Precession of Perihelion

Course Project for PHY 442: General Relativity

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Abstract

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1 Introduction

Perihelion, the closest point of Mercury the sun, in its plan of rotation around the sun appeared to shift its position in its plane.The problem was attempted through several techniques using Newtonian mechanics, as under that regime it was reasoned that precession occured solely due to the perturbation of the exterior planets, and solving a multi-body problem precisely is not possible. The Precession of Perihelion, see image 1, is the precession of Mercury's orbit while staying in the same plane. The closest point of approach takes a different value of the azimuthal angle ϕ compared to the previous approach of the perihelion. The blue dots in the image are the perihelia of Mercury as it orbits the sun.

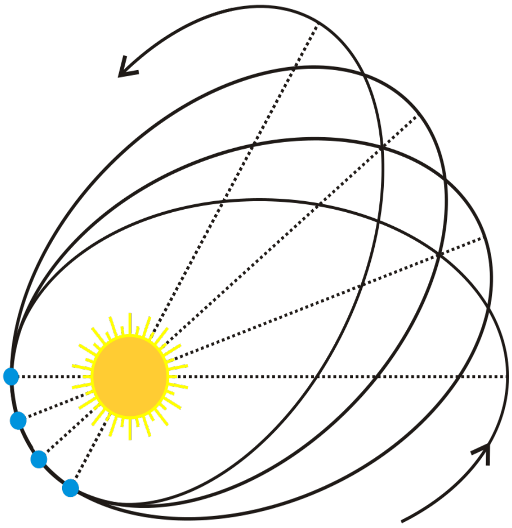

Figure 1: Precession of Perihelion(ref image)

The problem arose from the errors in Mercury's transit time calculated by the two-body problem's Kepler's orbits. Since the two-body problem is a simplified version of the planetary motion, the precession of perihelia was associated with the gravitational influence of the nearby planets. The perturbation of the outer orbits is known to precess the Perihelion. However, in the 19th century, Urbain Jean Le Verrier, after his Newtonian calculations, concluded that the error of 55 arc minutes in Mercury's transit time could not be resolved using Newtonian Mechanics(cite C pollock). The correction term that resolved this error came from the General Relativistic solution that corrected the potential in the central force problem, see more in:3.

2 The Incomplete Newtonian Approach

There is no shame in the inability to solve multi-body problems and "ignoring" the higher order terms and the interaction terms and their weak effects, as we can produce interesting results in line with the experimental evidence in these limits. However, several results become challenging to reason under this limit at the cost of arriving at the solution more easily.

The precessing orbit of Mercury in the Newtonian regime was reasoned to the perturbing effects of the outer planets. This would make sense as the orbiting planets could pull its elliptical orbit due to their gravitational effects. However, conventionally this approach would require us to solve the multi-body problem, which, as we know, can not be solved. Following Price and Rush(cite here), We will assume that outer planets are evenly distributed rings of masses, which, although far from experimental evidence, can produce good approximations in the Newtonian regime.

2.1 The Planetary Rings

To solve the problem in the Newtonian regime, we argue that the evenly distributed rings are a good approximation for two reasons. The first one is that since the planetary bodies follow the regime of classical particles, there is an equal probability of finding them along their orbit. Secondly, the gravitational pull of the Sun on Mercury is far stronger than that of other planets; consequently, the perturbation at any time would be small. Hence a linearly distributed ring sets up a solvable problem that can give a reasonable approximation with the deficit corrected later by the General Relativistic approach. This idea is illustrated in the image below, the grey planet in represents mercury, and the rings represent the exterior planets.

