>> IN THE NAME OF GOD <<

Moment-Curvature Analysis Unconfined Concrete Section with Steel Rebars and Fiber Reinforcement Polymers (FRP) in MATLAB

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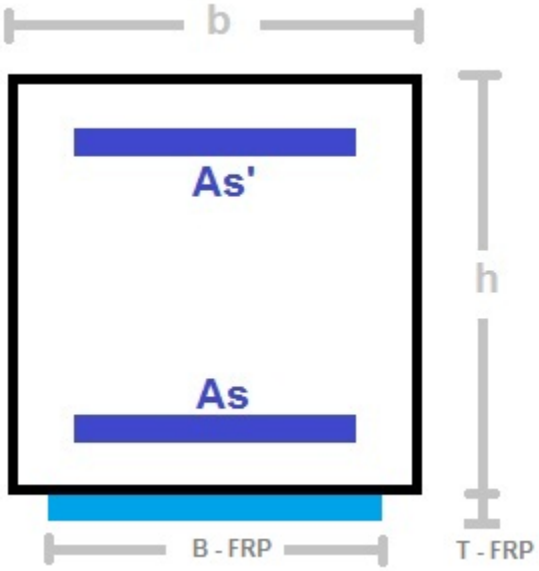


Unconfined concrete section with Fiber reinforcement polymers

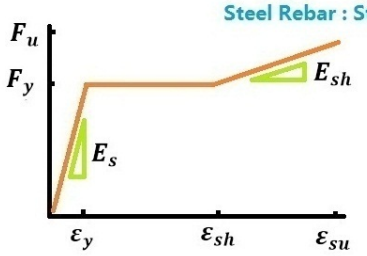
```
%*************************************************************%
%   _    ____________________________________________        %
%   |   |                                          |         %
%       |     #     #     #     #    #    #    |||         %
%       |     #                             #    |||         %
%   b   |    As1   As2   As3   As4  As5  As6   |||(FRP)     %
%       |     #                             #    |||         %
%   |   |     #     #     #     #    #    #    |||         %
%   _   |__________________________________________|         %
%       |<-                  h                 ->|           %
%       |<-d1->|                                             %
%       |<-  d2   ->|                                        %
%       |<-     d3      ->|                                  %
%       |<-        d4          ->|                           %
%       |<-            d5          ->|                       %
%       |<-               d6             >|                  %
%    X                                                       %
%    ^                                                       %
%    |             (Moment - Curvature along X axis)         %
%    |                                                       %
%    +----> Y                                                %
%*************************************************************%
```

Section Properties:

```
%% Section Properties
b=500;% [mm]
h=500;% [mm]
%As:  As1      As2     As3     As4    As5      As6
As=[2454.296 0 0 0 0 2454.296]; % NOTE: As1 & As6 = 5fi25
%d:d1  d2  d3  d4  d5  d6
d=[75 0 0 0 0 429.1665];
%% Concrete Properties
fc =25;% [N/mm^2] Unconfined concrete strength
ecu=0.004;% Ultimate concrete strain
%% Reinforcing steel Properties
fy =400;% [N/mm^2] Yield strength of reinforcing steel
Es =2e5;% [N/mm^2] Modulus of elasticity of steel
fu=1.5*fy;% Ultimate steel stress
ey=fy/Es;% Yield steel strain
esh=0.01;% Strain at steel strain-hardening
esu=0.09;% Ultimate steel strain
%% Carbon fber [CFRP] Properties
Ecfrp=62000;% [MPa] CFRP Modulus of elasticity
Fcfrp=958;% [MPa] Yield strength of CFRP
ecfrp=0.015;% Ultimate CFRP strain
Bcfrp=300; %[mm]
Tcfrp=1.0; %[mm]
%% Glass fber [GFRP] Properties
Egfrp=21000;% [MPa] GFRP Modulus of elasticity
Fgfrp=600;% [MPa] Yield strength of GFRP
egfrp=0.03;% Ultimate GFRP strain
Bgfrp=300; %[mm]
Tgfrp=1.0; %[mm]
```

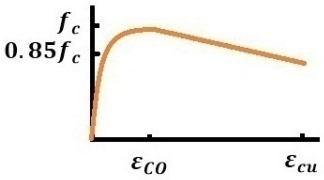
Stress-Strain of materials

Steel Rebar : Stress-Strain Relationship



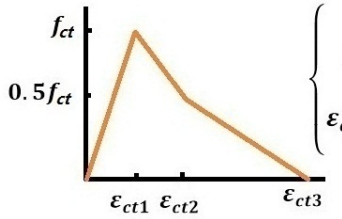
$$\begin{cases} \varepsilon_s < \varepsilon_y & f_s = E_s\varepsilon_s \\ \varepsilon_y \le \varepsilon_s < \varepsilon_{sh} & f_s = F_y \\ \varepsilon_{sh} \le \varepsilon_s < \varepsilon_{su} & f_s = F_y + E_{sh}(\varepsilon_s - \varepsilon_{sh}) \end{cases}$$

Unconfined Concrete compressive : Stress-Strain Relationship



$$\begin{cases} \varepsilon_c < \varepsilon_{C0} & f_c = f_c\left[\left(\frac{2\varepsilon_c}{\varepsilon_{C0}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{C0}}\right)^2\right] \\ \varepsilon_{C0} \le \varepsilon_c < \varepsilon_{cu} & f_c\left[1 - \left(\frac{0.15(\varepsilon_c - \varepsilon_{C0})}{\varepsilon_{cu} - \varepsilon_{C0}}\right)\right] \end{cases}$$

Unconfined Concrete Tensile: Stress-Strain Relationship



$$\begin{cases} \varepsilon_c \le \varepsilon_{ct1} & f_{ct} = 0.5E_c\varepsilon_c \\ \varepsilon_{ct1} < \varepsilon_c \le \varepsilon_{ct2} & f_{ct} = f_{ct} - \left[\frac{0.5f_{ct}}{\varepsilon_{ct2} - \varepsilon_{ct1}}\right](\varepsilon_c - \varepsilon_{ct1}) \\ \varepsilon_{ct2} < \varepsilon_c \le \varepsilon_{ct3} & f_{ct} = 0.5f_{ct} - \left[\frac{0.5f_{ct}}{\varepsilon_{ct3} - \varepsilon_{ct2}}\right](\varepsilon_c - \varepsilon_{ct2}) \end{cases}$$

