EE5600 Assignment 2

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***Abstract*—This document contains the solution to a Lines and planes problem.** Download all python codes(simulations) from

https://github.com/sahilsin/AI_ML/Assignment 2/codes

1 Problem

1.1. Three coins are tossed simultaneously. Consider the event E "three heads or three tails", F "at least two heads" and G "at most two heads". Of the pairs (E,F), (E,G) and (F,G), which are independent? which are dependent?

2 Solution

2.2. Let $X_i \subset \{0,1\}$ represent the toss of each coin, with 1 being a head Let

$$X = X_1 + X_2 + X_3 \tag{2.2.1}$$

$$P(E) = p(\{X = 3\} + \{X = 0\}) \tag{2.2.2}$$
$$= p(\{X = 3\}) + p(\{X = 0\}) \tag{2.2.3}$$
$$= {}^3C_3\left(\frac{1}{2}\right)^3 + {}^3C_0\left(\frac{1}{2}\right)^3 \tag{2.2.4}$$
$$= \frac{1}{4} \tag{2.2.5}$$
$$\tag{2.2.6}$$
$$P(F) = p(\{X >= 2\}) \tag{2.2.7}$$
$$= {}^3C_2\left(\frac{1}{2}\right)^3 + {}^3C_3\left(\frac{1}{2}\right)^3 \tag{2.2.8}$$
$$= \frac{1}{2} \tag{2.2.9}$$
$$\tag{2.2.10}$$
$$P(G) = p(\{X <= 2\}) \tag{2.2.11}$$
$$= 1 - p(\{X > 2\}) \tag{2.2.12}$$
$$= 1 - {}^3C_3\left(\frac{1}{2}\right)^3 \tag{2.2.13}$$
$$= \frac{7}{8} \tag{2.2.14}$$

$$P(EF) = p([\{X = 3\} + \{X = 0\}][\{X >= 2\}])$$
$$= p([\{X = 3\}][\{X >= 2\}]+[\{X = 0\}][\{X >= 2\}])$$
$$= p(\{X = 3\})$$
$$= \frac{1}{8}$$

$$P(EG) = p([\{X = 3\} + \{X = 0\}][\{X <= 2\}])$$
$$= p([\{X = 3\}][\{X <= 2\}]+[\{X = 0\}][\{X <= 2\}])$$
$$= p(\{X = 0\})$$
$$= \frac{1}{8}$$

$$P(FG) = p([\{X >= 2\}][\{X <= 2\}])$$
$$= p([\{X = 2\}])$$
$$= {}^3C_2\left(\frac{1}{2}\right)^3$$
$$= \frac{3}{8} \quad (2.2.15)$$

From the above equations we can clearly see that :

$$P(EF) = P(E)P(F) \quad (2.2.16)$$
$$P(GF) \neq P(G)P(F) \quad (2.2.17)$$
$$P(EG) \neq P(E)P(G) \quad (2.2.18)$$

Hence , the only pair of independent events are E and F , remaining are dependent.