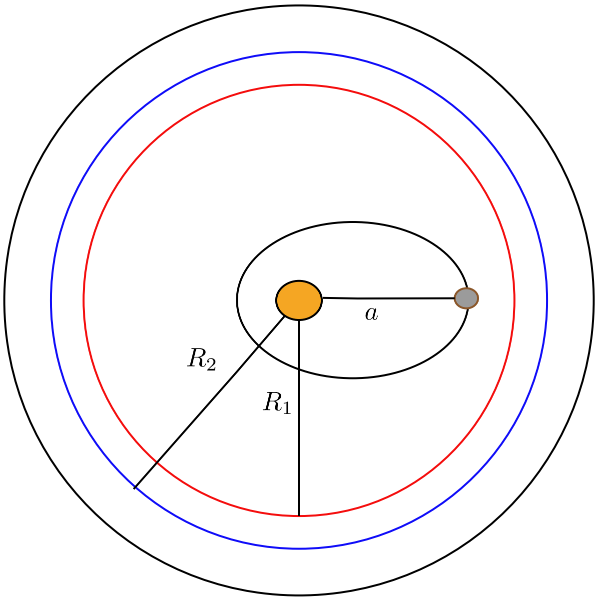

Figure 2: Concentric Planetary rings

To calculate the force due to these rings, we will have to integrate the Newtonian gravitational pull onto Mercury due to each mass element of these planetary rings. We take the mass distribution of each planet to be: $\lambda_i = \frac{M_i}{2\pi R_i}$, and we argue that each mass element dm_i on the ring has a pull equal to :

$$dF_i = GM\frac{Gdm}{r^2}. \tag{2.1}$$

Where, r, is the distance of the ring from the planet, and M is the planet's mass. To calculate the contribution of the whole ring, we will have to find the force as a function of Mercury's radial position. We will take Mercury to be initially at position $r = a$, as can be seen in the image below. From this point, we will measure the distance of each mass element of the outer ring and then integrate it over the whole ring.

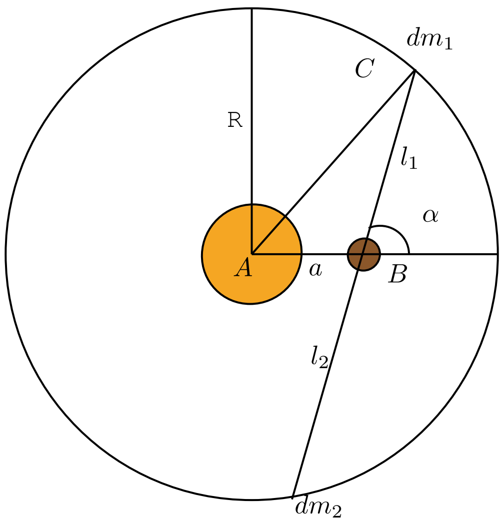

Figure 3: The circular distribution of Planets.

As it can be seen in the figure 3, dm_1's pull will be counteracted by dm_2, and so will every other point on the ring due to circular symmetry, (2.1) will become :

$$dF_i = GM\frac{dm_1}{l_1^2} - \frac{dm_2}{l_2^2},$$

upon substituting the mass density: $dm_j = \lambda_i ds_j$, where ds is the arc subtended by l_j and α. For small angular increments, the relationship can be approximated by: $dm_i \approx \lambda l_i d\alpha$, and the equation becomes:

$$dF_i = GM\lambda_i\left(\frac{l_2 - l_1}{l_1 l_2}\right)\hat{l}d\alpha,$$

in the $\hat{l}$ direction. Additionally, we will argue that because of the circular symmetry of the problem, only the components along the line AB will contribute, the perpendicular components will cancel. therefore the relationship will become radial, and the force will be :

$$F = GM\lambda\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\alpha\frac{l_2 - l_1}{l_1 l_2}cos(\alpha). \tag{2.2}$$

We know that l_1 and l_2 will be functions of alpha; substituting these functions and solving the integral in (2.2) will give the force acting on the planet by the rings of exterior planets. We can see the angle ABC in Fig 3 will be $\pi - \alpha$, and using the law of cosines, we have:

$$R^2 = a^2 + l_1^2 + 2al_1cos(\alpha)$$
$$l_1 = -acos(\alpha) \pm \sqrt{a^2cos^2(\alpha) - a^2 + R^2}.$$

The solution with the negative radical is unphysical; at $\alpha = 0$, we must have $l_1 = R - a$.Applying the same technique on l_2, the following result can be obtained:

$$l_2 = acos(\alpha) + \sqrt{a^2cos^2(\alpha) - a^2 + R^2}$$

Substituting these in (2.2):

$$F_i = GM\lambda_i\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\alpha\frac{2acos^2(\alpha)}{R^2 - a^2} = \frac{MG\lambda_i a\pi}{R_i^2 - a^2}$$

The relationship above is the force due to the i'th "ring-shaped" exterior planet onto the precessing planet. The total Newtonian Force will be :

$$\mathcal{F}(r) = -\frac{GM_sM}{r^2} + \sum_{i=1}^{8}\frac{MG\lambda_i\pi r}{R_i^2 - r^2}, \tag{2.3}$$

which will lead us to the solutions for the shift in Mercury's orbit.