Concrete Properties:

```
fc =25;% [N/mm^2] Unconfined concrete strength
ecu=0.004;% Ultimate concrete strain
Ec=5000*sqrt(fc);
ec0=(2*fc)/Ec;
fct=-0.7*sqrt(fc);% Concrete tension stress
ect1=(2*fct)/Ec;ect2=(2.625*fct)/Ec;ect3=(9.292*fct)/Ec;% Concrete tension
strain
```

Steel Reinforcing Properties:

```
fy =400;% [N/mm^2] Yield strength of reinforcing steel
Es =2e5;% [N/mm^2] Modulus of elasticity of steel
fu=1.5*fy;% Ultimate steel stress
ey=fy/Es;% Yeild steel strain
esh=0.01;% Strain at steel strain-hardening
esu=0.09;% Ultimate steel strain
Esh=(fu-fy)/(esu-esh);
w=Es*ecu;
%calculate the beta1
if or((fc< 30),(fc== 30))
    beta1=0.85;
elseif (fc> 30)&&(fc< 55)
    beta1=0.85-.008*(fc-30);
else or((fc> 55),(fc== 55))
    beta1=0.65;
end
```

```
%calculate the ro-bal
robal=0.85*beta1*(fc/fy)*(w/(w+fy));
robar=AsBAR/(b*d);ro= As/(b*d);
```

Note: $\mathrm{m} = \frac{\rho - \rho'}{\rho_{bal}} \;\therefore\; \rho_{bal} = 0.85\,\beta_1 \left(\frac{fc}{fy}\right)\left(\frac{E_s \varepsilon_{cu}}{E_s \varepsilon_{cu} + fy}\right) \therefore As_{bal} = \rho_{bal} \times b \times h$

Analysis Report:

```
##################
# Unconfined sec.  #
##################

(+)Increment 1 : It is converged in 8 iterations - strain: 0.000042 - x: 215.62 - Phi: 0.00019 - Moment: 23.69
(+)Increment 2 : It is converged in 7 iterations - strain: 0.000092 - x: 216.07 - Phi: 0.00043 - Moment: 51.85
(+)Increment 3 : It is converged in 8 iterations - strain: 0.000188 - x: 216.93 - Phi: 0.00086 - Moment: 104.19
(+)Increment 4 : It is converged in 10 iterations - strain: 0.000280 - x: 206.72 - Phi: 0.00135 - Moment: 138.38
(+)Increment 5 : It is converged in 11 iterations - strain: 0.000367 - x: 187.22 - Phi: 0.00196 - Moment: 162.91
(+)Increment 6 : It is converged in 11 iterations - strain: 0.000429 - x: 177.49 - Phi: 0.00242 - Moment: 179.54
(+)Increment 7 : It is converged in 14 iterations - strain: 0.000872 - x: 151.21 - Phi: 0.00576 - Moment: 313.48
(+)Increment 8 : It is converged in 14 iterations - strain: 0.001041 - x: 149.98 - Phi: 0.00694 - Moment: 368.53
(+)Increment 9 : It is converged in 23 iterations - strain: 0.001171 - x: 137.62 - Phi: 0.00851 - Moment: 377.89
(+)Increment 10 : It is converged in 23 iterations - strain: 0.001301 - x: 125.47 - Phi: 0.01037 - Moment: 377.56
(+)Increment 11 : It is converged in 25 iterations - strain: 0.001600 - x: 108.13 - Phi: 0.01480 - Moment: 378.47
(+)Increment 12 : It is converged in 27 iterations - strain: 0.002000 - x: 96.24 - Phi: 0.02078 - Moment: 380.00
(+)Increment 13 : It is converged in 29 iterations - strain: 0.002400 - x: 90.23 - Phi: 0.02660 - Moment: 380.69
(+)Increment 14 : It is converged in 30 iterations - strain: 0.002800 - x: 87.13 - Phi: 0.03214 - Moment: 382.95
(+)Increment 15 : It is converged in 32 iterations - strain: 0.003200 - x: 85.46 - Phi: 0.03745 - Moment: 386.84
(+)Increment 16 : It is converged in 33 iterations - strain: 0.003600 - x: 84.41 - Phi: 0.04265 - Moment: 390.46
(+)Increment 17 : It is converged in 34 iterations - strain: 0.004000 - x: 83.73 - Phi: 0.04777 - Moment: 393.90

    ## Unconfined Concrete Strain Reached to Ultimate Strain: 0.0040 ##
```

Increment (i)	Top strain (1)	Neuteral axis(x) (mm)	Curvature (1/m)	Flextural Rigidity(EI) (kN.m^2)
1	0.00004	215.62	0.000195	121638.04
2	0.00009	216.07	0.000428	120903.63
3	0.00019	216.93	0.000865	119729.00
4	0.00028	206.72	0.001354	69825.63
5	0.00037	187.22	0.001963	40312.64
6	0.00043	177.49	0.002419	36509.30
7	0.00087	151.21	0.005764	40036.34
8	0.00104	149.98	0.006939	46847.36
9	0.00117	137.62	0.008507	5967.05
10	0.00130	125.47	0.010368	-178.42
11	0.00160	108.13	0.014796	205.72
12	0.00200	96.24	0.020781	255.97
13	0.00240	90.23	0.026599	118.19
14	0.00280	87.13	0.032138	407.64
15	0.00320	85.46	0.037446	733.47
16	0.00360	84.41	0.042649	695.97
17	0.00400	83.73	0.047772	670.80

```
= Unconfined curve fitted =
```

