Mach Zehnder Interferometry Experiment

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1 Abstract

Quantum measurements in their nature have reductive capabilities, i.e they embody the uncertainty principle within themself. The uncertainty principle has a daunting limitation as it ensures that making a quantum measurement erases any information that we had before making it. Experiments that usually try to perform a quantum measurement se3 a total collapse of information; however, Mach Zehnder's interferometer deals with varying uncertainty. It dwells on the fact that performing a quantum measurement may not necessarily have such drastic consequences, and one could perform such measurements, where a slight measurement of their behaviour is measured and the remaining is left. In such cases, we observe a partial collapse in the information that was there before making the measurements.

2 Introduction

The Mach Zehnder Interferometer, unlike the Double slit experiment, measures the path from which the light is coming. The Double slit experiment requires one to block one of the slits in order to determine which path is taken by the light, which essentially destroys any interference pattern that could be observed.

Mach Zehnder's interferometer, on the contrary, does not have such binary consequences as it allows us to make partial measurements that correspond to the composition of light from each path. Smaller uncertainty of knowing the path the light has taken would have the greatest impact on the interference pattern. Furthermore, losing information about the path/introduces a greater uncertainty in the determination of the path taken by the light, allowing us to regain the interference pattern.
The experiment discussed in this report is the recreation of Mach Zehnder's interferometer which demonstrates these concepts in greater detail.

3 Theoretical Background

3.1 Polarization and Path Information

The polarization vector of Electromagnetic radiation, The vector along the wave's electric field, is the independent variable that allows the manipulation of the wave. The LASER fires photons of unknown polarization; hence it is impossible to write it as a state vector as soon as it is emitted by the source. Right after the source, a polarizer measures the polarization such that half of the photons are found to be vertically polarized, and the remaining half is horizontally polarized. The polarizer, as per its alignment, allows us to introduce a phase difference between the incoming beam as well, and with further polarizations, we can measure where the light is coming from. Once the polarization is measured, we have enough information to work with the light and measure the polarization across each path we split the beam. The image below shows a part of the experimental setup that shows how polarization allows us to guide the light of different polarizations across two different Paths:

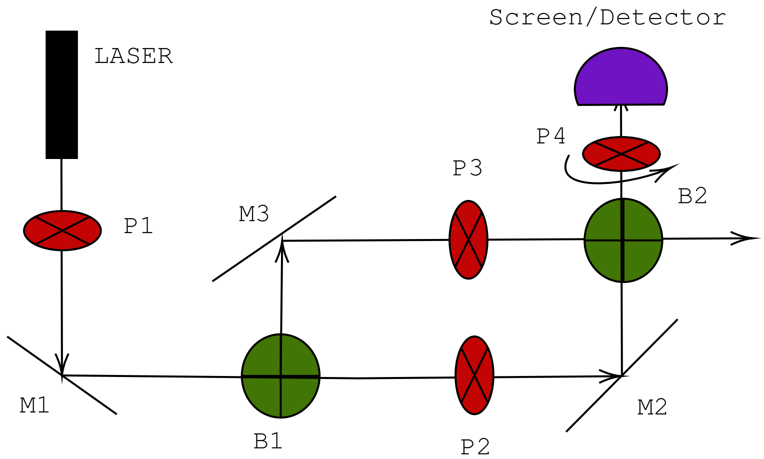

Figure 1: Inital setup for the Mach Zehnder Interferometer

The Polarizer P1 measures the polarization of the light, which allows us to write the Electric field vector as follows:

$$\vec{E} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{1}$$

After the Polarizer, the light will pass through a Beam splitter which will split the light into 2 beams, as seen in the figure. The beam splitter B1 is a non-polarizing kind and splits the beam in a 50:50 ratio[1], such that half of the polarization of either beam is vertical, and the other half is horizontal. The image above suggests how the light, once split, can be further polarized. Polarizers P2 and P3 introduce polarizations in their respective paths such that we are able to put a label on both of the paths. The control over polarizations allows us to ensure the similarity between the phase difference between the lights of two paths such that once the beams are merged again at beam splitter B2, we can determine the degree of mixing by using the polarizer P4.
Each optical instrument in the image above has a different operation on the incoming light from P1 as shown in (1), and we must perform the mathematical treatment to understand the form of the light as it approaches the detectors. As discussed earlier, the beam splitter splits the beam into 2 halves, whereas the polarizer's operation is

$$\hat{P}(\phi) = \begin{pmatrix} cos^2(\phi) & sin(\phi)cos(\phi) \\ cos(\phi)sin(\phi) & sin^2(\phi) \end{pmatrix}. \tag{2}$$

