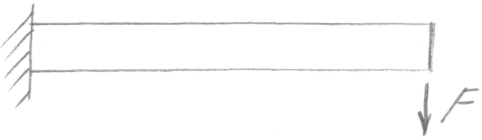
HTMs model for Lathe

Feb 27/2017

Good equations to reference

Cantilevered beam:

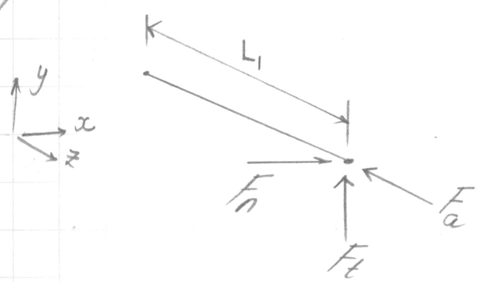
$\delta = \frac{FL^3}{3EI}$ $\theta = \frac{FL^2}{2EI}$



$\delta = \frac{ML^2}{2EI}$ $\theta = \frac{ML}{2EI}$

A. PART

Neglect torsion since that is a DOF



$\delta x = \frac{F_n L^3}{3EI}$ $\theta x = -\frac{F_t L^2}{2EI}$

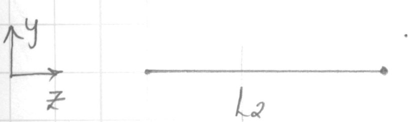
$\delta y = \frac{F_t L^3}{3EI}$ $\theta y = \frac{F_n L^2}{2EI}$

$\delta z = \frac{-F_a L}{EA} + \alpha L \Delta T$ $\theta z = 0$

Thermal and structural deformations.

⇒ Verify that contribution to moment-y from Fa is negligible.

B. CHUCK



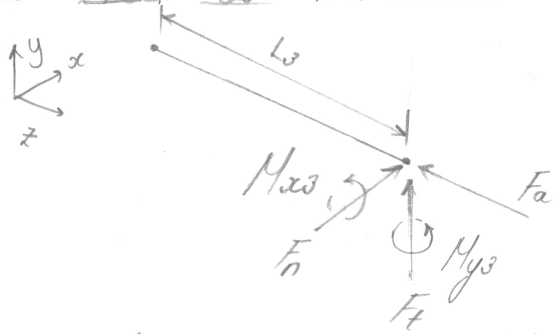
only have $\delta z = \alpha L_2 \Delta T$ due to thermal effects

⇒ Verify stiffness and compare to spindle shaft and part.

continued on following page

C. Spindle Shaft

Case a: Treat as cantilever



where $M_{y3} = F_n(L_1 + L_2)$
$M_{x3} = F_t(L_1 + L_2)$

$$\delta_x = \frac{F_n L_3^3}{3EI_3} + \frac{M_{y3} L_3^2}{2EI}$$

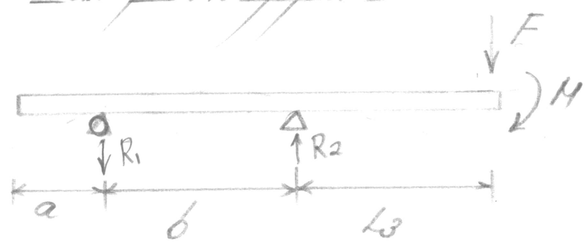
$$\delta_y = \frac{F_t L_3^3}{3EI_3} + \frac{M_{x3} L_3^2}{2EI}$$