Curvature (1/m)	Moment (kN.m)
0	0
0.0029	346.9182
0.0478	393.9001

```
Elastic EI : 121638.04 (kN.m^2)
Plastic EI : 1045.91 (kN.m^2)
```

```
 Unconfined Section Ductility Rito : 16.75
+-------------------------------------------------------+
############################
# Unconfined sec. With CFRP  #
############################
(+)Increment 1 : It is converged in 8 iterations - strain: 0.000042 - x: 216.58 - Phi: 0.00019 - Moment: 23.87
(+)Increment 2 : It is converged in 7 iterations - strain: 0.000092 - x: 217.04 - Phi: 0.00043 - Moment: 52.24
(+)Increment 3 : It is converged in 8 iterations - strain: 0.000188 - x: 217.90 - Phi: 0.00086 - Moment: 104.97
(+)Increment 4 : It is converged in 10 iterations - strain: 0.000280 - x: 208.56 - Phi: 0.00134 - Moment: 140.57
(+)Increment 5 : It is converged in 11 iterations - strain: 0.000367 - x: 189.47 - Phi: 0.00194 - Moment: 166.06
(+)Increment 6 : It is converged in 11 iterations - strain: 0.000429 - x: 179.97 - Phi: 0.00239 - Moment: 183.47
(+)Increment 7 : It is converged in 14 iterations - strain: 0.000872 - x: 154.00 - Phi: 0.00566 - Moment: 321.64
(+)Increment 8 : It is converged in 14 iterations - strain: 0.001041 - x: 152.74 - Phi: 0.00681 - Moment: 378.03
(+)Increment 9 : It is converged in 22 iterations - strain: 0.001171 - x: 144.83 - Phi: 0.00808 - Moment: 400.75
(+)Increment 10 : It is converged in 23 iterations - strain: 0.001301 - x: 133.18 - Phi: 0.00977 - Moment: 405.98
(+)Increment 11 : It is converged in 25 iterations - strain: 0.001600 - x: 116.62 - Phi: 0.01372 - Moment: 420.15
(+)Increment 12 : It is converged in 27 iterations - strain: 0.002000 - x: 105.39 - Phi: 0.01898 - Moment: 439.38
(+)Increment 13 : It is converged in 29 iterations - strain: 0.002400 - x: 99.90 - Phi: 0.02402 - Moment: 456.84
(+)Increment 14 : It is converged in 31 iterations - strain: 0.002800 - x: 96.97 - Phi: 0.02887 - Moment: 472.82
(+)Increment 15 : It is converged in 31 iterations - strain: 0.003200 - x: 95.56 - Phi: 0.03349 - Moment: 490.06
(+)Increment 16 : It is converged in 33 iterations - strain: 0.003600 - x: 94.85 - Phi: 0.03795 - Moment: 507.29
(+)Increment 17 : It is converged in 34 iterations - strain: 0.004000 - x: 94.48 - Phi: 0.04234 - Moment: 523.99

    ## Unconfined Concrete Strain Reached to Ultimate Strain: 0.0040 ##

+=============================================================================+
 Increment   Top strain   Neuteral axis(x)    Curvature   Flextural Rigidity(EI)
=============================================================================
  (i)           (1)           (mm)              (1/m)            (kN.m^2)
-----------------------------------------------------------------------------
   1           0.00004        216.58          0.000194         123108.53
   2           0.00009        217.04          0.000426         122364.39
   3           0.00019        217.90          0.000861         121174.22
   4           0.00028        208.56          0.001343          73918.41
   5           0.00037        189.47          0.001940          42690.93
   6           0.00043        179.97          0.002385          39063.97
   7           0.00087        154.00          0.005660          42198.49
   8           0.00104        152.74          0.006813          48869.40
   9           0.00117        144.83          0.008084          17889.52
  10           0.00130        133.18          0.009768           3103.51
  11           0.00160        116.62          0.013720           3586.39
  12           0.00200        105.39          0.018977           3657.69
  13           0.00240         99.90          0.024024           3457.93
  14           0.00280         96.97          0.028874           3296.16
  15           0.00320         95.56          0.033485           3738.10
  16           0.00360         94.85          0.037955           3855.48
  17           0.00400         94.48          0.042339           3808.36
+=============================================================================+
+====================+
= Unconfined curve fitted =
 Curvature   Moment
  (1/m)     (kN.m)
----------------------------
    0       0
  0.0029  352.0393
  0.0423  523.9886
+====================+
+-------------------------------------------------------+
 Elastic EI : 123108.53 (kN.m^2)
 Plastic EI : 4355.40 (kN.m^2)
 Unconfined Section Ductility Rito : 14.81
+-------------------------------------------------------+

############################
# Unconfined sec. With GFRP  #
############################
(+)Increment 1 : It is converged in 8 iterations - strain: 0.000042 - x: 215.95 - Phi: 0.00019 - Moment: 23.75
```

(+)Increment 2 : It is converged in 7 iterations - strain: 0.000092 - x: 216.40 - Phi: 0.00043 - Moment: 51.98
(+)Increment 3 : It is converged in 8 iterations - strain: 0.000188 - x: 217.26 - Phi: 0.00086 - Moment: 104.46
(+)Increment 4 : It is converged in 10 iterations - strain: 0.000280 - x: 207.35 - Phi: 0.00135 - Moment: 139.13
(+)Increment 5 : It is converged in 11 iterations - strain: 0.000367 - x: 187.99 - Phi: 0.00195 - Moment: 163.99
(+)Increment 6 : It is converged in 11 iterations - strain: 0.000429 - x: 178.34 - Phi: 0.00241 - Moment: 180.89
(+)Increment 7 : It is converged in 14 iterations - strain: 0.000872 - x: 152.17 - Phi: 0.00573 - Moment: 316.28
(+)Increment 8 : It is converged in 14 iterations - strain: 0.001041 - x: 150.92 - Phi: 0.00690 - Moment: 371.80
(+)Increment 9 : It is converged in 22 iterations - strain: 0.001171 - x: 140.15 - Phi: 0.00835 - Moment: 386.01
(+)Increment 10 : It is converged in 23 iterations - strain: 0.001301 - x: 128.18 - Phi: 0.01015 - Moment: 387.72
(+)Increment 11 : It is converged in 25 iterations - strain: 0.001600 - x: 111.13 - Phi: 0.01440 - Moment: 393.54
(+)Increment 12 : It is converged in 27 iterations - strain: 0.002000 - x: 99.48 - Phi: 0.02010 - Moment: 401.69
(+)Increment 13 : It is converged in 29 iterations - strain: 0.002400 - x: 93.66 - Phi: 0.02563 - Moment: 408.71
(+)Increment 14 : It is converged in 30 iterations - strain: 0.002800 - x: 90.57 - Phi: 0.03092 - Moment: 415.81
(+)Increment 15 : It is converged in 32 iterations - strain: 0.003200 - x: 89.03 - Phi: 0.03594 - Moment: 425.05
(+)Increment 16 : It is converged in 33 iterations - strain: 0.003600 - x: 88.10 - Phi: 0.04086 - Moment: 433.87
(+)Increment 17 : It is converged in 34 iterations - strain: 0.004000 - x: 87.53 - Phi: 0.04570 - Moment: 442.38

