Assignment 5

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Download all latex-tikz codes from

https://github.com/sahilsin/MatrixTheory/tree/master/Assignment5

1 Problem

find the QR decomposition of the following:

$$\mathbf{V} = \begin{pmatrix} 144 & -60 \\ -60 & 25 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Here, let the column vectors of $\mathbf{V}$ be α and β:

$$\alpha = \begin{pmatrix} 144 \\ -60 \end{pmatrix} \tag{2.0.1}$$

$$\beta = \begin{pmatrix} -60 \\ 25 \end{pmatrix} \tag{2.0.2}$$

To find $\mathbf{Q} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix}$, we will orthonormalize the columns of $\mathbf{V}$ using Gram Schmidt method:

$$\mathbf{u}_1 = \frac{\alpha}{k_1} \tag{2.0.3}$$

$$k_1 = \|\alpha\| = \sqrt{144^2 + (-60)^2} = 156 \tag{2.0.4}$$

$$\implies \mathbf{u}_1 = \frac{1}{156}\begin{pmatrix} 144 \\ -60 \end{pmatrix} = \begin{pmatrix} \frac{144}{156} \\ \frac{-60}{156} \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{u}_2 = \frac{\beta - r_1\mathbf{u}_1}{\|\beta - r_1\mathbf{u}_1\|} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T\beta}{\|\mathbf{u}_1\|^2} = \frac{\begin{pmatrix} \frac{144}{156} & \frac{-60}{156} \end{pmatrix}\begin{pmatrix} -60 \\ 25 \end{pmatrix}}{\sqrt{\frac{144}{156}^2 + (\frac{-60}{156})^2}} = -65 \tag{2.0.7}$$

$$\implies \mathbf{u}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.8}$$

$$\text{Also,} \quad k_2 = \mathbf{u}_2^T\beta = \begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} -60 \\ 25 \end{pmatrix} = 0 \tag{2.0.9}$$

The QR decomposition is given as:

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix}\begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

Where,

$$\mathbf{Q} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \tag{2.0.11}$$

$$\mathbf{R} = \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.12}$$

Putting the values of $\mathbf{u}_1$, $\mathbf{u}_2$, k_1, k_2 and r_1 in equation (2.0.10):

$$\boxed{\mathbf{V} = \begin{pmatrix} \frac{144}{156} & 0 \\ \frac{-60}{156} & 0 \end{pmatrix}\begin{pmatrix} 156 & -65 \\ 0 & 0 \end{pmatrix}} \tag{2.0.13}$$

Where,

$$\mathbf{Q} = \begin{pmatrix} \frac{144}{156} & 0 \\ \frac{-60}{156} & 0 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{R} = \begin{pmatrix} 156 & -65 \\ 0 & 0 \end{pmatrix} \tag{2.0.15}$$