$$\delta_z = \frac{-F_a L_3}{EA_3} + \alpha L_3 \Delta T$$

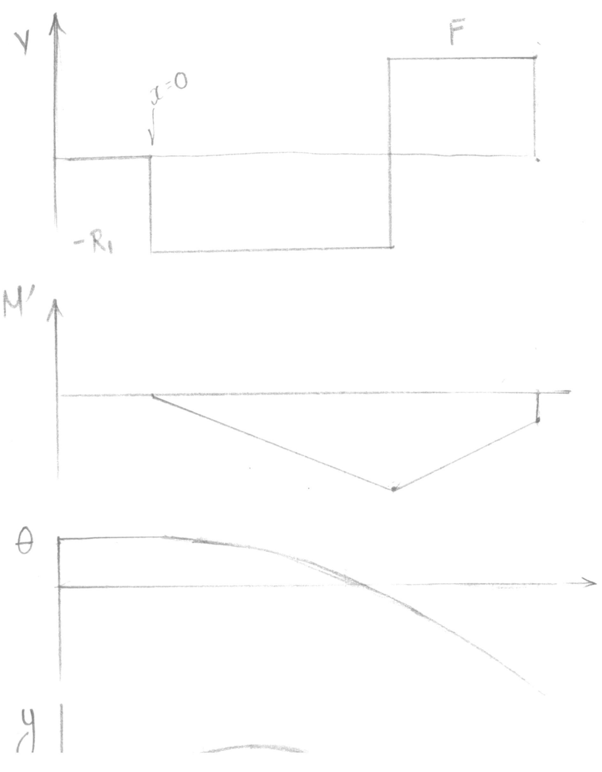
$$\theta_x = \frac{-F_t L_3^2}{2EI} - \frac{M_{x3} L_3}{EI}$$

$$\theta_y = \frac{F_n L_3^2}{2EI} + \frac{M_{y3} L_3}{EI}$$

Case b: Pin-pin support



$R_1 b = F L_3 + M$
$R_1 = F(L_3/b) + M(1/b)$
and $R_2 = F(L_3 + b)/b + M/b$



Know that
δ and θ due to F are

$$\delta_F = \frac{-F L_3^3}{3EI}\left(\frac{b}{L_3} + 1\right) \quad \text{[Shigley 1017]}$$

and

$$y_{BC} = \frac{F}{6EI}\left\{(x-b)^3 - L_3(3x^2 - 4xb + b^2)\right\}$$

$$\theta = \frac{F}{6EI}\left\{3(x-b)^2 - L_3(6x - 4b)\right\}$$

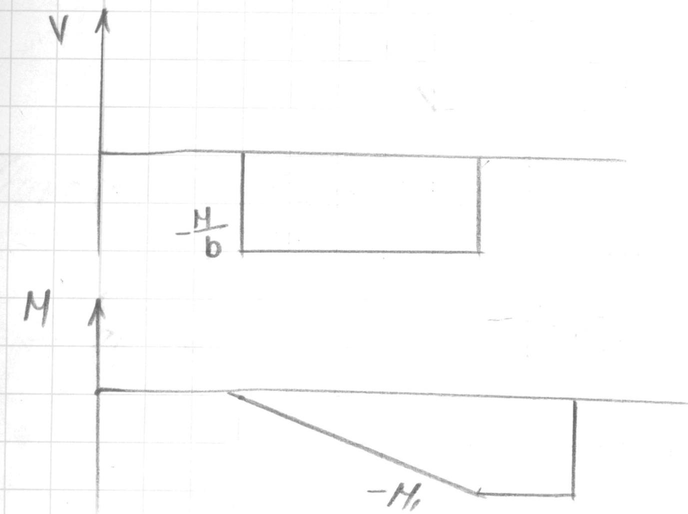
$$\theta_F = \frac{F}{6EI}\left\{3L_3^2 - L_3(6L_3 + 6b - 4b)\right\}$$

$$= \frac{F}{6EI}\left\{-3L_3^2 - 2bL_3\right\}$$

$$\theta_F = \frac{-F L_3^2}{2EI}\left\{1 + \frac{2}{3}\left(\frac{b}{L_3}\right)\right\}$$

important especially if $b > L_3$

Calculate δ_M and θ_M and superpose



$$M = -\frac{M}{b}x \quad x \le b$$
$$M = -M \quad x \ge b$$

$$EI\theta = -\frac{M}{2b}x^2 + A \quad x \le b$$
$$EI\theta = -M_1 x + B \quad x \ge b$$

$$EIy = -\frac{M}{6b}x^3 + Ax$$
$$EIy = -Mx^2/2 + Bx + C$$

but $\frac{1}{6}Mb^2 = Ab$

$$A = \frac{1}{6}Mb$$

and $-\frac{M}{2}b + \frac{Mb}{6} = -Mb + B$

$$B = \frac{2}{3}Mb$$

and $-\frac{Mb^2}{2} + \frac{2}{3}Mb^2 + C = 0$

$$C = -\frac{1}{6}Mb^2$$

$$\therefore \theta_M = \frac{1}{EI}\left\{-M(b+L_3) + \frac{2}{3}Mb\right\} \leftarrow$$