2.2 The Perturbed Central force Problem

For stable orbits, we know the moving body is supposed to be bound by the potential, and slight disturbance should not prevent it from retracing the path. Starting off with the Lagrangian of motion in polar coordinates(cite: Goldstein), we have :

$$\mathcal{L} = \frac{1}{2}(\dot{r}^2 + r^2\dot{\phi}^2) + V(r)$$

The Lagrangian equations of motion reduce our problem to:

$$M(\ddot{r} - r\dot{\phi}^2) = \frac{dV}{dr}$$

The above equation is known as the radial equation of motion. The $\frac{dv}{dr}$ term is the change in potential in the radial direction, also known as the force. Solving the azimuthal equation gives us the conservation of angular momentum: $L = Mr^2\dot{\phi}$. Substituting this in the radial equation, and replacing the derivative of potential with $\mathcal{F}$, reduces to :

$$M\left(\ddot{r} - \frac{L^2}{Mr^3}\right) = \mathcal{F}(r) \tag{2.4}$$

Here, $\mathcal{F}$ will be a combination of (2.2), and the Newtonian Gravity. As discussed above, small perturbations in the radial directions for stable orbits should not prevent the body from retracing its path. The (2.3) and (2.4) should remain valid. At this point, we will make another assumption that locally near a, the particle is performing the circular motion, so in the proximity of a, $\ddot{r} = 0$, which gives: $\mathcal{F}(a) = -\frac{L^2}{Ma^3}$. Now we will introduce perturbations near a, and argue that the planet's radial positions can be considered as oscillations in its radial position about $r = a$ during its elliptical orbit. We will introduce a new variable $x = r - a$, where r is the perturbations about a, in (2.4):

$$\mathcal{F}(r) = \mathcal{F}(x+a) = m\left(\ddot{x} - \frac{L^2}{M(x+a)^3}\right),$$

$$\mathcal{F}(x+a) = m\left(\ddot{x} - \frac{L^2}{Ma^3(1+\frac{x}{a})^3}\right)$$

We will further massage these equations by binomially expanding the denominator of the second equation and keeping the terms up to the first order as $x << a$. Furthermore, we will expand the left-hand side of the equation using the Taylor series about $r = a$ up to the first order, considering the higher order terms will have weaker effects, as x is small.

$$\mathcal{F}(r) = \mathcal{F}(a) + \mathcal{F}'(a)(r-a)$$

$$\mathcal{F}(a) + \mathcal{F}'(a)(r-a) = m\left(\ddot{x} - \frac{L^2}{Ma^3}\left(1 - \frac{3x}{a}\right)\right)$$

$$m\ddot{x} = \mathcal{F}(a) + \frac{L^2}{Ma^3} + \left(\mathcal{F}'(a) - \frac{L^2}{Ma^3}\frac{3}{a}\right)x$$

using the relation $\mathcal{F}(a) = -\frac{L^2}{Ma^3}$, the above equations will reduce to:

$$\ddot{x} = -\frac{1}{m}\left(-\mathcal{F}'(a) - \frac{3}{a}\mathcal{F}(a)\right)x. \tag{2.5}$$

With excessive massage to our equations, we have reduced our problem to a second-order differential equation with oscillatory solutions if the coefficient of x is less than zero. Assuming that the planet is indeed oscillating because it is, we know that the frequency of these oscillations will be the 2π times the root of the coefficient of x, hence the period will be:

$$T = 2\pi\sqrt{\frac{M}{-\mathcal{F}'(a) - \frac{3}{a}\mathcal{F}(a)}}. \tag{2.6}$$

The above expression is a very important result for calculating the precession of Perihelion in the Newtonian regime. Using this, we can calculate the ϕ angle swept in a period, as we expect the planet to have the same distance from the centre after every cycle.