The Electric field emerging out of B2 is :

$$\vec{E}_{out} = \hat{B}\hat{P}(\phi_2)\hat{B}\vec{E}_{in} + \hat{B}\hat{P}(\phi_3)\hat{B}\vec{E}_{in}, \tag{3}$$

where $\vec{E}_{in}$ is the state vector shown in (1).Expanding the expression above in terms of operators and vectors, we can write:

$$\vec{E}_{out} = \frac{1}{4\sqrt{2}} \begin{pmatrix} cos^2(\phi_2) + cos^2(\phi_3) & sin(\phi_2)cos(\phi_2) + sin(\phi_3)cos(\phi_3) \\ cos(\phi_2)sin(\phi_2) + cos(\phi_3)sin(\phi_3) & sin^2(\phi_2) + sin^2(\phi_3) \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \tag{4}$$

Usually, phase angles ϕ_2 and ϕ_3 are fixed, and polarization of P4 is varied, which brings us to the final state vector that will be incident on our detector :

$$\begin{aligned} \vec{E}_{in} = \frac{1}{4\sqrt{2}} \begin{pmatrix} cos^2(\phi_4) & sin(\phi_4)cos(\phi_4) \\ cos(\phi_4)sin(\phi_4) & sin^2(\phi_4) \end{pmatrix} \ldots \\ \begin{pmatrix} cos^2(\phi_2) + cos^2(\phi_3) + sin(\phi_2)cos(\phi_2) + sin(\phi_3)cos(\phi_3) \\ cos(\phi_2)sin(\phi_2) + cos(\phi_3)sin(\phi_3) + sin^2(\phi_2) + sin^2(\phi_3) \end{pmatrix}. \end{aligned} \tag{5}$$

As discussed earlier, angles ϕ_2, and ϕ_3 are fixed. Just like always, the top component of the vector represents the composition of the vertically polarized light, and the bottom component represents the horizontally polarized light. Now the goal is to align P4 so that we can measure the path from which the light has passed. This operation, as can be seen, requires that the phase difference between the beams coming across P2 and P3 is such that P4 can differentiate between them. At any particular instance, only one of the paths' light is incident on the detector/screen. P4 can also be adjusted such that there is perfect mixing of light and the light intercepted by the detectors is completely mixed incident from the 2 paths; hence the intensity will be greater, and a more contrasting interference pattern will be visible.

3.2 Path Erasure

Understanding path erasures requires an understanding of the consequences of quantum measurements. It is understood that the quantum nature of Particles suggests a beam of singular packets comprises what we understand as light. Meanwhile, the classical theory interprets light as a continuous wave. Performing quantum measurements such as the one described in the previous section should remove all of the localization that we observed prior to making any such measurements.

Path Erasure, as the name suggests is the erasure of the path information. Obtaining Path information is indeed a quantum measurement and precisely measuring the path the light has taken to the detector in figure 3.1 should essentially wipe out any classical certainties. Consequentially, knowing the path light has taken wipes away the interference pattern[2]. Intuition suggests that if knowing the path wipes away the interference pattern, not knowing the path should bring it back; however, it is not as simple as that. Path Erasure/ Having Path information is not as binary as one would think, especially in Mach Zehnder's interferometer. Since We are using polarizations to determine the path of the light incident on the screen, we can still introduce the mixing of the light from both of the paths, knowing the contributions of each path incident on the screen. This introduces a greater uncertainty in our Path information compared to precisely knowing the path. As uncertainty is increased, we started regaining our interference pattern, with the most distinct fringes appearing when we have a complete mixing of light from both of the paths.

