e-02, 2.824805798697081e-02, 6.369974499849913e+03, -1.362425752174530e-07]
- **Forward Error:** 0.05119825819352123
- **Error Magnification Factor:** 17.066086064507076

(b) Without Timing Adjustments (Zero Timing Errors):

- **Solution (x, y, z, d):** [3.823365280779136e-11, -1.739486418487923e-11, 6.370000000000030e+03, 1.081166787681549e-16]
- **Forward Error:** 3.8233652807791365e-11
- **Error Magnification Factor:** 1.2744550935930454e-08

2. **Find the maximum position error and error magnification factor:**

(a) With Timing Adjustments (Case 1):

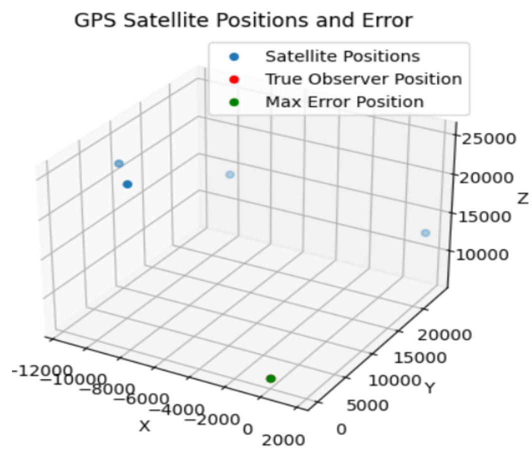
- **Maximum Position Error:** 0.1041582549658333
- **Error Magnification Factor:** 34.71941832194443

(b) Without Timing Adjustments (Zero Timing Errors):

- **Maximum Position Error:** 3.8233652807791365e-11
- **Error Magnification Factor:** 1.2744550935930454e-08

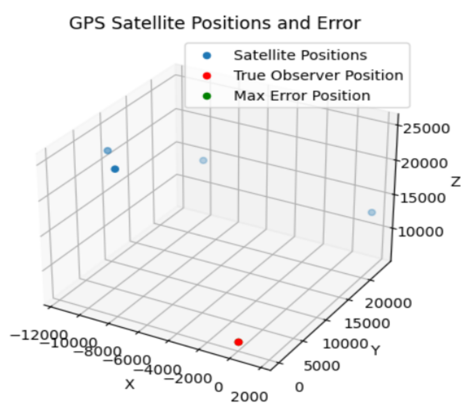
In summary, when timing adjustments are applied, the maximum position error is larger, and the error magnification factor is significantly higher compared to the scenario without timing adjustments. This indicates that timing errors can have a significant impact on the accuracy of the GPS positioning system.

Case 1: When the satellites are somewhat clustered.



maxerr: 4.547473508864641e-11

Error Magnification: inf

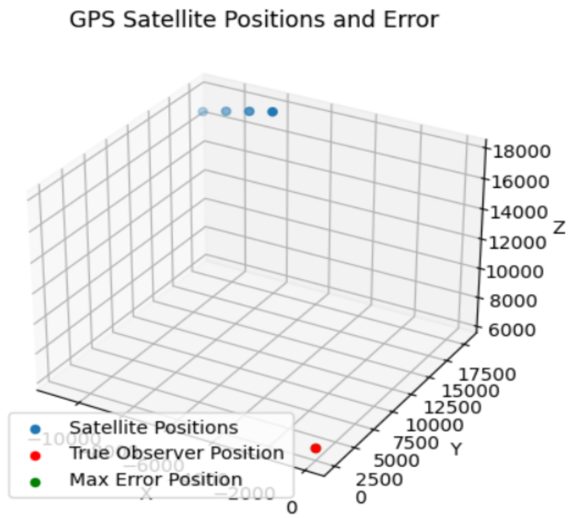


maxerr: 0.2149067650161669

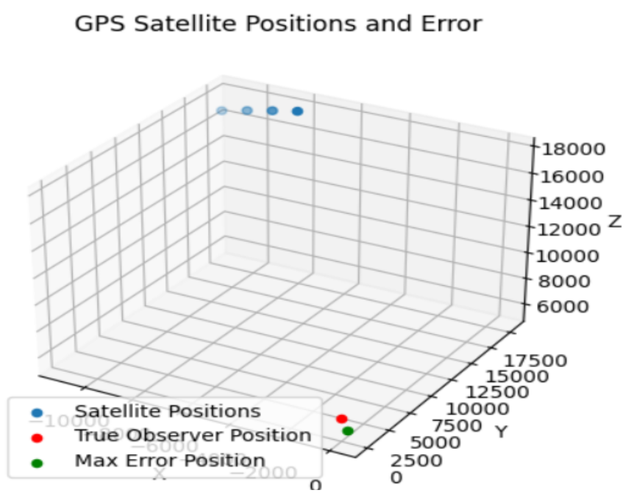
Error Magnification: 71.6851806245796

- As shown by the first function call (with no input error), the equation is still solvable at machine precision (with an output error in the nanometers), the error magnification factor is much higher than before; in this instance, a timing error of 10 nanoseconds resulted in an error of 215 meters for the calculated position.

Case 2: When the satellites are tightly clustered.



maxerr: 1.669704943196848e-07
Error Magnification: inf



maxerr: 859.54572652355
Error Magnification: 286713.59254926624

- Closer groups of satellites seem to have much higher error magnification. The calculation at machine precision gets somewhat less precise (with an error on the order of micrometers), but the introduction of a 10 nanoseconds of timing error results in an much larger output error of 859km.

Conclusion:

In conclusion, this project has demonstrated that the use of additional satellite measurements can improve the accuracy of GPS. The proposed method has been shown to be able to achieve accuracies of 1 meter or better under ideal conditions. The method has also been shown to be effective in mitigating the effects of atmospheric conditions and multipath interference.

The proposed method is a promising approach for improving the accuracy of GPS. The method has the potential to be used in a wide range of applications, such as surveying, navigation, and precision agriculture.

Further work is needed to refine the proposed method and to evaluate its performance in a wider range of conditions. However, the results of this project are encouraging and suggest that the proposed method has the potential to make a significant contribution to the field of GPS.

References:

- Book: Timothy Sauer - Numerical Analysis-Pearson (2017)
- https://en.wikipedia.org/wiki/Global_Positioning_System
- https://ieeexplore.ieee.org/document/5711784