2.3 Results in the Newtonian Regime

For a precessing orbit, we can expect that in one period, the planet will be at the same radial position from the Sun; however, the angle swept during that period will not be 2π. To calculate that, we will assume that Mercury's radial distance stays close to $r = a$, so the angular momentum stays constant during orbit. This assumption allows us to calculate the angle ϕ swept in 1 period:

$$\phi_T = T\dot{\phi} = 2\pi\sqrt{\frac{M}{-\mathcal{F}'(a) - \frac{3}{a}\mathcal{F}(a)}}\left(\frac{L}{Ma^2}\right). \tag{2.7}$$

Using $\mathcal{F}(a) = -\frac{L^2}{Ma^3}$, in the expression above, we will get:

$$\phi_T = 2\pi\left(3 + a\frac{\mathcal{F}'(a)}{\mathcal{F}(a)}\right)^{-\frac{1}{2}}$$

Plugging in (2.3), and its derivative in the equation above, we get:

$$\phi_T = 2\pi\left(3 + \frac{-2F_0 + K}{F_0 + \sum_{i=2}^{8} F_i}\right)^{-\frac{1}{2}} = 2\pi\left(\frac{F_0 + 3\sum_{i=2}^{8} F_i + K}{F_0 + \sum_{i=2}^{8} F_i}\right)^{-\frac{1}{2}}$$

Where F_0 is the Newtonian gravity, $K = \sum_{i=2}^{8} \frac{MG\lambda_i \pi a(R_i^2+a^2)}{(R_i^2-a^2)^2}$, an F_i is the force due to the planetary rings as discussed above. We will now divide the numerator and the denominator by F_0, and then further simplify the problem using binomial expansion as $F_0 >> \sum_{i=2}^{8} F_i$:

$$\phi_T = 2\pi \left(1 + \frac{3\sum_{i=2}^{8} F_i + K}{F_0}\right)^{-\frac{1}{2}} \left(1 + \frac{\sum_{i=2}^{8} F_i}{F_0}\right)^{\frac{1}{2}}$$
$$\phi_T = 2\pi \left(1 + \frac{\sum_{i=2}^{8} F_i}{2F_0}\right) \left(1 - \frac{3\sum_{i=2}^{8} F_i + K}{2F_0}\right).$$

We can calculate F_0, F_i's and K for each planet and subtract ϕ_T from 2π to get the azimuthal shift from the initial position in the plane of the orbit once it arrives at the same radial position.

Planet	Radius / $(10^{11}m)$	$\Delta\phi_{obs}$ arcsec/cen	$\Delta\phi_{pred}$arcsec/cen
Mercury	0.58	575	532
Venus	1.08	8.4	258
Earth	1.49	5.0	194

The above results (Cite scidirect)for Mercury show an error of 43 arcseconds per century, where arcseconds is $\frac{1}{3600}$ of a degree. Meanwhile, for Earth and Venus, the Newtonian calculation completely breaks down. The reason for the error in Mercury's Perihelion is the absence of the general relativistic correction that we will discuss in the next section. There were different ideas before general relativity, such as the existence of a dark planet, "Vulcan"; however, the general relativistic correction disproved any such possibility. The problem in the Newtonian predictions of Venus and Earth is that the perturbative model that we discussed in this section does not work well with orbits of low eccentricity as these become highly sensitive to perturbations as we know: $L^2 = ma(1 - e2)$ (cite d'inverno), and plugging this into (2.7) will give a $\Delta\phi$ that is extremely sensitive to perturbations.

3 The General Relativistic Correction

For the adoption of the General Relativistic model of Gravity, it needed to predict results that removed the discrepancies from the Predictions of Experimental results. These experiments were related to the observed solar system in the weak field limit, which means that the gravitational effects are fairly close to the Newtonian predictions. The error in Mercury's precession of perihelion was also suggested as a test for general relativity, and accurate predictions of these results allowed for General Relativity to be taken as the accepted model for gravity.
The general relativistic approach to the residual precession of the perihelion revisits the potential due to the central body, and due to the spherical symmetry in our problem, the metric that describes the spacetime is the Schwarzschild metric. From this point onwards, I will be following d'inverno(cite dinverno) to calculate the error in Mercury's Perihelion.

3.1 The geodesics of the Schwarzschild metric

Planets in stable orbit are moving along a geodesic, in order to solve for the orbits in our central mass system with spherically symmetric solutions, we will have to calculate the geodesics of the schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M_s}{r}\right) dt^2 + \left(1 - \frac{2M_s}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 sin^2(\theta) d\phi^2. \tag{3.1}$$