$$\boxed{\theta_M = \frac{-ML_3}{EI}\left\{1 + \frac{b}{3L}\right\}}$$

and

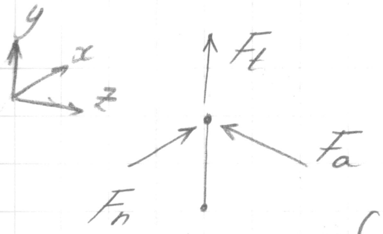
$$\delta_F = \frac{1}{EI}\left\{-\frac{M(b+L)^2}{2} + \frac{2}{3}Mb(b+L) - \frac{1}{6}Mb^2\right\}$$
$$= \frac{1}{EI}\left\{-\frac{1}{2}Mb^2 - MbL - \frac{ML^2}{2} + \frac{2}{3}Mb^2 + \frac{2}{3}MbL - \frac{1}{6}Mb^2\right\}$$

$$\delta_F = \frac{1}{EI}\left\{-\frac{ML^2}{2} - \frac{1}{3}MbL\right\} = \boxed{\frac{-ML_3^2}{2EI}\left\{1 + \frac{2}{3}\frac{b}{L_3}\right\}}$$

$\rightarrow$ Then:

$$\delta x = \frac{F_n L_3^3}{3EI}\left(1 + \frac{b}{L_3}\right) + \frac{M_{y3}L_3^2}{2EI}\left\{1 + \frac{2}{3}\frac{b}{L_3}\right\}$$
$$\delta y = \frac{F_t L_3^3}{3EI}\left(1 + \frac{b}{L_3}\right) + \frac{M_{x3}L_3^2}{2EI}\left\{1 + \frac{2}{3}\frac{b}{L_3}\right\}$$
$$\delta z = \frac{-F_a L_3}{EA_3} + \alpha L_3 \Delta T$$
$$\theta x = \frac{-F_t L_3^2}{2EI}\left(1 + \frac{2}{3}\frac{b}{L_3}\right) - \frac{M_{x3}L_3}{EI}\left(1 + \frac{b}{3L}\right)$$
$$\theta y = \frac{F_n L_3^2}{2EI}\left(1 + \frac{2}{3}\frac{b}{L_3}\right) + \frac{M_{y3}L_3}{EI}\left(1 + \frac{b}{3L}\right)$$

D. Bearing Stiffness



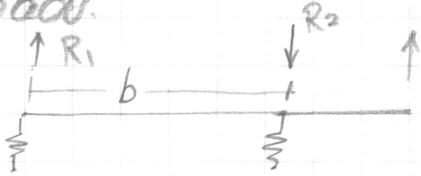
assume that deflections are due to structural loads on bearings and bearing housing.

$$\delta x = \delta y = \frac{R_2}{k_{bearing,2}} \quad \text{(see reaction load earlier)}$$

$$\delta z = 0 \quad \text{(assumed rigid in axial direction)}$$

$\Rightarrow$ verify!

→ Rotation due to stiffnesses of bearings with opposing loads.



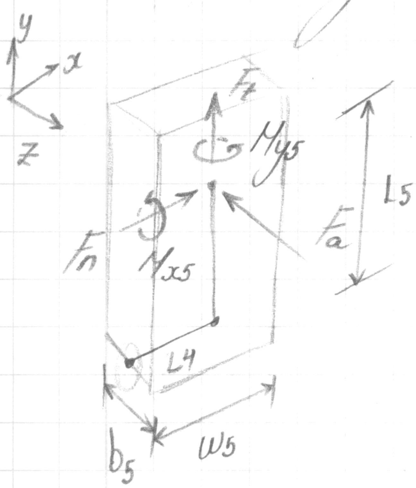
$$\theta = \frac{R_2/k_{b2} + R_1/k_{b1}}{b} \rightarrow \text{bearing stiffnesses}$$

$$\theta = \frac{R_2 + R_1}{k_b b} \rightarrow \text{if stiffnesses are equal}$$

∴ Calculate loads and reactions and sub into equations above.