The Mach Zehnder interferometer requires the measurements of the interference pattern's intensity as we introduce mixing in the incident light. It makes sense as having the path information requires a phase difference between the two paths, such that at any instance, the lights from both of the paths is not incident; hence a smaller intensity would be observed. As path information is erased, there is light from both of the paths with different compositions incident on the detector, and we should see a brighter intensity pattern. We can see a brighter and more distinct interference pattern in the

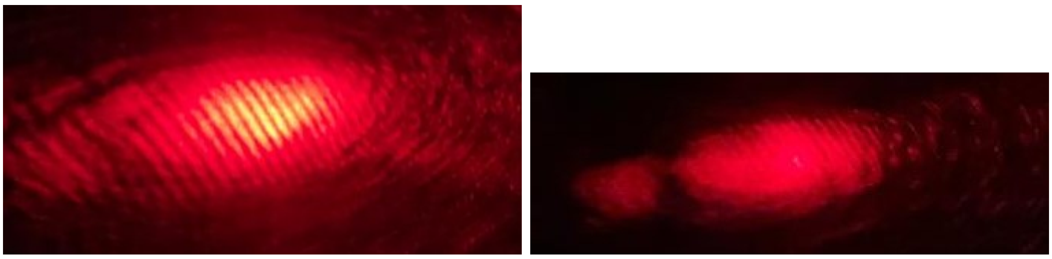

Figure 2: Interference patterns showing a negligible Phase difference vs a significant phase difference between the Paths.

image on the right, whereas on the left, the spot can merely be called an interference pattern. These two images show how due to an insignificant phase difference when we have no information about the path of the lights incident on the screen, whereas the images on the light show a significant phase difference, and at any instance, we can't say that light from both of the path's will be interfering with each other.

4 Experimental Setup

Figure 1 shows the complete experimental setup used in the latter half of the experiment. Initially, the Polarizer P4 is absent, and the light emergent from the 2 paths is incident on the screen. For this part, as can be seen in the figure below:

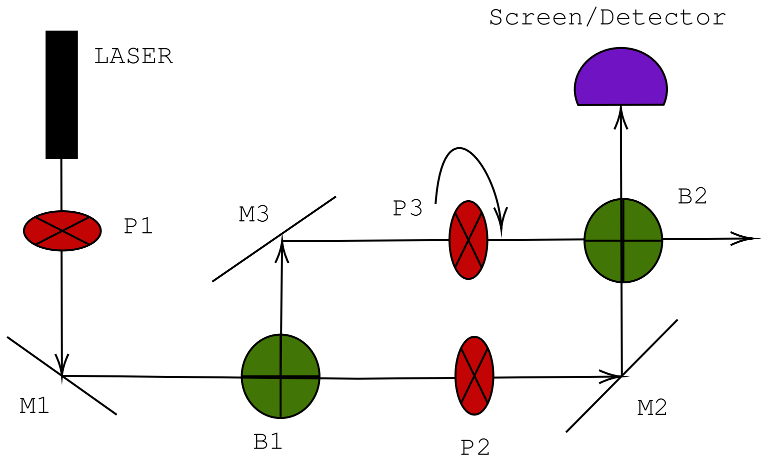

Figure 3: Initial Experimental setup

In this part, the P2 is set at $\frac{\pi}{2}$ and P3 is rotated to measure the intensity pattern as we change the phase difference. Since (4) is the vector before P4, let's set $\phi_2 = \frac{\pi}{2}$, and write the state vector in terms of ϕ_3:

$$\vec{E}_{out} = \frac{1}{4\sqrt{2}} \begin{pmatrix} cos^2(\phi_3) + sin(\phi_3)cos(\phi_3) \\ 1 + sin(\phi_3)cos(\phi_3) + sin^2(\phi_3) \end{pmatrix}. \tag{6}$$

The intensity of the light incident is calculated by taking the following product:

$$I = \vec{E}^*_{out}\vec{E}_{out},$$

which simplifies to:

$$I = 2 + 2sin(2\phi) + 2sin(\phi)^2. \tag{7}$$

. This is required to check how the interference pattern varies theoretically once we introduce the phase difference between the 2 paths.

4.1 Optical Alignment

The optical alignment for this experiment is fairly tedious, as it requires the alignment of 3 polarizers, 2 beam splitters, and 3 mirrors. The light's path is changed several times and hence requires accurate vertical and horizontal positioning of the beam across both paths to see an interference pattern. To ensure such alignment, irises are required at 2 points across each length. Placing 2 irises mounted around 10 cm apart allows us to ensure that the lights are parallel to the optical breadboard; slight deviations from the desired trajectory could fail to produce an interference pattern.