As per the variational approach, the geodesics can be solved for by taking the path of the stationary action, which means that we can solve for the geodesic by extremizing the action across two events in the space, as the shortest distance between them will be along the geodesic:

$$S = \int_a^b ds = \int_a^b \sqrt{g_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta} d\lambda,$$

which gives us the lagrangian:

$$L = \sqrt{g_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta}$$

We can solve for the geodesics using the Euler-Lagrange equations on this Lagrangian, however working with the square of the Lagrangian would be easier in this case to get rid of the square roots. We know that our Euler-Lagrange equations give us :

$$\frac{d}{d\lambda}\frac{\partial L}{\partial \dot{x}^{\alpha}} - \frac{\partial L}{\partial x^{\alpha}} = 0.$$

if instead we had applied these derivatives on L^2, it would have been equivalent of multiplying the whole expression with $2L$, and considering that is not zero, we will get the same equation with L^2 instead of L. To make the notation cleaner, I will be using $\mathcal{L}$ instead of L^2. The new Euler-Lagrange equations will be :

$$\frac{d}{d\lambda}\frac{\partial \mathcal{L}}{\partial \dot{x}^{\alpha}} - \frac{\partial \mathcal{L}}{\partial x^{\alpha}} = 0,$$

Where $\mathcal{L}$, will be :

$$\mathcal{L} = -\left(1 - \frac{2M_s}{r}\right)\dot{t}^2 + \frac{\dot{r}^2}{1 - \frac{2M_s}{r}} + r^2\dot{\theta}^3 + r^2 sin^2(\theta)\dot{\phi}^2. \tag{3.2}$$

For space-like geodesics, we hold the equation above equal to 1; for time-like geodesics, we hold it to be -1; and for null geodesics, it is equal to zero, as events on these geodesics are separated in that manner. By applying the Euler - Lagrange equations, we get the following set of differential equations:

$$\frac{dt}{d\lambda} = \frac{E}{1 - \frac{2GM_s}{r}}$$

$$\frac{d\phi}{d\lambda} = \frac{L}{M_s r^2 sin(\theta)}$$

$$\frac{d(r^2\dot{\theta})}{d\lambda} + r sin^2(\theta)\dot{\phi}^2 = 0$$

The first equation is known as Energy conservation, the second one is the conservation of Angular momentum. For the third equation if we assume that $\theta(0) = \frac{\pi}{2}$ and $\dot{\theta}(0) = 0$, which we can do as we can take any plane of orbit, and assume that the orbit is stationary in that plane initially, we can set all higher derivatives of θ to zero, which makes the second equation: $\dot{\phi} = \frac{L}{M_s r^2}$. Plugging these equations, and $\theta = \frac{\pi}{2}$ in (3.2), and since we are solving for time-like geodesics, we can set it equal to -1 to get:

$$\dot{r}^2 = E^2 - 1 + \frac{2M_s}{r} - \frac{L^2}{r^2} + \frac{2M_s L^2}{r^3}. \tag{3.3}$$

The $\frac{1}{r^3}$ term in this solution is unique to the General Relativistic gravitational potential, absent in the Newtonian solution. We can see that this term will have the most substantial effects at points closer to the central mass as it decays with the inverse cube of the radial coordinate. We can plot the potential, the negative of the right hand side of the equation, to see the points where we can get stable orbits for massive objects:

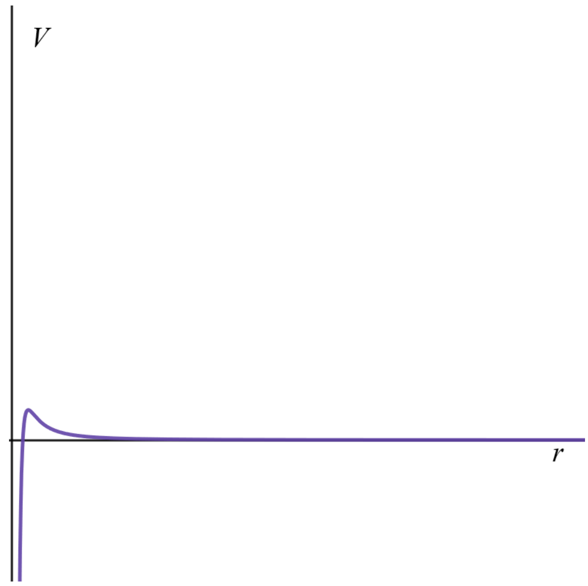

Figure 4: Effective Potential for massive objects

From here we can see that we will get stable orbits if the planets lie between the maxima and the point where the graph starts to asymptote. Calculating these critical points constrains the planets to lie between $3M_s < r < 6M_s$.