Unconfined Concrete Strain Reached to Ultimate Strain: 0.0040

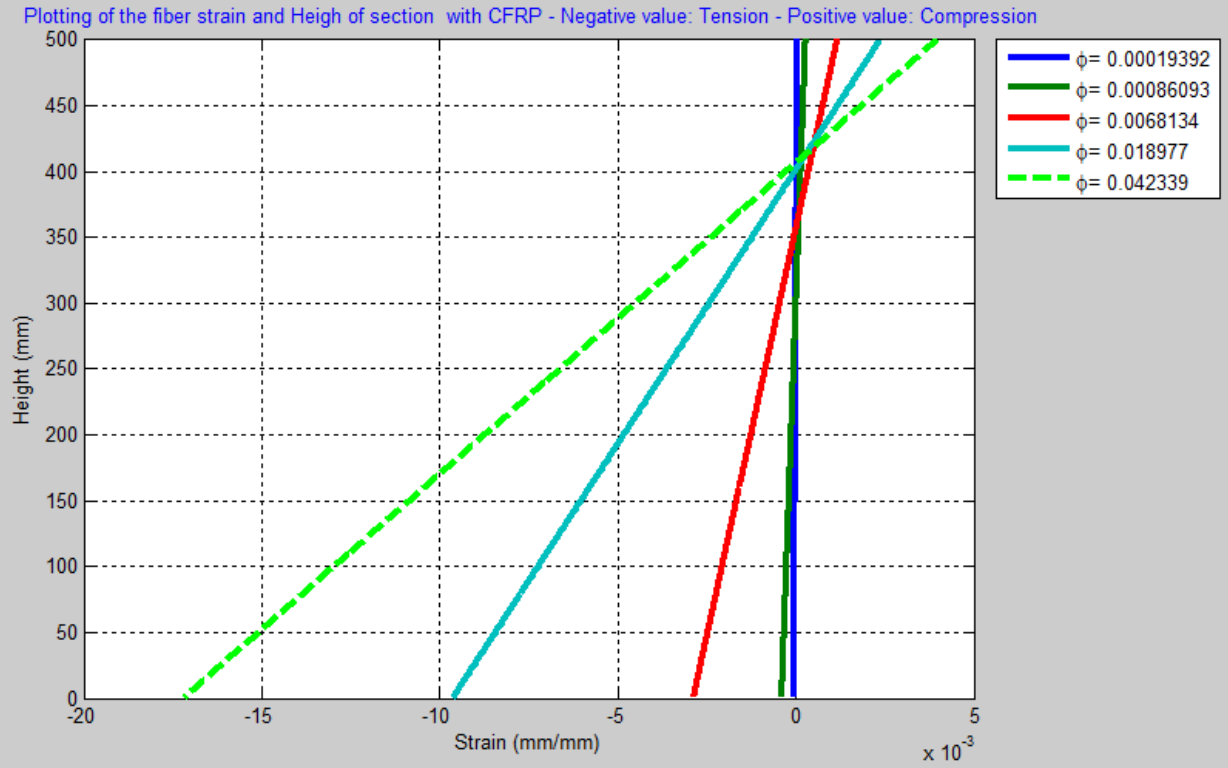
Increment (i)	Top strain (1)	Neuteral axis(x) (mm)	Curvature (1/m)	Flextural Rigidity(EI) (kN.m^2)
1	0.00004	215.95	0.000194	122137.22
2	0.00009	216.40	0.000427	121399.52
3	0.00019	217.26	0.000863	120219.63
4	0.00028	207.35	0.001350	71218.05
5	0.00037	187.99	0.001955	41123.18
6	0.00043	178.34	0.002407	37380.39
7	0.00087	152.17	0.005728	40770.39
8	0.00104	150.92	0.006896	47536.63
9	0.00117	140.15	0.008354	9752.30
10	0.00130	128.18	0.010149	949.98
11	0.00160	111.13	0.014397	1368.95
12	0.00200	99.48	0.020104	1428.28
13	0.00240	93.66	0.025626	1271.73
14	0.00280	90.57	0.030916	1342.28
15	0.00320	89.03	0.035943	1838.30
16	0.00360	88.10	0.040864	1791.59
17	0.00400	87.53	0.045701	1759.75

