EE5609: Matrix Theory
Assignment-7

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1 Problem

Let $W_1 = \{(u, v, w, x) \subset R^4 | u + v + w = 0, 2v + x = 0, 2u + 2w - x = 0\}$ and $W_2 = \{(u, v, w, x) \subset R^4 | u + v + x = 0, v - x = 0, u + w - 2x = 0\}$
Then which among the following is true?

$$1) dim(W_1) = 1 \tag{1.0.1}$$
$$2) dim(W_2) = 2 \tag{1.0.2}$$
$$3) dim(W_1 \cap W_2) = 1 \tag{1.0.3}$$
$$4) dim(W_1 + W_2) = 3 \tag{1.0.4}$$

2 Solution

1) Here we solve them one by one :

$$\begin{aligned}
&W_1 = \{(u, v, w, x) \subset R^4 \\
&|u + v + w = 0, 2v + x = 0, 2u + 2w - x = 0\} \\
&\implies \{(u, v, w, x) \subset R^4 \\
&|u + v + w = 0, x = -2v, 2u + 2w - (-2v) = 0\} \\
&\implies \{(u, v, w, x) \subset R^4 \\
&|u + v + w = 0, x = -2v, u + v + w = 0\} \\
&\implies \{(u, v, w, x) \subset R^4 \\
&|w = -u - v, x = -2v\} \\
&\implies \{(u, v, -u - v, -2v) \\
&|u, v \subset R\} \\
&\implies \{(u, 0, -u, 0) + (0, v, -v, -2v) \\
&|u, v \subset R\} \\
&\implies \{u(1, 0, -1, 0) + v(0, 1, -1, -2) \\
&|u, v \subset R\} \\
&\implies span\{(1, 0, -1, 0), (0, 1, -1, -2)\}
\end{aligned} \tag{2.0.1}$$

As these two are independent vectors,
$dim(W_1) = 2$

$$\begin{aligned}
&W_2 = \{(u, v, w, x) \subset R^4 | \\
&u + v + x = 0, v - x = 0, u + w - 2x = 0\} \\
&\implies \{(u, v, w, x) \subset R^4 | \\
&u + w = -x, v = x, u + w = 2x\} \\
&\implies \{(u, v, w, x) \subset R^4 | \\
&u + w = -x, v = x, -x = 2x\} \\
&\implies \{(u, v, w, x) \subset R^4 | \\
&u + w = -x, v = x, x = 0\} \\
&\implies \{(u, v, w, x) \subset R^4 | \\
&u + w = 0, v = 0, x = 0\} \\
&\implies \{(u, v, w, x) \subset R^4 | \\
&w = -u, v = 0, x = 0\} \\
&\implies \{(u, 0, -u, 0) | \\
&u \subset R\} \\
&\implies \{u(1, 0, -1, 0) | \\
&u \subset R\} \\
&\implies span\{(1, 0, -1, 0)\}
\end{aligned} \tag{2.0.2}$$

$dim(W_2) = 1$

$$\begin{aligned}
&W_1 \cap W_2 = span\{(1, 0, -1, 0)\} \\
&\implies dim(W_1 \cap W_2) = 1 \\
&dim(W_1 + W_2) \\
&= dim(W_1) + dim(W_2) - dim(W_1 \cap W_2) \\
&= 2 + 1 - 1 \\
&= 2
\end{aligned} \tag{2.0.3}$$

So, the answer is $3) dim(W_1 \cap W_2) = 1$