E Headstock

→ Assume high stiffness in y-axis (rotational) → $\theta_y \approx 0$



$$\delta x = \frac{F_n L_5^3}{3 E I_{zz}}$$

$$\delta y = \frac{F_t L_5}{E (\underbrace{b_5 w_5}_{A})}$$

$$\delta z = -\frac{F_a L_5^3}{3 E I_{xx}} - \frac{M_{x5} L_5^2}{2 E I_{xx}}$$

$$\theta_x = -\frac{M_{x5} L_5}{E I_{xx}} - \frac{F_a L_5^2}{2 E I_{xx}}$$

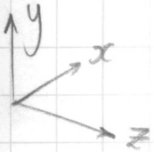
$$\theta_y = 0$$

$$\theta_z = -\frac{F_n L_5^2}{2 E I_{zz}}$$

$M_{y5} = F_n (L_1 + L_2)$
$M_{x5} = F_t (L_1 + L_2)$

Headstock aligned

E. RAIL GUIDES



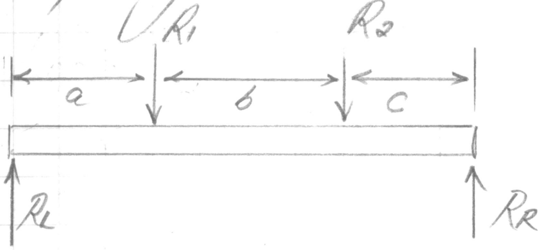
→ Stiffness in z - function of lead-screw and dancing man.

→ Stiffness in x-direction due to one-rail only.
→ Stiffness in y-direction due to two rails?

Consider sensitive direction first.

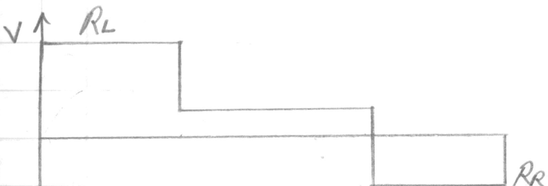
(a) Pin-pin joint

at bearings

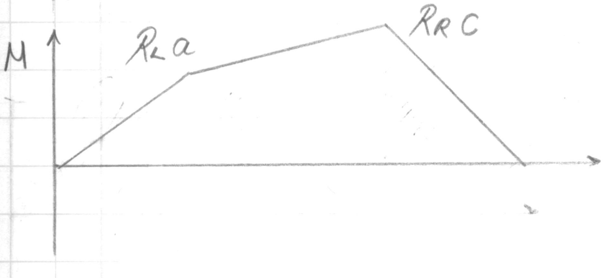


where $a+b+c=L$

$$R_L = {}^{c}/_{L}\, R_2 + \frac{b+c}{L} R_1$$
$$R_R = {}^{a}/_{L}\, R_1 + \frac{a+b}{L} R_2$$



$$\left.\begin{aligned} M &= R_L x \\ EI\theta &= \tfrac{1}{2} R_L x^2 + A \\ EI y &= \tfrac{1}{6} R_L x^3 + Ax \end{aligned}\right\} x \leq a$$



$$\left.\begin{aligned} M &= R_L a + (R_L - R_1)(x-a) \\ &= R_L x - R_1(x-a) \\ EI\theta &= \tfrac{1}{2} R_L x^2 - R_1(\tfrac{1}{2}x^2 - ax) + B \\ EI y &= \tfrac{1}{6} R_L x^3 - R_1(\tfrac{1}{6}x^3 - \tfrac{1}{2}ax^2) + Bx + C \end{aligned}\right\} a \leq x \leq a+$$

$$\left.\begin{aligned} M &= R_R(L-x) \\ EI\theta &= R_R(Lx - \tfrac{1}{2}x^2) + D \\ EI y &= R_R(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3) + Dx + E \end{aligned}\right\} x \geq a+$$