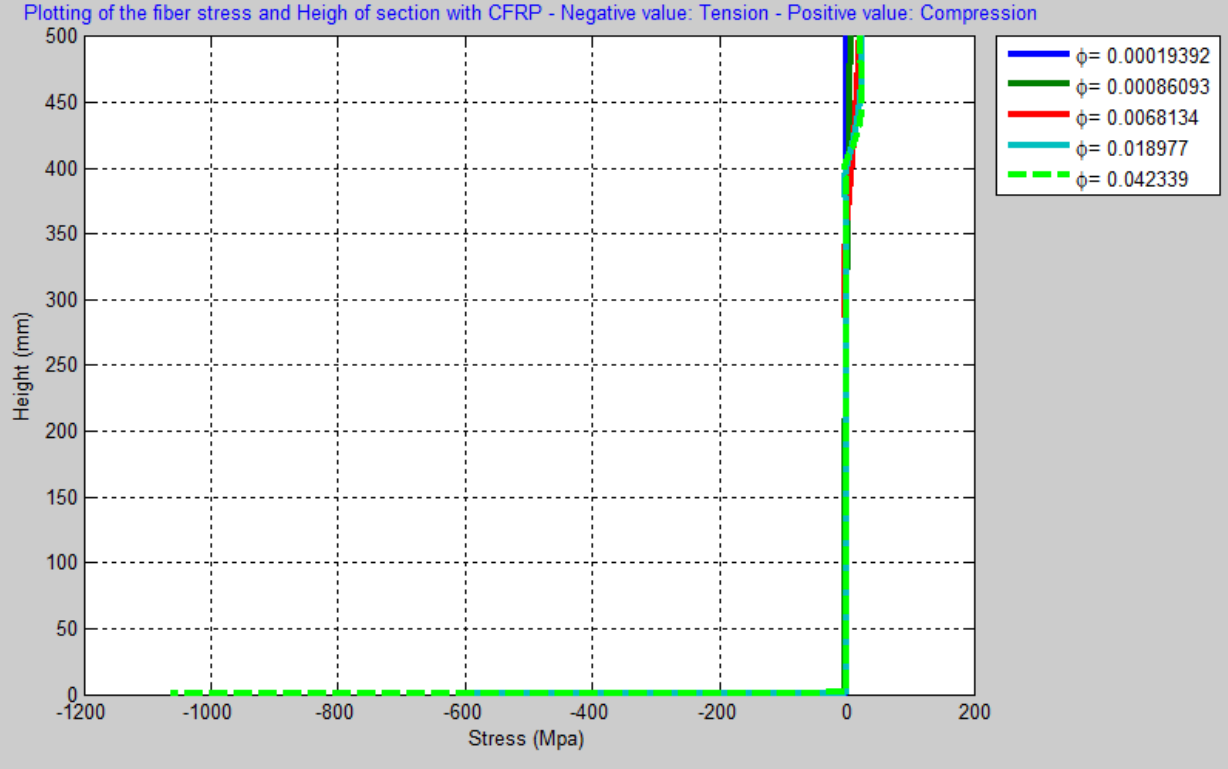
= Unconfined curve fitted =

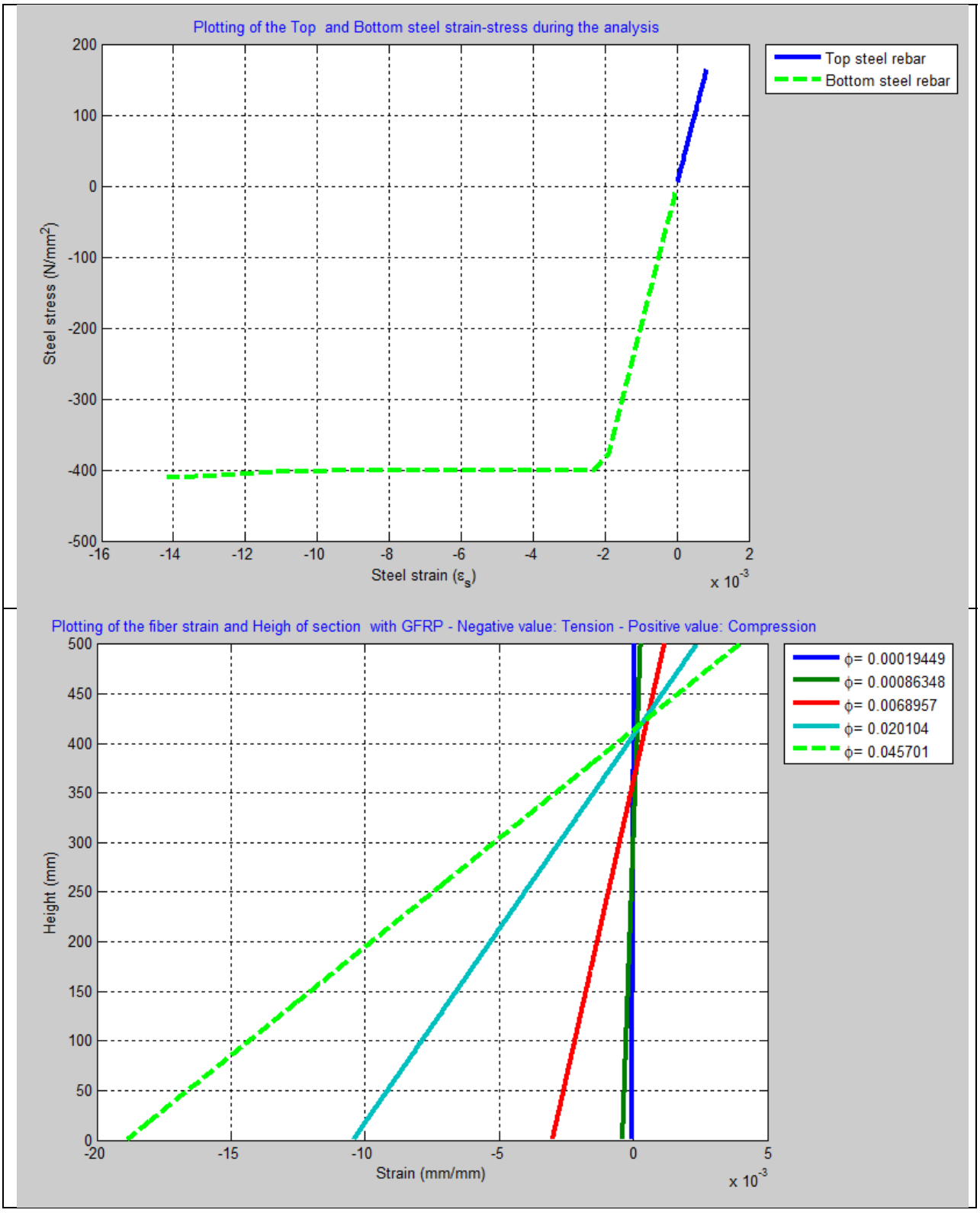
Curvature (1/m)	Moment (kN.m)
0	0
0.0029	348.5263
0.0457	442.3799