Once the irises were placed across each path, the mirror or the beam splitter from which the light had reflected before the Iris was adjusted until light passed through both of the irises. Once light passes through all of the optical board is parallel to the optical breadboard and is horizontally aligned as well, it is required to ensure the beams passing from polarizers P2 and P3 are incident on the same spot on the beam splitter and stay coalesced until they are incident on the screen. Any further optical alignment is not required for the polarizer P4 as it is introduced in an already aligned trajectory.

4.2 Data Acquisition

From this part of the report, the experiment will be discussed in 2 parts, the first one being the measurements taken for the variation in the intensity of the interference pattern when the phase difference of the lights is changed. The setup for this part can be seen in figure 3. As shown in (6), angle ϕ_2 is fixed at $\frac{\pi}{2}$ and P3 is rotated 2π radians, with a measurement taken after every rotation of $\frac{\pi}{12}$ radians.

The other part of the experiment measured the intensity of the light by introducing a different degree of mixing and obtaining the path information at any orientation of P4. For this part, (5) is valid; however, phase angles for P2 and P3 are fixed. $\phi_2$is fixed at 0 radians and ϕ_3 is set at $\frac{\pi}{2}$, whereas P4 is rotated and the variation in intensity is measured.The variation in intensity for any interference pattern is given by the expression below:

$$v = \frac{I_{max} - I_{min}}{I_{max} + I_{min}},$$

where I_{max} corresponds to the intensity of the brightest fringe, and I_{min} corresponds to the intensity of the darkest fringe.The greatest variation will correspond to the most distinct interference pattern, whereas a smaller variation would mean the interference pattern is fading away

The variation in intensities for both parts was taken by taking a close-up shot of the interference pattern after every rotation of $\frac{pi}{12}$of P3 and P4 for their relevant parts. Those pictures were then uploaded to ImageJ, which measured the grey value(the darkness/the absence of illumination) of each pixel starting from the centre of the image and then measured the change in grey value as the distance from the centre was increased. The interference pattern for this measurement was enlarged using a plano-convex lens. Furthermore, the intensity of each pattern was also measured using a photo-detector. The photo-detector emitted a current corresponding to the intensity pattern, which was then resisted by an I-V converter that alleviated the signal intensity in terms of the potential difference. A voltmeter or a multimeter intercepted the signal, and a reading was taken after rotating the polarizer every $\frac{\pi}{12}$ radians.

5 Experimental Results and Further Discussion

5.1 Variation of intensities due to the phase difference along the paths

The variation of intensities is shown in figures 19 -30. The variation graphs are only shown for rotation until π radians, as a repeating pattern emerges after that. This can be further confirmed by figure 17, which shows how the output from the photo-detector - analogous to the intensity of the pattern, shows how the pattern emerges after π radians. We can see in figure17 that there is a dip in intensity corresponding to $\frac{\pi}{4}$, moving over to 21, we can see that the variation of intensity is also reasonably uniform, that is reminiscent of a weak interference pattern. For this part of the experiment, P2 is aligned at $\frac{\pi}{2}$ and P4 is at $\frac{\pi}{4}$, which is when there is the least amount of mixing between the lights from either of the paths at an instant.

We observe a peak at $\frac{2\pi}{3}$, showing that we will have a maximum interference pattern. We can see in figure 13 that there is the maximum variation in the interference pattern corresponding to the peak as the graph is not smooth and its grey value fluctuates.

