Homework 2

CSCE 411
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16-1.3)

For selecting the activity of least duration from among those that are compatible with previously selected activities:

i i	1	2	3
S_i	0	2	3
F_i	3	4	6
Time	3	2	3

We get activity 2. But the optimal solution chooses activity 1, activity 3

For selecting the compatible activity that overlaps the fewest other remaining activities:

i i	1	2	3	4	5	6	7	8	9	10	11
S_i	0	1	1	1	2	3	4	5	5	5	6
F_i	2	3	3	3	4	5	6	7	7	7	8
Overlap	3	4	4	4	4	2	4	4	4	4	3

Activity 6 gets selected first. After that, each choice is only two other activities -> one from activity(1,2,3,4) and one from activity(8,9,10,11). Optimal solution gives us activity(1,5,7,11)

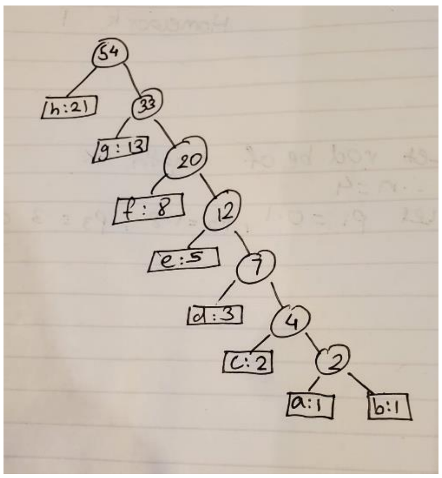
For choosing the compatible remaining activity with the earliest start time:

i i	1	2	3
S_i	0	2	4
Fi	20	4	8

The max is activity 2 and activity 3. But the earliest time is activity 1.

16.3-3)

h : 0
g : 1 0
f : 1 1 0
e : 1 1 1 0
d : 1 1 1 1 0
c : 1 1 1 1 1 0
b : 1 1 1 1 1 1 0
a : 1 1 1 1 1 1 1 0



This tree has n leaves. This will follow for the first n Fibonacci numbers.
Using 1,1,2,3 we have a basis for the first few Fibonacci numbers. Assuming this holds for all Fibonacci numbers smaller than $F_{n+2,}$
To prove for F_{n+2}:
$F_{n+2} = F_{n+1} + F_n = {}^{n-1}\sum_{i=0} F_i + 1 + F_n = {}^{n}\sum_{i=0} F_i + 1$
Therefore, $F_{n+2} < {}^{n}\sum_{i=0} F_i + 1$ and clearly $F_{n+2} < F_{n+1}$ so F_{n+2} is chosen after all smaller Fibonacci numbers have been merged.
Therefore this follows for F_{n+2}

4

16-1a)

c_q = floor(n/25), This is the largest number of quarters that can be used to make change for n cents.
$n_q = n - 25c_q$. This is the amount remaining after using c_q quarters
c_d = floor(n_q/10c) This is the largest number of dimes that can be used to make change for n cents.
$n_d = n_q - 10c_d$ This is the amount remaining after using c_q quarters.
c_n = floor(n_d/5c)
$c_p = n_p = n_d - 5c_n$.
Therefore, the solution uses c_q quarters, c_d dimes, c_n nickels and c_p pennies.

Proof: Assume Greedy solution G is not optimal. Let O be an optimal solution using o_q quarters, o_d dimes, o_n nickels and o_p pennies. If $o_p \geq 5$ we can replace every 5 pennies with one nickel, reducing the number of coins used, so we can assume $o_p < 5$. If $o_d \geq 3$, replace every three dimes with 1 quarter and 1 nickel without increasing the number of coins. Assume $o_d \leq 2$.
If $o_n \geq 2$ replace every 2 nickels with one dime, reducing the number of coins used, so we can assume on ≤ 1.
Suppose that $10o_d + 5o_n + o_p \geq 25$. The only way that this can happen is if $o_d = 2$ and $o_n = 1$.
We replace the two dimes and one nickel with one quarter, reducing the number of coins used, contradicting optimality of O. So this is impossible. This means that $10o_d + 5o_n + o_p < 25$. Since $n = 25o_q + 10o_d + 5o_n + o_p$ we have just shown that c_q = floor(n/25c) = o_q. After greedy takes off the $c_q = o_q$ quarters what remains is $n' = n - 25o_q = 10o_d + 5o_n + o_p$.
From $o_n \leq 1$ and $o_p \leq 4$, we see that $5o_n + o_p < 10$ so we choose $c_d = o_d$ dimes and then c_no_n nickels and then $c_p = o_p$ pennies. Therefore this greedy solution is optimal in **O(1)** time.