EE5600 Assignment 3

Sahil Kumar Singh

***Abstract*—This document contains the solution of geometry through linear algebra through the concept of optimization.**

Download latex and python codes from

https://github.com/sahilsin/AI_ML/blob/master/Assignment3/

1 Problem

Minimize and Maximize $Z = 5x + 10y$ subject to $x + 2y \leq 120,\ x + y \geq 60,\ x - 2y \geq 0,\ x, y \geq 0.$

2 Solution

First we will plot these lines which are the constraints and the area enclosed by is the region we are interested in.

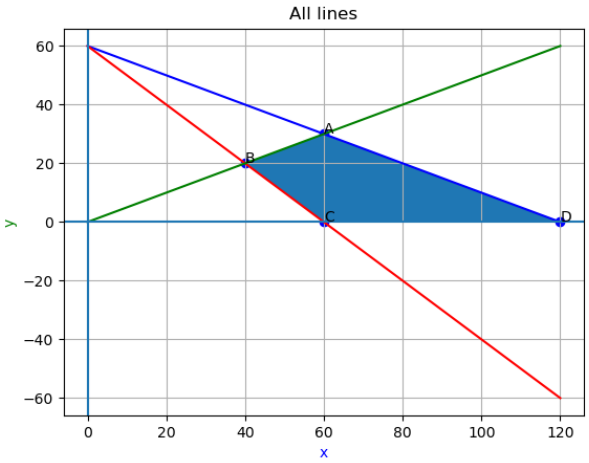

Fig. 0: optimal point through the intersection of various lines

The four points are the points which will maximize and minimize the function.These corner points are :

$$A = (60, 30)$$
$$B = (40, 20)$$
$$C = (60, 0)$$
$$D = (120, 0)$$

Value of Z at point

$$A = 5 \times 60 + 10 \times 30 = 600$$
$$B = 5 \times 40 + 10 \times 20 = 400$$
$$C = 5 \times 60 + 10 \times 0 = 300$$
$$D = 5 \times 120 + 10 \times 0 = 600$$

We can see that our function Z is maximum at points A and D that is (60,30) and (120,0)
and
Z is minimum at point C that is (60,0)
The given problem can be expressed in general as matrix inequality as:

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 5 & 10 \end{pmatrix} \mathbf{x} \tag{2.0.1}$$

$$s.t. \quad \begin{pmatrix} 1 & 2 \\ -1 & -1 \\ -1 & 2 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 120 \\ -60 \\ 0 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{x} \geq \mathbf{0} \tag{2.0.3}$$

$$\mathbf{y} \geq \mathbf{0} \tag{2.0.4}$$

$$\max_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \tag{2.0.5}$$

$$s.t. \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}, \tag{2.0.6}$$

$$\mathbf{x} \geq \mathbf{0} \tag{2.0.7}$$

$$\mathbf{y} \geq \mathbf{0} \tag{2.0.8}$$

where

$$\mathbf{c} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \tag{2.0.9}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -1 & -1 \\ -1 & 2 \end{pmatrix} \tag{2.0.10}$$

$$\mathbf{b} = \begin{pmatrix} 120 \\ -60 \\ 0 \end{pmatrix} \tag{2.0.11}$$

3 Verification

The given solution can be verified through the given code.

The given problem can be solved using *pulp* through the following code

https://github.com/sahilsin/AI_ML/blob/master/Assignment3/codes/Ai_ML_3.py