Homework 5

CSCE 411
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29.4-1)
The dual is –
minimize $20y_1 + 12y_2 + 16y_3$
subject to
$y_1 + y_2 \geq 18$
$y_1 + y_3 \geq 12.5$
$y_1, y_2, y_3 \geq 0$

29.5-5)
We will form an auxiliary linear program
Maximize -x0
subject to
$-x0 + x1 - x2 \leq 8$
$-x0 - x1 - x2 \leq -3$
$-x0 - x1 + 4x2 \leq 2$
$x0, x1, x2 \geq 0$
slack form:
$z = -x0$
$x3 = 8 + x0 - x1 + x2$
$x4 = -3 + x0 + x1 + x2$
$x5 = 2 + x0 + x1 - 4x2$
$x0, x1, x2, x3, x4, x5 \geq 0$

we can call PIVOT with x0 as the entering variable and x4 as leaving.
$z = -3 + x1 + x2 - x4$
$x0 = 3 - x1 - x2 + x4$
$x3 = 11 - 2x1 + x4$
$x5 = 5 - 5x2 + x4$
$x0, x1, x2, x3, x4, x5 \geq 0$

Now the basic solution is feasible, so we can call PIVOT to get the optimal solution to the auxiliary program. x1 is the entering and x0 is the leaving variable
$z = -x0$
$x1 = 3 - x0 - x2 + x4$
$x3 = 5 + 2x0 + 2x2 - x4$
$x5 = 5 - 5x2 + x4$
$x0, x1, x2, x3, x4, x5 \geq 0$

Basic solution is optimal for the aux program. We can return this to simplex with x0 = 0.
We get,

$z = 3 + 2x2 + x4$
$x1 = 3 - x2 + x4$
$x3 = 5 + 2x2 - x4$
$x5 = 5 - 5x2 + x4$
$x1, x2, x3, x4, x5 \geq 0$

x2 is entering and x5 is the leaving variable. Calling PIVOT gives,
$z = 5 + 7/5x4 - 2/5x5$
$x1 = 2 + 1/5x4 + 1/5x5$
$x2 = 1 + 4/5x4 - 1/5x5$
$x3 = 7 - 3/5x4 - 2/5x5$
$x1, x2, x3, x4, x5 \geq 0$

x4 is entering and x3 is leaving variable. Calling PIVOT gives,
$z = 64/3 - 7/3x3 - 4/3x5$
$x1 = 34/3 - 4/3x3 - 1/3x5$
$x2 = 10/3 - 1/3x3 - 1/3x5$
$x4 = 35/3 - 5/3x3 - 2/3x5$
$x1, x2, x3, x4, x5 \geq 0$
Since all coefficients are negative, the basic solution is the optimal solution. solution is (x1, x2) = (34/3, 10/3)

29.5-6)
Basic solution isn't feasible. Auxiliary linear program is
maximize -x0
subject to
$-x0 + x1 + 2x2 \leq 4$
$-x0 - 2x1 - 6x2 \leq -12$
$-x0 + x2 \leq 1$
$x0, x1, x2 \geq 0$

slack form:
$z = -x0$
$x3 = 4 + x0 - x1 - 2x2$
$x4 = -12 + x0 + 2x1 + 6x2$
$x5 = 1 + x0 - x2$
$x0, x1, x2, x3, x4, x5 \geq 0$

calling PIVOT once with x0 as entering and x4 as leaving variable
$z = -12 + 2x1 + 6x2 - x4$
$x0 = 12 - 2x1 - 6x2 + x4$
$x3 = 16 - 3x1 - 8x2 + x4$
$x5 = 13 - 2x1 - 8x2 + x4$
$x0, x1, x2, x3, x4, x5 \geq 0$

This gives the basic solution as $(x_0, x_1, x_2, x_3, x_4, x_5) = (12, 0, 0, 16, 0, 13)$
run SIMPLEX to find optimal value for the aux function. x_1 is entering and x_3 is leaving variable.
using PIVOT,

$z = -4/3 + 2/3x_2 – 2/3x_3 + 1/3x_4$
$x_0 = 4/3 – 2/3x_2 + 2/3x_3 + 1/3x_4$
$x_1 = 16/3 – 8/3x_2 + 1/3x_3 +1/3x_4$
$x_5 = 7/3 – 8/3x_2 + 2/3x_3 + 1/3x_4$
$x_0, x_1, x_2, x_3, x_4, x_5 \geq 0$

All coefficients are negative,
basic solution $(x_0, x_1, x_2, x_3, x_4, x_5) = (4/3, 16/3, 0, 0, 0, 7/3)$ which is optimal.
Since $x_0 \neq 0$, the original linear program is unfeasible.