3.2 The Azimuthal dependence of the orbits

Although we have not completely solved (3.3) for the geodesics, we can still calculate the azimuthal dependence of the radial coordinate. To further simplify our calculations, we will be using the substitution $u = r^{-1}$, and applying the chain rule using the conservation of angular momentum, we will get :

$$\left(\frac{du}{d\phi}\right)^2 = \frac{E^2 - 1}{L^2} + \frac{2M_s u}{L^2} - u^2 + 2M_s u^3.$$

This equation can be solved by taking the integral; however, that integral would be a horrible one we do not want to solve. To further simplify this equation, we will make the substitution: $\tilde{u} = \frac{L^2 u}{m}$, and take the second derivative with respect to ϕ, which simplifies our problem to a second-order differential equation:

$$\frac{d^2\tilde{u}}{d\phi^2} + \tilde{u} = 1 + \frac{3M_s^2}{L^2}\tilde{u}^2 \tag{3.4}$$

The quadratic term in this differential equation makes it difficult to solve directly, so we will use a perturbative method to solve this problem. We will substitute : $\tilde{u} = \tilde{u}_0 + \frac{3M_s}{L^2}\tilde{u}_1$, and $\epsilon = \frac{3M_s}{L^2}$.We know from the solutions for stable orbits that critical points only occur when: $\frac{L}{M} > \sqrt{12}$, making ϵ small. The substitution makes us our equation:

$$\frac{d^2\tilde{u}_0}{d\phi^2} + \epsilon\frac{d\tilde{u}_1}{d\phi^2} + \tilde{u}_0 + \epsilon\tilde{u}_1 = 1 + \epsilon\left(\tilde{u}_0 + \epsilon\tilde{u}_1\right)^2$$

We can solve the equation by breaking it into two separate equations by comparing the coefficients of the terms that are zeroth order in ϵ and first order in ϵ, ignoring the second order contributions. These equations allow us to reach fairly close approximate solutions as we are expecting the contributions due to $\frac{1}{r^3}$ term in the (3.3) will be small. The zeroth order differential equation is :

$$\frac{d^2\tilde{u}_0}{d\phi^2} = 1 - \tilde{u}_0 \tag{3.5}$$

This is also known as Binet's equation, which gives the solutions for Kepler's orbits(footnote Goldstein). The solutions of (3.5) will also be those of the conic sections that describe an elliptical orbit, this makes sense as the zeroth order perturbation only caters for the higher order terms:

$$\tilde{u}_0 = 1 + ecos(\phi), \tag{3.6}$$

where e is the eccentricity of the ellipse. The differential order with the first-order perturbation terms will be :

$$\frac{d^2\tilde{u}_1}{d\phi^2} + \tilde{u}_1 = \tilde{u_0}^2 = (1 + ecos(\phi))^2 \tag{3.7}$$

solving this equation would require us to consider the following second-degree derivatives:

$$\frac{d^2}{d\phi^2} + \phi sin(\phi) = 2cos(\phi)$$
$$\frac{d^2}{d\phi^2} cos(2\phi) + cos(2\phi) = -3cos(2\phi).$$

By plugging in the ansatz : $\tilde{u}_1 = A + B\phi sin(\phi) + Ccos(2\phi)$ in (3.7), we will get :

$$2Bcos(\phi) - 3Ccos(2\phi) + A = 1 + 2ecos(\phi) + \frac{e^2}{2}(1 + cos(2\phi)) \tag{3.8}$$

comparing the coefficients, we will get : $1 + \frac{e^2}{2}$, $B = e$ and, $C = -\frac{e^2}{6}$. Plugging the solutions for $\tilde{u}_0, \tilde{u}_1$, and plugging in the value for ϵ, we get :

$$\tilde{u} = (1 + ecos(\phi)) + \frac{3M_s}{L^2}\left(1 + \frac{e^2}{2} + e\phi sin(\phi) - \frac{e^2}{6}cos(2\phi)\right) \tag{3.9}$$

In the perturbed solution, the most important term is $\phi sin(\phi)$ as it grows when the planet rotates in its orbit, the remaining terms are oscillatory and will not have any noticeable contributions to the precession of the Perihelion. Ignoring those terms, the solution becomes:

$$u \approx \frac{M_s}{L^2}\left(1 + ecos(\phi) + \frac{3M_s}{L^2}(e\phi sin(\phi)\right) \tag{3.10}$$

The above equation shows the azimuthal dependence of the radial coordinate, and we can predict the radial position of the planet if we know its azimuthal position in its orbit.