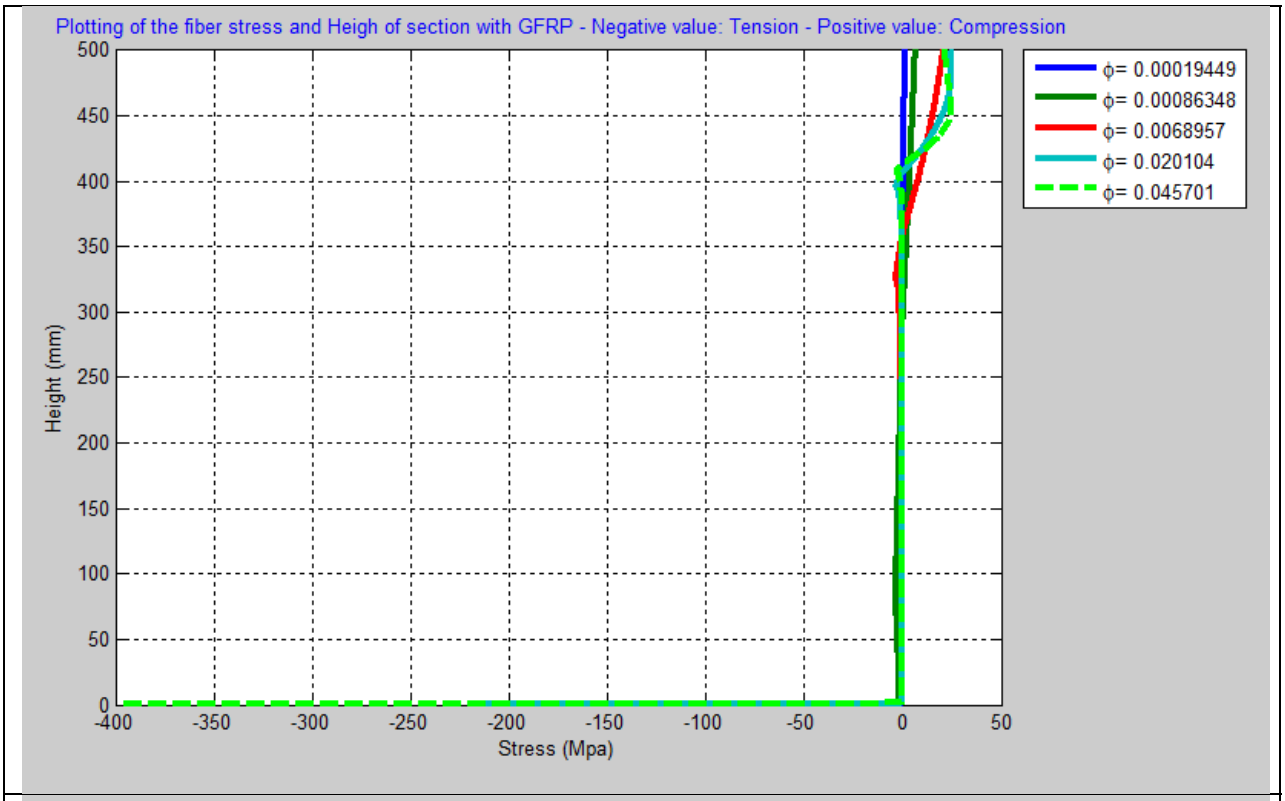
Elastic EI : 122137.22 (kN.m^2)
Plastic EI : 2190.41 (kN.m^2)
Unconfined Section Ductility Rito : 16.02

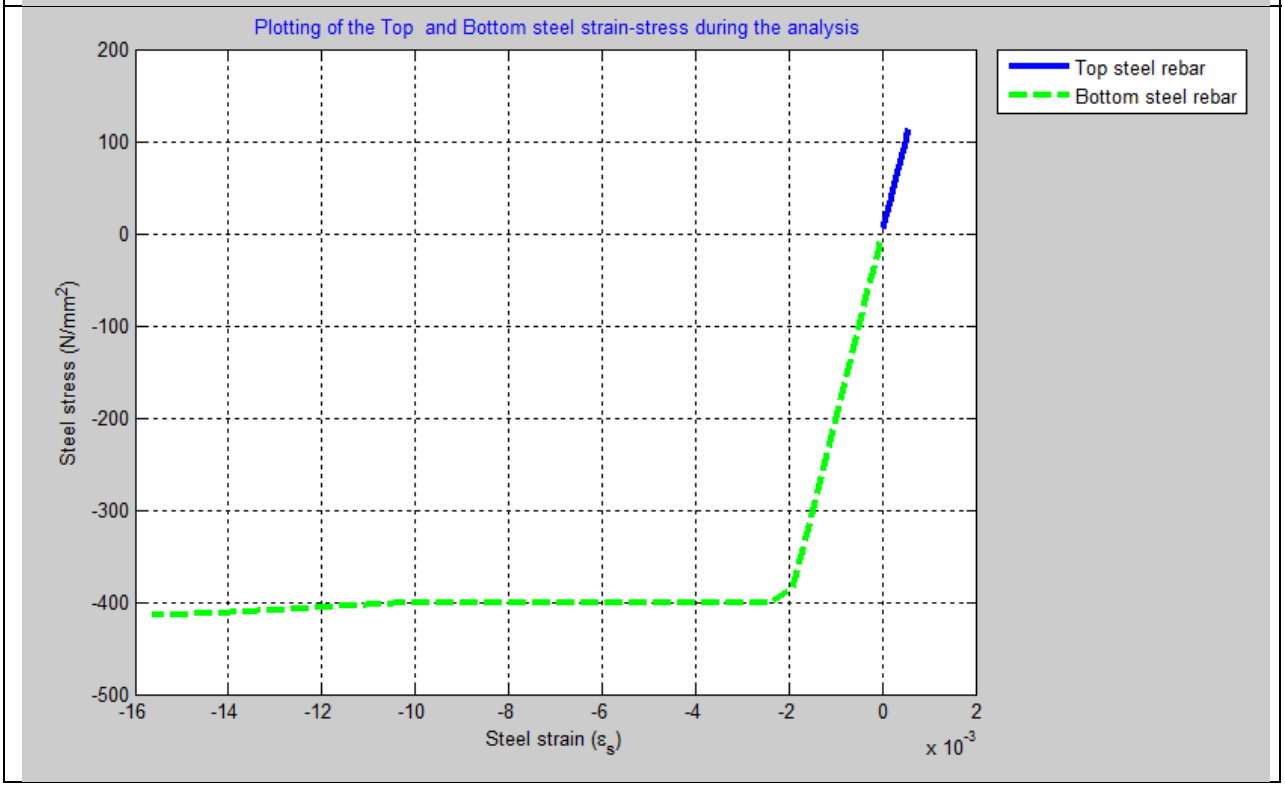
Plot:

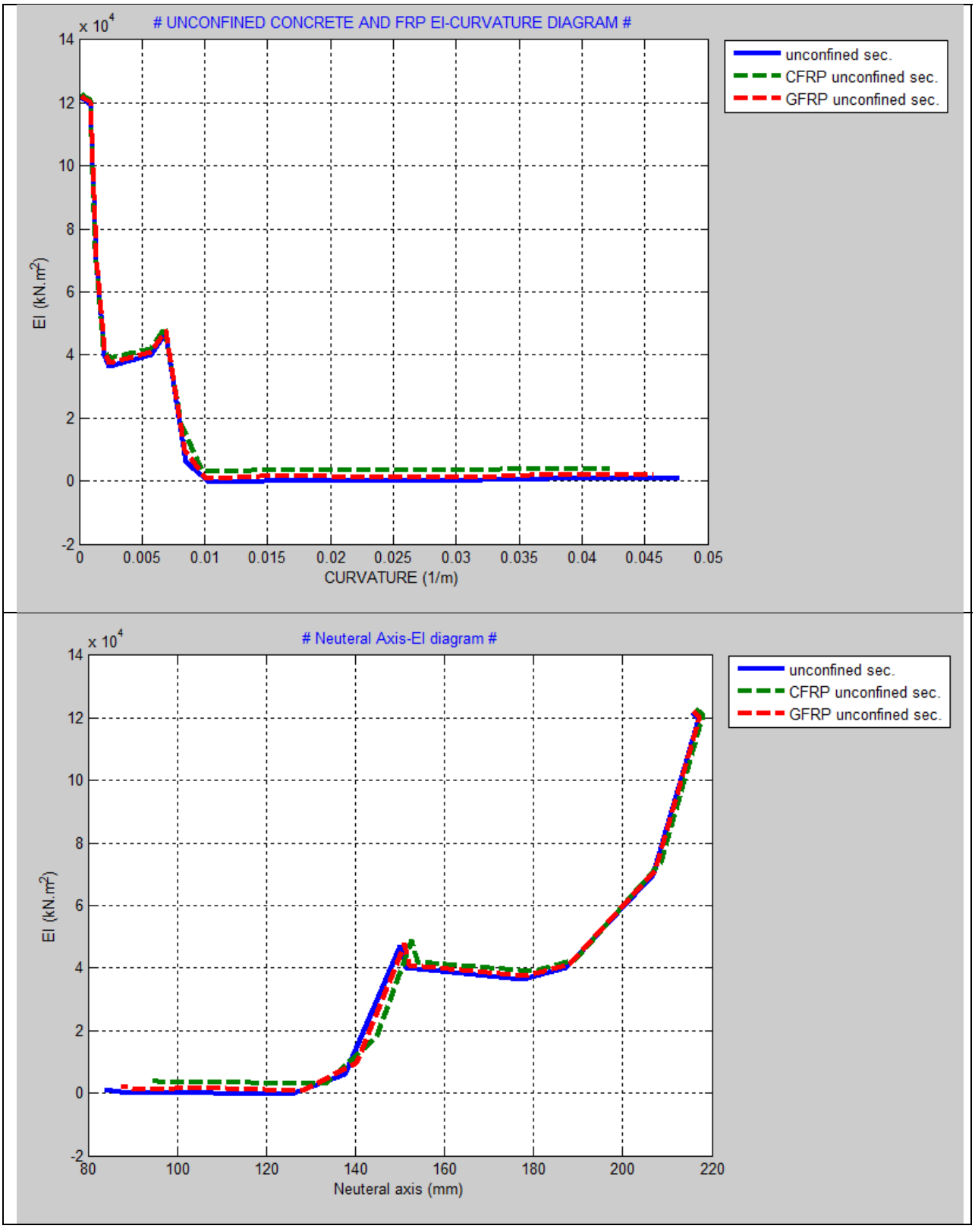

Plotting of the fiber strain and Heigh of section with CFRP - Negative value: Tension - Positive value: Compression
Height (mm)
Strain (mm/mm)
x 10^-3
ϕ= 0.00019392
ϕ= 0.00086093
ϕ= 0.0068134
ϕ= 0.018977
ϕ= 0.042339