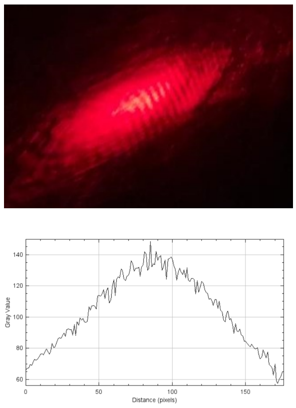

Figure 4: P3 rotated at 0 rad

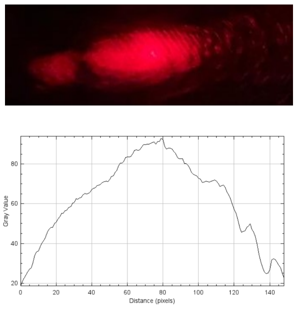

Figure 7: P3 rotated at $\frac{\pi}{4}$ rad

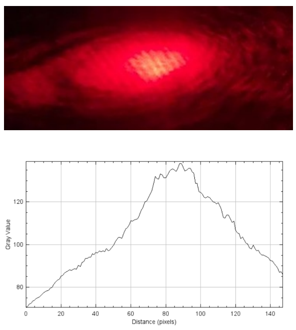

Figure 5: P3 rotated at $\frac{\pi}{12}$ rad

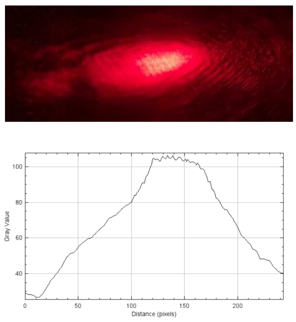

Figure 8: P3 rotated at $\frac{\pi}{3}$ rad

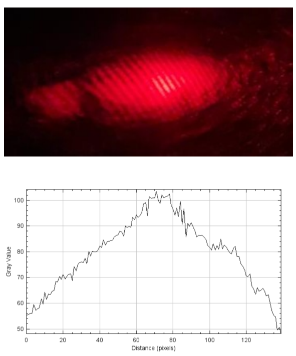

Figure 6: P3 rotated at $\frac{\pi}{6}$ rad

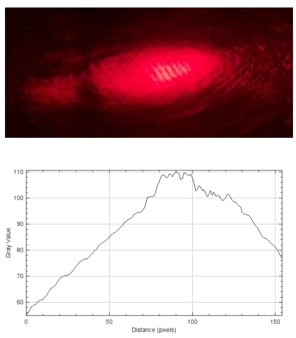

Figure 9: P3 rotated at $\frac{5\pi}{12}$ rad

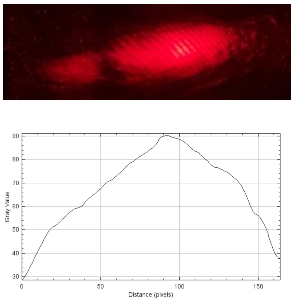

Figure 10: P3 rotated at $\frac{\pi}{2}$ rad

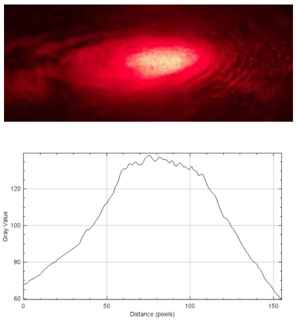

Figure 11: P3 rotated at $\frac{7\pi}{12}$ rad

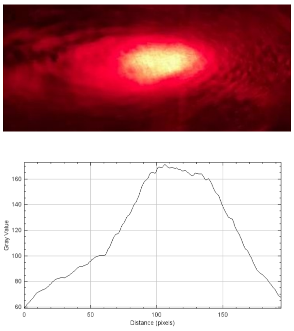

Figure 12: P3 rotated at $\frac{2\pi}{3}$ rad

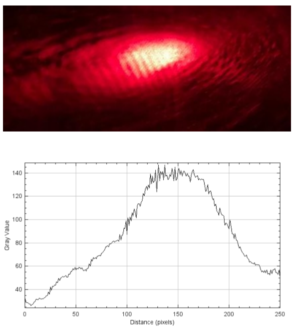

Figure 13: P3 rotated at $\frac{3\pi}{4}$ rad

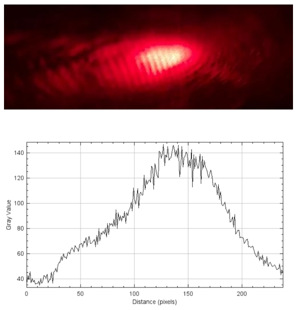

Figure 14: P3 rotated at $\frac{5\pi}{6}$ rad

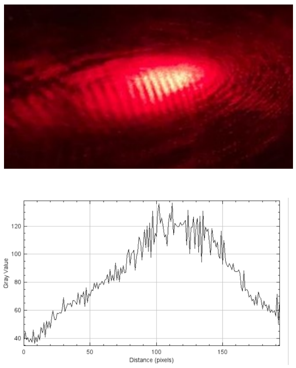

Figure 15: P3 rotated at $\frac{11\pi}{12}$ rad

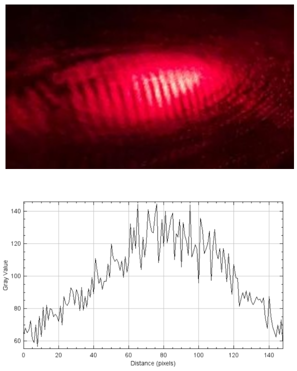

Figure 16: P3 rotated at π rad

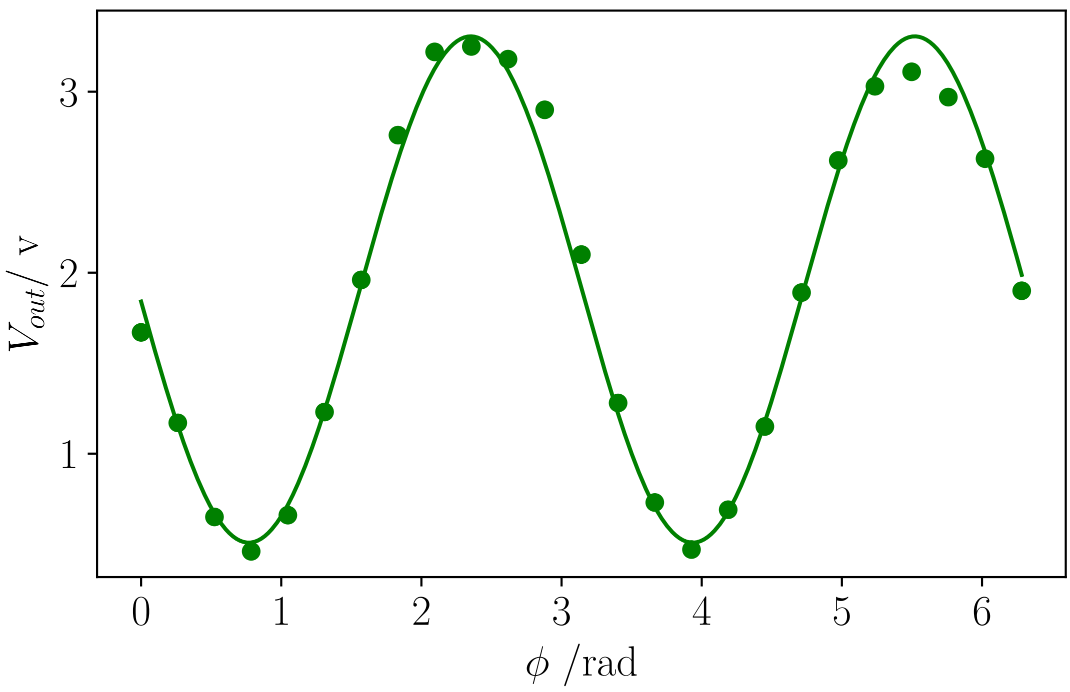

Figure 17: Photo-detector's output while rotating P3

5.2 Variation of Intensities While gaining Path information