3.3 The Residual Precession

With the help of the calculations in this section, we have derived the radial trajectory of the planet as a function of the azimuthal angle under the general relativistic gravitational potential. Without the loss of generality, we can take initially $\phi = 0$. Since we are expecting the perihelion to have shifted azimuthally, while keeping the radial position the same, we want to find the $\Delta\phi$, such that $u'(2\pi + \Delta\phi)$ will be zero. This is because the perihelion is the closest point of approach, it will have the minimum radial position, and the derivative of u should be zero at that point (cite Blau).

$$u'(\phi) = \frac{M_s}{L^2}\left(-esin(\phi) + \frac{3M_s}{L^2}esin(\phi) + \frac{3M_s}{L^2}e\phi cos(\phi)\right)$$

plugging in with $2\pi + \Delta\phi$, and equating the eqation to zero will give us :

$$sin(2\pi + \Delta\phi) = \frac{3M_s}{L^2}esin(2\pi + \Delta\phi) + \frac{3M_s}{L^2}e(2\pi + \Delta\phi)cos(2\pi + \Delta\phi)$$

Using the small angle approximations, as we are expecting the precession to be slow so $\Delta\phi$ will be small,and the fact that $\frac{3M_s}{L^2}$ is also small, we will get :

$$\Delta\phi = \frac{3M_s}{L^2}\Delta\phi + \frac{3M_s}{L^2}(2\pi + \Delta\phi)\Delta\phi, \tag{3.11}$$
$$\Delta\phi = \frac{6\pi M_s}{L^2}. \tag{3.12}$$

We have arrived at the final form of our solution. The expression above for $\Delta\phi$ represents the additional angle swept during one orbit as the planet approaches Perihelion again. We will use another result from Newtonian mechanics,and restoring the SI units, the Precession can be written as:

$$\Delta\phi = \frac{6\pi GM_s}{c^2(1 - e^2)a},$$

using : $L^2 = GM(1-e^2)a$ (cite Goldstein). Here a is the length of the semi-major axis. The trend with the length of the semi-major axis, as can be seen in the graph below, decays as we move for planets further away from the sun:

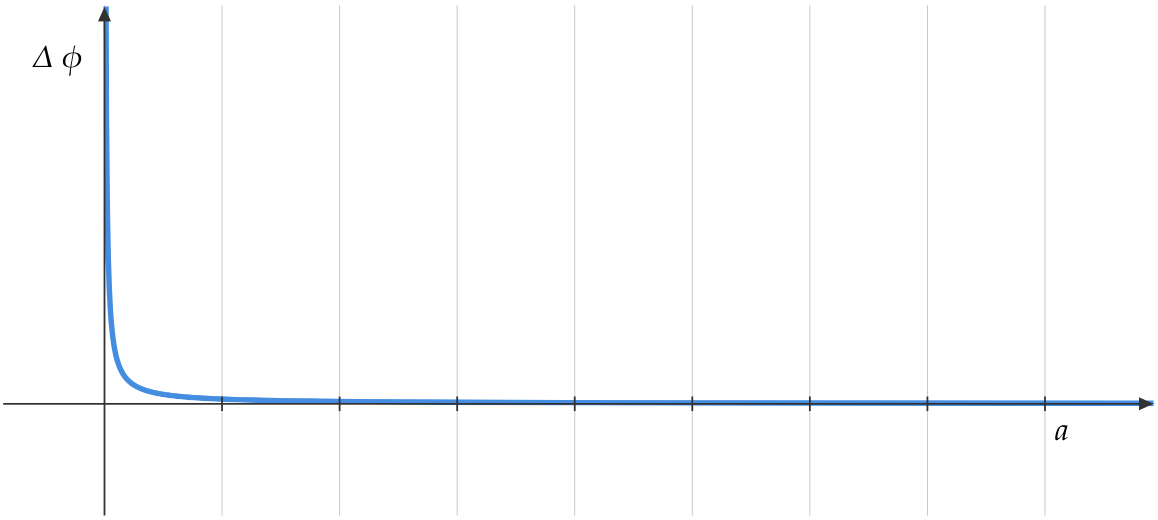

Figure 5: The Advance of Perihelion w.r.t the Semi Major axis

This phenomenon in not restricted to a particular planetary body, or a solar system, The Precession can take place in any Mass distribution that sets up the spacetime, like the Schwarzschild metric. The results of these calculations for the residual Precession, as seen in the table below (cite : dinverno) show how the additional term in the general relativistic gravitational potential corrected the errors in the Newtonian correction.

