

EE5609: Matrix Theory

Assignment-4

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Abstract—This document contains a problem based on conic problems.

Download the python codes from

<https://github.com/sahilsin/MatrixTheory/tree/master/Assignment4/codes>

1 PROBLEM

Trace the parabolas $144x^2 - 120xy + 25y^2 + 619x - 272y + 663 = 0$, and find its focus.

2 SOLUTION

$$144x^2 - 120xy + 25y^2 + 619x - 272y + 663 = 0 \quad (2.0.1)$$

$$(12x - 5y)^2 = 272y - 619x - 663 \quad (2.0.2)$$

Introducing a variable λ

$$(12x - 5y + \lambda)^2 = \quad (2.0.3)$$

$$272y - 619x - 663 - 10\lambda y + 24\lambda x + \lambda^2 \quad (2.0.4)$$

$$(12x - 5y + \lambda)^2 = \quad (2.0.5)$$

$$(272 - 10\lambda)y - (619 - 24\lambda)x + \lambda^2 - 663 \quad (2.0.6)$$

$$12x - 5y + \lambda = 0 \quad (2.0.7)$$

$$(272 - 10\lambda)y - (619 - 24\lambda)x + \lambda^2 - 663 = 0 \quad (2.0.8)$$

For λ we assume both lines to be perpendicular

$$\begin{aligned} m &= -\frac{12}{-5} = \frac{12}{5} \\ m' &= -\frac{-(619 - 24\lambda)}{272 - 10\lambda} \\ m \times m' &= -1 \\ \frac{12}{5} \times \frac{-(619 - 24\lambda)}{272 - 10\lambda} &= -1 \\ 7428 - 288\lambda &= 50\lambda - 1360 \\ \Rightarrow \lambda &= 26 \end{aligned}$$

$$\begin{aligned} &(12x - 5y + 26)^2 \\ &= (272 - 10 \times 26)y - (619 - 24 \times 26)x \\ &\quad + (26)^2 - 663 \end{aligned}$$

$$\Rightarrow (12x - 5y + 26)^2 = 5x + 12y + 13$$

$$\left(\frac{12x - 5y + 26}{13}\right)^2 = \frac{1}{13} \left(\frac{5x - 12y + 13}{13}\right)$$

$$Y = \frac{12x - 5y + 26}{13}$$

$$X = \frac{5x - 12y + 13}{13}$$

$$Y^2 = \frac{1}{13}X \quad (2.0.9)$$

Comparing with equation of parabola $y^2 = 4ax$:

$$4a = \frac{1}{13} \quad (2.0.10)$$

$$a = \frac{1}{52} \quad (2.0.11)$$

The focus of the parabola is obtained by solving

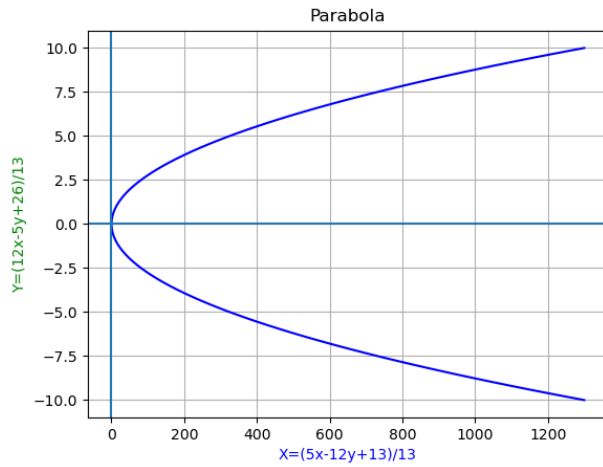


Fig. 0: Traced parabola

$X - a = 0$ and $Y = 0$.

$$\frac{5x - 12y + 13}{13} - a = 0 \quad (2.0.12)$$

$$\frac{5x - 12y + 13}{13} - \frac{1}{52} = 0 \quad (2.0.13)$$

$$20x + 48y + 51 = 0 \quad (2.0.14)$$

$$\frac{12x - 5y + 26}{13} = 0 \quad (2.0.15)$$

$$12x - 5y + 26 = 0 \quad (2.0.16)$$

Solving 2.0.14 and 2.0.16 we get :

$$x = -\frac{1503}{676} \quad (2.0.17)$$

$$y = -\frac{23}{169} \quad (2.0.18)$$

The focus is :

$$\left(-\frac{1503}{676}, -\frac{23}{169} \right) \quad (2.0.19)$$