Matrix theory Assignment 1

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***Abstract*—This document contains the solution complex numbers problem.** Download all python codes from

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https://github.com/sahilsin/MatrixTheory/
    Assignment1/codes
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1 Problem

Convert the following in Polar form:

$$a)\frac{\begin{pmatrix}1\\7\end{pmatrix}}{\begin{pmatrix}2\\-1\end{pmatrix}^2} \tag{1.0.1}$$

$$b)\frac{\begin{pmatrix}1\\3\end{pmatrix}}{\begin{pmatrix}1\\-2\end{pmatrix}} \tag{1.0.2}$$

2 Solution

The **Approach** is : For finding the polar form we will first simplify the expressions given in problem and after that the polar form of $\begin{pmatrix}x\\y\end{pmatrix}$ will be of two parts. Magnitude and angle. Magnitude will be $\sqrt{x^2+y^2}$ and angle will be $\arctan\frac{y}{x}$.

a) We first convert given problem into simpler form:

$$\frac{\begin{pmatrix}1\\7\end{pmatrix}}{\begin{pmatrix}2\\-1\end{pmatrix}^2} \tag{2.0.1}$$

$$\tag{2.0.2}$$

We know that :

$$\begin{pmatrix}a1\\a2\end{pmatrix}=\begin{pmatrix}a1 & -a2\\a2 & a1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.3}$$

The denominator can be simplified as :

$$\begin{pmatrix}2\\-1\end{pmatrix}^2=\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.4}$$

$$\tag{2.0.5}$$

$$\implies \begin{pmatrix}2\\-1\end{pmatrix}^2=\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.6}$$

$$\tag{2.0.7}$$

$$\implies \begin{pmatrix}2\\-1\end{pmatrix}^2=\begin{pmatrix}3\\-4\end{pmatrix} \tag{2.0.8}$$

Now our Expression becomes :

$$\begin{pmatrix}1\\7\end{pmatrix}\begin{pmatrix}3\\-4\end{pmatrix}^{-1} \tag{2.0.9}$$

Inverse can be calculated as:

$$\begin{pmatrix}x\\y\end{pmatrix}^{-1}=\frac{1}{x^2+y^2}\begin{pmatrix}x\\-y\end{pmatrix} \tag{2.0.10}$$

Our problem can now be written as :

$$\frac{1}{25}\begin{pmatrix}1\\7\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix} \tag{2.0.11}$$

$$\implies \frac{1}{25}\begin{pmatrix}1\\7\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix} \tag{2.0.12}$$

it can be further expanded as :

$$\implies \frac{1}{25}\begin{pmatrix}1 & -7\\7 & 1\end{pmatrix}\begin{pmatrix}3 & -4\\4 & 3\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.13}$$

$$\implies \frac{1}{25}\begin{pmatrix}-25 & -25\\25 & -25\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.14}$$

$$\implies \frac{25}{25}\begin{pmatrix}-1 & -1\\1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.15}$$

Dividing and multiplying by $\sqrt{2}$ we get:

$$\implies \sqrt{2}\begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \tag{2.0.16}$$

writing it into matrix polar form it becomes :

$$\implies \sqrt{2}\begin{pmatrix} \cos 135^\circ & -\sin 135^\circ \\ \sin 135^\circ & \cos 135^\circ \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.17}$$

$$\implies \sqrt{2}\begin{pmatrix} \cos 135^\circ \\ \sin 135^\circ \end{pmatrix} \tag{2.0.18}$$

polar form is :

$$\implies \sqrt{2}\angle 135^\circ \tag{2.0.19}$$

b) We first convert given problem into simpler form:

$$\frac{\begin{pmatrix} 1 \\ 3 \end{pmatrix}}{\begin{pmatrix} 1 \\ -2 \end{pmatrix}} \tag{2.0.20}$$

We know that :

$$\begin{pmatrix} a1 \\ a2 \end{pmatrix} = \begin{pmatrix} a1 & -a2 \\ a2 & a1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.21}$$

The denominator can be simplified as :

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.22}$$

Now our Expression becomes :

$$\begin{pmatrix} 1 \\ 7 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}^{-1} \tag{2.0.23}$$

Inverse can be calculated as:

$$\begin{pmatrix} x \\ y \end{pmatrix}^{-1} = \frac{1}{x^2+y^2}\begin{pmatrix} x \\ -y \end{pmatrix} \tag{2.0.24}$$

Our problem can now be written as :

$$\frac{1}{5}\begin{pmatrix} 1 \\ 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.25}$$

it can be further expanded as :

$$\implies \frac{1}{5}\begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.26}$$

$$\implies \frac{1}{5}\begin{pmatrix} -5 & -5 \\ 5 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.27}$$

$$\implies \frac{5}{5}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.28}$$

Dividing and multiplying by $\sqrt{2}$ we get:

$$\implies \sqrt{2}\begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \tag{2.0.29}$$

writing it into matrix polar form it becomes :

$$\implies \sqrt{2}\begin{pmatrix} \cos 135^\circ & -\sin 135^\circ \\ \sin 135^\circ & \cos 135^\circ \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.30}$$

$$\implies \sqrt{2}\begin{pmatrix} \cos 135^\circ \\ \sin 135^\circ \end{pmatrix} \tag{2.0.31}$$

polar form is :

$$\implies \sqrt{2}\angle 135^\circ \tag{2.0.32}$$

For both the part as our polar forms comes to be same , it will look like a line having magnitude as $\sqrt{2}$ and angle as 135° ,so it will look like a line $\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$ as shown :

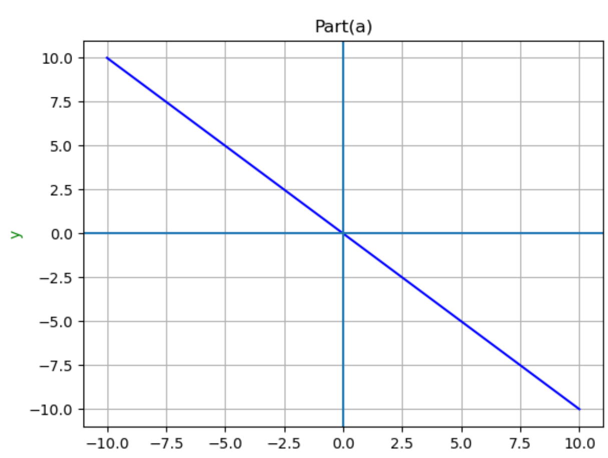

Fig. 2.0.1: part(a)