

# Matrix theory Assignment 1

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**Abstract**—This document contains the solution complex numbers problem. Download all python codes from

<https://github.com/sahilsin/MatrixTheory/Assignment1/codes>

## 1 PROBLEM

Convert the following in Polar form:

$$a) \frac{\begin{pmatrix} 1 \\ 7 \end{pmatrix}}{\begin{pmatrix} 2 \\ -1 \end{pmatrix}^2} \quad (1.0.1)$$

$$b) \frac{\begin{pmatrix} 1 \\ 3 \end{pmatrix}}{\begin{pmatrix} 1 \\ -2 \end{pmatrix}} \quad (1.0.2)$$

## 2 SOLUTION

The **Approach** is : For finding the polar form we will first simplify the expressions given in problem and after that the polar form of  $\begin{pmatrix} x \\ y \end{pmatrix}$  will be of two parts. Magnitude and angle.  
Magnitude will be  $\sqrt{x^2 + y^2}$  and angle will be  $\arctan \frac{y}{x}$ .

a) We first convert given problem into simpler form:

$$\frac{\begin{pmatrix} 1 \\ 7 \end{pmatrix}}{\begin{pmatrix} 2 \\ -1 \end{pmatrix}^2} \quad (2.0.1)$$

$$(2.0.2)$$

We know that :

$$\begin{pmatrix} a1 \\ a2 \end{pmatrix} = \begin{pmatrix} a1 & -a2 \\ a2 & a1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.3)$$

The denominator can be simplified as :

$$\begin{pmatrix} 2 \\ -1 \end{pmatrix}^2 = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$(2.0.5)$$

$$\Rightarrow \begin{pmatrix} 2 \\ -1 \end{pmatrix}^2 = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.6)$$

$$(2.0.7)$$

$$\Rightarrow \begin{pmatrix} 2 \\ -1 \end{pmatrix}^2 = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (2.0.8)$$

Now our Expression becomes :

$$\begin{pmatrix} 1 \\ 7 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix}^{-1} \quad (2.0.9)$$

Inverse can be calculated as:

$$\begin{pmatrix} x \\ y \end{pmatrix}^{-1} = \frac{1}{x^2 + y^2} \begin{pmatrix} x \\ -y \end{pmatrix} \quad (2.0.10)$$

Our problem can now be written as :

$$\frac{1}{25} \begin{pmatrix} 1 \\ 7 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (2.0.11)$$

$$\Rightarrow \frac{1}{25} \begin{pmatrix} 1 \\ 7 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (2.0.12)$$

it can be further expanded as :

$$\Rightarrow \frac{1}{25} \begin{pmatrix} 1 & -7 \\ 7 & 1 \end{pmatrix} \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \frac{1}{25} \begin{pmatrix} -25 & -25 \\ 25 & -25 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \frac{1}{25} \begin{pmatrix} -25 \\ 25 \end{pmatrix} \quad (2.0.15)$$

After further simplifying :

$$\Rightarrow \frac{25 \times \sqrt{2}}{25} \begin{pmatrix} -25 \\ 25 \end{pmatrix} \quad (2.0.16)$$

polar form is :

$$\Rightarrow \sqrt{2} \arctan \frac{25}{-25} \quad (2.0.17)$$

$$\Rightarrow \sqrt{2} \angle -45^\circ \quad (2.0.18)$$

b) We first convert given problem into simpler form:

$$\frac{\begin{pmatrix} 1 \\ 3 \end{pmatrix}}{\begin{pmatrix} 1 \\ -2 \end{pmatrix}} \quad (2.0.19)$$

We know that :

$$\begin{pmatrix} a1 \\ a2 \end{pmatrix} = \begin{pmatrix} a1 & -a2 \\ a2 & a1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.20)$$

The denominator can be simplified as :

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.21)$$

Now our Expression becomes :

$$\begin{pmatrix} 1 \\ 7 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix}^{-1} \quad (2.0.22)$$

Inverse can be calculated as:

$$\begin{pmatrix} x \\ y \end{pmatrix}^{-1} = \frac{1}{x^2 + y^2} \begin{pmatrix} x \\ -y \end{pmatrix} \quad (2.0.23)$$

Our problem can now be written as :

$$\frac{1}{5} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.24)$$

it can be further expanded as :

$$\Rightarrow \frac{1}{5} \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.25)$$

$$\Rightarrow \frac{1}{5} \begin{pmatrix} -5 & -5 \\ 5 & -5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.26)$$

$$\Rightarrow \frac{1}{5} \begin{pmatrix} -5 \\ 5 \end{pmatrix} \quad (2.0.27)$$

After further simplifying :

$$\Rightarrow \frac{5 \times \sqrt{2}}{5} \begin{pmatrix} -5 \\ 5 \end{pmatrix} \quad (2.0.28)$$

polar form is :

$$\Rightarrow \sqrt{2} \arctan \frac{5}{-5} \quad (2.0.29)$$

$$\Rightarrow \sqrt{2} \angle -45^\circ \quad (2.0.30)$$

For both the part as our polar forms comes to be same , it will look like a line having magnitude as  $\sqrt{2}$  and angle as  $-45^\circ$  ,so it will look like a line  $\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$  as shown :

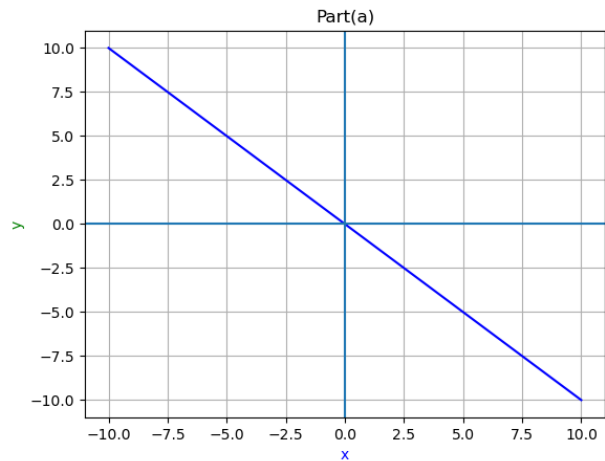


Fig. 2.0.1: part(a)