For this part of the experiment, Polarizer P2 was fixed at 0 radians and P3 was fixed at $\frac{\pi}{2}$. This ensured that before passing through P4, one polarization took the path across P3, and the other took P2. The degree of mixing of the light is determined by the interference pattern produced by changing the phase angle of P4.

For fixed P2 and P3, The state vector in (5) becomes:

$$\vec{E}_{out} = \frac{1}{4\sqrt{2}} \begin{pmatrix} cos^2\phi_4 + sin(\phi_4)cos(\phi_4) \\ \\ sin^2\phi_4 + sin(\phi_4) + cos(\phi_4) \end{pmatrix}. \tag{8}$$

The polarizer P4 can be aligned such that only vertical or horizontal polarisation hits the detector, which is essentially the measurement of the path. When P4 is aligned in such a manner, we see a minimum interference pattern as the wavefunction collapse occurs as soon as the measurement is taken.

We can see from 31 that a dip of intensity occurs at $\frac{\pi}{4}$ radians. The corresponding grey value plot also shows a minimal variation in the intensity, which means that the interference pattern faded away as soon as path information was determined. Similarly, When we allow mixing such that there is no information of the path taken by the light, we see a maximum in 31 at $\frac{2\pi}{3}$, and we can see a varying pattern in the corresponding grey value graph showing that corresponds to a more distinct interference patter,