Planet	Radius / $(10^{11}m)$	$\Delta\phi_{obs}$ arcsec/cen	$\Delta\phi_{pred}$arcsec/cen
Mercury	0.58	43.0	43.1
Venus	1.08	8.6	8.6
Earth	1.49	5.0	3.8

The correction term from Mercury fixed the error in the Newtonian calculations, and the observed Precession was predicted fairly well for Venus and Earth's Precessions. These corrections that aligned well with the Experimental observations were presented as evidence for General Relativity's adoption to understand the gravitational phenomenon.

3.4 Do Light orbits Precess?

In the Newtonian regime, without the concept of Matter curving space-time, the light follows the straight line paths; hence in Newtonian gravity, light does not orbit. In General Relativity, light travels along the null geodesics. Starting from (3.2) and equating it to zero for null geodesics, we can solve for the trajectory of light in an area governed by the Schwarzschild metric.

3.4.1 The null geodesics

Equating the (3.2) to zero will make our $2GM/r$ term zero due to the null path condition. This is a reasonable step as this term describes Newtonian gravity, and as we have discussed before, Newtonian gravity does not act on massless particles. The effective potential for the null geodesic becomes:

$$V_{eff} = \frac{L^2}{2r^2}\left(1 - \frac{2M_s}{r}\right), \tag{3.13}$$

When plotted against the radial coordinate:

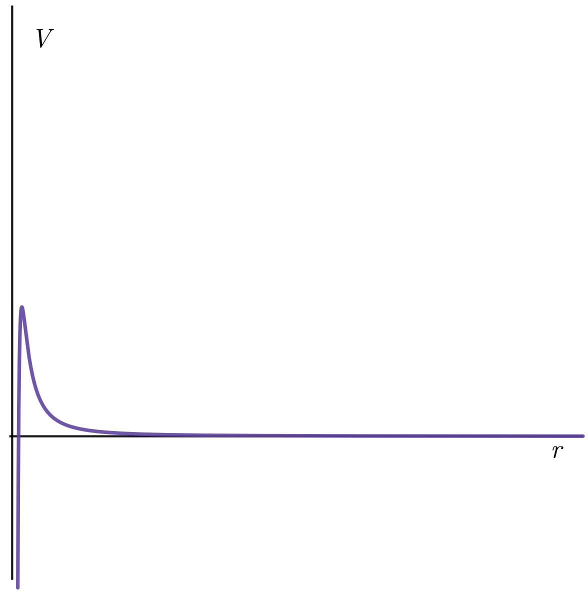

Figure 6: Effective potential on Light

We can see only one critical point on this potential, meaning only one stable orbit exists. Setting $\frac{dv}{dr} = 0$, we can find the only stable light orbit circular at $r = 3M$. We can calculate the effects of the additional $\frac{1}{r^3}$ term on the orbit; however, for the orbit to remain a stable circular orbit, it must stay a fixed distance away from the central Energy distribution. Any noticeable precession in orbit would mean the massless particle will fall into the singularity at $r = 0$ or move infinitely far away. Hence the orbits must be circular, such that the photons remain a fixed distance away from the central distribution. Hence any degree of the precession of the orbit about the centre will not be noticeable due to the circular symmetry of the orbit. If the precession takes place off-centre, that would require photons to be farther or closer from the central distribution, which would move the light out of its orbit.

4 Conclusion

The precession of Perihelion from the advent of Newtonian Mechanics was reasoned with the perturbative effects of the exterior planets and bodies disrupting the elliptical trajectories of the precessing orbits. It was concluded in the 19th century that Newtonian calculations were insufficient to eradicate the error observed in the precession of Mercury's Perihelion.

The general relativistic solution to the spherically symmetric metric set by a central Energy distribution corrected the gravitational potential, which, as discussed in detail above, corrected the error in the Newtonian calculations. Apart from serving as a test for General Relativity, this precession also has other interesting implications, as it is used as one of the reasons for the long-term climatic cycles as described by the Milvankovitch cycles(cite).

5 Bibliography

Cite all your sources.