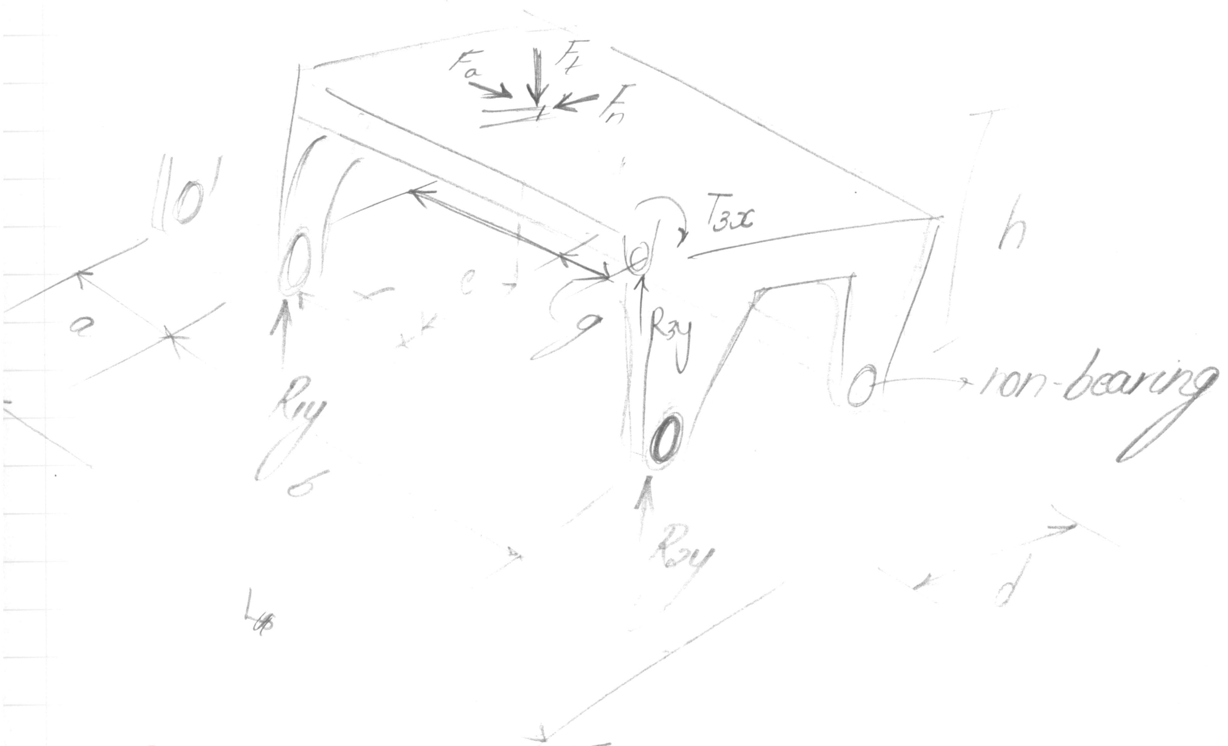
BCs and continuity

$$A - B = -R_1(-\tfrac{1}{2}a^2) = \tfrac{1}{2} R_1 a^2$$
$$aA - aB - C = \tfrac{1}{3} R_1 a^3$$
$$B - D = R_R(L-c)\{\tfrac{1}{2}L + \tfrac{1}{2}c\} - \tfrac{1}{2} R_L (L-c)^2 + R_1(L-c)\{\tfrac{1}{2}L - \tfrac{1}{2}c - a\}$$
$$= (L-c)\{\tfrac{1}{2}R_R(L+c) - \tfrac{1}{2}R_L(L-c) + \tfrac{1}{2}R_1(L-c-2a)\}$$
$$B(L-c) - D(L-c) + C - E = (L-c)^2\{\tfrac{1}{6}R_L(L-c) + R_1(\tfrac{1}{6}L - \tfrac{1}{6}c - \tfrac{1}{2}a) + R_R(\tfrac{1}{3}L + \tfrac{1}{6}c)\}$$
$$-DL - E = \tfrac{1}{3} R_R L^3$$

solve and plug-in above for δx and θy

was not

For y-deflection and rotation about z. Consider stiffness in flexure and other rail.



$R_{1y} + R_{2y} + R_{3y} = F_t$
$(R_{1y} + R_{2y})d = F_t(d-e) + F_n(h)$
$T_{3x} + R_{1y}(g) - R_{2y}(b-g) = F_t(g-f) - F_a h$

→ Assume bearing is torsionally more compliant than the beams.

CARRIAGE (rigid)
GT/I FLEXURES

(Victor will analyze and provide values)

III. Tool part