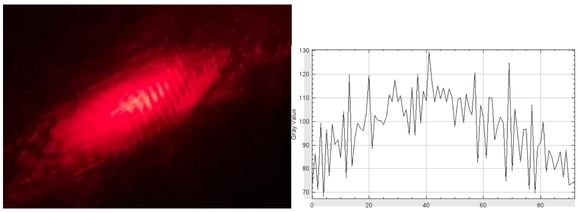

Figure 18: P4 rotated at 0 rad

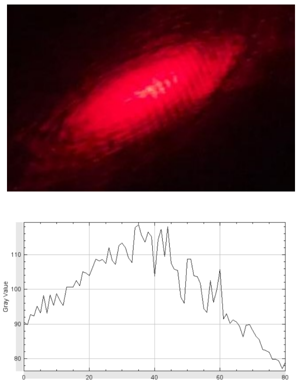

Figure 20: P4 rotated at $\frac{\pi}{6}$ rad

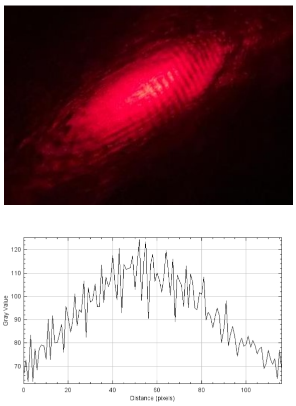

Figure 19: P4 rotated at $\frac{\pi}{12}$ rad

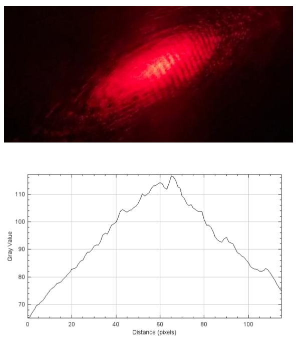

Figure 21: P4 rotated at $\frac{\pi}{4}$ rad

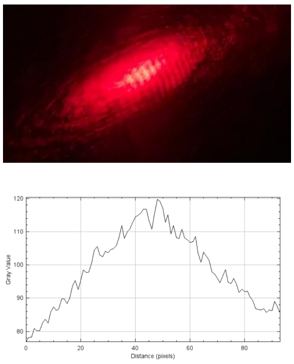

Figure 22: P4 rotated at $\frac{\pi}{3}$ rad

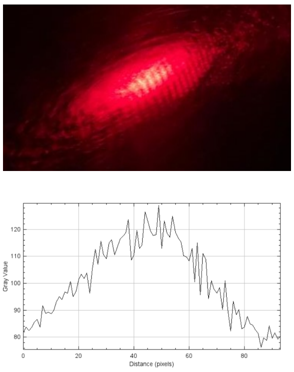

Figure 23: P4 rotated at $\frac{5\pi}{12}$ rad

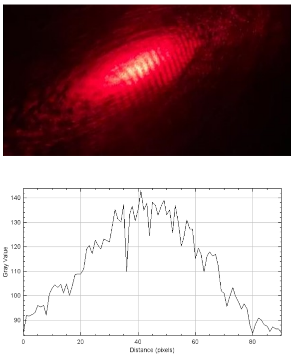

Figure 24: P4 rotated at $\frac{\pi}{2}$ rad

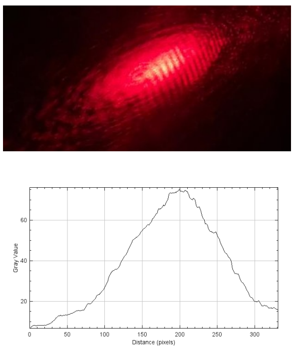

Figure 25: P4 rotated at $\frac{7\pi}{12}$ rad

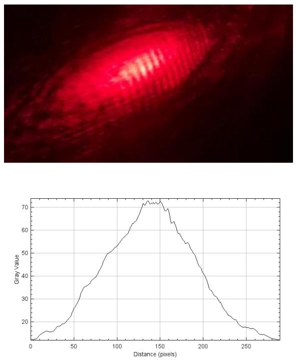

Figure 26: P4 rotated at $\frac{2\pi}{3}$ rad

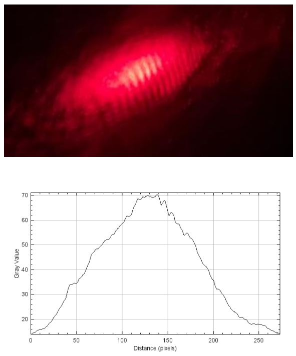

Figure 27: P4 rotated at $\frac{3\pi}{4}$ rad

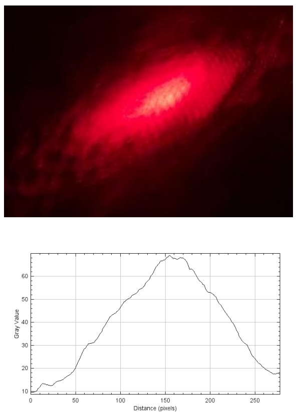

Figure 28: P4 rotated at $\frac{5\pi}{6}$ rad

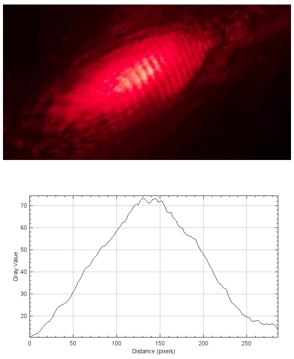