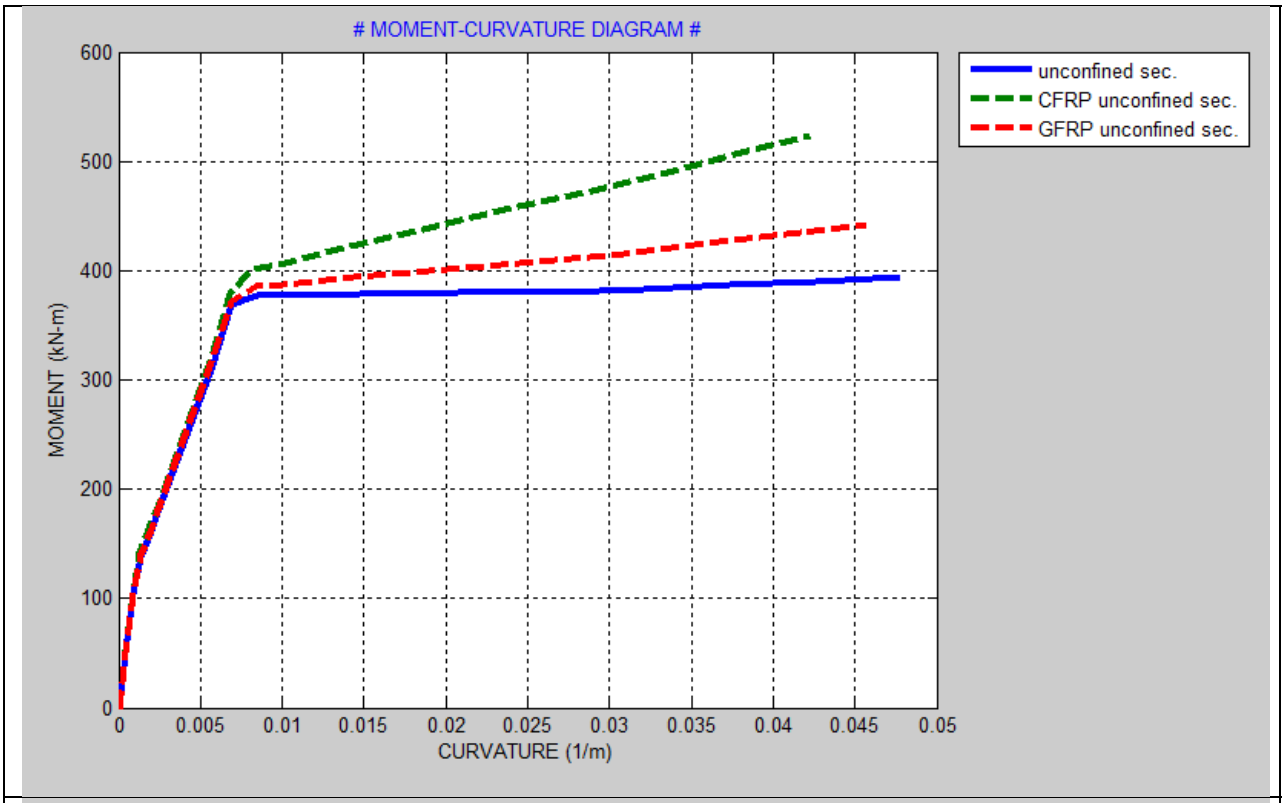

Plotting of the fiber stress and Heigh of section with CFRP - Negative value: Tension - Positive value: Compression
Height (mm)
Stress (Mpa)
ϕ= 0.00019392
ϕ= 0.00086093
ϕ= 0.0068134
ϕ= 0.018977
ϕ= 0.042339

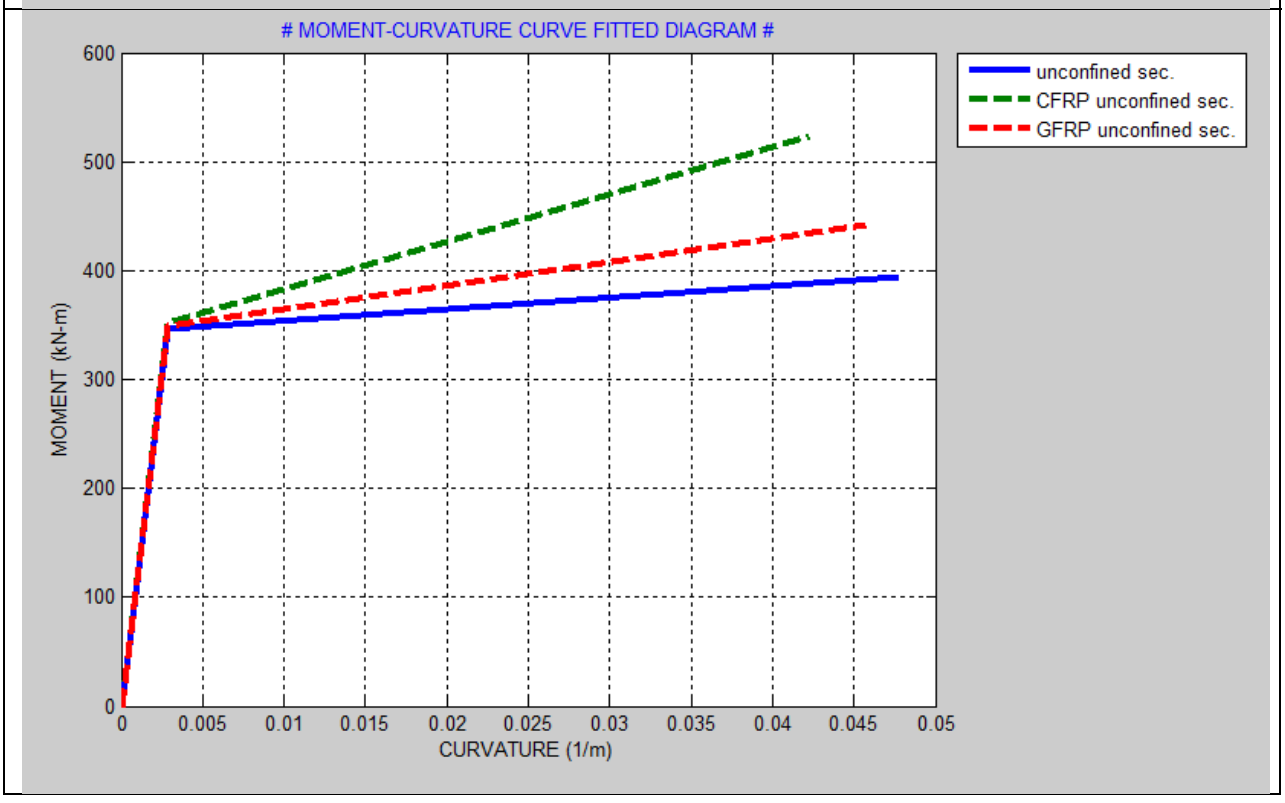

Plotting of the Top and Bottom steel strain-stress during the analysis
Top steel rebar
Bottom steel rebar
Steel stress (N/mm2)
Steel strain (εs)
x 10-3
200
100
0
-100
-200
-300
-400
-500
-16
-14
-12
-10
-8
-6
-4
-2
0
2
Plotting of the fiber strain and Heigh of section with GFRP - Negative value: Tension - Positive value: Compression
ϕ= 0.00019449
ϕ= 0.00086348
ϕ= 0.0068957
ϕ= 0.020104
ϕ= 0.045701
Height (mm)
Strain (mm/mm)
x 10-3
500
450
400
350
300
250
200
150
100
50
0
-20
-15
-10
-5
0
5


Plotting of the fiber stress and Heigh of section with GFRP - Negative value: Tension - Positive value: Compression
Height (mm)
Stress (Mpa)
ϕ= 0.00019449
ϕ= 0.00086348
ϕ= 0.0068957
ϕ= 0.020104
ϕ= 0.045701


Plotting of the Top and Bottom steel strain-stress during the analysis
Steel stress (N/mm2)
Steel strain (εs)
x 10-3
Top steel rebar
Bottom steel rebar


UNCONFINED CONCRETE AND FRP EI-CURVATURE DIAGRAM
x 10^4
EI (kN.m^2)
CURVATURE (1/m)
unconfined sec.
CFRP unconfined sec.
GFRP unconfined sec.
Neuteral Axis-EI diagram
Neuteral axis (mm)


MOMENT-CURVATURE DIAGRAM
MOMENT (kN-m)
CURVATURE (1/m)
unconfined sec.
CFRP unconfined sec.
GFRP unconfined sec.
0
100
200
300
400
500
600
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05


MOMENT-CURVATURE CURVE FITTED DIAGRAM
MOMENT (kN-m)
CURVATURE (1/m)
unconfined sec.
CFRP unconfined sec.
GFRP unconfined sec.
0
100
200
300
400
500
600
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05