EE5609: Matrix Theory Assignment-7

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1 Problem

Let

$$W_1 = \begin{pmatrix} u \\ v \\ w \\ x \end{pmatrix} \subset R^4$$

$$|u + v + w = 0, 2v + x = 0, 2u + 2w - x = 0$$

and

$$W_2 = \begin{pmatrix} u \\ v \\ w \\ x \end{pmatrix} \subset R^4$$

$$|u + v + x = 0, v - x = 0, u + w - 2x = 0$$

Then which among the following is true?

$$1) dim(W_1) = 1 \tag{1.0.1}$$

$$2) dim(W_2) = 2 \tag{1.0.2}$$

$$3) dim(W_1 \cap W_2) = 1 \tag{1.0.3}$$

$$4) dim(W_1 + W_2) = 3 \tag{1.0.4}$$

2 Solution

1) Here we solve them one by one :

$$W_1 = \{(u, v, w, x) \subset R^4$$

$$|u + v + w = 0, 2v + x = 0, 2u + 2w - x = 0\}$$

$$\implies \begin{pmatrix} u \\ v \\ w \\ x \end{pmatrix} \subset R^4$$

$$|u + v + w = 0, x = -2v, 2u + 2w - (-2v) = 0$$

$$\implies \begin{pmatrix} u \\ v \\ w \\ -2v \end{pmatrix} \subset R^3$$

$$|u + v + w = 0, x = -2v, u + v + w = 0$$

$$\implies \begin{pmatrix} u \\ v \\ -u - v \\ -2v \end{pmatrix} \subset R^2$$

$$|w = -u - v, x = -2v$$

$$\implies u \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + v \begin{pmatrix} 0 \\ 1 \\ -1 \\ -2 \end{pmatrix}$$

$$|u, v \subset R$$

$$\implies W_1 = span \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ 0 & -2 \end{pmatrix} \tag{2.0.1}$$

As these two are independent vectors, $dim(W_1) = 2$

$$W_2 = \begin{pmatrix} u \\ v \\ w \\ x \end{pmatrix} \subset R^4 |$$

$$u + v + x = 0, v - x = 0, u + w - 2x = 0$$

$$\implies \begin{pmatrix} u \\ v \\ w \\ v \end{pmatrix} \subset R^3 |$$

$$u + w = -x, v = x, u + w = 2x$$

$$\implies \begin{pmatrix} u \\ v \\ w \\ v \end{pmatrix} \subset R^3 |$$

$$u + w = -x, v = x, -x = 2x$$

$$\implies \begin{pmatrix} u \\ v \\ w \\ v \end{pmatrix} \subset R^3 |$$

$$u + w = -x, v = x, x = 0$$

$$\implies \begin{pmatrix} u \\ 0 \\ w \\ 0 \end{pmatrix} \subset R^2 |$$

$$u + w = 0, v = 0, x = 0$$

$$\implies \begin{pmatrix} u \\ 0 \\ -u \\ 0 \end{pmatrix} \subset R |$$

$$w = -u, v = 0, x = 0$$

$$\implies u \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \subset R |$$

$$u \subset R$$

$$\implies W_2 = span \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\implies dim(W_2) = 1 \quad (2.0.2)$$

$$W_1 \cap W_2 = span \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\implies dim(W_1 \cap W_2) = 1$$

$$\begin{aligned} &dim(W_1 + W_2) \\ &= dim(W_1) + dim(W_2) - dim(W_1 \cap W_2) \\ &= 2 + 1 - 1 \\ &= 2 \quad (2.0.3) \end{aligned}$$

So, the answer is 3)$dim(W_1 \cap W_2) = 1$