Figure 29: P4 rotated at $\frac{11\pi}{12}$ rad

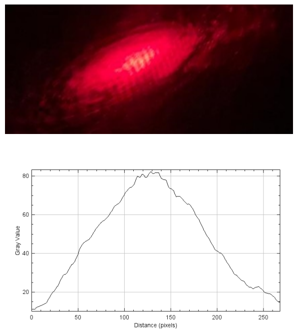

Figure 30: P4 rotated at π rad

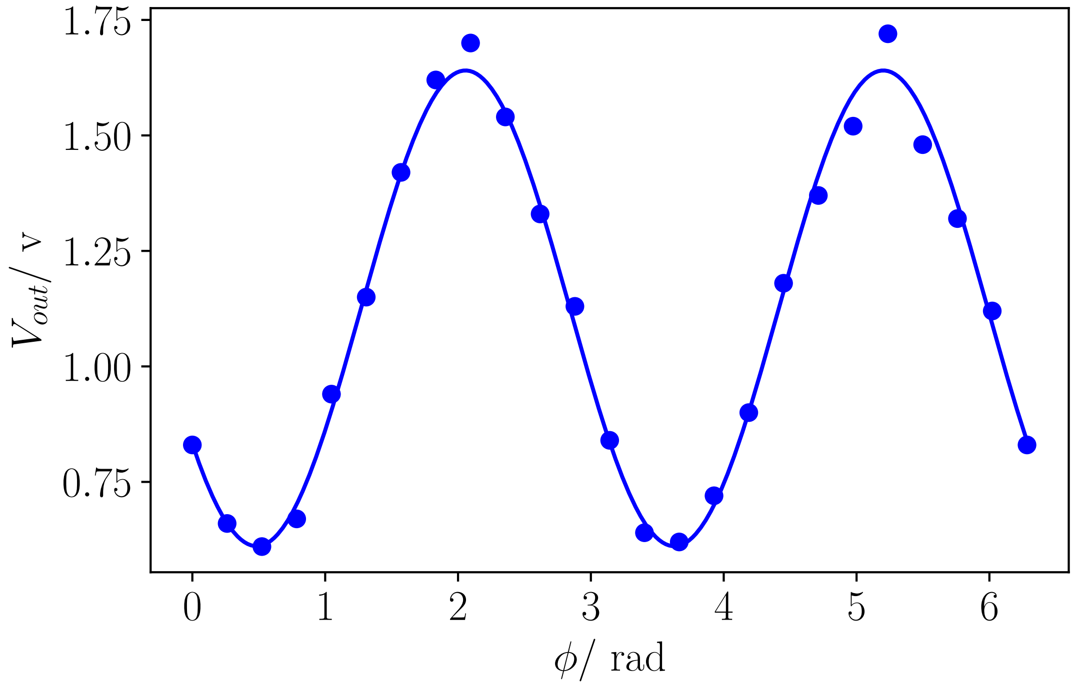

Figure 31: Photo-detector output while P4 was rotated

6 Conclusion

The experiment detailed above is a recreation of the sophisticated Mach Zehnder interferometry, which showed that the classical aspects of light could be observed if partial quantum measurements were being taken on the system. The experiment taught us how a quantum measurement can be made, and what are the physical consequences of making such measurements.

References

[1] O. K. Sabieh Anwar, Lahore University of Management Sciences.

[2] A. Maries, R. Sayer, and C. Singh, "Can students apply the concept of "which-path" information learned in the context of mach–zehnder interferometer to the double-slit experiment?" *American Journal of Physics*, vol. 88, no. 7, pp. 542–550, 2020. [Online]. Available: https://doi.org/10.1119/10.0001357